INDIANA UNIVERSITY
BLOOMINGTON

Jefferson Lab

Spectroscopy of light quarks

Emilie Passemar

Indiana University/Jefferson Laboratory

Photon 2019

LNF, Frascati, June 3rd, 2019

Outline

1. Introduction and Motivation
2. $\eta \to 3\pi$ and light quark masses
3. $\eta' \to \eta\pi\pi$ and chiral dynamics
4. Conclusion and Outlook

1. Introduction and Motivation

1.1 Light quark spectroscopy

- In the study of hadron spectroscopy, large amount of very precise data on meson physics have been and will be collected:

  ⇨ *KLOE & KLOEII, BES, A1, A2@MAMI, CLAS, GlueX, JEF, COMPASS, LHCb, PANDA,…*

  They are background for searches of new states

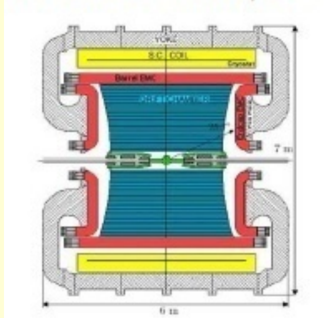
- Unique opportunity:
  - Test chiral dynamics at low energy
  - Extract fundamental parameters of the Standard Model: ex: light quark masses
  - Study of beyond Standard Model Physics

World competition in η decays

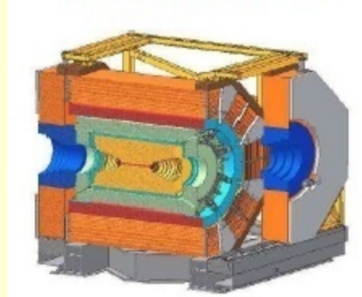
From L.Gan

e⁺e⁻ Collider

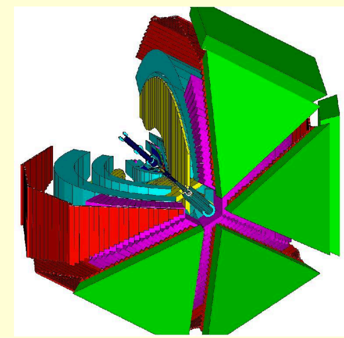
KLOE-2 at DAΦNE



BESIII at BEPCII

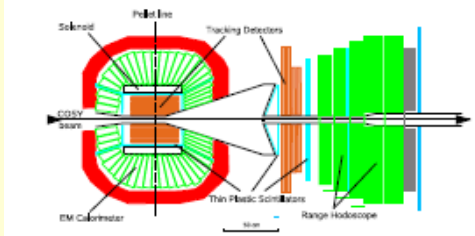


CLAS



Fixed-target

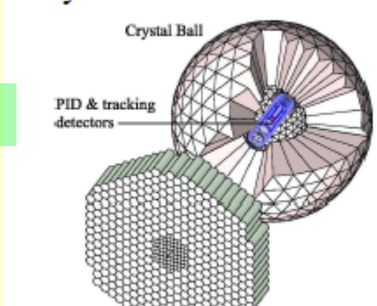
WASA at COSY



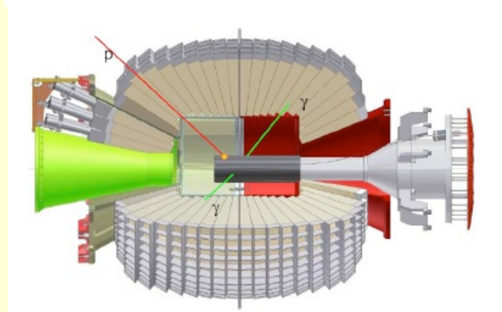
hadroproduction

photoproduction

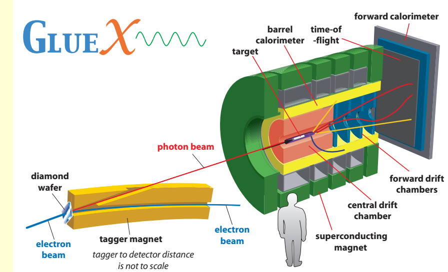
Crystall Ball at MAMI



CBELSA/TAPS at ELSA



JEF at Jlab



1.3 Study of η and η' physics

PDG'18
Gan, Kubis, E.P., Tulin
in progress

Channel	Expt. branching ratio	Discussion
$\eta \to 2\gamma$	$(39.3 \pm 0.2)\%$	anomaly, η–η' mixing
$\eta \to 3\gamma$	$< 1.6 \times 10^{-5}$	C violation
$\eta \to 4\gamma$	$< 2.8 \times 10^{-4}$	
$\eta \to e^+e^-\gamma$	$(6.9 \pm 0.4) \times 10^{-3}$	Theory input for $(g-2)_\mu$, dark photon (BSM)
$\eta \to 2\pi^0$	$< 3.5 \times 10^{-4}$	P, CP violation
$\eta \to \pi^+\pi^-$	$< 1.3 \times 10^{-5}$	P, CP violation
$\eta \to 3\pi^0$	$(32.6 \pm 0.2)\%$	$m_u - m_d$
$\eta \to \pi^+\pi^-\pi^0$	$(22.7 \pm 0.3)\%$	$m_u - m_d$, CP violation
$\eta \to \pi^0\gamma$	$< 9 \times 10^{-5}$	C violation, angular momentum nonconservation
$\eta \to \pi^0 e^+e^-$	$< 7.5 \times 10^{-6}$	C violation
$\eta \to \pi^0\gamma\gamma$	$(2.7 \pm 0.5) \times 10^{-4}$	χPT at $\mathcal{O}(p^6)$, leptophobic B boson (BSM)
$\eta \to 2\pi^0\gamma$	$< 5 \times 10^{-4}$	C, CP violation
$\eta \to \pi^+\pi^-\gamma$	$(4.22 \pm 0.08)\%$	chiral anomaly, C violation
$\eta \to 2\pi^0 e^+e^-$		
$\eta \to \pi^+\pi^- e^+e^-$	$(2.68 \pm 0.11) \times 10^{-4}$	CP violation
$\eta \to 3\pi^0\gamma$	$< 6 \times 10^{-5}$	C, CP violation
$\eta \to \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	
$\eta \to 4\pi^0$	$< 6.9 \times 10^{-7}$	P, CP violation

$$M_\eta = 547.862(17)\ \text{MeV}$$

1.3 Study of η and η' physics

PDG'19
Gan, Kubis, E. P., Tulin
in progress

Channel	Expt. branching ratio	Discussion
$\eta \to 2\gamma$	$(39.3 \pm 0.2)\%$	anomaly, η–η' mixing
$\eta \to 3\gamma$	$< 1.6 \times 10^{-5}$	C violation
$\eta \to 4\gamma$	$< 2.8 \times 10^{-4}$	
$\eta \to e^+e^-\gamma$	$(6.9 \pm 0.4) \times 10^{-3}$	Theory input for $(g-2)_\mu$, dark photon (BSM)
$\eta \to 2\pi^0$	$< 3.5 \times 10^{-4}$	P, CP violation
$\eta \to \pi^+\pi^-$	$< 1.3 \times 10^{-5}$	P, CP violation
$\eta \to 3\pi^0$	$(32.6 \pm 0.2)\%$	$m_u - m_d$
$\eta \to \pi^+\pi^-\pi^0$	$(22.7 \pm 0.3)\%$	$m_u - m_d$, CP violation
$\eta \to \pi^0\gamma$	$< 9 \times 10^{-5}$	C violation, angular momentum nonconservation
$\eta \to \pi^0 e^+e^-$	$< 7.5 \times 10^{-6}$	C violation
$\eta \to \pi^0\gamma\gamma$	$(2.7 \pm 0.5) \times 10^{-4}$	χPT at $\mathcal{O}(p^6)$, leptophobic B boson (BSM)
$\eta \to 2\pi^0\gamma$	$< 5 \times 10^{-4}$	C, CP violation
$\eta \to \pi^+\pi^-\gamma$	$(4.22 \pm 0.08)\%$	chiral anomaly, C violation
$\eta \to 2\pi^0 e^+e^-$		
$\eta \to \pi^+\pi^- e^+e^-$	$(2.68 \pm 0.11) \times 10^{-4}$	CP violation
$\eta \to 3\pi^0\gamma$	$< 6 \times 10^{-5}$	C, CP violation
$\eta \to \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	
$\eta \to 4\pi^0$	$< 6.9 \times 10^{-7}$	P, CP violation

1.3 Study of η and η' physics

$$M_{\eta'} = 957.78(6)\ \text{MeV}$$

PDG'19
Gan, Kubis, E. P., Tulin
in progress

$\eta' \to 2\gamma$	$(2.20 \pm 0.08)\%$	chiral anomaly
$\eta' \to 3\gamma$	$< 1.0 \times 10^{-4}$	C, CP violation
$\eta' \to e^+e^-\gamma$	$< 9 \times 10^{-4}$	χPT, dark photon (BSM)
$\eta' \to 2\pi^0$	$< 4 \times 10^{-4}$	P, CP violation
$\eta' \to \pi^+\pi^-$	$< 1.8 \times 10^{-5}$	P, CP violation
$\eta' \to 3\pi^0$	$(2.14 \pm 0.20)\%$	$m_u - m_d$
$\eta' \to \pi^+\pi^-\pi^0$	$(3.8 \pm 0.4) \times 10^{-3}$	$m_u - m_d$, CP violation
$\eta' \to \eta\pi^+\pi^-$	$(42.6 \pm 0.7)\%$	RχPT, anomaly, $\eta - \eta'$ mixing
$\eta' \to \eta\pi^0\pi^0$	$(22.8 \pm 0.8)\%$	RχPT, anomaly, $\eta - \eta'$ mixing
$\eta' \to \pi^0 e^+e^-$	$< 1.4 \times 10^{-3}$	C violation
$\eta' \to \pi^+\pi^-e^+e^-$	$(2.4^{+1.3}_{-1.0}) \times 10^{-3}$	P, CP violation
$\eta' \to \pi^0\gamma\gamma$	$< 8 \times 10^{-4}$	χPT, leptophobic B boson (BSM)
$\eta' \to \eta e^+e^-$	$< 2.4 \times 10^{-3}$	C violation

1.3 Study of η and η' physics

PDG'19
Gan, Kubis, E.P., Tulin in progress

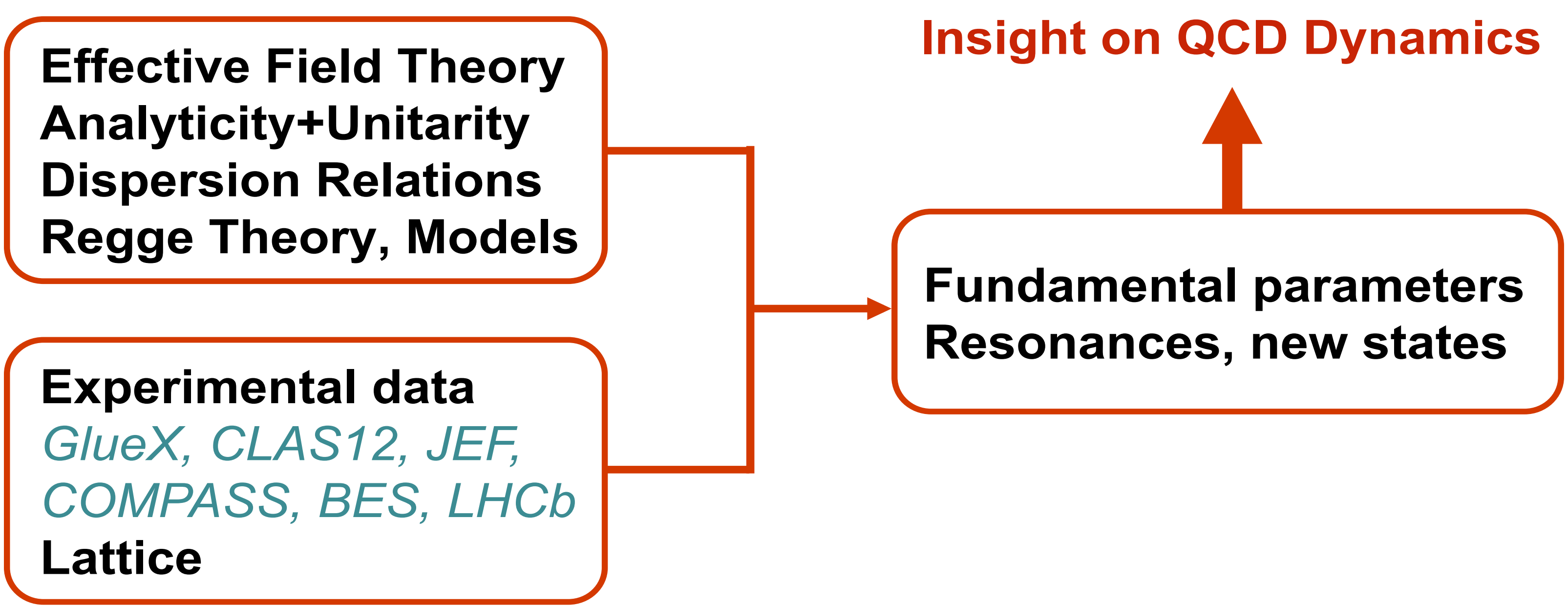
$\eta' \to 2\gamma$	$(2.20 \pm 0.08)\%$	chiral anomaly
$\eta' \to 3\gamma$	$< 1.0 \times 10^{-4}$	C, CP violation
$\eta' \to e^+e^-\gamma$	$< 9 \times 10^{-4}$	χPT, dark photon (BSM)
$\eta' \to 2\pi^0$	$< 4 \times 10^{-4}$	P, CP violation
$\eta' \to \pi^+\pi^-$	$< 1.8 \times 10^{-5}$	P, CP violation
$\eta' \to 3\pi^0$	$(2.14 \pm 0.20)\%$	$m_u - m_d$
$\eta' \to \pi^+\pi^-\pi^0$	$(3.8 \pm 0.4) \times 10^{-3}$	$m_u - m_d$, CP violation
$\eta' \to \eta\pi^+\pi^-$	$(42.6 \pm 0.7)\%$	RχPT, anomaly, $\eta - \eta'$ mixing
$\eta' \to \eta\pi^0\pi^0$	$(22.8 \pm 0.8)\%$	RχPT, anomaly, $\eta - \eta'$ mixing
$\eta' \to \pi^0 e^+e^-$	$< 1.4 \times 10^{-3}$	C violation
$\eta' \to \pi^+\pi^-e^+e^-$	$(2.4^{+1.3}_{-1.0}) \times 10^{-3}$	P, CP violation
$\eta' \to \pi^0\gamma\gamma$	$< 8 \times 10^{-4}$	χPT, leptophobic B boson (BSM)
$\eta' \to \eta e^+e^-$	$< 2.4 \times 10^{-3}$	C violation

