Symmetries and Conservation Laws

- Electric Charge
- Baryon Number
- Lepton Number
- Spin
- Parity
- Isospin
- Strangeness

A classification of symmetries in particle physics

Class	Invariance	Conserved quantity
Proper orthochronous Lorentz symmetry	translation in time (homogeneity)	energy
	translation in space (homogeneity)	linear momentum
	rotation in space (isotropy)	angular momentum
Discrete symmetry	P, coordinate inversion	spatial parity
	C, charge conjugation	charge parity
	T, time reversal	time parity
	CPT	product of parities
Internal symmetry (independent of spacetime coordinates)	U(1) gauge transformation	electric charge
	U(1) gauge transformation	lepton generation number
	U(1) gauge transformation	hypercharge
	$U(1)_Y$ gauge transformation	weak hypercharge
	U(2) [U(1) × SU(2)]	electroweak force
	SU(2) gauge transformation	Isospin
	$SU(2)_L$ gauge transformation	weak isospin
	P × SU(2)	G-parity
	SU(3) "winding number"	baryon number
	SU(3) gauge transformation	quark color
	SU(3) (approximate)	quark flavor
	S(U(2) × U(3)) [U(1) × SU(2) × SU(3)]	Standard Model

Lagrangian

It is a function that summarizes the dynamics of the system, starting from the principle of minimum action, valid also in quantum mechanics

$$\mathcal{L}(\dot{q}, q, t) = T(\dot{q}, q, t) - U(q, t)$$

Lagrangian

If the Lagrangian of a system is known, then the equations of motion of the system may be obtained by a direct substitution of the expression for the Lagrangian into the Euler–Lagrange Equation:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{q}} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} = 0$$

The principle of minimum action applies also to quantum mechanics.

$$\mathcal{S}[\mathbf{q}] \triangleq \int_{t_1}^{t_2} \mathcal{L}(\mathbf{q}(t), \dot{\mathbf{q}}(t), t)\, \mathrm{d}\, t$$

Action

$$\frac{\delta \mathcal{S}}{\delta \mathbf{q}(t)} = 0$$

Principle of minimum action

Hamiltonian

In Hamiltonian mechanics the time evolution of the system is uniquely defined by Hamilton's equation

$$\frac{d\boldsymbol{p}}{dt} = -\frac{\partial \mathcal{H}}{\partial \boldsymbol{q}}$$
$$\frac{d\boldsymbol{q}}{dt} = +\frac{\partial \mathcal{H}}{\partial \boldsymbol{p}}$$

where is the Hamiltonian, which corresponds to the **total energy** of the system. For a closed system, it is the sum of the kinetic and potential energy of the system.

$$\mathcal{H} = T + V, \quad T = \frac{p^2}{2m}, \quad V = V(q).$$

Hamiltonian from Lagrangian

Given a Lagrangian in terms of the generalized coordinates and generalized velocities and time:

momenta are calculated by differentiating the Lagrangian with respect to the eneralized) velocities:

$$p_i(q_i, \dot{q}_i, t) = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

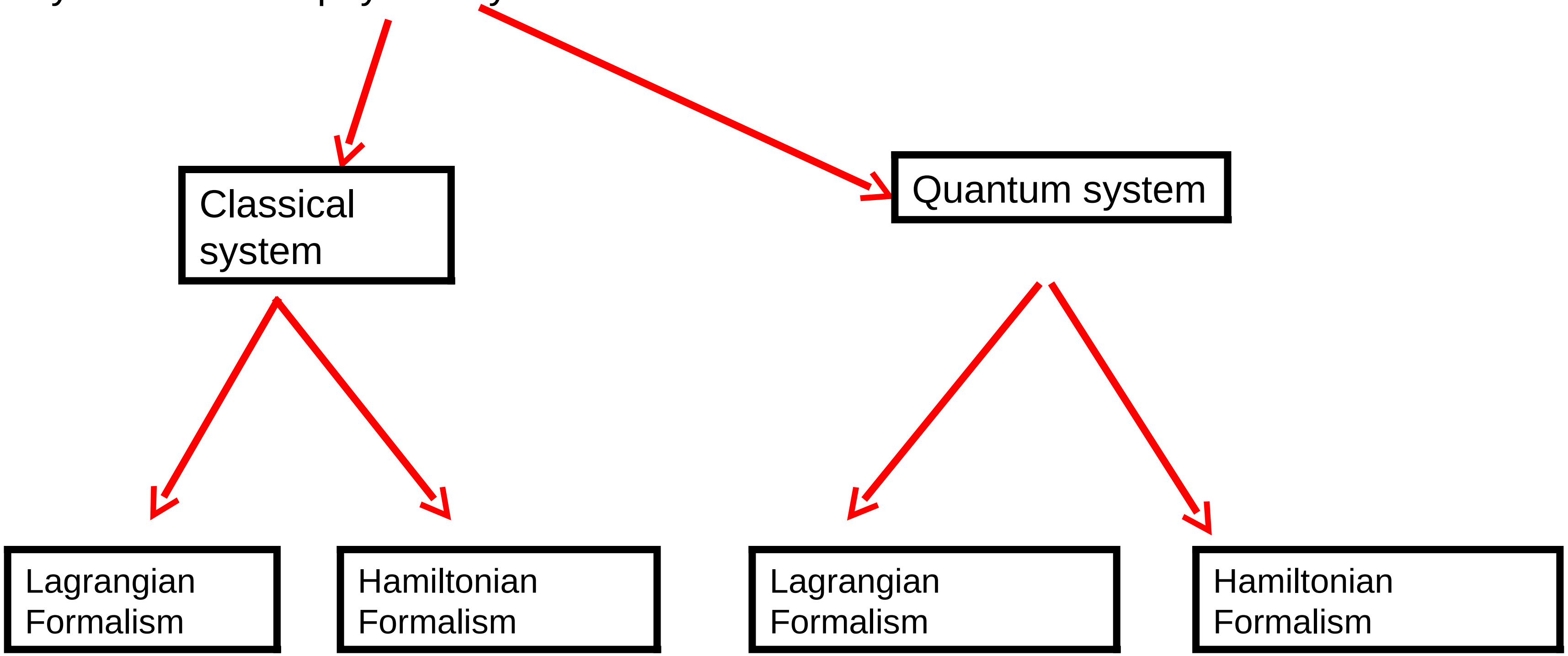
Velocities qi are expressed in terms of the momenta pi by inverting the expressions the previous step.

Hamiltonian is calculated using the usual definition of as the Legendre transformation :

$$\mathcal{H} = \sum_i \dot{q}_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \mathcal{L} = \sum_i \dot{q}_i p_i - \mathcal{L}$$

nen the velocities are substituted for using the previous results.

