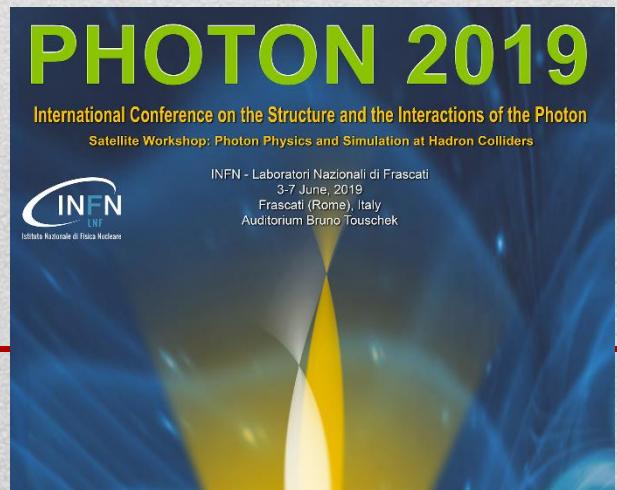


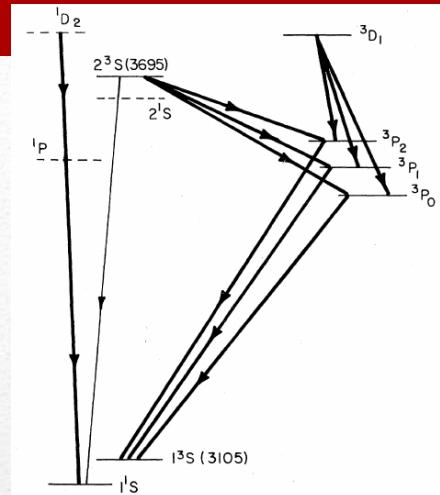
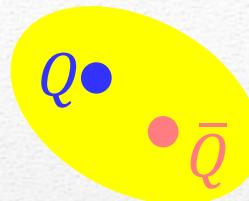
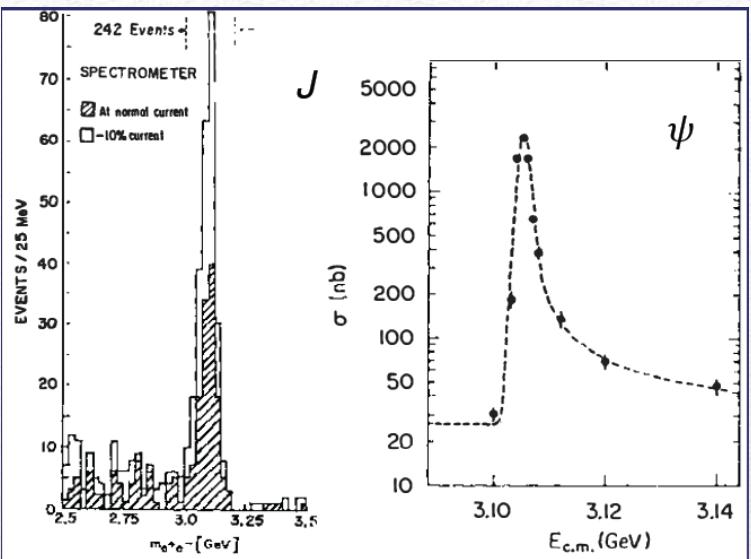
Heavy quark Spectroscopy

Alessandro Pilloni

PHOTON 2019, June 3rd, 2019



Quarkonium orthodoxy



Potential models

(meaningful when $M_Q \rightarrow \infty$)

$$V(r) = -\frac{C_F \alpha_s}{r} + \sigma r \quad (\text{Cornell potential})$$

Solve NR Schrödinger eq. → spectrum

Effective theories

(HQET, NRQCD, pNRQCD...)

Integrate out heavy DOF



(spectrum), decay & production rates

$\alpha_s(M_Q) \sim 0.3$
(perturbative regime)
OZI-rule, QCD multipole

Heavy quark spin flip suppressed by quark mass,
approximate heavy quark spin symmetry (HQSS)

Multiscale system

Systematically integrate
out the heavy scale,

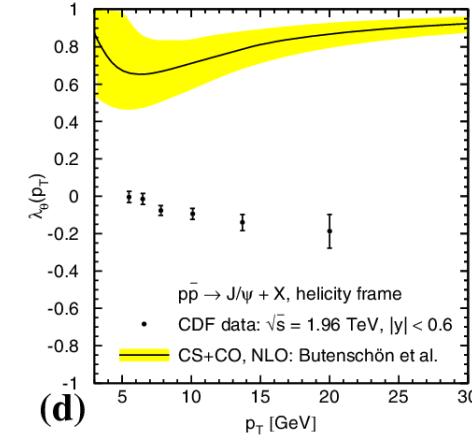
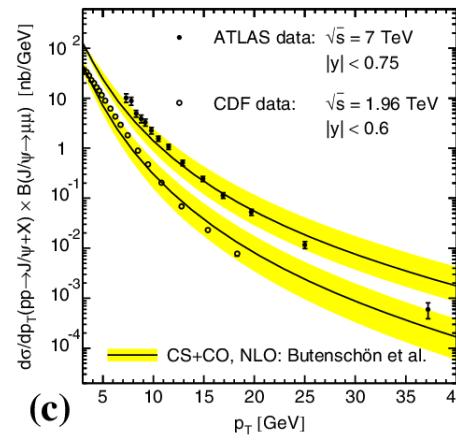
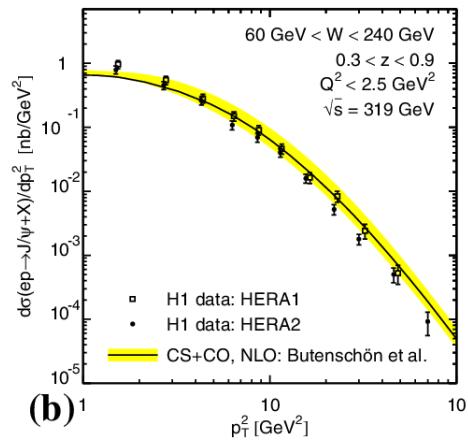
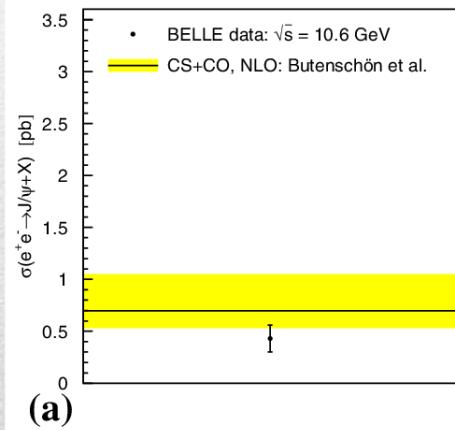
$$m_Q \gg \Lambda_{QCD}$$

$$m_Q \gg m_Q v \gg m_Q v^2$$

Full QCD → NRQCD → pNRQCD

$$m_b \sim 5 \text{ GeV}, m_c \sim 1.5 \text{ GeV}$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

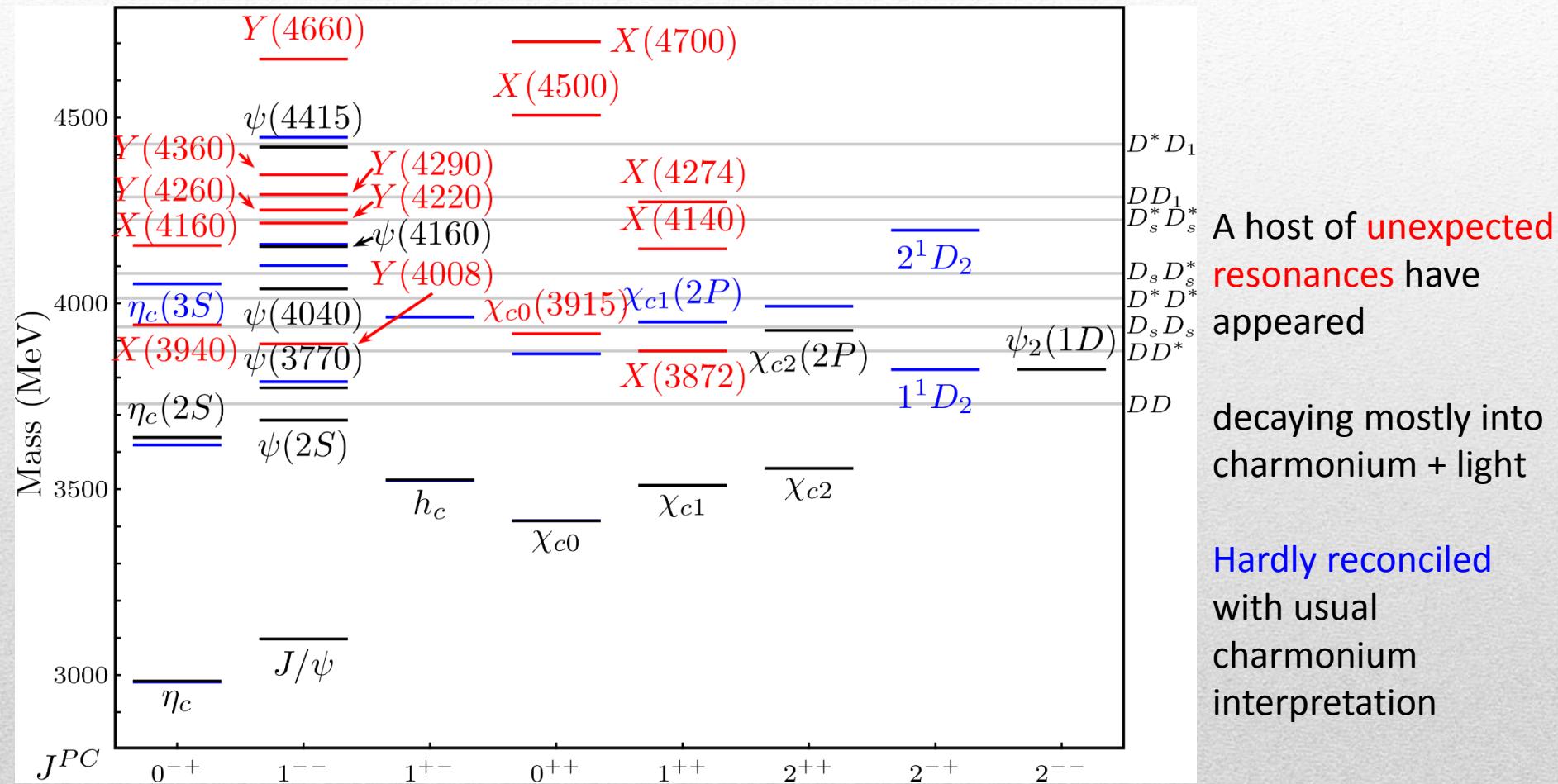


Factorization (to be proved)
of universal LDMEs

Good description of many production channels,
some known puzzles (polarizations)

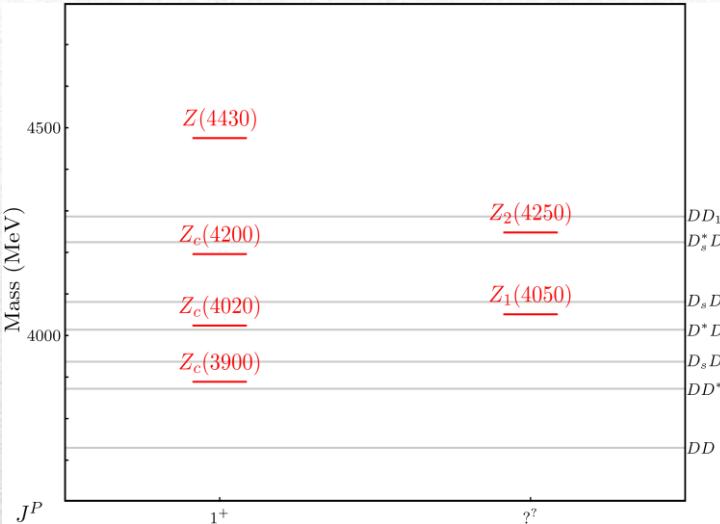
Exotic landscape

Esposito, AP, Polosa, Phys.Rept. 668



Charged Z states: $Z_c(3900)$, $Z'_c(4020)$

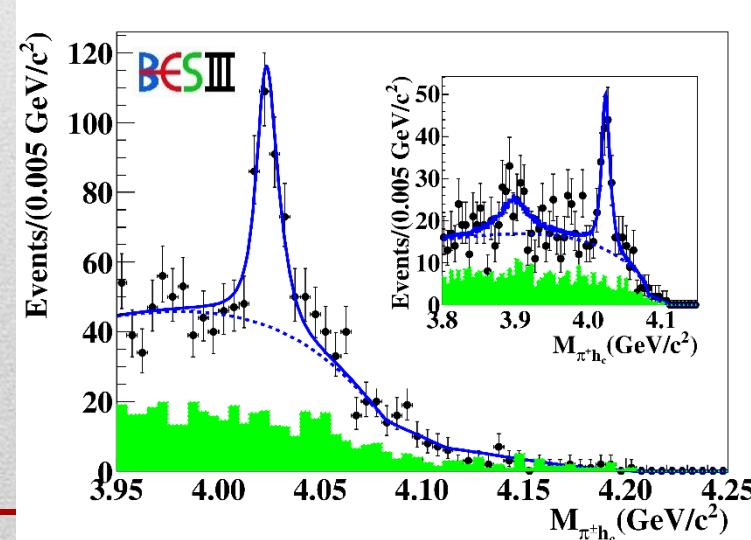
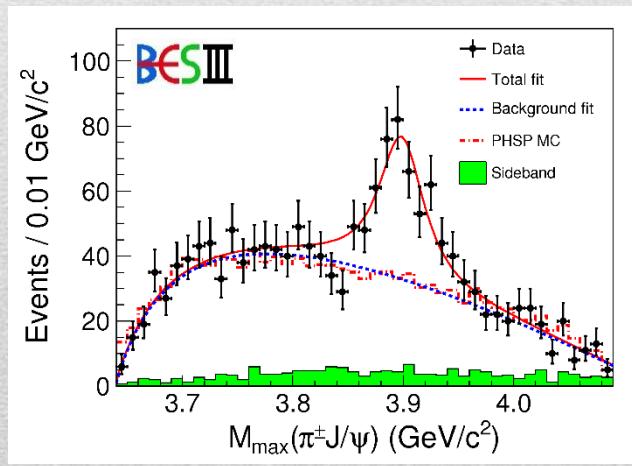
Charged quarkonium-like resonances have been found, **4q needed**



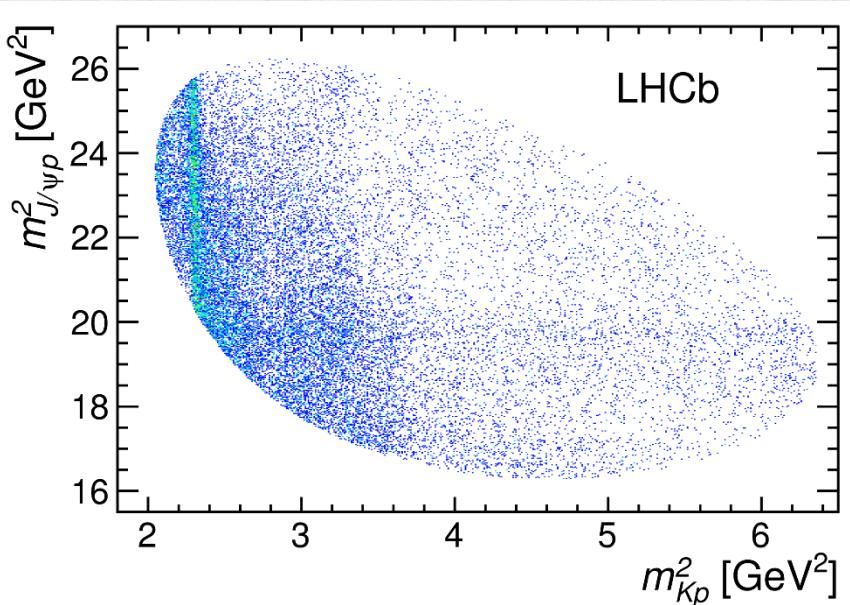
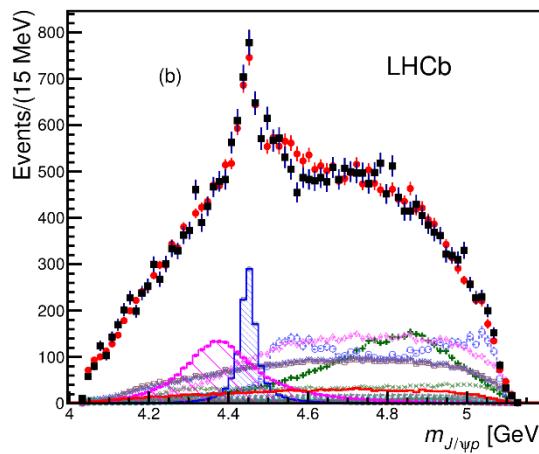
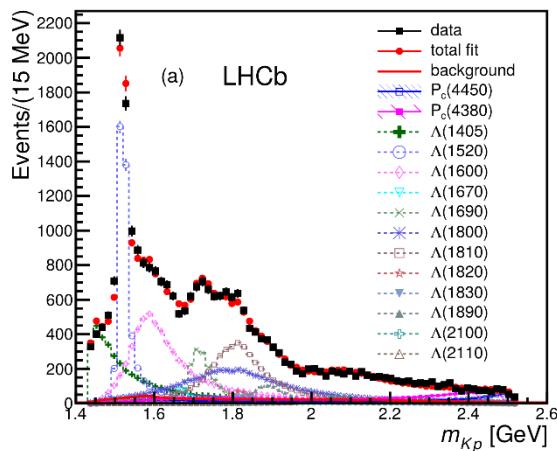
Two states $J^{PC} = 1^{+-}$ appear slightly above $D^{(*)}D^*$ thresholds

$e^+e^- \rightarrow Z_c(3900)^+\pi^- \rightarrow J/\psi\pi^+\pi^-$ and $\rightarrow (DD^*)^+\pi^-$
 $M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$

$e^+e^- \rightarrow Z'_c(4020)^+\pi^- \rightarrow h_c\pi^+\pi^-$ and $\rightarrow \bar{D}^{*0}D^{*+}\pi^-$
 $M = 4023.9 \pm 2.4 \text{ MeV}, \Gamma = 10 \pm 6 \text{ MeV}$



Pentaquarks!



LHCb, PRL 115, 072001
 LHCb, PRL 117, 082003

Two states seen in $\Lambda_b \rightarrow (J/\psi p) K^-$,
 evidence in $\Lambda_b \rightarrow (J/\psi p) \pi^-$
 $M_1 = 4380 \pm 8 \pm 29$ MeV
 $\Gamma_1 = 205 \pm 18 \pm 86$ MeV
 $M_2 = 4449.8 \pm 1.7 \pm 2.5$ MeV
 $\Gamma_2 = 39 \pm 5 \pm 19$ MeV

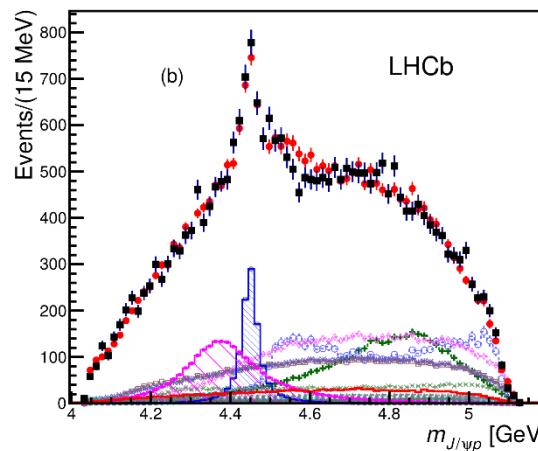
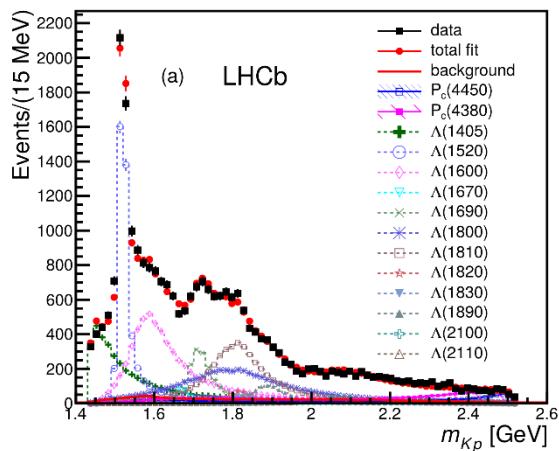
Quantum numbers

$$J^P = \left(\frac{3}{2}^-, \frac{5}{2}^+\right) \text{ or } \left(\frac{3}{2}^+, \frac{5}{2}^-\right) \text{ or } \left(\frac{5}{2}^+, \frac{3}{2}^-\right)$$

Opposite parities needed for the
 interference to correctly describe angular
 distributions, low mass region
 contaminated by Λ^* (model dependence?)

No obvious threshold nearby

Pentaquarks!



LHCb, PRL 115, 072001
LHCb, PRL 117, 082003

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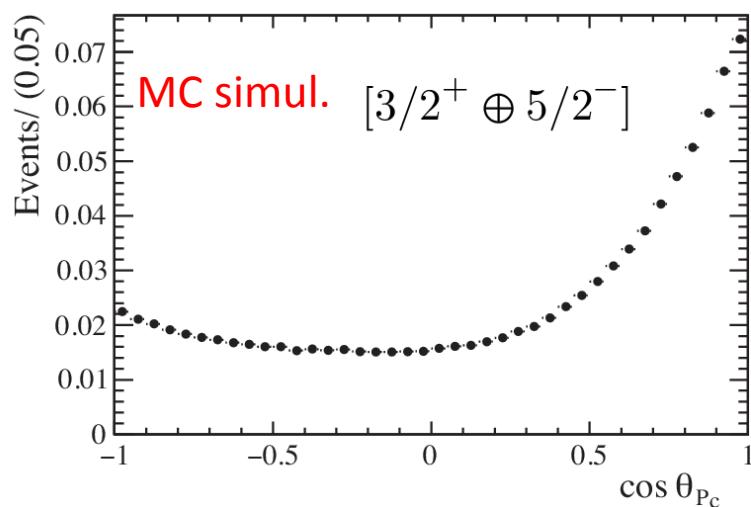
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Opposite parities needed for the interference to correctly describe angular distributions, low mass region contaminated by Λ^* (model dependence?)

No obvious threshold nearby



Life is not easy...

*a*₁(1260) WIDTH

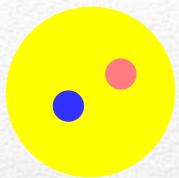
INSPIRE search

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
250 to 600	OUR ESTIMATE			
$367 \pm 9^{+28}_{-25}$	420k	ALEKSEEV	2010 COMP	$190 \pi^- \rightarrow \pi^- \pi^- \pi^+ Pb'$
... We do not use the following data for averages, fits, limits, etc. ...				
$410 \pm 31 \pm 30$		1 AUBERT	2007AU BABR	$10.6 e^+ e^- \rightarrow \rho^0 \rho^\pm \pi^\mp \gamma$
520 - 680	6360	2 LINK	2007A FOCS	$D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$
480 ± 20		3 GOMEZ-DUMM	2004 RVUE	$\tau^+ \rightarrow \pi^+ \pi^+ \pi^- \nu_\tau$
580 ± 41	90k	SALVINI	2004 OBLX	$\bar{p} p \rightarrow 2 \pi^+ 2 \pi^-$
460 ± 85	205	4 DRUTSKOY	2002 BELL	$B^{(*)} K^- K^{*0}$
$814 \pm 36 \pm 13$	37k	5 ASNER	2000 CLE2	$10.6 e^+ e^- \rightarrow \tau^+ \tau^- , \tau^- \rightarrow \pi^- \pi^0 \pi^0 \nu_\tau$

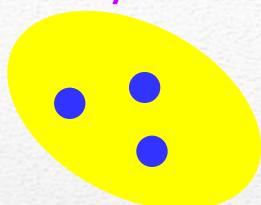


Building Hadrons

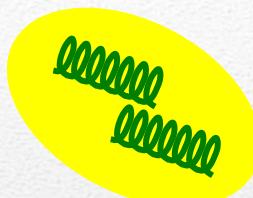
Meson



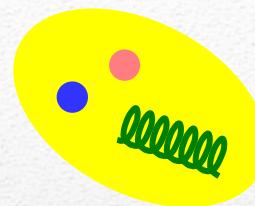
Baryon



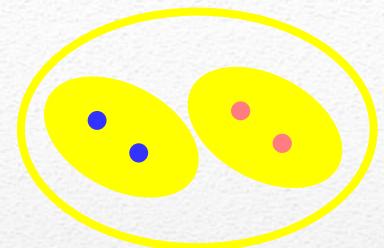
Glueball



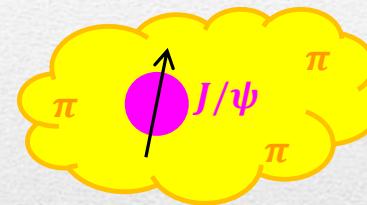
Hybrids



Tetraquark



Molecule



Hadroquarkonium

Experiment

Lattice QCD

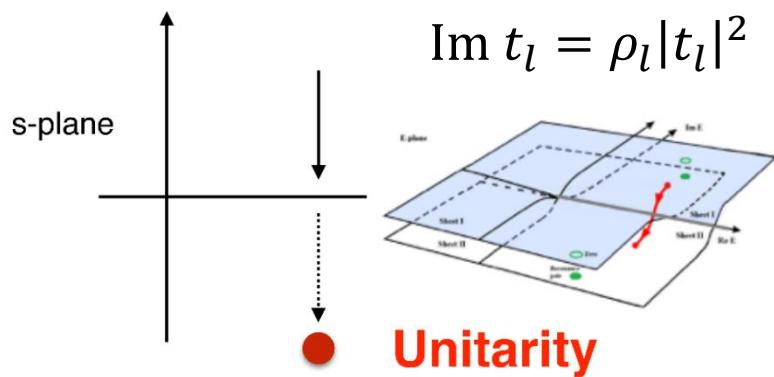
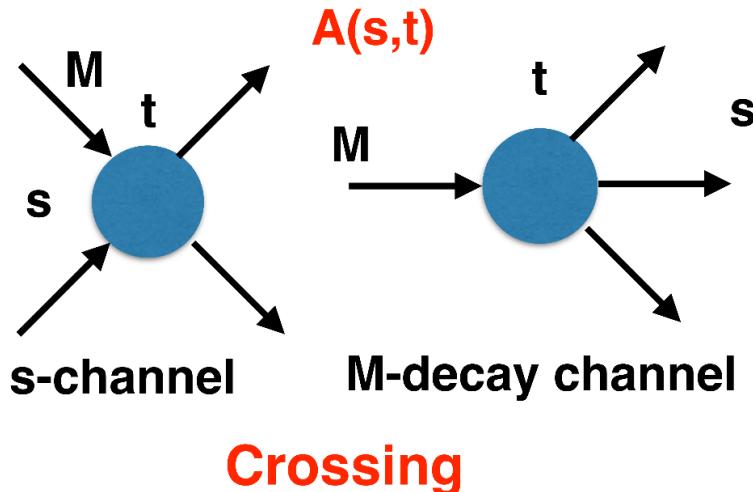
Data

Amplitude analysis

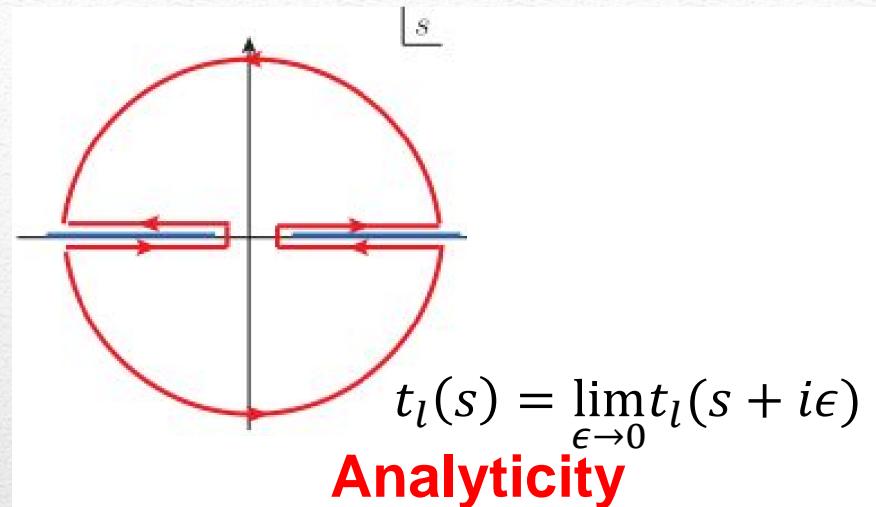
Properties,
Model building

Interpretations on the spectrum leads to
understanding fundamental laws of nature

S -Matrix principles



+ Lorentz, discrete & global symmetries

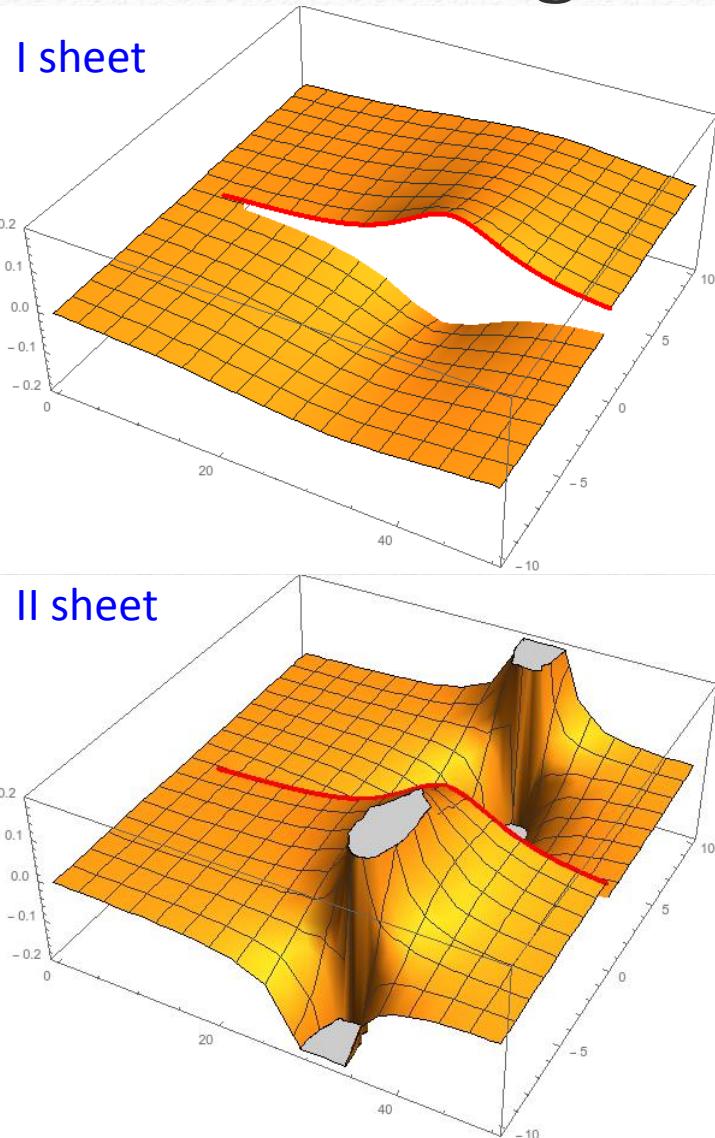


These are **constraints** the amplitudes have to satisfy, but **do not fix the dynamics**

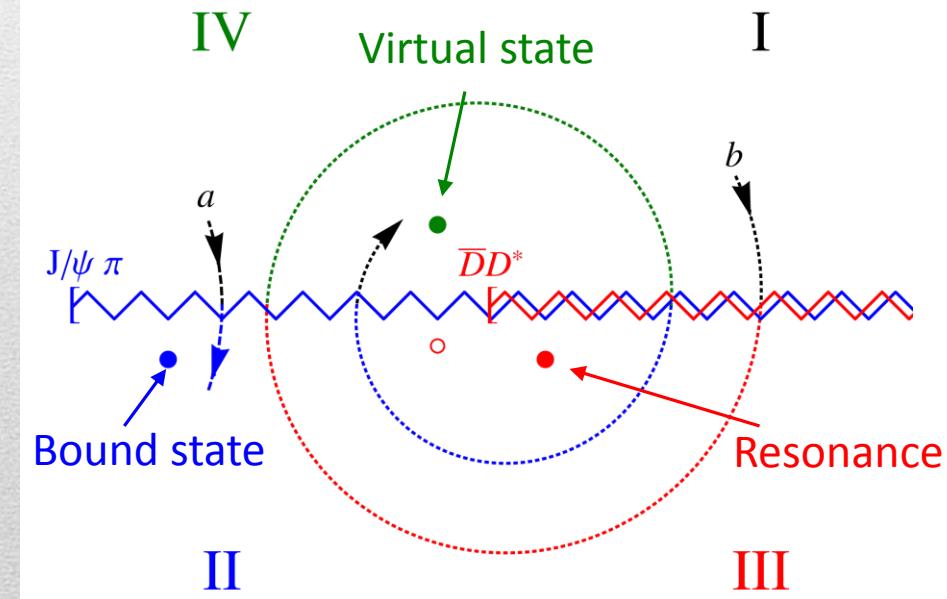
They can be imposed with an **increasing amount of rigor**, to extract robust physics information

The «background» phenomena can be effectively parameterized in a **controlled way**

Pole hunting



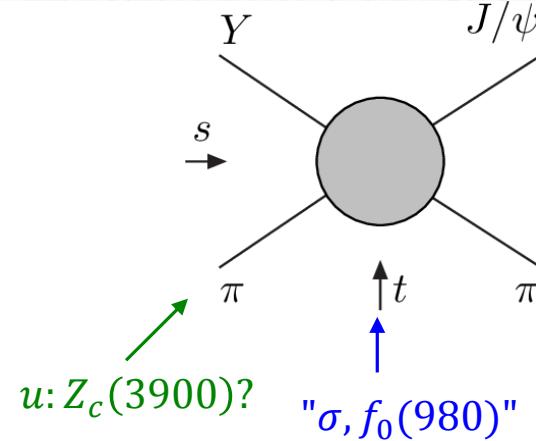
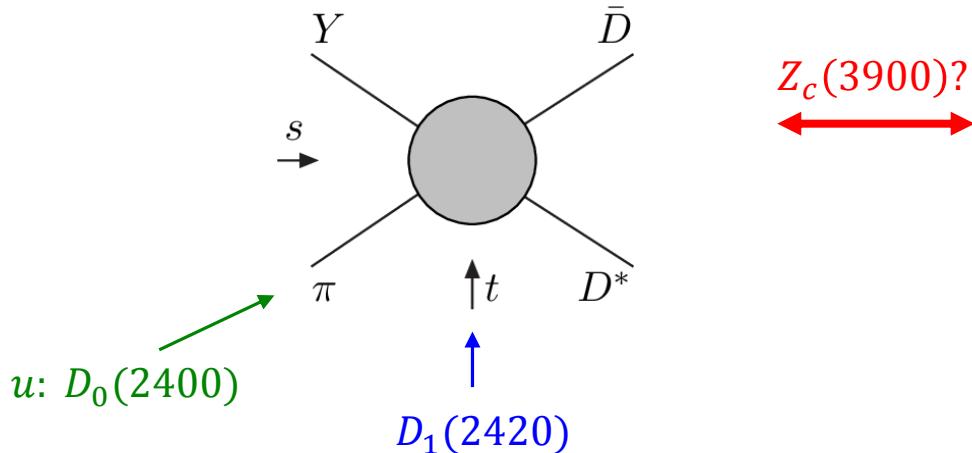
- Resonances are identified as **poles** in the unphysical Riemann sheets.
- The analytic structure (lineshape) can also provide **hints** about their **nature**
- Analytic continuation is always **model-dependent**, and requires careful systematic checks