1.4 Analytical methods for light quark spectroscopy

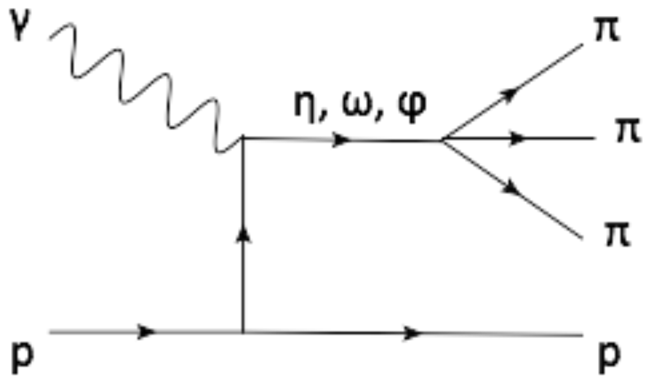
- In the study of hadron spectroscopy, large amount of very precise data on meson physics have been and will be collected:

  ⇨ *KLOE & KLOE-II, BES, A1, A2@MAMI, CLAS, GlueX, JEF, COMPASS, LHCb, PANDA,…*

  They are background for searches of new states



1.4 Light Meson Decays



- If E > 1 GeV: ChPT not valid anymore to describe dynamics of the processes

  ⇨ Resonances appear :

  For ππ: *I=1*: ρ(770), ρ(1450), ρ(1700), …,

  Especially true for ϕ (M_ϕ=1020 MeV)

- Use Isobar model to describe the data

  ⇨ Improve to include FSI

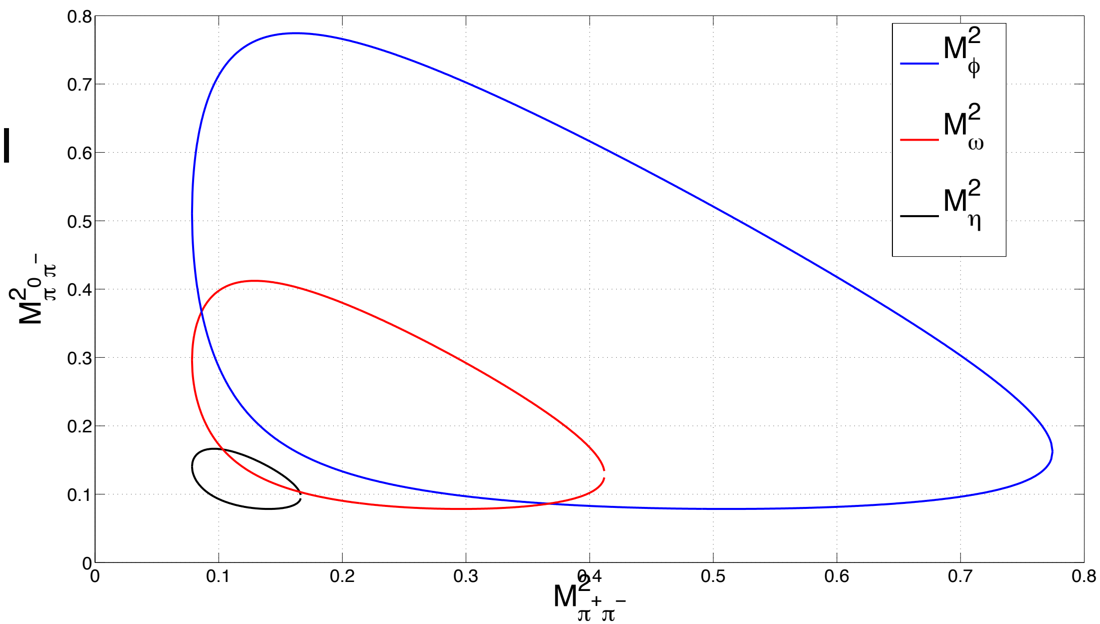
- Build an amplitude with physical properties:

  → Analyticity, Unitarity and Crossing Symmetry:

  ⇨ *Dispersion Relations*

  → Chiral constraints at LE

  → Regge behavior at HE

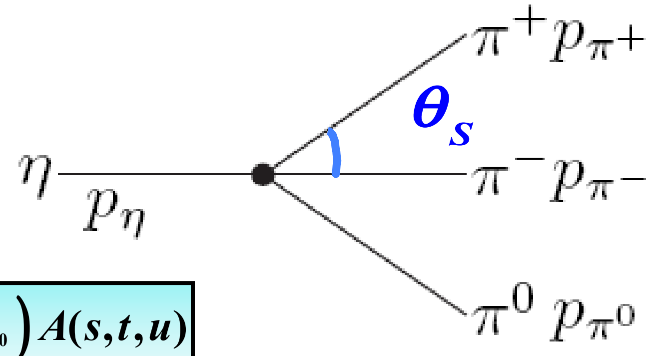


2. $\eta \to 3\pi$ and light quark mass extraction

In collaboration with G. Colangelo, S. Lanz and H. Leutwyler (ITP-Bern)

Phys. Rev. Lett. 118 (2017) no.2, 022001
Eur.Phys.J. C78 (2018) no.11, 947

2.1 Definitions



- η decay: $\eta \to \pi^+ \pi^- \pi^0$

$$\langle \pi^+\pi^-\pi^0{}_{out} | \eta \rangle = i(2\pi)^4 \delta^4 \left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right) A(s,t,u)$$

- Mandelstam variables $s = \left(p_{\pi^+} + p_{\pi^-}\right)^2,\ t = \left(p_{\pi^-} + p_{\pi^0}\right)^2,\ u = \left(p_{\pi^0} + p_{\pi^+}\right)^2$

  ➡ only two independent variables

$$s+t+u = M_\eta^2 + M_{\pi^0}^2 + 2M_{\pi^+}^2 \equiv 3s_0$$

- 3 body decay ➡ Dalitz plot

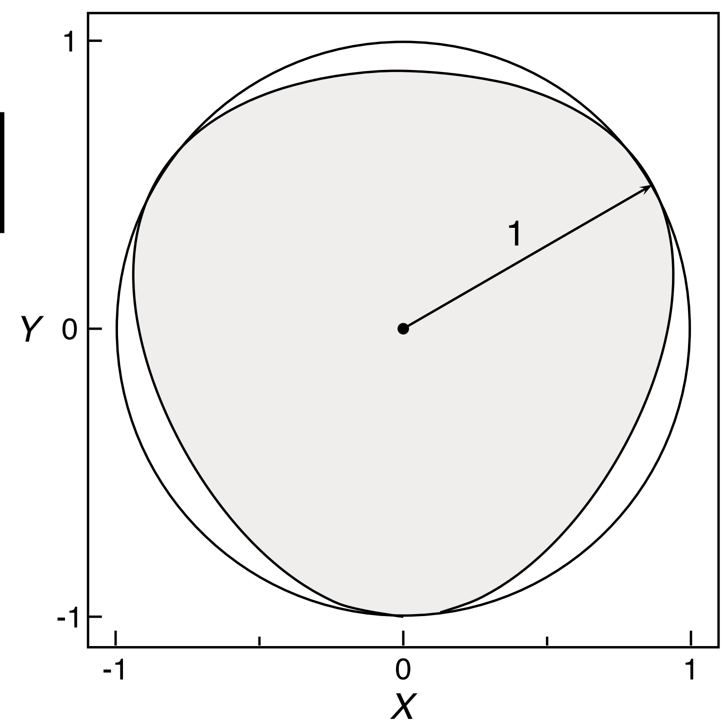
$$\left|A(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3 + \ldots\right)$$

Expansion around X=Y=0

$$X = \sqrt{3}\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$



2.1 Why is it interesting to study $\eta \to 3\pi$?

- Decay forbidden by isospin symmetry

$$A = (m_u - m_d) A_1 + \alpha_{em} A_2$$

- α_{em} effects are small

  Sutherland'66, Bell & Sutherland'68
  Baur, Kambor, Wyler'96, Ditsche, Kubis, Meissner'09

- Decay rate measures the size of isospin breaking ($m_u - m_d$) in the SM:

$$L_{QCD} \to L_{IB} = -\frac{m_u - m_d}{2}\left(\bar{u}u - \bar{d}d\right)$$

Unique access to ($m_u - m_d$)

2.2 Quark mass ratio

- In the following, extraction of *Q* from $\eta \to \pi^+ \pi^- \pi^0$

$$\Gamma_{\eta\to\pi^+\pi^-\pi^0} = \frac{1}{Q^4}\frac{M_K^4}{M_\pi^4}\frac{\left(M_K^2 - M_\pi^2\right)^2}{6912\pi^3 F_\pi^4 M_\eta^3}\int_{s_{\min}}^{s_{\max}} ds \int_{u_-(s)}^{u_+(s)} du \; \left|M(s,t,u)\right|^2$$

Determined from experiment

Determined from:
- Dispersive calculation
- ChPT

Fit to Dalitz distr.

$$\left[Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \right] \qquad \left[\hat{m} \equiv \frac{m_d + m_u}{2} \right]$$

- Aim: Compute M(s,t,u) with the *best accuracy*

2.3 Computation of the amplitude

- What do we know?

- Compute the amplitude using ChPT : the effective theory that describe dynamics of the Goldstone bosons (kaons, pions, eta) at low energy

- Goldstone bosons interact weakly at low energy and $m_u, m_d \ll m_s < \Lambda_{QCD}$
  Expansion organized in external momenta and quark masses

Weinberg's power counting rule

$$\mathcal{L}_{eff} = \sum_{d\geq 2} \mathcal{L}_d \ , \mathcal{L}_d = \mathcal{O}\left(p^d\right) \ , p \equiv \left\{q, m_q\right\}$$

$$\mathrm{p} << \Lambda_H = 4\pi F_\pi \sim 1 \ \mathrm{GeV}$$

2.3 Computation of the amplitude

- What do we know?
- Compute the amplitude using ChPT :

$$\Gamma_{\eta \to 3\pi} = \left(66 + 94 + ... + ...\right) \text{eV} = \left(300 \pm 12\right) \text{eV}$$

LO NLO NNLO

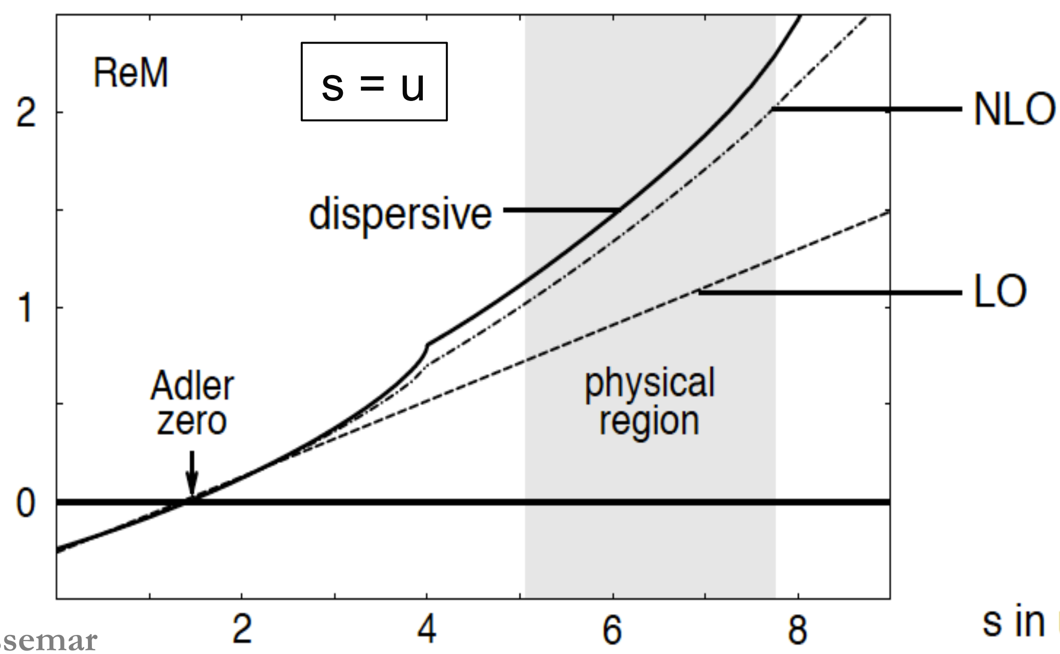
PDG'16

LO: *Osborn, Wallace'70*

NLO: *Gasser & Leutwyler'85*

NNLO: *Bijnens & Ghorbani'07*

The Chiral series has convergence problems



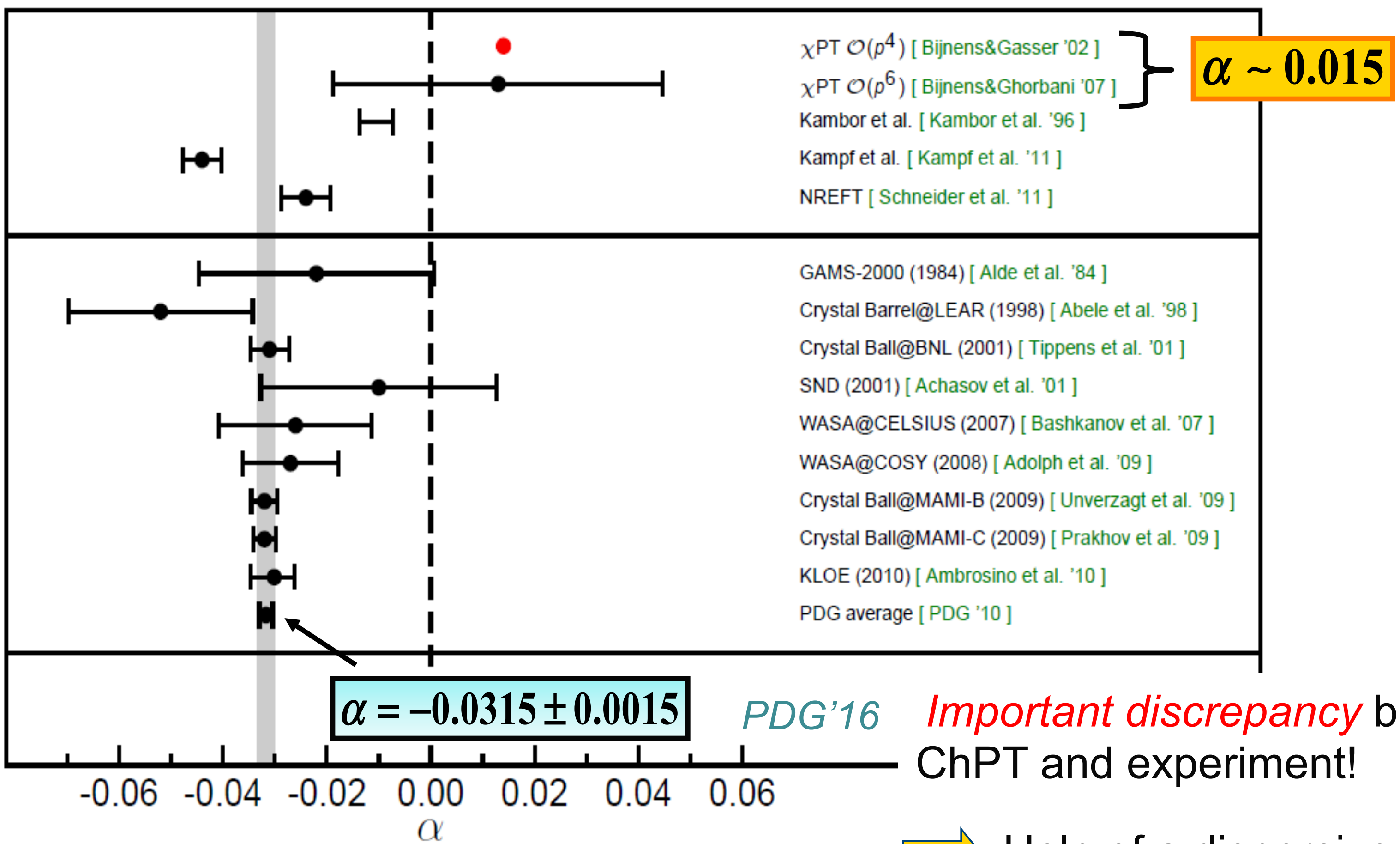
s in units of M_π

Anisovich & Leutwyler'96

2.4 Neutral Channel : $\eta \to \pi^0 \pi^0 \pi^0$

- Decay amplitude $\Gamma_{\eta\to 3\pi} \propto \left|\bar{A}\right|^2 \propto 1 + 2\alpha Z$ with $Z = \frac{2}{3}\sum_{i=1}^{3}\left(\frac{3T_i}{Q_n} - 1\right)^2$

