



# Analysis of states in $^{13}\text{C}$ and their cluster structure

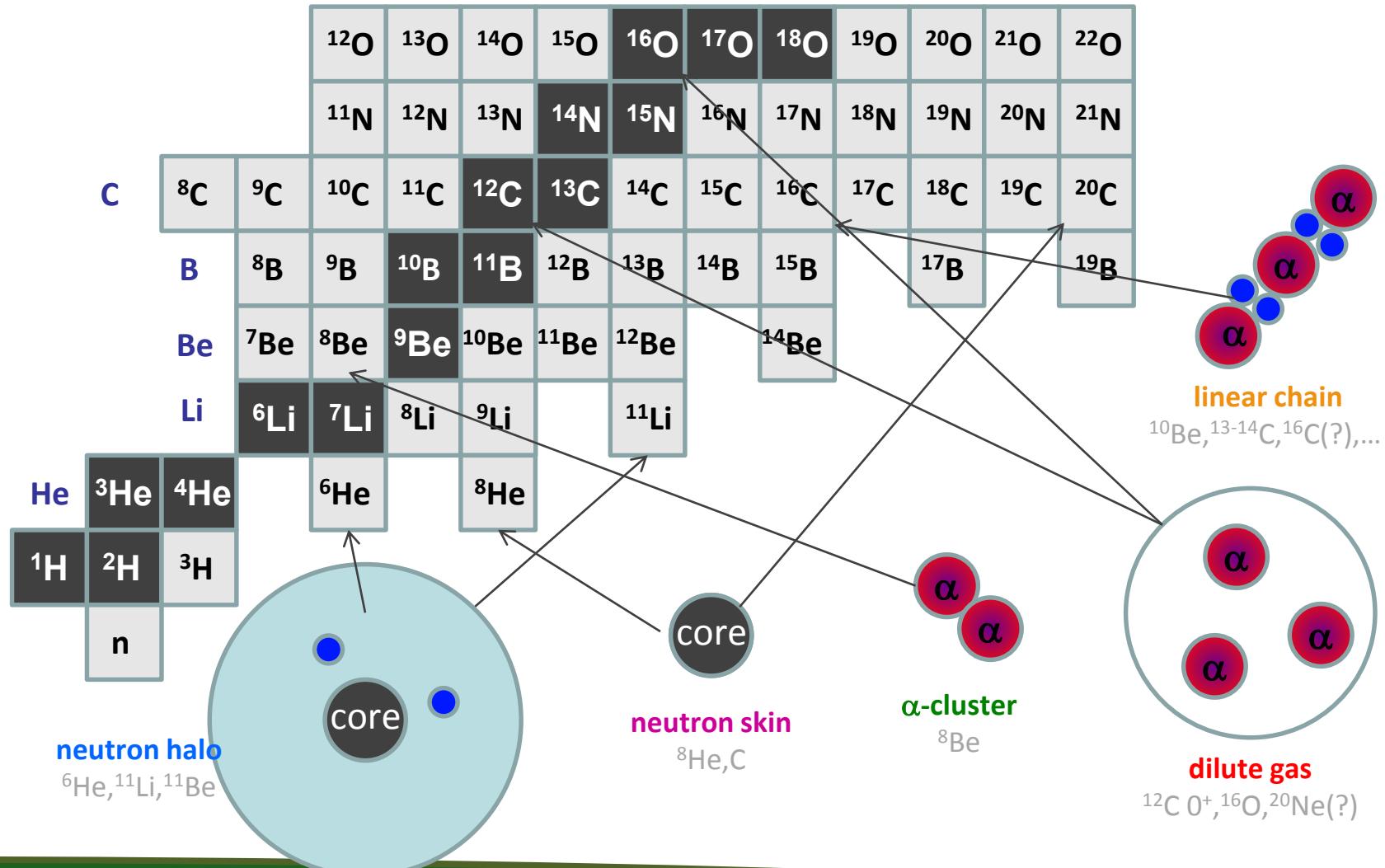
Ivano Lombardo<sup>1</sup>

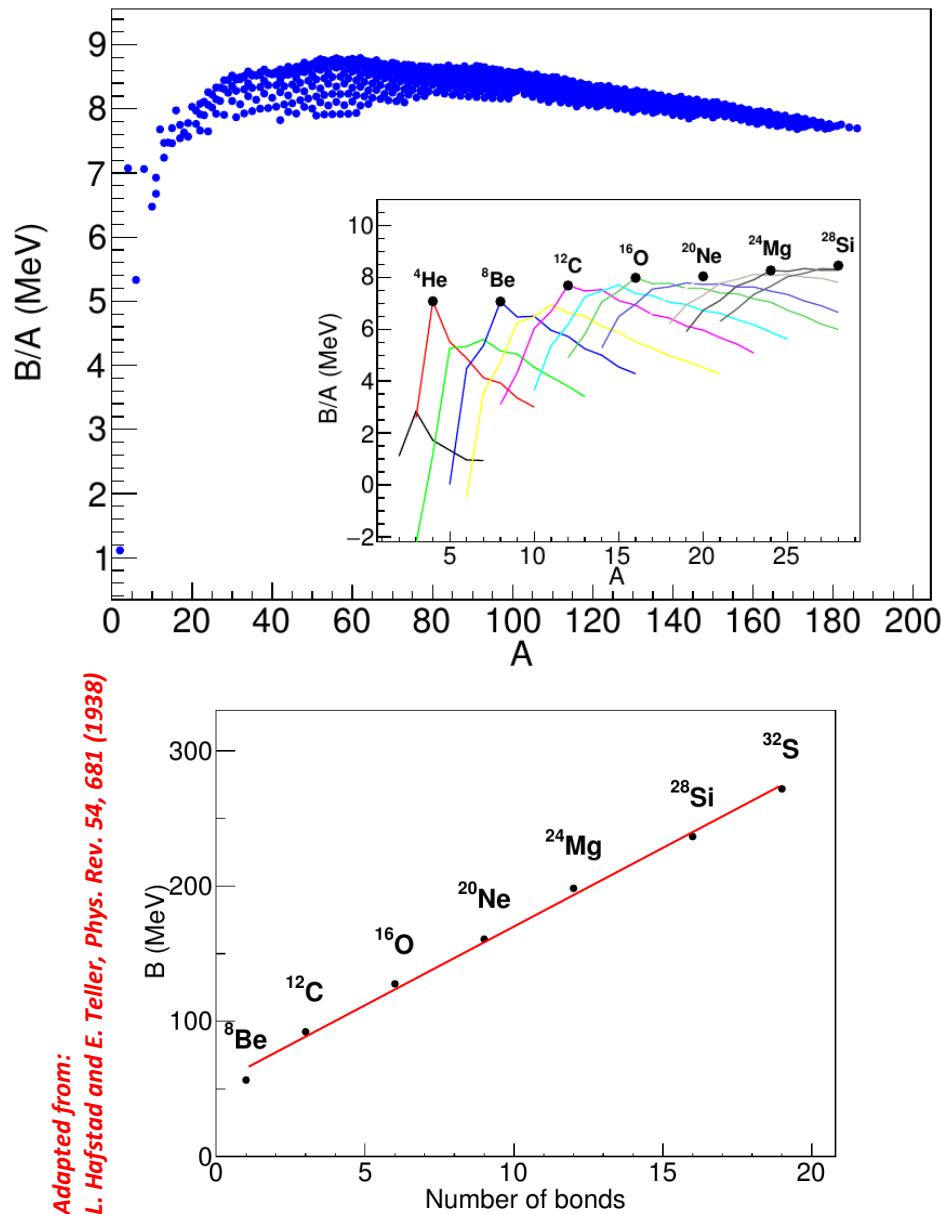
<sup>1</sup> INFN – Sezione di Catania



[ivano.lombardo@ct.infn.it](mailto:ivano.lombardo@ct.infn.it)

**Complexity of nuclear force** → long-range **correlations**: spatial re-organization of nucleons in bounded **sub-units (cluster model)**.



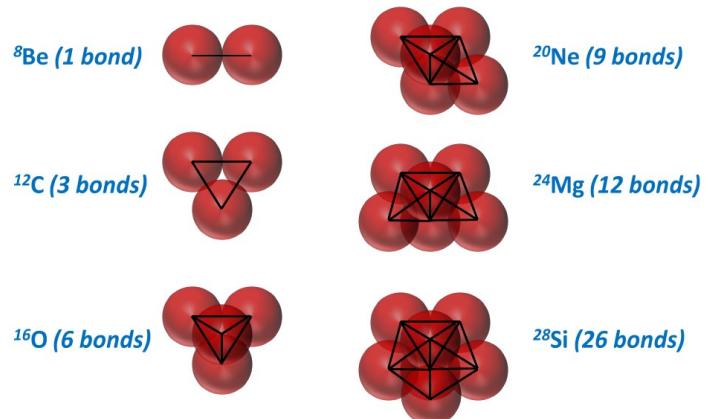


Binding energy of nuclei → self-conjugated nuclei are exceptionally bound → *long range* correlations → **clustering phenomena**.

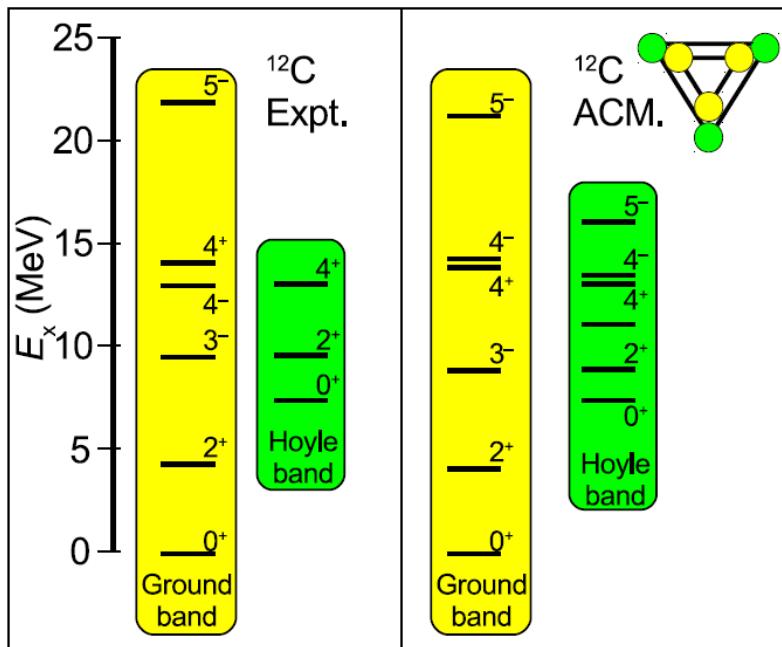
1930's : *Hasfstad and Teller* point out the existence of a linear correlation between the number of  $\alpha$ - $\alpha$  bonds and the binding energy of self-conjugated nuclei:

*L. Hafstad and E. Teller, Phys. Rev. 54, 681 (1938)*

The beginning of the  $\alpha$ -cluster model.



