Theory of Heavy Ion Charge Exchange Reactions as Probes for Beta-Decay

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Agenda:

- **Heavy ion Single Charge Exchange (SCE) reactions**
- **SCE reactions and $1\nu1\beta$ decay**
- **2-step Double Single Charge Exchange reactions (DSCE)**
- **1-step DCE Mechanism: „Majorana" DCE reactions (MDCE)**
- **Connection to NME of $2\nu2\beta$ and $0\nu2\beta$ double beta decay**
- **Summary and Outlook**

Single Charge Exchange Reactions

Separation of Reaction and Nuclear Dynamics

$$M_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \int \frac{d^3p}{(2\pi)^3}\langle\chi_\beta^{(-)}|e^{-i\mathbf{p}\cdot\mathbf{r}}\mathcal{U}_{\alpha\beta}(\mathbf{p})|\chi_\alpha^{(+)}\rangle$$

$$\mathcal{U}_{\alpha\beta} \sim \langle J_b M_b J_B M_B | T_{NN}^{(C)} + T_{NN}^{(Tn)} \ldots | J_a M_a J_A M_A\rangle$$

$$N_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta,\mathbf{p}) = \frac{1}{(2\pi)^3}\langle\chi_\beta^{(-)}|e^{-i\mathbf{p}\cdot\mathbf{r}}|\chi_\alpha^{(+)}\rangle,$$

$$M_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \int d^3p\,\mathcal{U}_{\alpha\beta}(\mathbf{p})N_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta,\mathbf{p})$$

Distortion Coefficient and S-Matrix:

$$N_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta,\mathbf{p})_{p=0} \rightarrow N_D = \frac{1}{(2\pi)^3}\langle\chi_\alpha^{(-)}|\chi_\alpha^{+}\rangle = \frac{1}{k_\alpha^2}\delta\left(k_\alpha - k'_\alpha\right)\frac{1}{4\pi}S_\alpha$$

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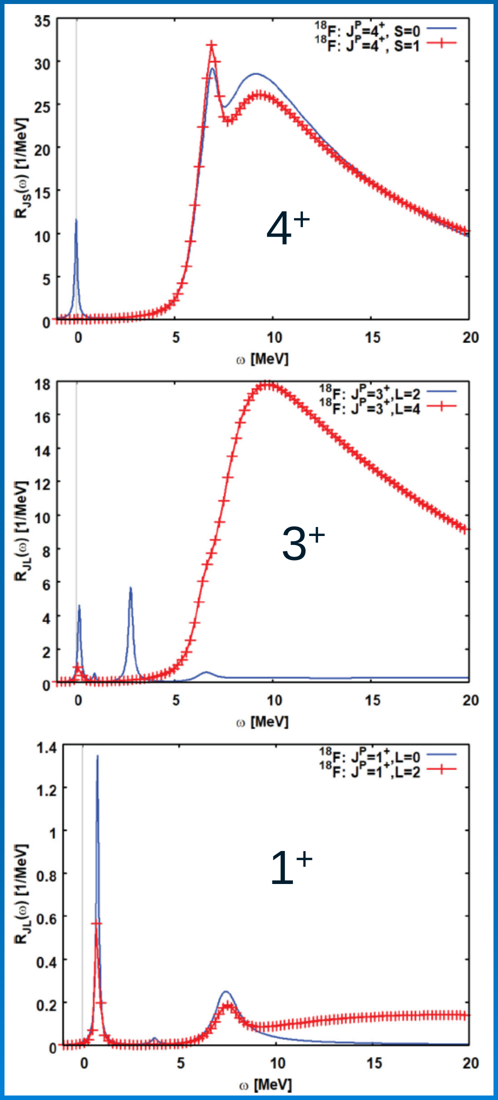
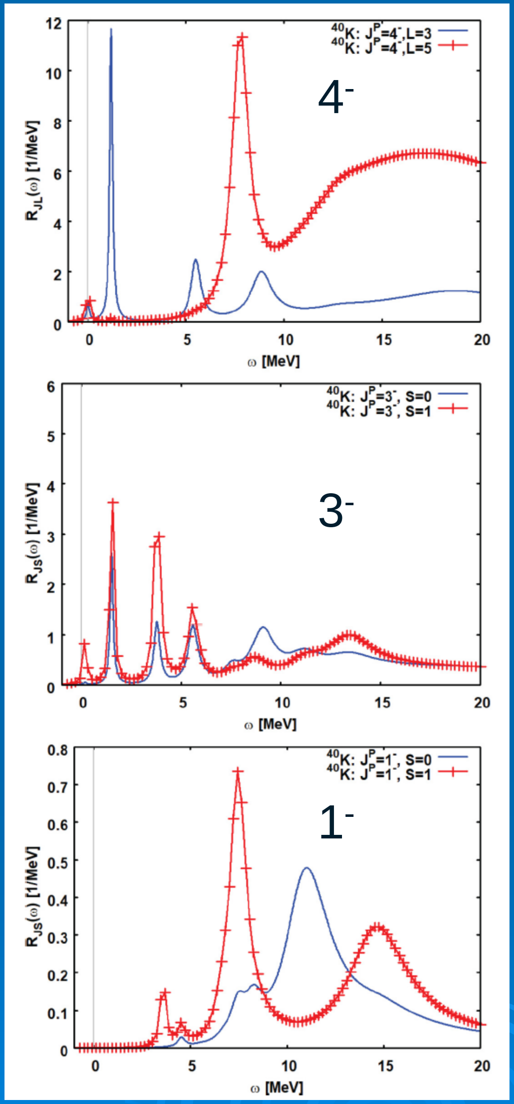
QRPA Response Functions

$^{18}O \rightarrow {}^{18}F$



$^{40}Ca \rightarrow {}^{40}K$



Transition Operator:

$$T_{LSJM} = \left(\frac{r}{R_d}\right)^L [\boldsymbol{\sigma}^S \otimes Y_L]_{JM}\tau_\pm$$

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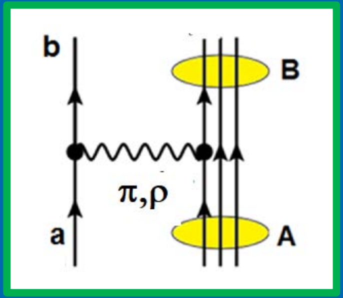
Nuclear Interactions and beta-Decay



