

## Single and Double Charge exchange excitations of Spin-Isospin mode

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1. Introduction
2. HF+BCS+QRPA for Double Gamow-Teller excitations
3. Quenching of GT, Spin-Dipole and spin M1 transitions
4. Summary



# Spin-Isospin Excitations

Isospin triplet and quintet symmetry

$0^+ \sim 4^+$

DSD

$\sigma Y^{(l=1)} \tau_-$

Spin-Dipole(SD)

$0^- \ 1^- \ 2^-$

$0^+ \ 1^+ \ 2^+$

DGT

$\sigma \tau_-$

$1^+$

GT

$\sigma Y^{(l=1)} \tau_-$

$1^-$

$(p,p')$   
 $(e,e')$

QRPA

$0^+$

DIAS

$\tau_-$

$0^+$

IAS

CXQRPA

QRPA

GDR

CXQRPA

$\sigma \tau_-$

$\tau_-$

$1^+$

Shell model

IS and IV  
Spin M1

$0^+$

$\beta_-$

$\beta_+$

$2\beta_-$

$0^+$

$(N-1, Z+1)$

$(p,n), (^3\text{He}, t)$

$(n,p), (t, ^3\text{He}), (^{12}\text{C}, ^{12}\text{N}), (^{16}\text{O}, ^{16}\text{F})$

$(N, Z)$

$(N+1, Z-1)$

$(N-2, Z+2)$

$(N+2, Z-2)$

$(^{12}\text{C}, ^{12}\text{Be}^*), (^{20}\text{Ne}, ^{20}\text{O}), (^{11}\text{C}, ^{11}\text{Be}^*), (\pi^+, \pi^-)$

$(^{18}\text{O}, ^{18}\text{Ne}), (^8\text{He}, ^8\text{Be}), (^{14}\text{C}, ^{14}\text{O})$

## DBD

- 0v mode  $(A, Z) \rightarrow (A, Z + 2) + 2e^-$
- 2v mode  $(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$

## 0v mode

- Majorana vs. Dirac
- absolute mass : nuclear matrix element(NME) needed (depends on theory)

## ● Double GT(DGT) states : particular interest

### Connection to double beta decay

→ Possible calibration standard of nuclear structure model for  $2\nu\beta\beta$  decay

*The ( $\beta\beta$ -decay) matrix element, however, still remains very small and accounts for only a  $10^{-4}$  to  $10^{-3}$  of the total DGT sum rule. A precise calculation of such hindered transition is, of course, very difficult and is inherently a subject of large percent uncertainties. At present there is no direct way to “calibrate” such complicated nuclear structure calculations involving miniature fractions of the two-body DGT transitions. By studying the stronger DGT transitions and, in particular, the giant DGT states experimentally and as we do here, theoretically, one may be able to “calibrate” the calculations of  $\beta\beta$ -decay nuclear elements.*

by N. Auerbach, L. Zamick, and D. Zheng, Annals of Physics 192, 77 (1989).

$$S_- - S_+ = 3(N-Z) = 24$$

## Sum rule study for double Gamow-Teller states

H. Sagawa<sup>1,2</sup> and T. Uesaka<sup>1</sup>

$$D_{\pm}^J = \frac{1}{2J_i + 1} \sum_{J_f} |(J_f || [\hat{O}_{\pm} \times \hat{O}_{\pm}]^J || J_i)|^2,$$

$$\hat{O}_{\pm}(\text{GT}) = \sum_{\alpha} \sigma(\alpha) t_{\pm}(\alpha).$$

Initial state	(J = 0)	(J = 2)	DIAS	DIAS(DGT)	$\Sigma_{\text{sum}}$
<sup>6</sup> He	12.0(12)	0.0 (0)	4	3.70(3.70)	$0.211 \times 10^{-3}$ (0.667)
<sup>8</sup> He	39.7(40)	80.7 (80)	24	7.71(2.47)	0.052 (1.333)
<sup>14</sup> C	8.98(12)	7.55 (0)	4	4.56 (0.15)	0.566 (1.333)
<sup>18</sup> O	10.4(12)	3.96 (0)	4	7.72 (2.61)	0.297 (0.80)
<sup>20</sup> O	35.5(40)	91.3 (80)	24	4.13 (1.74)	0.845 (1.60)
<sup>42</sup> Ca	8.50(12)	8.75 (0)	4	3.80 (2.18)	0.66 (0.86)
<sup>44</sup> Ca	32.6(40)	98.5(80)	24	2.31 (1.47)	1.38 (1.71)
<sup>46</sup> Ca	72.3(84)	269.3(240)	60	1.89 (1.32)	2.20 (2.57)
<sup>48</sup> Ca	135.5(144)	501.2(480)	112	3.70 (1.25)	1.59 (3.43)
<sup>90</sup> Zr	196.3(220)	859.2(800)	180	(1.11)	(4.44)

$$\begin{aligned}
 D_-^{(J=0)} - D_+^{(J=0)} &= \langle i | [[\hat{O}_+ \times \hat{O}_+]^{(J=0)}, [\hat{O}_- \times \hat{O}_-]^{(J=0)}] | i \rangle \\
 &= 2(N-Z)(N-Z+1) + \frac{4}{3}[(N-Z)S \\
 &\quad - \langle i | [i \hat{\Sigma} \cdot (\hat{O}_- \times \hat{O}_+) + \hat{\Sigma} \cdot \hat{\Sigma}] | i \rangle], \quad (
 \end{aligned}$$

$$\begin{aligned}
 D_-^{(J=2)} - D_+^{(J=2)} &= \langle i | \sum_{\mu} (-1)^{\mu} [[\hat{O}_+ \times \hat{O}_+]_{\mu}^{(J=2)}, [\hat{O}_- \times \hat{O}_-]_{-\mu}^{(J=2)}] | i \rangle \\
 &= 10(N-Z)(N-Z-2) \\
 &\quad + \frac{10}{3}[2(N-Z)S_+ + \langle i | [i \hat{\Sigma} \cdot (\hat{O}_- \times \hat{O}_+) + \hat{\Sigma} \cdot \hat{\Sigma}] | i \rangle].
 \end{aligned}$$

$$\hat{\Sigma} = \sum_{\alpha} \sigma(\alpha)$$

$$\begin{aligned}
 B(2\nu 2\beta) &\sim 0.02 \\
 &\leq 10^{-5} S_{\text{DGT}}
 \end{aligned}$$