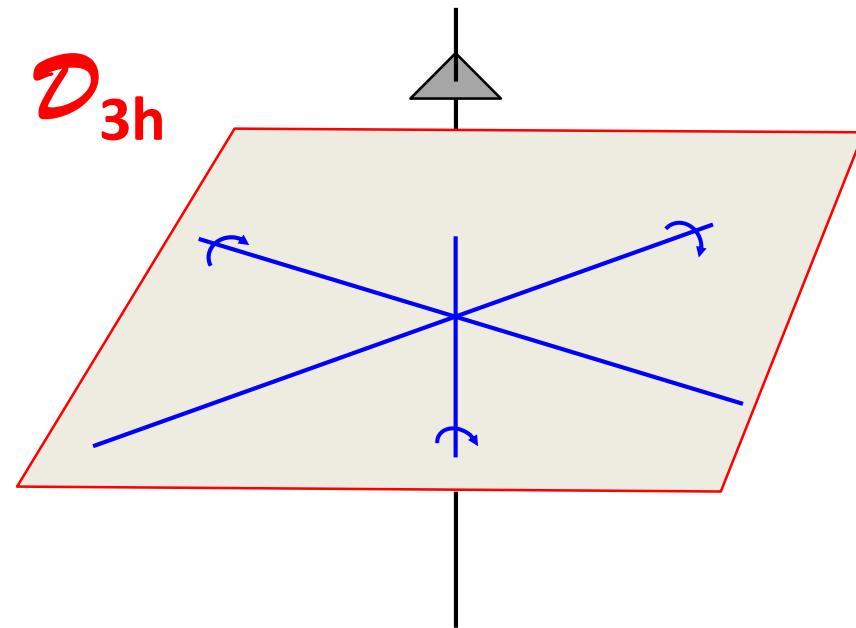
Recently → applications of *point group symmetries* to clusterized nuclei, as  $^{12}\text{C}$



R. Bijker & F. Iachello, Phys. Rev. C 61, 067305 (2000).  
D. M. Lambarri et al, Phys. Rev. Lett. 113 (2014) 012502

**Symmetry** considerations → important  
benchmark for *microscopic calculations*

Probe of the geometry of  $^{12}\text{C}$  → *nuclear fluorescence* experiments with *linearly polarized  $\gamma$  beams*, see e.g. L. Fortunato, PRC 99 (2019) 031302(R)



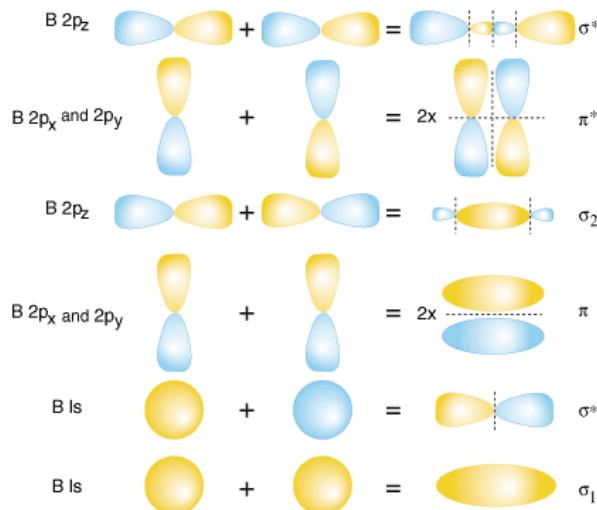
- 3-fold principal symmetry axis ( $C_3$ )
- 3 two-fold axes perpendicular to  $C_3$
- Horizontal mirror plane

**Symmetry** considerations apply  
for *other self-conjugate* nuclei!  
e.g.  $^8\text{Be}$  with  $Z_2$  symmetry, see  
next talk by M. Gai

And for *non self-conjugate* nuclei?

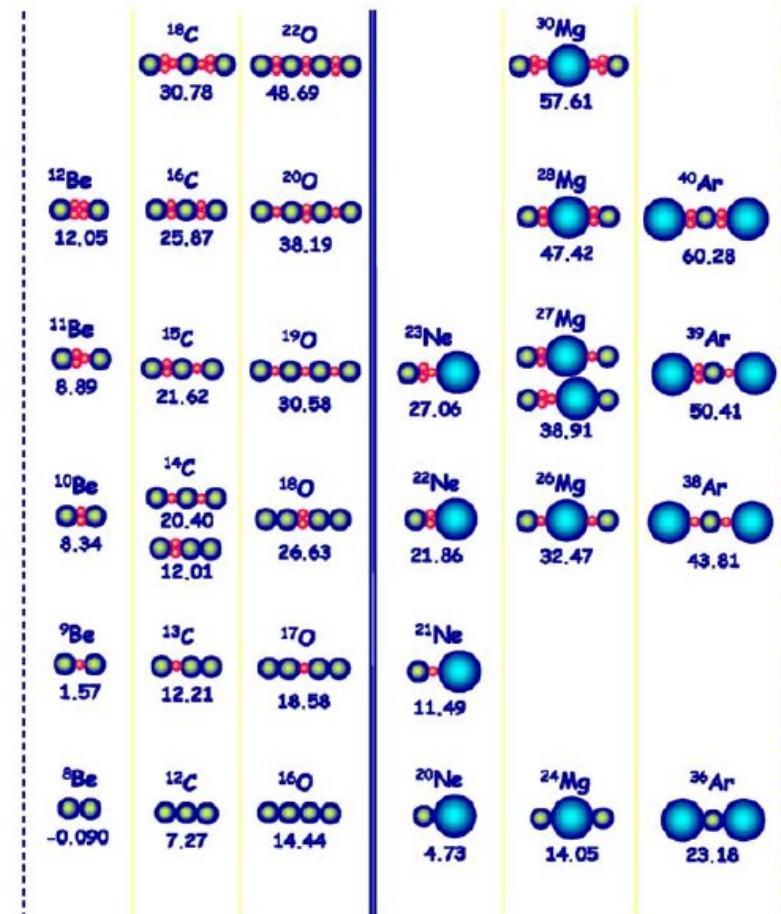
Even if the occurrence of **clustering** in non-self-conjugated nuclei has been reported many decades ago (see, e.g. Rosenfeld & Blatt-Weisskopf) → renewed interest in more recent times

Some ideas taken from **physical chemistry**  
→ extra neutrons acting as **covalence particles**



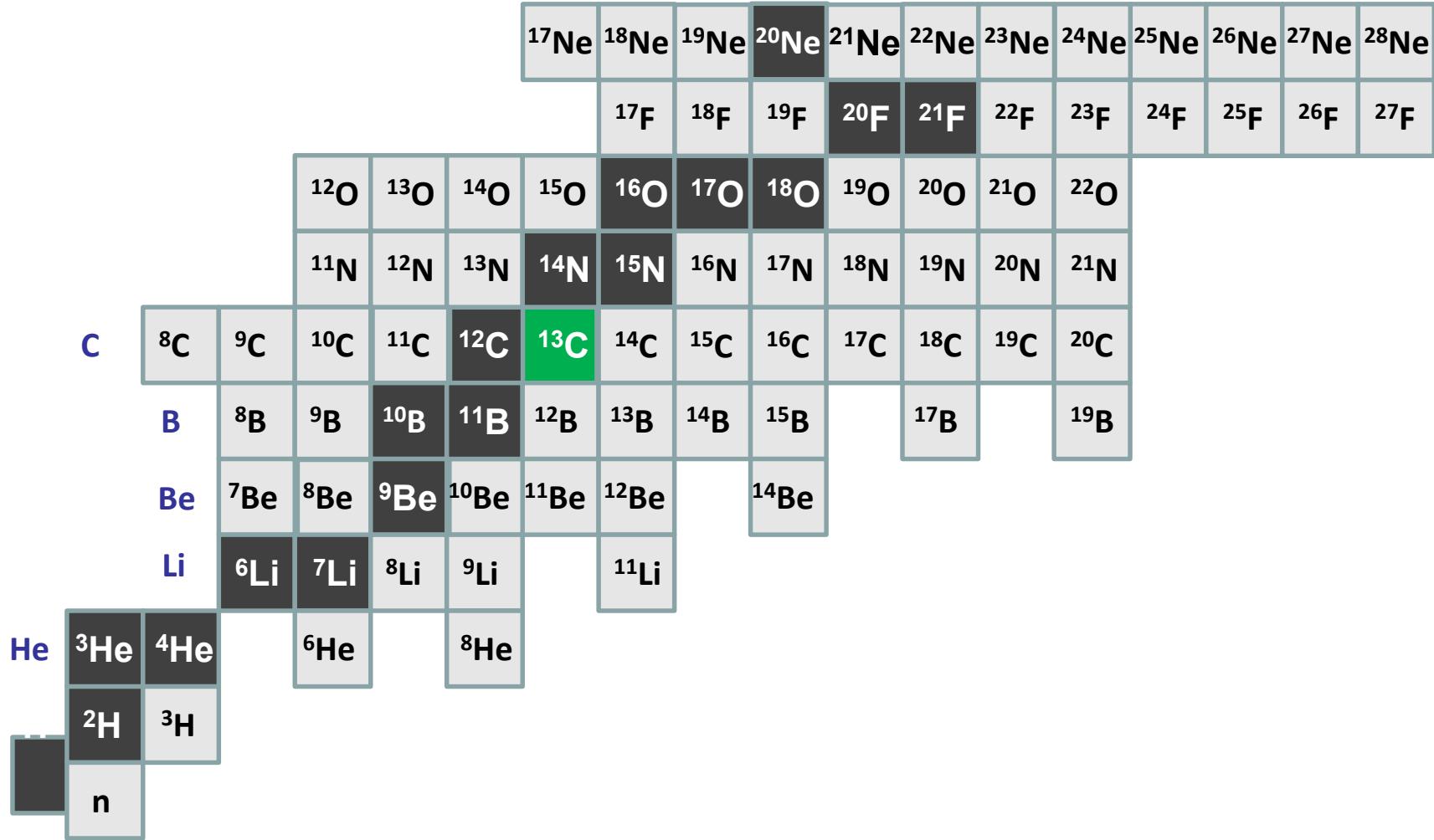
**Energy** considerations → molecular states close to **Nα+n** disintegration thresholds

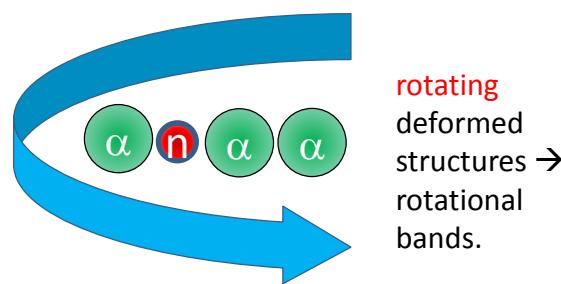
Signals of clusterization also in **p-rich** nuclei



«**Modified** Ikeda Diagram»

