Studies of ISR process $e^+e^- \to \pi^+\pi^-\pi^0\gamma$ at $\sqrt{s} \approx m_\phi$

Status report

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Overview

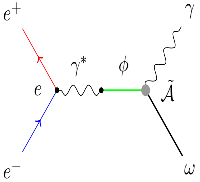
Physics

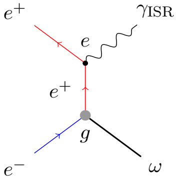
Rare decays

meson (J^{PC})	quark content	mass MeV/c^2	decay modes (>5%)
ϕ (1^{--})	$s\bar{s}$	1019.461±0.019	K^+K^- ($48.9 \pm 0.5\%$) $K^0_L K^0_S$ ($34.2 \pm 0.4\%$) $\rho\pi + \pi^+\pi^-\pi^0$ ($15.32 \pm 0.32\%$)
ω (1^{--})	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	782.65±0.12	$\pi^+\pi^-\pi^0$ ($89.2 \pm 0.7\%$) $\pi^0\gamma$ ($8.28 \pm 0.28\%$)

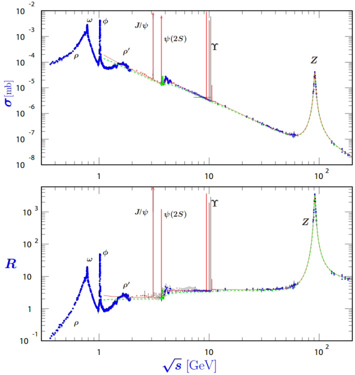
BR($\phi \to \omega\gamma$)<5%, CL=84% J.S. Lindsey et al, PR 147 (1966) 913 (bubble chamber).

Hadron production





Hadron cross section



Total hadronic cross section (top), hadronic ratio (bottom).

$$R_{hadron} = \frac{\sigma\left(e^+e^- \to \gamma^* \to hadrons\right)}{\sigma_0\left(e^+e^- \to \mu^+\mu^-\right)} \tag{1}$$

Data & MC status

$\pi^+\pi^-\pi^0\gamma$ MC sample from $\phi \to \rho\pi$ physics, MC sample "all_physics" is dedicated for $K_S K_L$ analysis

Year	First run	Last run	$\sqrt{s}$	$\int \mathcal{L}$ [pb^{-1}]
2004	30300	34169	M_ϕ	535.42
2005	34280	41902	M_ϕ	1201.68

Streams Code	MC_CARD_ID	MC_Code	Stream_ID	Stream_Code	Version	LSF
Bhabha Monte Carlo DST	415	"eeg100"	67	"mba"	26	0.5
ksl Monte Carlo DST	2	"all_phys"	62	"mk0"	26	1
rpi Monte Carlo DST	2	"all_phys"	63	"m3p"	26	1
ksl DST	-	-	42	"dk0"	26	-

Data sample from 2004/2005 with a total Int.Lumi$\sim$ 1724 pb^{-1}. Current analysis covers $\sim$ 80% of total Int.Lumi.

Physics	Run nr. range	Total runs	$\int \mathcal{L}$ [pb^{-1}]
$\phi \to K_L K_S$	30300 - 41902	8373	1724.47
$\phi \to \rho\pi$	30300 - 41902	8375	1724.88
Bhabha	30300 - 41902	8342	1752.54

Event Selection

Pre-selections

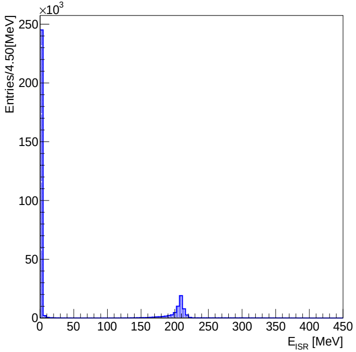
- At lest two tracks with opposite curvature.
- Neutral clusters with $|\cos\theta| < 0.92$, $\theta_{\min} = 23°$, number of in time clusters $n_{\text{clus}} \geq 2$ with $t_{\text{clu}} - \frac{R_{\text{clu}}}{c} < \min(2, 5\sigma_t)$ ns. $\sigma_t = \frac{0.057}{\sqrt{E(\text{GeV})}} \oplus 0.14$ ns.