Strong Interaction **Weak Interaction**

$$V_{NN} \sim V_{01}\left(q^2\right)\tau_\pm\tau_\mp \quad \leftrightarrow \quad g_F\left(q^2\right)\tau_\pm \quad \text{"Fermi"}$$

$$+V_{11}\left(q^2\right)\sigma_1\cdot\sigma_2\ \tau_\pm\tau_\mp \quad \leftrightarrow \quad g_A\left(q^2\right)\sigma\ \tau_\pm \quad \text{"Gamow-Teller"}$$

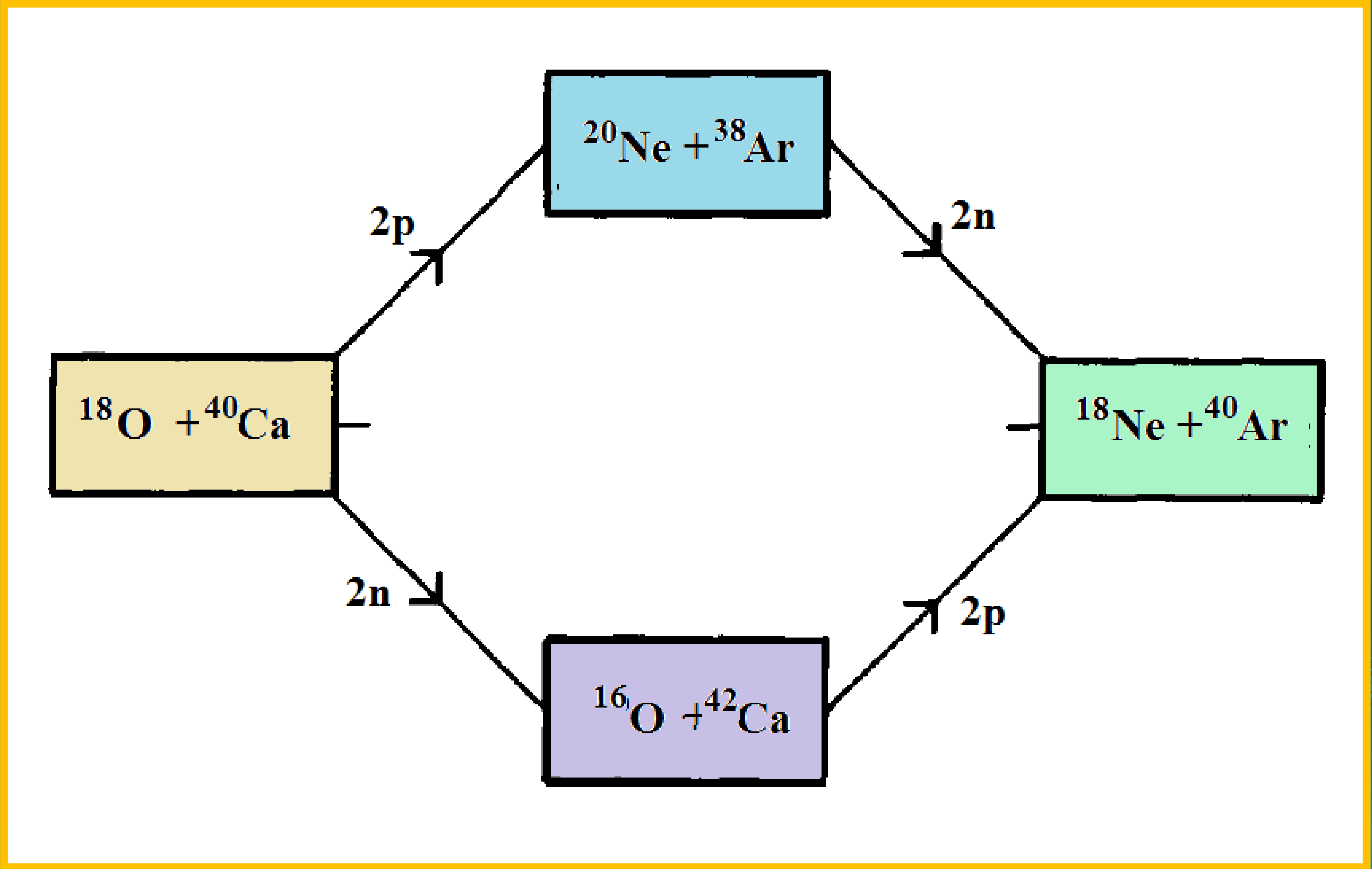
$$+V_{T1}\left(q^2\right)S_{12}\ \tau_\pm\tau_\mp \quad \leftrightarrow \quad g_M\left(q^2\right)\sigma\times \mathrm{q}\ \tau_\pm \quad \text{"weak magnetic"}$$

$$+\ldots$$

Rank-2 tensor operator: $S_{12} = \frac{1}{q^2}\left[3\vec{\sigma}_1\cdot\vec{q}\sigma_2\cdot\vec{q} - \sigma_1\cdot\sigma_2 q^2\right]$

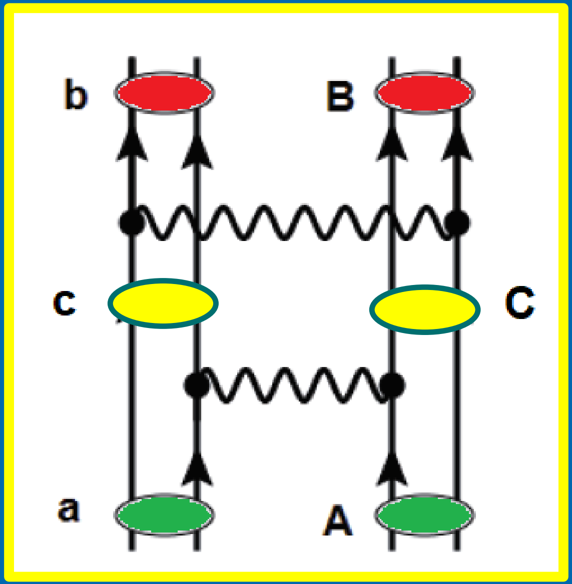
Nuclear Double Charge Exchange Processes

The NUMEN Scheme of a Double Charge Exchange Reaction: Double-SCE (DSCE) plus Majorana-DCE (MDCE)