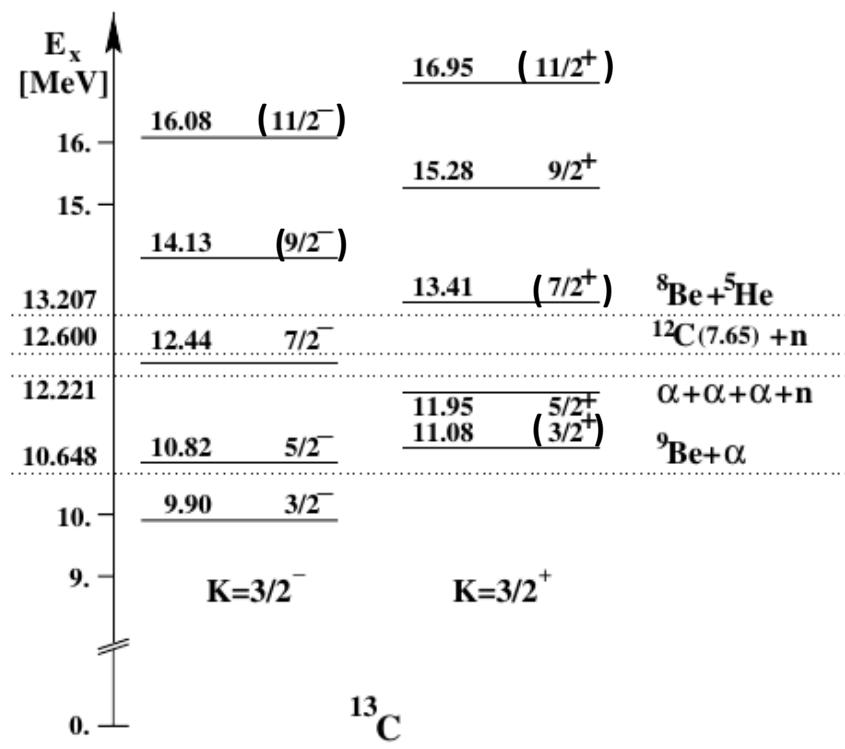
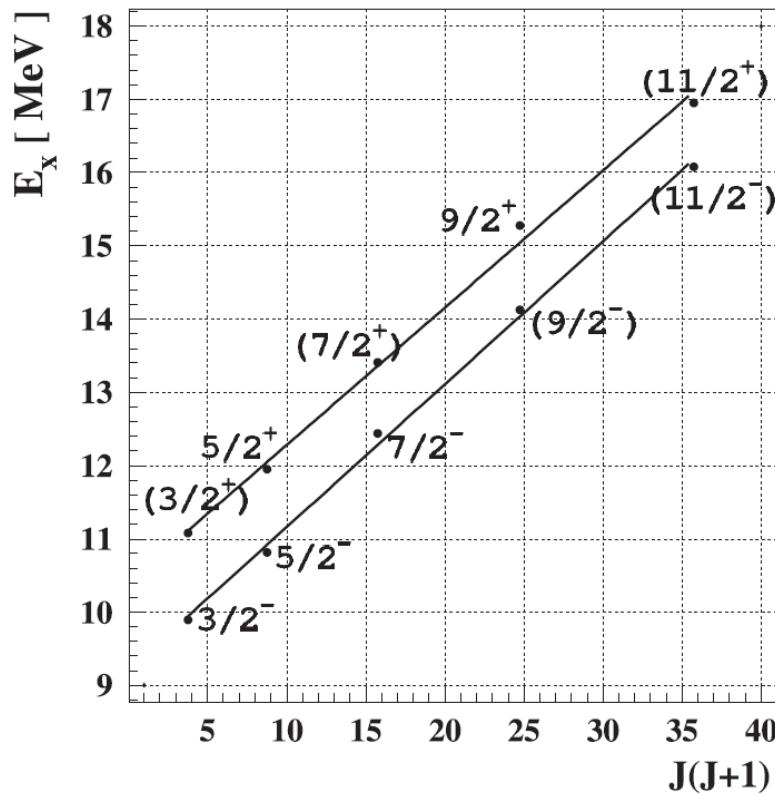
# Clustering in $^{13}\text{C}$ with $\alpha + ^9\text{Be}$ reactions





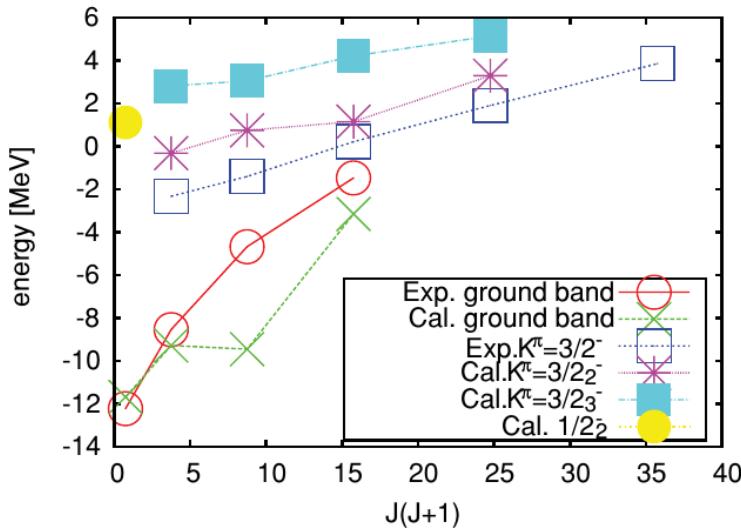
Near and above the  $\alpha$ -threshold → different  $\alpha$ -cluster configurations proposed for  $^{13}\text{C}$  → theoretical works:

M. Milin and W. Von Oertzen EPJ A 14 (2002) 295  
proposed parity doublet band of  $^9\text{Be}_{\text{gs}} + \alpha$  cluster prolate configuration →  $J^\pi$  assignments based on the rotational bands ( $K=3/2^\pm$ ).



M. Milin and W. Von Oertzen EPJ A 14 (2002) 295

Near and above the  $\alpha$ -threshold  $\rightarrow$  different  $\alpha$ -cluster configurations proposed for <sup>13</sup>C



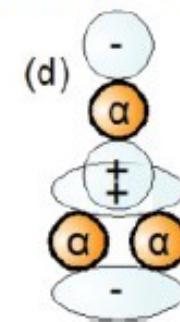
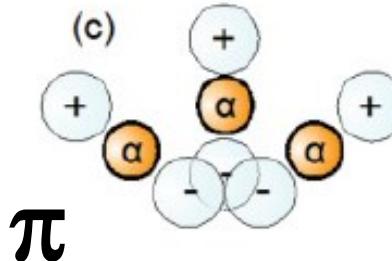
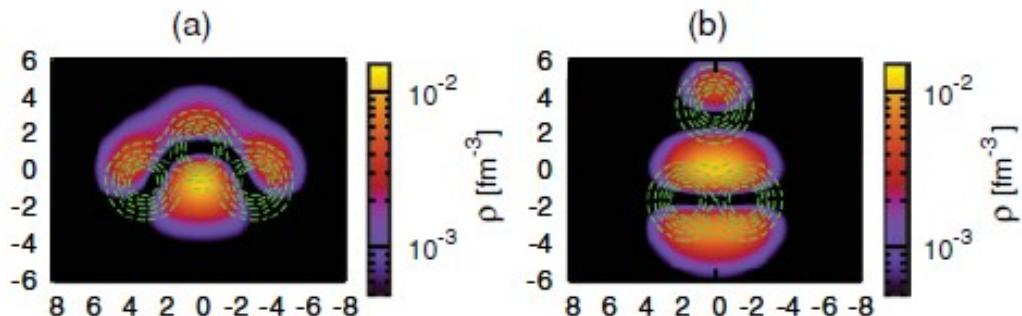
“By adding the valence neutron in this ( $\pi$ ) orbital, the  $\alpha$ - $\alpha$  **distance shrinks** and the **binding energy** of the system **increases**. As a consequence, the **stability** of the **bent linear-chain** configuration is **achieved**.”

N. Furutachi and M. Kimura, Phys. Rev. C 83 (2011)

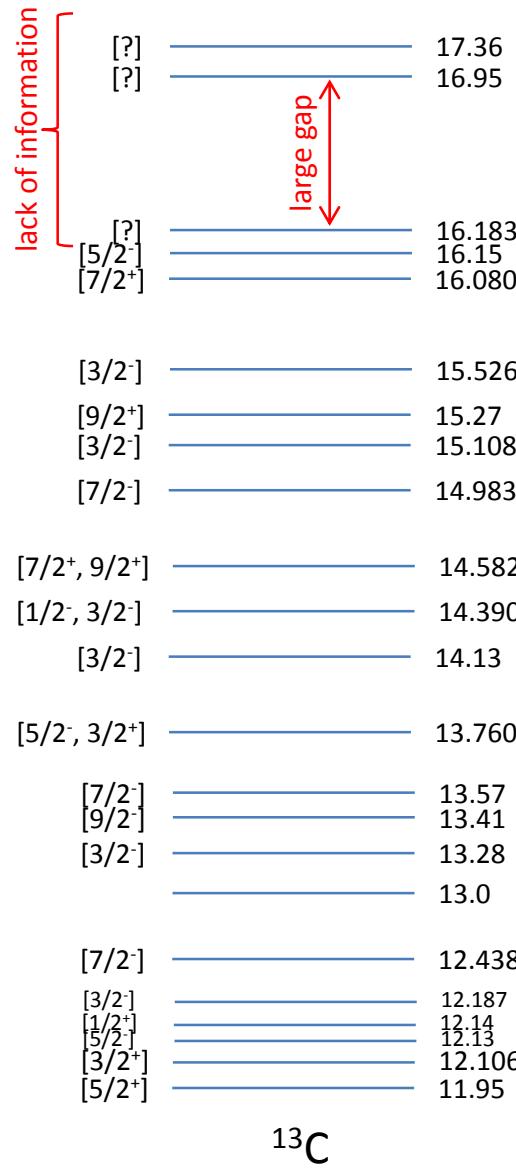
Microscopic GCM  $3\alpha + n$  model  $\rightarrow$  proposed two new rotational bands ( $K=3/2^-_2$  and  $K=3/2^-_3$ ).

$3/2^-_2$

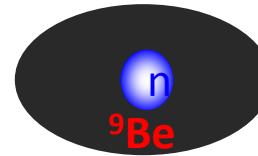
$3/2^-_3$



$\pi$

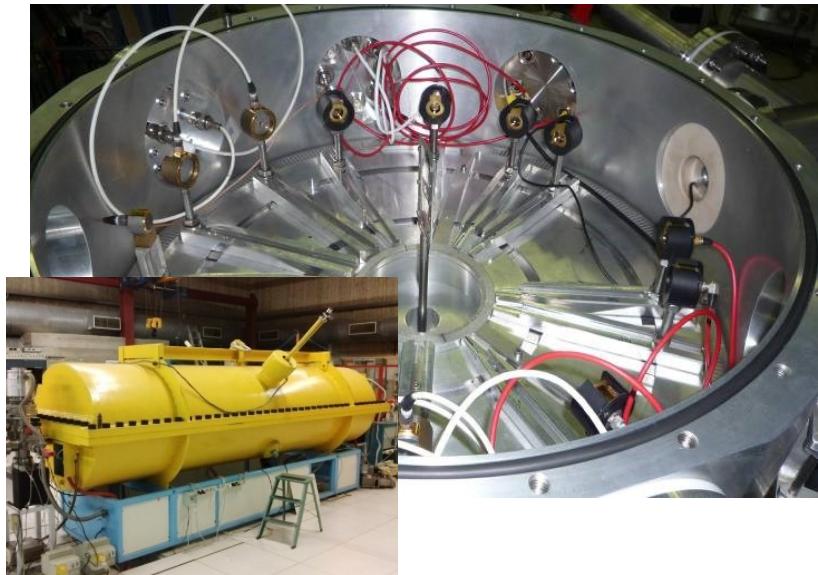
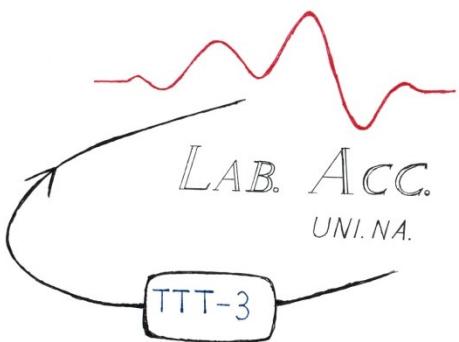


To investigate the structure of  $^{13}\text{C}$  above the  $\alpha$ -threshold (10.651 MeV)  $\rightarrow {}^9\text{Be}(\alpha, n), {}^9\text{Be}({}^6\text{Li}, d), {}^{13}\text{C}(\alpha, \alpha'), {}^9\text{Be}(\alpha, \alpha)$  etc.



The **Resonant Elastic Scattering** is particularly suited for investigating states in the compound  $^{13}\text{C}$  with  $\alpha$ -cluster nature since the  ${}^9\text{Be}$  presents a well developed **molecular nature**.