Additional selections

- $K_L K_S$ stream cuts, FILFO
- Exact two prompt neutral cluster (pnc) with $E_{\text{pnc}} \geq 15$ MeV.
- Exact two tracks with opposite curvature that are extrapolated inside a cylinder with $\rho = \sqrt{x^2 + y^2} < 5$ mm and $|z| < 5$ mm.

Selection of $e^+e^- \to \pi^+\pi^-\pi^0\gamma$

- π^+, π^- and π^0 in final states.
- Select ISR photon candidates which are not associated with primary vertex.



Simulated ISR photon energy distribution. Total number of generated events $N_{\text{gen}} = 310559$ generated events.

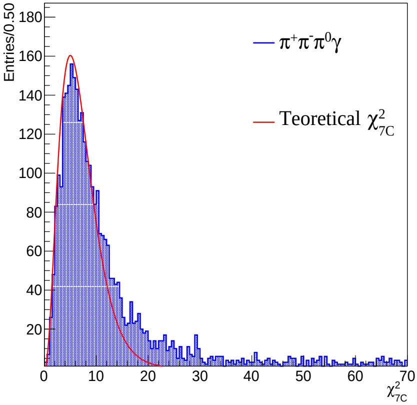
7C Kinematic fit

7C Kinematic fit

- Energy and momentum conservation with additional Time-of-Fight (ToF) resulting 7 constraints on final state particles.
- χ^2_{7C} distribution with ndf $= 7$, fitted observables are cluster energy E_{clu}, position x, y, z and ToF t_{clu}. Total 15 observables.

Error parametrization

- If $\sqrt{x^2+y^2} > 200$ cm (barrel), $\sigma_{xy} = 1.2$ cm & $\sigma_z = \frac{1.4}{\sqrt{E(\mathrm{GeV})}}$ cm,
- If $\sqrt{x^2+y^2} < 200$ cm (end-cap), $\sigma_{xz} = 1.2$ cm & $\sigma_y = \frac{1.4}{\sqrt{E(\mathrm{GeV})}}$ cm.
- $\frac{\sigma_{E_\gamma}}{E_\gamma} = \frac{5.7\%}{\sqrt{E(\mathrm{GeV})}}$.
- $\sigma_t = \frac{57 \text{ ps}}{\sqrt{E(\mathrm{GeV})}} \oplus 147$ ps.



χ^2_{7C} distribution, all selection cuts are applied with χ^2_{7C} less than 500.

π^0 photon selection

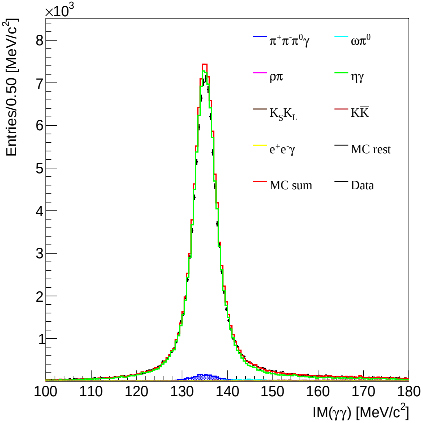
The $\chi^2_{\gamma\gamma}$ is calculated from objective function

$$\chi^2_{\gamma\gamma} = \frac{(m_{\gamma\gamma} - m_0)^2}{\sigma^2_{\gamma\gamma}}, \quad (2)$$

where m_0 is the true value of π^0 mass. Energy dependent relative error $\frac{\sigma_{\gamma\gamma}}{m_{\gamma\gamma}}$ is calculated using unconstrained energies $E_{1,2}$ and corresponding uncertainties $\sigma_{1,2}$

$$\frac{\sigma_{\gamma\gamma}}{m_{\gamma\gamma}} = \frac{1}{2}\sqrt{\left(\frac{\sigma_1}{E_1}\right)^2 + \left(\frac{\sigma_2}{E_2}\right)^2}. \quad (3)$$

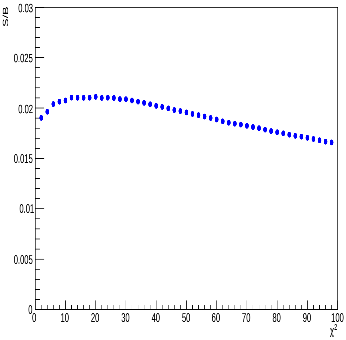
The test is performed event wise, all three combinations are tested and photon pair with the lowest $\chi^2_{\gamma\gamma}$ is chosen to be the best π^0 photon pair candidates. A study of $\phi \to \eta\gamma$ MC sample with high statistics shows shift on reconstructed 3π and paired photon 2γ invariant mass spectrum due to miss matched π^0 photon pairs.