The QRPA state and QRPA phonon for standard (non-charge exchange) excitations are defined as

$$|k, I^\pi M\rangle = Q_k^\dagger(I^\pi M)|\hat{0}\rangle, \quad (3)$$

$$\begin{aligned} Q_k^\dagger(I^\pi M) = & \sum_{a \leq a'} \frac{1}{\sqrt{1 + \delta_{aa'}}} [Z_{aa'}^k(I^\pi) A_{aa'}^\dagger(I^\pi M) \\ & - W_{aa'}^k(I^\pi) \tilde{A}_{aa'}(I^\pi M)] \end{aligned} \quad (4)$$

The quasi-particle pair creation and annihilation operators read

$$\begin{aligned} A_{aa'}^\dagger(I^\pi M) &= [\alpha_a^\dagger \alpha_{a'}^\dagger]_{IM} \\ \tilde{A}_{aa'}(I^\pi M) &= (-1)^{I+M} A(I^\pi - M) = -[\tilde{\alpha}_a \tilde{\alpha}_{a'}]_{IM} \end{aligned} \quad (5)$$

the Bogoliubov-Valatin transformation,

$$\begin{pmatrix} \alpha_a^\dagger \\ \tilde{\alpha}_a \end{pmatrix} = \begin{pmatrix} u_a & v_a \\ u_a & -v_a \end{pmatrix} \begin{pmatrix} c_a^\dagger \\ \tilde{c}_a \end{pmatrix}$$

Fully self-consistent calculations  
No closure approximation

The charge exchange QRPA phonon state is given by

$$|m, J^\pi M\rangle = \Gamma_m^\dagger(J^\pi M)|\hat{0}\rangle \quad (8)$$

where  $m$  labels the  $m$ -th excited state referred to the QRPA vacuum and the phonon operator is defined by

$$\Gamma_m^\dagger(J^\pi M) = \sum_{pn} [X_{pn}^m(J^\pi) A_{pn}^\dagger(J^\pi M) - Y_{pn}^m(J^\pi) \tilde{A}_{pn}(J^\pi M)] \quad (9)$$

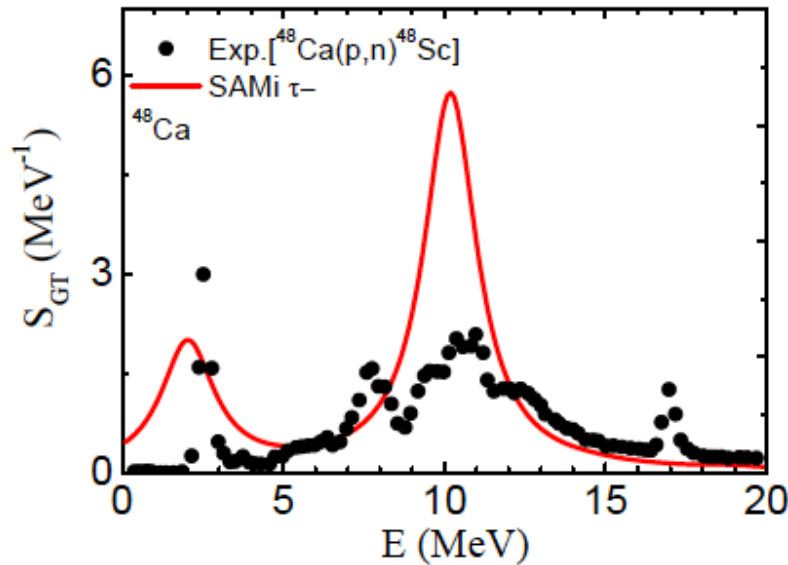
$$\begin{aligned} M^{DGTR}(0^+ \rightarrow 2^+) &= \frac{1}{\sqrt{3}} \sum_{m,m'} \langle f, 2^+ || \sigma \tau_- || m, 1^+ \rangle \\ &\times \langle m, 1^+ | m', 1^+ \rangle \langle m', 1^+ || \sigma \tau_- || GS, 0 \rangle, \end{aligned} \quad (10)$$

$$\begin{aligned} M^{DGTR}(0^+ \rightarrow 0^+) &= \sum_{m,m'} \langle f, 0^+ || \sigma \tau_- || m, 1^+ \rangle \\ &\times \langle m, 1^+ | m', 1^+ \rangle \langle m', 1^+ || \sigma \tau_- || GS, 0 \rangle. \end{aligned} \quad (11)$$

$$\langle I^\pi || \sigma \tau_- || 1^+ \rangle = \langle QRPA || [[\Gamma^+(1^+), \sigma \tau_-], Q(I^\pi)] || QRPA \rangle$$

## Quenching of spin-isospin excitations

1. Gamow-teller and Spin-dipole: the same quenching factor?  
(renormalization of  $0\nu\beta\beta$  and  $2\nu\beta\beta$  matrix elements)
2. IS and IV spin modes: quenching or enhancement?
3. Delta-hole excitation and/or two-body meson exchange currents