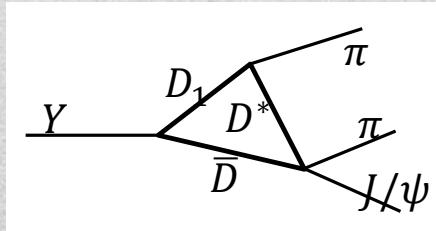
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to different singularities → different natures

AP et al. (JPAC), PLB772, 200



Triangle rescattering,
logarithmic branching point



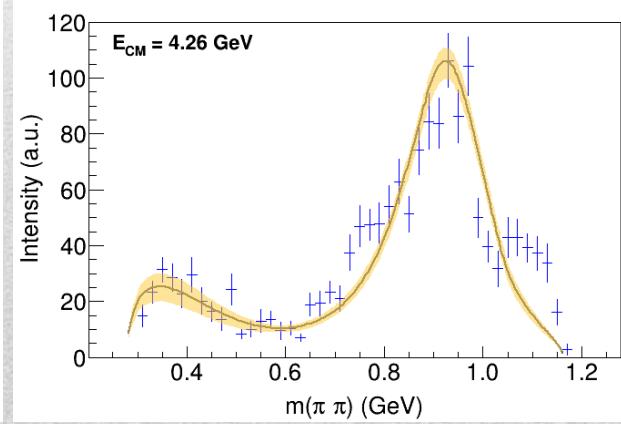
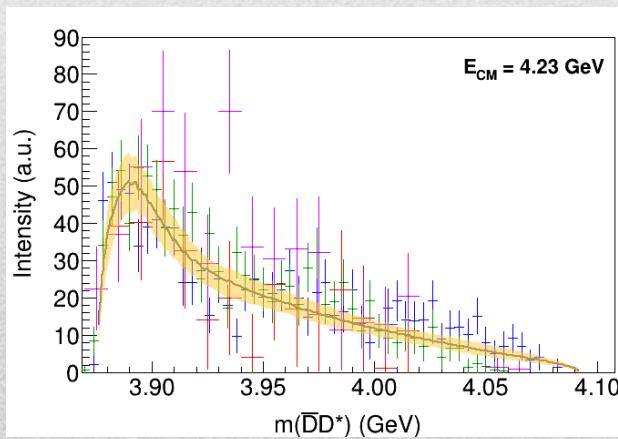
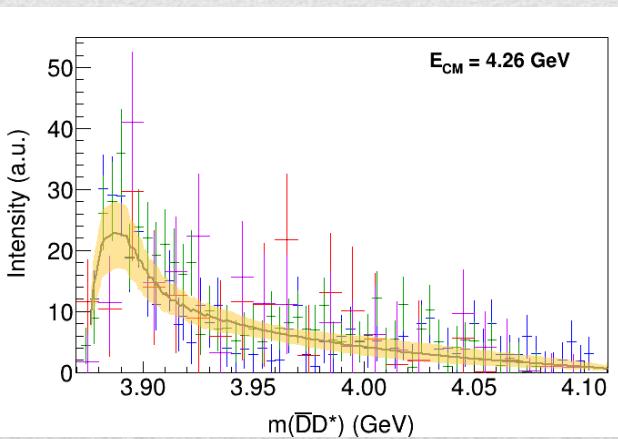
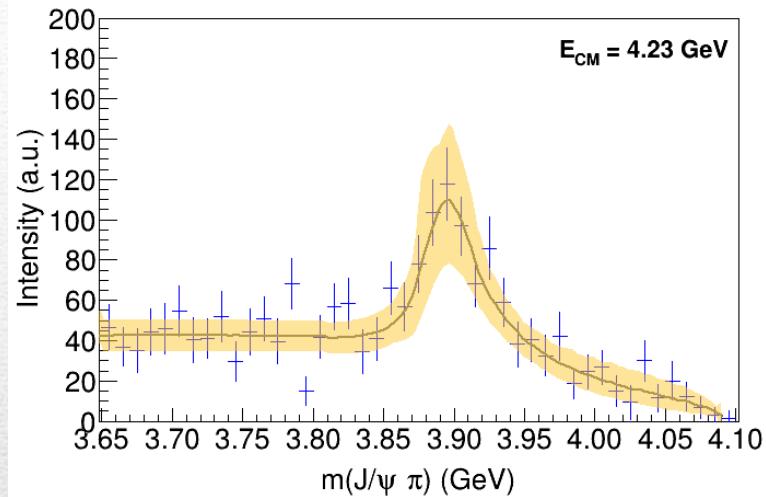
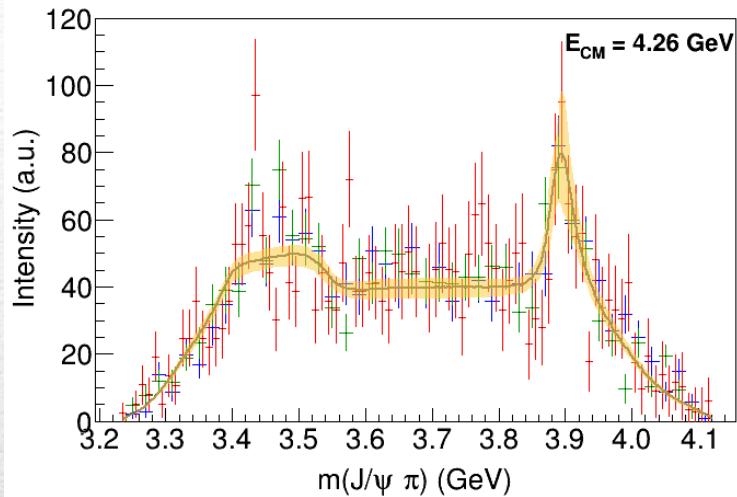
Szczepaniak, PLB747, 410

(anti)bound state,
II/IV sheet pole
("molecule")
Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart et al. PRL111, 132003

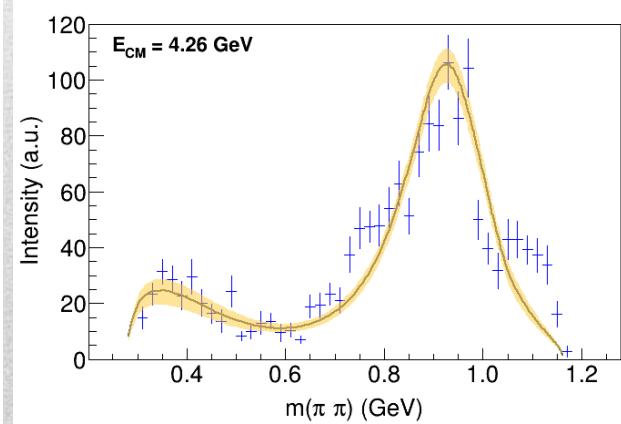
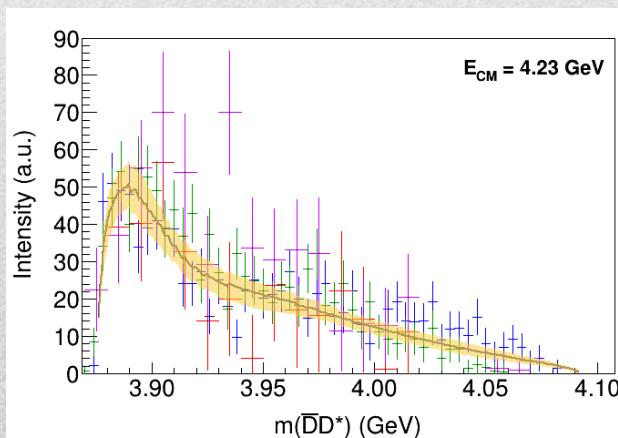
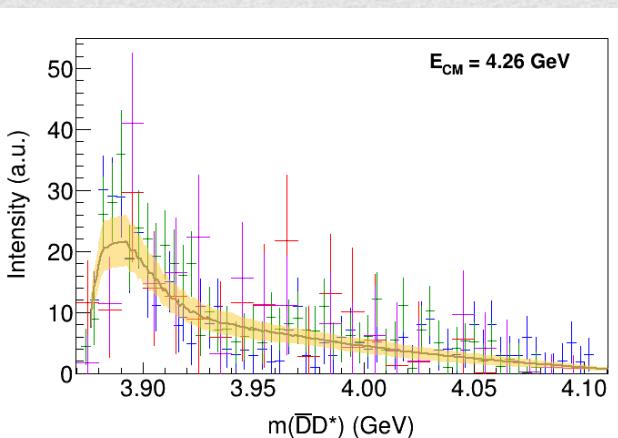
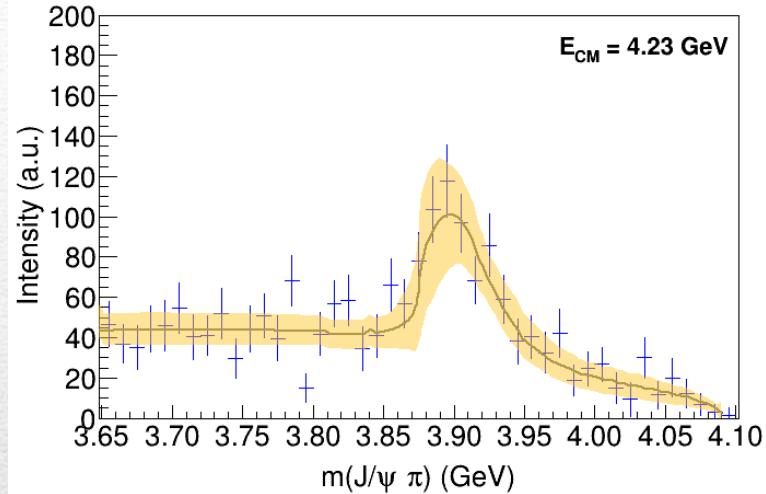
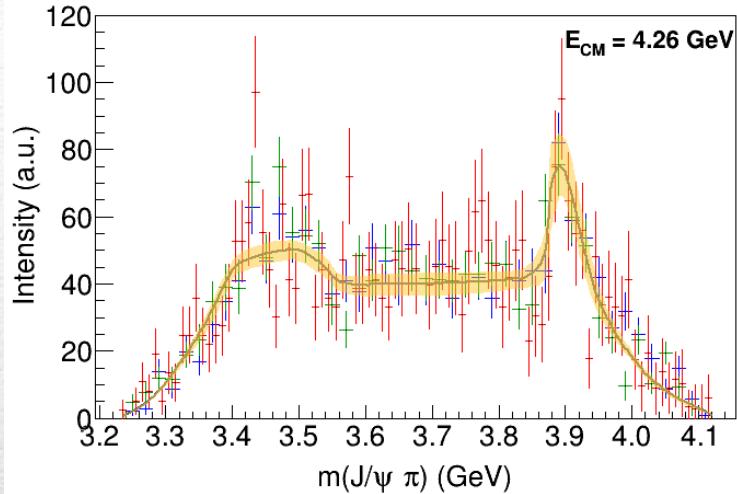
Resonance,
III sheet pole
("compact state")

Maiani et al., PRD71, 014028
Faccini et al., PRD87, 111102
Esposito et al., Phys.Rept. 668

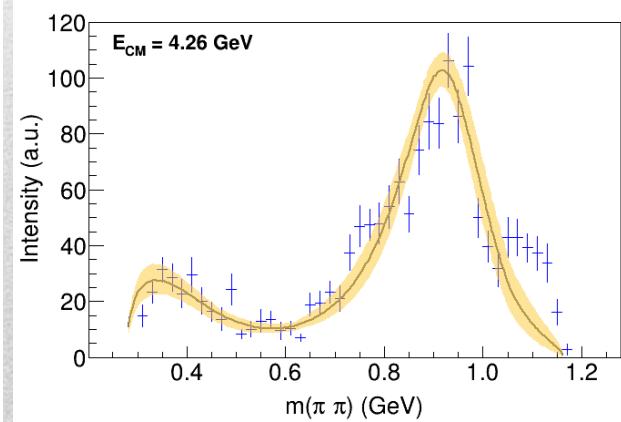
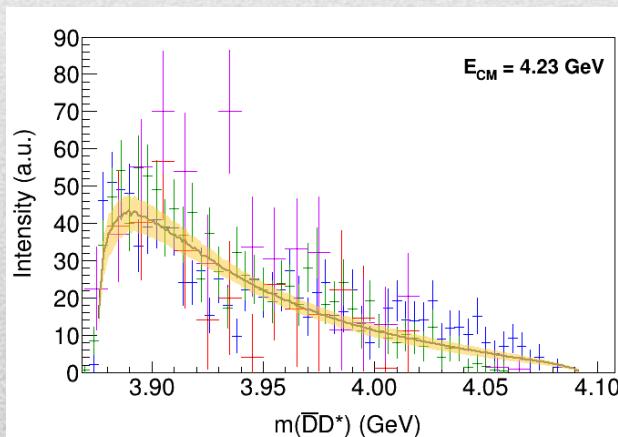
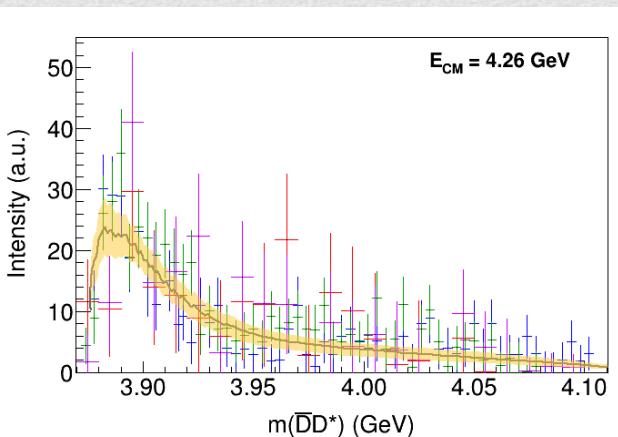
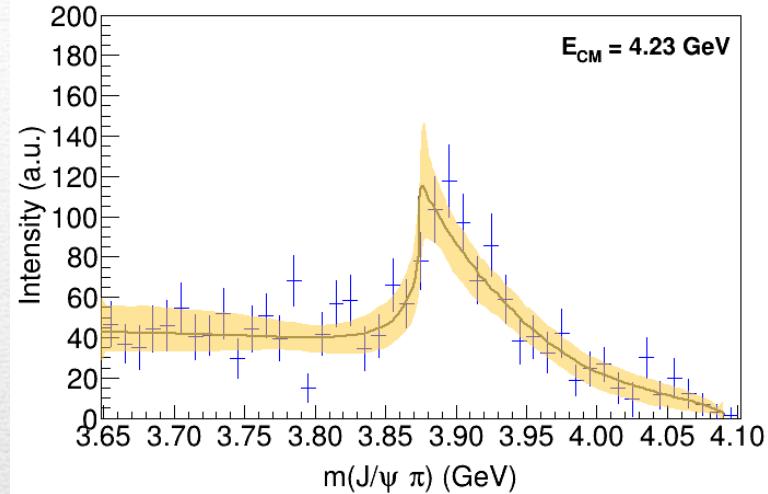
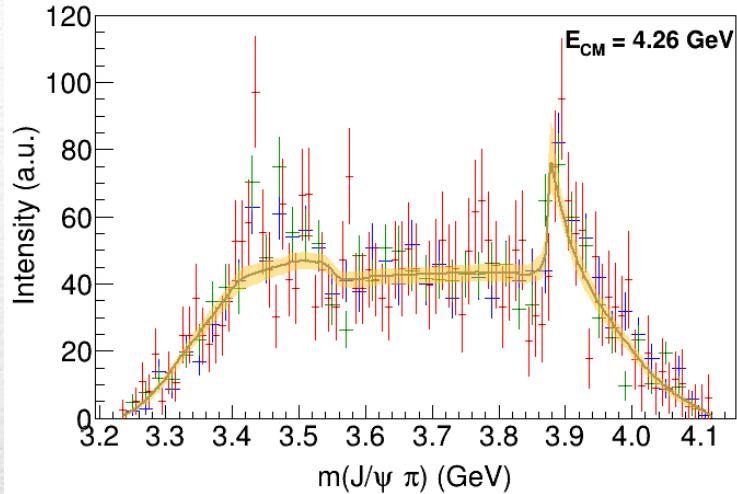
Fit: III



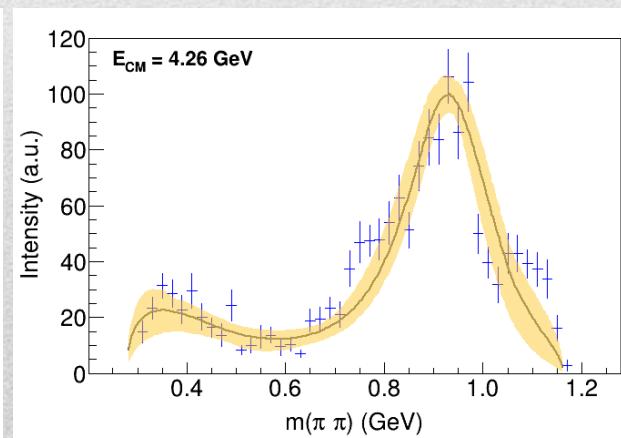
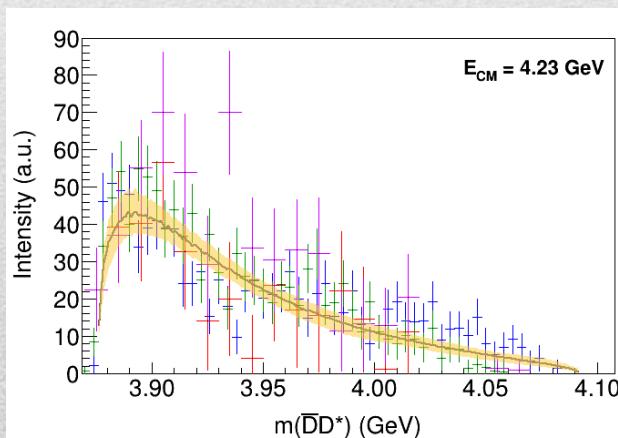
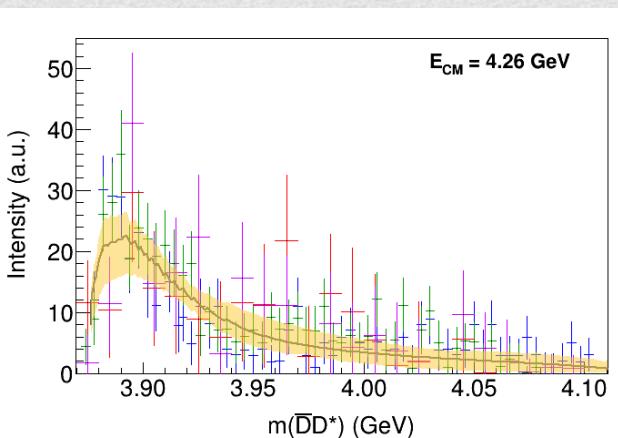
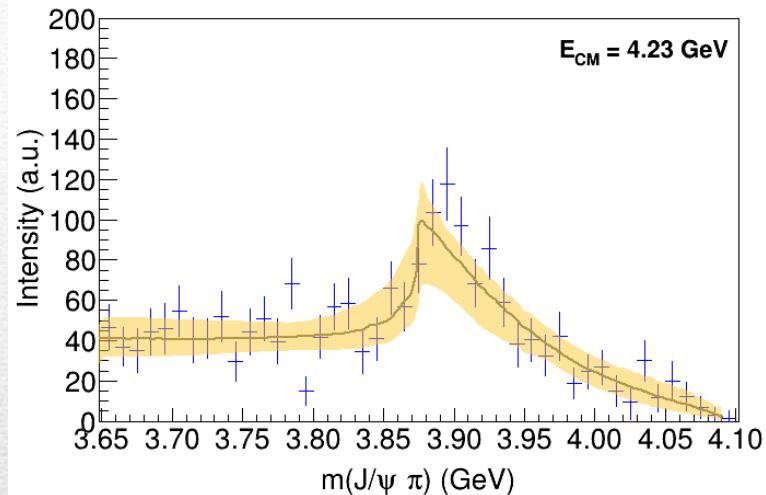
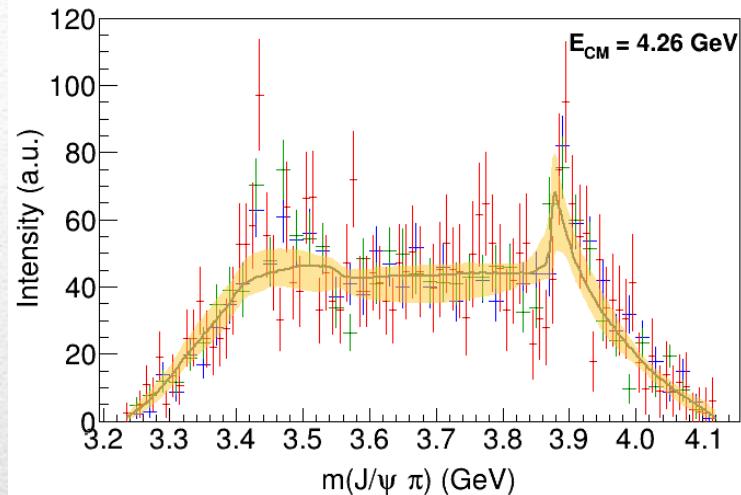
Fit: III+tr.



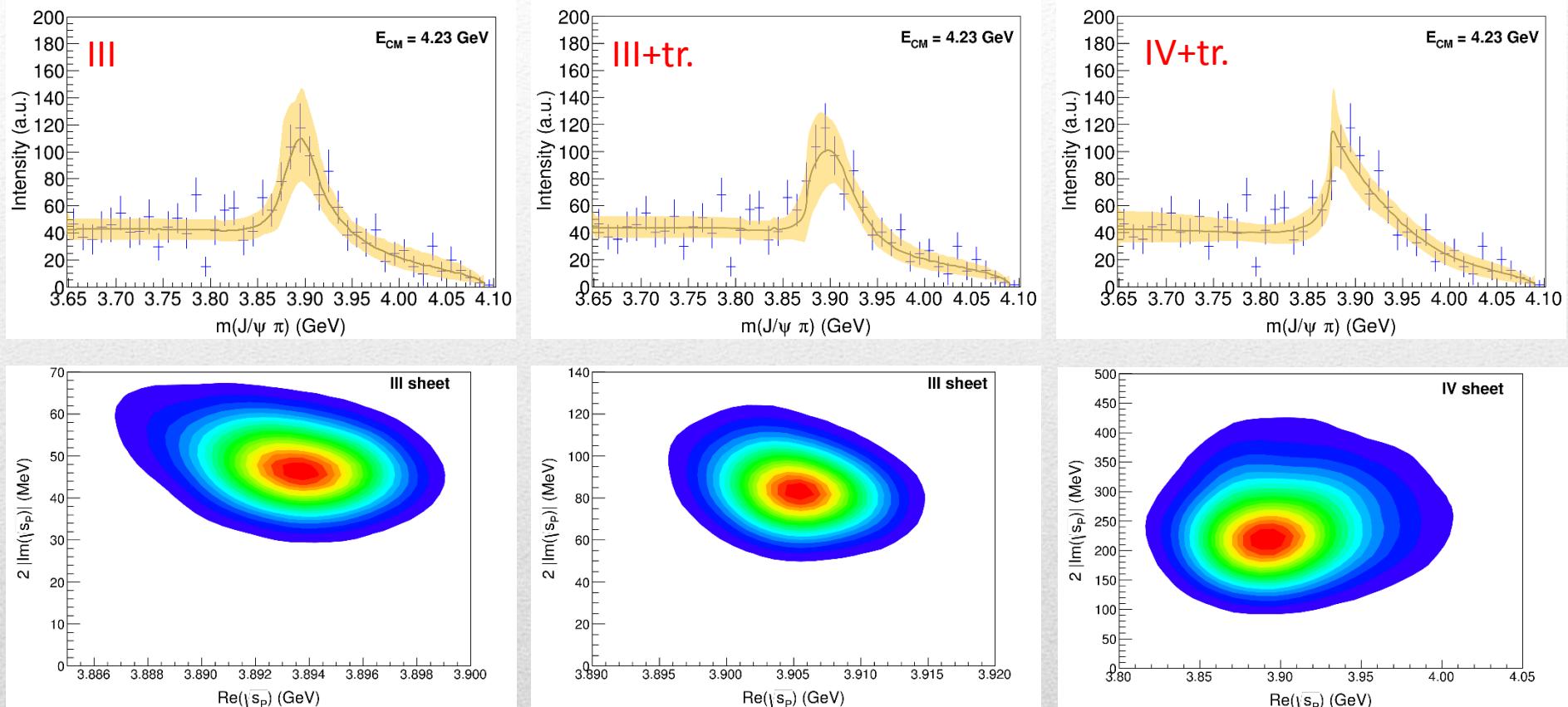
Fit: IV+tr.



Fit: tr.



Pole extraction

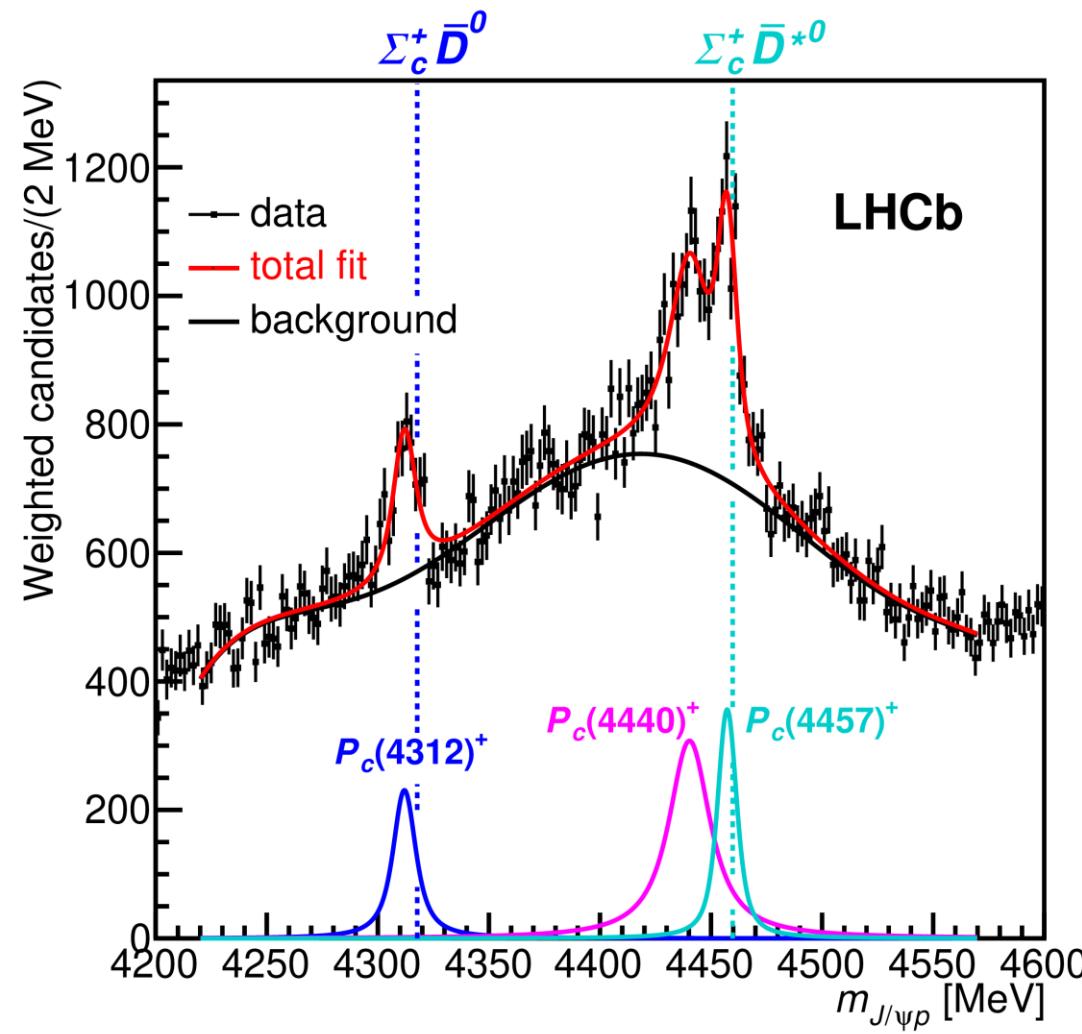


Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	“ 2.4σ ” (“ 1.4σ ”)
III+tr.	–	1.5σ (3.1σ)	“ 2.6σ ” (“ 1.3σ ”)
IV+tr.	–	–	“ 2.1σ ” (“ 0.9σ ”)

	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

Not conclusive at this stage

New pentaquarks discovered



LHCb, 1904.03947

The lowest $P_c(4312)$ appears as an isolated peak at the $\Sigma_c^+ \bar{D}^0$ threshold

A detailed study of the lineshape provides insight on its nature

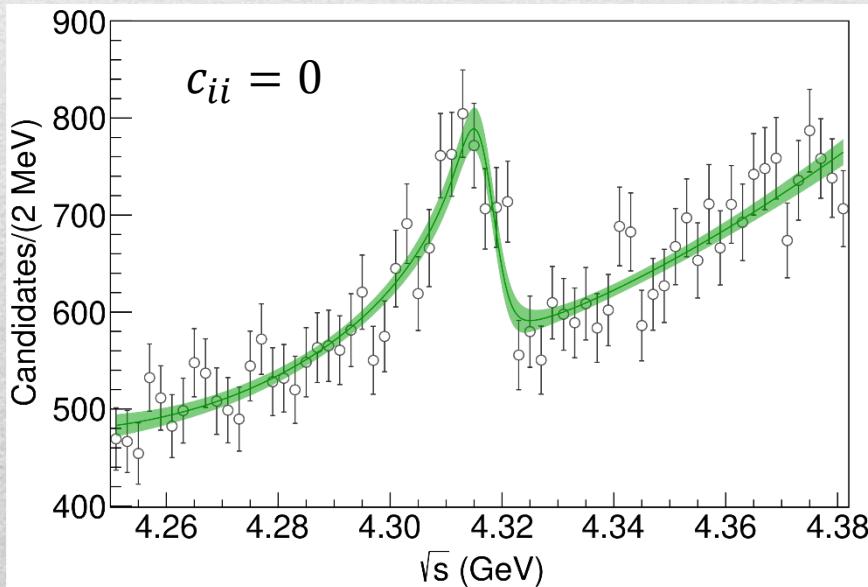
Minimal(istic) model

Fernandez-Ramirez, AP et al. (JPAC), 1904.10021

$$\frac{dN}{d\sqrt{s}} = \rho(s) [|F(s)|^2 + b_0 + b_1 s]$$

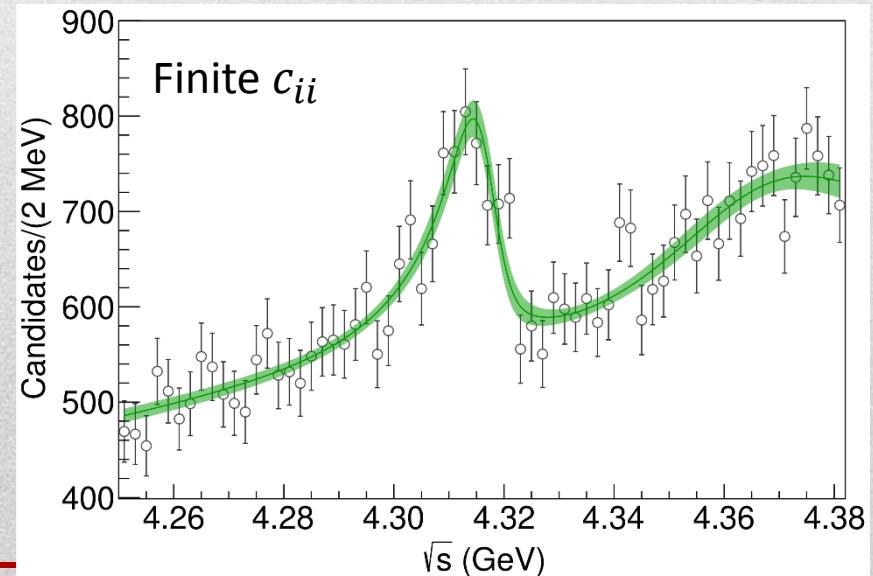
$$F(s) = (N_1 + N_2 s) T_{11}(s)$$

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$



We can set $c_{ii} = 0$ to reduce to the scattering length approximation

Effective range expansion



Minimal(istic) model

Fernandez-Ramirez, AP et al. (JPAC), 1904.10021

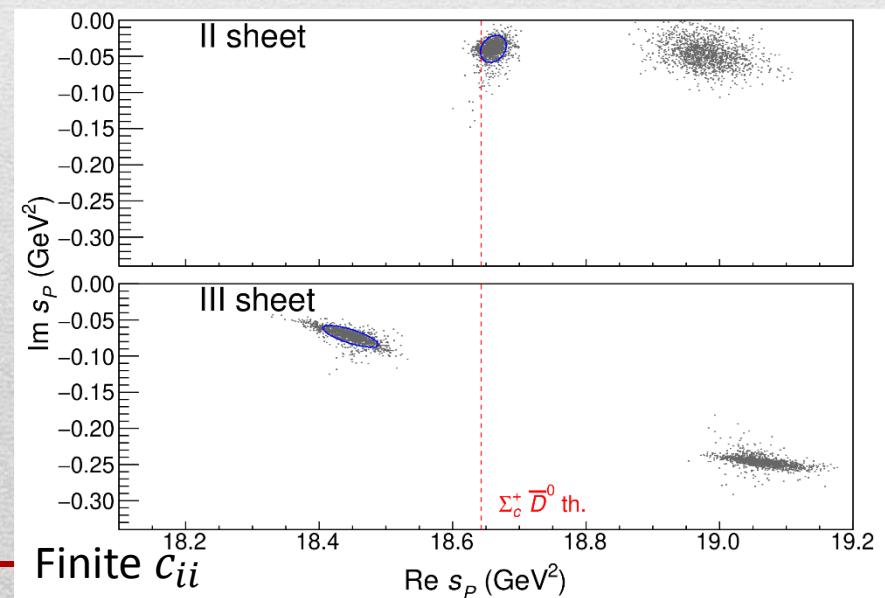
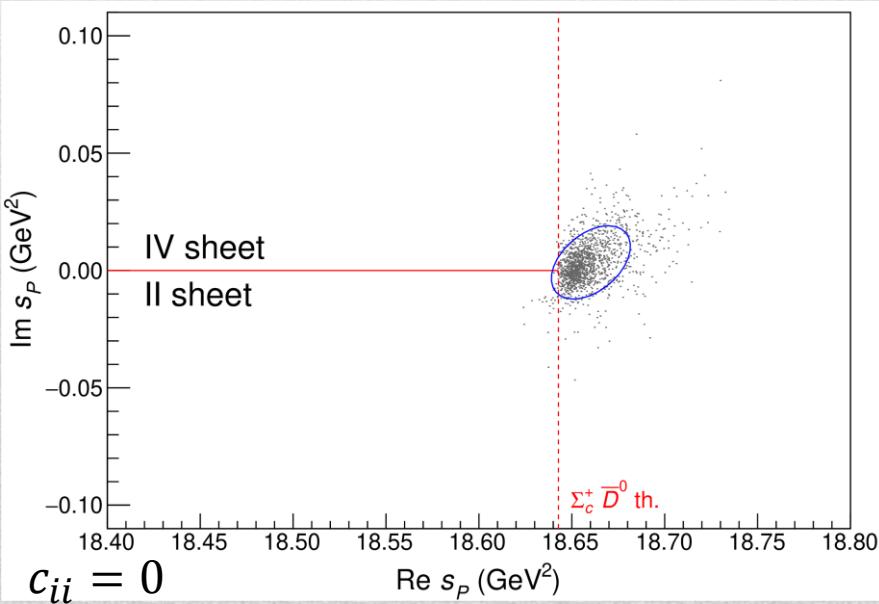
$$\frac{dN}{d\sqrt{s}} = \rho(s) [|F(s)|^2 + b_0 + b_1 s]$$