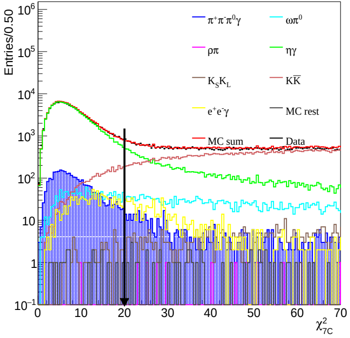


Invariant mass of paired photons $M_{\gamma\gamma}$. $\sigma_{\gamma\gamma} = 2.75$ MeV/c^2 is determined from $\pi^+\pi^-\pi^0\gamma$ sample. App.1.

Background rejection

$\chi^2 < 20$

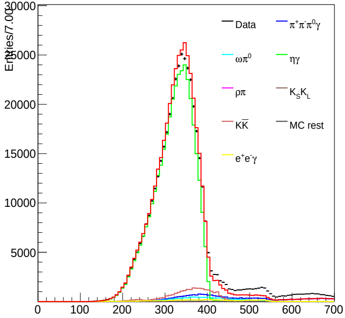


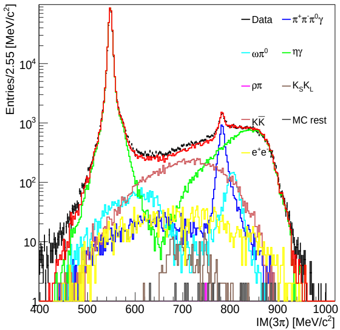


Signal background ratio S/B vs χ^2_{7C} cuts (left), χ^2_{7C} distribution (right).

Background rejection

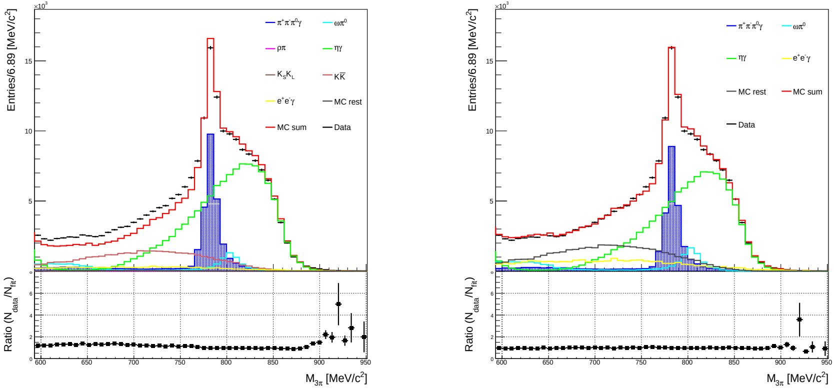
$$|p_+| + |p_-| < 390 \text{ MeV/c}$$





Momenta sum of charge tracks $|p_+| + |p_-|$ after χ^2_{7C} cut (left). Invariant mass of $\pi^+\pi^-\pi^0$ distribution after χ^2_{7C} and track momentum cuts (right).