Symmetries of a physical system:



Invariance of Equations of Motion

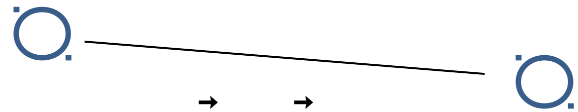
- Invariance of dynamica equations
- Invariance of commutation relations

(Invariance of probability)

E. Noether's Theorem (valid for any lagrangian theory, classical or quantum) relates symmetries to conserved quantities of a physical system

A “classical” example :

$$\begin{cases} T = \frac{1}{2} m_1 \vec{r}_1^{\,2} + \frac{1}{2} m_2 \vec{r}_2^{\,2} \\ V = V(\vec{r}_1 - \vec{r}_2) \end{cases}$$



$\vec{r}_1 - \vec{r}_2$

$$m_1 \ddot{\vec{r}}_1 = -\frac{\partial}{\partial \vec{r}_1} V(\vec{r}_1 - \vec{r}_2) \qquad m_2 \ddot{\vec{r}}_2 = -\frac{\partial}{\partial \vec{r}_2} V(\vec{r}_1 - \vec{r}_2)$$

Let us do a translation : $\vec{r}_i \rightarrow \vec{r}_i^{\,'} = \vec{r}_i + \vec{a}$

$$V(\vec{r}_1 - \vec{r}_2) \rightarrow V(\vec{r}_1 + \vec{a} - \vec{r}_2 - \vec{a}) = V(\vec{r}_1 - \vec{r}_2)$$

$$\Rightarrow \quad m_i \ddot{\vec{r}}_i^{\,'} = -\frac{\partial}{\partial \vec{r}_i^{\,'}} V(\vec{r}_1 - \vec{r}_2)$$

The equations of motion are translation Invariant !

If one calculates the forces acting on 1 and 2:

$$\vec{F}_{TOT} = \vec{F}_1 + \vec{F}_2 = -\frac{\partial}{\partial \vec{r}_1} V\left(\vec{r}_1 - \vec{r}_2\right) - \frac{\partial}{\partial \vec{r}_2} V\left(\vec{r}_1 - \vec{r}_2\right) = \frac{\partial}{\partial \vec{r}_2} V\left(\vec{r}_1 - \vec{r}_2\right) - \frac{\partial}{\partial \vec{r}_2} V\left(\vec{r}_1 - \vec{r}_2\right) = 0$$

$$m_i \ddot{\vec{r}}_i' = -\frac{\partial}{\partial \vec{r}_i'} V\left(\vec{r}_1 - \vec{r}_2\right)$$

In the classical Lagrangian formalism :

$$L = L(q_i, \dot{q}_i) \qquad p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

$$\frac{dp_i}{dt} = \frac{\partial L}{\partial q_i}$$

L invariant with respect to q

p conserved

In the Hamiltonian formalism

$$\dot{q}_i = \{ q_i , H \}$$

$$\dot{p}_i = \{ p_i , H \}$$

$$\frac{d\omega(q_i, p_i)}{dt} = \{ \omega, H \}$$

Possible conservation of a dinamical quantity

Possible symmetry

This formalism can easily be used in Quantum Mechanics

In Quantum Mechanics, starting from the Schroedinger Equation :

$$i \hbar \frac{\partial}{\partial t} \psi_s(t) = H \psi_S(t)$$

$$\psi_s(t) = \exp\left[-i (t - t_0) H / \hbar \right] \psi_S(t_0)$$

$T(t, t_0)$ Time evolution (unitary operator)

Schroedinger and Heisenberg Pictures:

$$\vec{r}_i \rightarrow \vec{r}_i^{\,\prime} = \vec{r}_i + \vec{a}$$

$$\psi_S(t_0)^* Q(t)\, \psi_S(t_0) = \psi_S(t)^*\, Q_0\, \psi_S(t)$$

Heisenberg

Schroedinger

$$\psi_S(t_0)^* Q(t)\, \psi_S(t_0) = \psi_S(t_0)^*\, T^{-1} Q_0\, T\psi_S(t_0)$$

$$Q(t) = T^{-1} Q_0\, T$$ Operators in the Heisenberg picture

Taking the derivatives:

$$i\hbar \frac{d}{dt} Q(t) = i\hbar \frac{dT^{-1}}{dt} Q_0\, T + i\hbar\, T^{-1} Q_0 \frac{dT}{dt} = -HT^{-1} Q_0 T + T^{-1} Q_0 TH$$

$$i\hbar \frac{d}{dt} Q(t) = -HQ + QH = [Q, H]$$ Conserved quantities: commute with H

$$i\hbar \frac{d}{dt} Q(t) = i\hbar \frac{\partial Q}{\partial t} + [Q, H]$$ In the case when there is an explicit time dependence (non-isolated systems)

Translational invariance: a continuous spacetime symmetry

$$\psi(r+\delta r) = \psi(r) + \delta r \frac{\partial \psi}{\partial r} = \left(1+\delta r \frac{\partial}{\partial r}\right)\psi = D\,\psi$$

The translation operator is naturally associated to the linear momentum

$$D(\delta r) = \left(1+\frac{i}{\hbar}p\,\delta r\right)$$

For a finite translation :

$$D(\Delta r) = \lim_{n\to\infty}\left(1+\frac{i}{\hbar}p\,\delta r\right)^n = \exp\left(\frac{i}{\hbar}p\,\Delta r\right) \qquad (\Delta r = n\,\delta r)$$

unitary

the generator of space translations

If H does not depend on coordinates $[D,H]=0$

⇓

momentum is conserved $[p,H]=0$

Rotational invariance: a continuous spacetime symmetry

$$T = \frac{1}{2} m_1 \vec{r}_1^{\,2} + \frac{1}{2} m_2 \vec{r}_2^{\,2}$$

The rotation operator is naturally associated with angular momentum

$$J_z = -i\hbar \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) = -i\hbar \frac{\partial}{\partial \varphi}$$

Angular momentum (z-comp.) operator (angle phi)

A finite rotation

unitary

Self-adjoint: rotation generator

$$R(\Delta\varphi) = \lim_{n \to \infty} \left(1 + \frac{i}{\hbar} J_z \, \delta\varphi \right)^n = \exp\left(\frac{i}{\hbar} J_z \, \Delta\varphi \right) \qquad \Delta\varphi = n\,\delta\varphi$$

If H does not depend on the rotation angle φ around the z-axis

$$[R, H] = 0 \quad \Longrightarrow \quad [J_z, H] = 0$$

The angular momentum is conserved

Time invariance (a continuous symmetry)

The generator of time translation is actually the energy!

$$\psi_s(t) = \exp\left[-i\left(t-t_0\right) H / \hbar\right] \psi_S(t_0)$$

Using the equation of motion of the operators :

$$i\hbar \frac{d}{dt} H(t) = i\hbar \frac{\partial Q}{\partial t} + \left[Q, H\right] \Rightarrow i\hbar \frac{d}{dt} H(t) = i\hbar \frac{\partial H}{\partial t} + \left[H, H\right]$$

If H does not dipend from t, the energy is conserved

The continous spacetime symmetries:

Space translation	→	Linear momentum
Space rotation	→	Angular momentum
Time translation	→	Energy

Electric charge Q

- Charge conservation can also be understood as a consequence of symmetry through Noether's theorem. The symmetry that is associated with charge conservation is the global gauge invariance of the electromagnetic field.