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Effective range expansion

We can set $c_{ii} = 0$ to reduce to the scattering length approximation



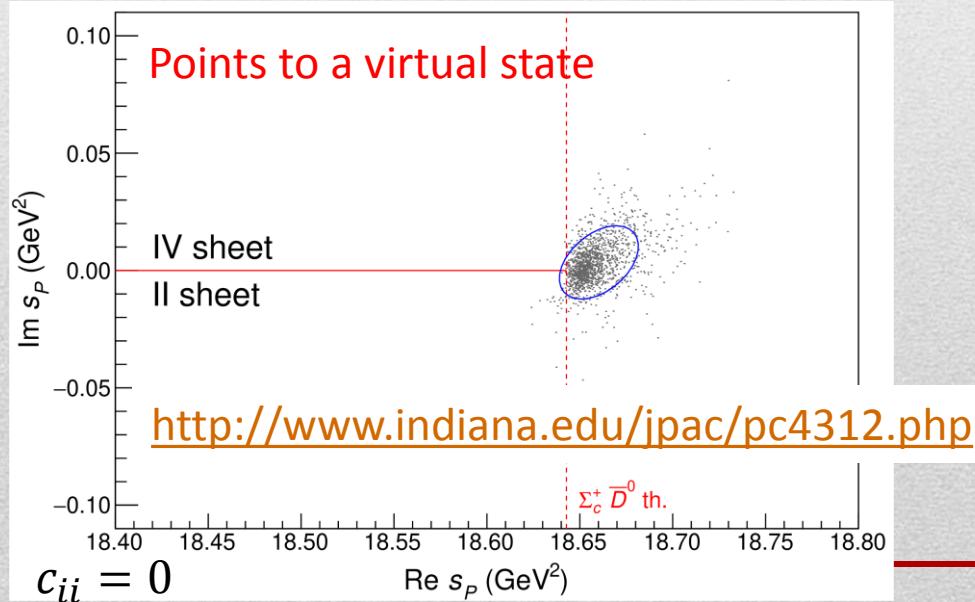
Minimal(istic) model

Fernandez-Ramirez, AP et al. (JPAC), 1904.10021

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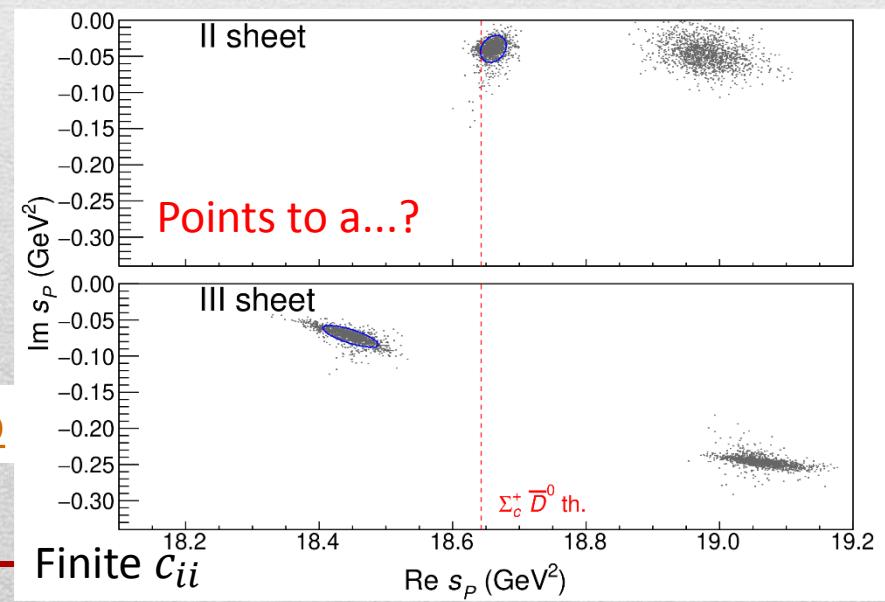
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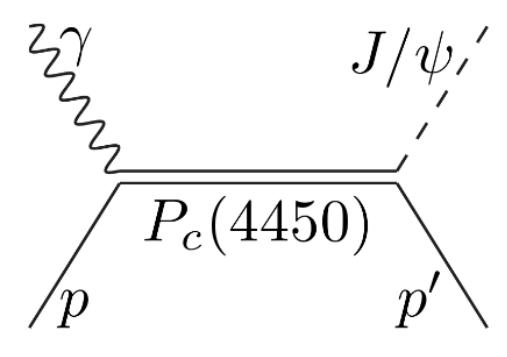
Effective range expansion

We can set $c_{ii} = 0$ to reduce to the scattering length approximation



P_c photoproduction

To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in photoproduction.



$$\langle \lambda_\psi \lambda_{p'} | T_r | \lambda_\gamma \lambda_p \rangle = \frac{\text{Hadronic vertex}}{M_r^2 - W^2 - i\Gamma_r M_r} \frac{\text{EM vertex}}{\langle \lambda_R | T_{\text{em}}^\dagger | \lambda_\gamma \lambda_p \rangle}$$

Hadronic part

- 3 independent helicity couplings,
→ approx. equal, $g_{\lambda_\psi, \lambda_{p'}} \sim g$
- g extracted from total width and (unknown)
branching ratio

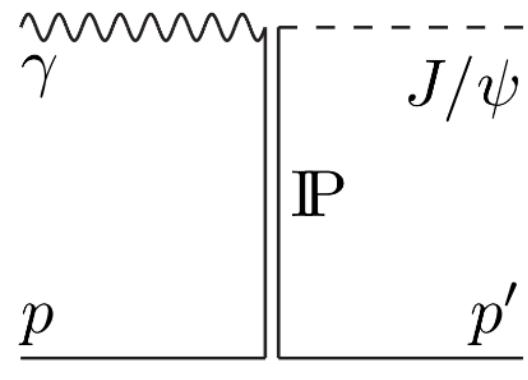
Vector meson dominance
relates the radiative width to the
hadronic width

$$\Gamma_\gamma = 4\pi\alpha \Gamma_{\psi p} \left(\frac{f_\psi}{M_\psi}\right)^2 \left(\frac{\bar{p}_i}{\bar{p}_f}\right)^{2\ell+1} \times \frac{4}{6}$$

Hiller Blin, AP et al. (JPAC), PRD94, 034002

Background parameterization

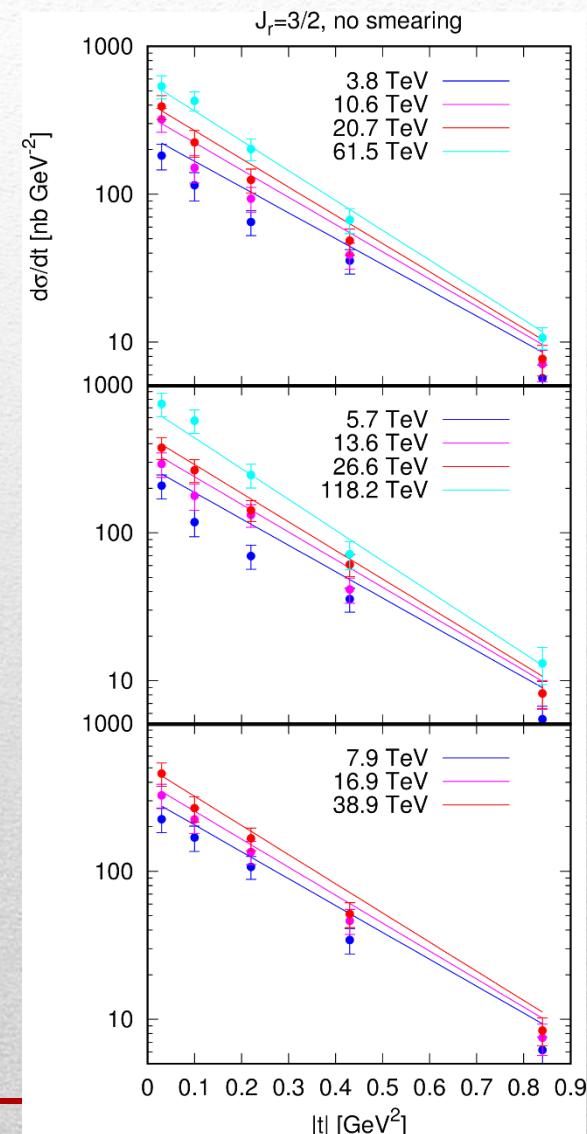
The background is described via an **Effective Pomeron**, whose parameters are fitted to high energy data from Hera



$$\langle \lambda_\psi \lambda_{p'} | T_P | \lambda_\gamma \lambda_p \rangle = iA \left(\frac{s - s_t}{s_0} \right)^{\alpha(t)} e^{b_0(t-t_{\min})} \delta_{\lambda_p \lambda_{p'}} \delta_{\lambda_\psi \lambda_\gamma}$$

Asymptotic + Effective threshold

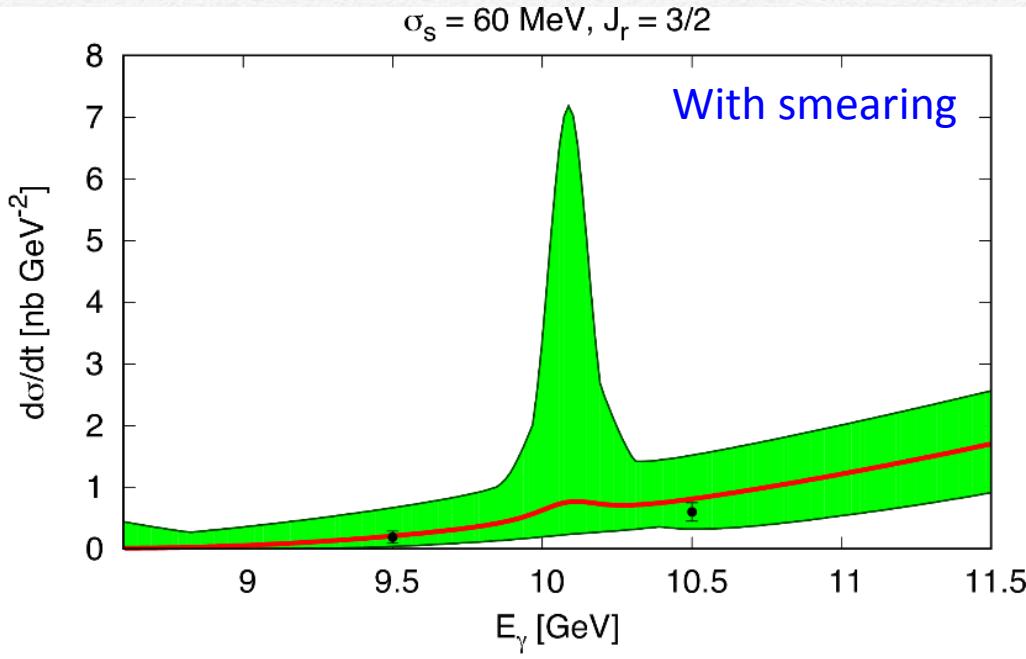
Helicity conservation



Hiller Blin, AP et al. (JPAC), PRD94, 034002

Pentaquark photoproduction

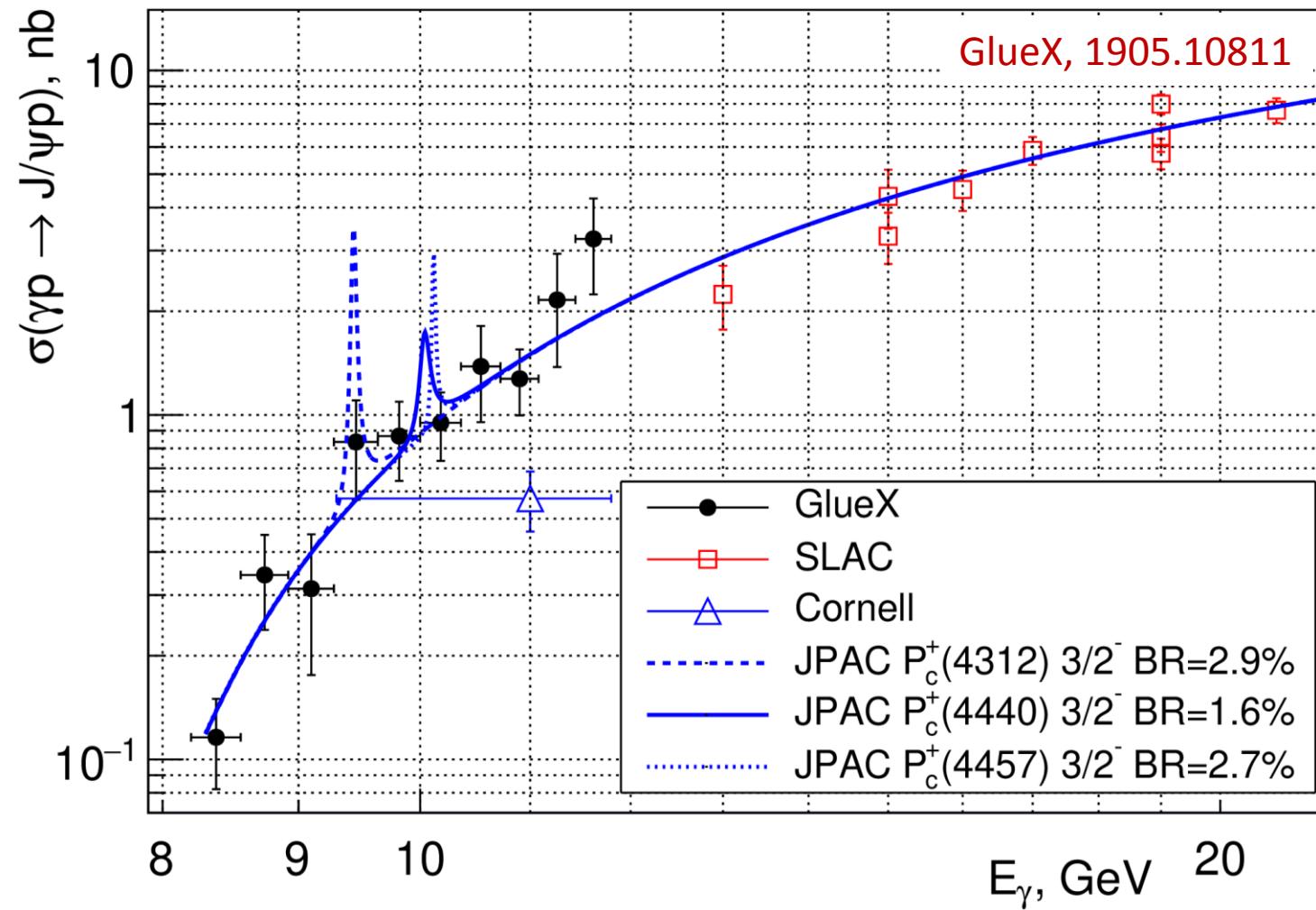
$$J^P = (3/2)^-$$



σ_s (MeV)	0	60
A	$0.156^{+0.029}_{-0.020}$	$0.157^{+0.039}_{-0.021}$
α_0	$1.151^{+0.018}_{-0.020}$	$1.150^{+0.018}_{-0.026}$
α' (GeV^{-2})	$0.112^{+0.033}_{-0.054}$	$0.111^{+0.037}_{-0.064}$
s_t (GeV^2)	$16.8^{+1.7}_{-0.9}$	$16.9^{+2.0}_{-1.6}$
b_0 (GeV^{-2})	$1.01^{+0.47}_{-0.29}$	$1.02^{+0.61}_{-0.32}$
$\mathcal{B}_{\psi p}$ (95% CL)	$\leq 29\%$	$\leq 30\%$

Hiller Blin, AP et al. (JPAC), PRD94, 034002

Photoproduction at GlueX



No evidence of pentaquark signals has been seen in GlueX, upper limits have been set

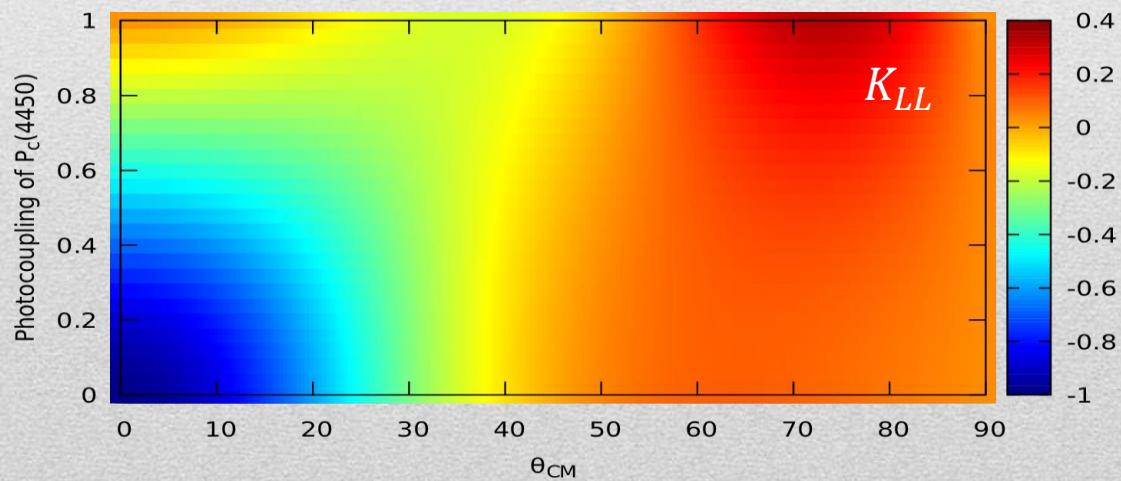
Polarization observables

One can take advantage of the polarized beam at JLab

High intensity beam in SBS (Hall A) looks promising

Need polarized target (A_{LL}) or polarization of recoiling proton (K_{LL})

$$A(K)_{LL} = \frac{1}{2} \left[\frac{d\sigma(++) - d\sigma(+-)}{d\sigma(++) + d\sigma(+-)} - \frac{d\sigma(-+) - d\sigma(--)}{d\sigma(-+) + d\sigma(--)} \right]$$



Fanelli, Pentchev, Wojtsekhowski,
Lol12-18-001
Winney, AP *et al.* (JPAC),
to appear

Epilogue

Studying exotic hadrons is challenging

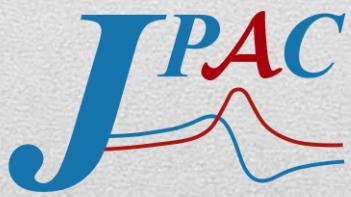
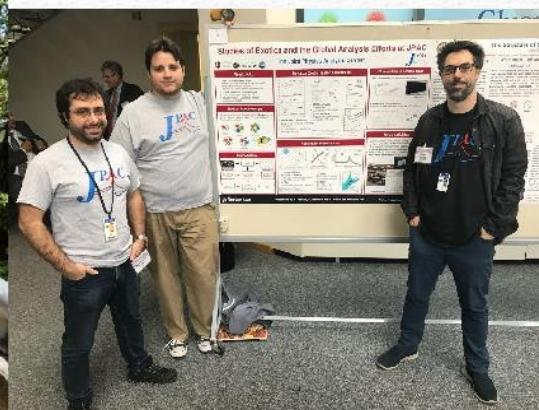
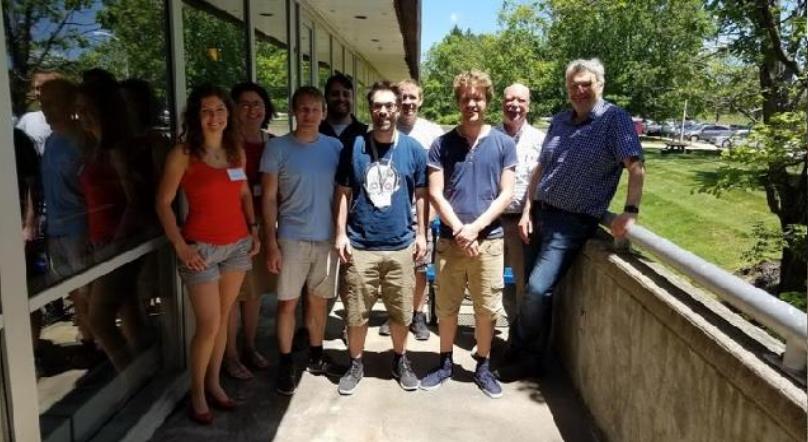
- They are related to **dynamics of QCD building blocks**
- They often appear as small fraction of cross sections, hard (but mandatory!) to find **consistency between different channels**
- Dispersive methods can improve the consistency, and offer insights into their nature

Thank you!

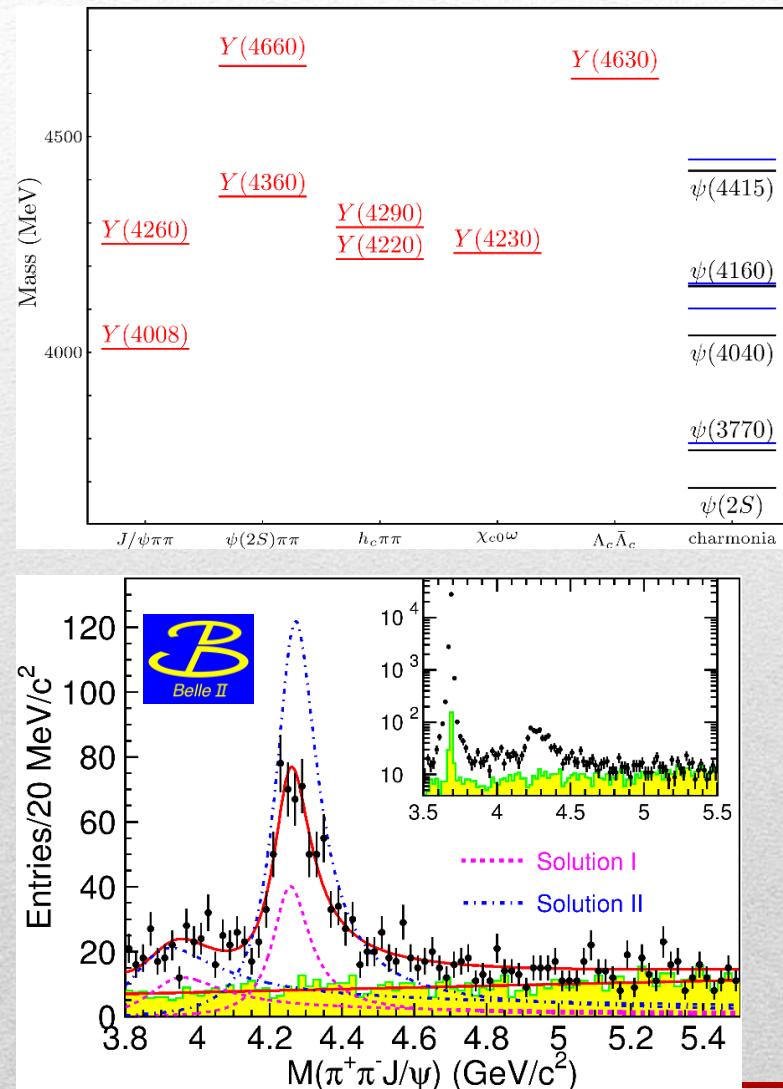
BACKUP



Joint Physics Analysis Center

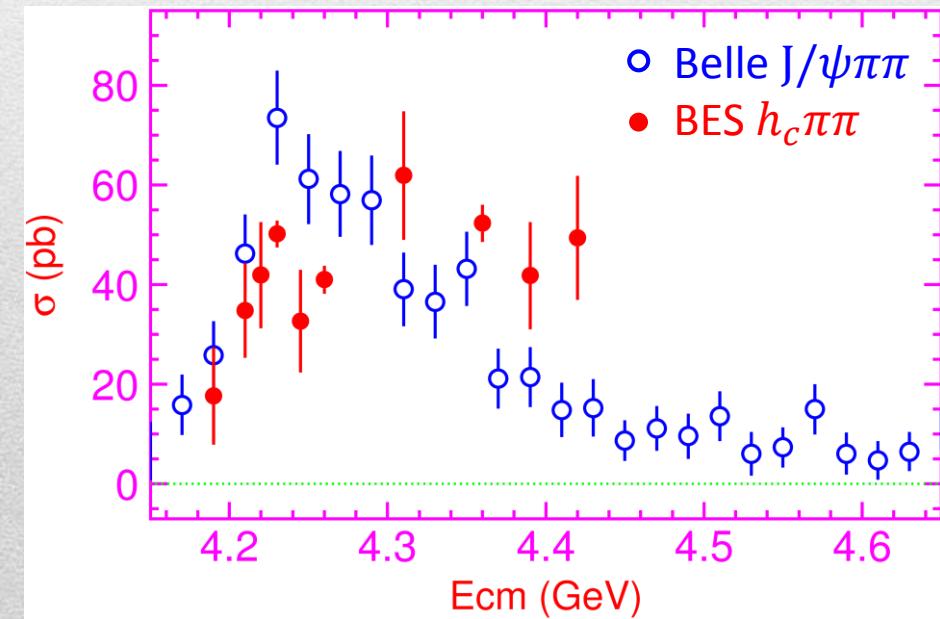


Vector Y states

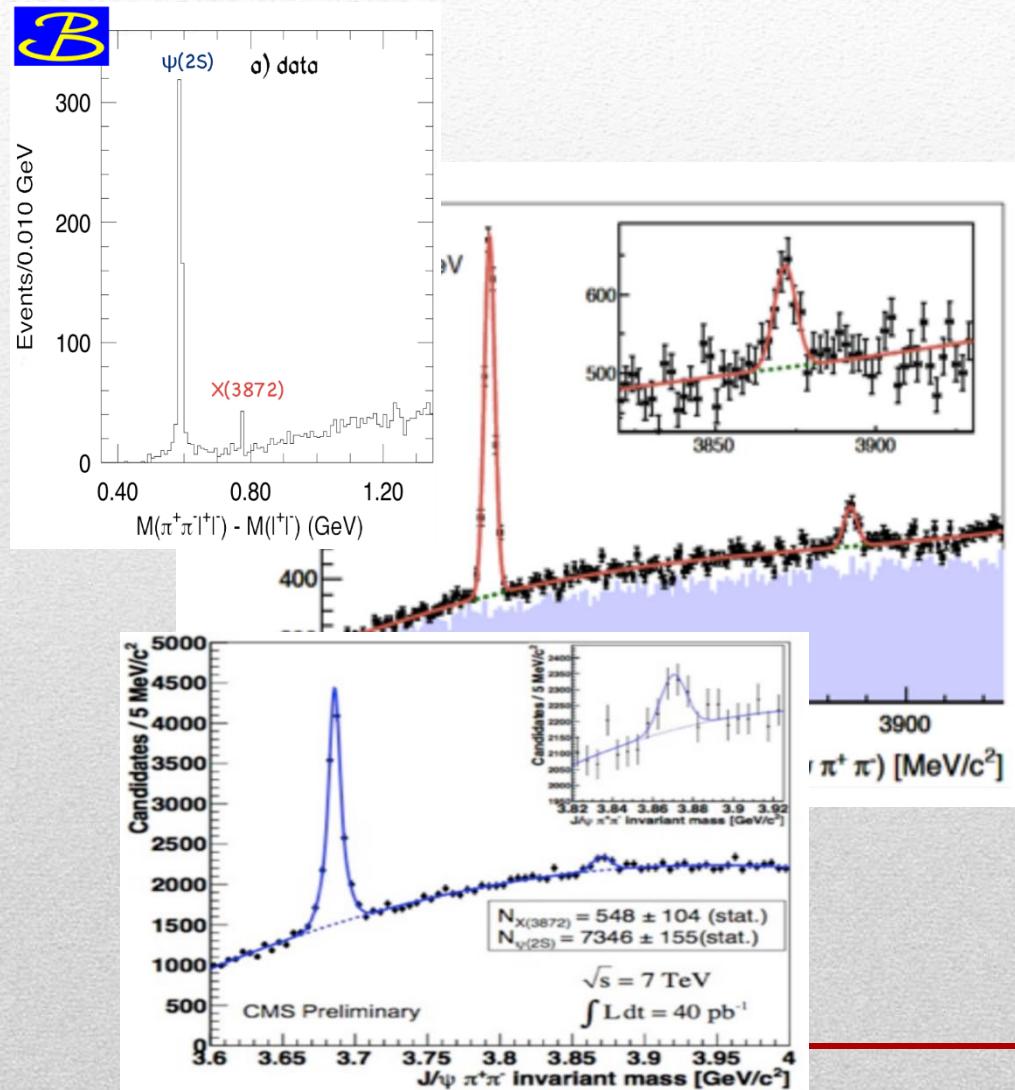


Lots of unexpected $J^{PC} = 1^{--}$ states found in ISR/direct production (and nowhere else!) Seen in few final states, mostly $J/\psi \pi\pi$ and $\psi(2S) \pi\pi$

Not seen decaying into open charm pairs
Large HQSS violation



$X(3872)$



- Discovered in $B \rightarrow K X \rightarrow K J/\psi \pi\pi$
- Quantum numbers 1^{++}
- Very close to DD^* threshold
- Too narrow for an above-threshold charmonium
- Isospin violation too big

$$\frac{\Gamma(X \rightarrow J/\psi \omega)}{\Gamma(X \rightarrow J/\psi \rho)} \sim 0.8 \pm 0.3$$
- Mass prediction not compatible with $\chi_{c1}(2P)$

$$M = 3871.68 \pm 0.17 \text{ MeV}$$

$$M_X - M_{DD^*} = -3 \pm 192 \text{ keV}$$

$$\Gamma < 1.2 \text{ MeV @90\%}$$

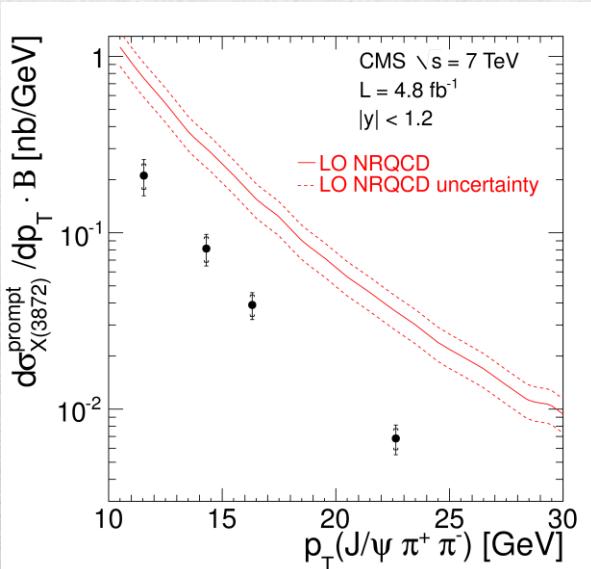
$X(3872)$

Large prompt production
at hadron colliders

$$\sigma_B/\sigma_{TOT} = (26.3 \pm 2.3 \pm 1.6)\%$$

$$\begin{aligned}\sigma_{PR} \times B(X \rightarrow J/\psi \pi\pi) \\ = (1.06 \pm 0.11 \pm 0.15) \text{ nb}\end{aligned}$$