$Q_n \equiv M_\eta - 3M_{\pi^0}$



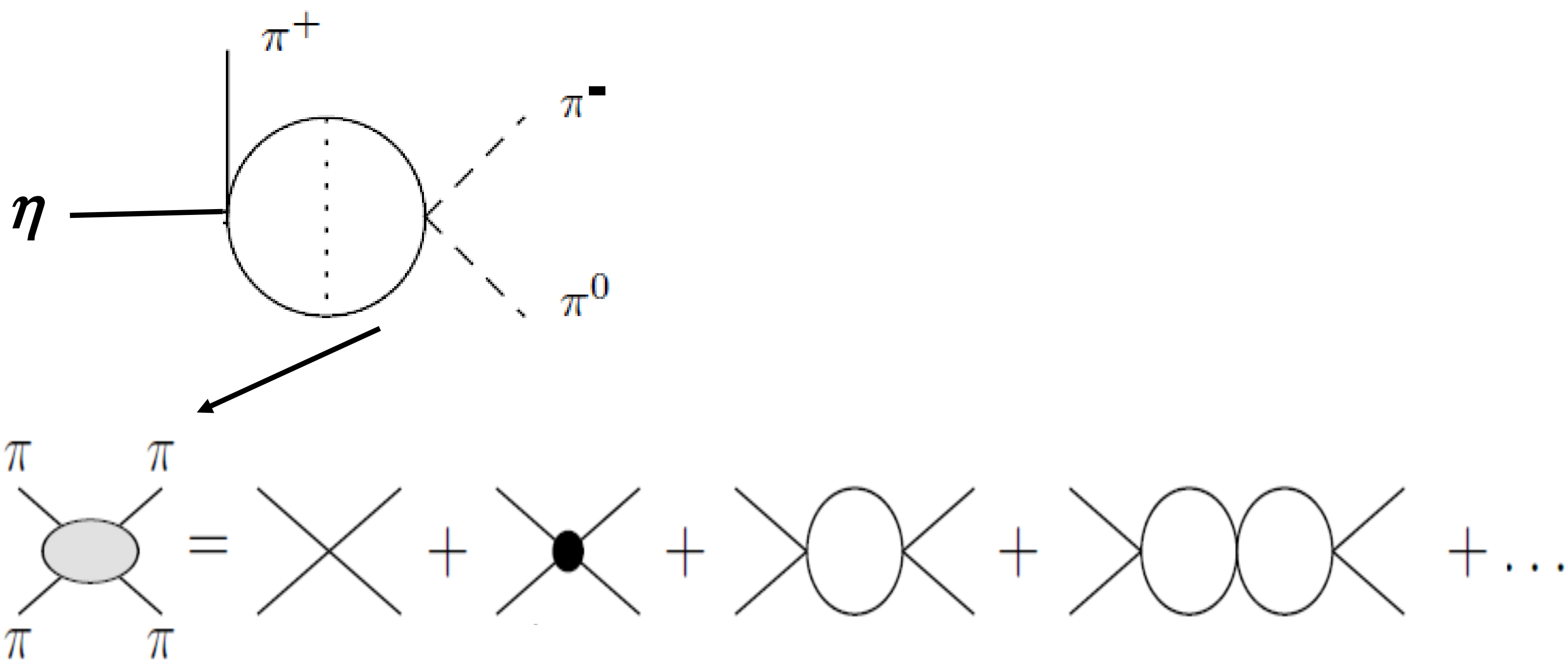
Important discrepancy between ChPT and experiment!

⇨ Help of a dispersive treatment?

2.5 Dispersive treatment

- The Chiral series has convergence problems

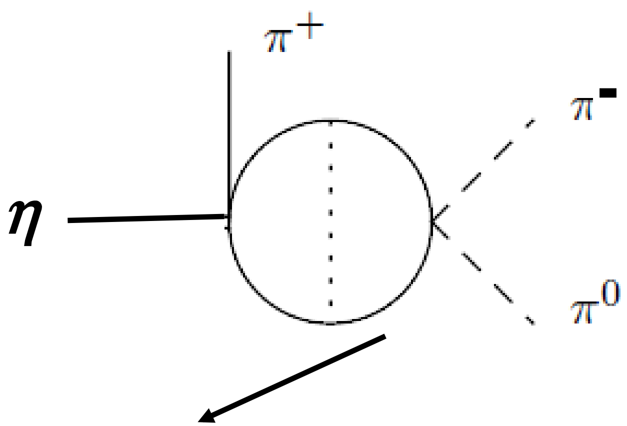
  ➡ Large *ππ final state interactions* *Roiesnel & Truong'81*

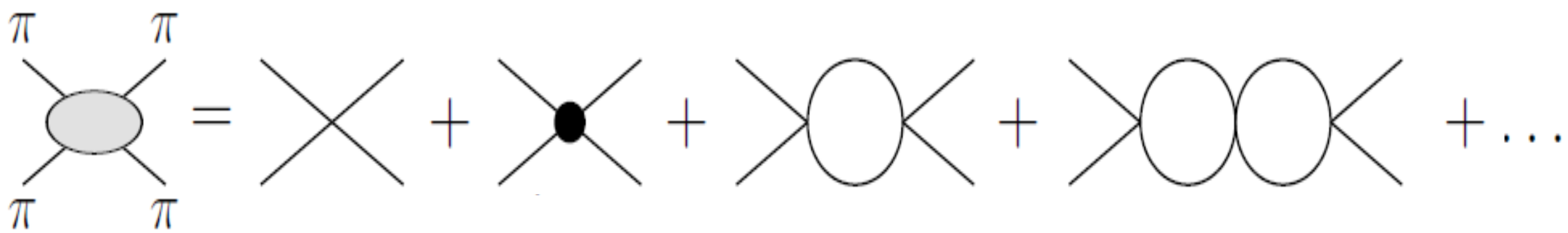


2.5 Dispersive treatment

- The Chiral series has convergence problems

  ➡ Large *ππ final state interactions* *Roiesnel & Truong'81*





- *Dispersive treatment :*
  - analyticity, unitarity and crossing symmetry
  - Take into account all the rescattering effects

2.6 Why a new dispersive analysis?

- Several new ingredients:
  - New inputs available: extraction $\pi\pi$ phase shifts has improved

    Ananthanarayan et al'01, Colangelo et al'01

    Descotes-Genon et al'01

    Kaminsky et al'01, Garcia-Martin et al'09

  - New experimental programs, precise Dalitz plot measurements

    TAPS/CBall-MAMI (Mainz), WASA-Celsius (Uppsala), WASA-Cosy (Juelich)

    CBall-Brookhaven, CLAS, GlueX (JLab), KLOE I-II (Frascati)

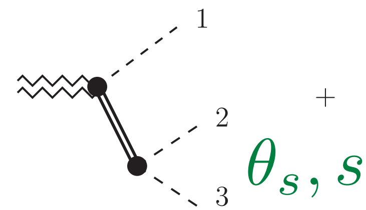
    BES III (Beijing)

  - Many improvements needed in view of very precise data: inclusion of
    - Electromagnetic effects ($\mathcal{O}(e^2m)$) *Ditsche, Kubis, Meissner'09*
    - Isospin breaking effects

2.7 Method

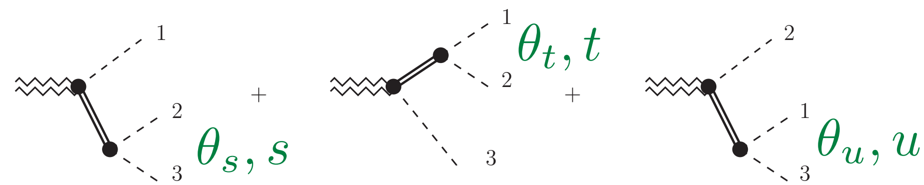
- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ➡ Isobar approximation

$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$



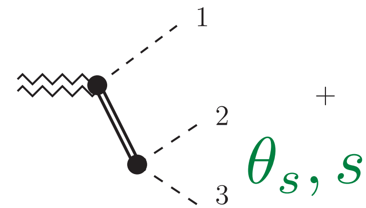
3 BWs (ρ^+, ρ^-, ρ^0) + background term

➡ Improve to include final states interactions

2.7 Method

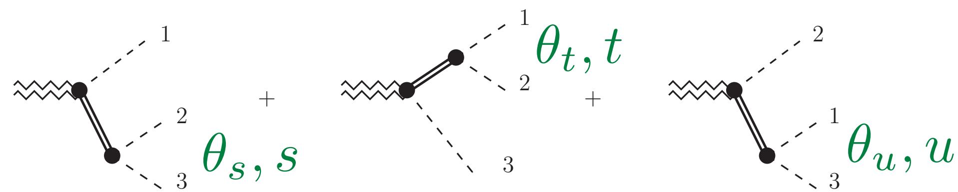
- S-channel partial wave decomposition

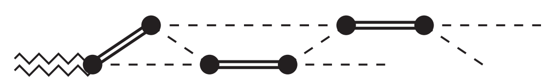
$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ⇨ Isobar approximation

$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$





- Use a Khuri-Treiman approach or dispersive approach

  ⇨ Restore 3 body unitarity and take into account the final state interactions in a systematic way

2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

Fuchs, Sazdjian & Stern'93
Anisovich & Leutwyler'96

  - M_I isospin *I* rescattering in two particles
  - Amplitude in terms of S and P waves ⇒ exact up to NNLO ($\mathcal{O}(p^6)$)
  - Main two body rescattering corrections inside M_I

2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0^0(s) + (s-u)M_1^1(t) + (s-t)M_1^1(u) + M_0^2(t) + M_0^2(u) - \frac{2}{3}M_0^2(s)$$

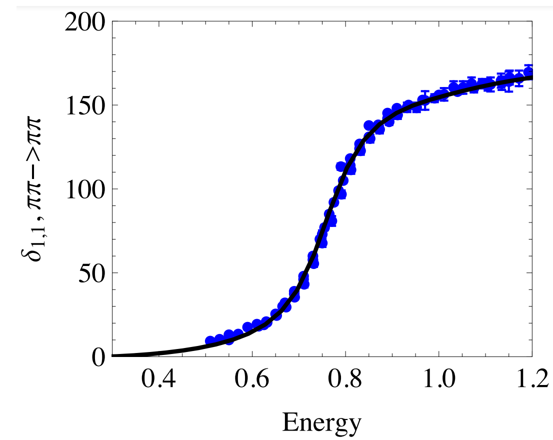
- Unitarity relation:

$\pi\pi \to \pi\pi$

$$disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)$$

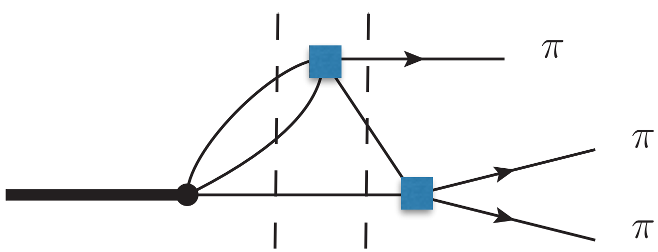
input

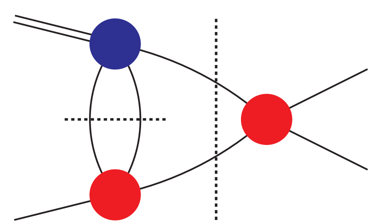
Roy analysis
Colangelo et al.'01



right-hand cut

left-hand cut





2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

- Unitarity relation:

$$disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)$$

- Relation of dispersion to reconstruct the amplitude everywhere:

$$M_I(s) = \Omega_I(s)\left(P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^n} \frac{\sin\delta_I(s')\,\hat{M}_I(s')}{|\Omega_I(s')|(s'-s-i\varepsilon)} \right)$$