- The full statement of gauge invariance is that the physics of an electromagnetic field are unchanged when the scalar and vector potential are shifted by the gradient of an arbitrary scalar field χ

$$\phi' = \phi - \frac{\partial \chi}{\partial t} \qquad \mathbf{A}' = \mathbf{A} + \nabla \chi.$$

Electric charge Q

Quantum Numbers are quantised properties of particles that are subject to constraints. They are often related to symmetries.

Electric Charge Q is conserved in all interactions.

✓

	π^-	+	p	$\rightarrow$	K^0	+	Λ^0
Q:	-1		+1		0		0

Strong Interaction

✓

	K^0	$\rightarrow$	π^+	+	π^-
Q:	0		+1		-1

Weak Interaction

Electric charge Q

- In quantum mechanics the scalar field is equivalent to a phase shift in the wavefunction of the charged particle:

$$\psi' = e^{i\chi}\psi$$

- so gauge invariance is equivalent to the well known fact that changes in the phase of a wavefunction are unobservable, and only changes in the magnitude of the wavefunction result in changes to the probability function $|\psi|^2$.

Barion Number B

There is no field theory that would imply the existence of a conserved quantity such as lepton number and baryon number. For that reason, it is believed that baryon and lepton number are only approximately conserved. No evidence is yet seen for baryon or lepton number violation.

Baryons have $B = +1$ Antibaryons have $B = -1$ Everything else has $B = 0$	or	Quarks have $B = +\frac{1}{3}$ Antiquarks have $B = -\frac{1}{3}$ Everything else has $B = 0$

Baryons = $qqq = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ Mesons = $q\bar{q} = \frac{1}{3} + -\frac{1}{3} = 0$

Baryon Number B is conserved in Strong, EM and Weak interactions.

Total (quarks - antiquarks) is constant.

Barion Number B

✓

	π^-	+	p	$\rightarrow$	K^0	+	Λ^0
B:	0		+1		0		+1

Strong Interaction

✓

	Λ^0	$\rightarrow$	p	+	π^-
B:	+1		+1		0

Weak Interaction

Since the proton is the lightest baryon it cannot decay if B is conserved e.g:

✗

	p	$\not\rightarrow$	e^+	+	π^0
B:	+1		0		0

Lepton Number

Pair Production ✓

	γ	+	N	$\rightarrow$	e^+	+	e^-	+	N
L_e:	0		0		-1		+1		0

Pion Decay ✓

	π^+	$\rightarrow$	μ^+	+	ν_μ
L_μ:	0		-1		+1

Muon Decay ✓

	μ^+	$\rightarrow$	e^+	+	ν_e	+	$\bar{\nu}_\mu$
L_e :	0		-1		+1		0
L_μ :	-1		0		0		-1
L :	-1		-1		+1		-1

✗

	μ^+	$\not\to$	e^+	+	γ
L_e :	0		-1		0
L_μ :	-1		0		0
L :	-1		-1		0

L_e, L_μ ← Forbidden

L ← OK

L is conserved but neither L_e or L_μ separately.

The decay $\mu^+ \to e^+ + \gamma$ has not been observed and has a "Branching Ratio" $< 10^{-9}$.

Barion-lepton conservation

- Gauge invariance → conservation law (i.e. charge)

- In field theories with local gauge simmetry: absolutely conserved quantity implies long-range field (i.e Em field) coupled to the charge

- If baryon number were absolutely conserved (from local gauge simmetry), a long-range field coupled to it should exist.

- No evidence for such a field! However:

charge conservation → $\tau\left(n \to p \nu_e \bar{\nu}_e\right) > 10^{18}\ yr$

lepton conservation → $\tau\left({}^{76}Ge \to {}^{76}Se + e^- + e^+\right) > 10^{26}\ yr$

baryon conservation → $\tau\left(p \to e^+ \pi^0\right) > 10^{33}\ yr$

- Highest limits are on the lepton and baryon nr conservation, even if not protected by any gauge principle

- Other reasons for baryon non-conservation:

huge baryon-antibaryon asimmetry in the Universe (N_B today $\approx 10^{79}$!)

- For practical purposes, we will assume that baryon and lepton nr are conserved, even if there is no deep theoretical reasons to suppose this conservation rule as absolute.

- While total lepton number seems to be conserved, weak transition between leptons of different flavours (e.g.: $\nu_e \rightarrow \nu_\mu$) can be possible (see: experiments on neutrino oscillations)

Spin S

- W. Paulі introduced for the 1st time a fourth quantic number - the **spin** - to completely describe the electron state inside the atomic orbitals

- No physics meaning was assigned to the spin until 1927, when the experiment of Phipps ad Taylor associated to the spin a magnetic moment of the electron.

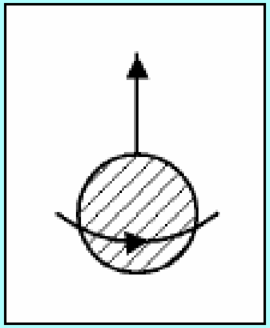
- The electron spin can assume only two values: +1/2 and –1/2: it is an intrinsic attribute of the electron and it appears only in a relativistic scenario

- Later, it was possible to attribute the spin to other particles (m, p e n) by applying the law of the angular momentum conservation or the principle of the detailed balance

Spin is an intrinsic property of all particles:

$0\hbar, 1\hbar, 2\hbar, 3\hbar, \ldots$ Bosons

$\frac{1}{2}\hbar, \frac{3}{2}\hbar, \frac{5}{2}\hbar, \frac{7}{2}\hbar, \ldots$ Fermions



Spin is like angular momentum but a Quantum Mechanical effect.

For spin S there are $2S+1$ states of different S_z (like $2J+1$ in Angular Momentum) e.g.

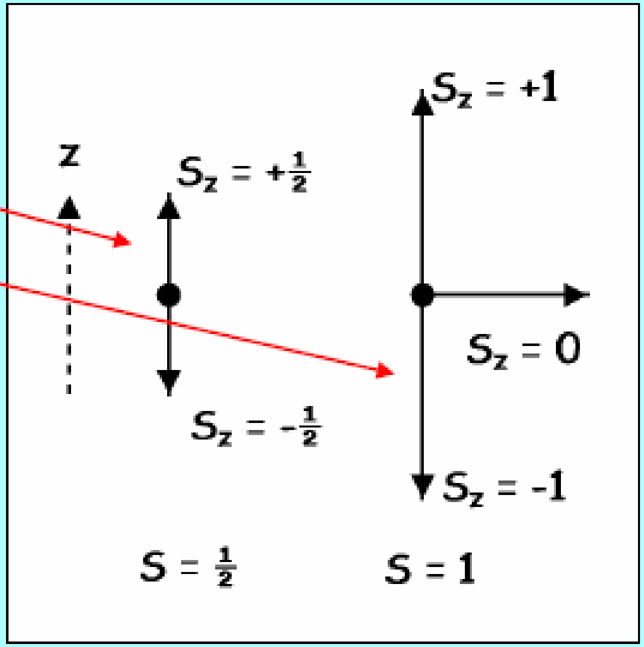
For Spin $S = \frac{1}{2}$, S_z can be $+\frac{1}{2}$ or $-\frac{1}{2}$ (2 states).