CMS, JHEP 1304, 154



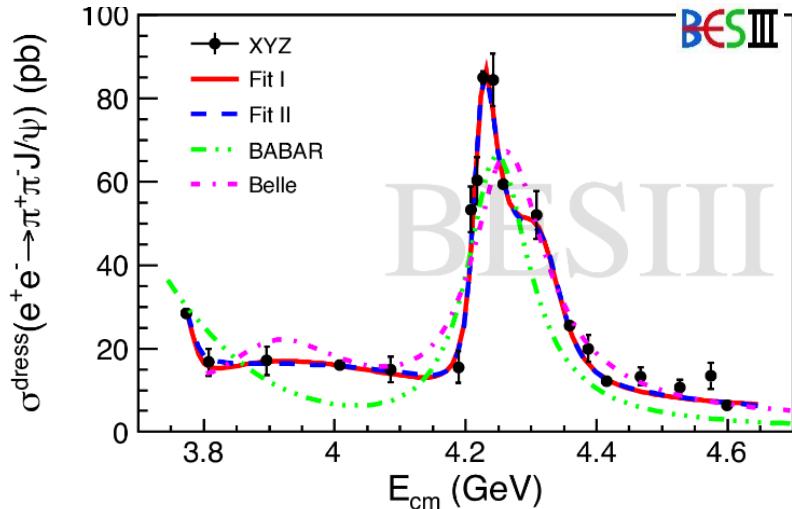
B decay mode	X decay mode	product branching fraction ($\times 10^5$)	B_{fit}	R_{fit}
$K^+ X$	$X \rightarrow \pi\pi J/\psi$	0.86 ± 0.08	(BABAR, ^[26] Belle ^[25])	0.081 ^{+0.019} _{-0.031}
		0.84 ± 0.15 ± 0.07		
		0.86 ± 0.08 ± 0.05		
$K^0 X$	$X \rightarrow \pi\pi J/\psi$	0.41 ± 0.11	(BABAR, ^[26] Belle ^[25])	0.77 ^{+0.28} _{-0.32}
		0.35 ± 0.19 ± 0.04		
		0.43 ± 0.12 ± 0.04		
$(K^+\pi^-)_{NR} X$	$X \rightarrow \pi\pi J/\psi$	$0.81 \pm 0.20^{+0.11}_{-0.14}$	Belle ^[106]	
$K^{*0} X$	$X \rightarrow \pi\pi J/\psi$	< 0.34, 90% C.L.	Belle ^[106]	
$K X$	$X \rightarrow \omega J/\psi$	$R = 0.8 \pm 0.3$	BABAR ^[33]	0.061 ^{+0.024} _{-0.036}
		0.6 ± 0.2 ± 0.1		
		0.6 ± 0.3 ± 0.1		
$K X$	$X \rightarrow \pi\pi\pi^0 J/\psi$	$R = 1.0 \pm 0.4 \pm 0.3$	Belle ^[32]	
$K^+ X$	$X \rightarrow D^{*0} \bar{D}^0$	8.5 ± 2.6	(BABAR, ^[38] Belle ^[37])	0.614 ^{+0.166} _{-0.074}
		16.7 ± 3.6 ± 4.7		
		7.7 ± 1.6 ± 1.0		
$K^0 X$	$X \rightarrow D^{*0} \bar{D}^0$	12 ± 4	(BABAR, ^[38] Belle ^[37])	8.2 ^{+2.3} _{-2.8}
		22 ± 10 ± 4		
		9.7 ± 4.6 ± 1.3		
$K^+ X$	$X \rightarrow \gamma J/\psi$	0.202 ± 0.038	(BABAR, ^[35] Belle ^[34])	0.019 ^{+0.005} _{-0.009}
$K^+ X$		0.28 ± 0.08 ± 0.01		
$K^0 X$		$0.178^{+0.048}_{-0.044} \pm 0.012$	Belle ^[34]	0.24 ^{+0.05} _{-0.06}
		0.26 ± 0.18 ± 0.02		
		$0.124^{+0.076}_{-0.061} \pm 0.011$		
$K^+ X$	$X \rightarrow \gamma\psi(2S)$	0.44 ± 0.12	BABAR ^[35]	0.04 ^{+0.015} _{-0.020}
		0.95 ± 0.27 ± 0.06		
		$0.083^{+0.198}_{-0.183} \pm 0.044$		
$K^0 X$		$R' = 2.46 \pm 0.64 \pm 0.29$	LHCb ^[36]	0.51 ^{+0.13} _{-0.17}
		1.14 ± 0.55 ± 0.10		
		$0.112^{+0.357}_{-0.290} \pm 0.057$		
$K^+ X$	$X \rightarrow \gamma\chi_{c1}$	< 9.6×10^{-3}	Belle ^[23]	< 1.0×10^{-3}
$K^+ X$	$X \rightarrow \gamma\chi_{c2}$	< 0.016		
$K X$	$X \rightarrow \gamma\gamma$	< 4.5×10^{-3}	Belle ^[111]	< 4.7×10^{-4}
$K X$	$X \rightarrow \eta J/\psi$	< 1.05		
$K^+ X$	$X \rightarrow p\bar{p}$	< 9.6×10^{-4}	LHCb ^[110]	< 2.2×10^{-3}

Vector Y states in BESIII

BESIII, PRL118, 092002 (2017)

BESIII, PRL118, 092001 (2017)

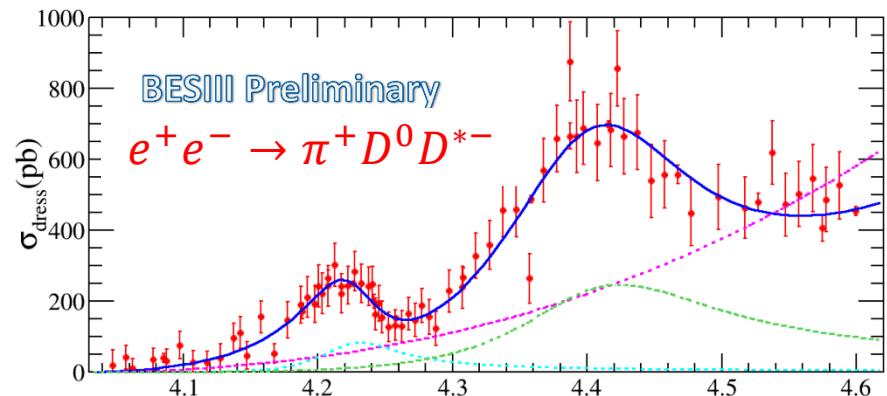
$e^+e^- \rightarrow J/\psi \pi\pi$



Parameters	Solution I	Solution II
$\Gamma_{e^+e^-} \mathcal{B}[\psi(3770) \rightarrow \pi^+\pi^- J/\psi]$	$0.5 \pm 0.1 (0)$	$0.5 \pm 0.1 (0)$
$\Gamma_{e^+e^-} \mathcal{B}(R_1 \rightarrow \pi^+\pi^- J/\psi)$	$8.8^{+1.5}_{-2.2} (\dots)$	$6.8^{+1.1}_{-1.5} (\dots)$
$\Gamma_{e^+e^-} \mathcal{B}(R_2 \rightarrow \pi^+\pi^- J/\psi)$	$13.3 \pm 1.4 (12.0 \pm 1.0)$	$9.2 \pm 0.7 (8.9 \pm 0.6)$
$\Gamma_{e^+e^-} \mathcal{B}(R_3 \rightarrow \pi^+\pi^- J/\psi)$	$21.1 \pm 3.9 (17.9 \pm 3.3)$	$1.7^{+0.8}_{-0.6} (1.1^{+0.5}_{-0.4})$
ϕ_1	$-58 \pm 11 (-33 \pm 8)$	$-116^{+9}_{-10} (-81^{+7}_{-8})$
ϕ_2	$-156 \pm 5 (-132 \pm 3)$	$68 \pm 24 (107 \pm 20)$

New BESIII data show a peculiar lineshape for the $Y(4260)$

The state appear lighter and narrower, compatible with the ones in $h_c\pi\pi$ and $\chi_{c0}\omega$. A broader old-fashioned $Y(4260)$ is appearing in $\bar{D}D^*\pi$, maybe indicating a $\bar{D}D_1$ dominance

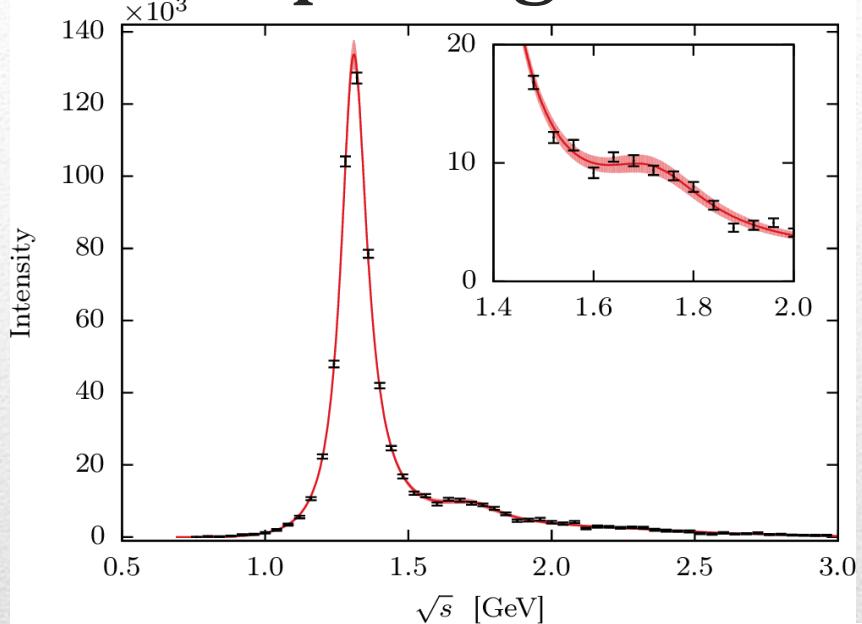


Fit with a constant (pink dashed triple-dot line) and two constant width relativistic BW functions (green dashed double-dot line and aqua dashed line).

$$M(Y(4220)) = (4224.8 \pm 5.6 \pm 4.0) \text{ MeV}/c^2, \Gamma(Y(4220)) = (72.3 \pm 9.1 \pm 0.9) \text{ MeV}$$

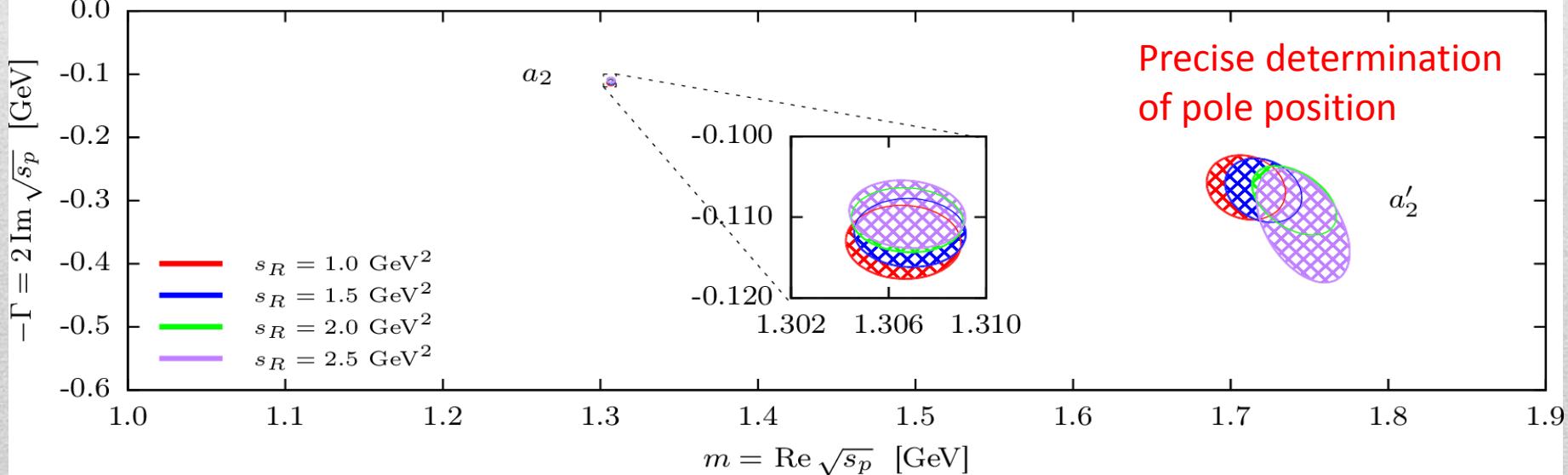
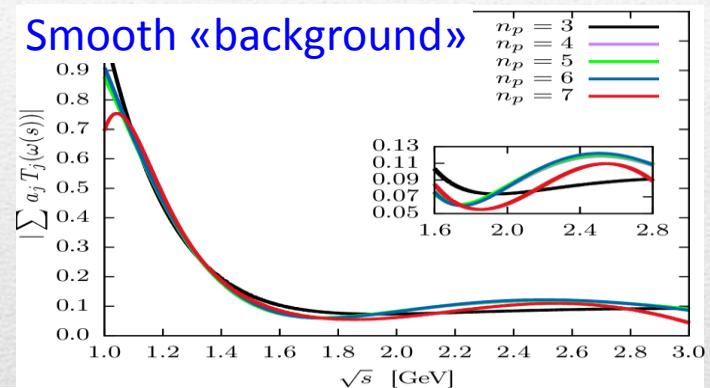
$$M(Y(4390)) = (4400.1 \pm 9.3 \pm 2.1) \text{ MeV}/c^2, \Gamma(Y(4220)) = (181.7 \pm 16.9 \pm 7.4) \text{ MeV}$$

Recap: single channel $\eta\pi$



Test against the D -wave $\eta\pi$ data, where the a_2 and the a'_2 show up

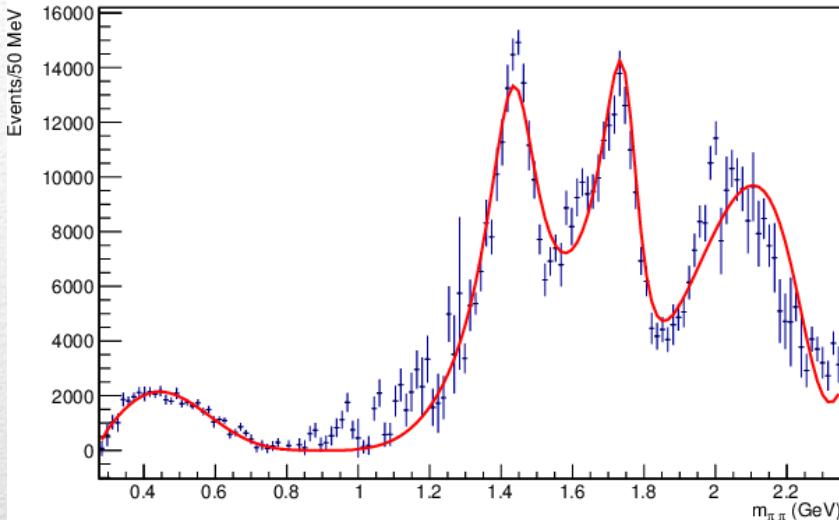
A. Jackura, M. Mikhasenko, AP et al. (JPAC & COMPASS), PLB779, 464-472



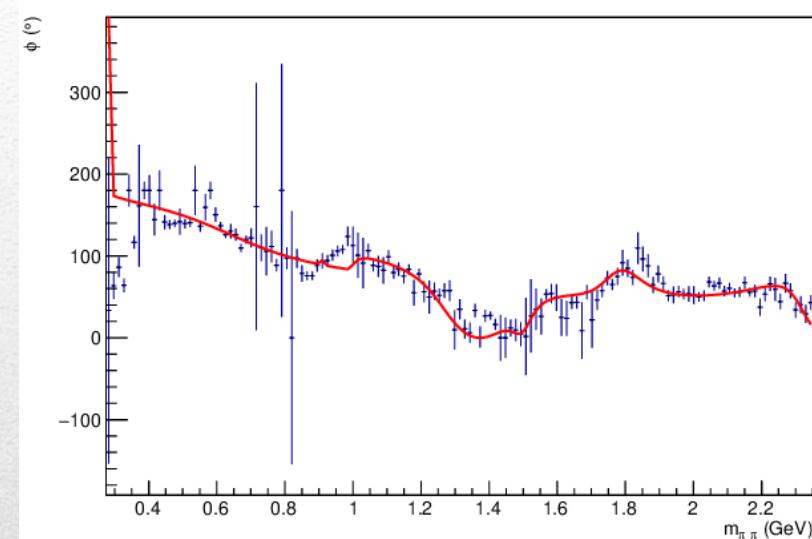
Precise determination
of pole position

What about the σ ?

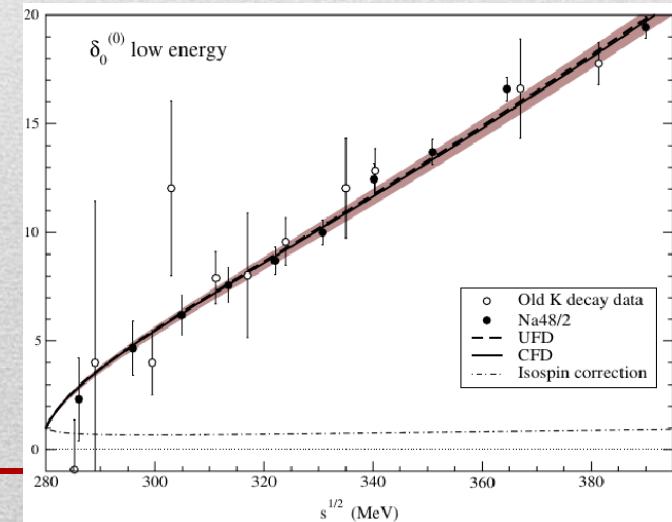
$J/\psi \rightarrow \gamma \pi \pi$ (S wave)



$J/\psi \rightarrow \gamma \pi \pi$ (phase)

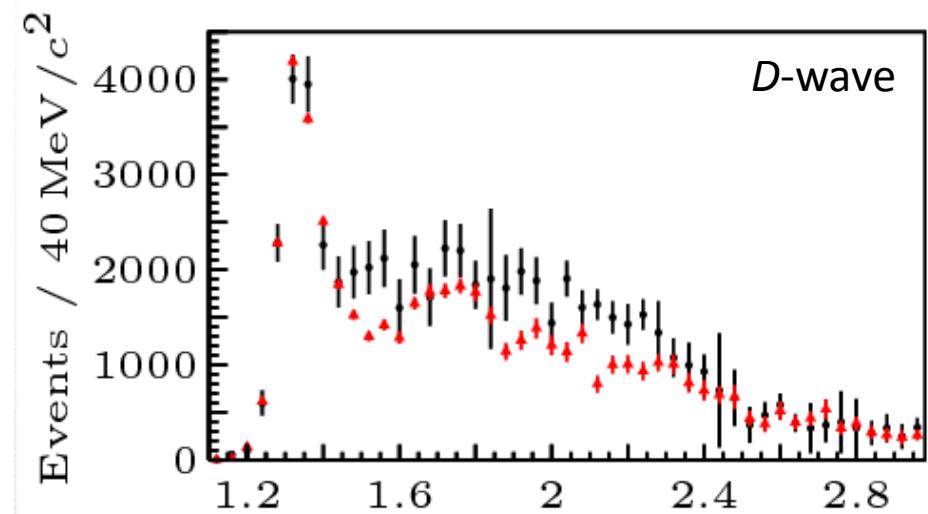
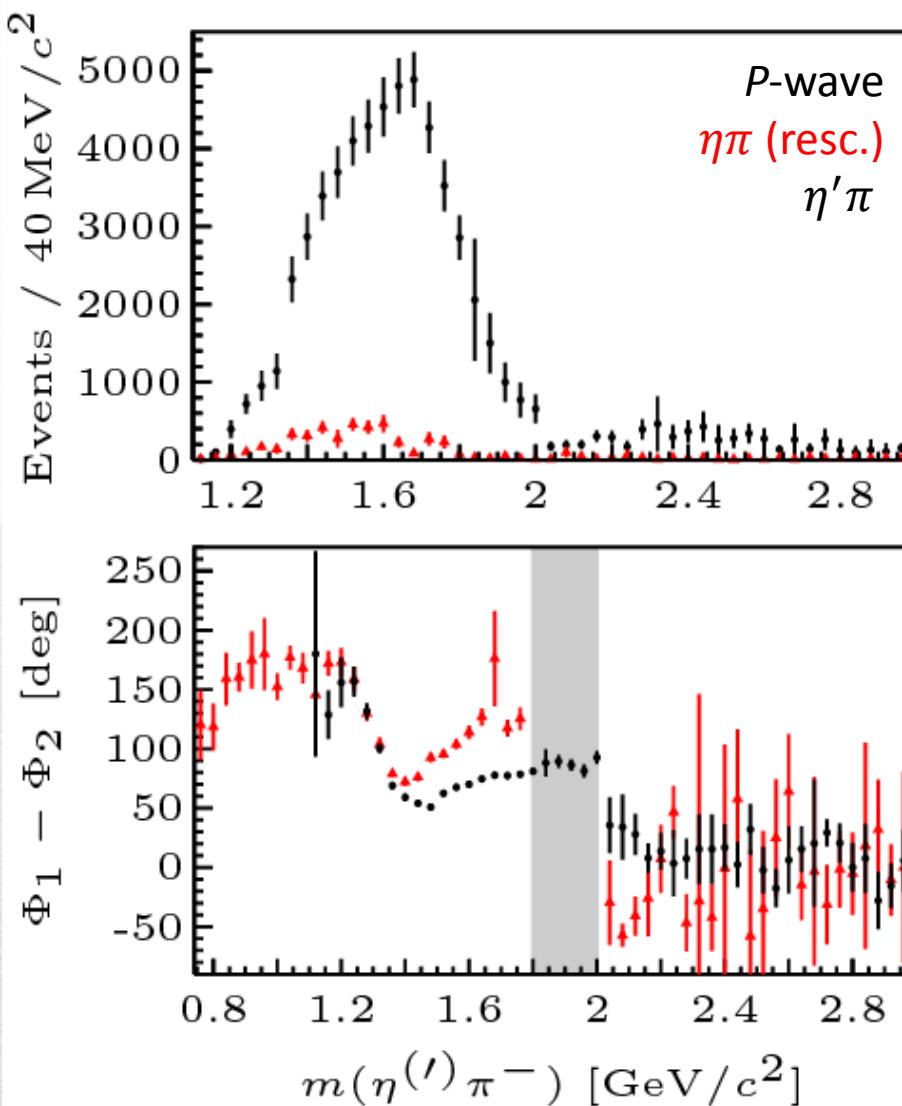


Even if an Adler zero is added,
hard to obtain a reasonable behavior of the phase
in the σ region
We are exploring new parametrization



Data

COMPASS, PLB740, 303-311



A sharp drop appears at 2 GeV in *P*-wave intensity and phase

No convincing physical motivation for it

It affects the position of the $a'_2(1700)$

We decided to fit up to 2 GeV only

Systematic studies

- Change of functional form and parameters in the denominator

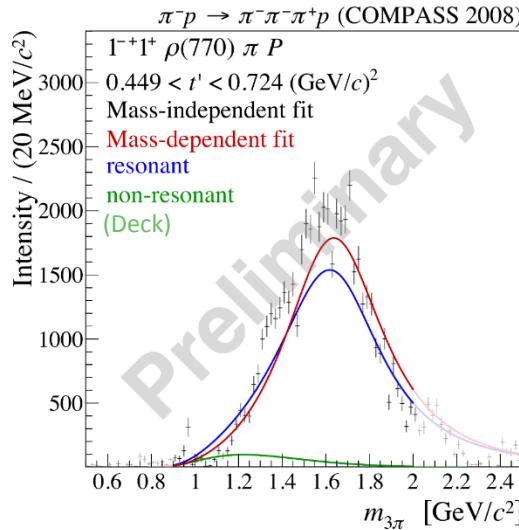
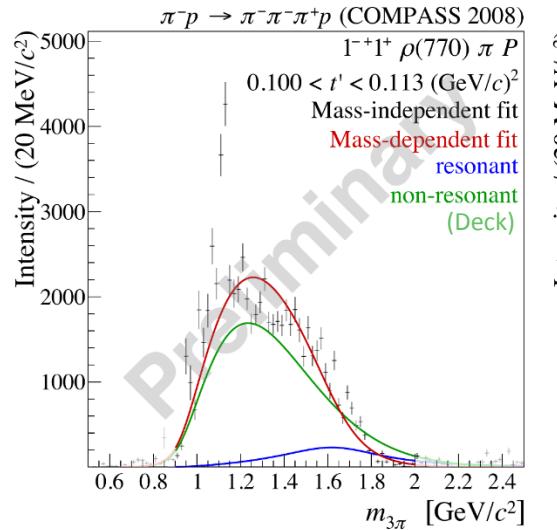
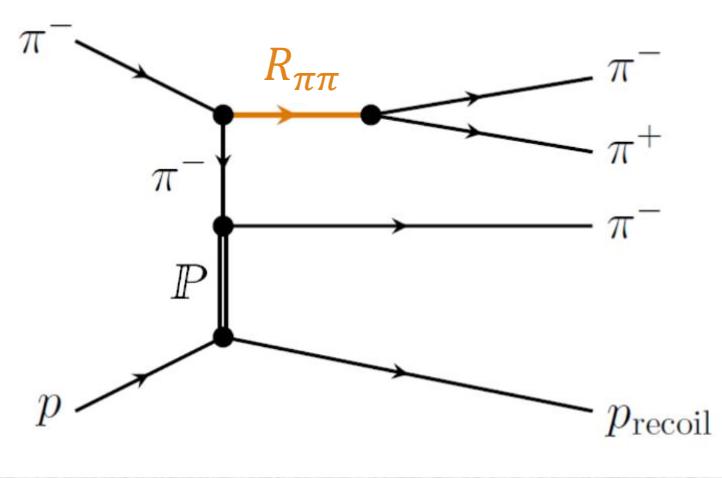
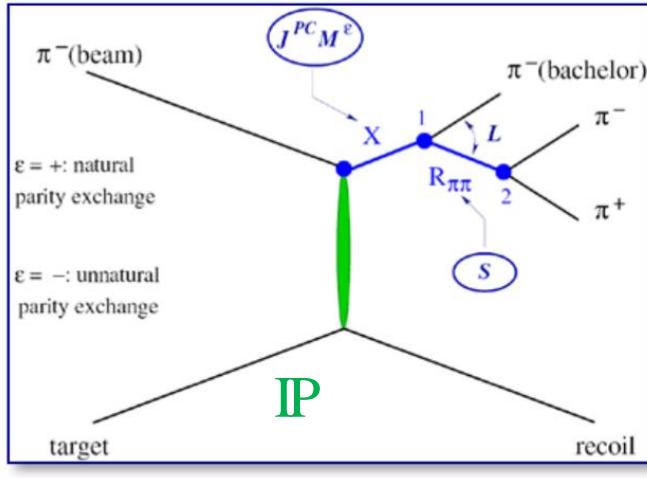
$$\rho N_{ki}^J(s') = g \delta_{ki} \frac{\lambda^{J+1/2} \left(s', m_{\eta^{(\prime)}}^2, m_\pi^2 \right)}{(s' + s_R)^{2J+1+\alpha}}$$

- Default: $s_R = 1 \text{ GeV}^2$. We try $s_R = 0.8, 1.8 \text{ GeV}^2$
- Default: $\alpha = 2$. We try $\alpha = 1$
- We also try a different function: $\rho N_{ki}^J(s') = g \delta_{ki} \frac{Q_J(z_{s'})}{s'^\alpha \lambda^{1/2}(s', m_{\eta^{(\prime)}}, m_\pi)}$ with $\alpha = 2, 1.5, 1$

- Change of parameters in the numerator

- Default: $t_{\text{eff}} = -0.1 \text{ GeV}^2$. We try $t_{\text{eff}} = -0.5 \text{ GeV}^2$
- Default: 3rd order polynomial. We try 4th

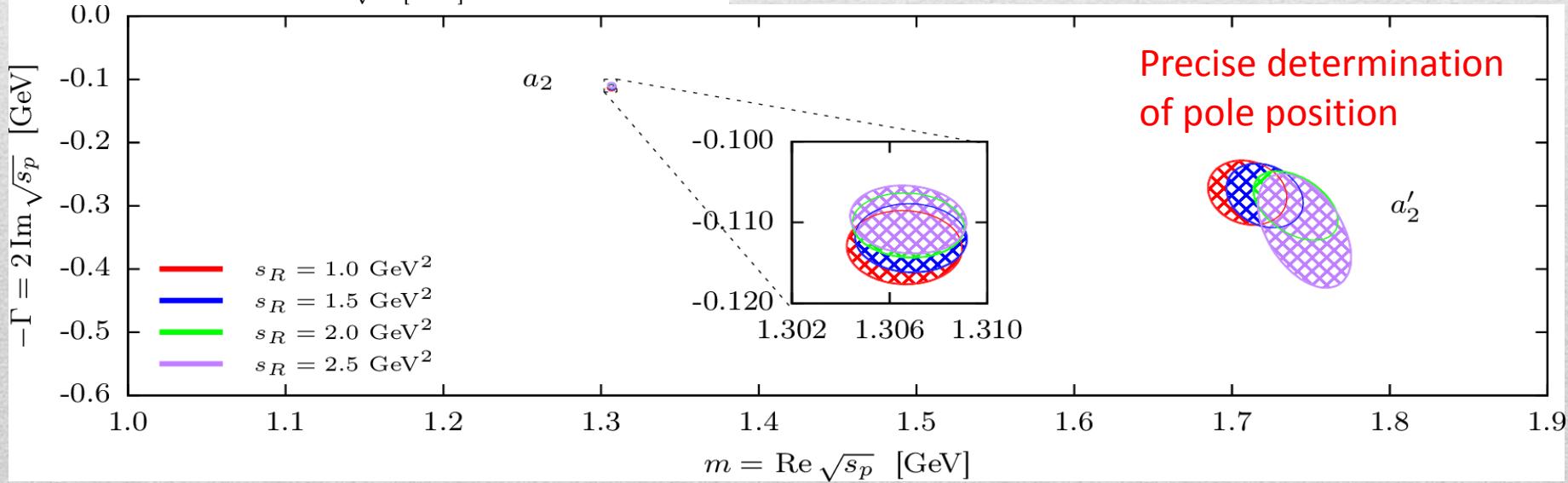
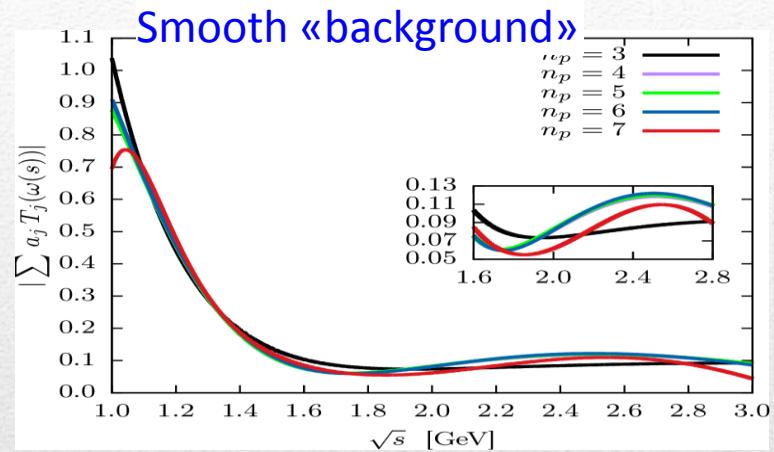
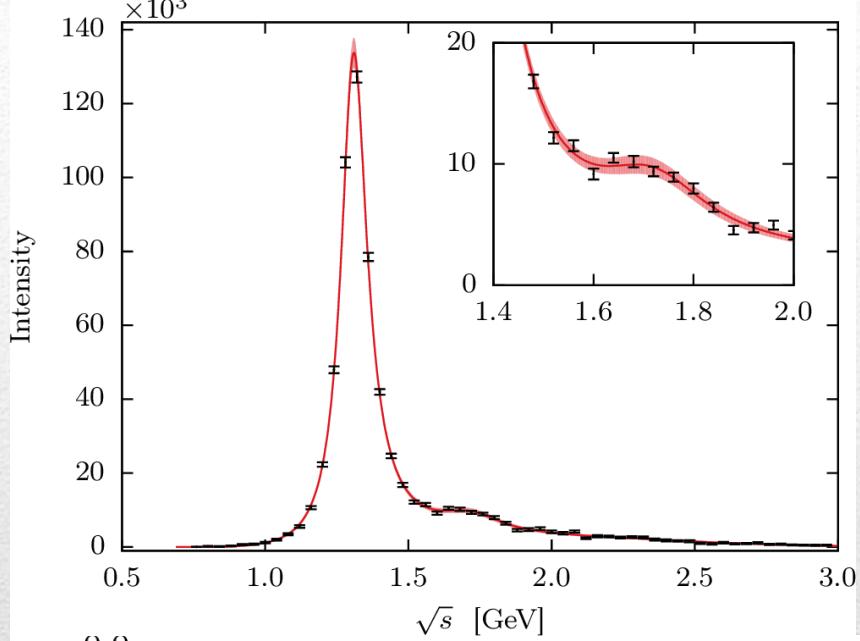
$\rho\pi$ channel and Deck amplitude



The production allows for a nonresonant component (**Deck effect**)
The singularity is close to the physical region, **peaking background**

We do not include this channel

Searching for resonances in $\eta\pi$



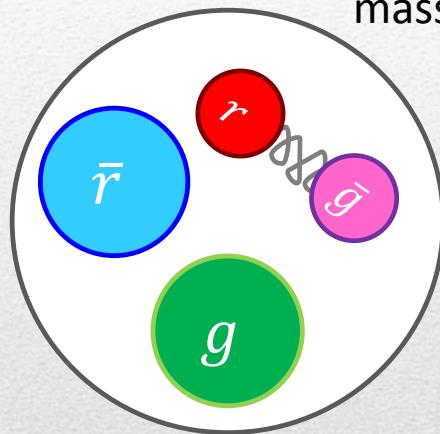
The hybrid π_1

A. Rodas, AP *et al.* (JPAC) PRL122, 042002

HV
Hybrid Vehicle



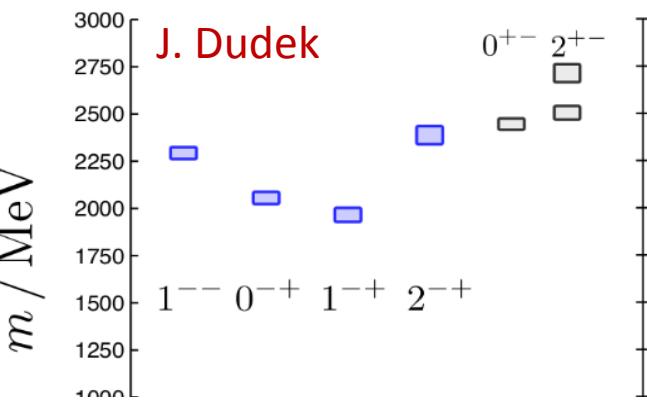
Hybrid hunting



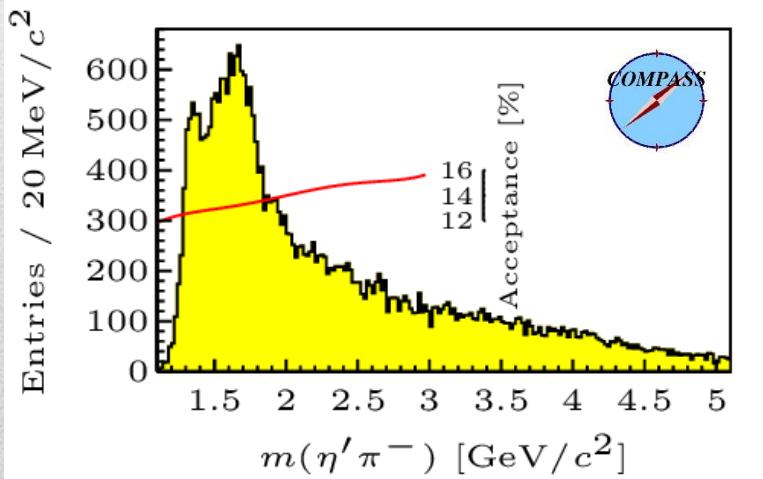
Excited gluon,
 $J^{PC} = 1^{+-}$
mass $\sim 1.0\text{--}1.5 \text{ GeV}$

Look for a π_1 state with $J^{PC} = \mathbf{1}^{-+}$

decaying into $\left\{ \begin{array}{l} \eta \pi \text{ and } \eta' \pi \\ \rho \pi \rightarrow 3\pi \\ b_1 \pi \rightarrow 5\pi \end{array} \right.$

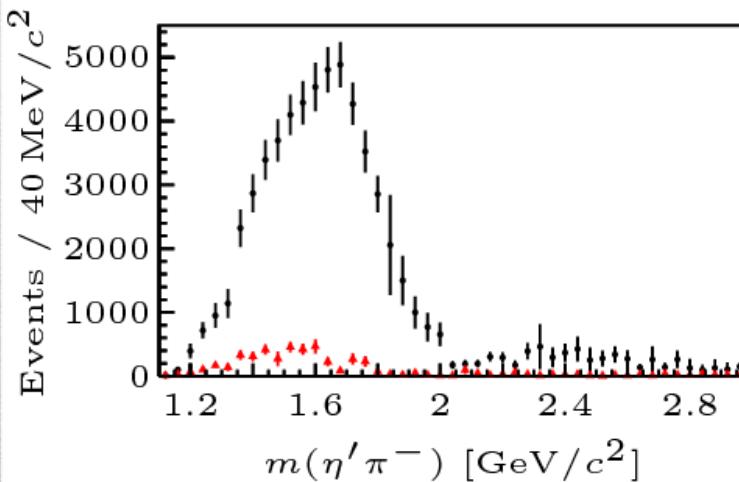
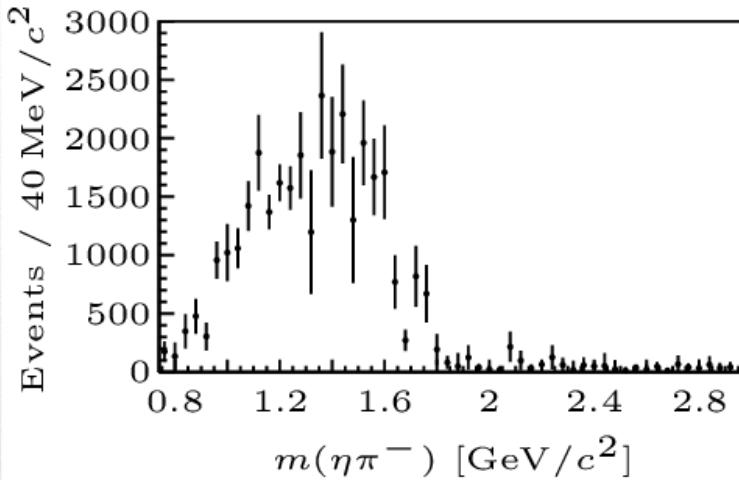


excitation of a quasiparticle?
→ degenerate $J^{PC} = (0, \mathbf{1}, 2)^{-+}, 1^{--}$



Small signal in data

Two hybrid states???