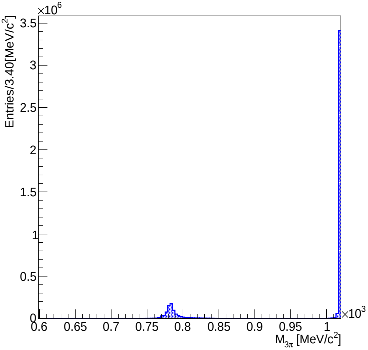
MC-Data comparison



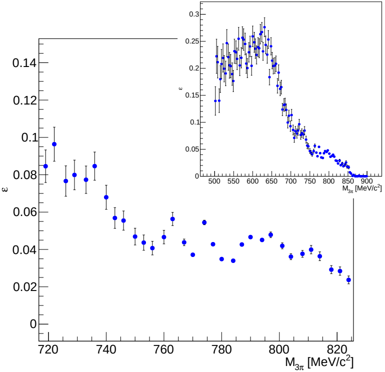
Invariant mass of $\pi^+\pi^-\pi^0$ distribution $M_{3\pi}$ in ω mass region before scaling (left), after scaling (right), residual MC background includes $K_S K_L$, $K\bar{K}$ and $\rho\pi$. Fitting range $M_{3\pi}$ $\in$[400 1020] MeV/c^2. Maximum likelihood fitting is applied to obtain scaling factor $\hat{w}$.

Efficiencies

Selection efficiency at i^{th} mass bin $\Delta M_{3\pi,i}$ is calculated using $\varepsilon_i = \frac{N_{\text{rec},i}}{N_{\text{gen,i}}}$ where $N_{\text{rec},i}$ and $N_{\text{gen},i}$ are number of reconstructed and generated $\pi^+\pi^-\pi^0\gamma$ events, the error is calculated using $\sigma_\varepsilon = \sqrt{\varepsilon\,(1-\varepsilon)\,/N_{\text{gen}}}$. The global efficiency ε ~0.0090±0.0002 due to rejection of small ISR energy photons associated to $e^+e^- \to \phi\gamma \to \pi^+\pi^-\pi^0\gamma$ events Fig.8.



Simulated $\pi^+\pi^-\pi^0$ invariant mass $\text{M}_{3\pi}$.



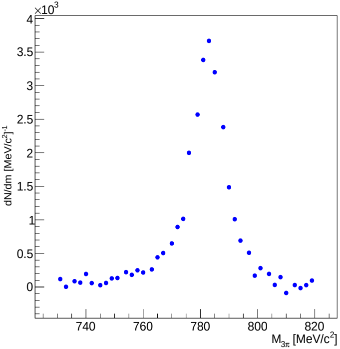
Efficiency ε as a function of $\pi^+\pi^-\pi^0$ invariant mass $\text{M}_{3\pi}$.

Cross Section

Differential cross section is measured according to

$$\frac{\mathrm{d}\sigma\left(e^+e^- \to \pi^+\pi^-\pi^0\gamma\right)}{\mathrm{d}m} = \frac{\mathrm{d}N/\mathrm{d}m}{\varepsilon L}. \quad (4)$$

$N_{\mathrm{obs}}/\Delta M_{3\pi}$ is measured $dN/\mathrm{d}m$ corrected by efficiency ε where $N_{\mathrm{obs}} = N_{\mathrm{sel}} - N_{\mathrm{bkg}}$ is observed number of $\pi^+\pi^-\pi^0\gamma$ in mass interval $\Delta M_{3\pi}$ with a choice $1.4\sigma_{3\pi} \sim 3.4$ MeV/c^2. N_{sel} and N_{bkg} are number of selected data and MC back ground events survived after all selection cuts. L is integrated total luminosity.



Observed number of $\sigma\left(e^+e^- \to \pi^+\pi^-\pi^0\right)$ events N_{obs} as function of invariant mass of $\pi^+\pi^-\pi^0$ system.

Cross Section

The visible cross section

$$\sigma_{\text{vis}} = \int_0^{x_{\max}} \epsilon(s,x) W(s,x) \sigma_0 \left(s\left(1-x\right)\right) dx \tag{5}$$

$\sigma_{\text{vis}} \equiv \sigma\left(e^+e^- \to \pi^+\pi^-\pi^0\right)$ measurement of Born cross section σ_0 associates to differential cross section in Eq.4 through