Double-Single Charge Exchange (DSCE) Reactions

DSCE:
Double Charge Exchange by Sequential Single Charge Exchange

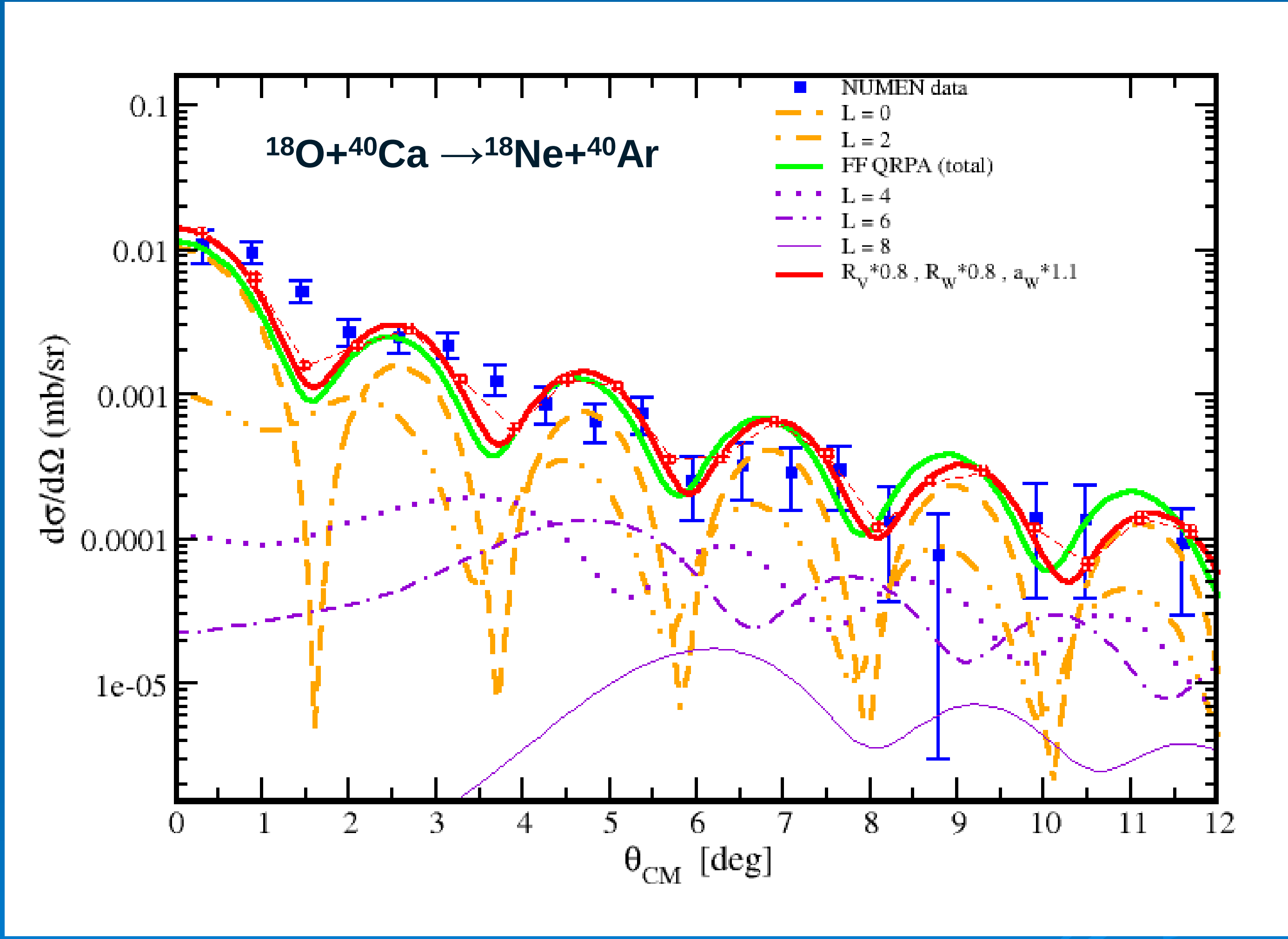


$$^{18}O + {}^{40}Ca \rightarrow {}^{18}F + {}^{40}K \rightarrow {}^{18}Ne + {}^{40}Ar$$

Reaction Amplitude

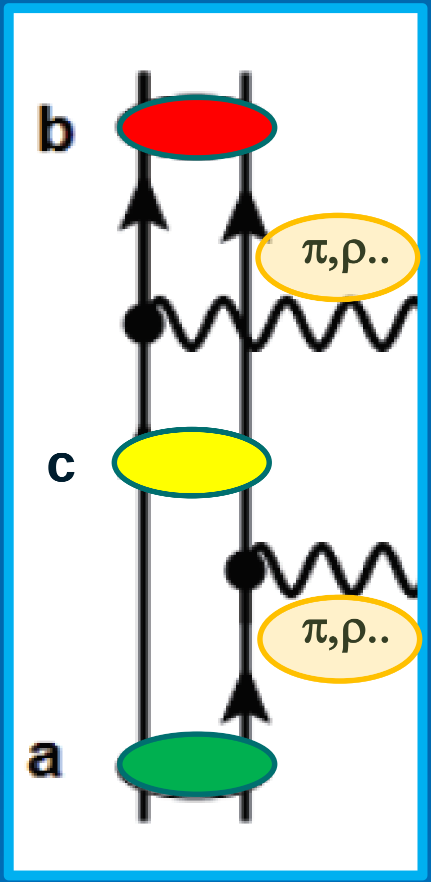
$$M^{(DSCE)}_{\alpha\beta}(\mathbf{k}_{bB}, \mathbf{k}_{\alpha}) = \langle \chi^{(-)}_{\beta}, bB | T_{NN} \mathcal{G}^{(+)}(\omega) T_{NN} | aA, \chi^{(+)}_{aA} \rangle .$$