$\pi_1(1400) \quad I^G(J^{PC}) = 1^-(1^{-+})$

See also the mini-review under non- $q\bar{q}$ candidates in PDG 2006 , Journal of Physics G33 1 (2006).

$\pi_1(1400)$ MASS

$1354 \pm 25 \text{ MeV} (\text{S} = 1.8)$

$\pi_1(1400)$ WIDTH

$330 \pm 35 \text{ MeV}$

Decay Modes

Mode

$\Gamma_1 \quad \eta\pi^0$

Fraction (Γ_i / Γ)

seen

$\Gamma_2 \quad \eta\pi^-$

seen

$\Gamma_3 \quad \eta'\pi^-$

Scale Factor/
Conf. Level

Neither lattice nor models predict two

1^{-+} states in this region!

$\pi_1(1600) \quad I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1600)$ MASS

$1662_{-9}^{+8} \text{ MeV}$

$\pi_1(1600)$ WIDTH

$241 \pm 40 \text{ MeV} (\text{S} = 1.4)$

Decay Modes

Mode

$\Gamma_1 \quad \pi\pi\pi$

Fraction (Γ_i / Γ)

seen

$\Gamma_2 \quad \rho^0\pi^-$

seen

$\Gamma_3 \quad f_2(1270)\pi^-$

not seen

$\Gamma_4 \quad b_1(1235)\pi$

seen

$\Gamma_5 \quad \eta'(958)\pi^-$

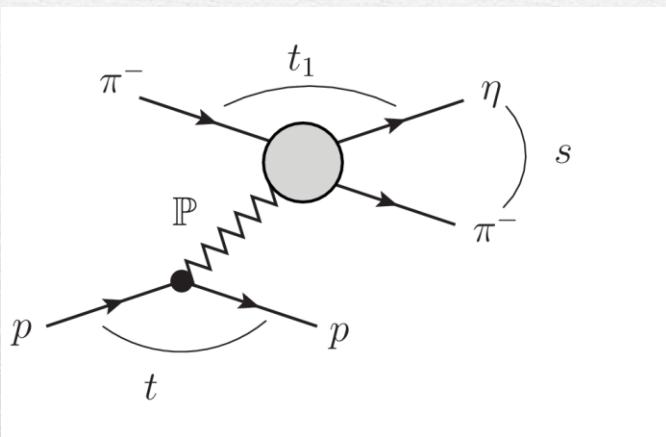
seen

$\Gamma_6 \quad f_1(1285)\pi$

seen

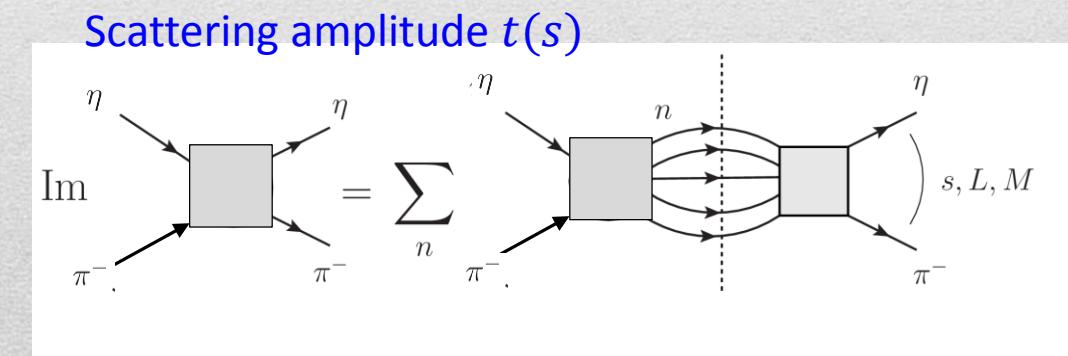
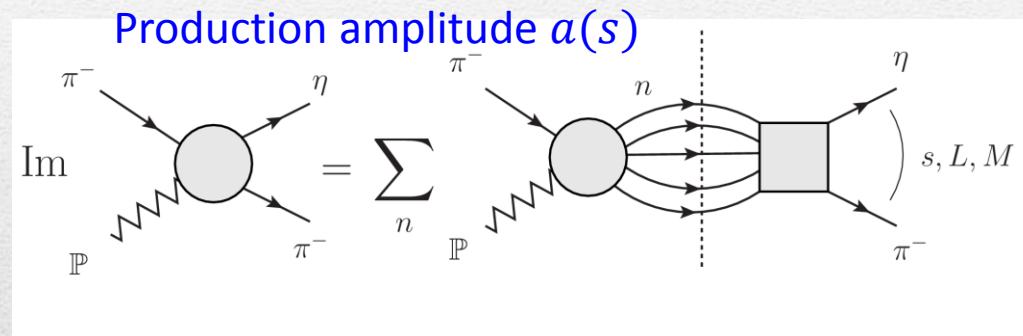
Amplitudes for $\eta^{(\prime)}\pi$

We build the partial wave amplitudes according to the ***N/D* method**



$$\text{Im } a(s) = \rho a(s) t^*(s)$$

$$\frac{d\sigma}{d\sqrt{s}} \propto \frac{\rho}{\sqrt{s}} |p^L q^{L-1} a(s)|^2$$



Amplitudes for $\eta^{(\prime)}\pi$

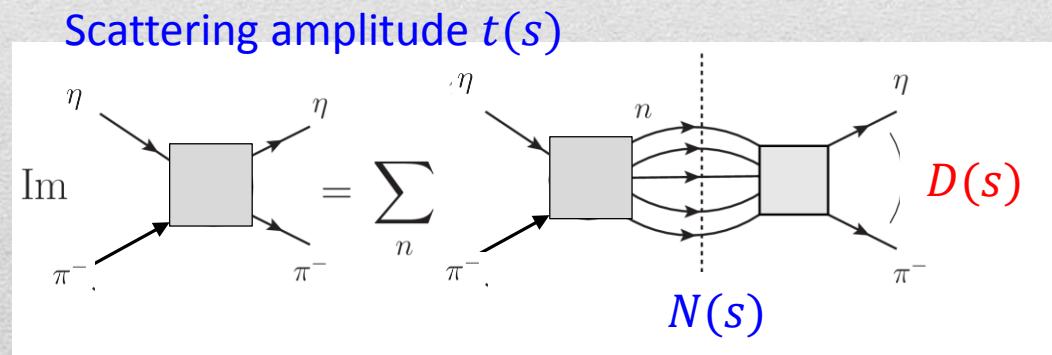
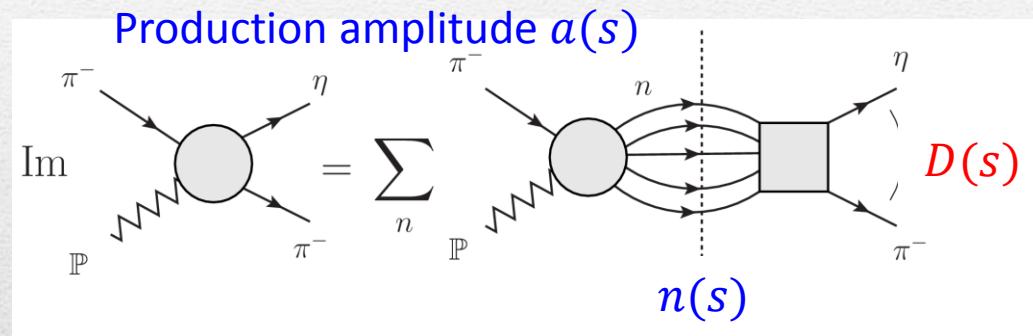
We build the partial wave amplitudes according to the ***N/D* method**

$$t(s) = \frac{N(s)}{D(s)}, a(s) = \frac{n(s)}{D(s)}$$



The $D(s)$ has **only right hand cuts**;
it contains all the Final State Interactions
constrained by unitarity \rightarrow **universal**

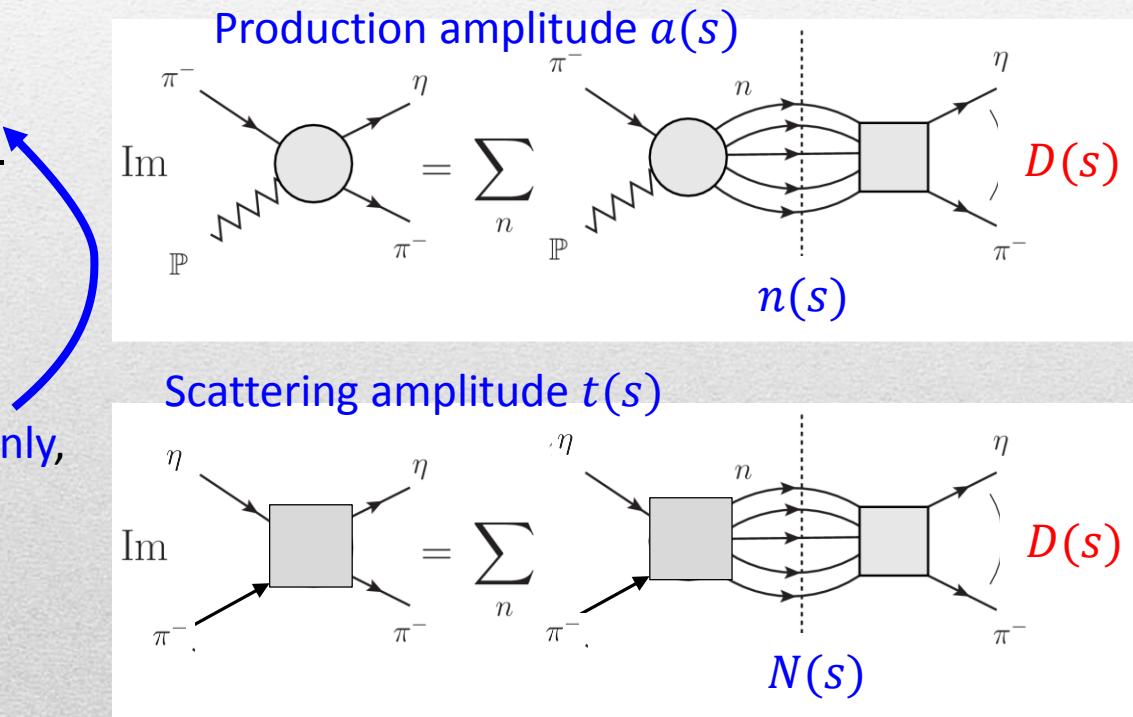
$$\text{Im } D(s) = -\rho N(s)$$



Amplitudes for $\eta^{(\prime)}\pi$

We build the partial wave amplitudes according to the ***N/D* method**

$$t(s) = \frac{N(s)}{D(s)}, a(s) = \frac{n(s)}{D(s)}$$



The $n(s), N(s)$ have **left hand cuts only**, they depend on the exchanges \rightarrow process-dependent, smooth

Coupled channel: the model

A. Rodas, AP et al. (JPAC) PRL122, 042002

Two channels, $i, k = \eta\pi, \eta'\pi$

Two waves, $J = P, D$

37 fit parameters

$$D_{ki}^J(s) = \left[K^J(s)^{-1} \right]_{ki} - \frac{s}{\pi} \int_{s_k}^{\infty} ds' \frac{\rho N_{ki}^J(s')}{s'(s' - s - i\epsilon)}$$

$$K_{ki}^J(s) = \sum_R \frac{g_k^{(R)} g_i^{(R)}}{m_R^2 - s} + c_{ki}^J + d_{ki}^J s$$

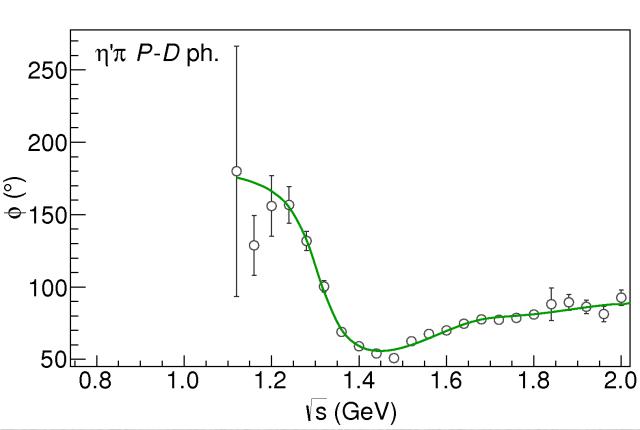
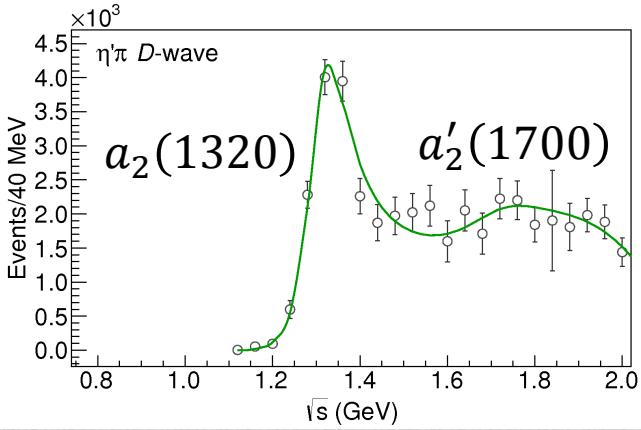
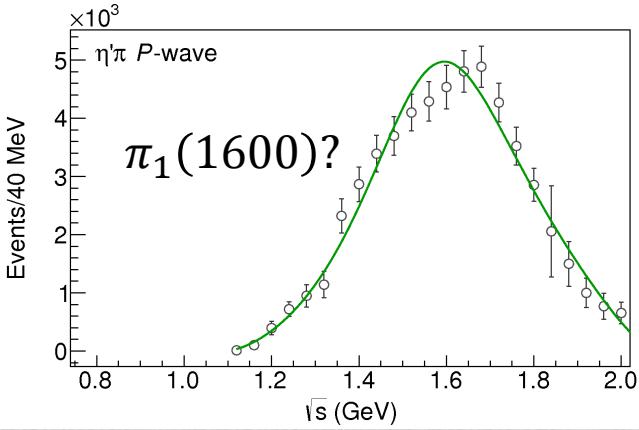
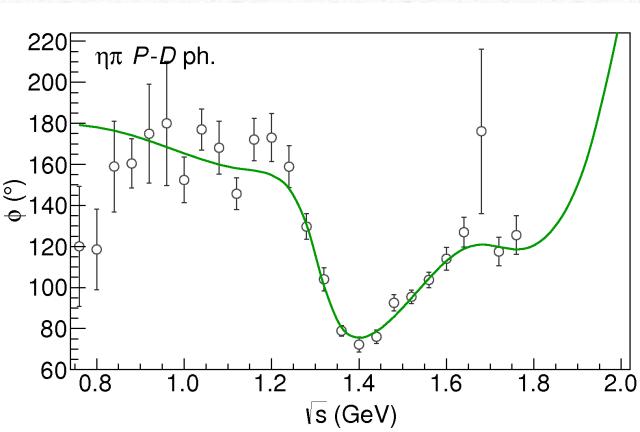
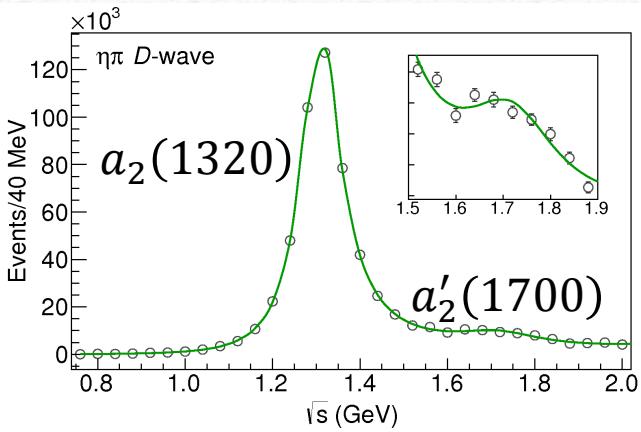
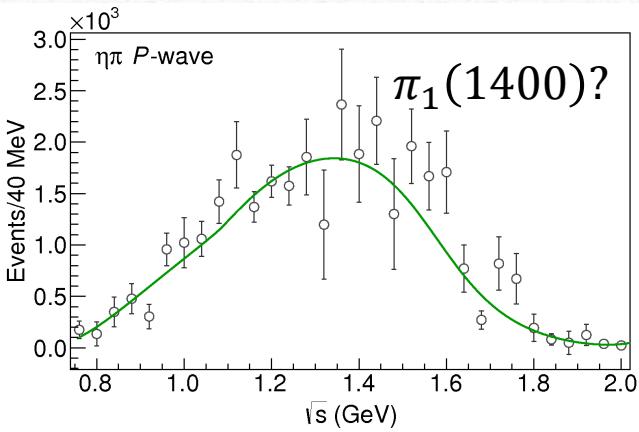
1 K-matrix pole for the P-wave
2 K-matrix poles for the D-wave

$$\rho N_{ki}^J(s') = \delta_{ki} \frac{\lambda^{J+1/2} \left(s', m_{\eta'}^2, m_\pi^2 \right)}{(s' + s_R)^{2J+1+\alpha}}$$

$$n_k^J(s) = \sum_{n=0}^3 a_n^{J,k} T_n \left(\frac{s}{s + s_0} \right)$$

Left-hand scale (Blatt-Weisskopf radius) $s_R = s_0 = 1 \text{ GeV}^2$
 $\alpha = 2, 3\text{rd order polynomial for } n_k^J(s)$

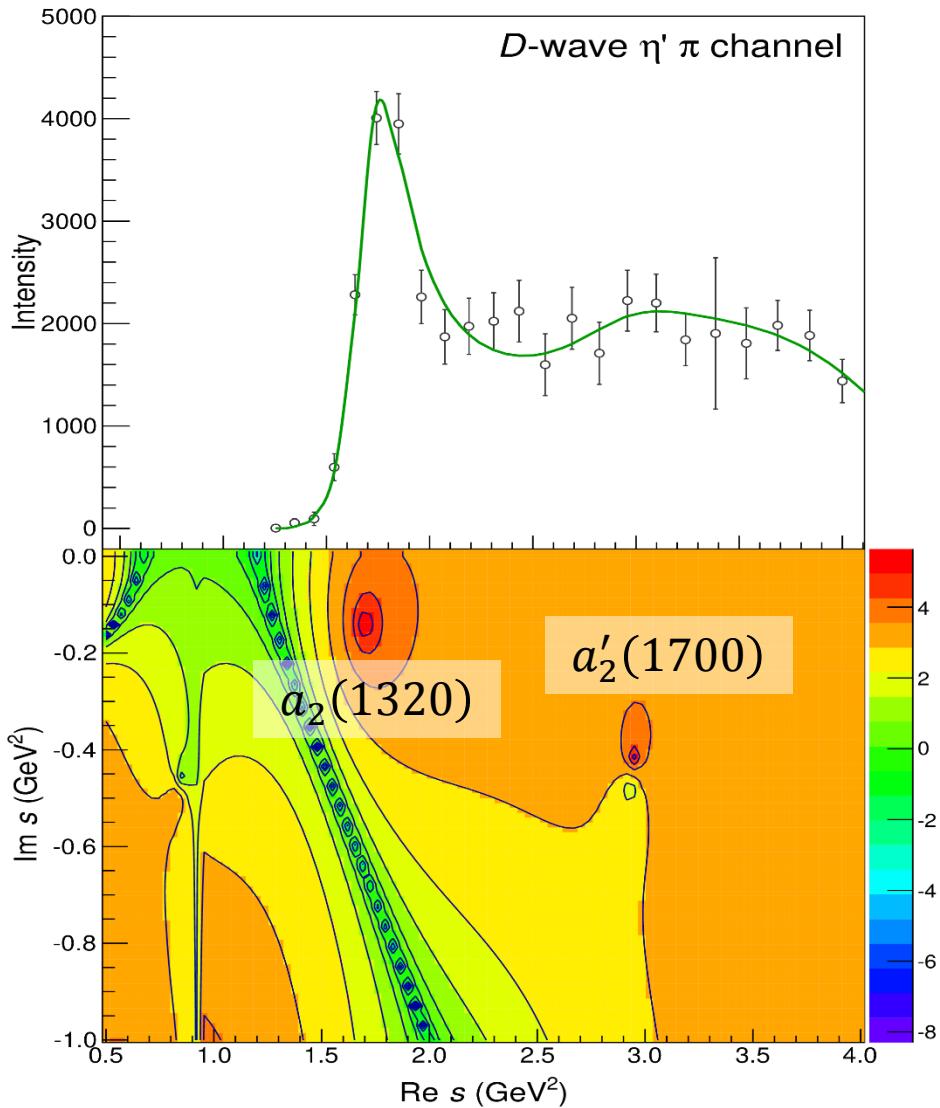
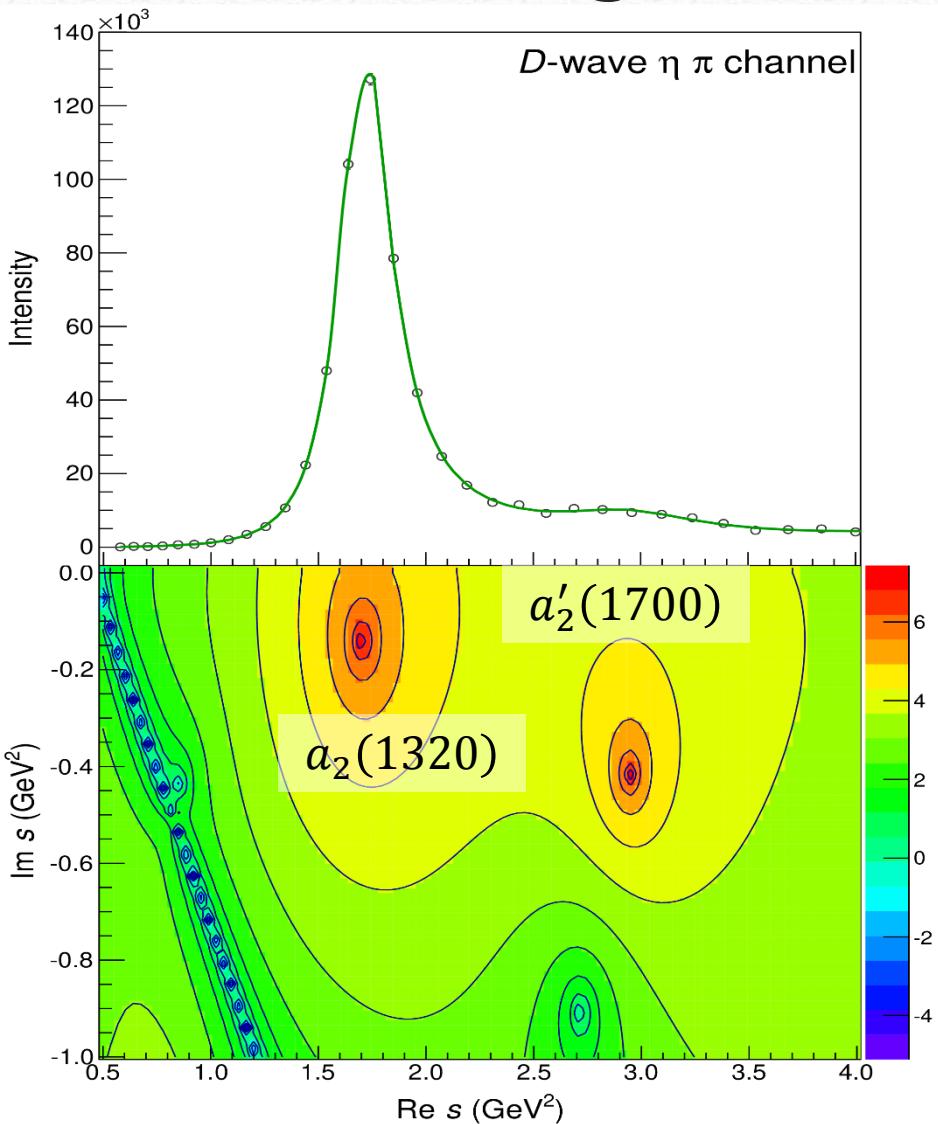
Fit to $\eta^{(\prime)}\pi$



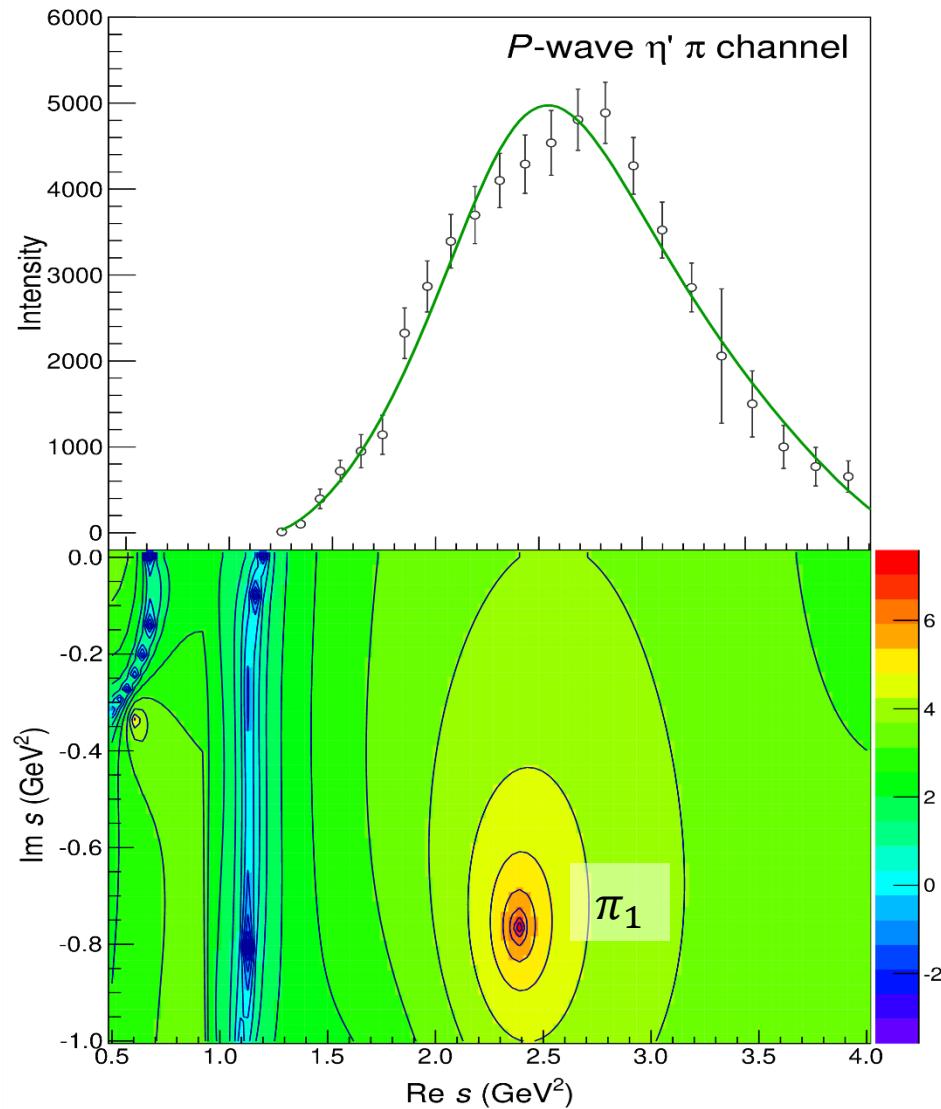
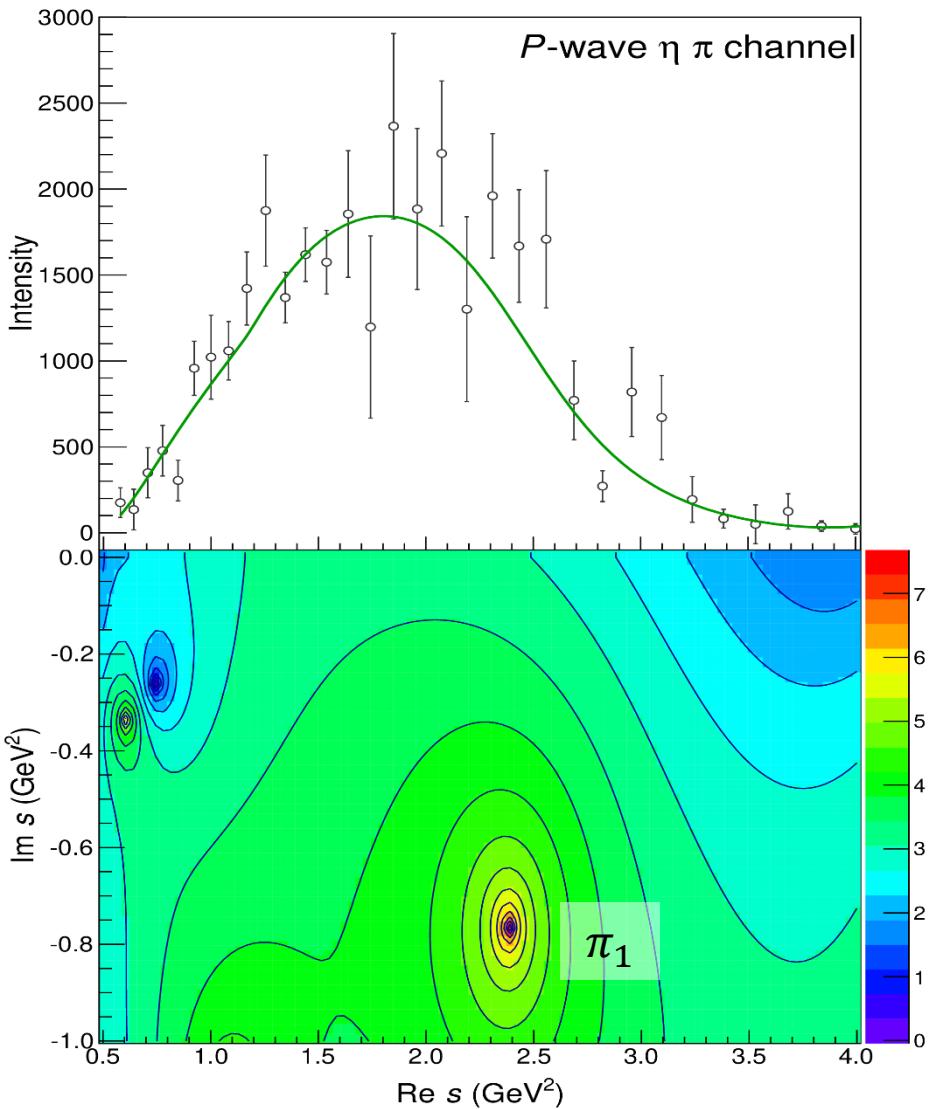
$$J^{PC} = 1^{-+}$$