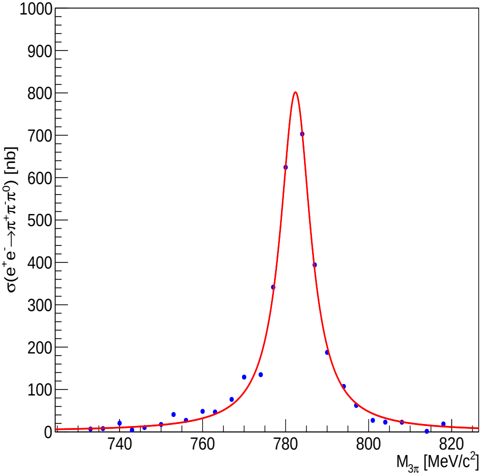
$$\sigma_{\text{vis}} = \frac{\mathrm{d}N/\mathrm{d}m}{\varepsilon \mathrm{d}L/\mathrm{d}m} \tag{6}$$

where $\mathrm{d}L/\mathrm{d}m$ is measured using $W\left(s, M_{3\pi}\right) \frac{2M_{3\pi}}{s} L$, $x \equiv 2E_\gamma/\sqrt{s}$ is ISR emission fraction

$$E_\gamma = \frac{\sqrt{s}}{2}\left(1 - \frac{M_{3\pi}^2}{s}\right) \tag{7}$$

with $\sqrt{s} \approx 1.02$ GeV, $E_\gamma \approx 209$ MeV when ω is produced. $M_{3\pi}$ is known as effective c.m. energy $\sqrt{s'}$. $W\left(s,x\right)$ is the radiator function.

Cross Section



Observed total cross section σ for $e^+e^- \to \pi^+\pi^-\pi^0$ (dot), Breit–Wigner Parametrization (red solid).

Total cross section

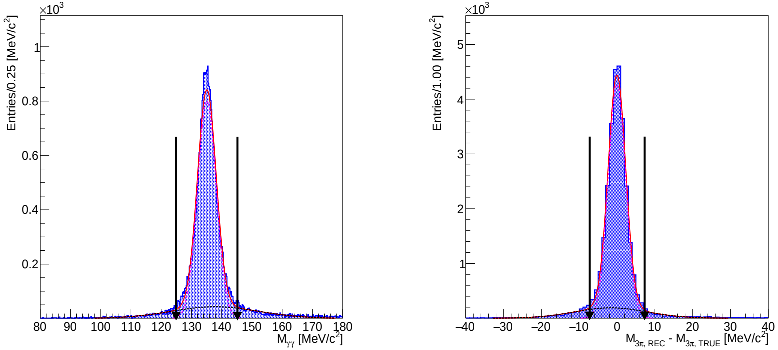
TABLE III. e^+e^- effective c.m. energy $\sqrt{s'}$, number of observed events $N_{\rm obs}$, detection efficiency ε, differential ISR luminosity L and measured total cross section σ for $e^+e^- \to \pi^+\pi^-\pi^0$. All values are calculated in the mass range $M_{3\pi} \in [730\ 820]$ MeV/c^2 with a bin width 3.4 MeV/c^2. No statistical or systematic errors are quoted.

$M_{3\pi}$ [MeV]	$N_{\rm obs}$ -	ε [%]	L [nb^{-1}]	σ nb	$M_{3\pi}$ [MeV]	$N_{\rm obs}$ -	ε [%]	L [nb^{-1}]	σ nb
733	88	7.73	169.65	6.71	777	3167	4.27	216.71	341.48
736	113	8.46	172.35	7.75	780	4801	3.4	220.60	624.46
740	245	6.79	176.01	20.48	784	5395	3.40	225.99	703.04
743	46	5.68	178.91	4.52	787	3869	4.27	230.12	394.18
746	102	5.54	181.83	10.11	790	2050	4.66	234.39	187.574
750	152	4.69	185.84	17.44	794	1162	4.51	240.28	107.245
753	338	4.37	188.93	40.97	797	731	4.79	244.84	62.39
756	213	4.07	192.10	27.23	801	286	4.19	251.13	27.16
760	443	4.66	196.45	48.37	804	212	3.61	256.01	22.91
763	528	5.63	199.82	46.93	808	225	3.77	262.74	22.74
767	686	4.38	204.43	76.52	811	-60	3.99	267.98	-5.61
770	999	3.72	208.00	129.25	814	13	3.64	273.27	1.31
774	1565	5.44	212.91	135.01	818	153	2.92	280.83	18.66

Summary

- More statistics ✓, full statistics ✗
- MC generation of $\phi \to \omega\gamma$, $\pi^+\pi^-\gamma$ and $\mu^+\mu^-\gamma$. ✗
- Calculate differential and total cross section ✓, correct physics ✗.
- Efficiency. ✗, errors ✗.
- Systematics. ✗