$0\nu\beta\beta$   
 $2\nu\beta\beta$

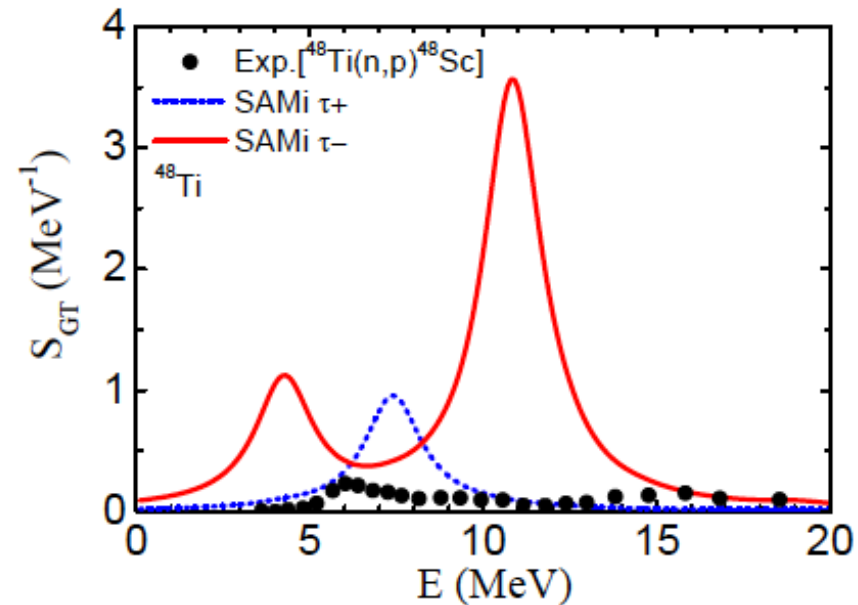


FIG. 1: (Color online) RPA response function for the GT transition operator  $\hat{O}_-(GT) = \sum_{\alpha} \sigma(\alpha) t_-(\alpha)$  to  $^{48}\text{Ca}$  calculated by HF+RPA model with a Skyrme parameter set SAMi. Experimental data are taken from ref. [8]. Calculated results are averaged by a Lorentzian weighting function with the width of 1 MeV.

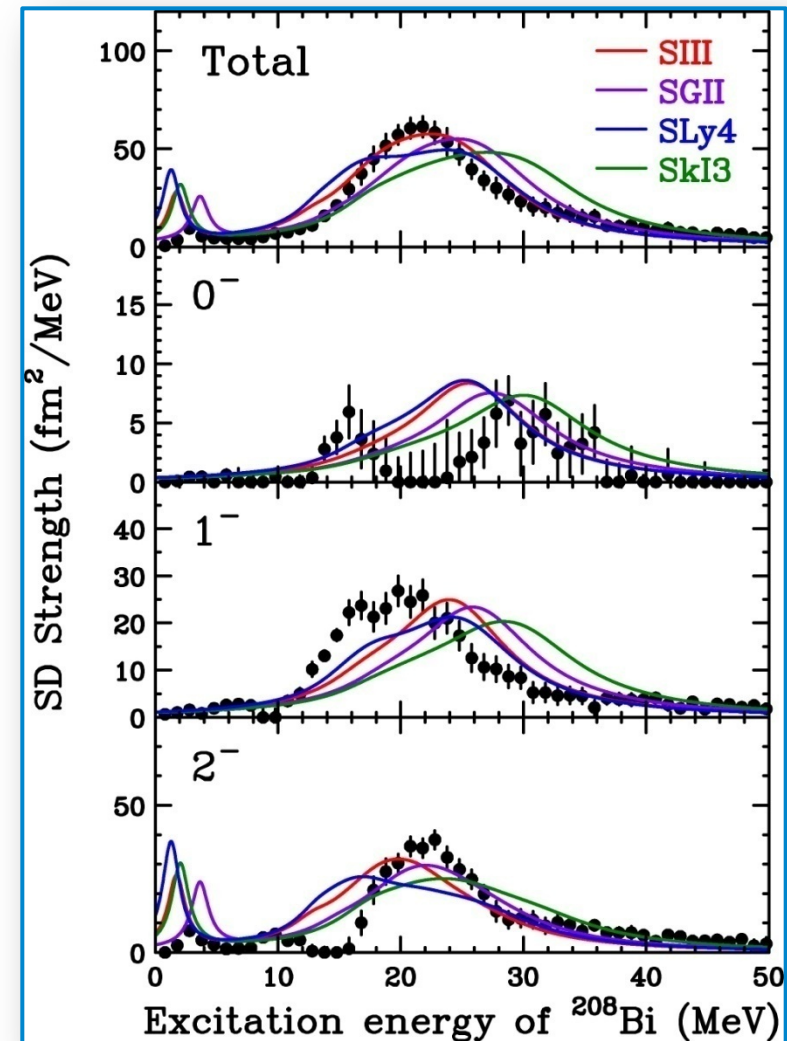
FIG. 3: (Color online) RPA response function for the GT transition operator  $\hat{O}_{\pm}(GT) = \sum_{\alpha} \sigma(\alpha) t_{\pm}(\alpha)$  to  $^{48}\text{Ti}$  calculated by HF+RPA model with a Skyrme parameter set SAMi. Experimental data are taken from ref. [8].