$$J^{PC} = 2^{++}$$

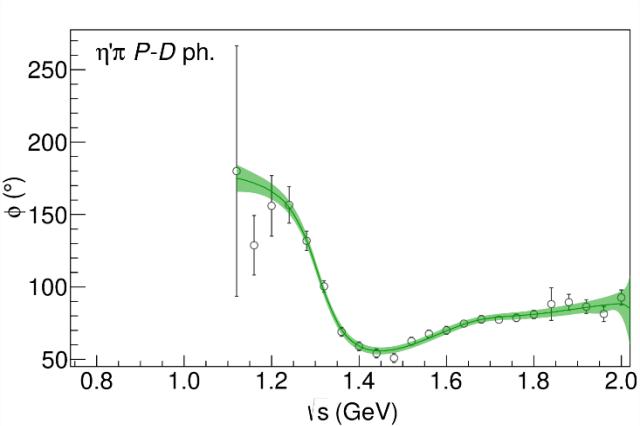
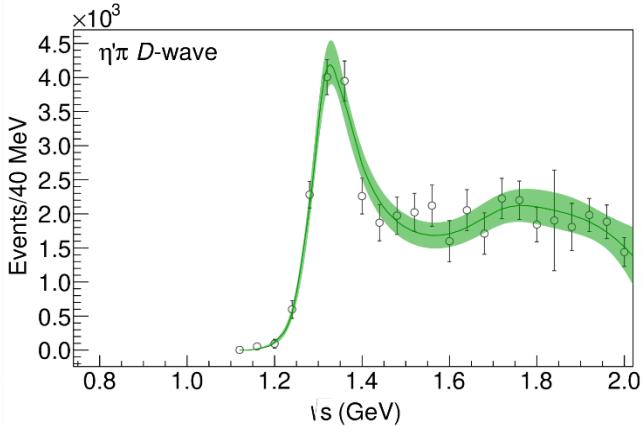
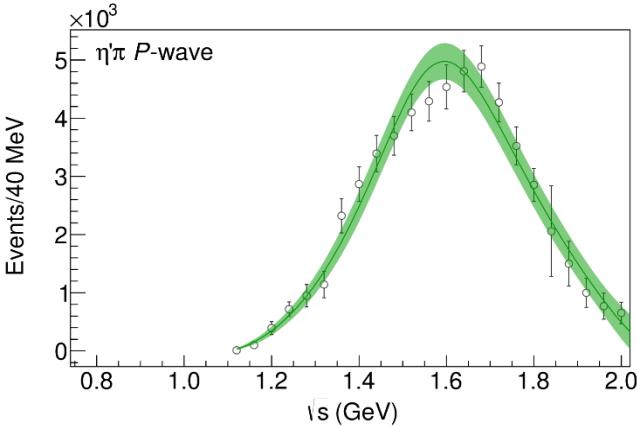
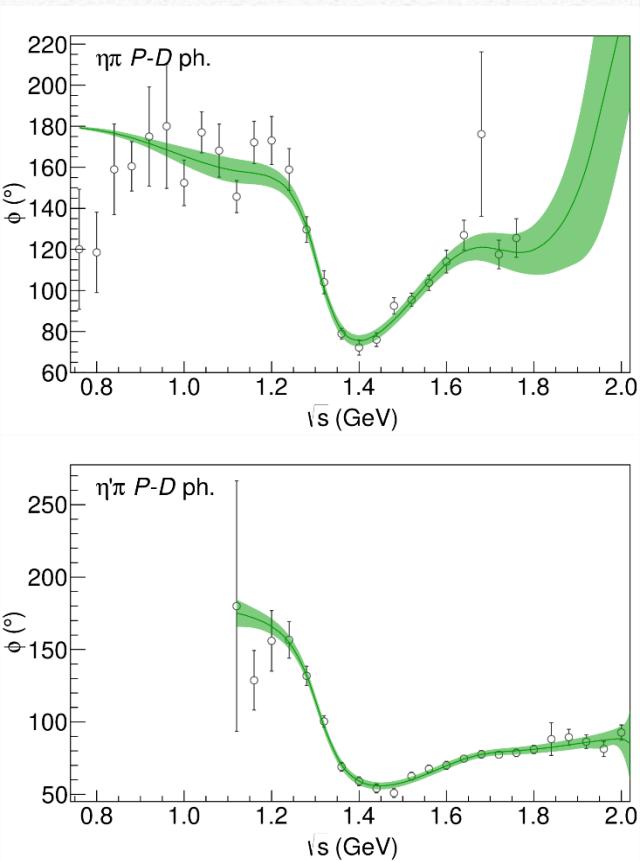
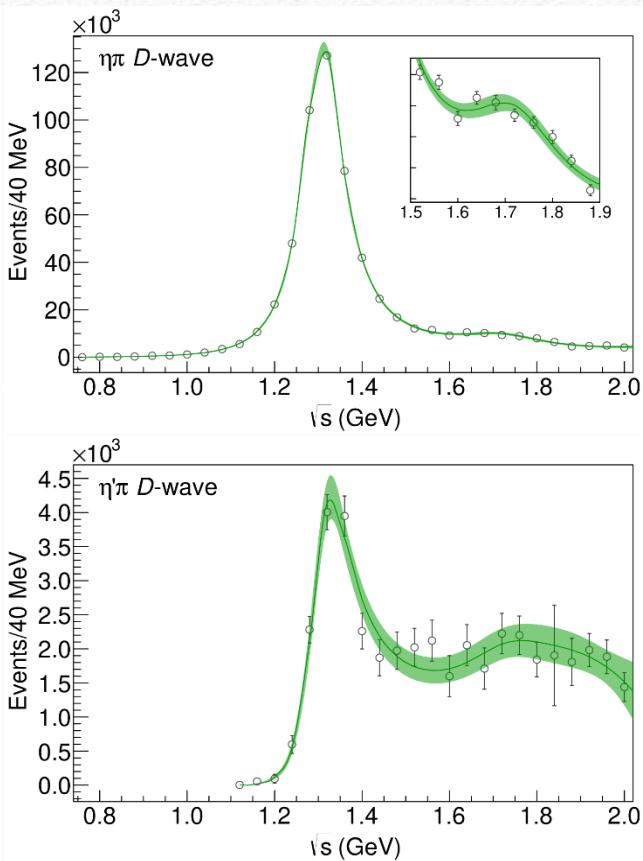
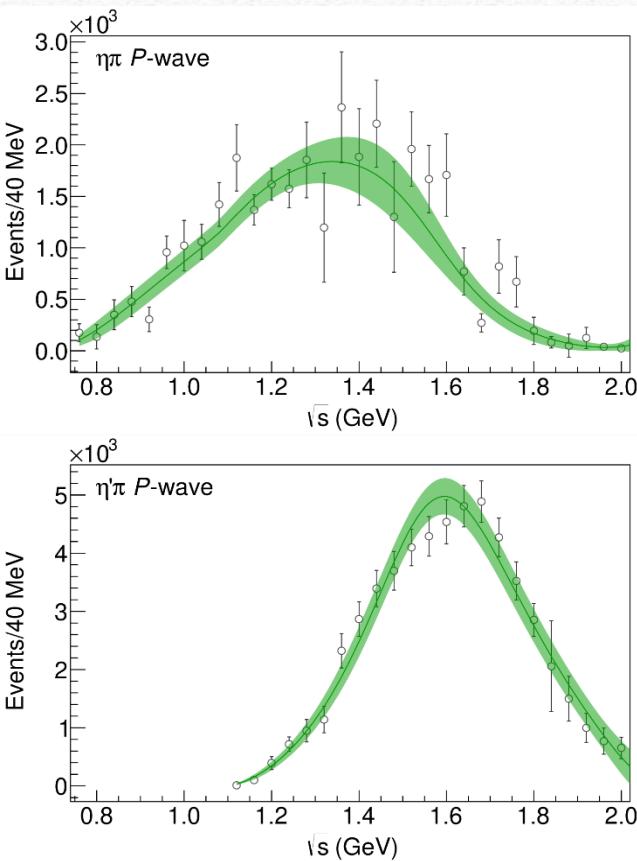
Pole hunting



Pole hunting



Statistical Bootstrap



Correlations

Denominator parameters uncorrelated with the numerator ones ✓

Production (numerator) parameters

	$a_0^P\pi\pi$	$a_1^P\pi\pi$	$a_2^P\pi\pi$	$a_3^P\pi\pi$	$a_0^D\pi\pi$	$a_1^D\pi\pi$	$a_2^D\pi\pi$	$a_3^D\pi\pi$	$a_0^P\eta\pi$	$a_1^P\eta\pi$	$a_2^P\eta\pi$	$a_3^P\eta\pi$	$a_0^D\eta\pi$	$a_1^D\eta\pi$	$a_2^D\eta\pi$	$a_3^D\eta\pi$
$m_{D,2}^2$	16	-16	16	-15	-20	16	-19	1	-5	5	-5	6	15	-19	6	-7
$g_{\eta'\pi}^{D,2}$	45	-45	44	-43	-8	-3	-5	-8	-40	41	-41	41	-4	-2	-4	-6
$g_{\eta\pi}^{D,2}$	13	-13	13	-13	-3	-8	-2	-8	-1	2	-3	7	-10	5	-8	-4
$m_{D,1}^2$	24	-23	21	-15	-4	5	-15	1	-25	20	-9	-12	5	-4	-13	2
$g_{\eta'\pi}^{D,1}$	9	-9	10	-12	18	4	-27	32	-5	7	-11	19	9	10	-24	35
$g_{\eta\pi}^{D,1}$	23	-22	20	-15	-0	1	-13	1	-24	20	-9	-12	1	-0	-16	4
$m_{P,1}^2$	25	-24	24	-23	-21	12	-9	-6	-26	28	-31	36	2	-10	7	-12
$g_{\eta'\pi}^{P,1}$	-12	11	-11	9	8	-7	7	-0	14	-13	12	-9	5	-5	7	-2
$g_{\eta\pi}^{P,1}$	-6	6	-7	10	-6	10	-3	3	5	-5	8	-11	11	-11	11	-4
Γ_{π_1}	22	-23	23	-25	-4	5	-9	3	-3	2	1	-5	6	-6	0	-1
m_{π_1}	-10	9	-8	4	12	-8	3	3	6	-6	7	-7	-6	11	-9	8
$\Gamma_{a'_2}$	-17	17	-16	14	-21	25	-8	6	17	-15	8	4	26	-27	28	-10
$m_{a'_2}$	8	-9	9	-11	-17	21	-13	8	-3	4	-8	16	19	-21	17	-7
Γ_{a_2}	-3	3	-4	4	-6	4	1	-4	2	-3	4	-7	5	-7	6	-4
m_{a_2}	-6	6	-5	5	-12	14	-5	3	7	-6	4	-0	13	-15	15	-7

K-matrix «pole» parameters

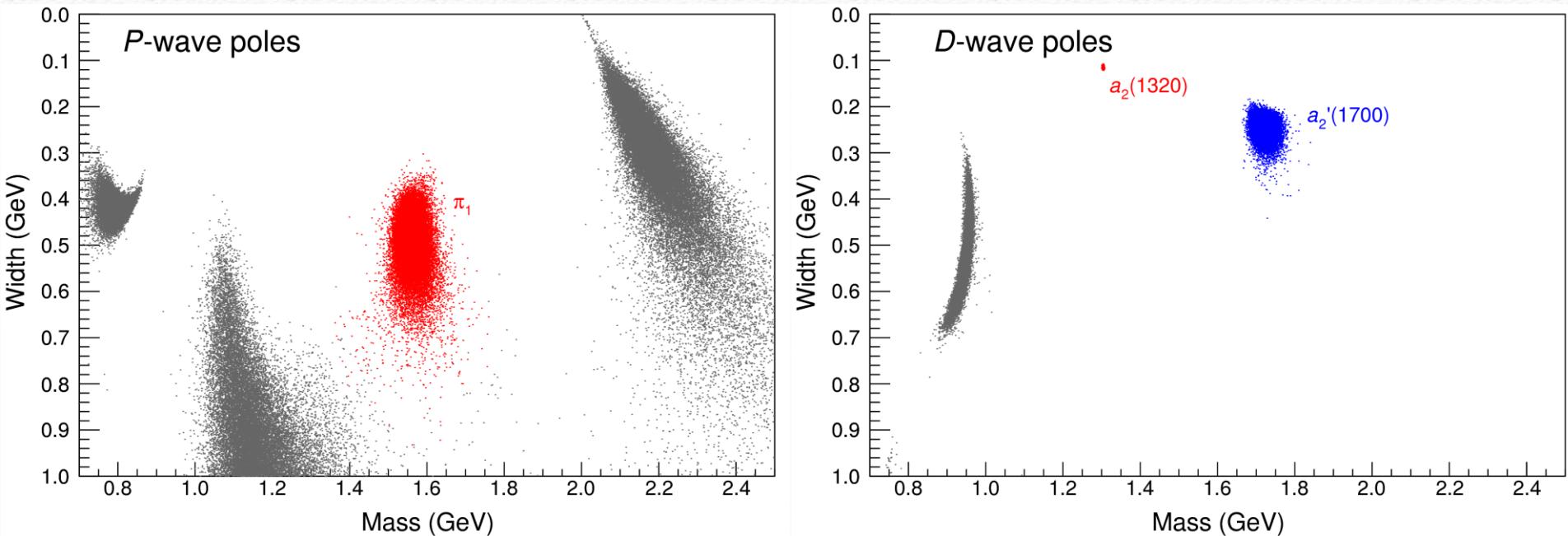
	$C_{\eta'\pi,\eta'\pi}^P$	$C_{\eta\pi,\eta\pi}^P$	$C_{\pi\pi,\pi\pi}^P$	$C_{\eta\pi,\eta\pi}^D$	$C_{\pi\pi,\pi\pi}^D$	$C_{\eta'\pi,\eta'\pi}^D$	$C_{\eta\pi,\eta\pi}^D$	$d_{\eta'\pi,\eta'\pi}^P$	$d_{\eta\pi,\eta\pi}^P$	$d_{\pi\pi,\pi\pi}^P$	$d_{\eta'\pi,\eta'\pi}^D$	$d_{\eta\pi,\eta\pi}^D$	$d_{\pi\pi,\pi\pi}^D$
$d_{\eta'\pi,\eta'\pi}^P$	-27	0	-22	-57	-53	84	22	-3	20	-20	-91	100	100
$d_{\eta\pi,\eta\pi}^P$	32	-4	22	32	63	-76	-29	4	-18	-18	100	-91	-91
$d_{\pi\pi,\pi\pi}^P$	-22	11	1	67	-15	-17	24	-4	-4	100	-18	-20	-20
$d_{\eta'\pi,\eta'\pi}^D$	-28	-74	-90	-8	-17	20	23	72	100	-4	-18	20	20
$d_{\eta\pi,\eta\pi}^D$	-45	-94	-60	2	-5	2	45	100	72	-4	4	-3	-3
$d_{\pi\pi,\pi\pi}^D$	-92	-30	-24	-8	-9	13	100	45	23	24	-29	22	22
$C_{\eta'\pi,\eta'\pi}^D$	-18	-7	-19	-26	-84	100	13	2	20	-17	-76	84	84
$C_{\eta\pi,\eta\pi}^D$	10	6	16	-19	100	-84	-9	-5	-17	-15	63	-53	-53
$C_{\pi\pi,\pi\pi}^D$	14	3	10	100	-19	-26	-8	2	-8	67	32	-57	-57
$C_{\eta'\pi,\eta'\pi}^P$	31	67	100	10	16	-19	-24	-60	-90	1	22	-22	-22
$C_{\eta\pi,\eta\pi}^P$	41	100	67	3	6	-7	-30	-94	-74	11	-4	0	0
$C_{\pi\pi,\pi\pi}^P$	100	41	31	14	10	-18	-92	-45	-28	-22	32	-27	-27

Denominator parameters uncorrelated between P - and D -wave ✓

K-matrix «bkg» parameters

K-matrix «bkg» parameters

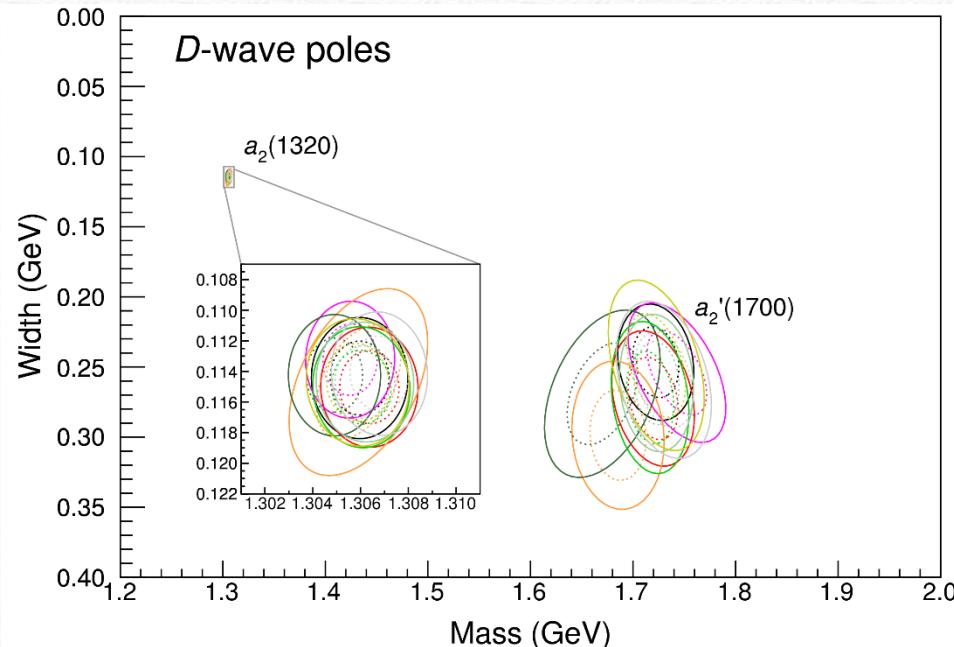
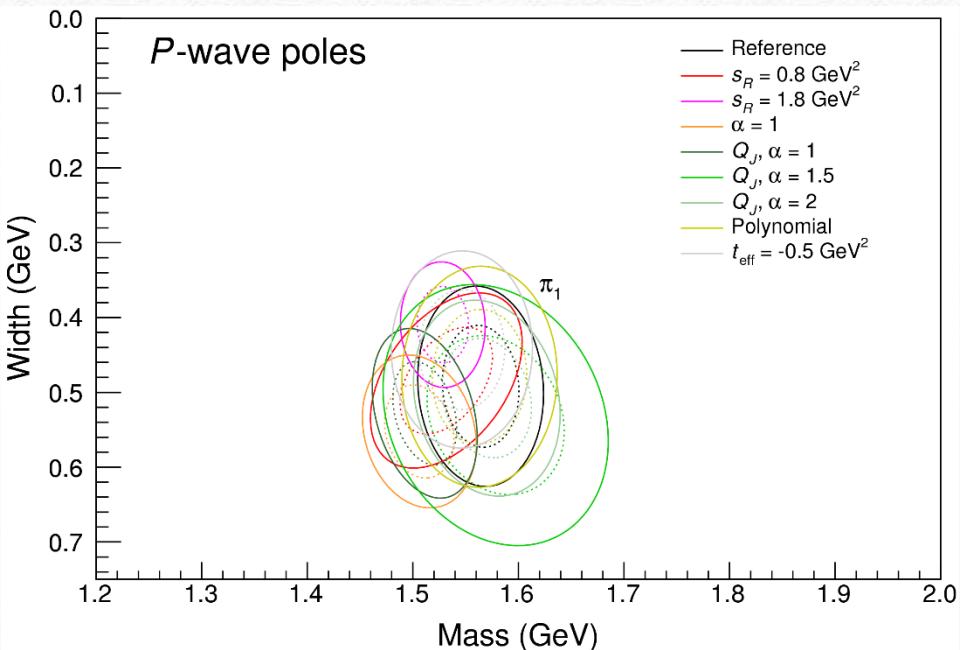
Bootstrap



We can identify the poles in the region $m \in [1.2, 2] \text{ GeV}, \Gamma \in [0, 1] \text{ GeV}$

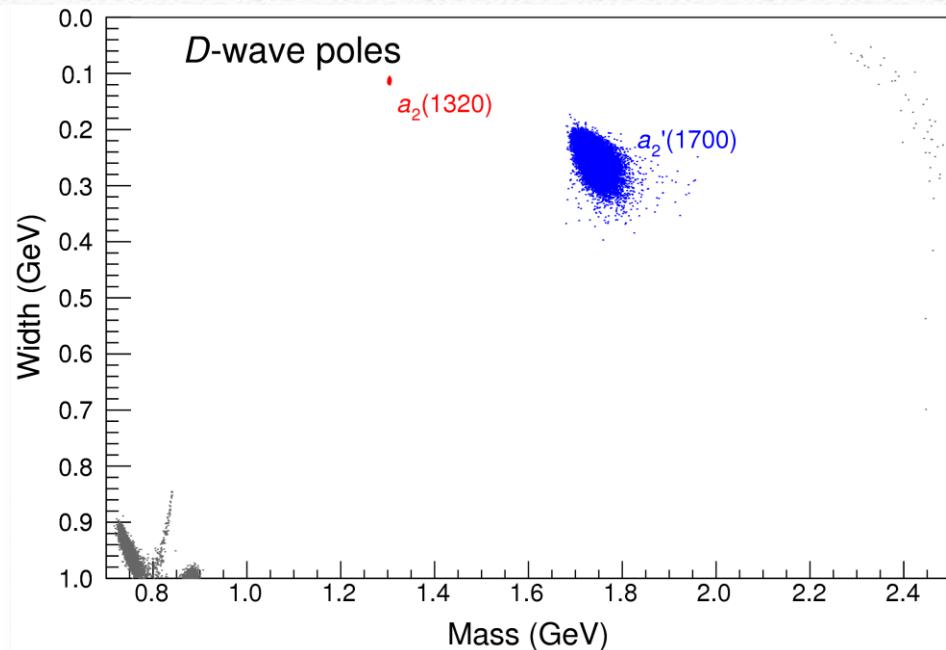
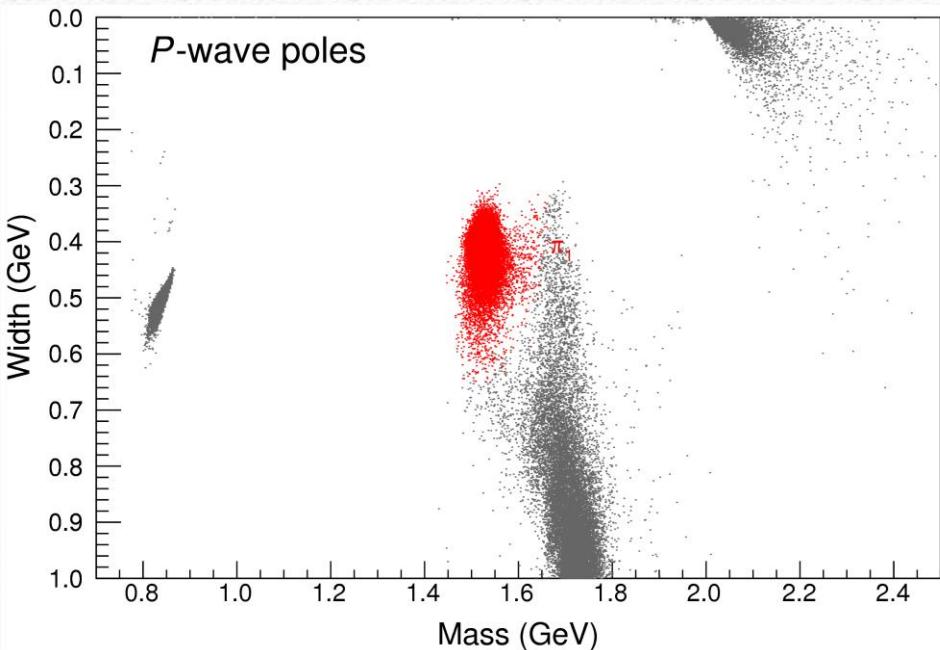
Two stable isolated poles are identifiable in the *D*-wave
Only one is stable in the *P*-wave

Systematic studies



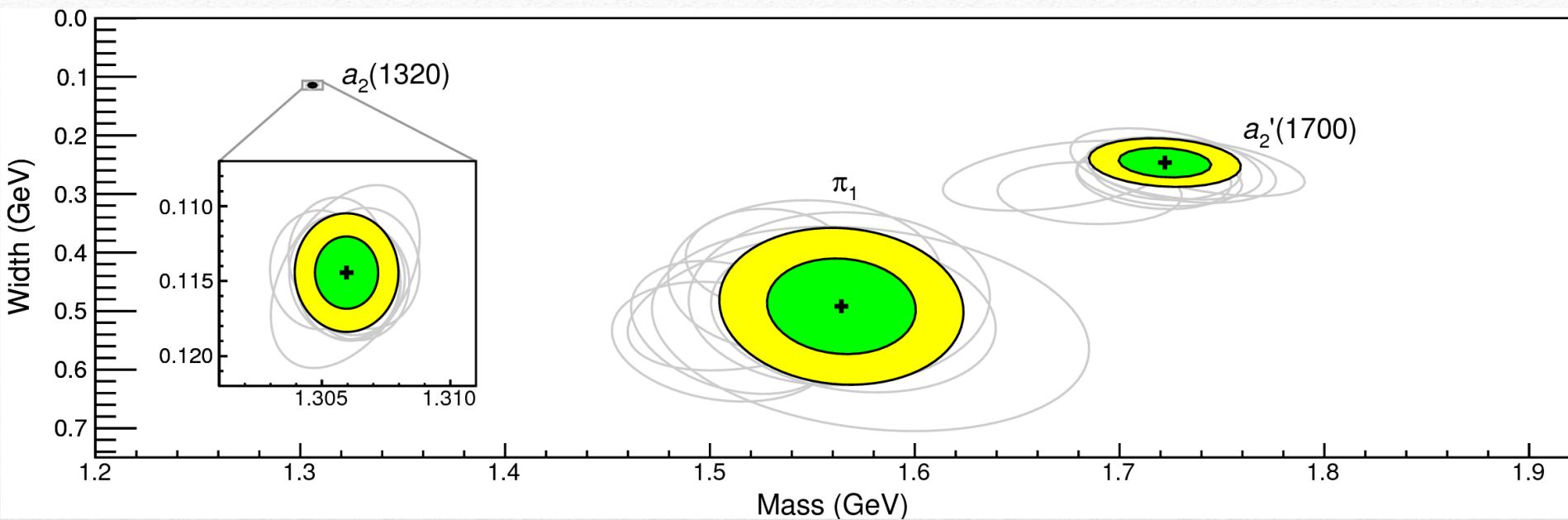
For each class, the maximum deviation of mass and width is taken as a systematic error
Deviation smaller than the statistical error are neglected
Systematic of different classes are summed in quadrature

Bootstrap for $s_R = 1.8 \text{ GeV}^2$



Our skepticism about a second pole in the relevant region is confirmed:
It is unstable and not trustable

Final results



Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	$1306.0 \pm 0.8 \pm 1.3$	$114.4 \pm 1.6 \pm 0.0$
$a'_2(1700)$	$1722 \pm 15 \pm 67$	$247 \pm 17 \pm 63$
π_1	$1564 \pm 24 \pm 86$	$492 \pm 54 \pm 102$

Agreement with Lattice is restored

That's the **most rigorous** extraction
of an exotic meson available so far!

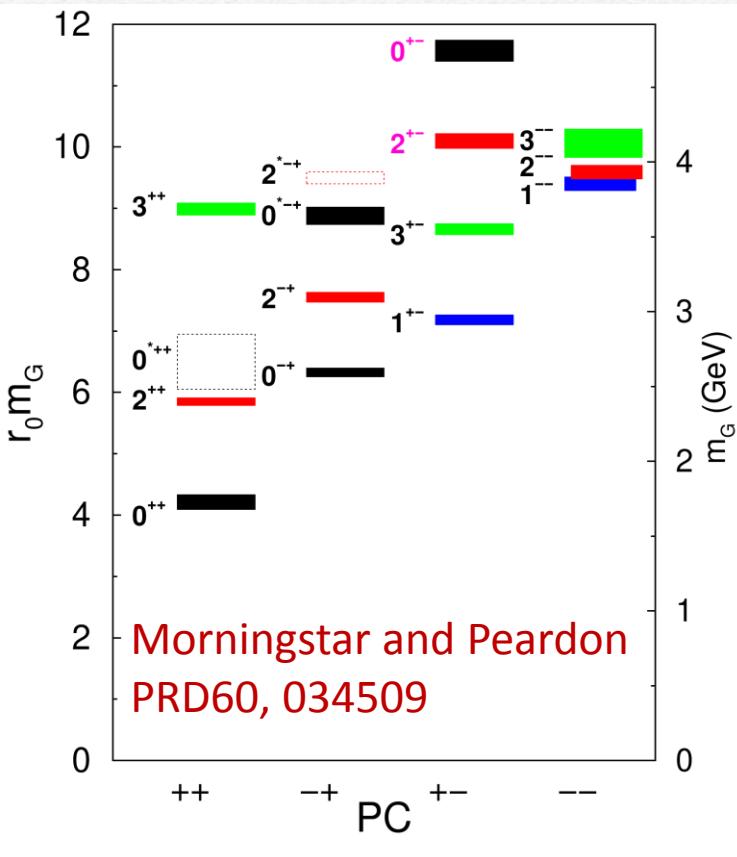
The scalar glueball

A. Rodas, AP *et al.* (JPAC) in progress



Glueballs

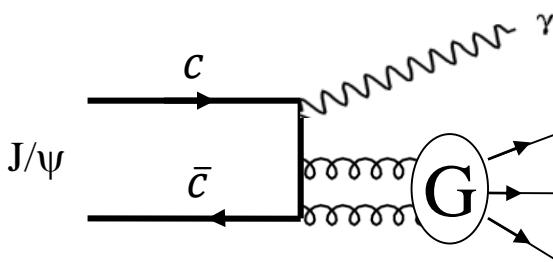
The **clearest** sign of confinement in pure Yang-Mills
The **worst** state to search in real life



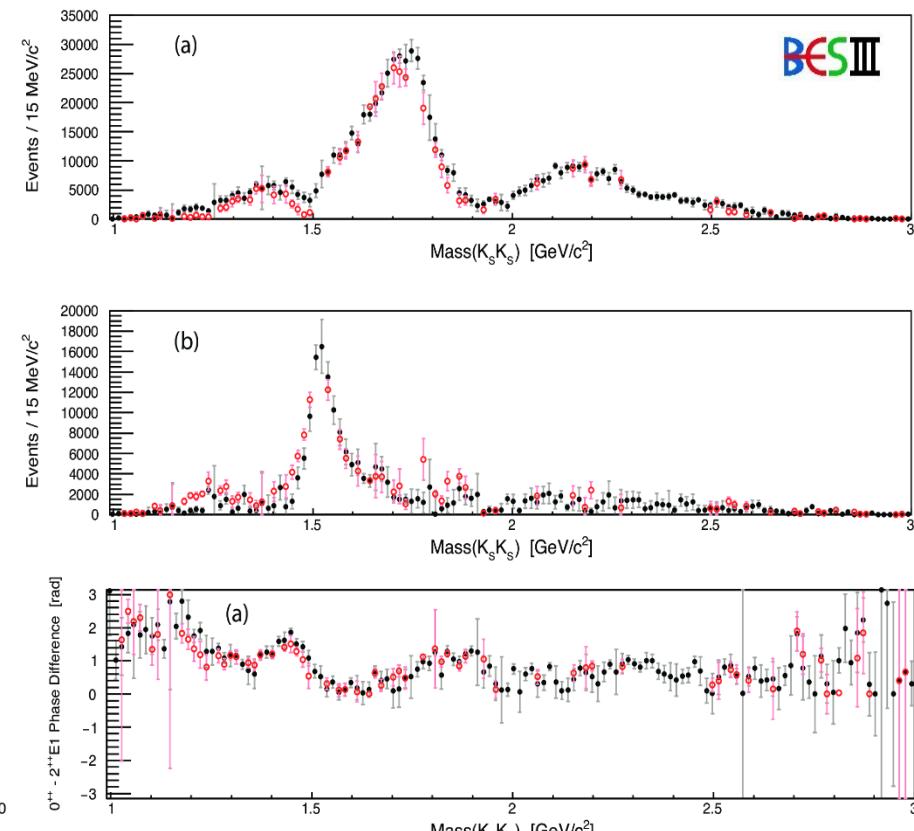
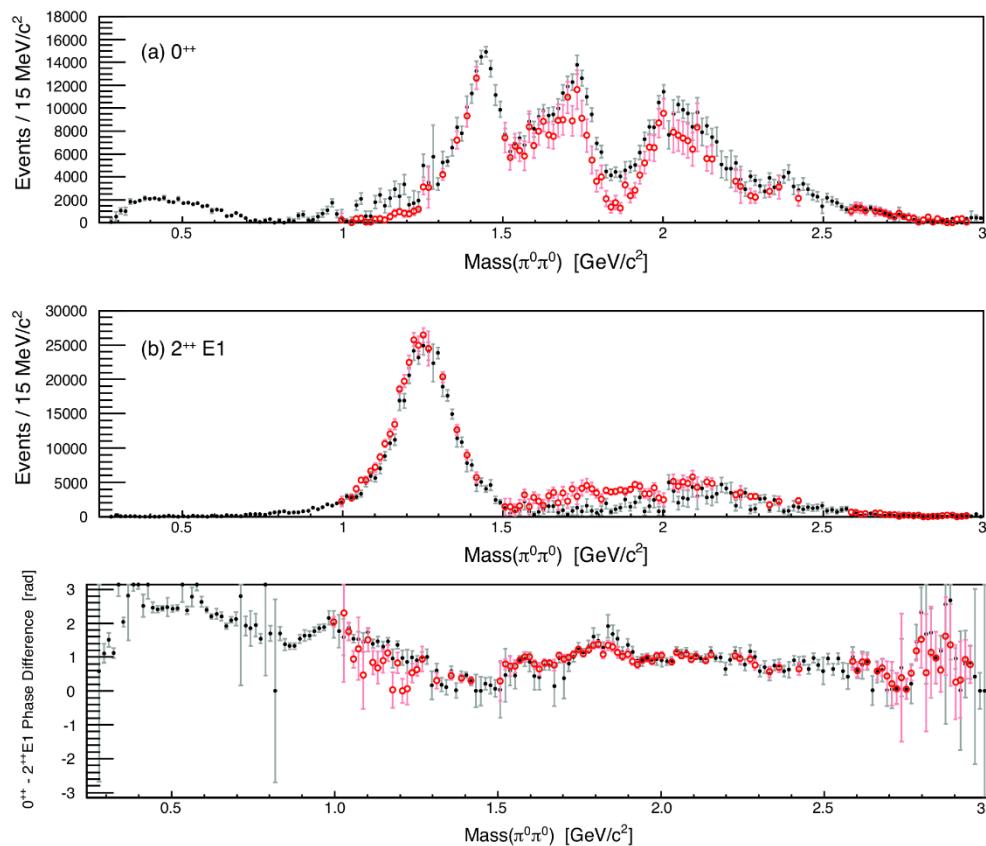
J^{PC}	Mass MeV			
	Unquenched This work	Quenched		
		M&P	Ky	Meyer
0 ⁻⁺		2590(40)(130)	2560(35)(120)	2250(60)(100)
2 ⁻⁺	3460(320)	3100(30)(150)	3040(40)(150)	2780(50)(130)
0 ⁻⁺	4490(590)	3640(60)(180)		3370(150)(150)
2 ⁻⁺				3480(140)(160)
5 ⁻⁻				3942(160)(180)
0 ⁻⁻ (exotic)	5166(1000)			
1 ⁻⁻		3850(50)(190)	3830(40)(190)	3240(330)(150)
2 ⁻⁻	4590(740)	3930(40)(190)	4010(45)(200)	3660(130)(170)
2 ⁻⁻				3.740(200)(170)
3 ⁻⁻		4130(90)(200)	4200(45)(200)	4330(260)(200)
1 ⁻⁻	3270(340)	2940(30)(140)	2980(30)(140)	2670(65)(120)
3 ⁻⁻	3850(350)	3550(40)(170)	3600(40)(170)	3270(90)(150)
3 ⁻⁻				3630(140)(160)
2 ^{+ -} (exotic)		4140(50)(200)	4230(50)(200)	
0 ^{+ -} (exotic)	5450(830)	4740(70)(230)	4780(60)(230)	
5 ^{- +}				4110(170)(190)
0 ⁺⁺	1795(60)	1730(50)(80)	1710(50)(80)	1475(30)(65)
2 ⁺⁺	2620(50)	2400(25)(120)	2390(30)(120)	2150(30)(100)
0 ⁺⁺	3760(240)	2670(180)(130)		2755(30)(120)
3 ⁺⁺		3690(40)(180)	3670(50)(180)	3385(90)(150)
0 ⁺⁺				3370(100)(150)
0 ⁺⁺				3990(210)(180)
2 ⁺⁺				2880(100)(130)
4 ⁺⁺				3640(90)(160)
6 ⁺⁺				4360(260)(200)