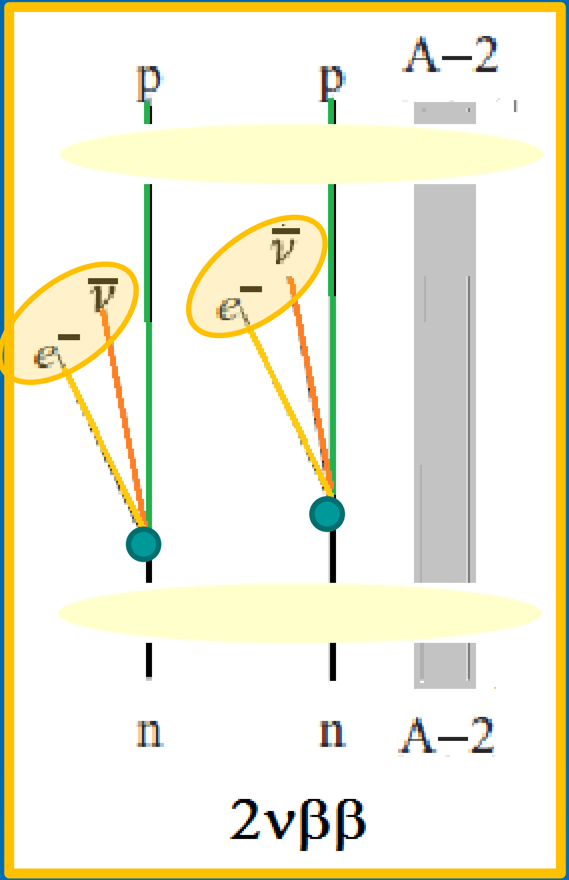
DSCE Results in Pole Approximation



Theory: J. **Bellone, M. Collona, H.L. (2019);**
Data: F. Cappuzzello et al., EPJ A51 (2015)

DSCE and 2ν2β Beta Decay





2nd order
Strong Interaction

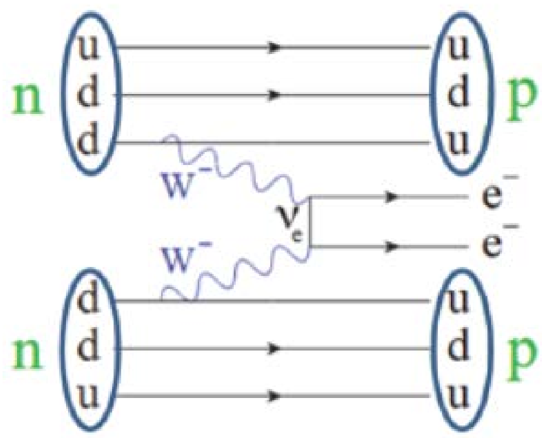
2nd order
Weak Interaction

Double Charge Exchange through 2-Body Interactions:

„Majorana“ DCE

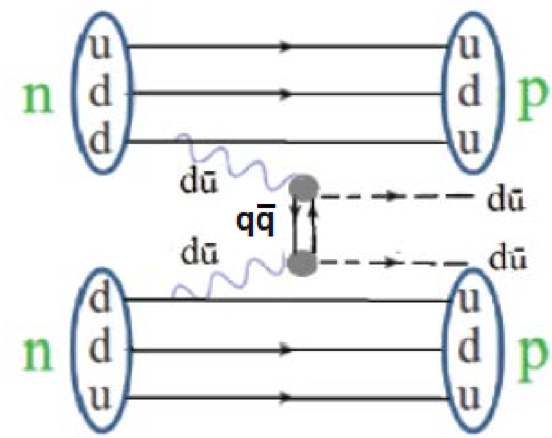
Weak 0ν2β Decay and Strong Interaction Analogue

0ν2β Decay



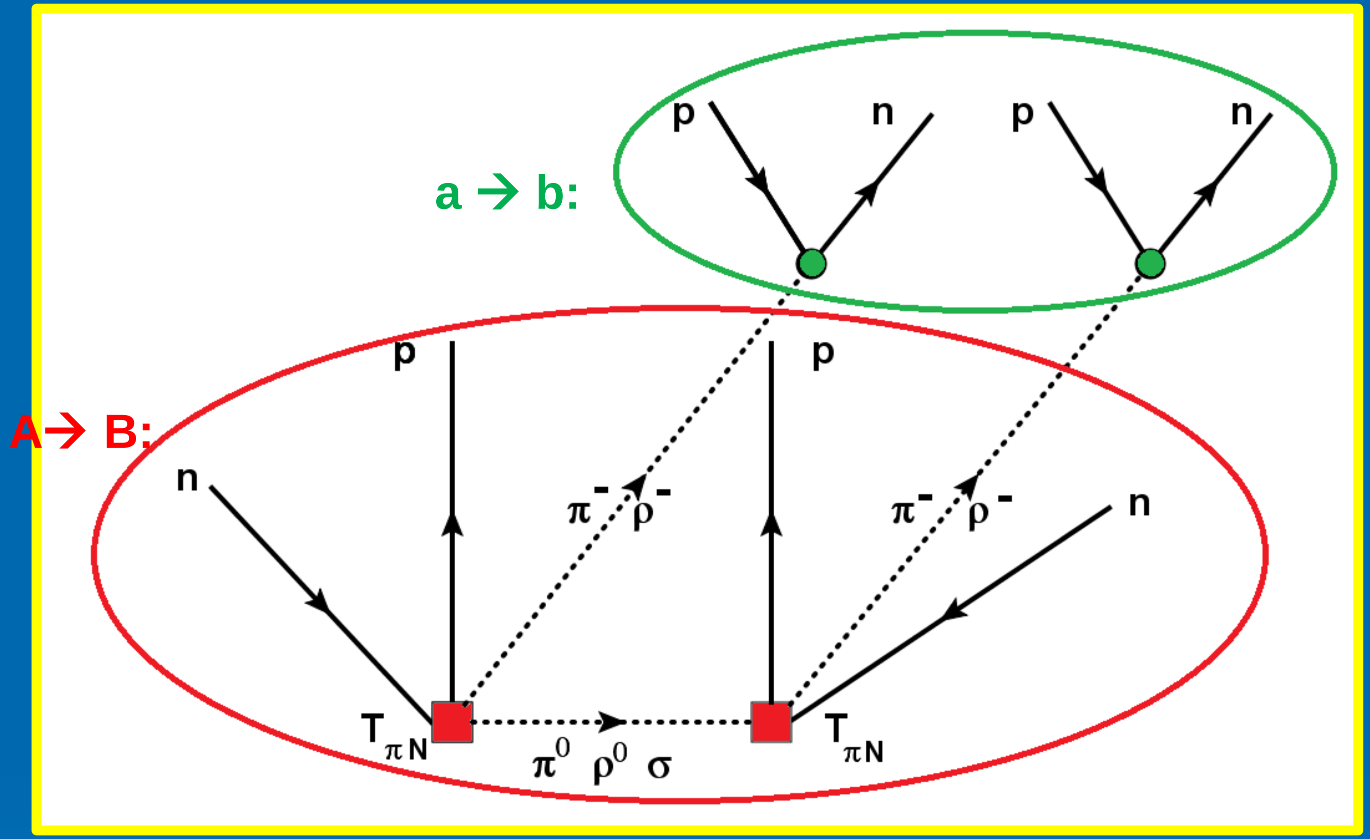
- simultaneous d→u Δq=+1 transitions by emission of a virtual weak gauge boson W⁻
- $W^- \rightarrow e^- + \bar{\nu}_e / \nu_e$: decay into electron and Majorana neutrino
- Correlation of the two events by exchange of the virtual $\nu_e \bar{\nu}_e$ pair
- Emission of two electrons ON their mass-shell: $p^2_e = m^2_e$
- Direct observation (in principle)