# Puzzle in SD Strength Distributions (Wakasa, SIR2010, 18-21 Feb., 2010)

*H. Sagawa et al., PRC 76, 024301 (2007).*

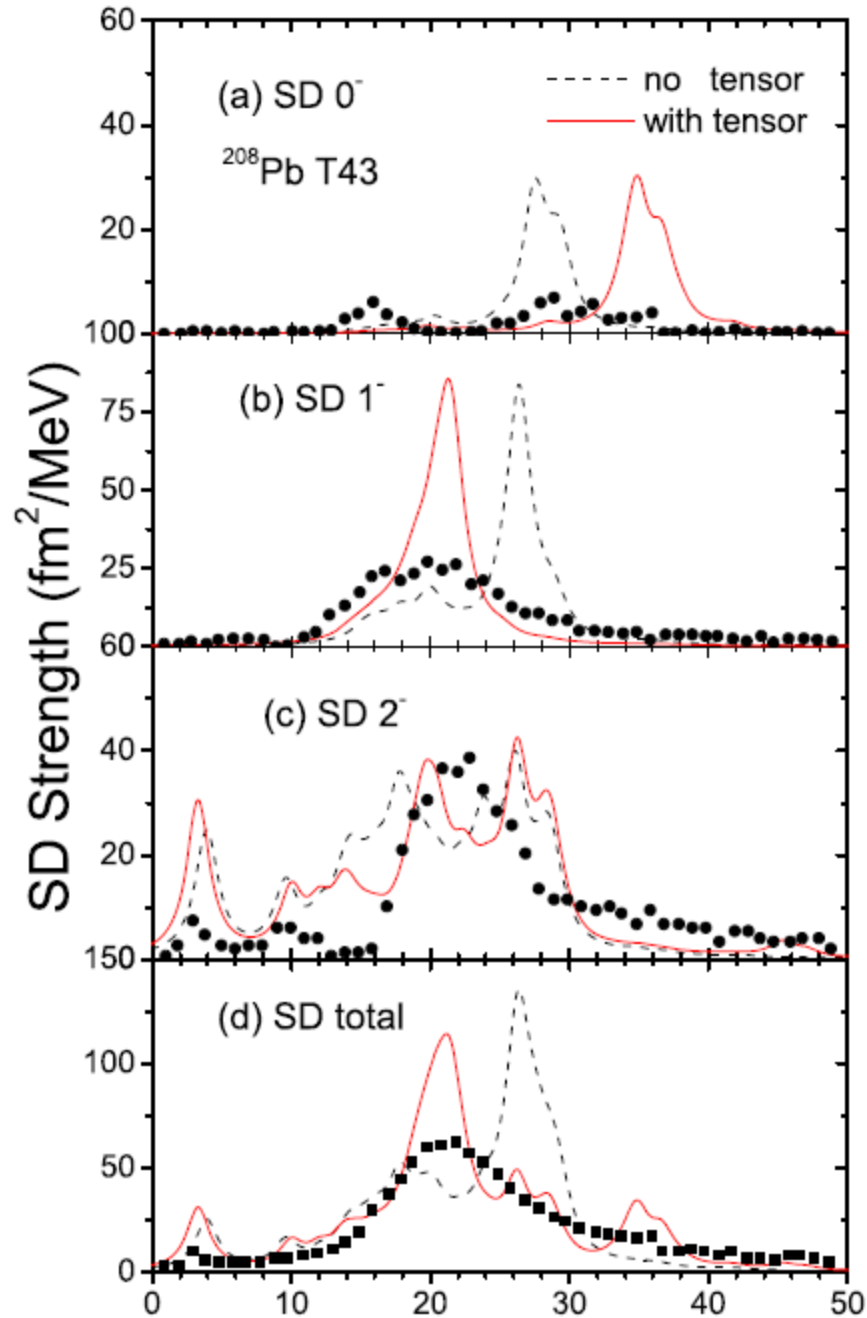
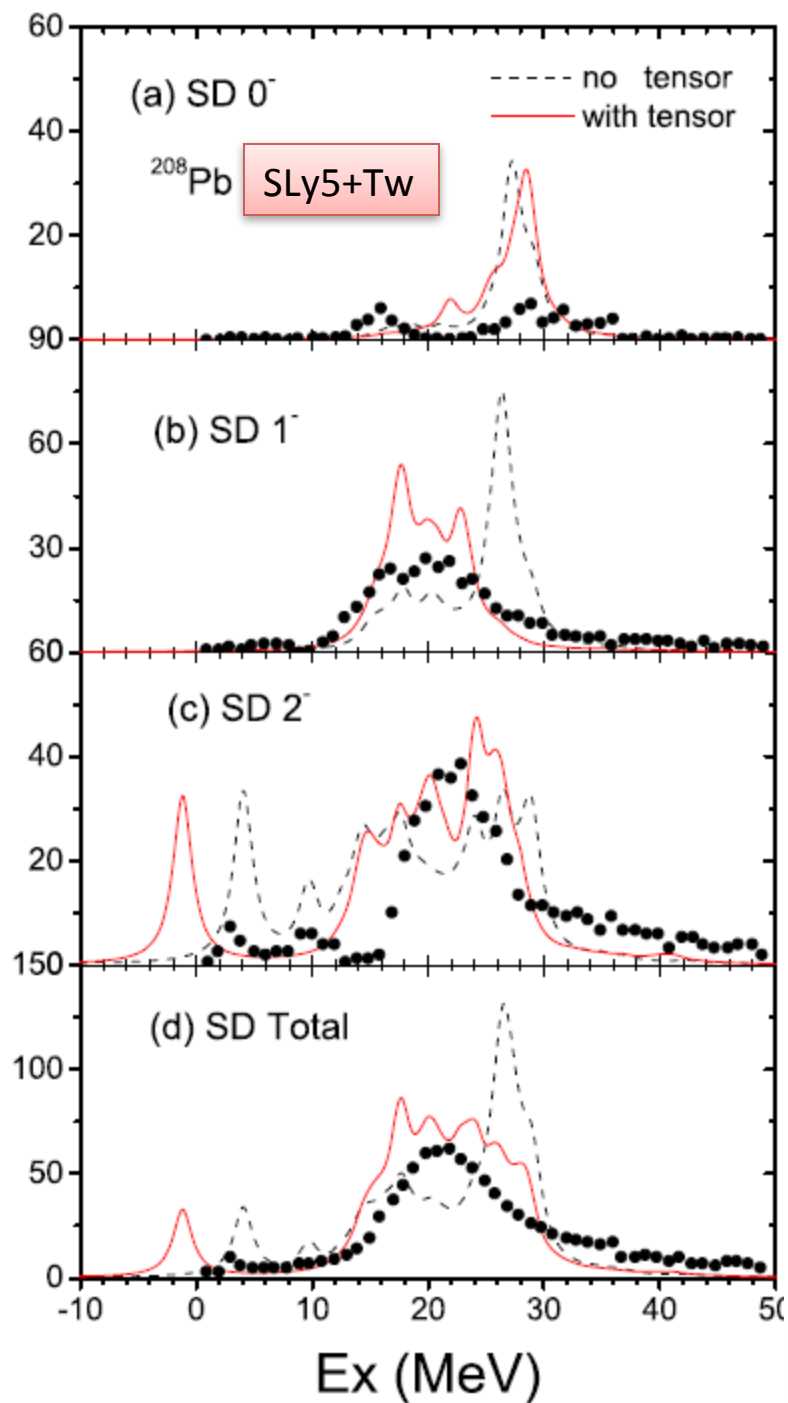
- Total strength
  - Asymmetric single bump
    - Extend up to  $\sim 50$  MeV
    - Same as  $^{90}\text{Zr}(p,n)$  results
  - SIII provides better description
- $0^-$  strength
  - Quenched
    - Seems to be fragmented
- $1^-$  strength
  - Softened compared with theory
    - Peak shift to lower  $E_x$
- $2^-$  strength
  - Hardened compared with theory
    - Peak shift to higher  $E_x$