$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

Omnès function

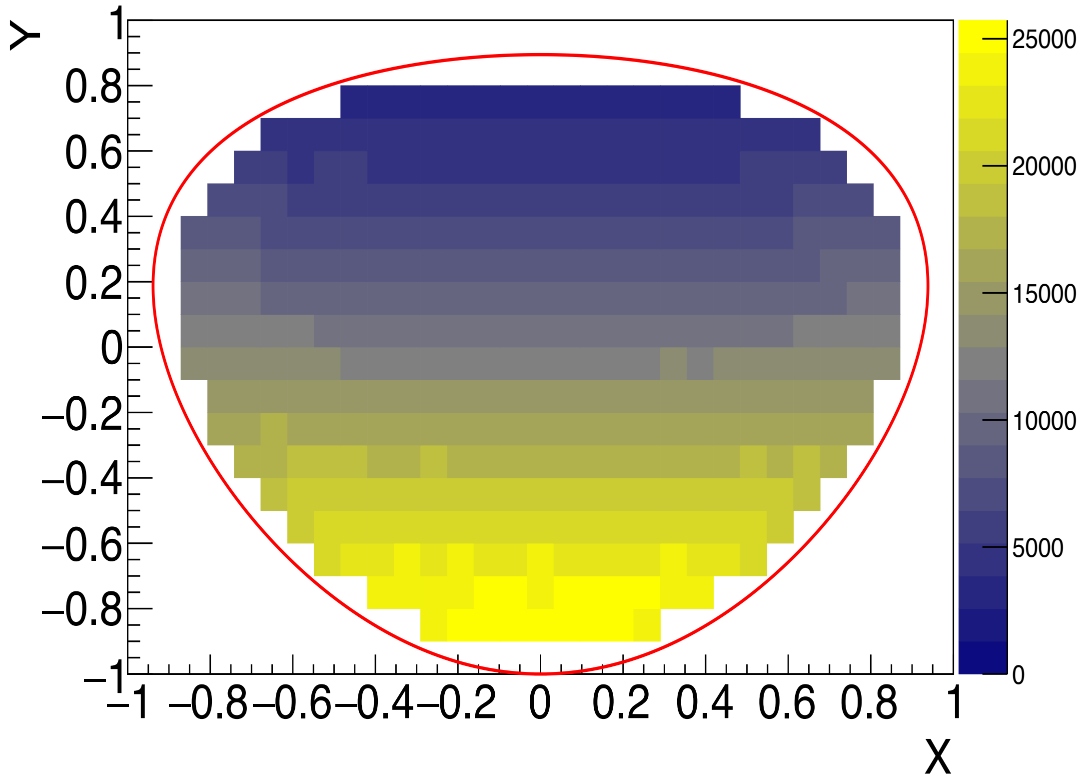
Gasser & Rusetsky'18

- $P_I(s)$ determined from a fit to NLO ChPT + experimental Dalitz plot

2.9 $\eta \to 3\pi$ Dalitz plot

- In the charged channel: experimental data from *WASA, KLOE, BESIII*

KLOE'16



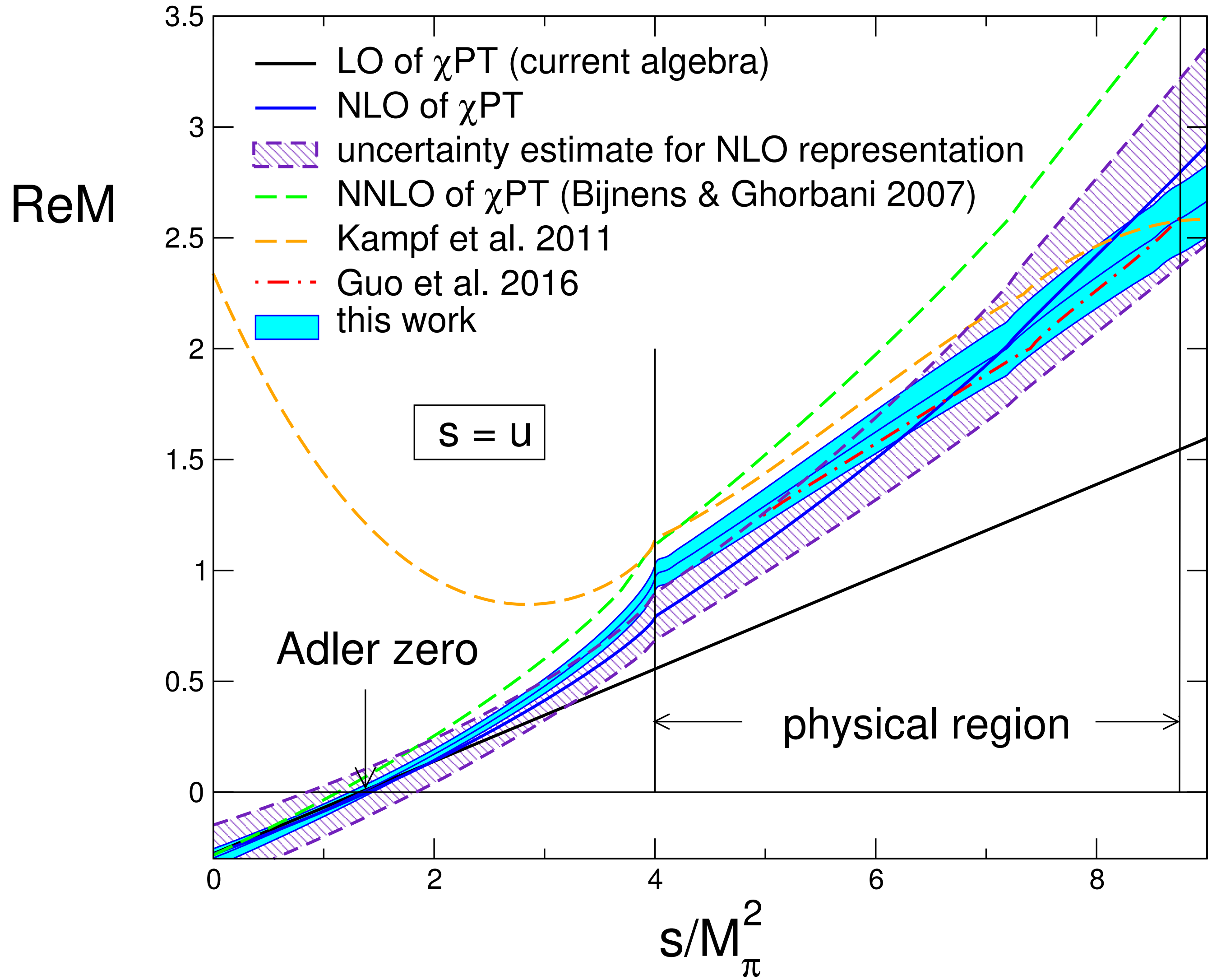
$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

- New data expected from *CLAS* and *GlueX* with very different systematics

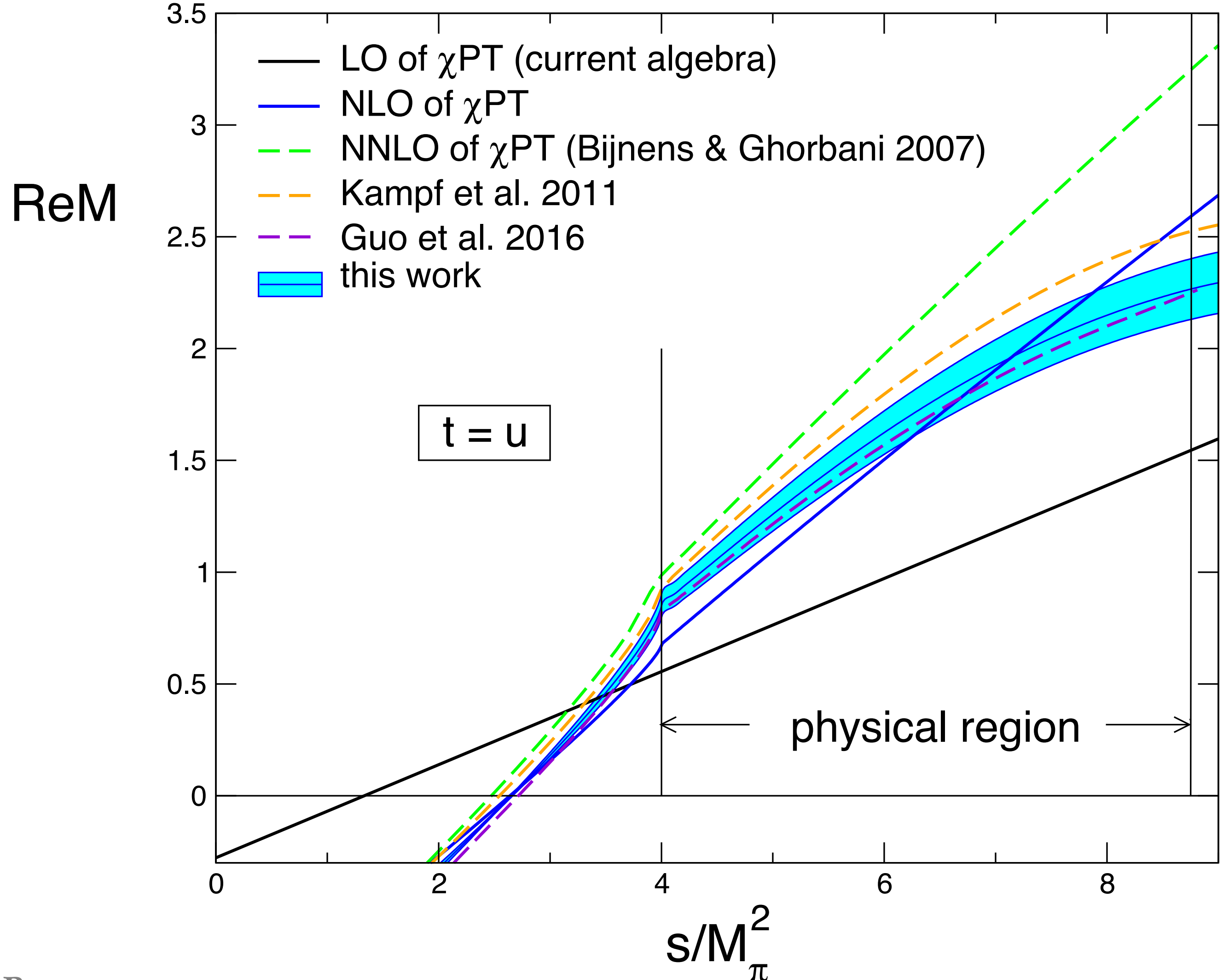
2.10 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line s = u :



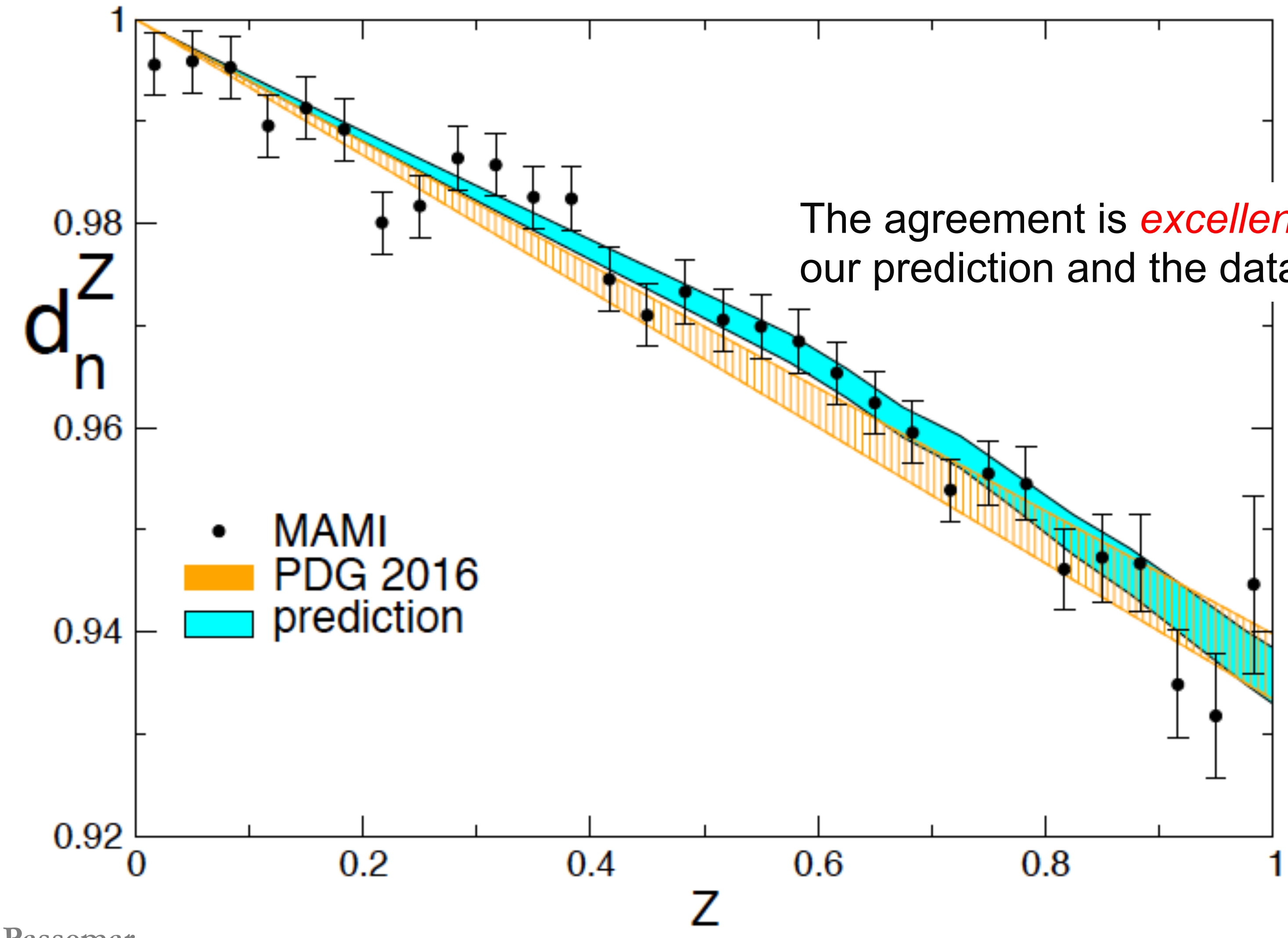
2.10 Results: Amplitude for $\eta \to \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line t = u :



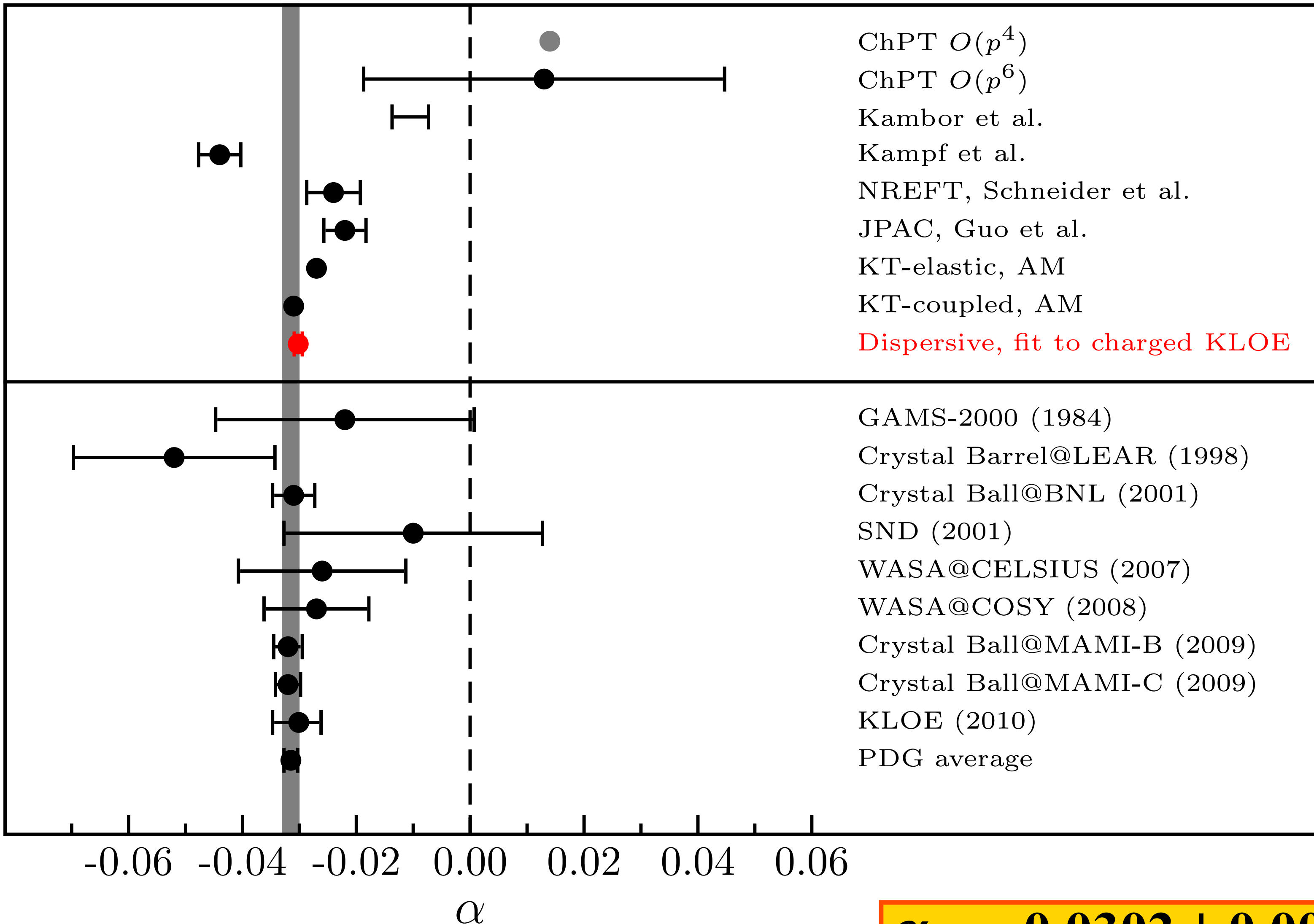
2.11 Z distribution for $\eta \rightarrow \pi^0 \pi^0 \pi^0$ decays