Hadronic Analogue

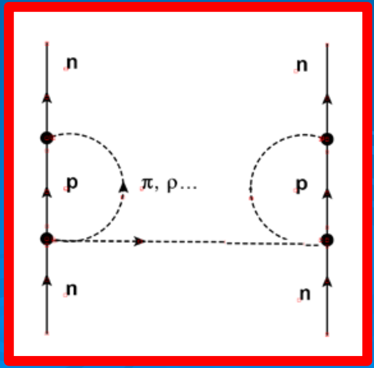


- simultaneous d→u Δq=+1 transitions by emission of a virtual $d\bar{u}$ vector pair ↔ ρ⁻ meson
- $\rho^- \rightarrow \pi^- + \pi^0$: decay into a pair of pions
- Heavy vector mesons ρ^{-*}
- Correlation of the two events by exchange of the virtual $q\bar{q}$ pair as contained in $\pi^0 \cong (d\bar{d}+u\bar{u})/\sqrt{2}$
- Emission of two π^- OFF their mass-shell: $p^2_\pi \neq m^2_\pi$
- No direct observation

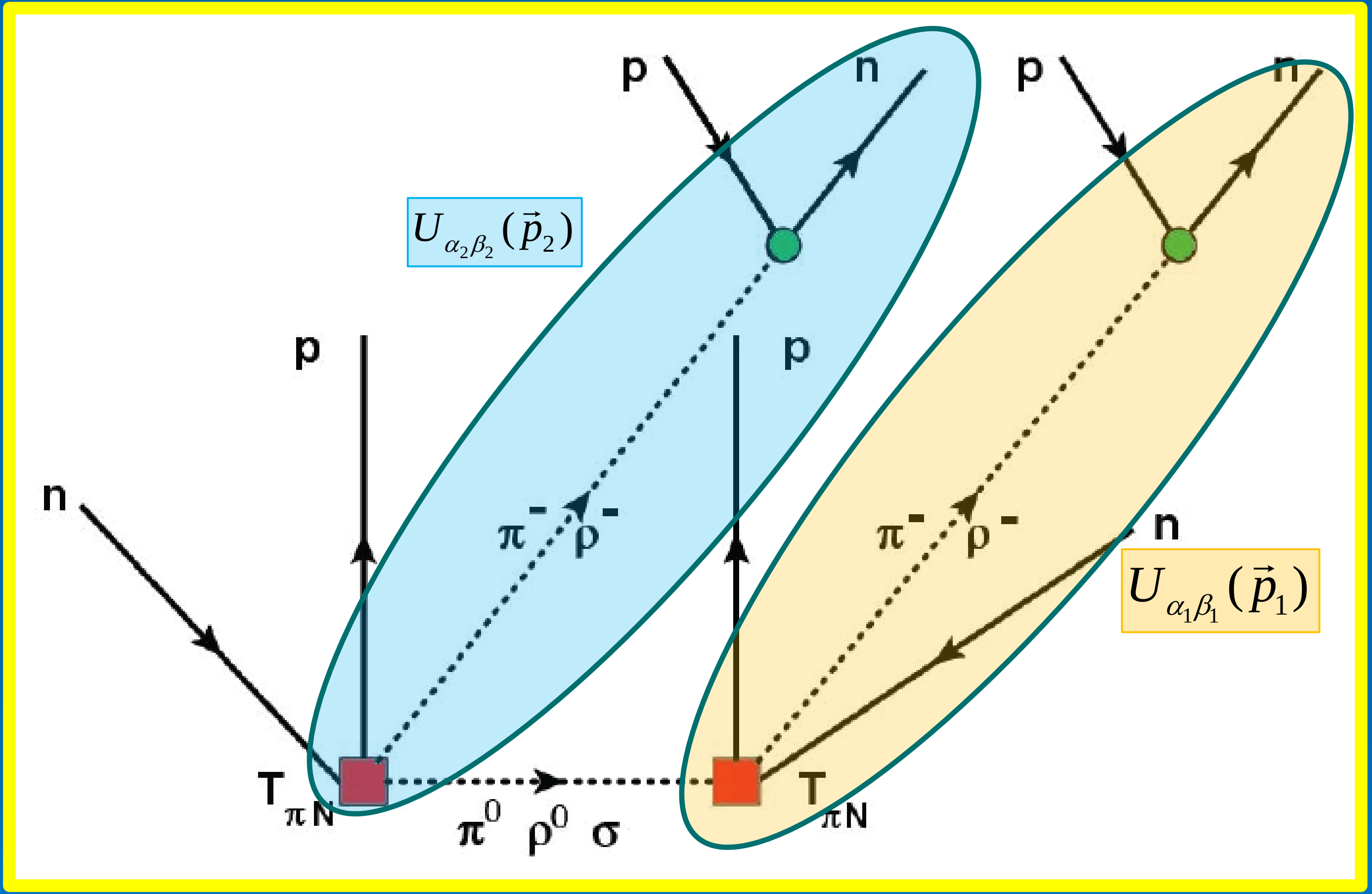
Majorana DCE Transitions by Meson Exchange