For Spin $S = 1$, S_z can be $+1, 0, -1$ (3 states).

For a process $a + b \rightarrow c + d$ the cross section

$$\sigma \sim (2S_c + 1)(2S_d + 1) \times \text{Other Factors}$$

This can be used to determine the spin of unknown particles.



• Spin and cross sections

Suppose the initial-state particles are unpolarised.
Total number of final spin substates available is:
$g_f = (2s_c+1)(2s_d+1)$
Total number of initial spin substates: $g_i = (2s_a+1)(2s_b+1)$

One has to average the transition probability over all possible initial states, all equally probable, and sum over all final states
⇒ Multiply by factor g_f / g_i

$$a+b \rightarrow c+d$$
$$a+\bar{c} \rightarrow \bar{b}+d$$
$$a+\bar{d} \rightarrow c+\bar{b}$$
$$a \rightarrow \bar{b}+c+d$$
$$c+d \rightarrow a+b$$

• All the so-called crossed reactions are allowed as well, and described by the same matrix-elements (but different kinematic constraints)

Good quantum numbers:
if associated with a conserved observables
(= operators commute with the Hamiltonian)

Spin: one of the quantum numbers which characterise any particle
(elementary or composite)

Spin $\mathbf{S}_p$ of the particle, $\vec{J}$ is the total angular momentum **J** of its costituents in their centre-of-mass-frame

Quarks are spin-1/2 particles → the spin quantum number $S_p = J$ can be integer or half integer

The spin projection on the z-axis – Jz- can assume any of 2J+ 1 values, from –J to J, with steps of 1, depending on the particle's spin orientation

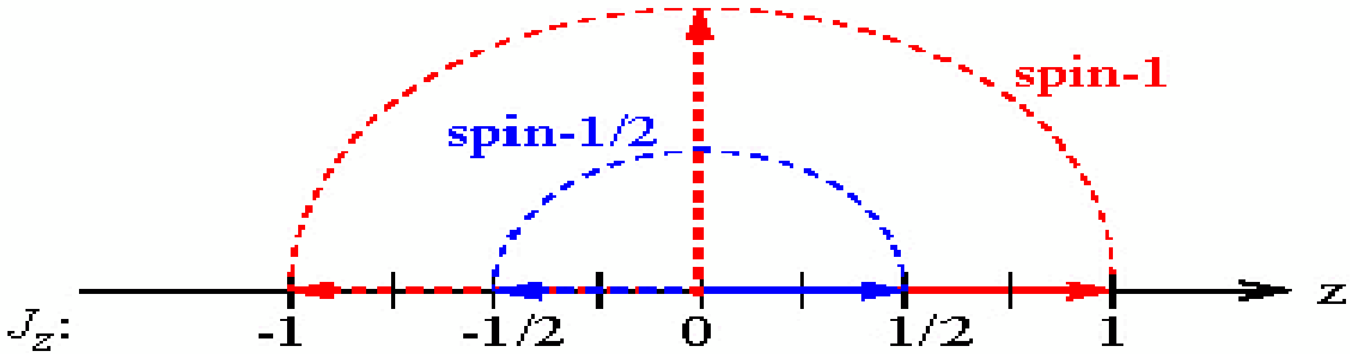


Illustration of possible J_z values for Spin-1/2 and Spin-1 particles

It is assumed that L and S are "good" quantum numbers with $J = S_p$

J_z depends instead on the spin orientation

- Using "good"quantum numbers, one can refer to a particle using the **spectroscopic notation**

$$^{(2S+1)}L_J$$

Following chemistry traditions, instead of numerical values of L = 0,1,2,3.....letters S,P,D,F are used

In this notation, the lowest-lying (L=0) bound state of two particles of spin-1/2 will be $^1S_0$ or $^3S_1$

$L=0$ $\quad$ $^1S_0$ $\quad$ $^3S_1$

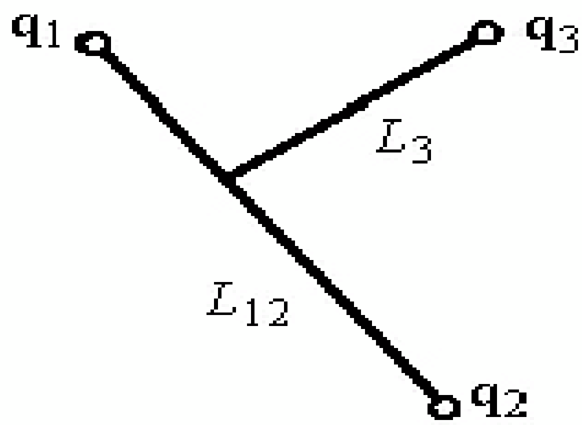
$S=1/2-1/2=0$ $\quad$ $S=1/2+1/2=1$

$J=L+S=0$ $\quad$ $J=L+S=1$

$$\tau\left(n\rightarrow p\nu_e\bar{\nu}_e\right)>10^{18}\,yr$$
$$\tau\left({}^{76}Ge\rightarrow{}^{76}Se+e^-+e^+\right)>10^{26}\,yr$$
$$\tau\left(p\rightarrow e^+\pi^0\right)>10^{33}\,yr$$

- For **mesons** with L >= 1, possible states are:

- Baryons are bound states of 3 quarks → two orbital angular momenta connected to the relative motions of quarks



Internal orbital angular momenta of a 3-quarks state

Possible baryon states:

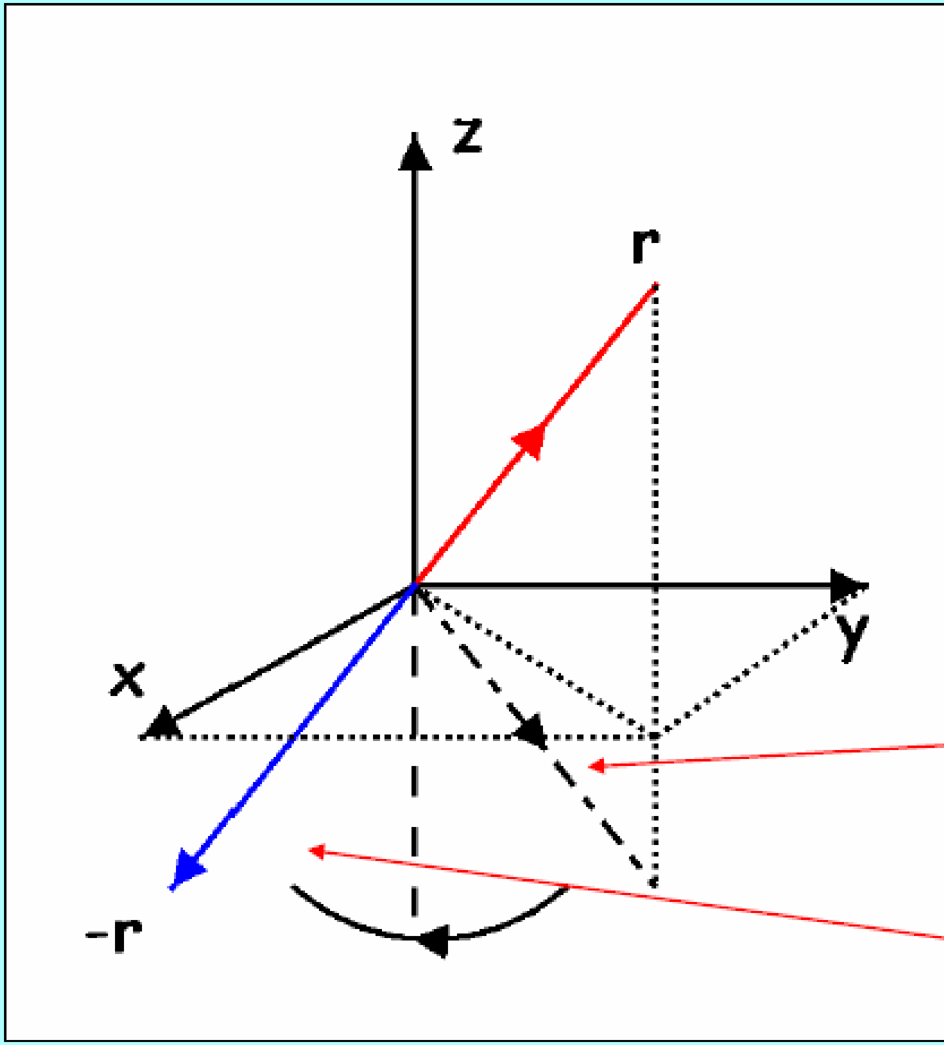
$$^2S_{1/2}, {}^4S_{3/2} \qquad (L=0)$$
$$^2P_{1/2}, {}^2P_{3/2}, {}^4P_{1/2}, {}^4P_{3/2}, {}^4P_{5/2} \qquad (L=1)$$
$$^2L_{L+1/2}, {}^2L_{L-1/2}, {}^4L_{L-3/2}, {}^4L_{L-1/2}, {}^4L_{L+1/2}, {}^4L_{L+3/2} \qquad (L=2)$$

Discrete symmetries: P,C,T

Discrete symmetries describe non-continuous changes in a system.
They cannot be obtained by integrating infinitesimal transformations.

These transformations are associated to discrete symmetry groups

Parity P



The Parity Operator reverses the coordinates r to -r.

Equivalent to a reflection in the x-y plane followed by a rotation about the z axis

Reflection in x-y plane

Rotation about z axis

Parity P

Parity is a Quantum Mechanical concept.

For a wavefunction $\Psi(r)$ and Parity operator P, the Parity Operator reverses the coordinates r to $-r$.

$$P\,\Psi(r) \rightarrow \Psi(-r) = \lambda\,\Psi(r)$$

But $P^2\,\Psi(r) \rightarrow \Psi(r) = \lambda^2\,\Psi(r)$

i.e. $\lambda^2 = 1$ so that $\lambda = \pm 1$

If an operator O acts on a wavefunction Ψ such that Ψ is unchanged

$$O\,\Psi = \lambda\,\Psi$$

Ψ is an Eigenfunction of O and λ is the Eigenvalue.

Hence the eigenvalues of Parity are $+1$ (even) and -1 (odd).

Parity P

Parity is conserved in a system when $[H, P] = 0$

The case of the central potential: $PH(\vec{r}) = H(-\vec{r}) = H(\vec{r})$

Bound states of a system with radial symmetry have definite parity

Example: the hydrogen atom

Hydrogen atom: wavefunction (no spin)

$$\vec{F}_{TOT} = \vec{F}_1 + \vec{F}_2 = -\frac{\partial}{\partial \vec{r}_1} V(\vec{r}_1 - \vec{r}_2) - \frac{\partial}{\partial \vec{r}_2} V(\vec{r}_1 - \vec{r}_2) = \frac{\partial}{\partial \vec{r}_2} V(\vec{r}_1 - \vec{r}_2) - \frac{\partial}{\partial \vec{r}_2} V(\vec{r}_1 - \vec{r}_2) = 0$$

Radial part

Angular part

$$V(\vec{r}_1 - \vec{r}_2) \rightarrow V(\vec{r}_1 + \vec{a} - \vec{r}_2 - \vec{a}) = V(\vec{r}_1 - \vec{r}_2)$$

So the parity P of a spherical armonics is $(-1)^l$

Parity is a multiplicative quantum number. The parity of a composite system is equal to the product of the parities of the parts:

$$\Psi = \Phi_A \Phi_B \ldots \qquad P_\Psi = P_A \times P_B$$

One can show that a state with angular momentum ℓ has parity

$$P = (-1)^{\ell}$$

For a system of particles:

$$P\,(\text{overall}) = P\,(\text{relative motion}) \times P\,(\text{intrinsic})$$

For Fermions $P\,(\text{antiparticle}) = (-1) \times P\,(\text{particle})$

For Bosons $P\,(\text{antiparticle}) = P\,(\text{particle})$

Arbitrarily assign $n, p \rightarrow P = +1 \qquad \bar{p}, \bar{n} \rightarrow P = -1$

Others determined from experiment (angular distributions)

Parity of $\pi^+, \pi^-, \pi^0 \rightarrow P = -1$

The intrinsic parity is defined as the eigenstate of the P operator, in the frame where the particle is at rest. It can be P= +1 or P=-1 .

Fermions have half-integer spin and angular momentum conservation requires their **production in pairs**. You can definethetherefore just relative parity. By convention, P (p) = +1

The Dirac equation and, more generally, the field theory, imply that the parity of a **fermion and its antiparticle** are opposite, of a boson and its anti-boson are equal. So, in particular, P ($\bar{p}$) = -1, P (e^+) = -1

The strange hyperons are produced by strong interactions paired with another strange particle, which prevents to establish the equality of both. You can not use the L pπ-decay, as the weak interaction does not conserve parity. By convention, P (L) = +1

By definition**, all the quarks** have parity equal +1

-The intrinsic parities of e- and e+ are related, namely: $P_{e+}P_{e-} = -1$

This is true for all fermions (spin-1/2 particles): $P_{f+}\ P_{f-} = -1$

Experimentally this can be confirmed by studying the reaction: $e+e- \rightarrow \gamma\gamma$
where initial state has zero orbital momentum and parity of $P_{e+}P_{e}$

If the final state has relative orbital angular momentum l_γ, its parity is: $P_\gamma^2(-1)^{l\gamma}$

Since $P_\gamma^2=1$, from the parity conservation law:

$$P_{e+}P_{e-} = (-1)^{l\gamma}$$

Experimental measurement of l_γ confirm this result

- However, it is impossible to determine P_{e-} or P_{e+}, since these particles are created or destroyed in pairs

-Conventionally, defined parities of leptons are:

$$P_{e-} = P_{\mu-} = P_{t-} = 1$$

Consequently, parities of anti-leptons have opposite sign

-Since quarks and anti-quarks are also created only in pairs, their parities are also defined by convention:

$$P_u = P_d = P_s = P_c = P_b = P_t = 1$$

With parities of antiquarks being –1

-For a meson $M=(a,\bar{b})$ parity is calculated as: $P_M = P_a P_{\bar{b}} (-1)^L = (-1)^{L+1}$

For L=0 that means P = -1, confirmed by observations.