- No Skyrme int. which reproduces both total and separated strengths
- $\Delta J^\pi$ -dependent correlation ?  $\rightarrow$  Require further investigations



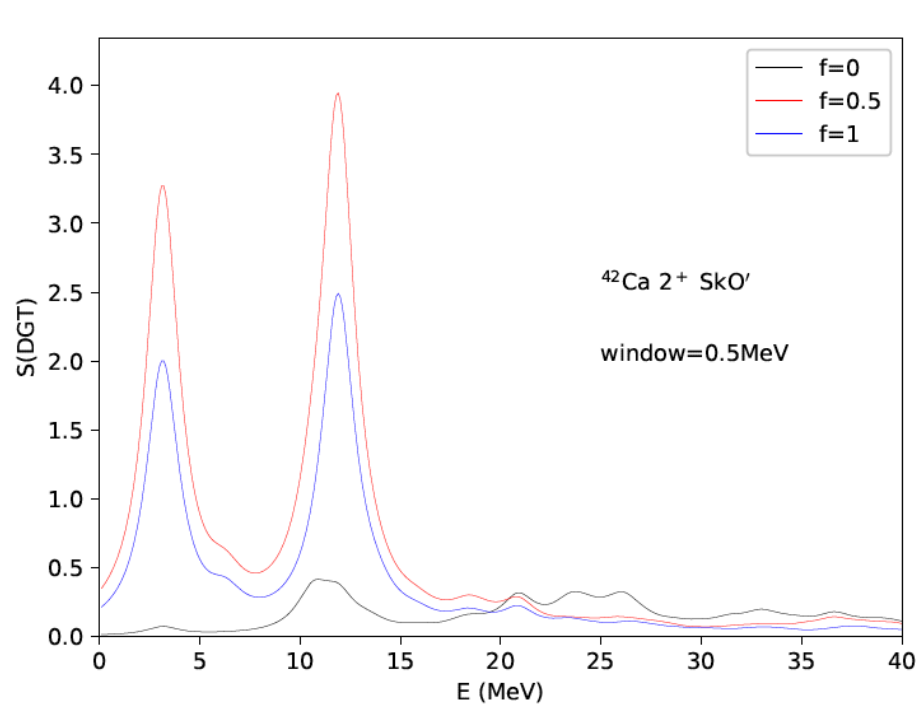
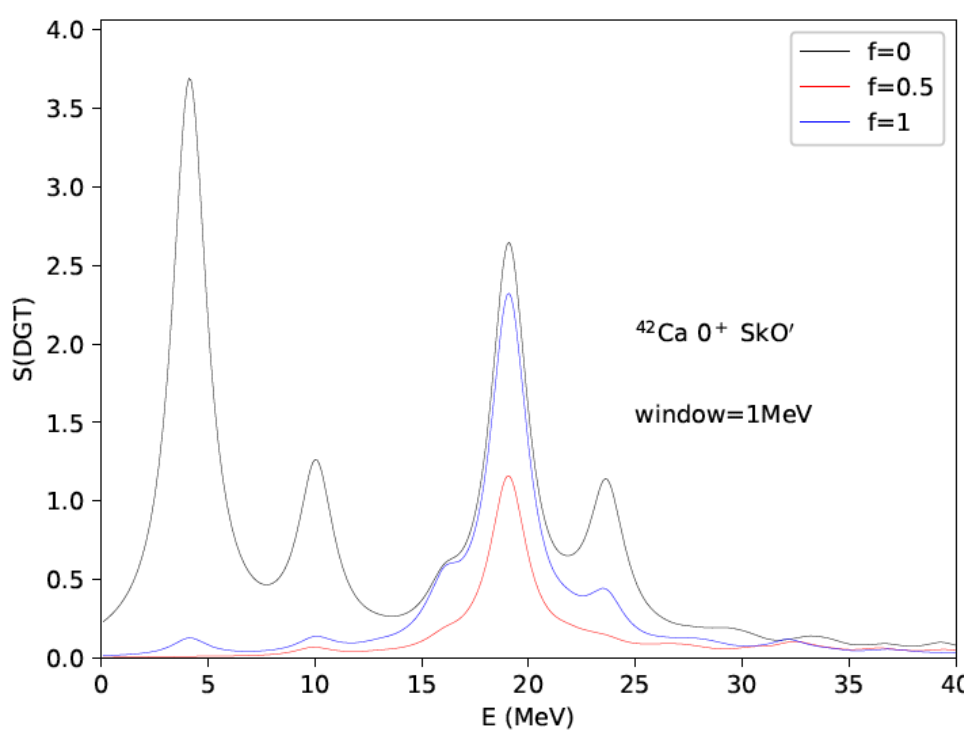
SD strength ( $\text{fm}^2/\text{MeV}$ )