…two-nucleon DCE mechanism ↔ SRC:
~ 20% effect in nuclear ground states



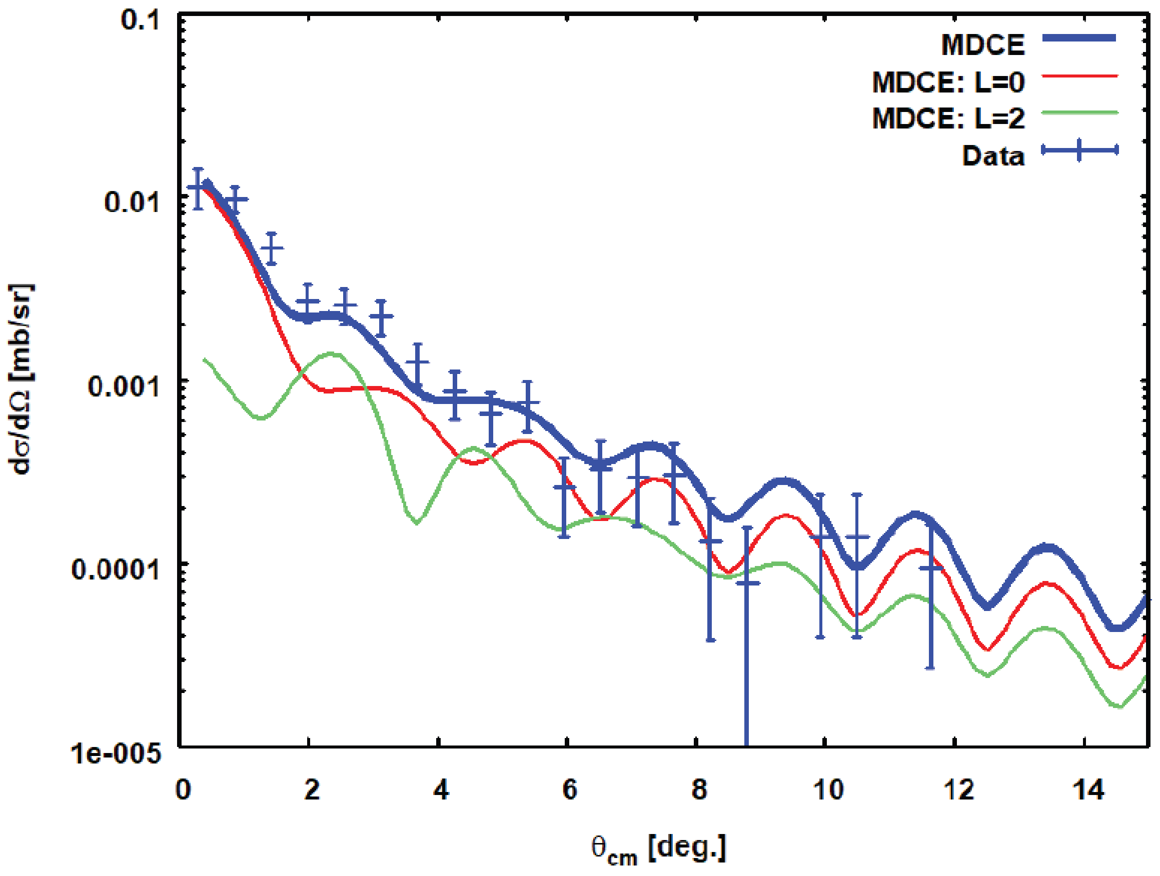
The Majorana DCE Transition Form factor



$$F_{\alpha\beta}(\vec{q}) \approx g_{\pi N}^2 \int d^3p_1 \int d^3p_2 U_{\alpha_2\beta_2}(\vec{p}_2) D_{\pi^0}(\vec{p}_1 - \vec{p}_2) U_{\alpha_1\beta_1}(\vec{p}_1) \delta(\vec{p}_1 + \vec{p}_2 - \vec{q}) + \ldots$$

MDCE Cross Section

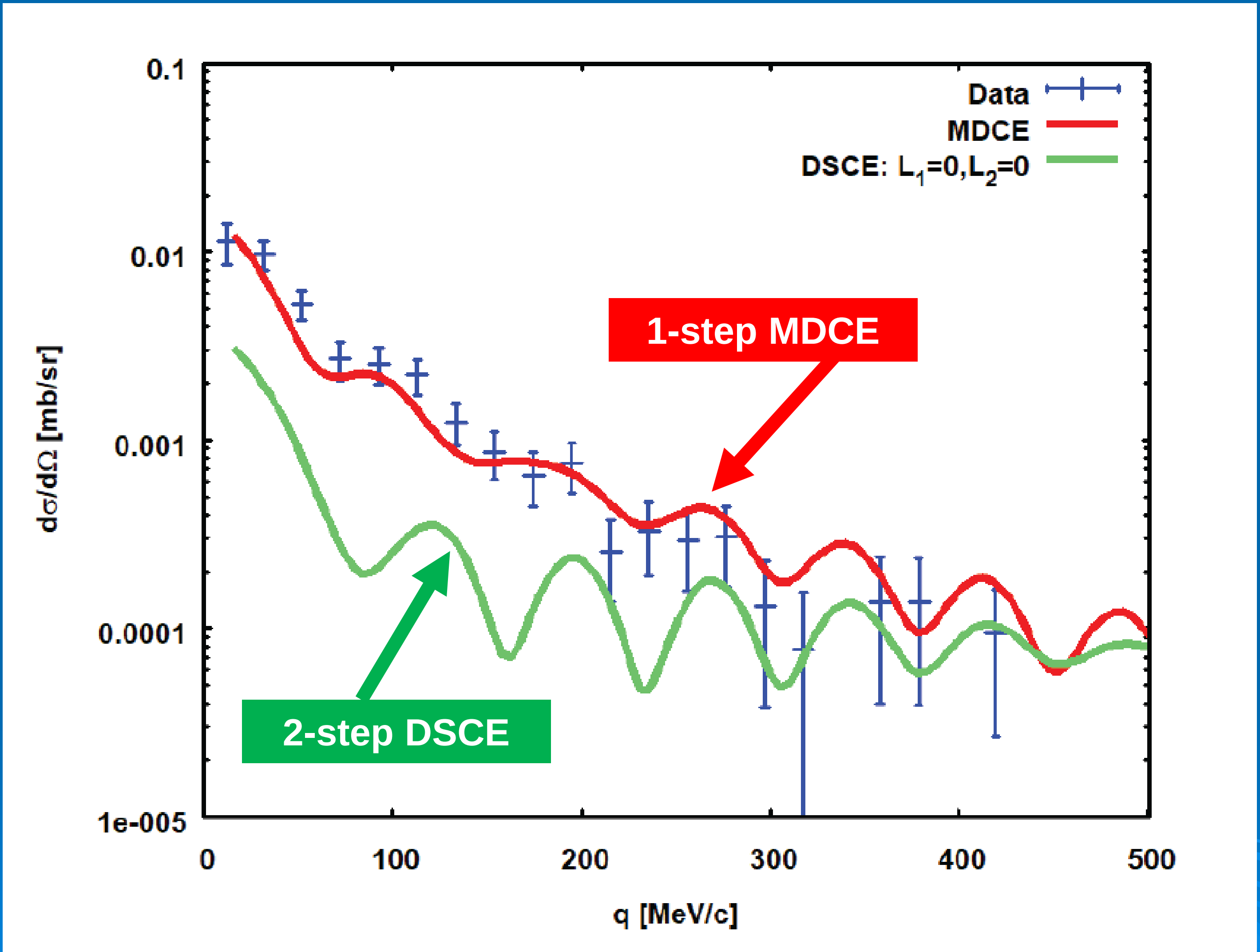
$^{18}O + ^{40}Ca \rightarrow ^{18}Ne + ^{40}Ar$