- R.B. Taylor et al, NPA 65 (1965) 318      DK       $6.3 < E_{\text{lab}} < 19.7$
- J.D. Goss et al, PRC 7 (1973)      DK       $1.7 < E_{\text{lab}} < 5.6$
- Z.A. Saleh et al, Ann. Phys. 7 (1974) 76      DK       $1.4 < E_{\text{lab}} < 2.5$
- J. Leavitt et al, NIM B 85 (1994) 37      DK       $0.6 < E_{\text{lab}} < 4.2$
- J. Liu et al, NIM B 108 (1996) 247      DK       $0.9 < E_{\text{lab}} < 5.3$
- M. Zadro et al, NIM B 259 (2007) 836      IK       $2.0 < E_{\text{cm}} < 4.3$
- M. Freer et al, PRC 84 (2011) 034317      IK       $1.6 < E_{\text{cm}} < 6.0$



**Thin  ${}^9\text{Be}$  target** → self supporting,  $122 \mu\text{g}/\text{cm}^2$

**C** and **O** contamination

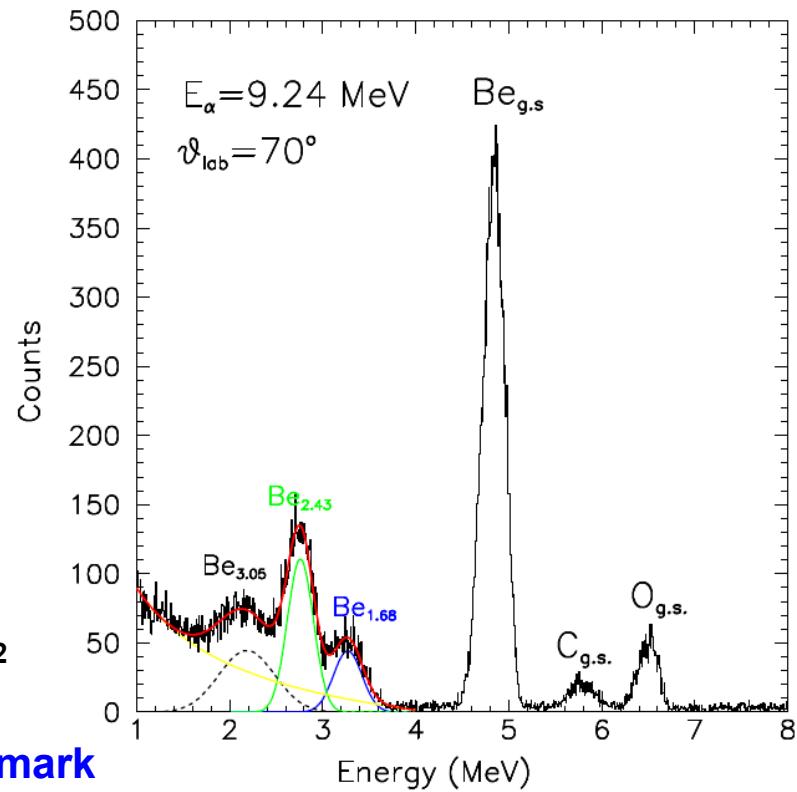
**Thin target absolute cross sections** → **benchmark**  
with a dedicated **thick target experiment**

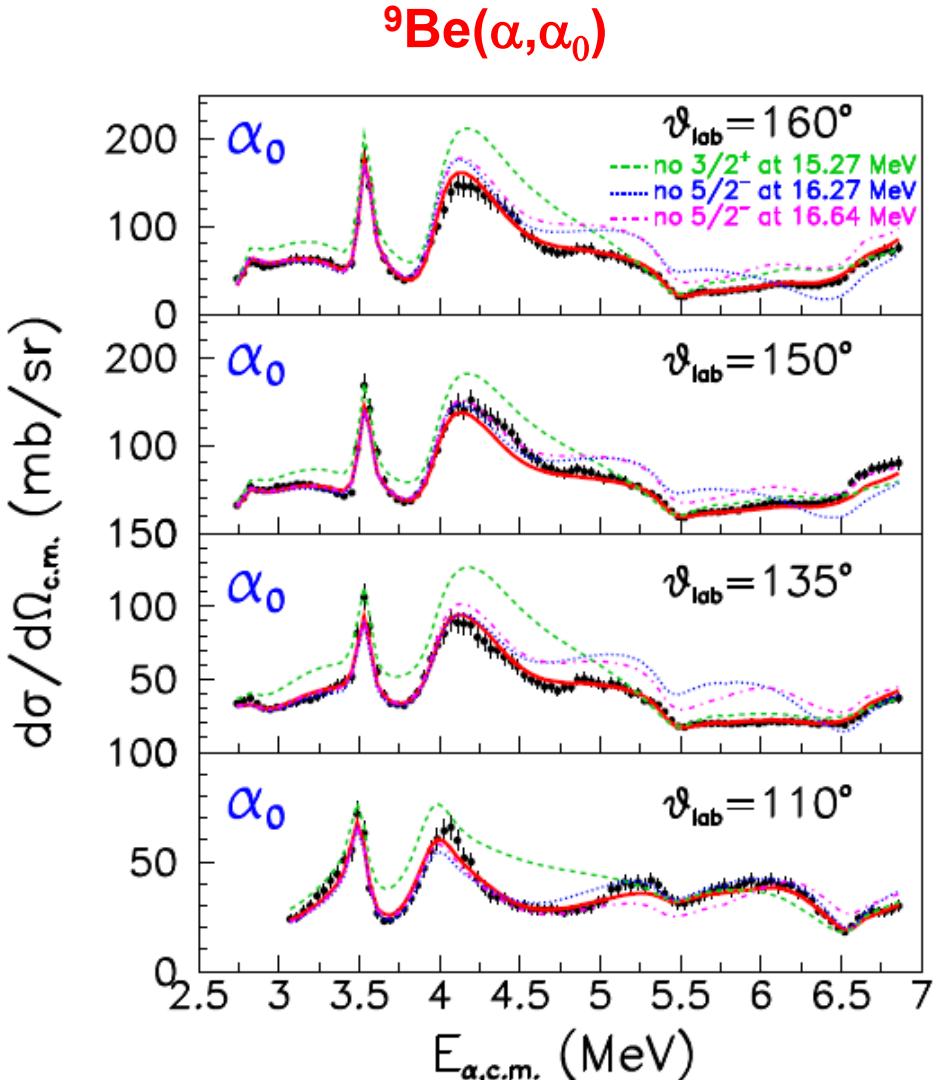
I. Lombardo et al, NIM B 302 (2013) 19-23

**Beam:**  ${}^4\text{He}^{++} \rightarrow E_{\text{Lab}}$ : **3.6 – 10 MeV**

(**60 keV step, 110 energy changes, energy spread  $\leq 3 \text{ keV}$** ) TTT-3MV of Lab. Acceleratore – Naples, Italy