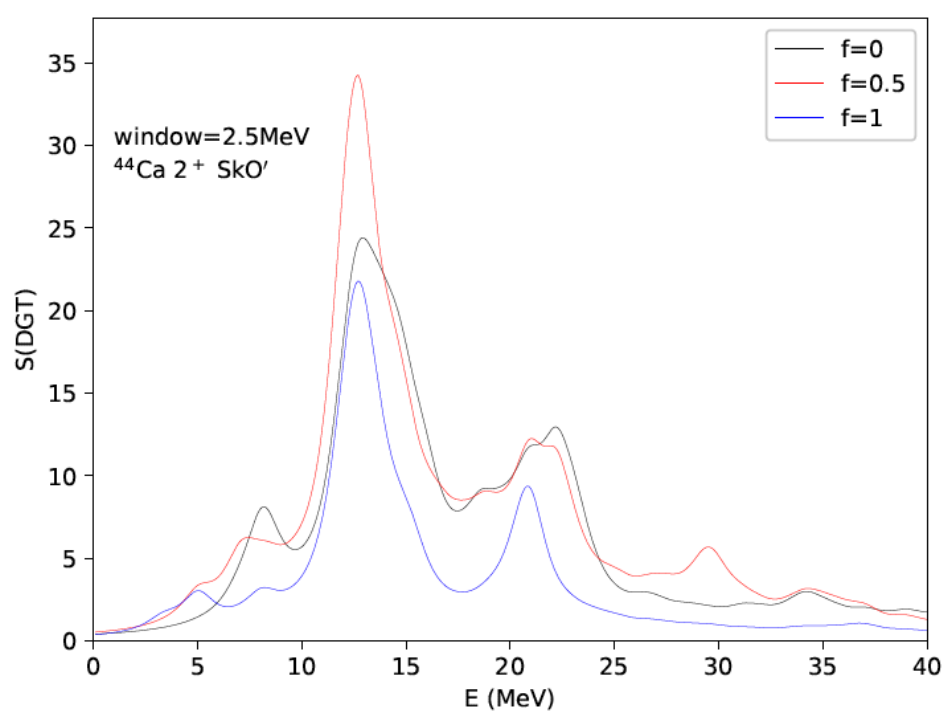
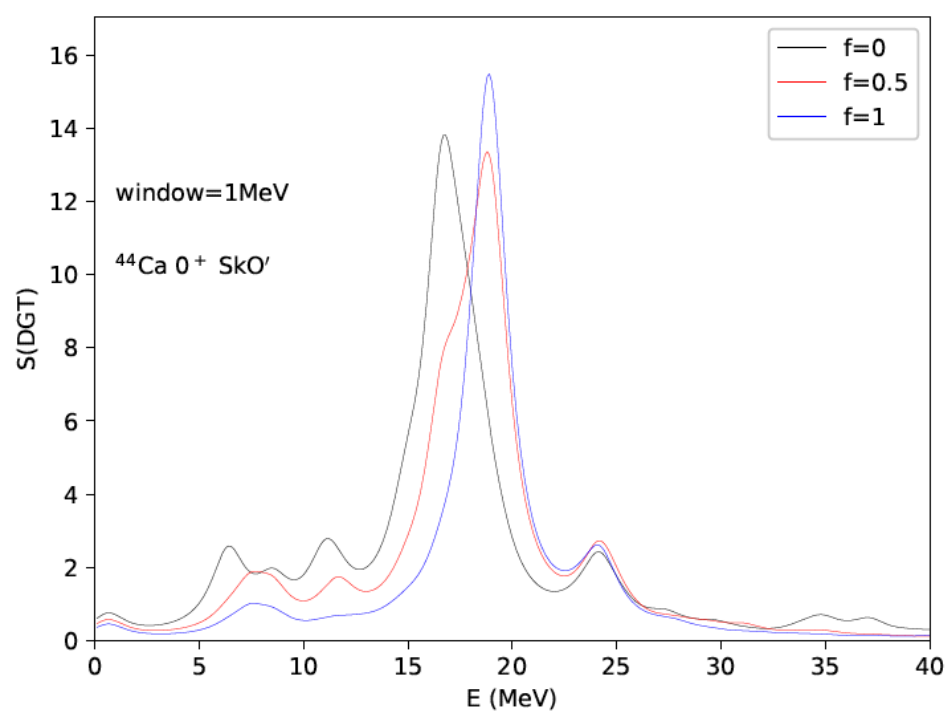
## Nuclear correlations study on DGT and DIAS

1. n-n and p-p pairing correlations
2. n-p pairing
3. Tensor correlations
4. CSB and CIB interactions

to be continued.