Data: F. Cappuzzello et al., EPJ A51 (2015)

MDCE and DSCE Cross Sections

$^{18}O + ^{40}Ca \rightarrow ^{18}Ne + ^{40}Ar$



Data: F. Cappuzzello et al., EPJ A51 (2015)

Summary and Outlook

- Theory of heavy ion SCE reactions: direct 1-step, transfer 2-step
- 2-step DSCE reaction mechanism ↔ $2\nu2\beta$ – decay analogue
- 1-step MDCE ↔ $0\nu2\beta$ – decay analogue
- Effective rank-2 IsoTensor ion-ion interaction
- Investigations of rare processes :
  - Probing nuclear 2-body CC currents and short range correlations
  - Probing $2\nu2\beta$ & $0\nu2\beta$-type NME in a hadronic analogue process
- Gateway to precision physics with heavy ions

…together with the NUMEN theory group J. Bellone, S. Burello, M. Colonna (Catania), J.-A. Lay (Sevilla), E. Santopinto (Genova) and the NUMEN@LNS collaboration

Backups

Effective **rank-2** Iso-Tensor Interaction $\sim [\tau_j \times \tau_k]_2$

DCE Reaction Amplitude

$$M_{\alpha\beta} \sim \langle \chi_\beta^{(-)\dagger}, bB | V^{(MDCE)} + V^{(DSCE)} | aA, \chi_\alpha^{(+)} \rangle = M_{\alpha\beta}^{(MDCE)} + M_{\alpha\beta}^{(DSCE)}$$

DSCE Interaction

$$V^{(DSCE)}(\mathbf{13}, \mathbf{24}) \sim \sum_{cC} T_{NN}(\mathbf{3}, \mathbf{4}) \mathcal{G}_{cC}(\mathbf{2} - \mathbf{4}, \mathbf{1} - \mathbf{3}) T_{NN}(\mathbf{2}, \mathbf{1})$$

MDCE Interaction

$$V^{(MDCE)}(\mathbf{13}, \mathbf{24}) \sim T_{\pi^- p, \pi^0 n}(\mathbf{1}, \mathbf{3}) D_{\pi^0}(\mathbf{1} - \mathbf{2}) T_{\pi^0 n, \pi^- p}(\mathbf{2}, \mathbf{4})$$

→ Rank-2 iso-tensor interactions with spin operators of rank S=0,1,2

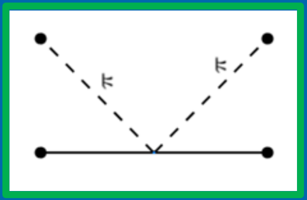
The $0\nu2\beta$ $0^+ \rightarrow 0^+$ Nuclear Matrix Element

$$M^{0\nu} = \frac{4\pi R}{g_A^2(0)} \sum_{L} \int d^3x_1 \int d^3x_2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q}\cdot(\mathbf{x}_1-\mathbf{x}_2)}}{q(q+E_d)} \langle 0_F^+ | \mathcal{J}_{L,\mu}^{\dagger}(\mathbf{x}_1) \mathcal{J}_L^{\mu\dagger}(\mathbf{x}_2) | 0_I^+ \rangle$$

Nuclear charge-changing (CC) Currents $\mathcal{J}_L$:

- **Vector**
- **Pseudo-Vector**
- **Axial-Vector**
- **Magnetic-Tensor**

Work in Progress:
EFT Approach to MDCE in Time-Ordered Perturbation Theory



Axial Vector

$$\mathcal{L}_{AV} = -\frac{g_A}{2f_\pi}\,\bar{\Psi}\,\gamma^\mu\,\gamma_5\,\boldsymbol{\tau}\,\Psi\cdot\partial_\mu\boldsymbol{\pi}\,.$$

covariant

→

$$\widehat{\mathcal{L}}_{AV} = -\frac{g_A}{2f_\pi}\,\bar{N}\,\boldsymbol{\tau}\cdot(\vec{\sigma}\cdot\vec{\nabla})\boldsymbol{\pi}\,N$$

Non-relativistic

Weinberg-Tomozawa

$$\mathcal{L}_{WT} = -\frac{1}{4f_\pi^2}\,\bar{\Psi}\,\gamma^\mu\,\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\,\Psi$$

covariant

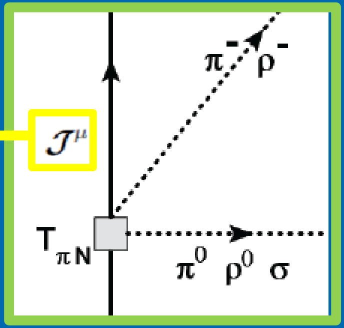
→

$$\widehat{\mathcal{L}}_{WT} = -\frac{1}{4f_\pi^2}\,\bar{N}\,\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_0\boldsymbol{\pi})\,N$$

Non-relativistic

Hadronic CC Currents and Transition Amplitude

$$\mathcal{J}_V^\mu = \bar{\Psi}_N \gamma^\mu \boldsymbol{\tau} \Psi_N$$
$$\mathcal{J}_A^\mu = \bar{\Psi}_N \gamma^\mu \gamma_5 \boldsymbol{\tau} \Psi_N$$
$$\mathcal{J}_S = \bar{\Psi}_N \gamma_5 \boldsymbol{\tau} \Psi_N .$$



…requires πN T-matrix!

$$m_\pi \mathcal{T}_{\pi N}^{(CC)} = T_V(s,t)\,\mathcal{J}_V^\mu \cdot \partial_\mu \left(\phi_\pi \times \phi_\pi\right) + T_A(s,t)\,\mathcal{J}_A^\mu \cdot \left(\phi_{\mu,\rho} \times \phi_\pi\right) + T_P(s,t)\,\mathcal{J}_A^\mu \cdot \partial_\mu \left(\phi_\sigma \phi_\pi\right) + T_S(s,t)\,\mathcal{J}_S \cdot \left(\phi_\sigma \phi_\pi\right) .$$