**Silicon detectors:** energy resolution  $\approx 5.0 \%$

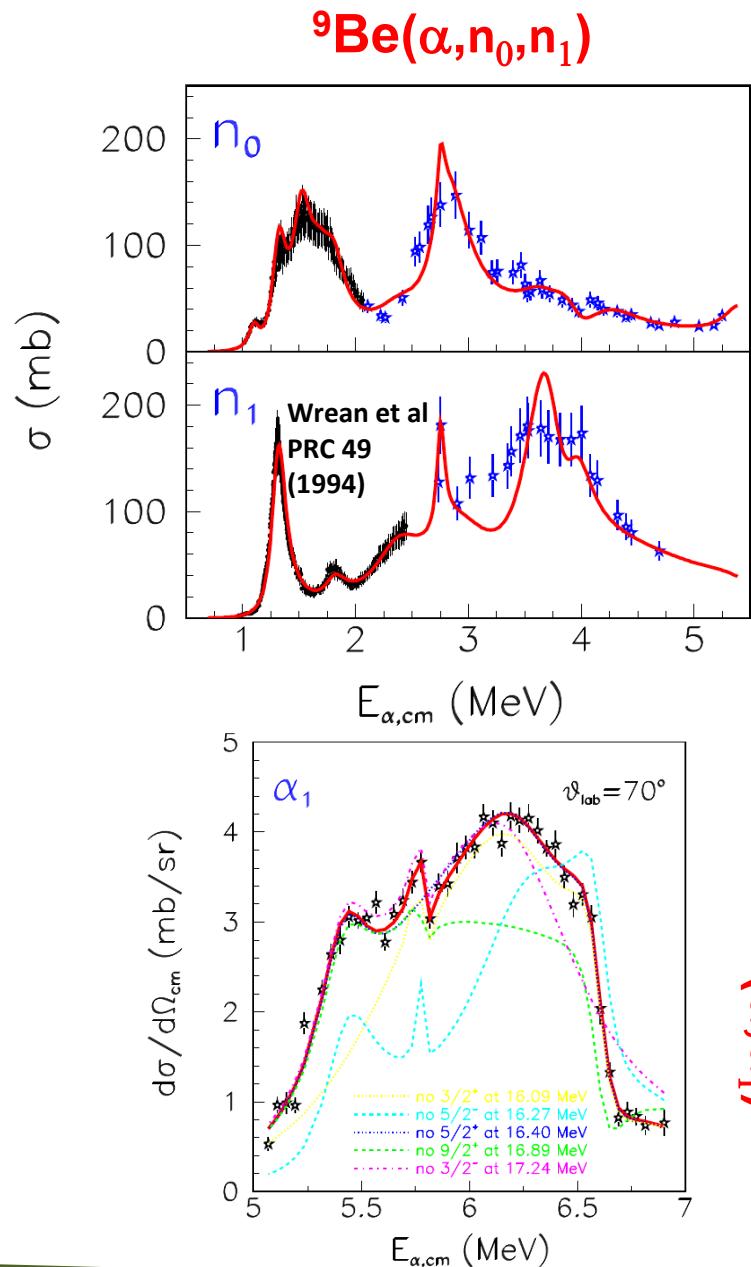




AZURE R-Matrix Program

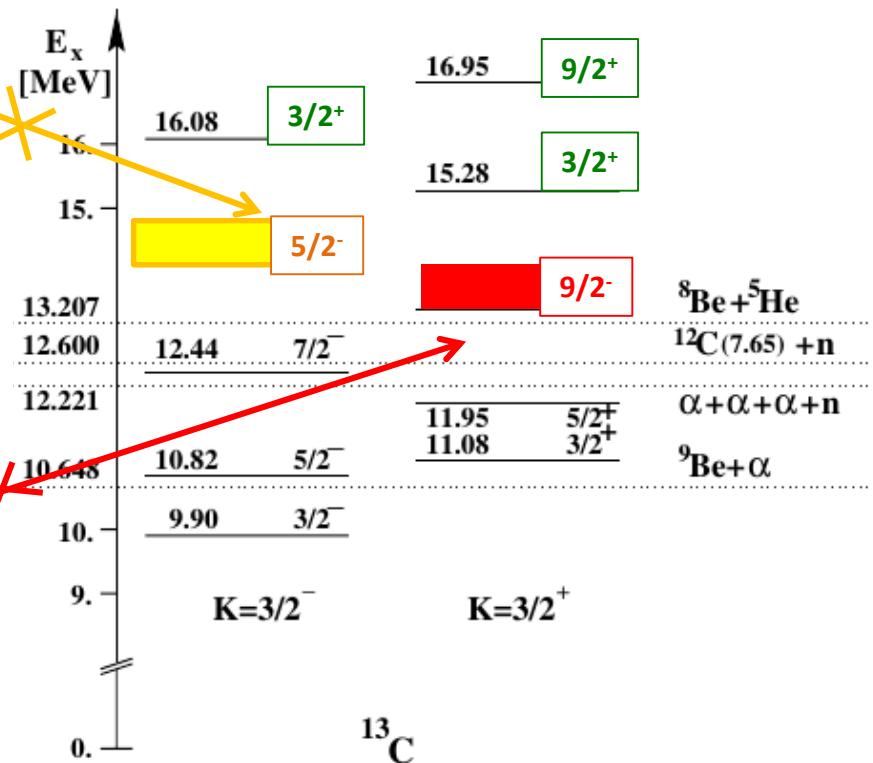
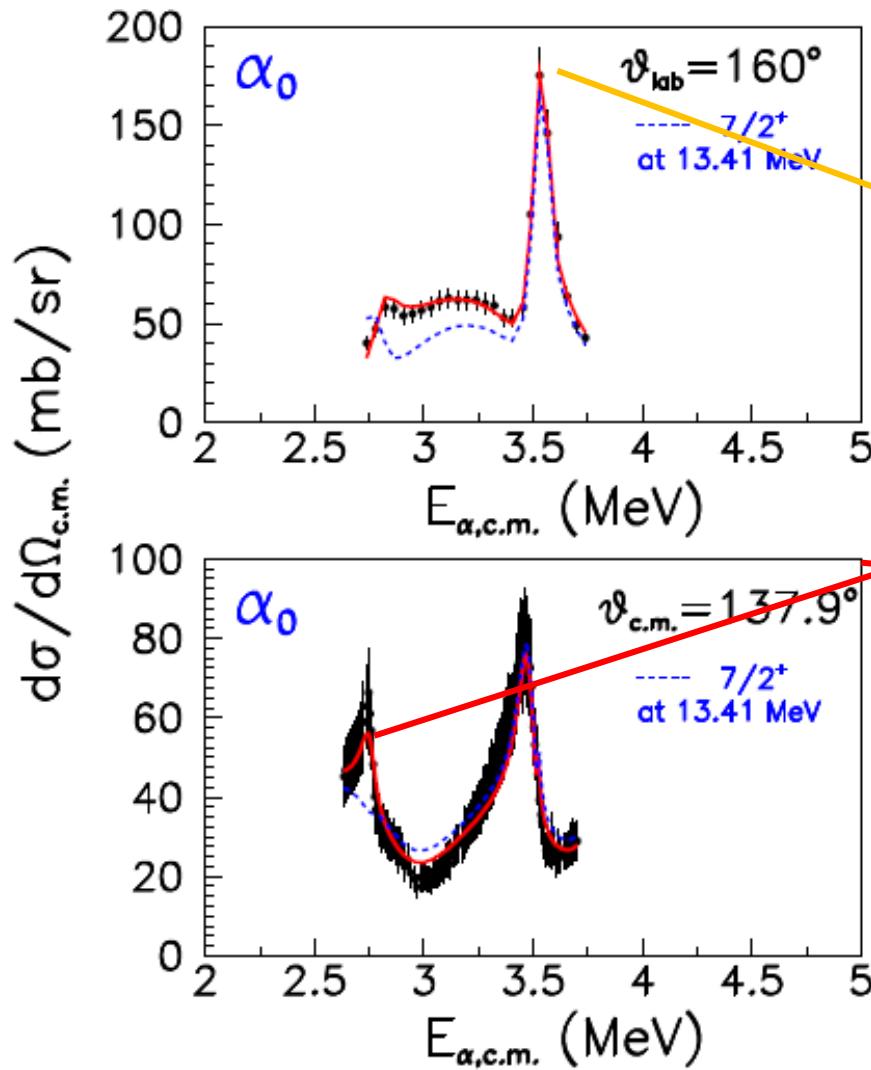
AZURE2

R. Azuma et al, Phys. Rev. C 81, 045805 (2010)



L. van der Zwan and K.W. Geiger,  
Nucl. Phys. A 152 (1970) 481

${}^9\text{Be}(\alpha, \alpha_1)$

«Differential diagnosis» on some «critical»  $J^\pi$  assignments

# $^{13}\text{C}$ structure via $\alpha + {}^9\text{Be}$ : R-matrix calculations

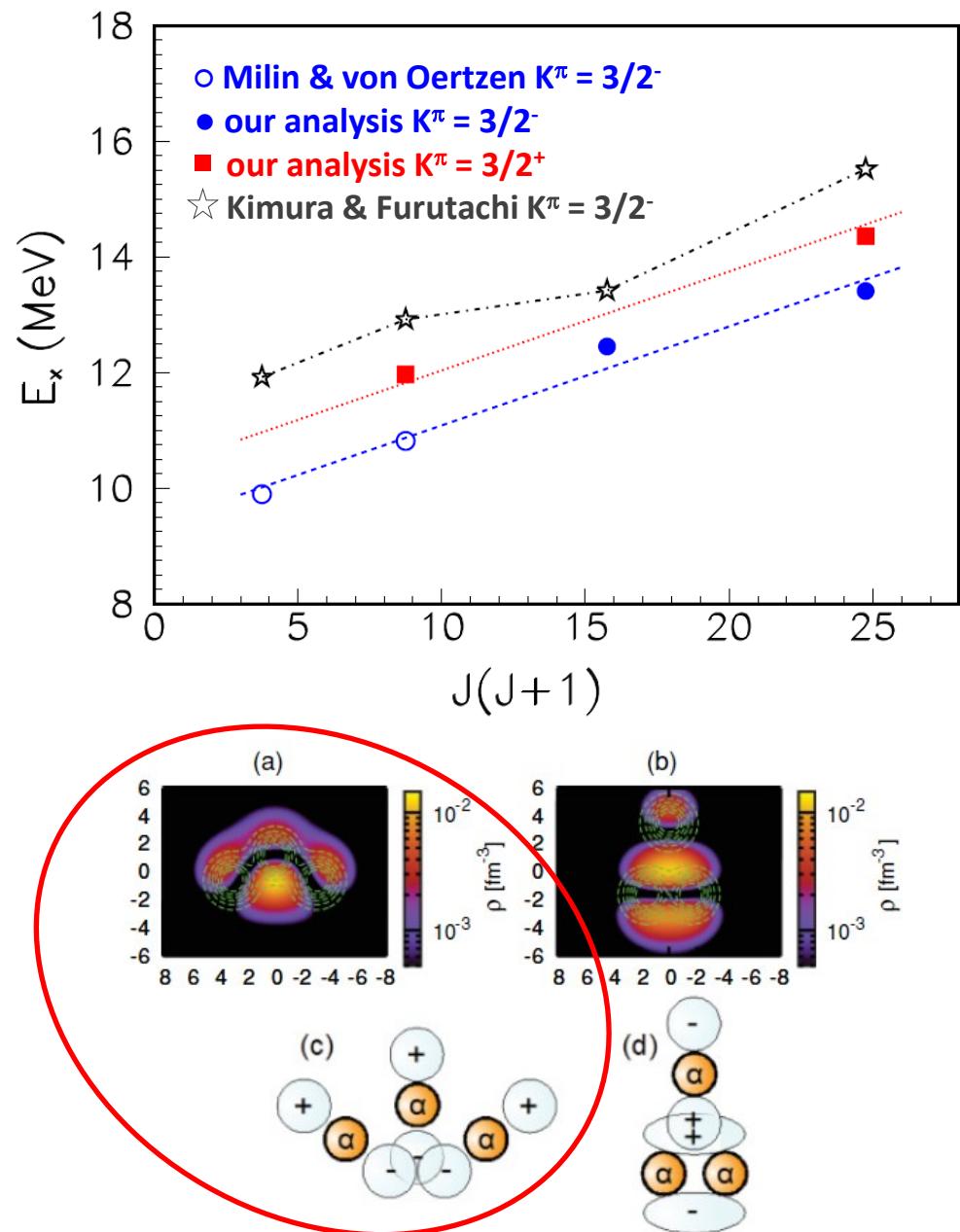
literature

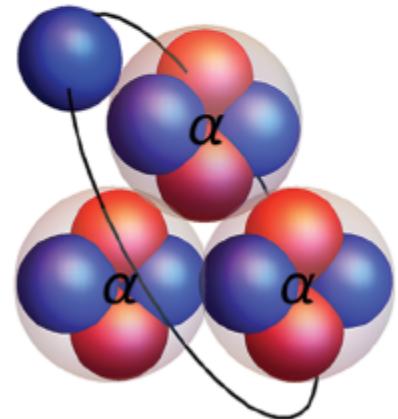
our data from the Napoli exp.

$E_x^{\text{lit}}$	$J_x^{\pi}$	$E_x$	$J_x^{\pi}$	$\Gamma$	$\Gamma_{\alpha_0}$	$\Gamma_{\alpha_1}$	$\Gamma_n$
11.75	$3/2^-$	11.75	$3/2^-$	116(27)	3(1)	—	113(26)
11.97	$5/2^+$						
12.13	$5/2^-$	12.17	$5/2^-$	199(28)	28(7)	—	171(21)
12.14	$1/2^+$	12.33	$1/2^+$	230(37)	40(7)	—	190(30)
12.44	$7/2^-$						
13.28	$3/2^-$	13.05	$3/2^-$	546(112)	153(29)	—	393(83)
13.41	$9/2^-$						
13.57	$7/2^-$	13.49	$7/2^-$	417(116)	114(51)	—	303(65)
13.76	$(3/2, 5/2)^+$	13.63	$5/2^+$	743(51)	623(30)	—	120(21)
14.13	$3/2^-$	14.13	$5/2^-$	94(12)	94(12)	—	—
		14.17	$7/2^+$	6(1)	6(1)	—	—
14.39	$(1/2, 5/2)^-$	14.28	$7/2^-$	392(93)	185(51)	—	207(42)
14.58	$(7/2^+, 9/2^+)$						
14.98	$(7/2^-)$	14.64	$7/2^-$	361(47)	279(28)	—	82(19)
		15.04	$5/2^+$	965(377)	831(355)	—	134(22)
15.27	$9/2^+$	15.27	$3/2^+$	1201(280)	1061(260)	—	140(20)
16.08	$(7/2^+)$	16.09	$3/2^+$	365(55)	233(29)	55(14)	77(12)
		16.27	$5/2^-$	1596(142)	1503(130)	87(10)	6(2)
16.18		16.40	$5/2^+$	17(4)	2(1)	14(2)	1(1)
		16.64	$5/2^-$	1502(156)	1294(110)	10(6)	153(40)
		16.67	$7/2^+$	904(100)	633(100)	2(1)	—
16.95		16.89	$9/2^+$	635(60)	501(40)	86(10)	4(2)
		16.91	$3/2^-$	1079(200)	702(100)	257(50)	120(50)
		17.23	$3/2^+$	393(120)	280(80)	—	113(40)
17.36		17.24	$3/2^-$	216(75)	185(60)	20(9)	11(6)
		17.52	$5/2^+$	2153(290)	1834(170)	86(50)	233(70)
17.92		17.86	$7/2^-$	477(210)	457(200)	—	20(10)

**Some (naive) speculations:**

- **2 opposite parity group of states could match the rule of *rotational band***; for some of them,  $\gamma_\alpha^2$  is a sizable fraction of the WL.
- **Large separation** between the two opposite parity bands ( $\approx 1$  MeV);
- Almost *parallel* bands;
- **Short bands ( $J < 9/2$ ): centrifugal stress?**
- **Very large values of moment of inertia**: similar to the ones calculated with the GCM by Kimura & Furutachi





## On the Cover

A  $^{13}\text{C}$  nucleus modeled as a triangle of three  $\alpha$  particles plus an additional neutron.