Gregory et al.
JHEP1210, 170

$J/\psi \rightarrow \gamma \pi^0\pi^0$ and $\rightarrow \gamma K_S^0 K_S^0$



This is a gluon-rich process, expected to be one of the golden channels for the search of the scalar glueball



Same model as before

Two channels, $i, k = \pi\pi, KK$

Two waves, $J = S, D$

52 parameters

$$D_{ki}^J(s) = \left[K^J(s)^{-1} \right]_{ki} - \frac{s}{\pi} \int_{s_k}^{\infty} ds' \frac{\rho N_{ki}^J(s')}{s'(s' - s - i\epsilon)}$$

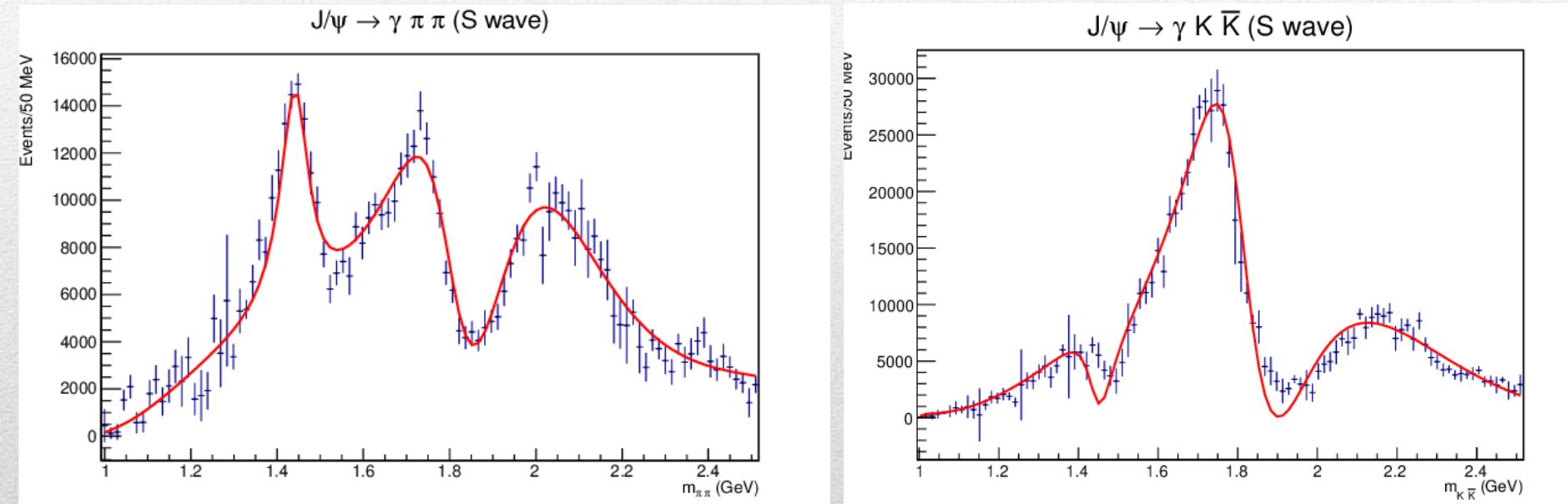
$$K_{ki}^J(s) = \sum_R \frac{g_k^{(R)} g_i^{(R)}}{m_R^2 - s} + c_{ki}^J + d_{ki}^J s$$

3 *K*-matrix pole for the S-wave
3 *K*-matrix poles for the D-wave

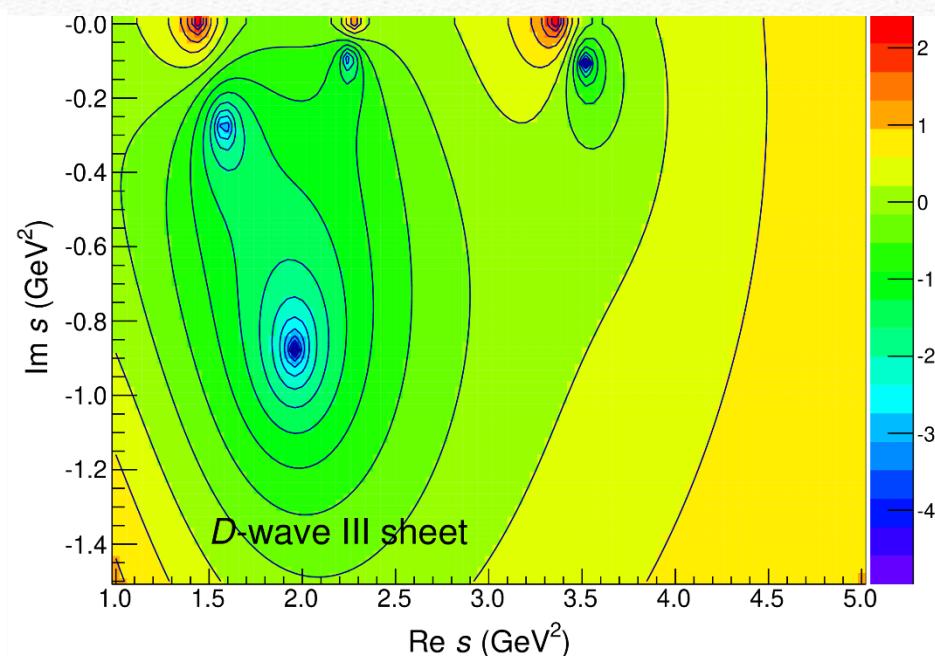
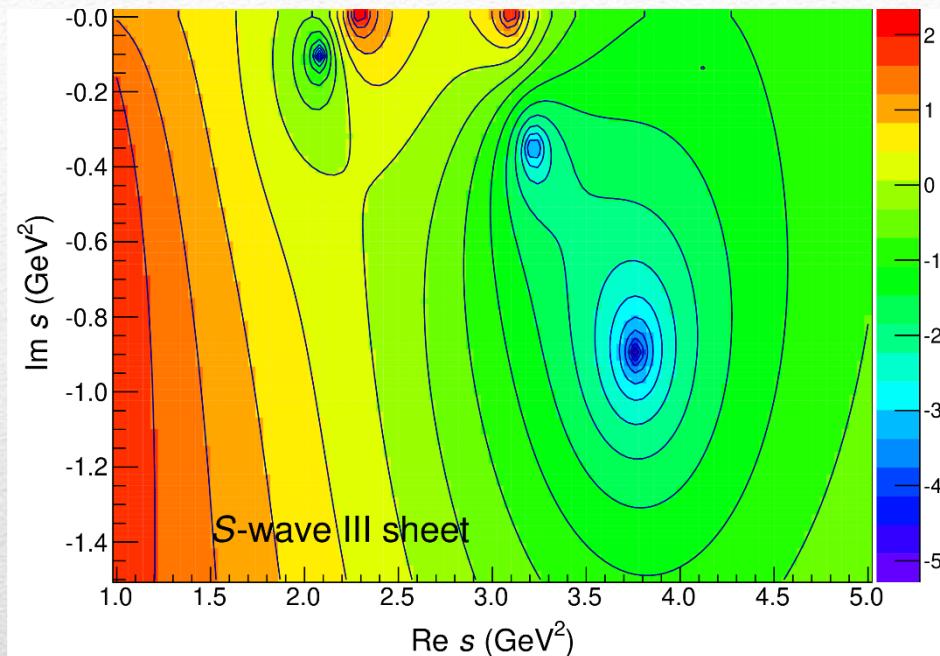
$$\rho N_{ki}^J(s') = \delta_{ki} \frac{\lambda^{J+1/2} \left(s', m_{\eta^{(\prime)}}^2, m_\pi^2 \right)}{(s' + s_R)^{2J+1+\alpha}}$$

$$n_k^J(s) = \sum_{n=0}^3 a_n^{J,k} T_n \left(\frac{s}{s + s_0} \right)$$

Fit results (preliminary)



Pole position (preliminary)



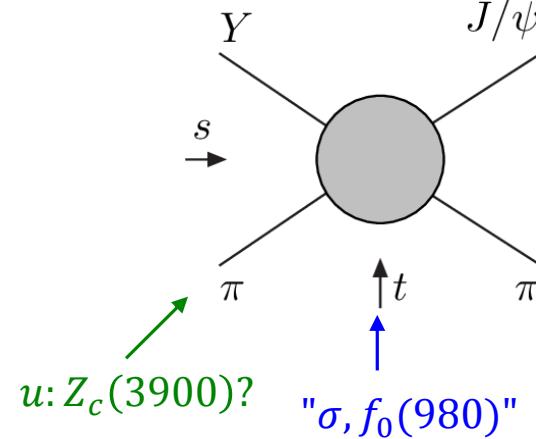
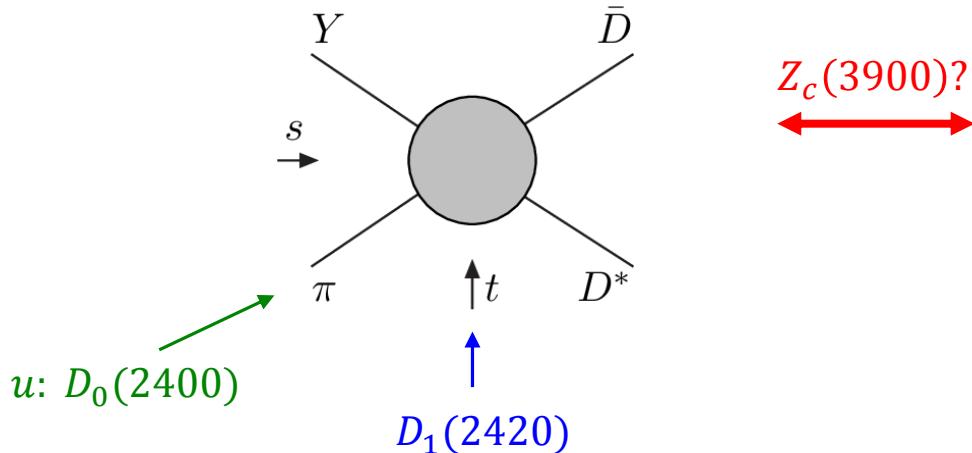
$$\begin{aligned} M(f_0(1500)) &= 1460 \text{ MeV} \\ M(f_0(1710)) &= 1800 \text{ MeV} \\ M(f_0(2020)) &= 1970 \text{ MeV} \end{aligned}$$

$$\begin{aligned} \Gamma(f_0(1500)) &= 85 \text{ MeV} \\ \Gamma(f_0(1710)) &= 190 \text{ MeV} \\ \Gamma(f_0(2020)) &= 490 \text{ MeV} \end{aligned}$$

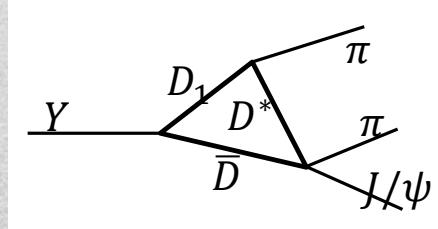
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to different singularities → different natures

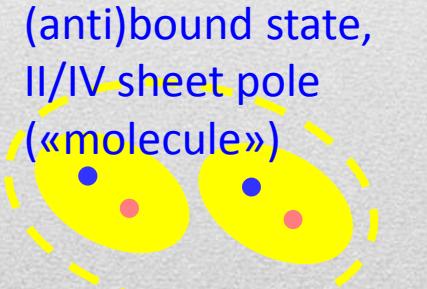
AP et al. (JPAC), PLB772, 200



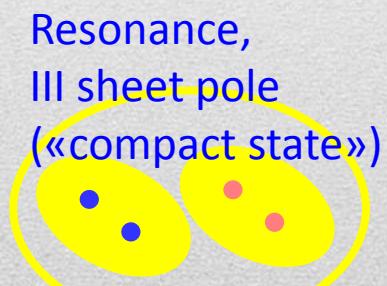
Triangle rescattering,
logarithmic branching point



Szczepaniak, PLB747, 410

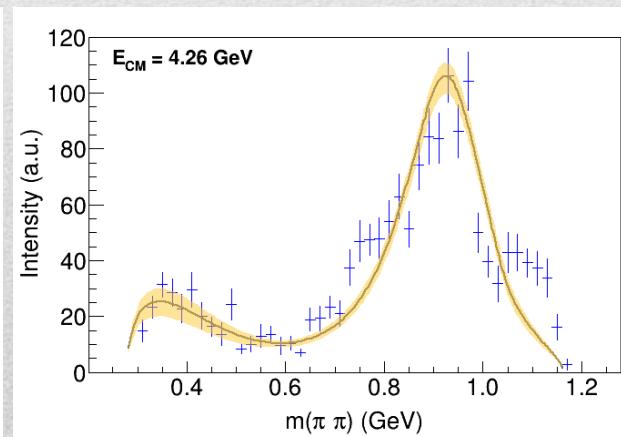
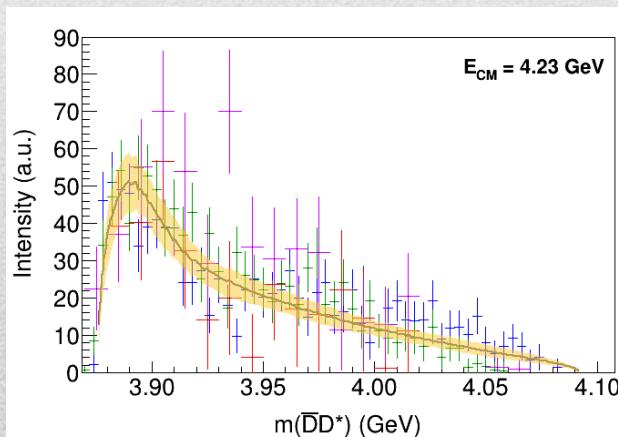
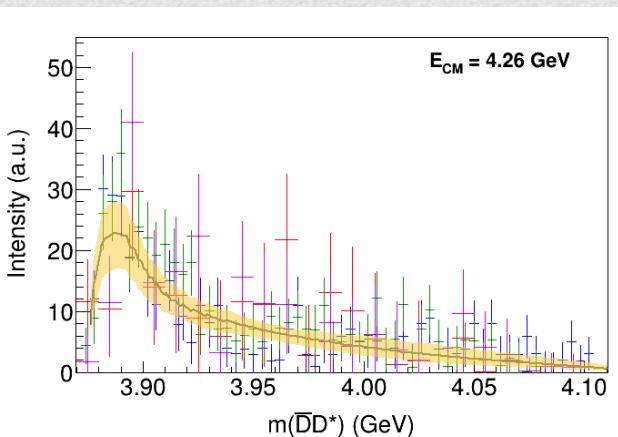
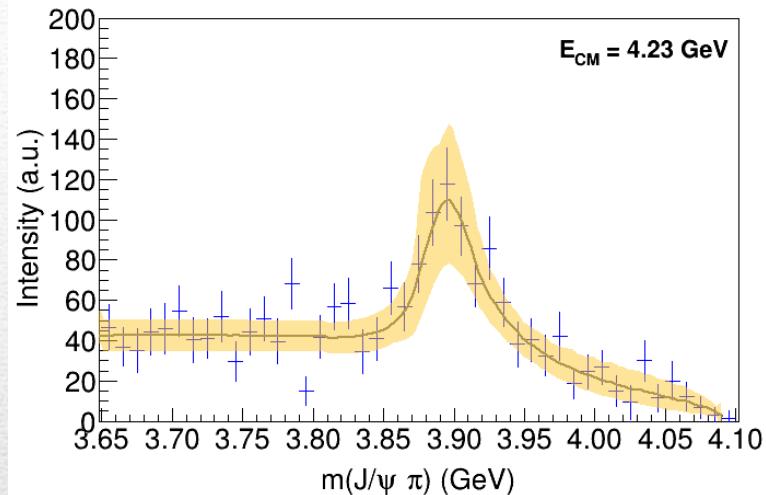
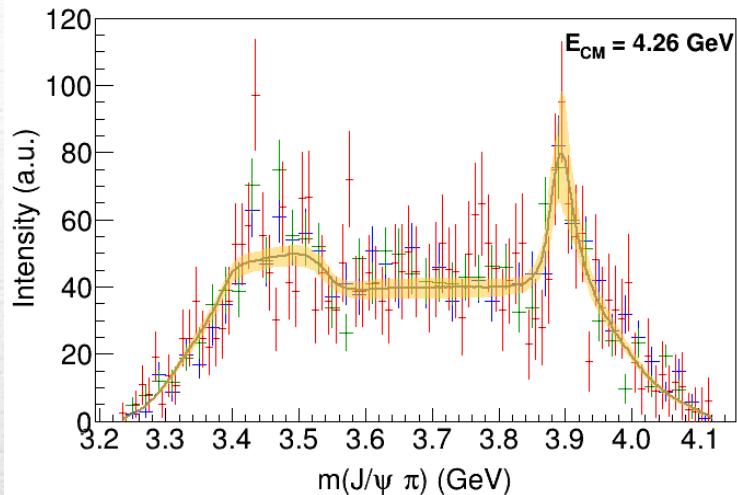


Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart et al. PRL111, 132003

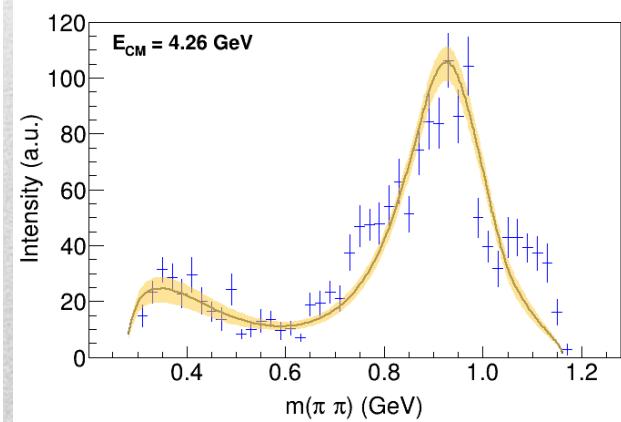
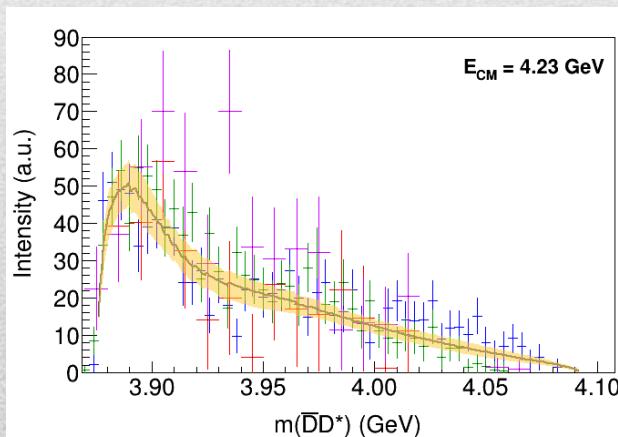
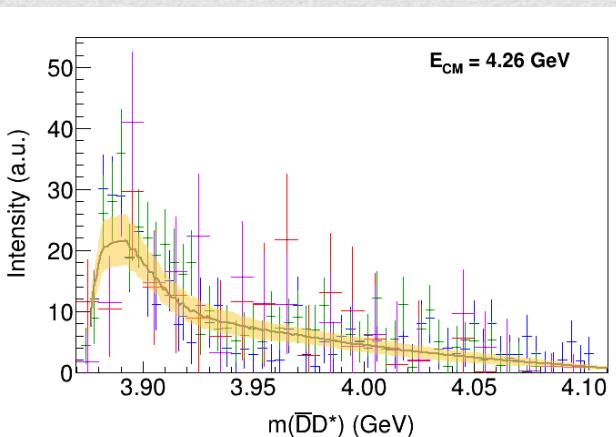
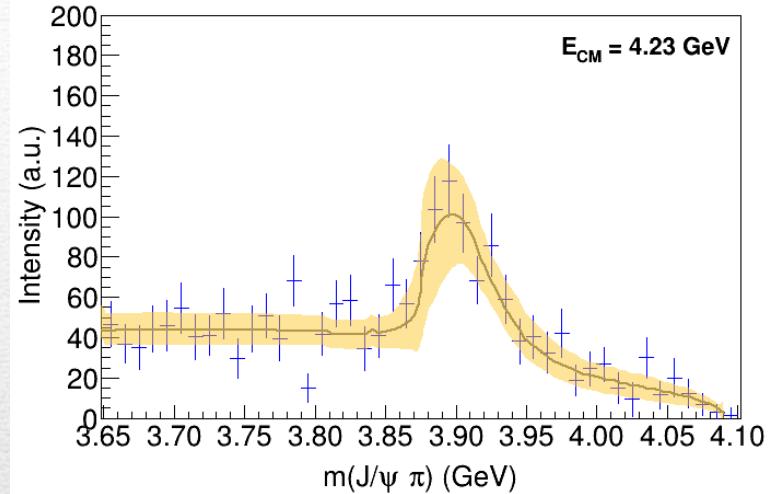
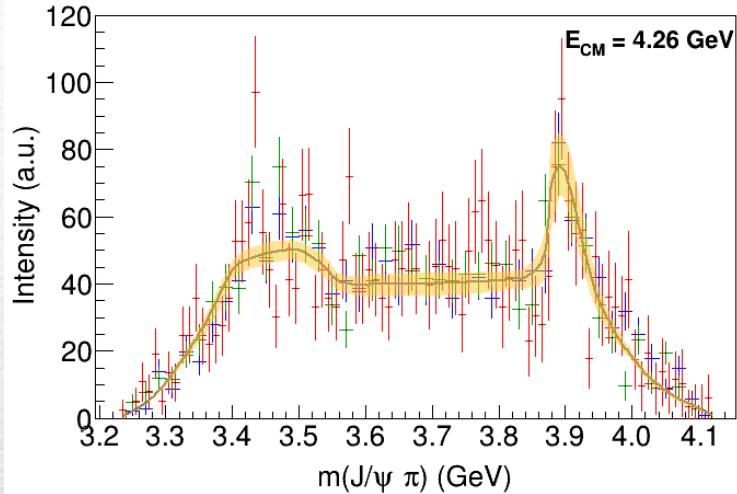


Maiani et al., PRD71, 014028
Faccini et al., PRD87, 111102
Esposito et al., Phys.Rept. 668

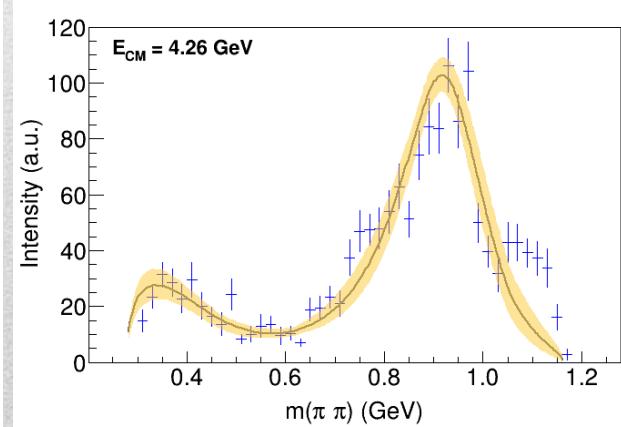
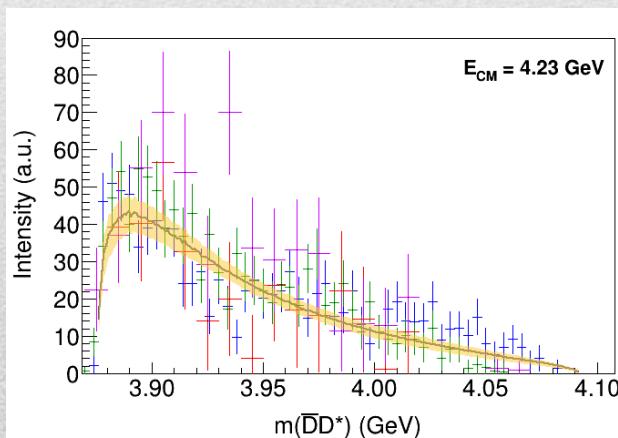
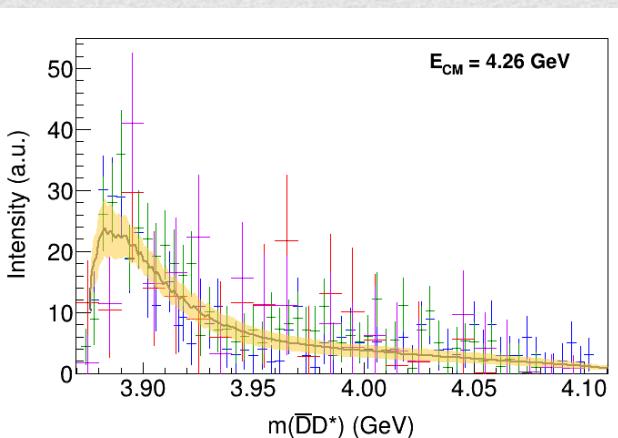
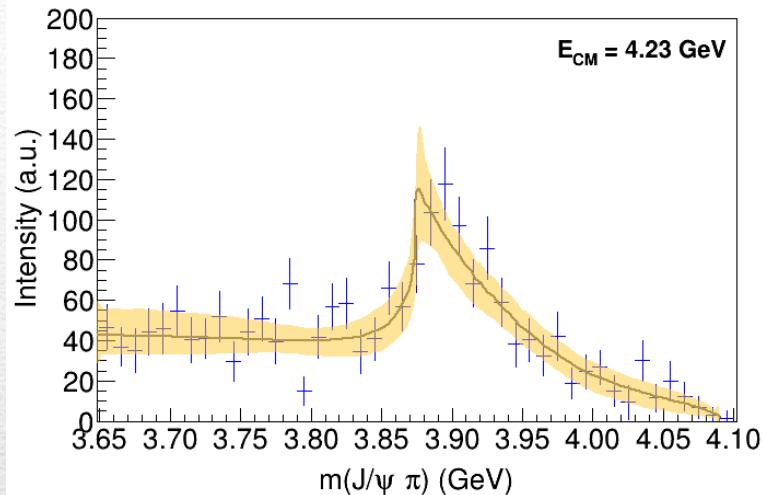
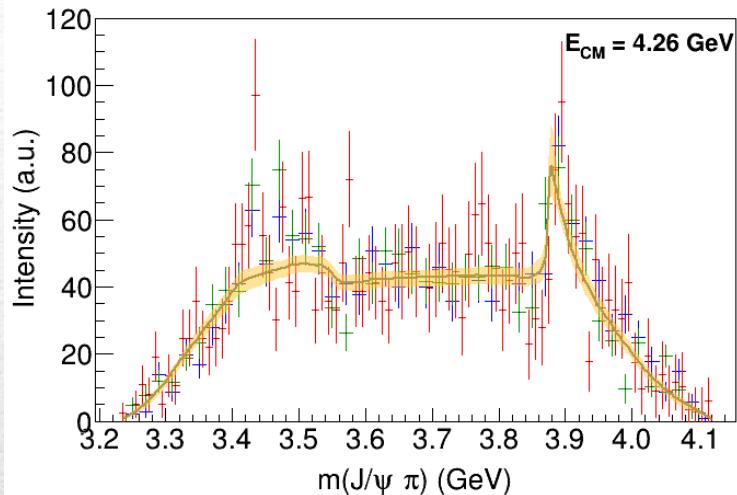
Fit: III



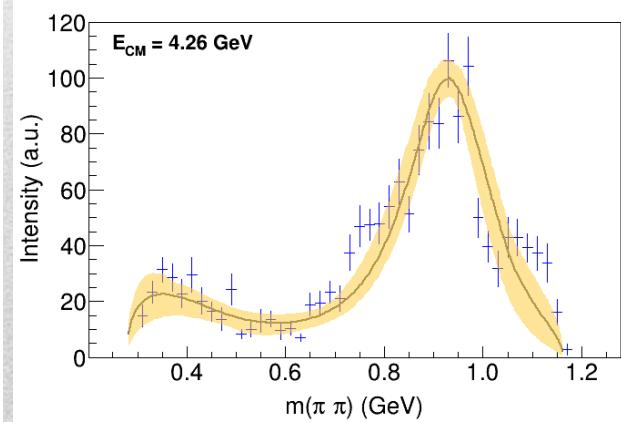
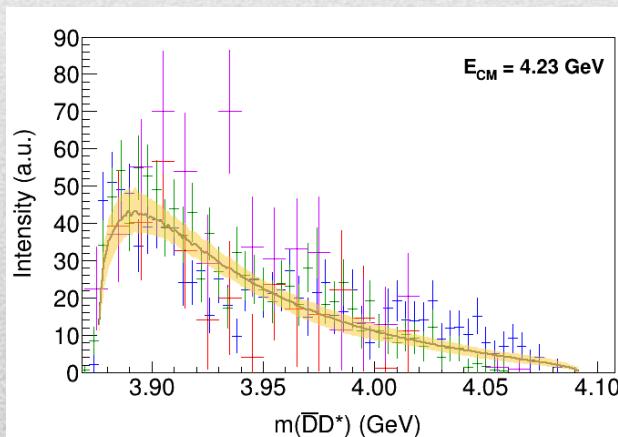
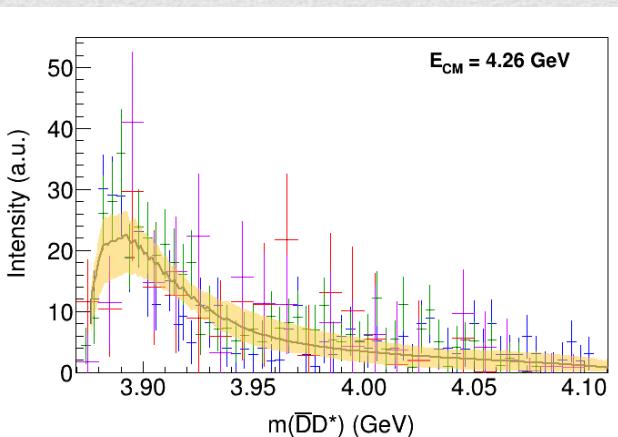
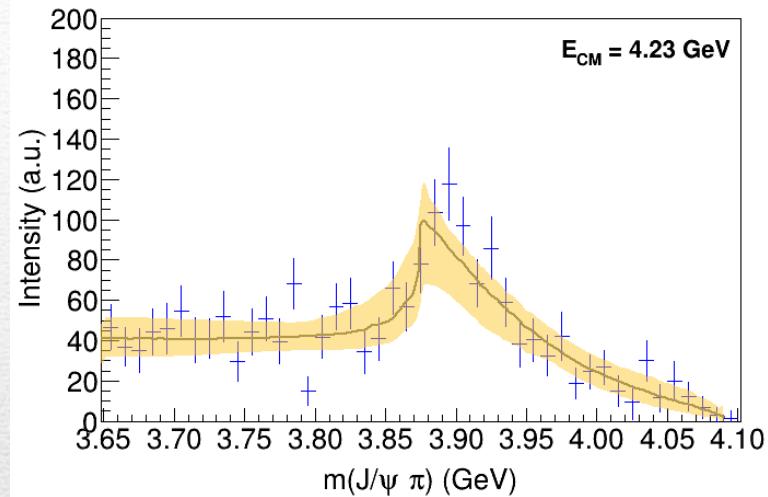
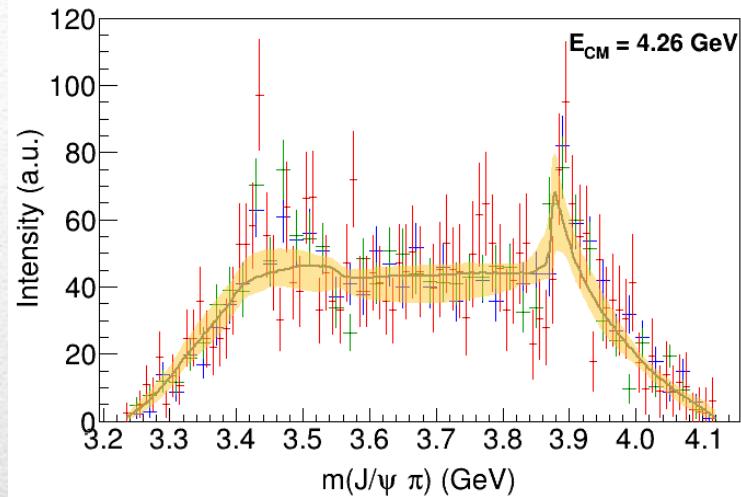
Fit: III+tr.



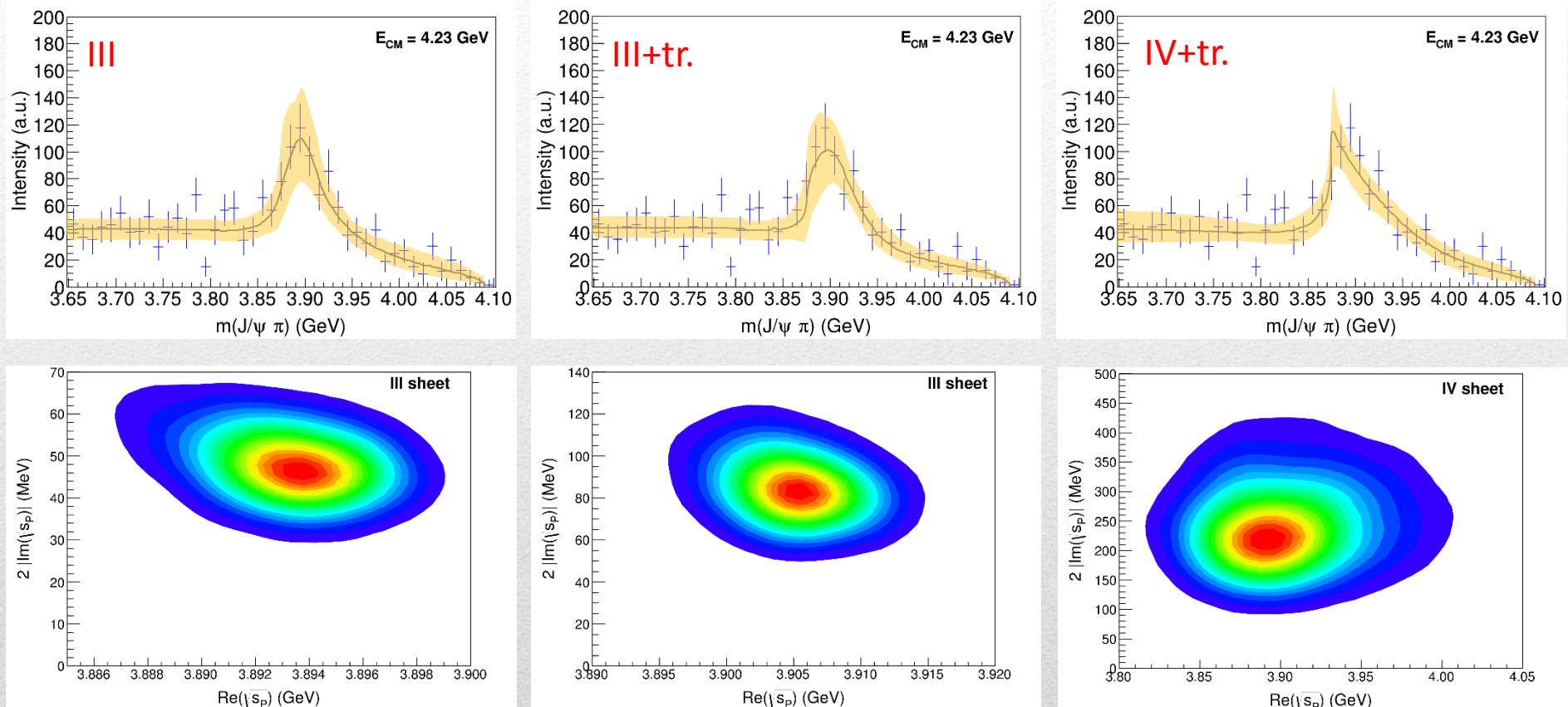
Fit: IV+tr.