Resolutions



$M_{\gamma\gamma}$ plot from $\pi^+\pi^-\pi^0\gamma$ (left), a double guassian fit in mass range [80, 180] MeV/c^2. $\sigma_{\gamma\gamma} \sim 2.75$ MeV/c^2. $\pi^+\pi^-\pi^0$ invariant mass difference $M_{3\pi,\mathrm{rec}} - M_{3\pi,\mathrm{true}}$ from $\pi^+\pi^-\pi^0\gamma$ (right), a double guassian fit in range [-100, 100] MeV/c^2, $\sigma_{3\pi} \sim 2.42$ MeV/c^2. Arrows indicate 3-σ region.

Chi-square Fitting

At the i^{th} bin, given selected data events N_{d_i}, MC channels of "signal" N_{s_i} and set of background $N_{b_{j,i}}$ with indices s_i and $b_{j,i}$ respectively, $j = 1, 2, 3, 4$ gives number of MC channels. The prediction of number of data N_{p_i} is given by a superposition of MC events

$$N_{p_i} = N_D \left(f_s \frac{N_{s_i}}{N_S} + \sum_{j=1}^{4} f_{b_j} \frac{N_{b_{j,i}}}{N_{B_j}} \right) \equiv \omega_s N_{s_i} + \sum_{j=1}^{4} \omega_{b_j} N_{b_{j,i}} \tag{8}$$

where $N_D = \sum_{i=1} N_{d_i}$ is total number of selected data events N_S and N_{B_j} are total number of MC signal and MC background, w_s and w_{b_j} are the normalized weights for signal and background known as Scaling Factor (SF). f_s and f_{b_j} are fraction of MC signal and MC background events satisfy constraint

$$f_s + \sum_{j=1}^{4} f_{b_j} = 1 \tag{9}$$

Fit N_{p_i} to the observation N_{d_i} with error $\sigma_i = \sqrt{N_{d_i}}$ by minimizing the chi-square

$$\chi^2 = \sum_{i=1} \left(\frac{N_i - N_{p_i}}{\sigma_i} \right)^2 \tag{10}$$

to obtain fitted parameters $\hat{f}_s$ and $\hat{f}_{b_j}$ and calculate corresponding scaling factor $\hat{w}_s$ and $\hat{w}_{b_j}$ according Eq.8.

Error Propagation

Recall predicted number of events at i^{th} bin N_{p_i} from Eq.8 $N_{p_i} = \omega_s N_{s_i} + \sum_{j=1}^{4} \omega_{b_j} N_{b_{j,i}}$. It is straight forward to write down error propagation formula

$$\sigma_{N_{p_i}} = \sqrt{N_{s_i}^2 \sigma_{w_s}^2 + w_s^2 N_{s_i} + \sum_{j=1}^{4} N_{b_i}^2 \sigma_{w_{b_{j,i}}}^2 + w_{b_j}^2 N_{b_{j,i}}} \tag{11}$$

Here we have used possion distribution assumption that $\sigma_{s_i}^2$ or $\sigma_{b_{j,1}}^2$ equal the event number N_{s_i} or $N_{b_{j,i}}$. It is sufficient to derive uncertainty formula for "signal" channel σ_{w_s}. From definition $w_s = \frac{N_D f_s}{N_S}$, one could write down

$$\begin{aligned} \sigma_{w_s} &= \sqrt{\left|\frac{\partial w_s}{\partial N_D}\right|^2 \sigma_{N_D}^2 + \left|\frac{\partial w_s}{\partial N_S}\right|^2 \sigma_{N_S}^2 + \left|\frac{\partial w_s}{\partial f_s}\right|^2 \sigma_{f_s}^2} \\ &= w_s \sqrt{\frac{1}{N_D} + \frac{1}{N_S} + \frac{1}{f_s}} \end{aligned} \tag{12}$$

similarly, for background channels

$$\sigma_{b_j} = w_{b_j} \sqrt{\frac{1}{N_D} + \frac{1}{N_{B_j}} + \frac{1}{f_{b_j}}} \tag{13}$$