preliminary

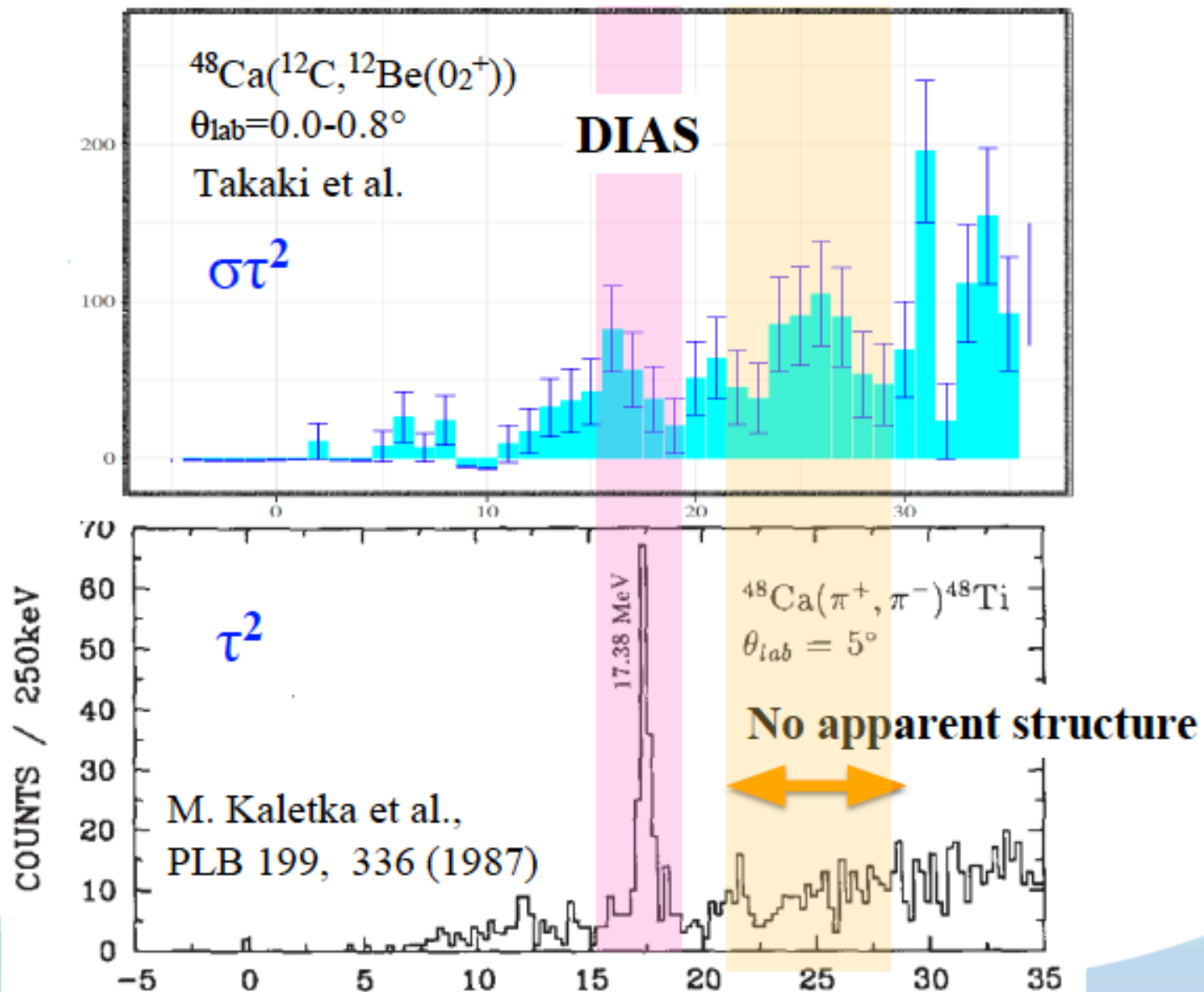


preliminary

(Ca isotopes, shell model calculations were performed by N. Shimizu et al. and N. Auerbach et al. (2018))

# DCX Spectrum and comparison with $(\pi^+, \pi^-)$

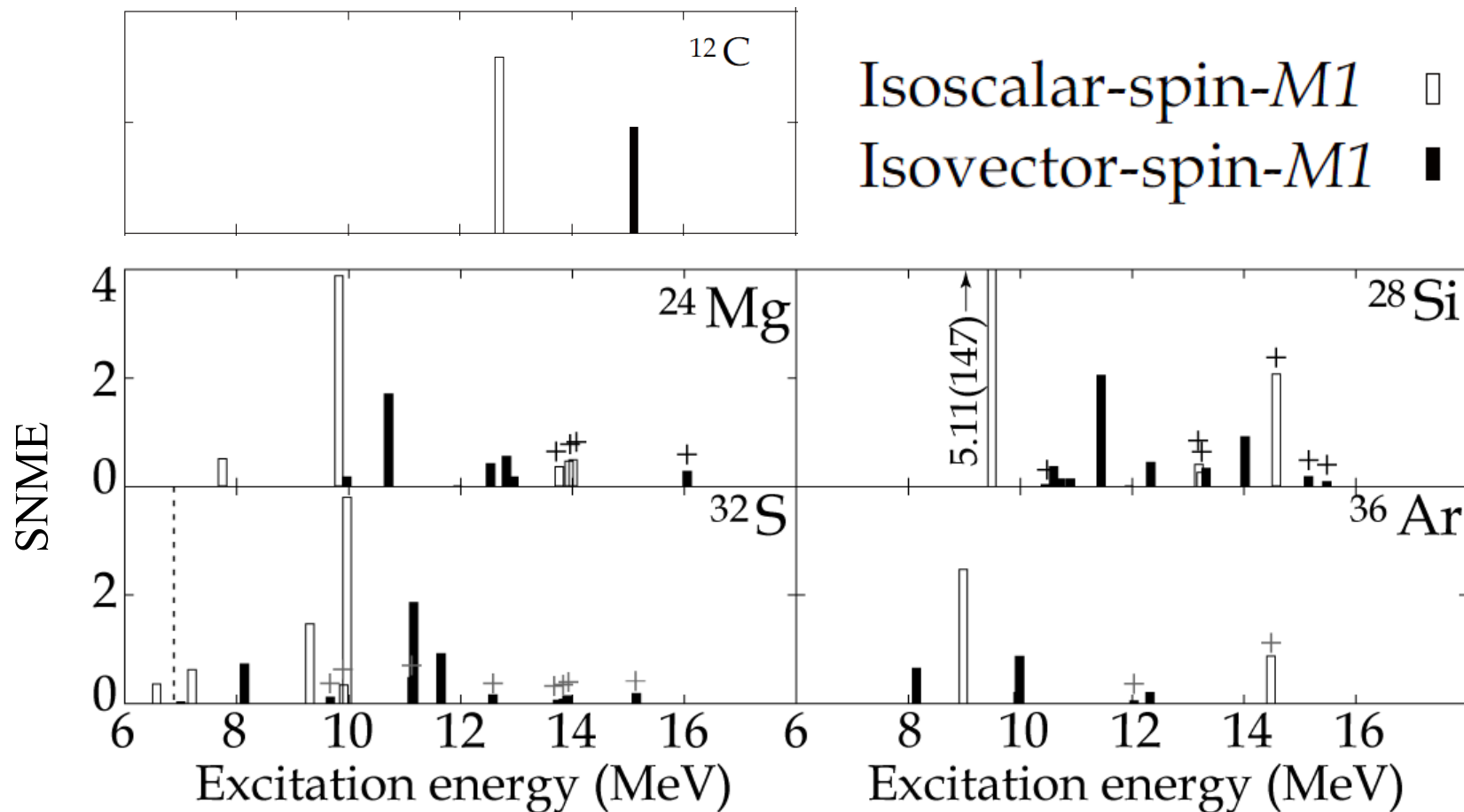
Courtesy of K. Yako (2019, RCNP experiment)