- The amplitude squared in the neutral channel is



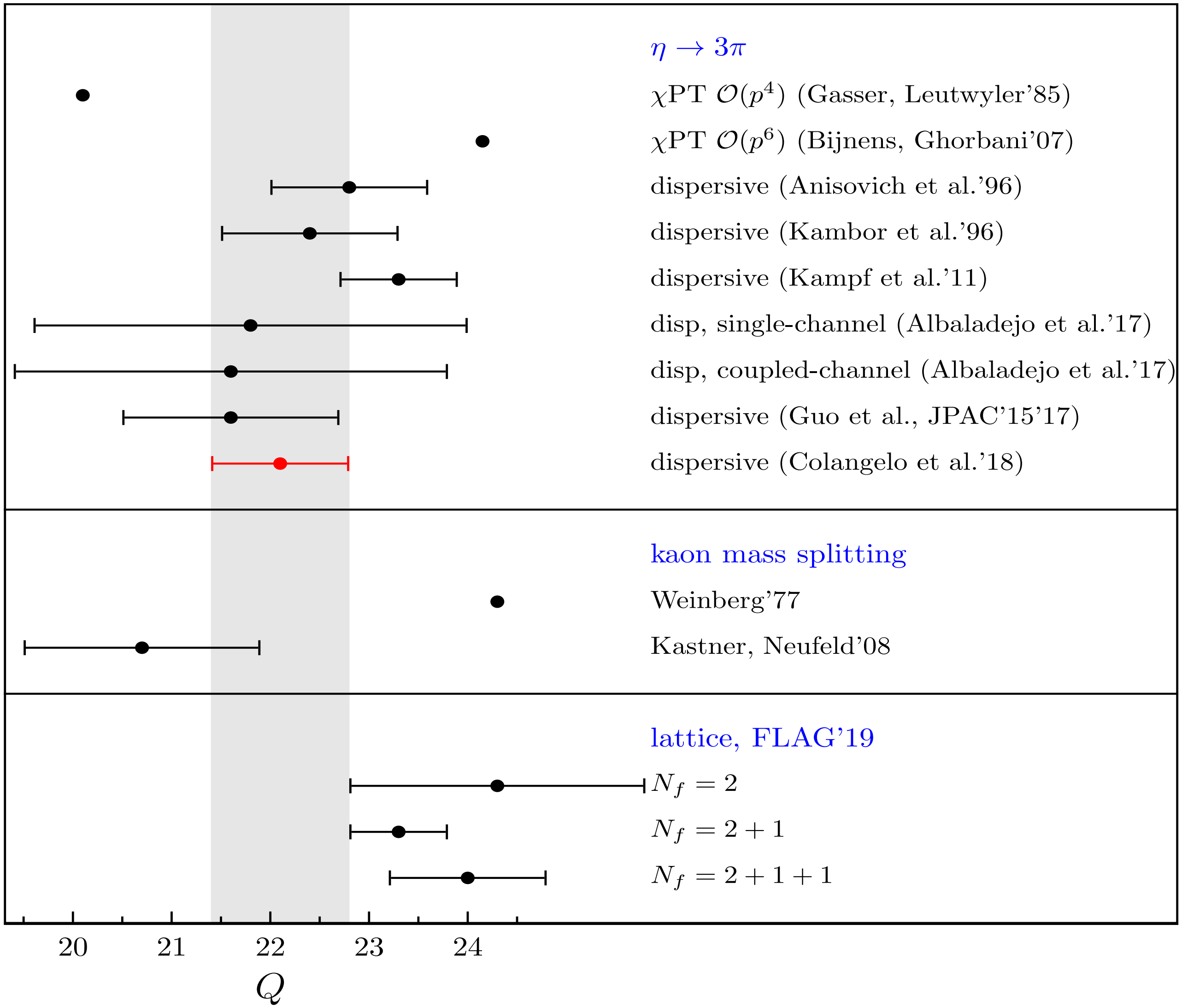
The agreement is *excellent* between our prediction and the data!

2.12 Comparison of results for α



$$\alpha = -0.0302 \pm 0.0011$$

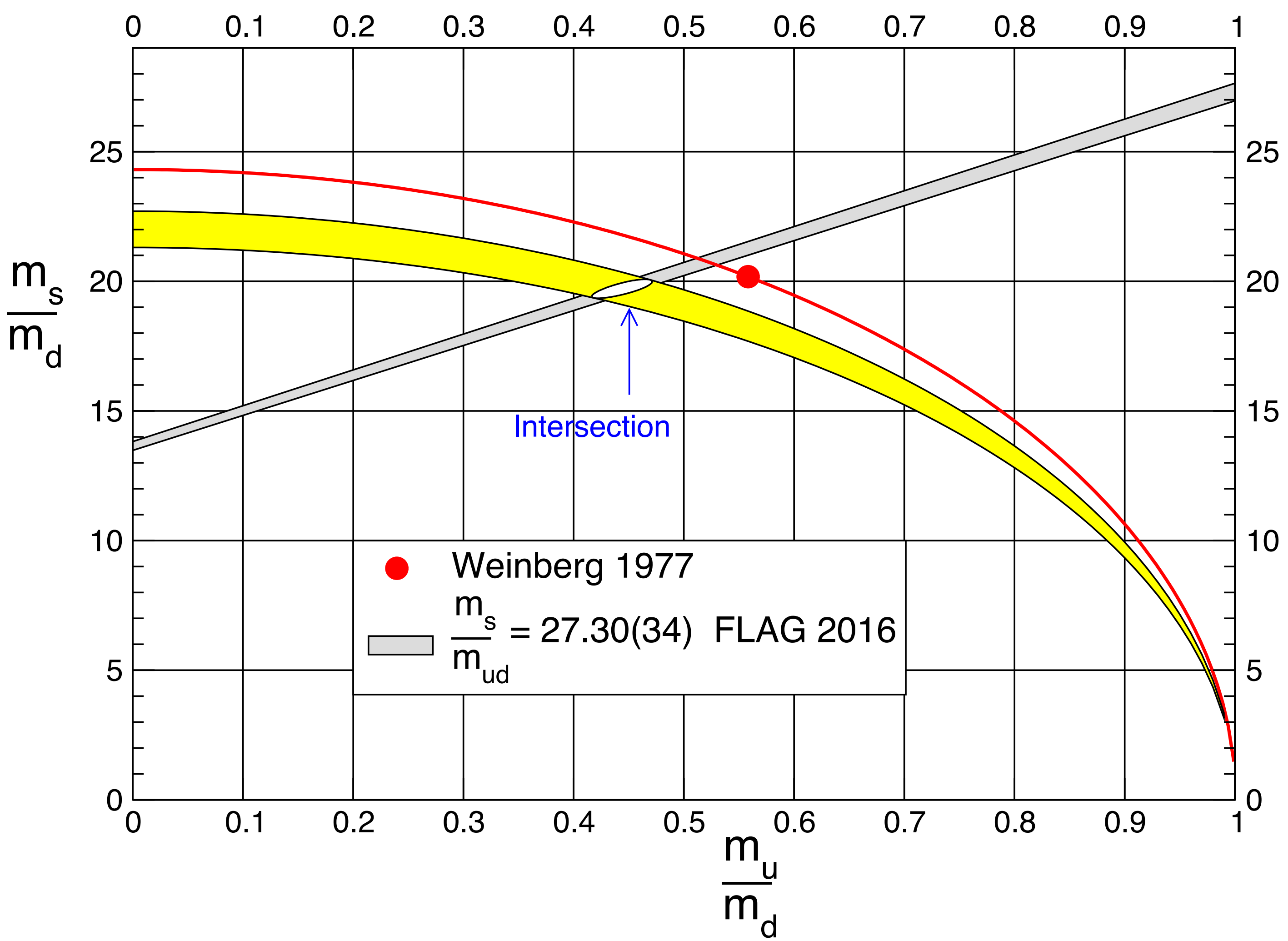
2.13 Quark mass ratio



$$Q = 22.1 \pm 0.7$$

- No systematics taken into account ⇨ collaboration with experimentalists

2.14 Light quark masses

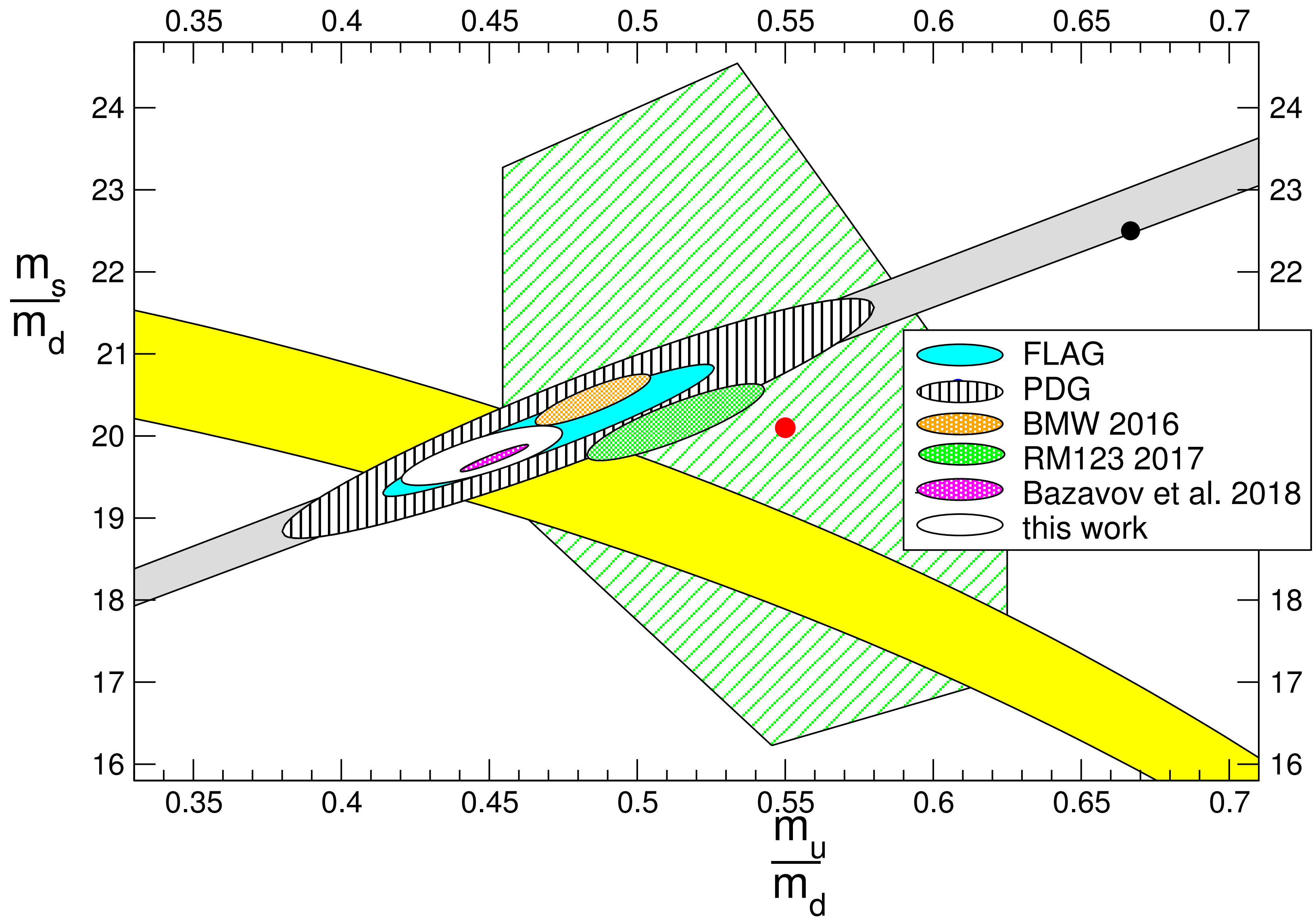


$Q = 22.1 \pm 0.7$

$\frac{m_u}{m_d} = 0.44 \pm 0.03$

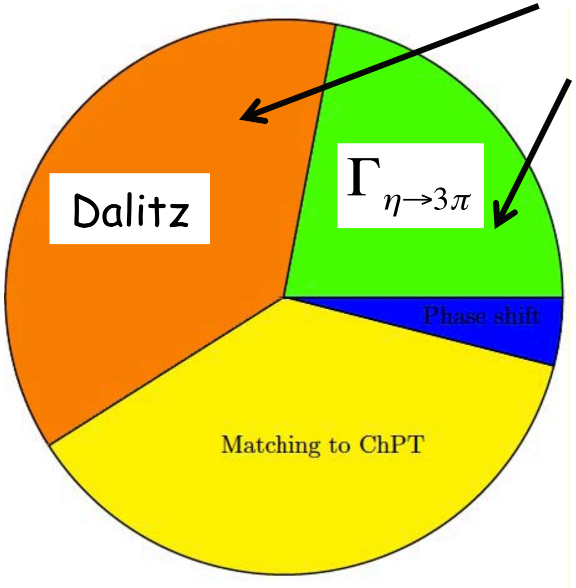
- Smaller values for Q ⇨ smaller values for m_s/m_d and m_u/m_d than LO ChPT