-For a **baryon** B=(abc), parity is given as:

$$P_B = P_a P_b P_c (-1)^{L_{12}} (-1)^{L_3} = (-1)^{L_{12}+L_3}$$

and for antibaryon $P_{\bar{B}} = -P_B$ as for leptons

For the low-lying baryons, the formula predicts positive parities (confirmed by experiments).

In the **electric dipole transition** with the selection rule $\Delta l = \pm 1$ the atom's parity changes. So the parity of the emitted radiation must be odd, so that the parity of the whole system (atom+photon) is conserved.

$$P(\gamma) = 1$$

Parity of the photon from a classical analogy

A classical E field obeys :

$$\nabla \vec{E}(\vec{x},t) = \rho(\vec{x},t)$$

Let us take the P:

$$(-\nabla)\left[P\,\vec{E}(\vec{x},t)\right] = \rho(-\vec{x},t)$$

To keep the Poisson Equation invariant, we need to have the following law for E :

$$\left[P\,\vec{E}(\vec{x},t)\right] = -E(-\vec{x},t)$$

On the other hand, in vacuum:

$$\vec{E}(\vec{x},t) = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t} = -\frac{\partial\vec{A}}{\partial t}$$

$$\vec{A}(\vec{x},t) = N\,\vec{\varepsilon}(\vec{k})\exp\left[i(\vec{k}\vec{x}-Et)\right]$$

And the parity operation would give :

$$\vec{A}(\vec{x},t) \rightarrow P_\gamma\,\vec{A}(-\vec{x},t)$$

In order to make it consistent with the electric field transformationt:

$$P_\gamma = -1$$

Parity of $\pi^{\pm}$

Il π è un mesone di spin 0. Consideriamo la reazione

$$\pi^- + d \rightarrow n + n$$

(il d è uno stato legato *pn*).

Nello stato iniziale *l=0*; essendo $s_\pi=0$, $s_d=1$ si deve avere momento angolare totale J=1 (**J=L+S**). Quindi *anche nello stato finale* deve essere J=1. La simmetria della funzione d'onda nello stato finale (per lo scambio dei due neutroni) è data da:

$$K = \underbrace{(-1)^{S+1}}_{spin}\underbrace{(-1)^{L}}_{orbitale} = (-1)^{L+S+1}$$

Trattandosi di due fermioni identici deve essere K=-1, che implica L+S pari.

Dovendo essere J=1 ci sono le seguenti possibilità:

L=0 S=1 no L=1 S=0 no L=2 S=1 no

L=1 S=1 OK

Quindi la parità spaziale dello stato finale è $P=(-1)^L=-1$.Essendo la parità del deuterio $P_d=+1$ otteniamo per la parità intrinseca del π $P_\pi = -1$.

Il π è dunque un mesone pseudoscalare.

Parity of π^0

$\pi^0 \rightarrow \gamma\gamma$ B.R. = (99.798 ± 0.032) %

Siano **k** e **-k** gli impulsi spaziali dei γ; $\mathbf{e}_1$ ed $\mathbf{e}_2$ i rispettivi vettori di polarizzazione. Le due funzioni d'onda più semplici per lo stato finale di due fotoni con (simmetria di scambio pari) sono:

$$\psi_1(2\gamma) = A(\vec{e}_1 \cdot \vec{e}_2) \propto \cos\phi$$

$$\psi_2(2\gamma) = B(\vec{e}_1 \times \vec{e}_2) \cdot \vec{k} \propto \sin\phi$$

ψ_1 è pari sotto inversione spaziale, ψ_2 è dispari. Quindi:

$$P_{\pi^0} = +1 \quad |\psi|^2 \propto \cos^2\phi \qquad\qquad P_{\pi^0} = -1 \quad |\psi|^2 \propto \sin^2\phi$$

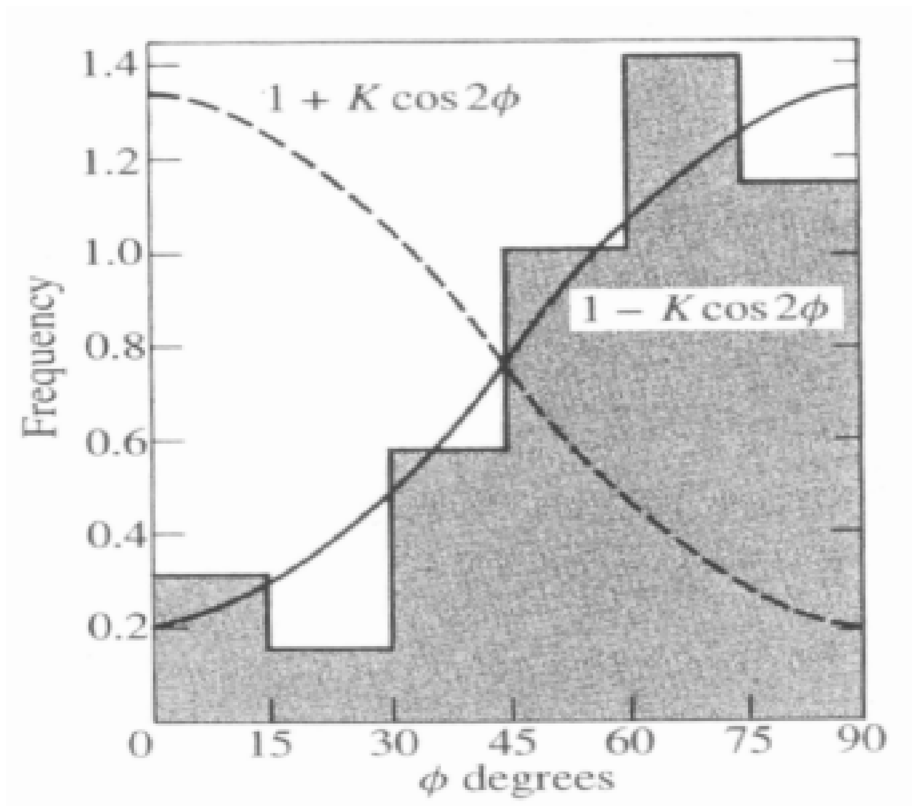
dove ϕ è l'angolo fra i piani di polarizzazione dei γ. L'esperimento è stato fatto usando il decadimento:

$$\pi^0 \rightarrow e^+ + e^- + e^+ + e^-$$

(doppio Dalitz; B.R. = $(3.14 \pm 0.30)\times 10^{-5}$) in cui ciascuna coppia di Dalitz si trova prevalentemente nel piano di polarizzazione del fotone che "converte" internamente. Si trova $P_{\pi 0}$=-1.