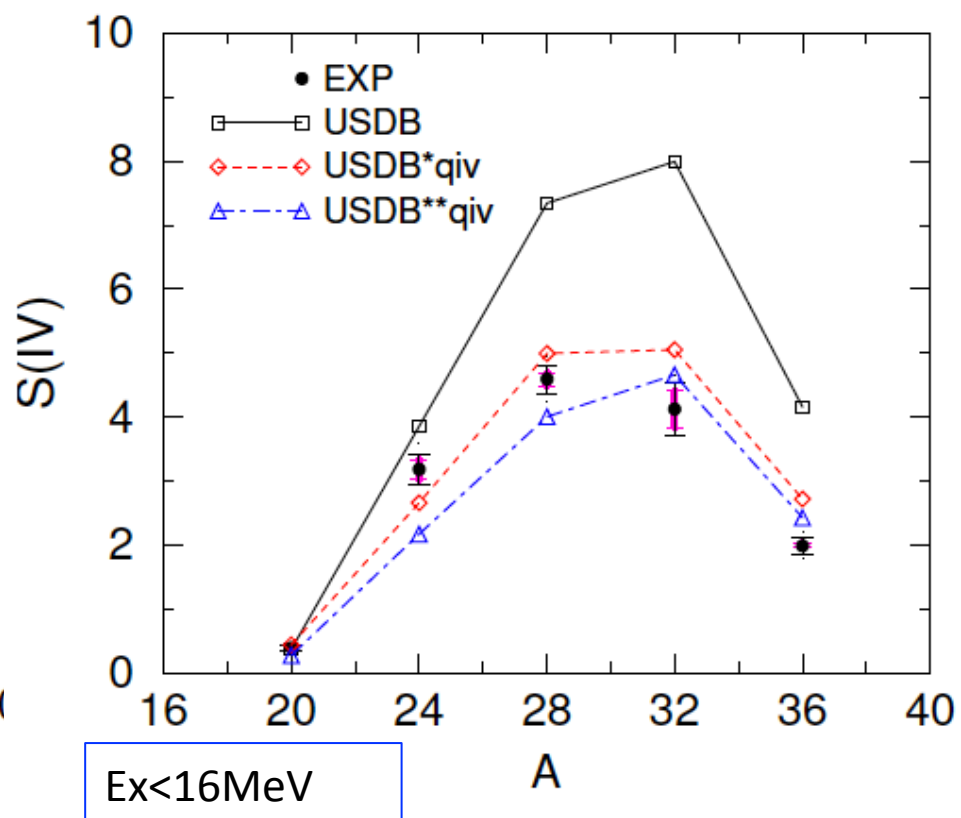
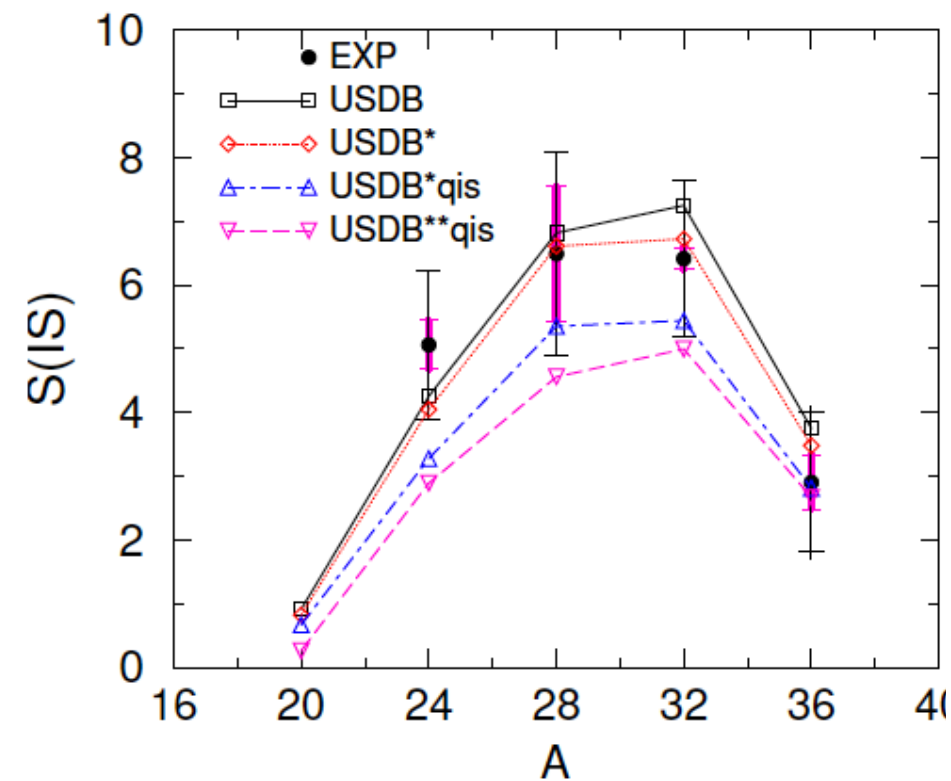


# IS/IV-spin-M1 Squared Nuclear Matrix Elements (SNMEs)

Exp. Data, Matsubara, et al., PRL115, 102501(2015)

High energy resolution proton inelastic scattering with  $E_p=295\text{MeV}$





$$S(\vec{\sigma}) = \sum_f \frac{1}{2J_i + 1} |\langle J_f || \hat{O}_{IS} || J_i \rangle|^2,$$

$$S(\vec{\sigma}\tau_z) = \sum_f \frac{1}{2J_i + 1} |\langle J_f || \hat{O}_{IV} || J_i \rangle|^2.$$



## Summary and future perspectives

1. GT states: 50% quenching of strength in  $^{48}\text{Ca}$ :  
SD states in  $^{208}\text{Pb}$ : 20% quenching.
2. DGTR is a new Double phonon state and provide a key calibration unit for 2nu double beta decay cross section.
3. HF+HFB(BCS)+QRPA calculations are performed for Ca isotopes to Ti DGT transitions with n-n, p-p and n-p pairings.
4. IS spin M1 transitions are observed in N=Z sd-shell nuclei and give useful information to find out the IS and IV quenching of spin transitions.
5. No quenching of IS spin transitions => No delta-isobar coupling, IS spin-triplet pairing.

1. Extend calculations to medium-heavy and heavy nuclei
2. Double spin-dipole excitations