2.14 Light quark masses



2.15 Prospects

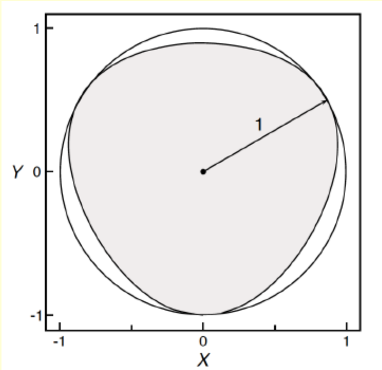
- Uncertainties in the quark mass ratio (rough attempt)



Can be investigated and reduced at *future facilities*

Experimental Measurements of $\eta \to 3\pi$

From L. Gan



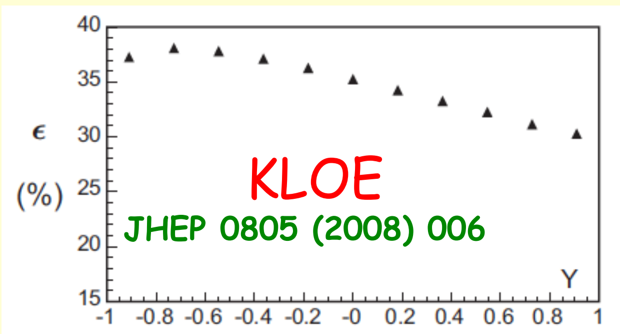
$$X = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

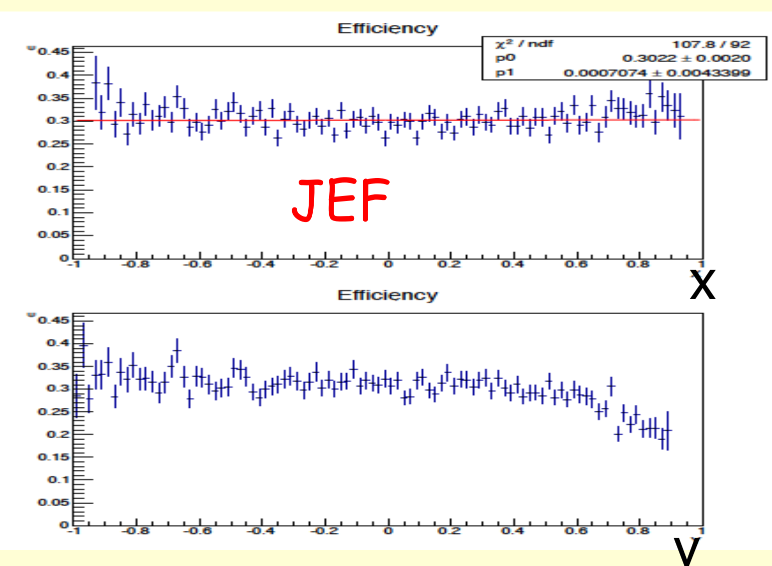
$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

$$Y = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

$$Z = X^2 + Y^2$$

Exp.	$3\pi^0$ Events (10^6)	$\pi^+\pi^-\pi^0$ Events (10^6)
Total world data (include prel. WASA and prel. KLOE)	6.5	6.0
GlueX+PrimEx-η +JEF	20	19.6





- Existing data from the low energy facilities are sensitive to the detection threshold effects
- JEF at high energy has uniform detection efficiency over Dalitz phase space
- JEF will offer large statistics and improved systematics

3. $\eta' \to \eta\pi\pi$ and chiral dynamics

In collaboration with
S. Gonzalez-Solis (Indiana University)
Eur. Phys. J. C78 (2018) no.9, 758

3.1 Why is it interesting to study $\eta' \to \eta\pi\pi$?

- Main decay channel of the η′: *PDG'19*

$$\mathrm{BR}(\eta' \to \eta\pi^0\pi^0) = 22.8(8)\%$$ and $$\mathrm{BR}(\eta' \to \eta\pi^+\pi^-) = 42.6(7)\%$$

- Precise meaurements became available: recent results on
  - neutral channel by *A2 collaboration* : 1.2 x 10^5 events
  - neutral and charged channel by *BESIII* collaboration: 351 016 events

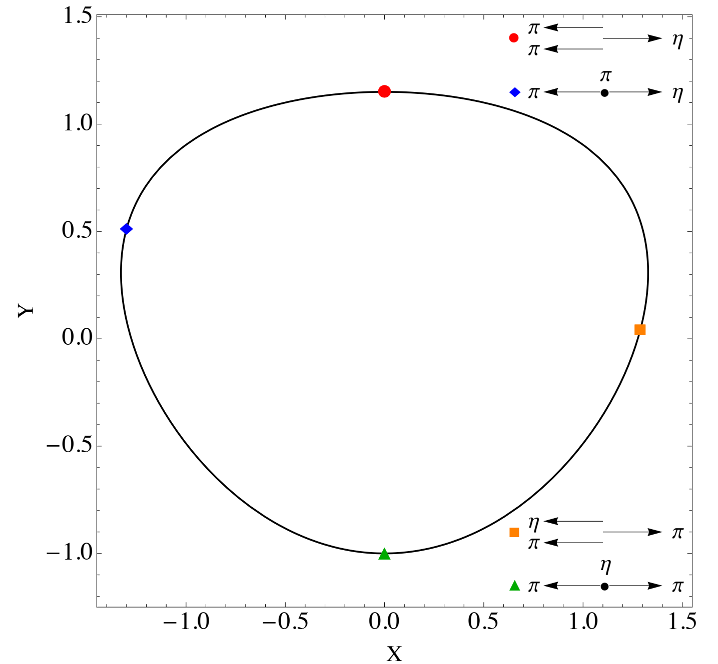
$$\left|A(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3 + \ldots\right)$$

$$s = \left(p_{\eta'} - p_{\eta}\right)^2,\ t = \left(p_{\eta'} - p_{\pi^+}\right)^2,\ u = \left(p_{\eta'} - p_{\pi^-}\right)^2$$

Expansion around X=Y=0

$$X = \sqrt{3}\,\frac{T_- - T_+}{Q_{\eta'}} = \frac{\sqrt{3}}{2M_{\eta'}Q_{\eta'}}\left(t-u\right)$$

$$Y = \frac{\left(M_\eta + 2M_\pi\right)}{M_\pi}\frac{T_\eta}{Q_{\eta'}} - 1 = \frac{\left(M_\eta + 2M_\pi\right)}{M_\pi}\frac{\left(\left(M_{\eta'} - M_\eta\right)^2 - s\right)}{2M_{\eta'}Q_{\eta'}} - 1$$



$$Q_{\eta'} \equiv M_{\eta'} - M_\eta - 2M_\pi$$

3.1 Why is it interesting to study $\eta' \to \eta\pi\pi$?

- Main decay channel of the η′:

PDG'19

$$\mathbf{BR}(\eta' \to \eta\pi^0\pi^0) = 22.8(8)\%$$ and $$\mathbf{BR}(\eta' \to \eta\pi^+\pi^-) = 42.6(7)\%$$

- Precise meaurements became available: recent results on
  - neutral channel by *A2 collaboration* : 1.2 x 10^5 events
  - Neutral and charged channel by *BESIII* collaboration: 351 016 events

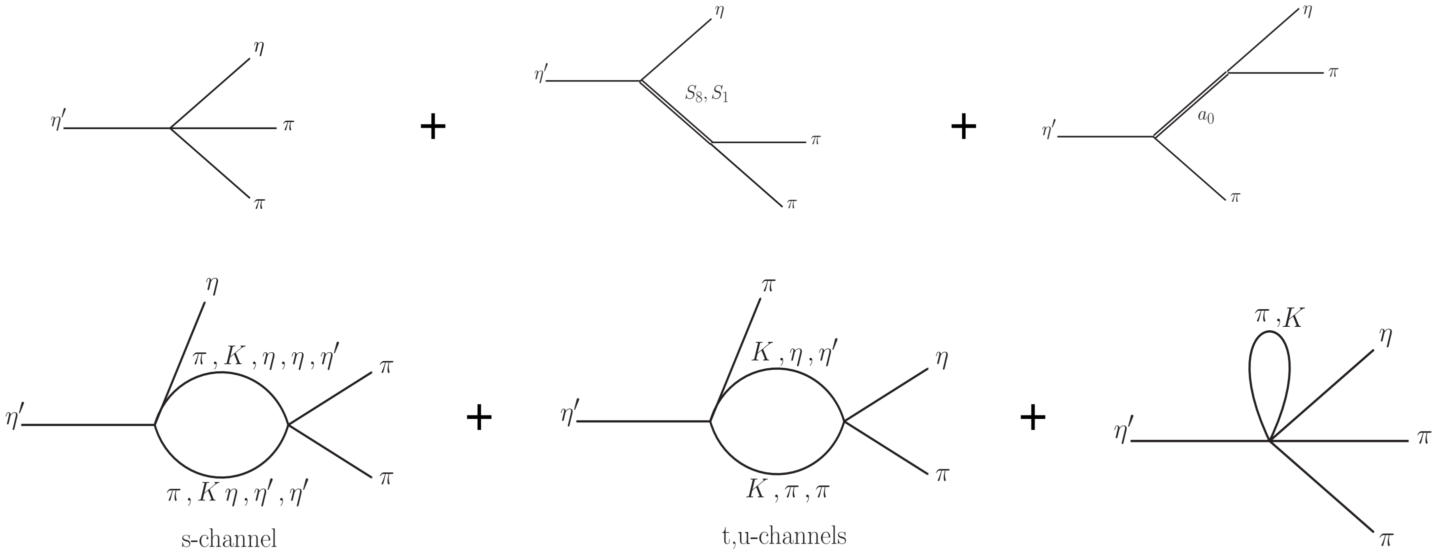
$$\left|A(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3 + \ldots\right)$$

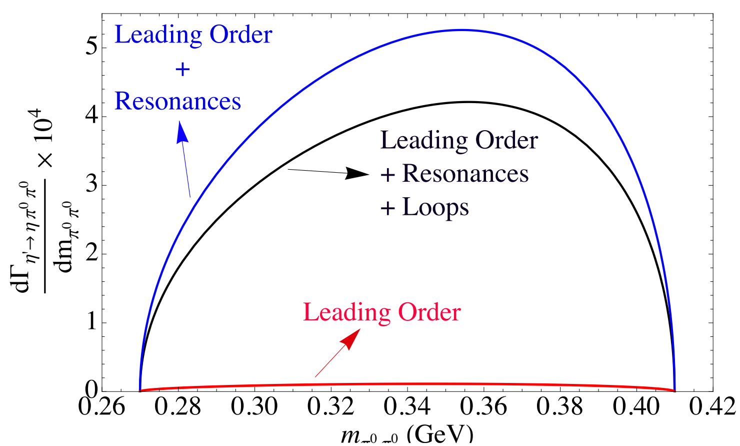
- Studying this decay allows
  - to test any of the extensions of ChPT e.g. resonance chiral theory, Large-N_C U(3) ChPT etc
  - to study the effects of the ππ and πη final-state interactions