Parity of π^0

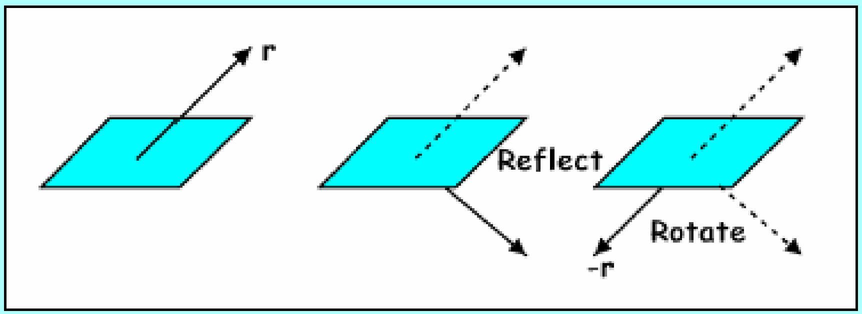
$$\pi^0 \rightarrow e^+ + e^- + e^+ + e^-$$



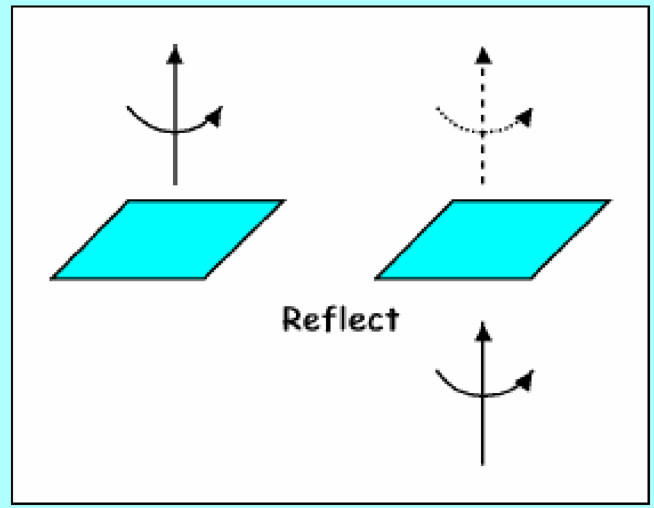
We label mesons by J^P – Spin^{Parity} corresponding to how their wavefunctions behave:

Examples of things that have these properties

J^P	Name	Examples
$J^P = 0^-$	Pseudoscalar	(Pressure...)
0^+	Scalar	(Mass, time, wavelength...)
1^-	Vector	(Momentum, position...)
1^+	Axial Vector	(Spin, angular momentum...)
2^+	Tensor	



Vector $r \rightarrow -r \therefore P = -1$



Axial Vector $r \rightarrow r \therefore P = +1$

Parity is conserved Strong and EM Interactions but NOT Weak.

The action of parity on relevant physical quantities

Position: $P_M = P_a P_{\bar{b}} (-1)^L = (-1)^{L+1}$

Time: $P:\ t \rightarrow t$

Momentum: $P:\ \vec{p} = m\frac{d\vec{r}}{dt} \rightarrow -\frac{d\vec{r}}{dt} = -\vec{p}$

Angular Momentum: $[\boldsymbol{R}, \boldsymbol{H}] = \mathbf{0}$

Charge: $P:\ q \rightarrow q$

Current: $P:\ \vec{J} = \rho\,\vec{v} \rightarrow -\rho\,\vec{v} = -\vec{J}$

E field: $P:\ \vec{E} = k\,q\,\frac{\vec{r}}{r^3} \rightarrow k\,q\,\frac{-\vec{r}}{r^3} = -\vec{E}$

B field: $P:\ \vec{B} = k\,\frac{\vec{s}\times\vec{r}}{r^2}\,I \rightarrow k\,\frac{(-\vec{s})\times(-\vec{r})}{r^2}\,I = \vec{B}$

Spin: $P:\ \sigma \rightarrow \sigma$

Parity Conservation

Most sensitive tests *P* conservation in **strong interactions:** search for nuclear states or mesons decays which could happen through strong interaction if the latter violated *P*

Example: decay of a pseudovector in two identical scalars, $1^+ \rightarrow 0^+ + 0^+$, not possible when conserving *P*

decay speed and cross sections proportional to $|M|^2$, which is a scalar regardless of *M* being a scalar or a pseudoscalar. In order to see an effect **both must contribute:**

$$M = M_S + M_P \quad \Longrightarrow \quad |M|^2 = |M_S|^2 + 2\, M_S^* M_P + |M_P|^2$$

Decay of the exited state $^{20}\mathrm{Ne}$: $^{20}\mathrm{Ne}^*(1^+) \rightarrow {}^{16}\mathrm{O}\,(0^+) + a$

One looks for a resonance in the process $p + {}^{19}\mathrm{F} \rightarrow [{}^{20}\mathrm{Ne}^*(1^+)] \rightarrow {}^{16}\mathrm{O}\,(0^+) + a$

Not found ⟹ $|T_P / T_S|^2 \leq 10^{-8}$

Helicity

Projection of the spin in the momentum direction $\lambda = \frac{\vec{\sigma}\,\vec{p}}{|\sigma|\,E}$

$$\begin{array}{lll} right\ \ handed & H = +1 \\ left\ \ \ handed & H = -1 \\ scalar & H = 0 \end{array}$$

An approximate quantum number for massive particles
So much better inasmuch the particle is relativistic
Exact for photons

The advantages of a description $\vec{p}, \lambda$ over a description $\vec{p}, m$

- The Helicity is unchanged by rotation

- Since $\vec{\sigma}\,\vec{p} = \vec{J}\,\vec{p}$, the helicity can be defined in a relativistic context

Invariance laws in action:

An example: E.M. interactions conserve Parity
One can then build a Parity-violating quantity, like:

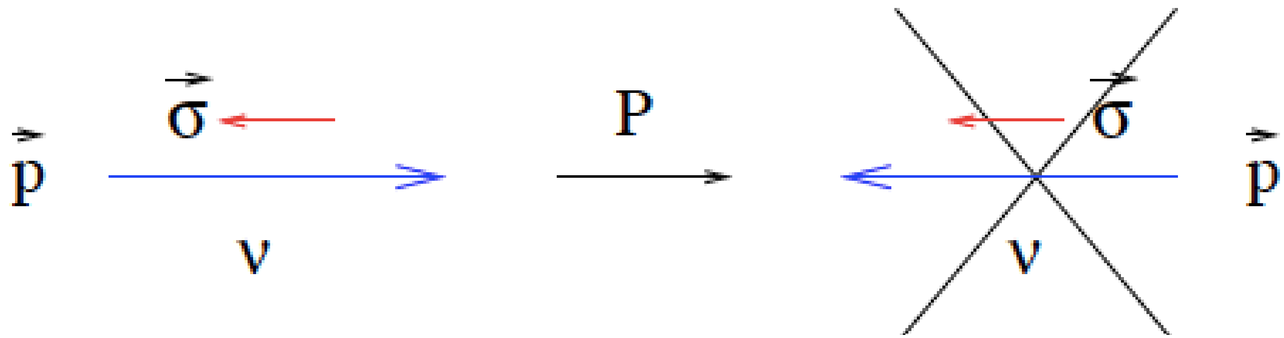
$$P:\ \vec{\sigma}\,\vec{p} \rightarrow \vec{\sigma}\,(-\vec{p}) = -\vec{\sigma}\,\vec{p}$$

Then, in E.M. interactions this quantity must be zero. And one can test this!
In E.M. Interactions right-handed and left-handed photons appear with equal amplitudes. In this way they compensate to the result of conserved Parity.