## From the article

Evidence for Triangular  $\mathcal{D}'_{3\text{h}}$  Symmetry in  $^{13}\text{C}$ 

R. Bijker and F. Iachello

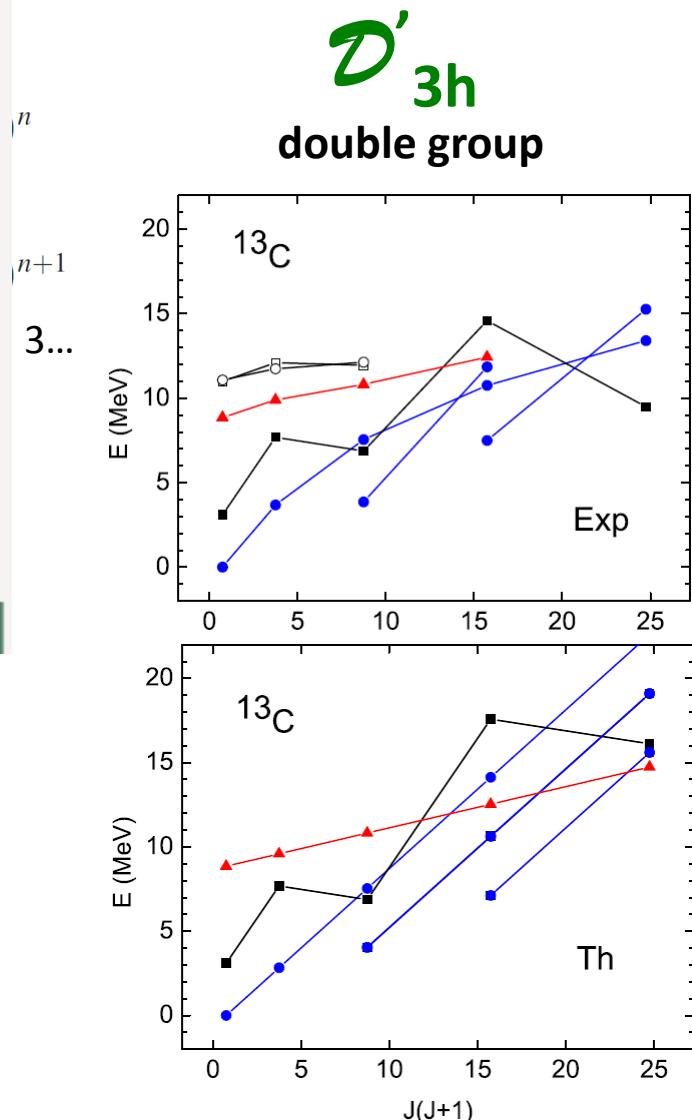
Phys. Rev. Lett. **122**, 162501 (2019)[View Issue](#)

the so-called decoupling parameter with  $g_\Omega(J) = \delta_{K,1/2}(-1)^{J+1/2}(J + 1/2)$ . The latter term applies only to representations  $\Omega \equiv E_{1/2}^{(\pm)}$  and  $K^P = 1/2^\pm$ .

Symmetry  $\rightarrow$  **recurrences** in the rotational bands

e.g.  $1/2^-$  coupled to  $5/2^+$  and  $7/2^+$ , etc...

e.g.  $1/2^+$  coupled to  $5/2^-$  and  $7/2^-$ , etc...

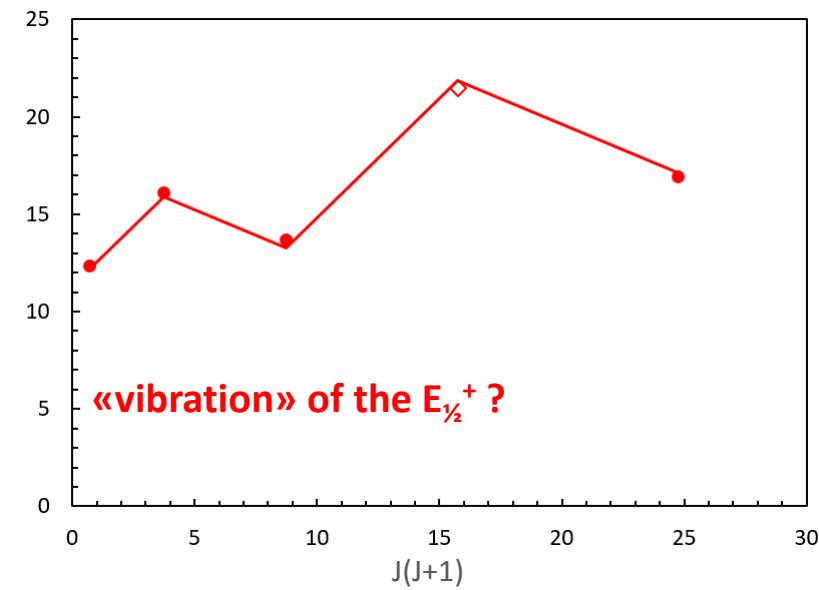
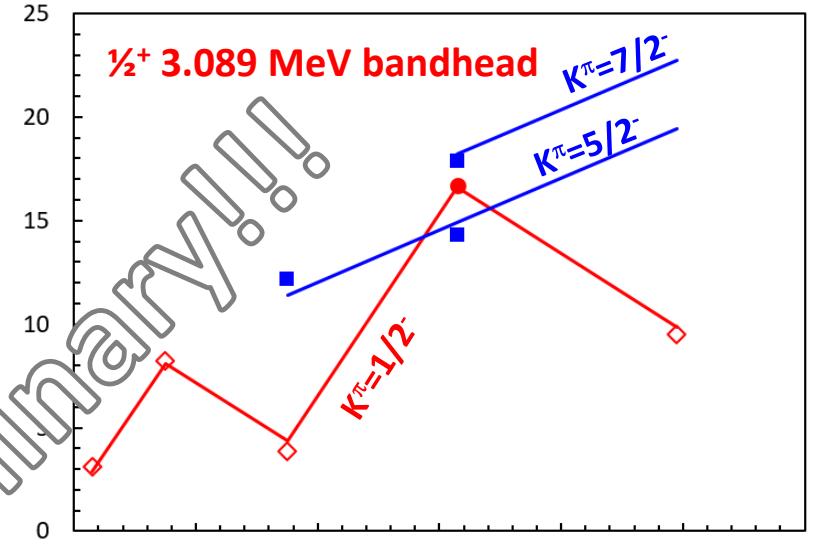
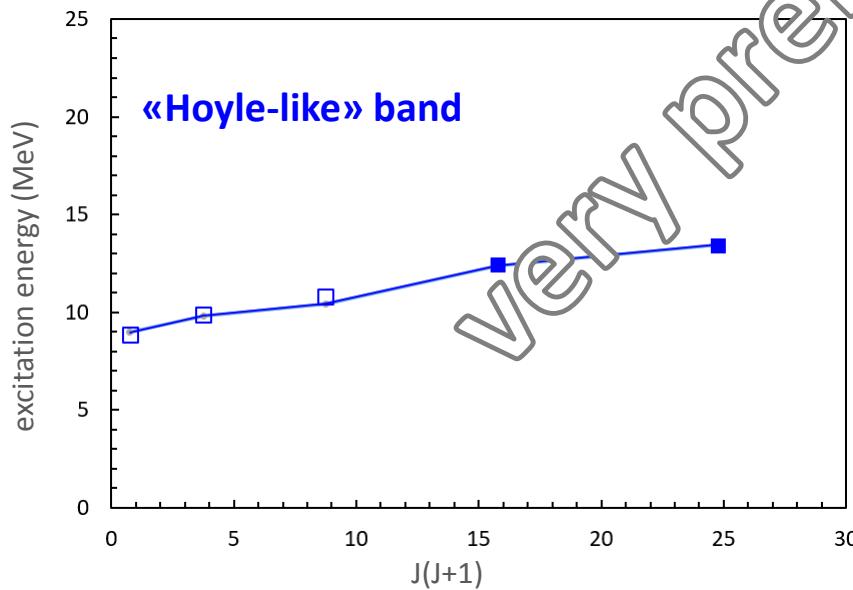
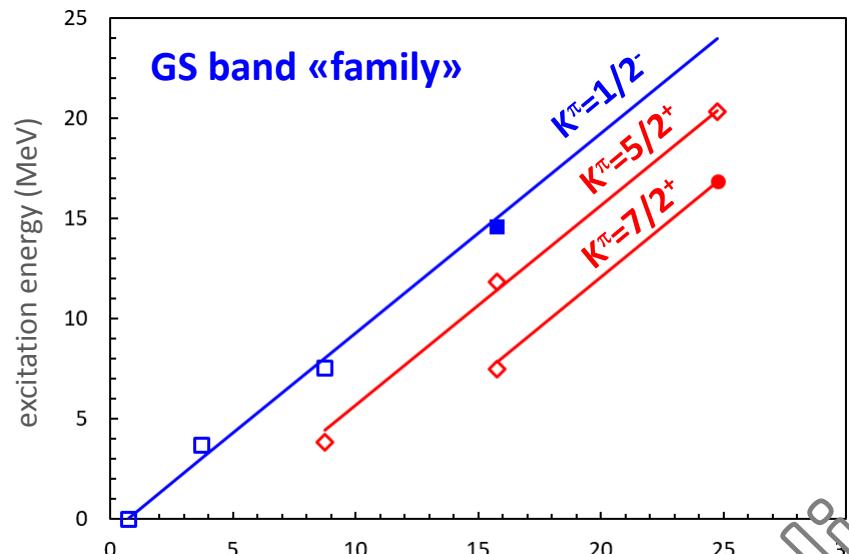
description of  $^{13}\text{C}$  spectrumR. Bijker & F. Iachello, Phys. Rev. Lett. **122** (2019) 162501

Open symbols: Ajzenberg-Selove A 523 (1991)