3.2 Theoretical Framework

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$$

- U(3) ChPT with resonances at one-loop

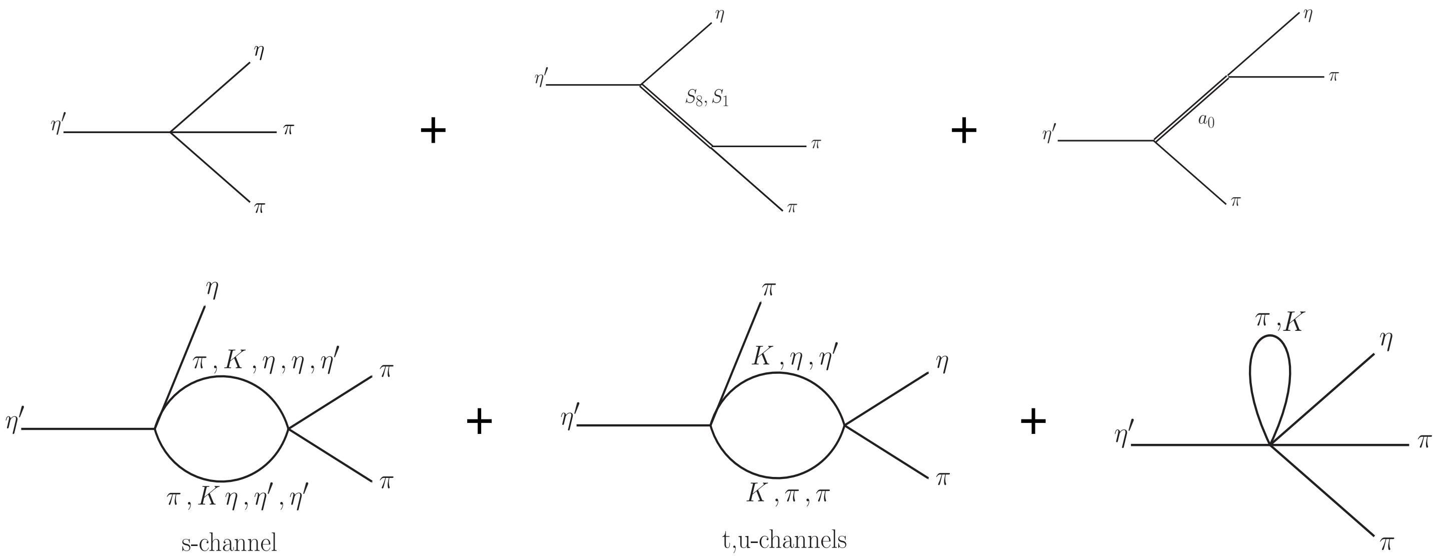


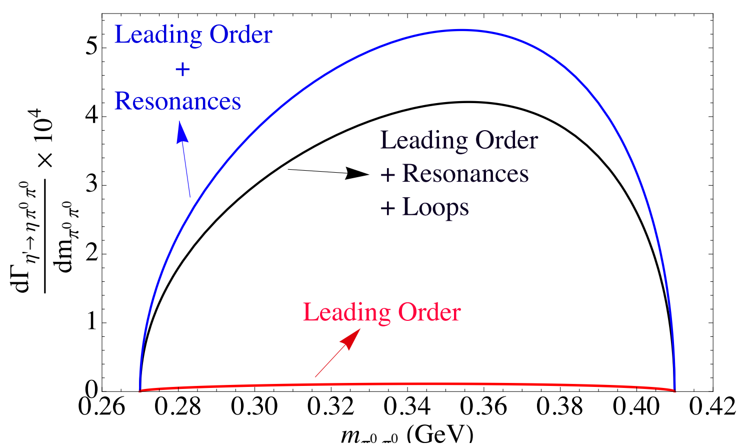


3.2 Theoretical Framework

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$$

- U(3) ChPT with resonances at one-loop



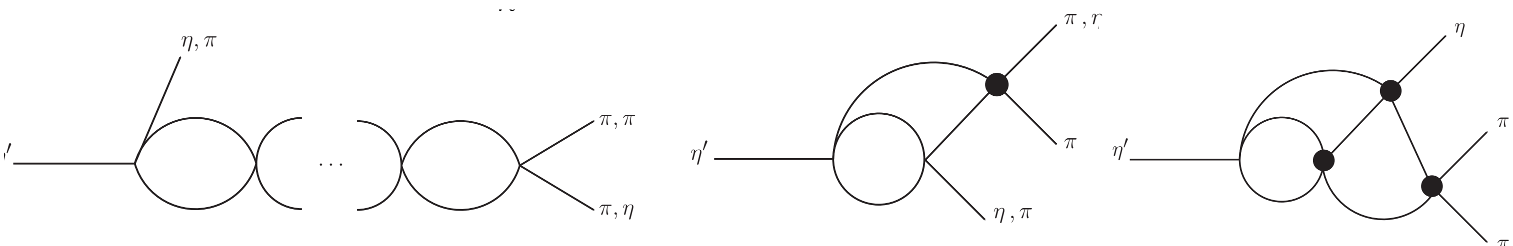


Final-state interaction through the N/D unitarization method

3.2 Theoretical Framework

- Unitarity relations

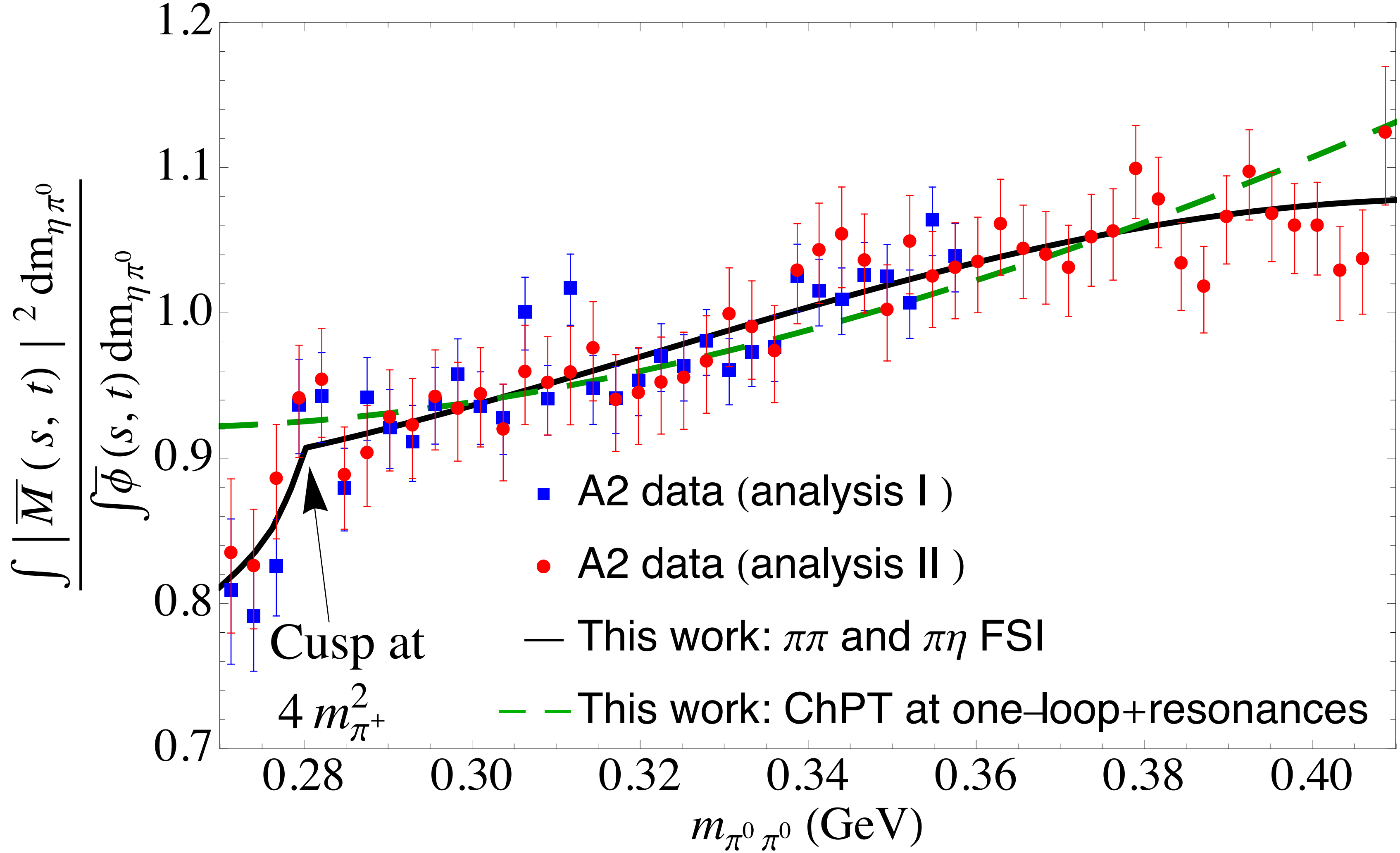
$$\mathrm{Im}\mathcal{M}_{\eta'\to\eta\pi\pi} = \frac{1}{2}\sum_n (2\pi)^4\,\delta^4\left(p_\eta + p_1 + p_2 - p_n\right)\mathcal{T}^*_{n\to\eta\pi\pi}\mathcal{M}_{\eta'\to n}$$



not included

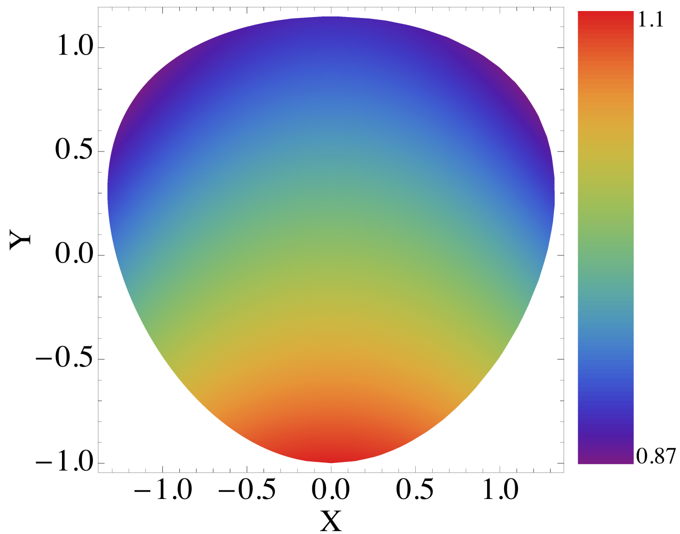
- A dispersive analysis also exists by *Isken et al.'17* but here we include D waves as well as kaon loops

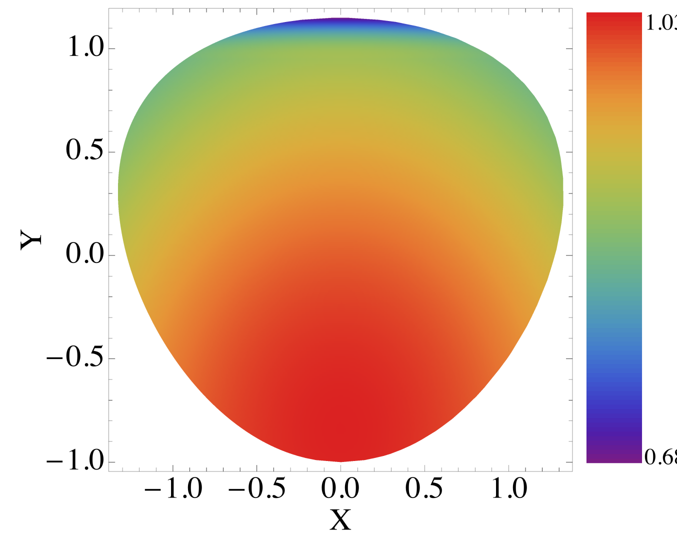
3.3 Results



$\frac{\int |\overline{M}(s,t)|^2 dm_{\eta\pi^0}}{\int \overline{\phi}(s,t)\, dm_{\eta\pi^0}}$
1.2
1.1
1.0
0.9
0.8
0.7
0.28
0.30
0.32
0.34
0.36
0.38
0.40
$m_{\pi^0\pi^0}$ (GeV)
Cusp at
$4\,m_{\pi^+}^2$
A2 data (analysis I)
A2 data (analysis II)
This work: $\pi\pi$ and $\pi\eta$ FSI
This work: ChPT at one-loop+resonances

3.3 Results



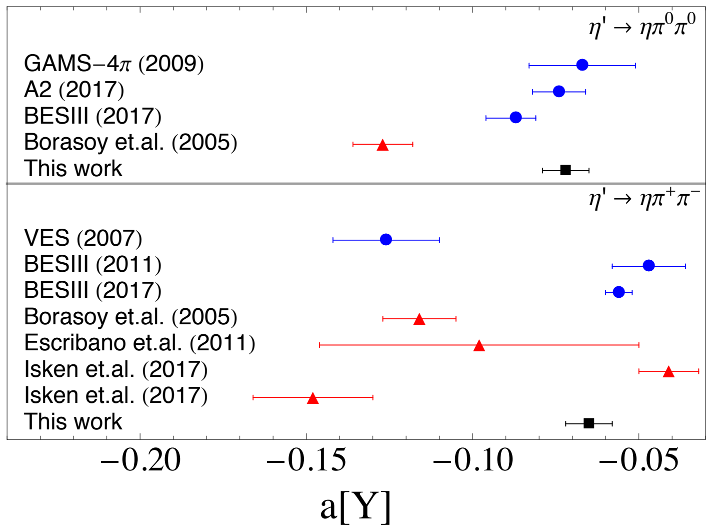


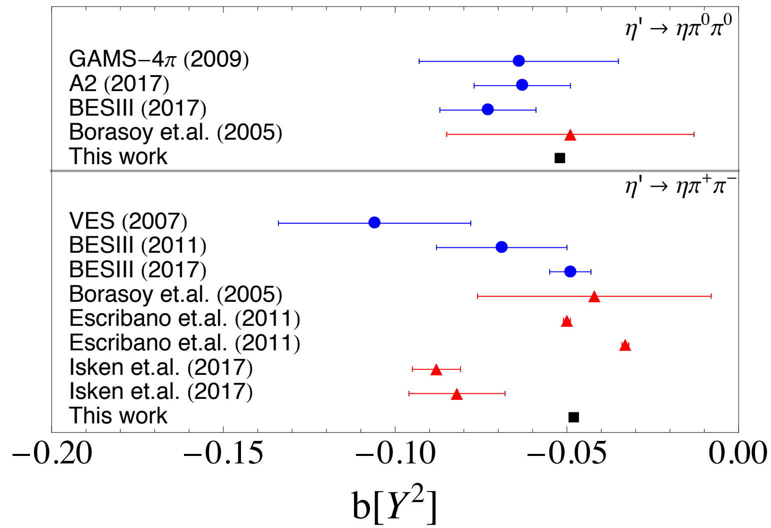
Dalitz slope parameters

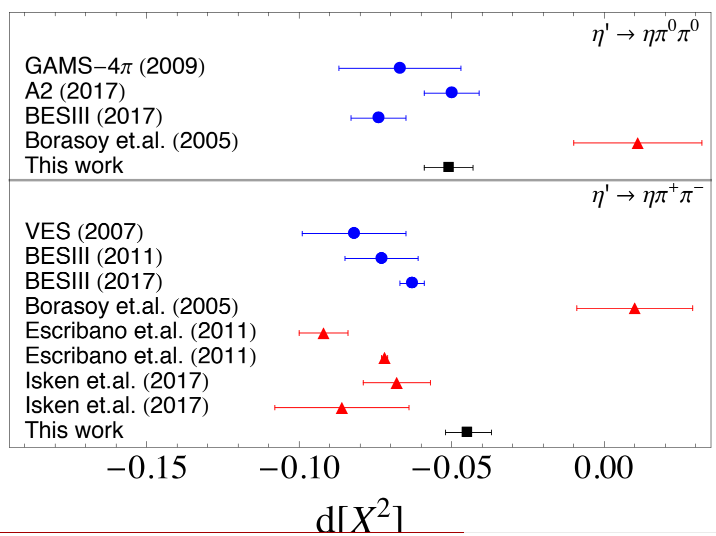
ChPT		Final-state interactions
$a[Y] = -0.095(6)$		$a[Y] = -0.073(7)(5)$
$b[Y^2] = 0.005(1)$	$\Rightarrow$	$b[Y^2] = -0.052(1)(2)$
$d[X^2] = -0.037(5)$		$d[X^2] = -0.052(8)(5)$

$$|A(s,t,u)|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3 + \ldots\right)$$

3.3 Results





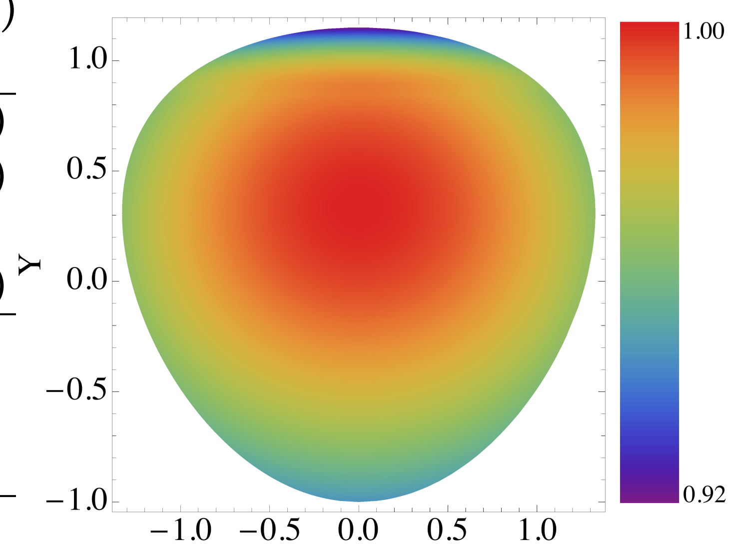


$$\left|A(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3 + \ldots\right)$$