Fit: tr.



Pole extraction

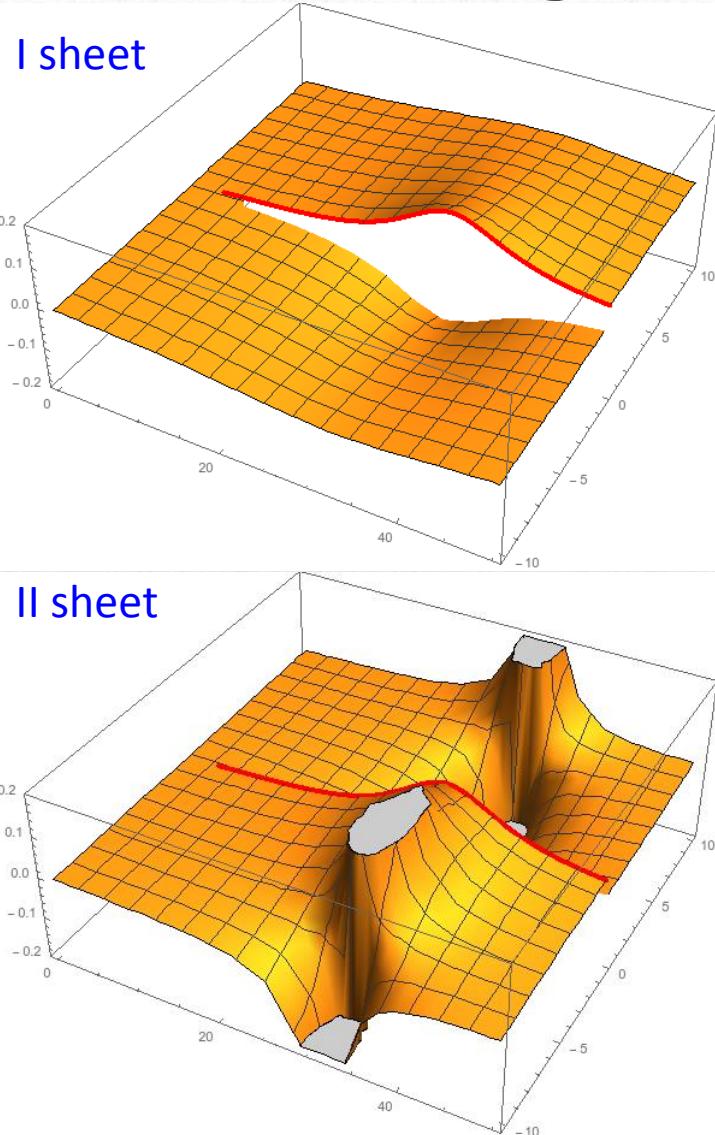


Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	" 2.4σ " (" 1.4σ ")
III+tr.	—	1.5σ (3.1σ)	" 2.6σ " (" 1.3σ ")
IV+tr.	—	—	" 2.1σ " (" 0.9σ ")

	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

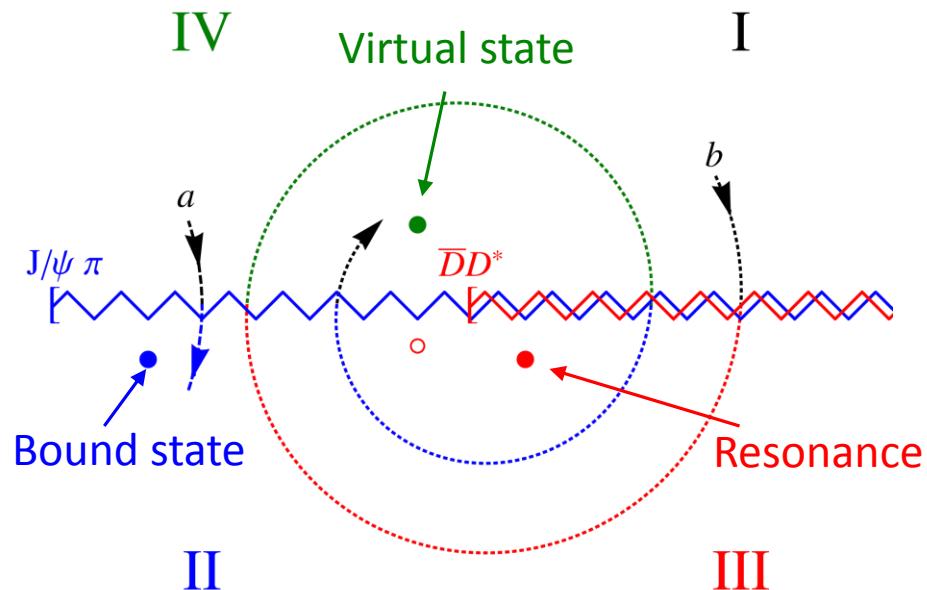
Not conclusive at this stage

Pole hunting



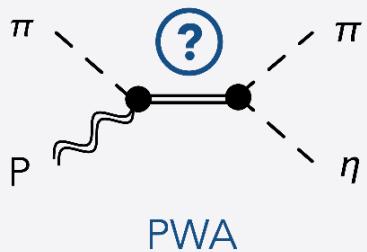
Extracting physics information
means to hunt for poles in the
complex plane

Pole position → Mass and width
Residues → Couplings

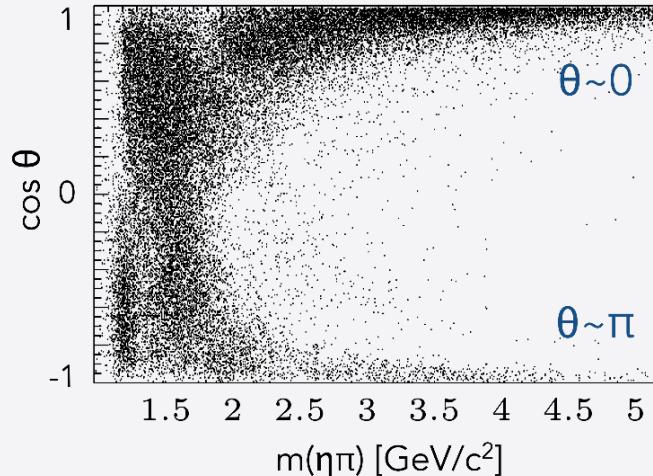
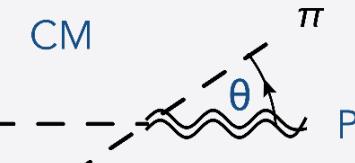


Finite energy sum rules

$$m(\eta\pi) < 3 \text{ (GeV/c}^2)^2$$

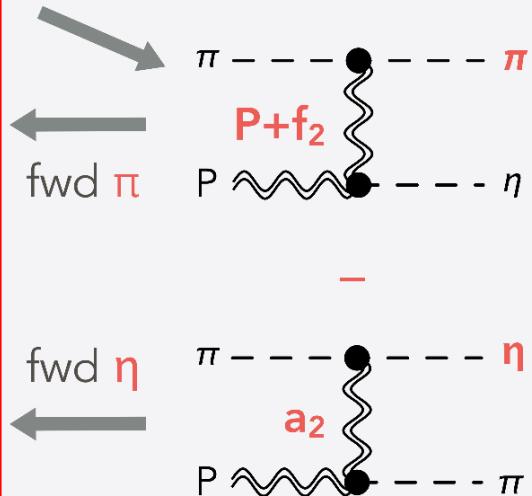


PWA in the low energy region
Resonance extraction



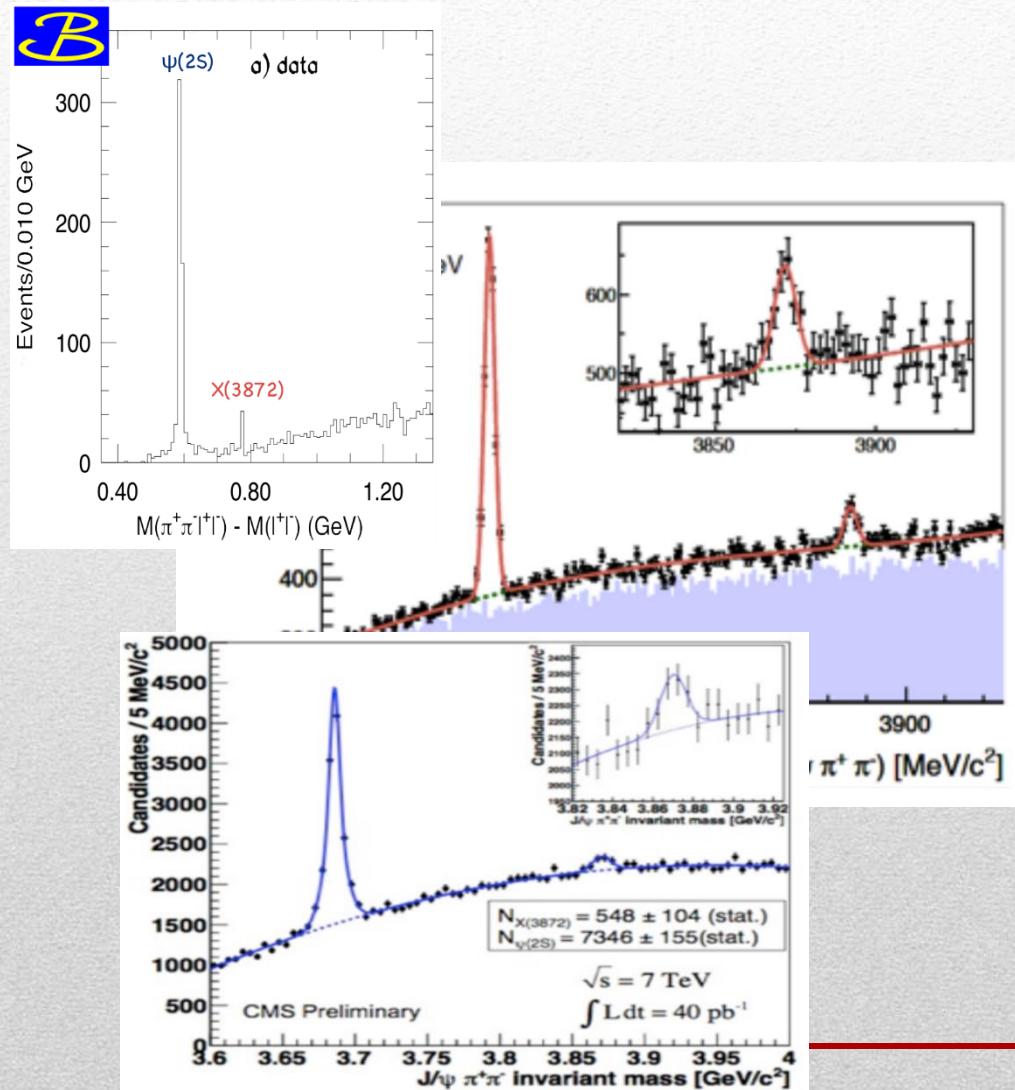
Analytically connected

$$m(\eta\pi) \in [5-6] \text{ (GeV/c}^2)^2$$



Regge exchanges at high energy

$X(3872)$



- Discovered in $B \rightarrow K X \rightarrow K J/\psi \pi\pi$
- Quantum numbers 1^{++}
- Very close to DD^* threshold
- Too narrow for an above-threshold charmonium
- Isospin violation too big

$$\frac{\Gamma(X \rightarrow J/\psi \omega)}{\Gamma(X \rightarrow J/\psi \rho)} \sim 0.8 \pm 0.3$$
- Mass prediction not compatible with $\chi_{c1}(2P)$

$$M = 3871.68 \pm 0.17 \text{ MeV}$$

$$M_X - M_{DD^*} = -3 \pm 192 \text{ keV}$$

$$\Gamma < 1.2 \text{ MeV} @ 90\%$$

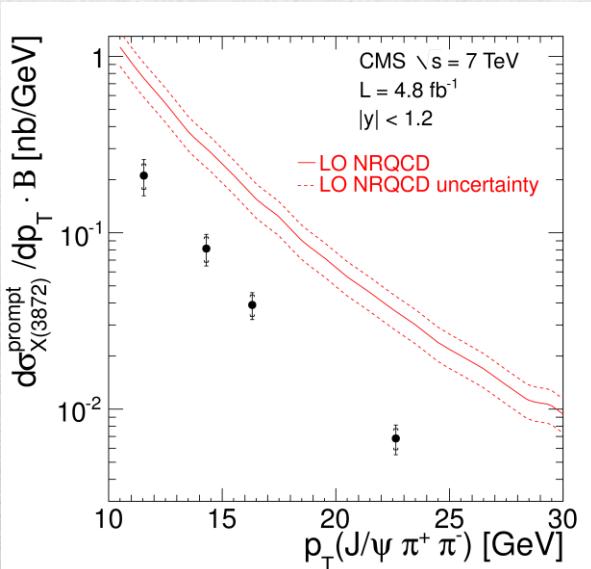
$X(3872)$

Large prompt production
at hadron colliders

$$\sigma_B/\sigma_{TOT} = (26.3 \pm 2.3 \pm 1.6)\%$$

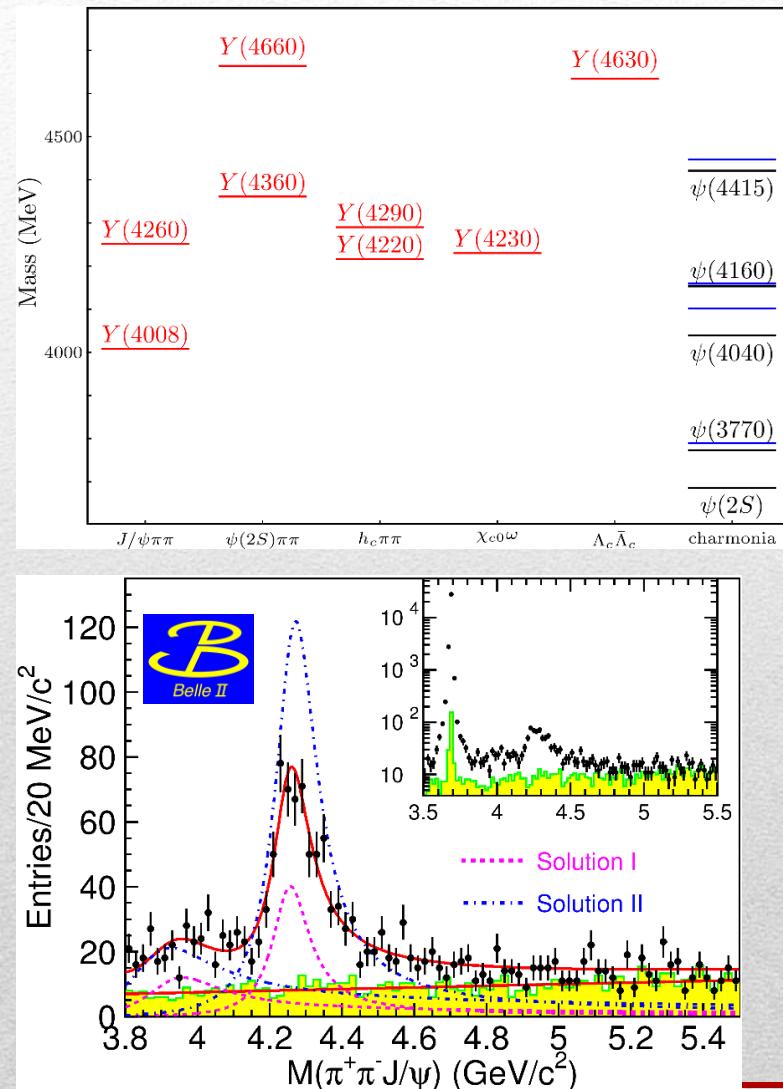
$$\begin{aligned}\sigma_{PR} \times B(X \rightarrow J/\psi \pi\pi) \\ = (1.06 \pm 0.11 \pm 0.15) \text{ nb}\end{aligned}$$

CMS, JHEP 1304, 154



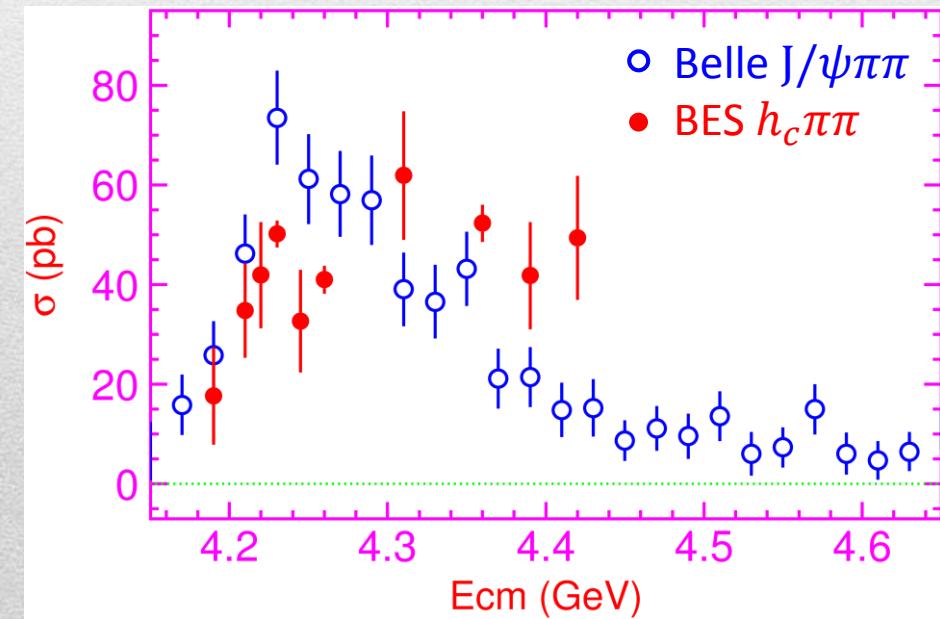
B decay mode	X decay mode	product branching fraction ($\times 10^5$)	B_{fit}	R_{fit}
$K^+ X$	$X \rightarrow \pi\pi J/\psi$	0.86 ± 0.08	(BABAR, ^[26] Belle ^[25])	0.081 ^{+0.019} _{-0.031}
		0.84 ± 0.15 ± 0.07		
		0.86 ± 0.08 ± 0.05		
$K^0 X$	$X \rightarrow \pi\pi J/\psi$	0.41 ± 0.11	(BABAR, ^[26] Belle ^[25])	0.77 ^{+0.28} _{-0.32}
		0.35 ± 0.19 ± 0.04		
		0.43 ± 0.12 ± 0.04		
$(K^+\pi^-)_{NR} X$	$X \rightarrow \pi\pi J/\psi$	$0.81 \pm 0.20^{+0.11}_{-0.14}$	Belle ^[106]	
$K^{*0} X$	$X \rightarrow \pi\pi J/\psi$	< 0.34, 90% C.L.	Belle ^[106]	
$K X$	$X \rightarrow \omega J/\psi$	$R = 0.8 \pm 0.3$	BABAR ^[33]	0.061 ^{+0.024} _{-0.036}
		0.6 ± 0.2 ± 0.1		
		0.6 ± 0.3 ± 0.1		
$K X$	$X \rightarrow \pi\pi\pi^0 J/\psi$	$R = 1.0 \pm 0.4 \pm 0.3$	Belle ^[32]	
$K^+ X$	$X \rightarrow D^{*0} \bar{D}^0$	8.5 ± 2.6	(BABAR, ^[38] Belle ^[37])	0.614 ^{+0.166} _{-0.074}
		16.7 ± 3.6 ± 4.7		
		7.7 ± 1.6 ± 1.0		
$K^0 X$	$X \rightarrow D^{*0} \bar{D}^0$	12 ± 4	(BABAR, ^[38] Belle ^[37])	8.2 ^{+2.3} _{-2.8}
		22 ± 10 ± 4		
		9.7 ± 4.6 ± 1.3		
$K^+ X$	$X \rightarrow \gamma J/\psi$	0.202 ± 0.038	(BABAR, ^[35] Belle ^[34])	0.019 ^{+0.005} _{-0.009}
$K^+ X$		0.28 ± 0.08 ± 0.01		
$K^0 X$		$0.178^{+0.048}_{-0.044} \pm 0.012$	Belle ^[34]	0.24 ^{+0.05} _{-0.06}
		0.26 ± 0.18 ± 0.02		
		$0.124^{+0.076}_{-0.061} \pm 0.011$		
$K^+ X$	$X \rightarrow \gamma\psi(2S)$	0.44 ± 0.12	BABAR ^[35]	0.04 ^{+0.015} _{-0.020}
		0.95 ± 0.27 ± 0.06		
		$0.083^{+0.198}_{-0.183} \pm 0.044$		
$K^0 X$		$R' = 2.46 \pm 0.64 \pm 0.29$	LHCb ^[36]	0.51 ^{+0.13} _{-0.17}
		1.14 ± 0.55 ± 0.10		
		$0.112^{+0.357}_{-0.290} \pm 0.057$		
$K^+ X$	$X \rightarrow \gamma\chi_{c1}$	$< 9.6 \times 10^{-3}$	Belle ^[23]	< 1.0 × 10 ⁻³
$K^+ X$	$X \rightarrow \gamma\chi_{c2}$	< 0.016		
$K X$	$X \rightarrow \gamma\gamma$	$< 4.5 \times 10^{-3}$	Belle ^[111]	< 4.7 × 10 ⁻⁴
$K X$	$X \rightarrow \eta J/\psi$	< 1.05		
$K^+ X$	$X \rightarrow p\bar{p}$	$< 9.6 \times 10^{-4}$	LHCb ^[110]	< 1.6 × 10 ⁻⁴

Vector Y states



Lots of unexpected $J^{PC} = 1^{--}$ states found in ISR/direct production (and nowhere else!) Seen in few final states, mostly $J/\psi\pi\pi$ and $\psi(2S)\pi\pi$

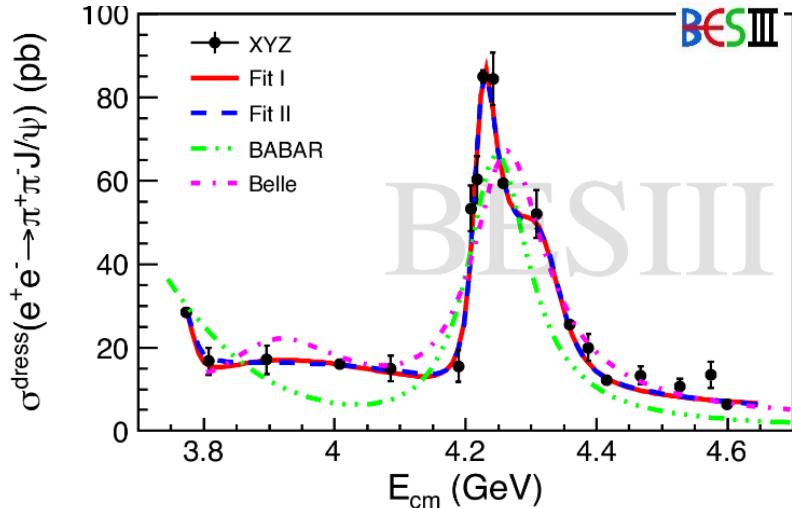
Not seen decaying into open charm pairs
Large HQSS violation



Vector Y states in BESIII

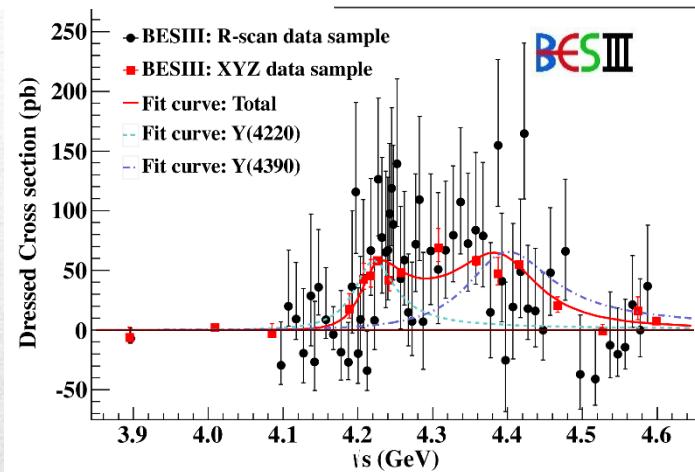
BESIII, PRL118, 092001 (2017)

$e^+e^- \rightarrow J/\psi \pi\pi$



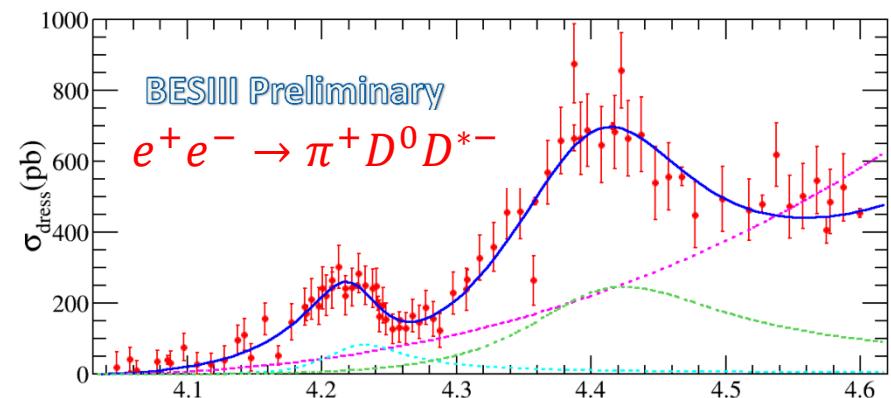
BESIII, PRL118, 092002 (2017)

$e^+e^- \rightarrow h_c \pi\pi$



New BESIII data show a peculiar lineshape for the $Y(4260)$

The state appear lighter and narrower, compatible with the ones in $h_c \pi\pi$ and $\chi_{c0} \omega$. A broader old-fashioned $Y(4260)$ is appearing in $\bar{D}D^*\pi$, maybe indicating a $\bar{D}D_1$ dominance



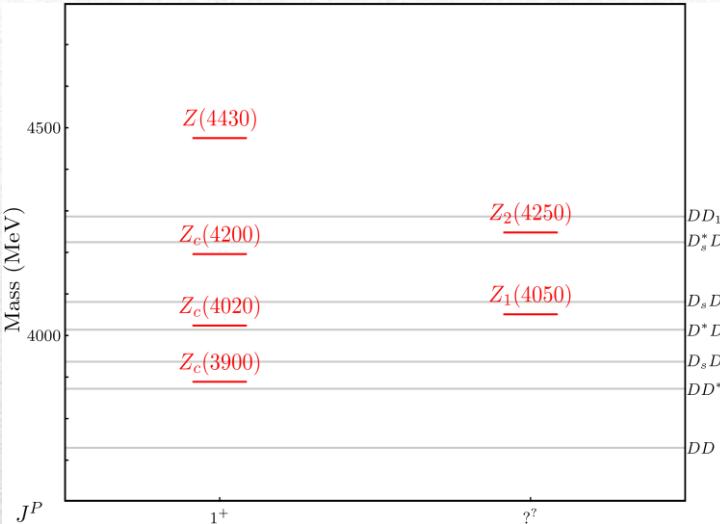
Fit with a constant (pink dashed triple-dot line) and two constant width relativistic BW functions (green dashed double-dot line and aqua dashed line).

$$M(Y(4220)) = (4224.8 \pm 5.6 \pm 4.0) \text{ MeV}/c^2, \Gamma(Y(4220)) = (72.3 \pm 9.1 \pm 0.9) \text{ MeV}$$

$$M(Y(4390)) = (4400.1 \pm 9.3 \pm 2.1) \text{ MeV}/c^2, \Gamma(Y(4220)) = (181.7 \pm 16.9 \pm 7.4) \text{ MeV}$$

Charged Z states: $Z_c(3900)$, $Z'_c(4020)$

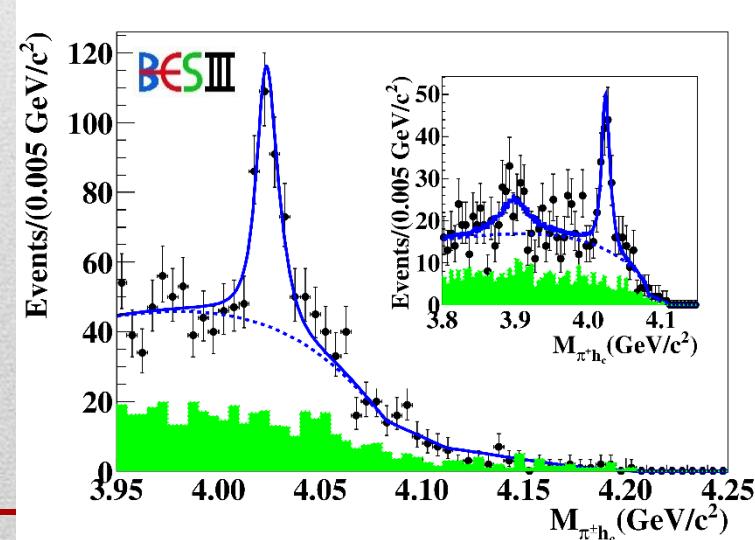
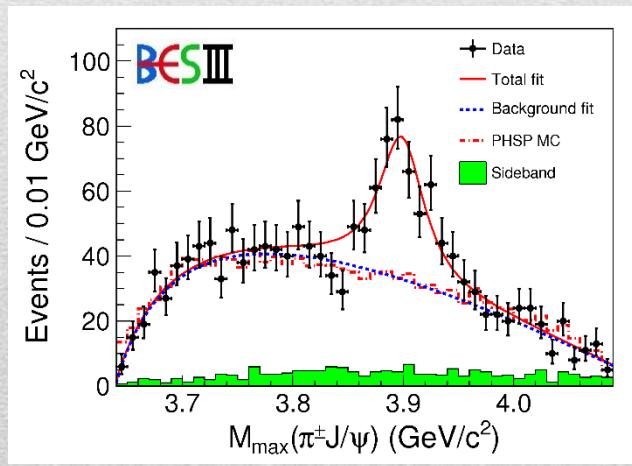
Charged quarkonium-like resonances have been found, **4q needed**



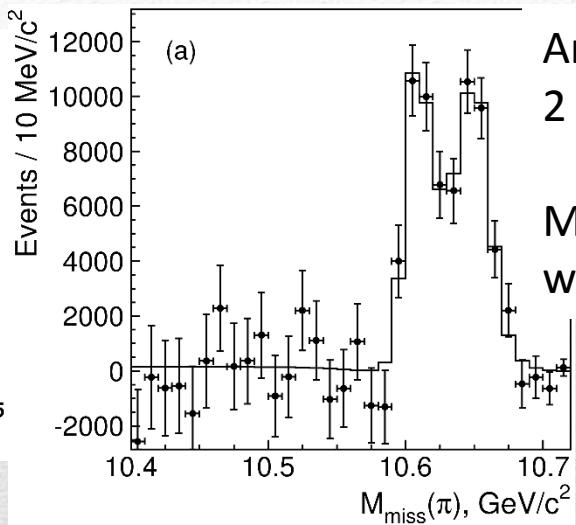
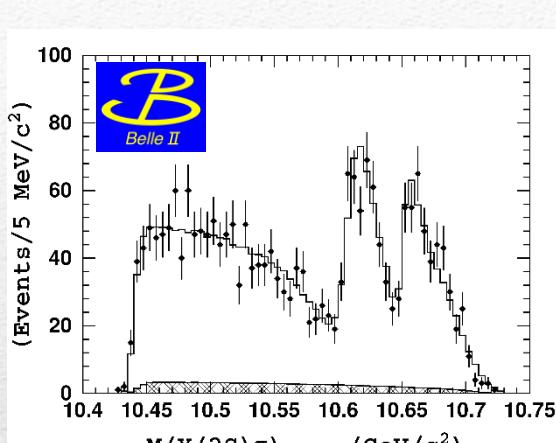
Two states $J^{PC} = 1^{+-}$ appear slightly above $D^{(*)}D^*$ thresholds

$e^+e^- \rightarrow Z_c(3900)^+\pi^- \rightarrow J/\psi\pi^+\pi^-$ and $\rightarrow (DD^*)^+\pi^-$
 $M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$

$e^+e^- \rightarrow Z'_c(4020)^+\pi^- \rightarrow h_c\pi^+\pi^-$ and $\rightarrow \bar{D}^{*0}D^{*+}\pi^-$
 $M = 4023.9 \pm 2.4 \text{ MeV}, \Gamma = 10 \pm 6 \text{ MeV}$



Charged Z states: $Z_b(10610)$, $Z'_b(10650)$



Moreover, observed $\Upsilon(5S) \rightarrow h_b(nP)\pi\pi$
which violates HQSS

2 twin resonances!

$$\Upsilon(5S) \rightarrow Z_b(10610)^+\pi^- \rightarrow \Upsilon(nS)\pi^+\pi^-, h_b(nP)\pi^+\pi^-$$

and $\rightarrow (BB^*)^+\pi^-$

$$M = 10607.2 \pm 2.0 \text{ MeV}, \Gamma = 18.4 \pm 2.4 \text{ MeV}$$

$$\Upsilon(5S) \rightarrow Z'_b(10650)^+\pi^- \rightarrow \Upsilon(nS)\pi^+\pi^-, h_b(nP)\pi^+\pi^-$$

and $\rightarrow \bar{B}^{*0}B^{*+}\pi^-$

$$M = 10652.2 \pm 1.5 \text{ MeV}, \Gamma = 11.5 \pm 2.2 \text{ MeV}$$