Blue symbols: negative parity states

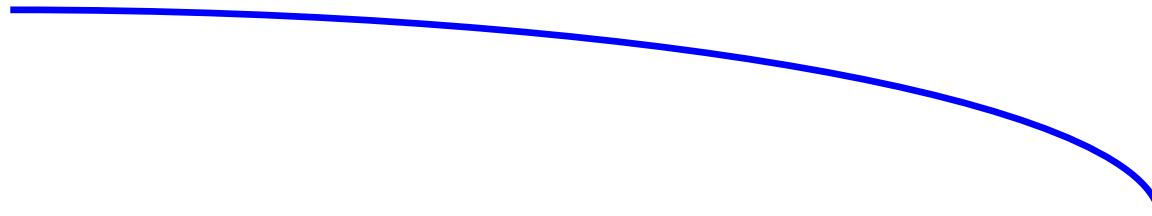
Full symbols: present work

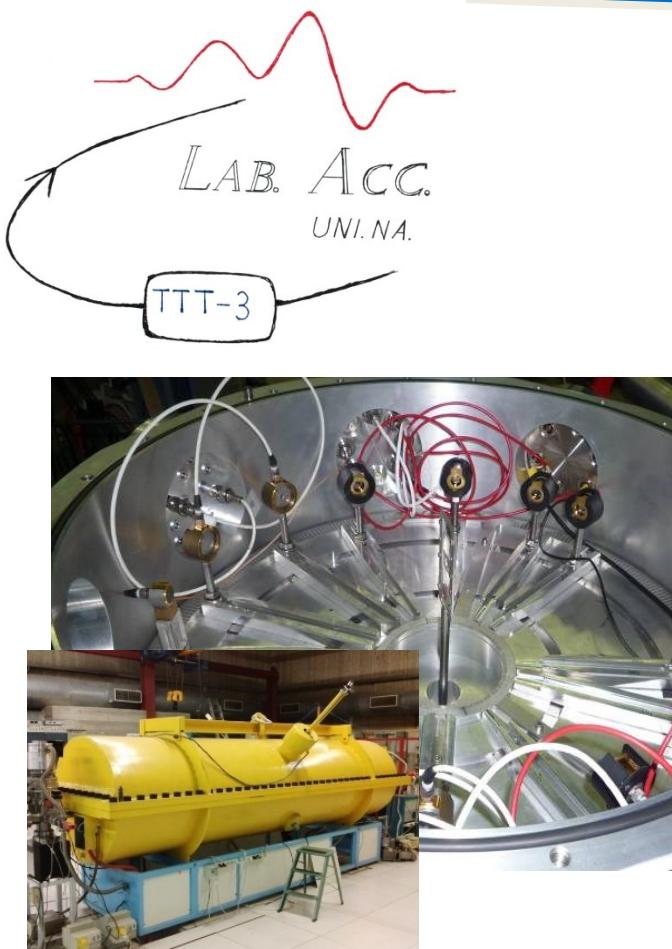
Red symbols: positive parity states



- Investigations of  $^{13}\text{C}$  spectroscopy at  $E_x$  larger than the  $\alpha$  emission threshold → useful method to unveil *cluster structures* in *non self-conjugated nuclei*.
- A possible reaction →  $^9\text{Be}(\alpha, \alpha)$  RES → *few data* available in the literature (many uncertainties in  $J^\pi$  assignments)
- A new experiment carried out at Laboratorio dell'Acceleratore (TTT-3MV) of the University of Naples "Federico II" → direct kinematics → excitation functions of  $^9\text{Be}(\alpha, \alpha_0)$  EBS at  $\vartheta_{\text{lab}} = 110^\circ, 135^\circ, 150^\circ, 160^\circ$  in  $E_\alpha = 3.6 - 10$  MeV energy range → inelastic  $\alpha_1$  at  $\vartheta_{\text{lab}} = 70^\circ$
- Comprehensive *R-matrix fit* of the data with the AZURE2 code by including the  $^9\text{Be}(\alpha, \alpha_0)$  data ( $160^\circ, 150^\circ, 135^\circ$  and  $110^\circ$ ), the  $^9\text{Be}(\alpha, \alpha_1)$  data ( $70^\circ$ ), the low energy points from *Saleh et al.* and *Goss et al* and the *neutron* channels.
- Strong efforts to fit data at various angles → reproduction of data with a reasonable set of parameters in a wide energy range → *revised spectroscopy* in the region  $E_x = 12$  MeV –  $17$  MeV (above the  $\alpha + ^9\text{Be}$  and  $\alpha + \alpha + \alpha + n$  cluster thresholds)
- *Hints* for the existence of *rotational bands* with molecular nature. Preliminary comparison with *models* seems to confirm this point.

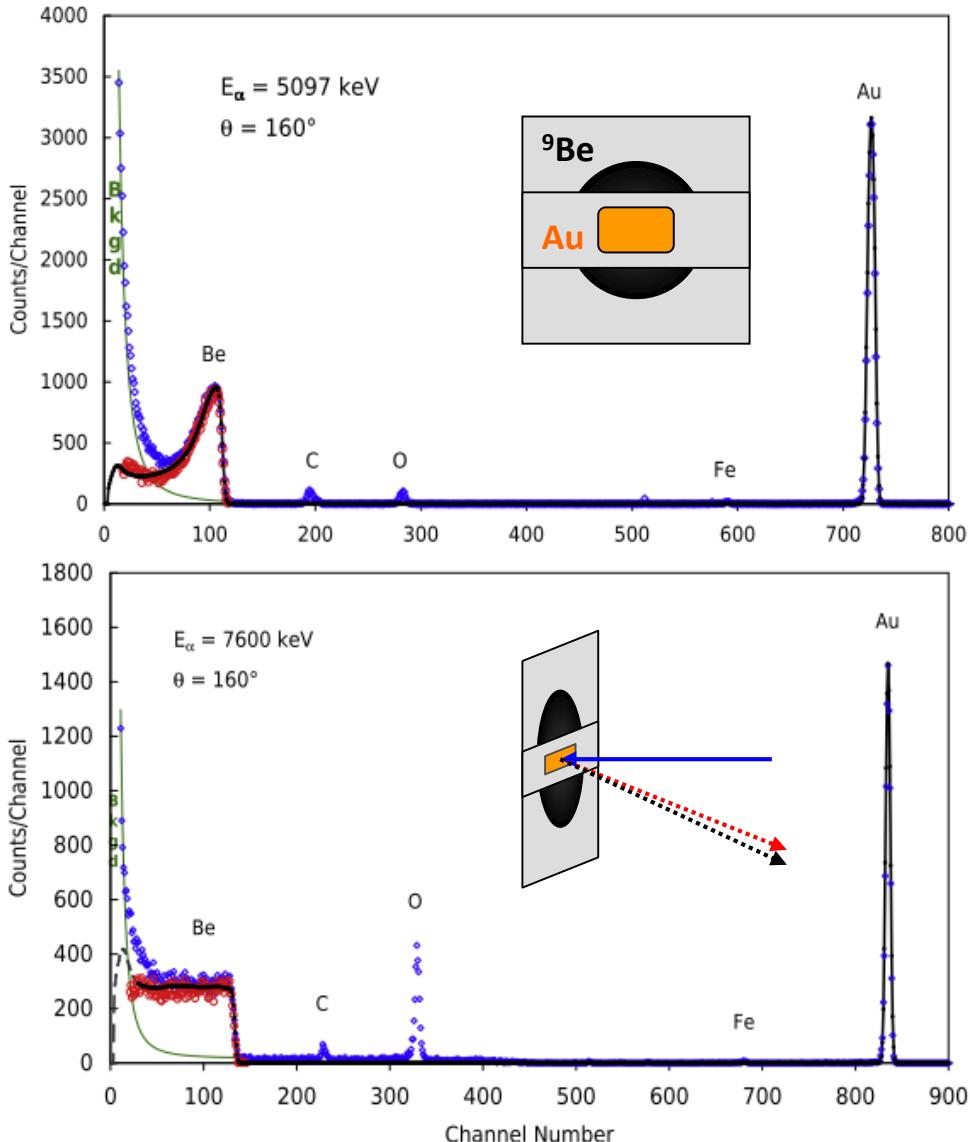
# Further Slides

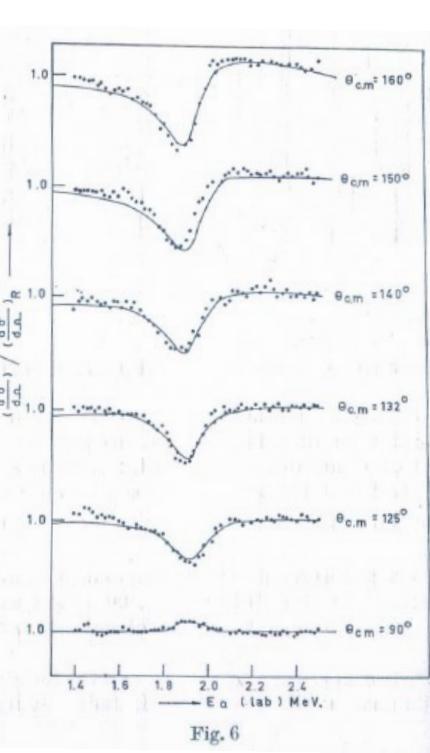




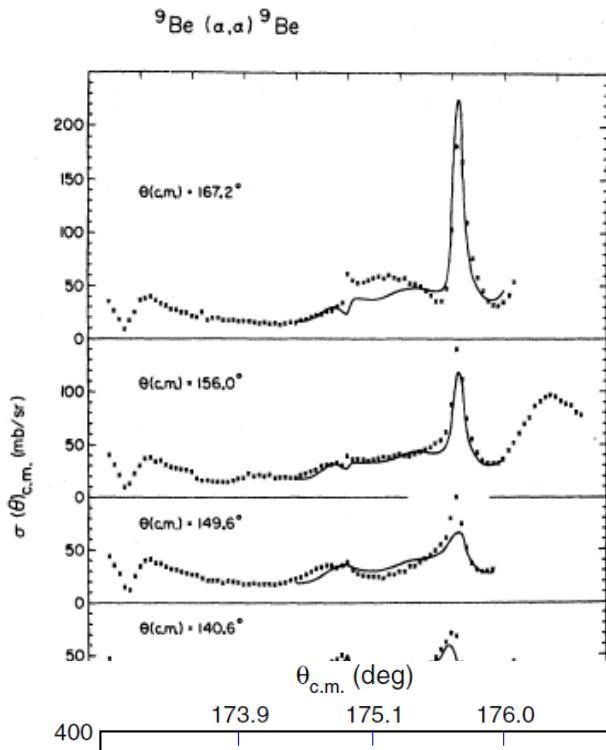
Thin  ${}^9\text{Be}$  target → self support  
 C and O contamination  
 Thin target absolute cross sec  
 with a dedicated thick target ex  
 I. Lombardo et al, NIM B 302 (2013) 19-23

### cross section check with thick target method

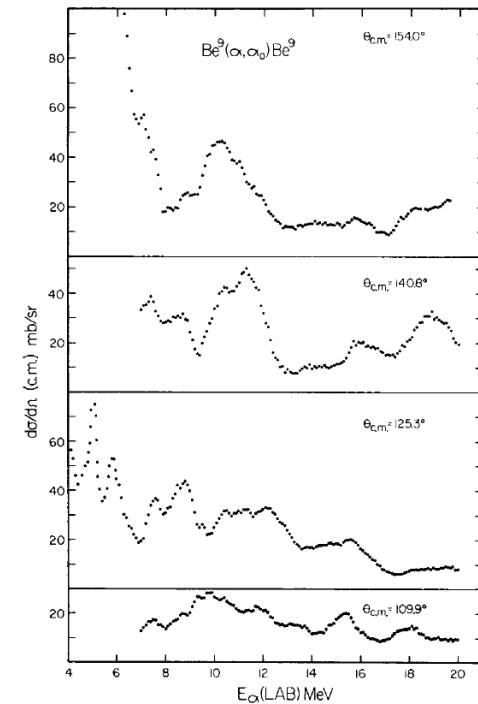




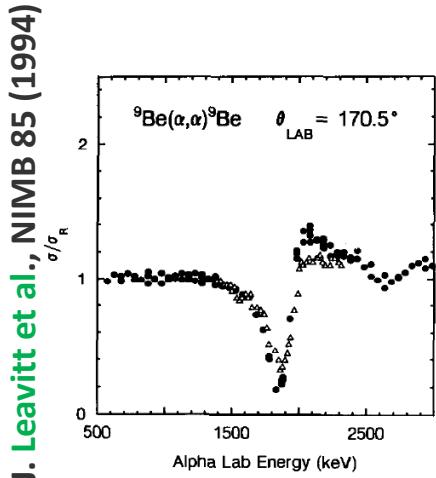
Z.A. Saleh et al., Ann. Phys. (1974)



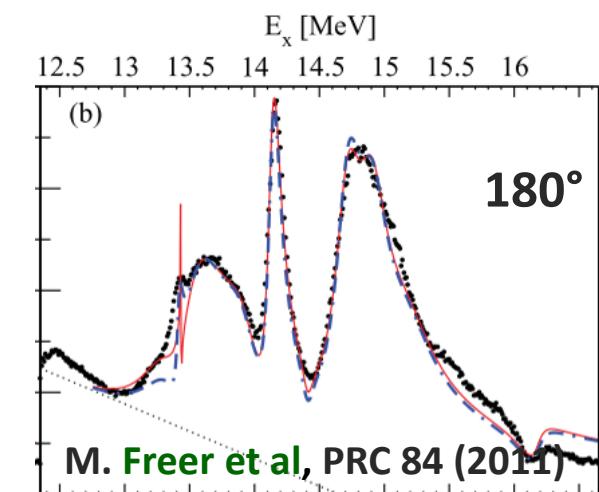
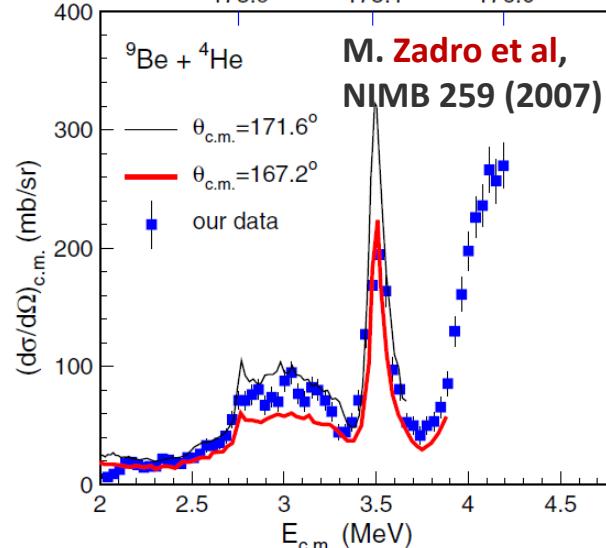
J.D. Goss et al, PRC 7 (1973)