3.4 Role of the D-wave ππ FSI

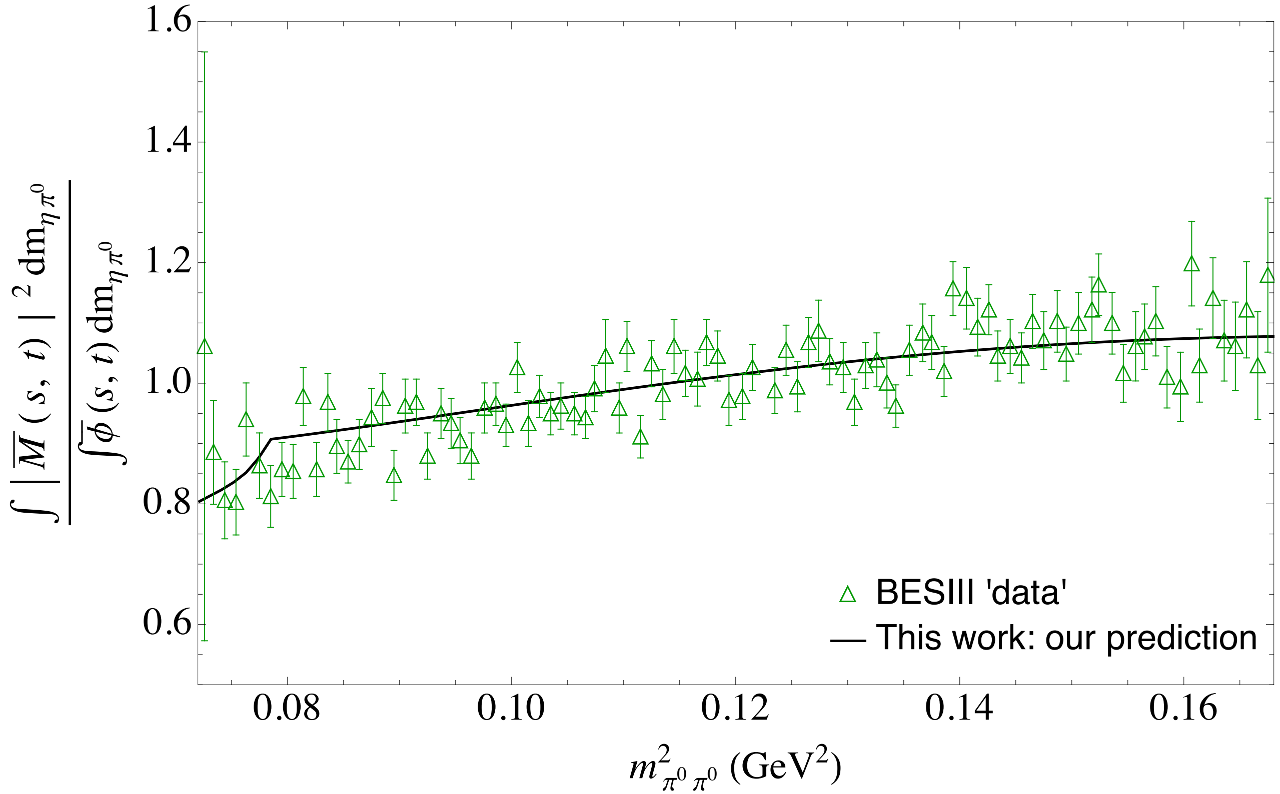
Parameter	Analysis I	
	Fit 1 (with D-wave)	Fit 1 (w/o D-wave)
M_S	$1017(68)(24)$	$996(66)(25)$
c_d	$30.4(4.8)(9)$	$23.3(3.5)(1.5)$
c_m	$= c_d$	$= c_d$
$\tilde{c}_d$	$17.6(2.8)(5)$	$13.5(2.0)(9)$
$\tilde{c}_m$	$= \tilde{c}_d$	$= \tilde{c}_d$
$a_{\pi\pi}$	$0.76(61)(6)$	$2.01(1.61)(71)$
$\chi^2_{\rm dof}$	1.12	1.24
$a[Y]$	$\mathbf{-0.074(7)(8)}$	$\mathbf{-0.091(9)(4)}$
$b[Y^2]$	$\mathbf{-0.049(1)(2)}$	$\mathbf{-0.013(1)(5)}$
$c[X]$	$\mathbf{0}$	$\mathbf{0}$
$d[X^2]$	$\mathbf{-0.047(8)(4)}$	$\mathbf{-0.031(6)(3)}$
$\kappa_{03}[Y^3]$	0.001	0.001
$\kappa_{21}[YX^2]$	-0.004	-0.001
$\kappa_{22}[Y^2X^2]$	0.001	0.0004

$|M(X,Y)_{\rm Full}|^2/|M(X,Y)_{\rm D-wave=0}|^2$



3.5 Prospects

- Comparison to BESIII data



- Simultaneous fit by experimental collaborations to the neutral and charged channels etc

4. Conclusion and Outlook

4.1 Conclusion

- Light Meson component very important for spectroscopy
- Knowing conventional modes important for studies of background for looking for exotics
- Study of fundamental properties of QCD:
  - Extraction of fundamental parameters of the SM,
    ⇨ e.g. light quark masses
  - Study of chiral dynamics
- To studies meson modes with the best precision: Development of amplitude analysis techniques consistent with analyticity, unitarity, crossing symmetry ⇨ dispersion relations allow to take into account *all rescattering effects* being as model independent as possible combined with ChPT ⇨ Provide parametrization for experimental studies
- In this talk, illustration with $\eta \to 3\pi$ and extraction of the light quark masses and $\eta' \to \eta\pi\pi$
- Other illustrations in the talks of *A. Pilloni* and *P. Masjuan*

4.2 Outlook:

- Apply dispersion relations + (R)ChPT to other modes in the light meson sector
  - $\omega/\varphi \to 3\pi, \pi\gamma$: *Niecknig, Kubis, Schneider'12*, *Danilkin et al. JPAC'15,'16*
  - $\varphi \to \eta\pi\gamma$: *Moussallam, Shekhovtsova in progress*
  - $\eta' \to 3\pi$: *Isken, Kubis and Stoffer in progress*
  - etc…

5. Back-up

Experimental Facilities and Role of JLab 12

M. J. Amaryan et al.
CLAS Analysis Proposal, (2014)

π	$e^+e^-\gamma$			
η	$e^+e^-\gamma$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$, $\pi^+\pi^-$	$\pi^+\pi^-e^+e^-$
η'	$e^+e^-\gamma$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$, $\pi^+\pi^-$	$\pi^+\pi^-\eta$, $\pi^+\pi^-e^+e^-$
ρ		$\pi^+\pi^-\gamma$		
ω	$e^+e^-\pi^0$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$	
φ			$\pi^+\pi^-\pi^0$	$\pi^+\pi^-\eta$

η' → ηππ at leading order

- ChPT Lagrangian at $\mathcal{O}(p^2)$

$$\mathcal{L}^{p^2} = \frac{F_\pi^2}{4}\langle u_\mu u^\mu \rangle + \frac{F_\pi^2}{4}\langle \chi_+ \rangle + \frac{F_\pi^2}{3} m_1^2 \ln^2 \det u$$

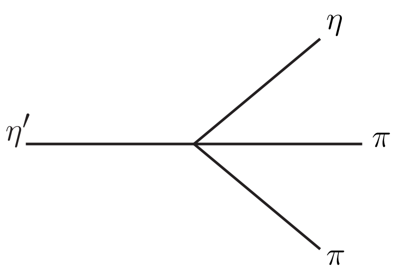
- Expanding in powers of Φ

$$\mathcal{L}^{p^2} = \frac{1}{2}\langle \partial_\mu \Phi \partial^\mu \Phi \rangle + \frac{1}{12 f^2}\underline{\langle (\Phi(\partial_\mu \Phi) - (\partial_\mu \Phi)\Phi)(\Phi(\partial^\mu \Phi) - (\partial^\mu \Phi)\Phi)\rangle}$$

$$+ B_0 \left\{ -\langle \mathcal{M}\Phi^2 \rangle + \underline{(1/6 f^2)\langle \mathcal{M}\Phi^4 \rangle} \right\} + \mathcal{O}\left(\frac{\Phi^6}{f^4}\right)$$

no contribution

$$\Downarrow$$

- 

$$= \boxed{\mathcal{M}^{LO}_{\eta' \to \eta\pi\pi} = \frac{M_\pi^2}{6 F_\pi^2}\left(2\sqrt{2}\cos 2\theta - \sin 2\theta\right)}$$

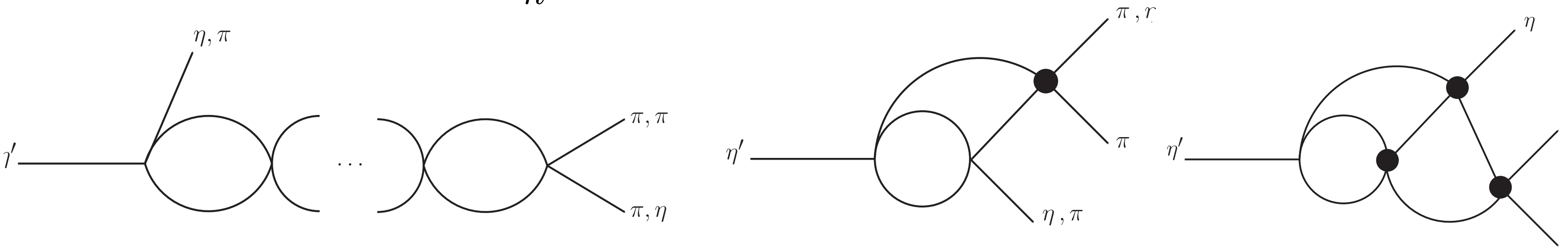
	$BR(\eta' \to \eta\pi^+\pi^-)$	$BR(\eta' \to \eta\pi^0\pi^0)$
Leading Order	1.1%	0.6%
PDG 2018	$42.6(7)$	$22.8(8)\%$

- Reason for this difference: amplitude is chirally suppressed (vanishes when $M_\pi^2 \to 0$)
- Higher order effects? • Resonances exchanges (a_0, f_0, σ) • $\pi\pi$, $\pi\eta$ final state interactions

Unitarity in $\eta' \to \eta\pi\pi$

- Unitarity relation

$$\mathrm{Im}\mathcal{M}_{\eta'\to\eta\pi\pi} = \frac{1}{2}\sum_n (2\pi)^4\,\delta^4\left(p_\eta + p_1 + p_2 - p_n\right)\mathcal{T}^*_{n\to\eta\pi\pi}\mathcal{M}_{\eta'\to n}$$



- Two-particle unitarity relation for the partial-wave decay amplitude

$$\mathrm{Im}\left(m^{00}_{\eta'\to\eta\pi\pi}(s)\right) = \sigma_\pi(s)\left(t^{00}_{\pi\pi\to\pi\pi}(s)\right)^* m^{00}_{\eta'\to\eta\pi\pi}(s)\times\theta(s-4m_\pi^2)\,,$$

$$\mathrm{Im}\left(m^{02}_{\eta'\to\eta\pi\pi}(s)\right) = \sigma_\pi(s)\left(t^{02}_{\pi\pi\to\pi\pi}(s)\right)^* m^{02}_{\eta'\to\eta\pi\pi}(s)\times\theta(s-4m_\pi^2)\,,$$

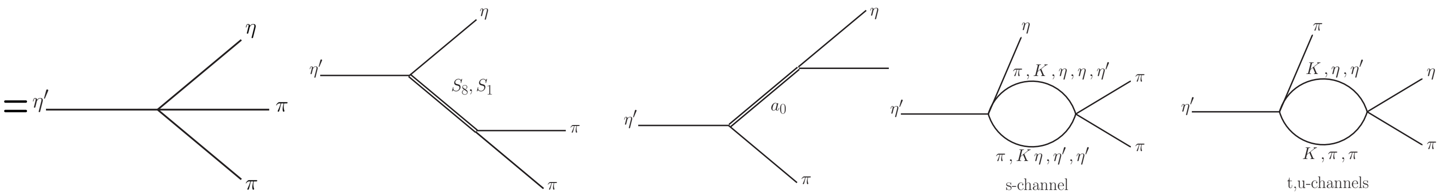
$$\mathrm{Im}\left(m^{10}_{\eta'\to\eta\pi\pi}(t)\right) = \frac{\lambda^{1/2}(t,m_\pi^2,m_\eta^2)}{t}\left(t^{10}_{\pi\eta\to\pi\eta}(t)\right)^*\times m^{10}_{\eta'\to\eta\pi\pi}(t)\theta(t-(m_\pi+m_\eta)^2)\,,$$

$$\mathrm{Im}\left(m^{10}_{\eta'\to\eta\pi\pi}(u)\right) = \frac{\lambda^{1/2}(u,m_\pi^2,m_\eta^2)}{u}\left(t^{10}_{\pi\eta\to\pi\eta}(u)\right)^*\times m^{10}_{\eta'\to\eta\pi\pi}(u)\theta(u-(m_\pi+m_\eta)^2)\,,$$

N/D unitarisation in $\eta' \to \eta\pi\pi$

- Amplitude at one-loop in Large-N_C $U(3)$ ChPT with resonances

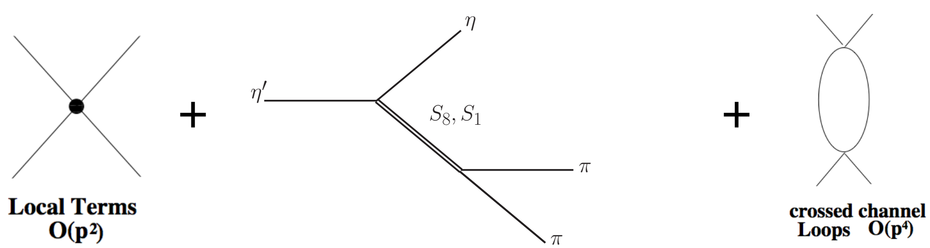
$\mathcal{M}^{\eta'\to\eta\pi\pi}(s) =$



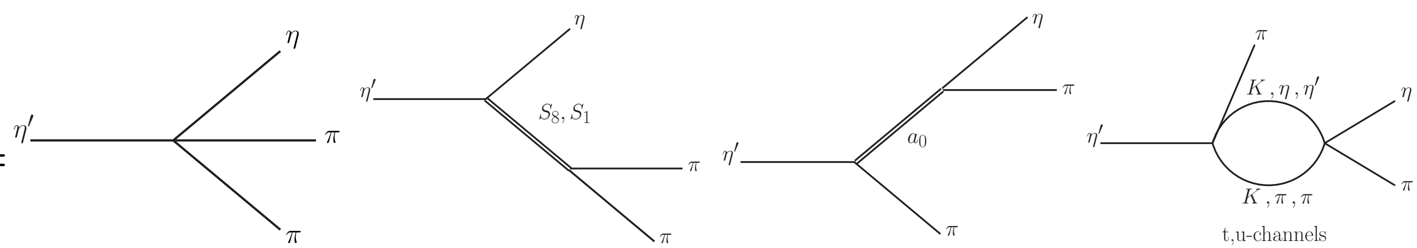
- N/D representation of $\mathcal{M}^{\eta'\to\eta\pi\pi}(s)$

$$m^{IJ}_{\eta'\to\eta\pi\pi}(s) = \left[1 + N^{IJ}_{\pi\pi}(s)\, g_{\pi\pi}(s)\right]^{-1} R^{IJ}_{\eta'\to\eta\pi\pi}(s)$$

$N^{IJ}_{\pi\pi\to\pi\pi}(s) =$



$R^{IJ}_{\eta'\to\eta\pi\pi}(s) =$



$g_{\pi\pi}(s) = a_{\pi\pi}(\mu) -$  $= \frac{1}{16\pi^2}\left(a_{\pi\pi}(\mu) + \log\frac{m_\pi^2}{\mu^2} - \sigma(s)\log\frac{\sigma(s)-1}{\sigma(s)+1}\right)$

ηπ phase shift extraction from η' → ηππ