Nucleon Iso-spinor Fields: $\Psi_N \equiv (\psi_p, \psi_n)^T$

Meson Iso-vector Fields: $\phi_\pi = (\phi_{\pi^-}, \phi_{\pi^0}, \phi_{\pi^+})^T,\ \phi_\rho^\mu = \left(\phi_{\rho^-}^\mu, \phi_{\rho^0}^\mu, \phi_{\rho^+}^\mu\right)^T$

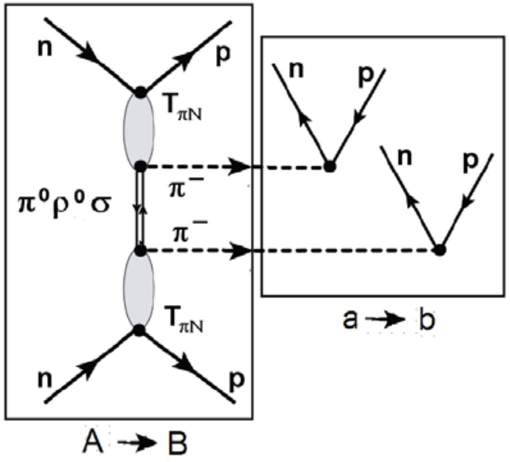
Meson Iso-scalar Field: ϕ_σ

The MDCE Reaction Amplitude

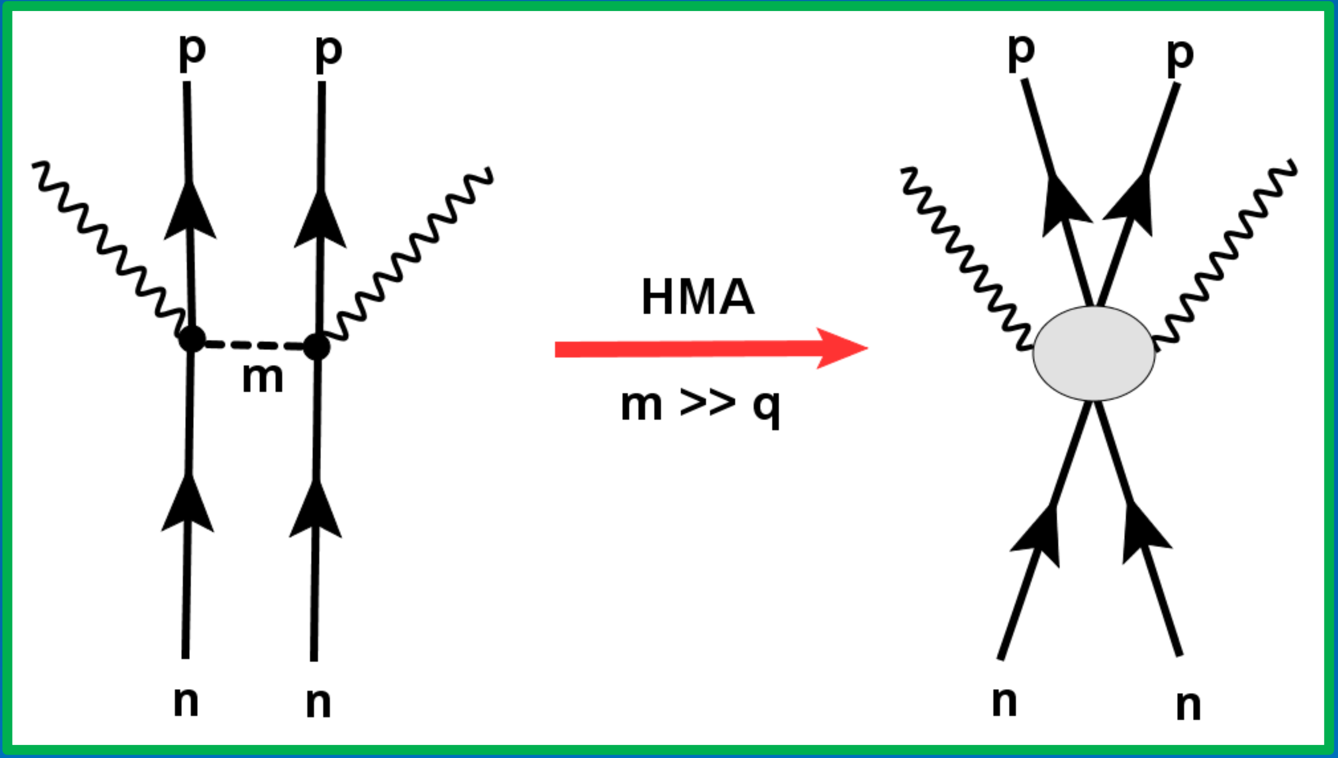
$$M_{\alpha\beta}^{(MDCE)} = \left\langle \chi_\beta^{(-)} \middle| U_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_\beta) \middle| \chi_\alpha^{(+)} \right\rangle$$

$$U_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_\beta) = \int \frac{d^3 p_\beta}{(2\pi)^3} \int \frac{d^3 p_\alpha}{(2\pi)^3} e^{ip_\beta \bullet r_\beta} e^{ip_\alpha \bullet r_\alpha} K_{\alpha\beta}(\boldsymbol{p}_\alpha, \boldsymbol{p}_\beta)$$

$$\mathcal{K}_{\alpha\beta}(\mathbf{p}_\alpha, \mathbf{p}_\beta) = \int \frac{d^3k}{(2\pi)^3} D_{\pi^0}(k) \int \frac{d^3k_1}{(2\pi)^3} D_{\pi^-}(k_1) \int \frac{d^3k_2}{(2\pi)^3} D_{\pi^-}(k_2)$$
$$\delta(\mathbf{k}_1 + \mathbf{k}_2 - (\mathbf{p}_\alpha - \mathbf{p}_\beta))$$
$$T_{\pi N}^{(1)}(\mathbf{k} - \mathbf{k}_1) T_{\pi N}^{(2)}(\mathbf{k}_2 - \mathbf{k})$$
$$\langle B | \mathcal{J}_+^{(1)}(\mathbf{k} - \mathbf{k}_1) \mathcal{J}_+^{(2)}(\mathbf{k} + \mathbf{k}_2 | A \rangle \langle b | \mathcal{S}_{--}(\mathbf{k}_1), \mathbf{k}_2) | a \rangle$$



The MDCE Vertex
Heavy Meson Approximation



…being measured on-shell at HADES@GSI via $pp \rightarrow pp\pi^+\pi^-$