R.B. Taylor et al, Nucl. Phys. 65 (1965)

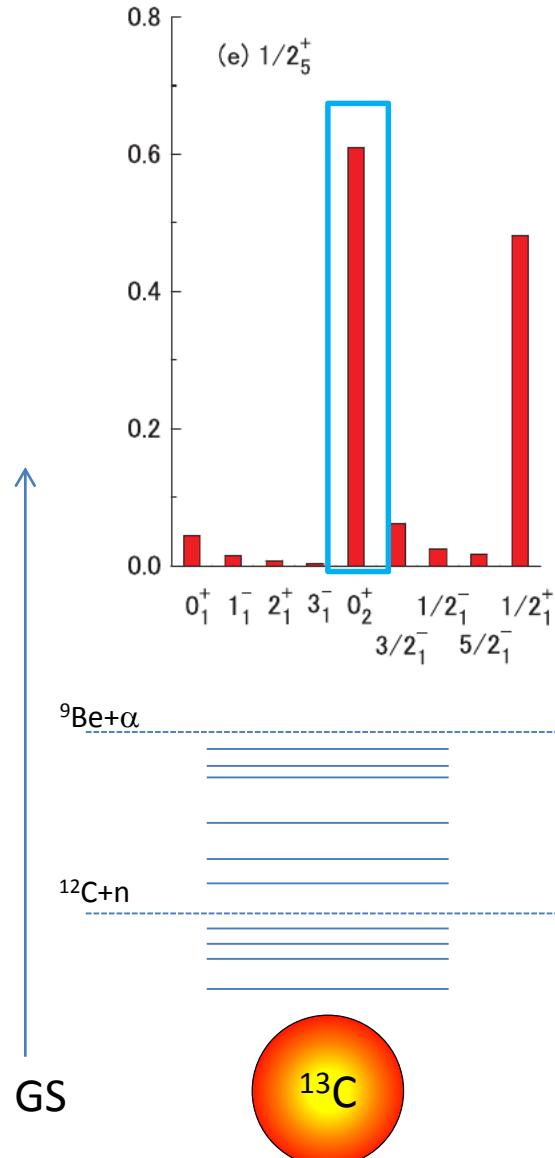


J. Leavitt et al., NIMB 85 (1994)



Limited  $\theta_{cm}$  and  $E_\alpha$  ranges → new experiment

T. Yamada and Y. Funaki, Phys. Rev. C 92 (2015) 034326



Near and above the  $\alpha$ -threshold  $\rightarrow$  different  $\alpha$ -cluster configurations proposed for <sup>13</sup>C  $\rightarrow$  theoretical works:

M. Milin and W. Von Oertzen EPJ A 14 (2002) 295

proposed parity doublet band of <sup>9</sup>Be<sub>gs</sub> +  $\alpha$  cluster prolate configuration  $\rightarrow J^\pi$  assignments based on the rotational bands ( $K=3/2^\pm$ ).

T. Yoshida, N. Itagaki and T. Otsura, Phys. Rev. C 79 (2009)

N. Furutachi and M. Kimura, Phys. Rev. C 83 (2011)

microscopic 3 $\alpha$ +n model  $\rightarrow$  proposed two new rotational bands ( $K=3/2_{-2}^\pm$  and  $K=3/2_{-3}^\pm$ ).

**Coupling of one neutron with a <sup>12</sup>C\* core  $\rightarrow$  possible gas-like states analog of Hoyle state in <sup>13</sup>C:**

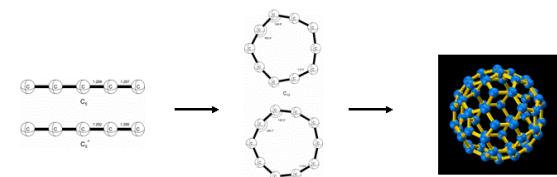
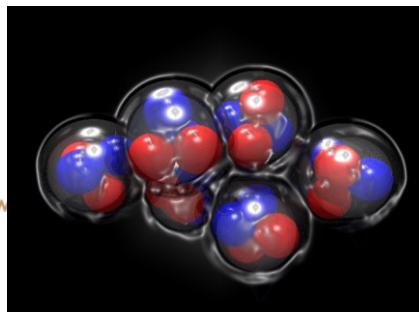
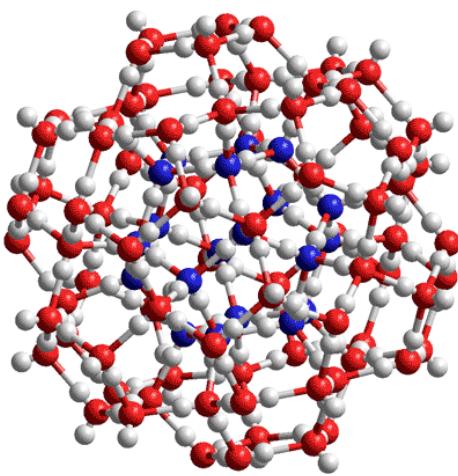
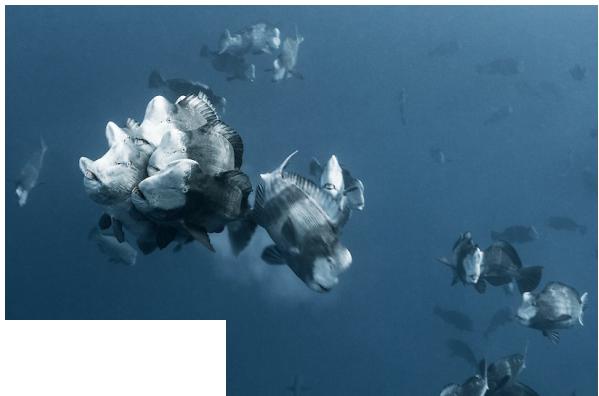
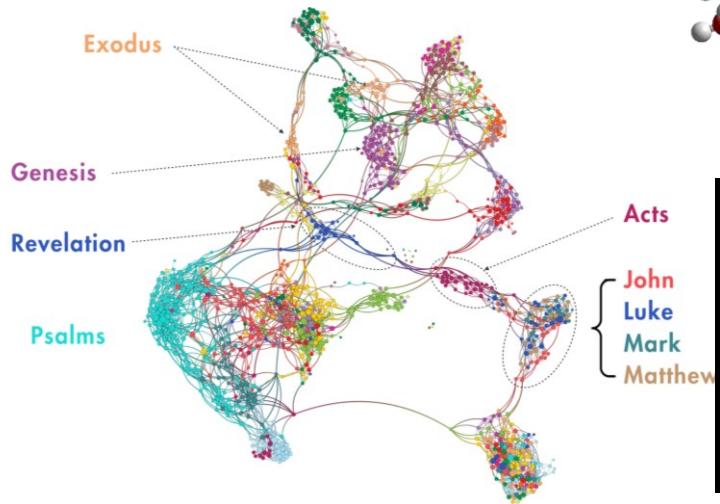
Y. Chiba and M. Kimura, J. Phys. Conf. Ser. 569 (2014) 012047

molecular bands ( $K=1/2^\pm$ ).

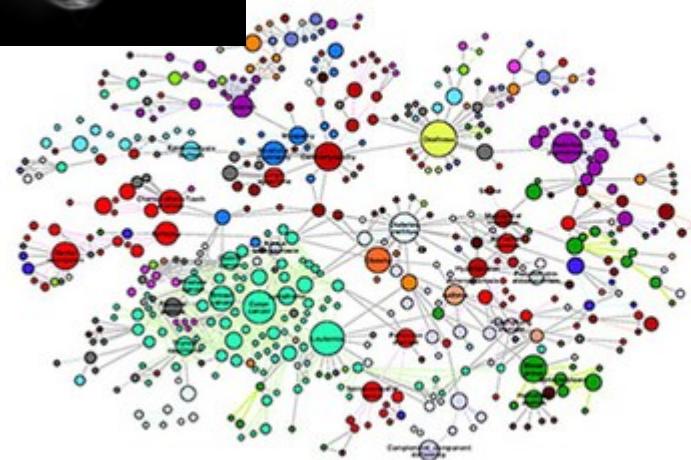
T. Yamada and Y. Funaki, Phys. Rev. C 92 (2015) 034326

$1/2_{-5}^+$  state predicted at 14.9 MeV with a strong <sup>12</sup>C( $0_2^+$ ) + n spectroscopic factor  $\rightarrow$  analog of Hoyle state in <sup>13</sup>C.

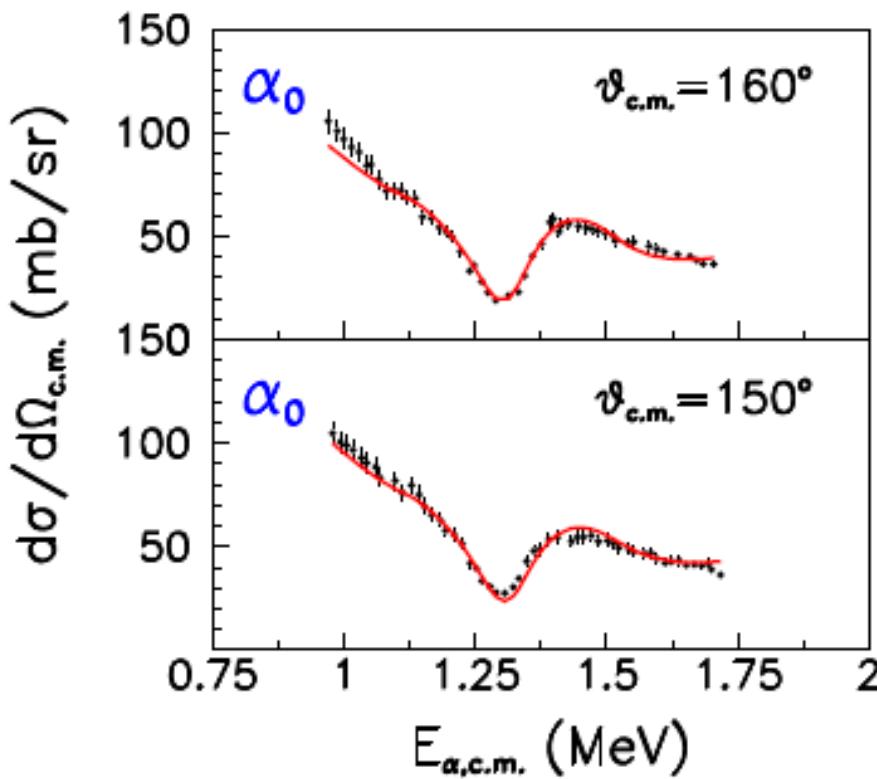
# Clustering: an *ubiquitous* phenomenon



Disease Network



- Disorder Class**
- Bone
  - Cancer
  - Cardiovascular
  - Connective tissue
  - Dermatological
  - Developmental
  - Ear, Nose, Throat
  - Endocrine
  - Gastrointestinal
  - Hematological
  - Immunological
  - Metabolic
  - Muscular
  - Neurological
  - Nutritional
  - Ophthalmological
  - Psychiatric
  - Renal
  - Respiratory
  - Skeletal
  - multiple
  - Unclassified



Saleh et al (Ann. Phys. 7) to study the spectroscopy of states at lower energies

Lack of high accuracy data at  $E_{cm} = 1.75 - 2.5$  MeV → Goss et al