Parity Violation

Parity is conserved in strong and electromagnetic interactions, while it is violated in weak interactions (V-A theory, maximal violation of parity)

Example:



Experimentally, strongs and electromagnetic interactions can show tiny parity violating effects, due to the weak interaction contribution: $H = H_S + H_{em} + H_W$. Atomic reactions:

$$^{16}O^{*} \rightarrow {}^{12}C + \alpha$$

$$J^P = 2^- \qquad J^P = 2^+$$

With width $\Gamma_\alpha = (1.0 \pm 0.3) \times 10^{-10}$ eV, to be compared with $^{16}O^{*} \rightarrow {}^{16}O + \gamma$ of width 3×10^{-3} eV.

The parity operation (P) changes the direction (sign) of each of the spatial coordinates. Hence, it changes the sign of momentum. Since spin is like angular momentum (the cross product of a vector direction and a vector momentum, both of which change sign under the parity operation), spin does not change direction under the parity operation.

	n	$\bar{\nu}$	p	e
momentum direction		$\leftarrow$		$\rightarrow$
spin direction	$\Leftarrow$	$\Leftarrow$	$\Rightarrow$	$\Leftarrow$

Parity operation:

	n	$\bar{\nu}$	p	e
momentum direction		$\rightarrow$		$\leftarrow$
spin direction	$\Leftarrow$	$\Leftarrow$	$\Rightarrow$	$\Leftarrow$

The world would look different under the parity operation, since now the electron's spin would be in the same direction as its momentum.

The world is not symmetric under the parity operation!
Parity violation occurs only in the weak interaction.

Charge conjugation

An internal discrete symmetry

The Charge Conjugation operator reverses the sign of electric charge and magnetic moment (μ).

This implies particle ⇌ antiparticle.

proton	⇌	antiproton
$Q = +e$	C	$Q = -e$
$B = +1$		$B = -1$
μ		$-\mu$

$|X\rangle$ *is (Dirac) bra/ket notation for* Ψ_X *i.e.* $|\pi^+\rangle \equiv \Psi_{\pi^+}$

$C\,|\pi^+\rangle \rightarrow |\pi^-\rangle$ Hence $\pi^\pm$ not eigenstates of C.

C only has definite eigenvalues for neutral systems such as the π^0.

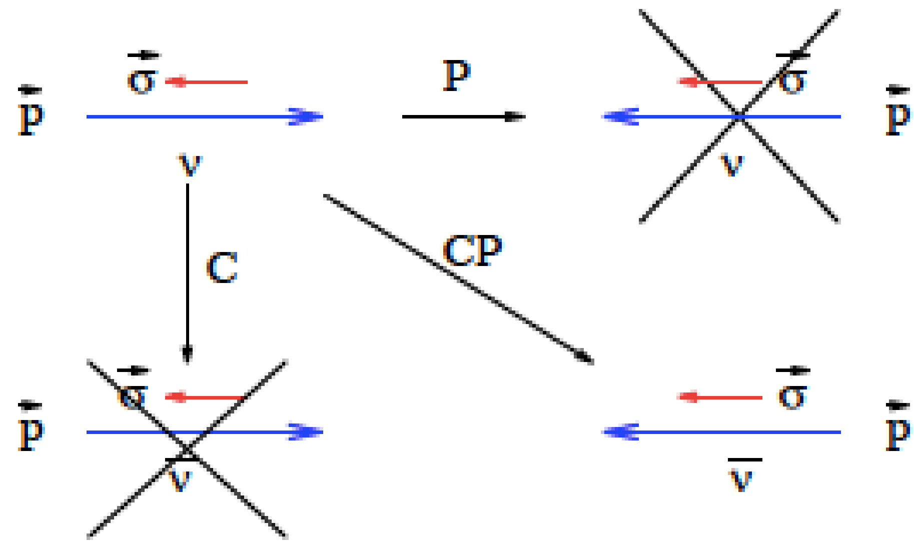
$$C\,|\pi^0\rangle = \lambda|\pi^0\rangle \qquad C^2\,|\pi^0\rangle = \lambda^2|\pi^0\rangle = |\pi^0\rangle$$

$$\therefore \lambda = \pm 1$$

Charge conjugation

$$p \xrightarrow{C} \bar{p}$$

	p	$\bar{p}$
Q	+e	-e
B	+1	-1
μ	+2.79(e$\hbar$/2mc)	-2.79(e$\hbar$/2mc)
σ	½$\hbar$	½$\hbar$



EM fields come from moving charges which change sign under Charge Conjugation $\therefore C_\gamma = -1$.

$$\therefore n \text{ photons have } C = (-1)^n$$

Since $\pi^0 \to \gamma\gamma$ this implies $C_{\pi^0} = +1$ (assuming C invariance in EM decays).

Note $\pi^0 \to \gamma\gamma\gamma$ is then forbidden.

The η (eta) meson (mass 550 MeV/c^2)

$$\eta \to \gamma\gamma$$

$$\eta \not\to \gamma\gamma\gamma$$

i.e. $C_\eta = +1$

$$\frac{BR(\pi^0 \to 3\gamma)}{BR(\pi^0 \to 2\gamma)} < 3.1\times10^{-8}$$

C is conserved Strong and EM Interactions but NOT Weak.

Let's denote particle which have distinct anti-particles by "α", and otherwise by "a". Then:

$$\hat{C}|\alpha,\Psi\rangle = C_\alpha|\alpha,\Psi\rangle$$

That is, the final state acquires a phase factor. Otherwise:

$$\hat{C}|a,\Psi\rangle = |\bar{a},\Psi\rangle$$

That is, from the particle in the initial state, we arrive to **the antiparticle in the final state**

Applying the transformation once more, turns antiparticles back to particles, and hence:

$$\hat{C}^2 = 1 \longrightarrow C_\alpha = \pm 1$$

For multiparticle states the transformation is:

$$\hat{C}|\alpha_1,\alpha_2,\ldots.a_1,a_2,..;\Psi\rangle = C_{\alpha_1,}C_{\alpha_1,}\ldots|\alpha_1,\alpha_2,\ldots.\bar{a}_1,\bar{a}_2,..;\Psi\rangle$$

It is clear that particles α = γ, π^0,.etc, are C eigenstates with eigenvalues $C_\alpha = \pm 1$

-Other eigenstates can be constructed from particle-antiparticle pairs:

$$P_{\bar{B}} = -P_B$$

For a state of definite L, interchanging between particle and anti-particle reverses their relative position vector. For example:

$$\hat{C}^2 = 1$$

For fermion-antifermion pairs theory predicts:

$$\hat{C}|\bar{f}f;J,L,S\rangle = (-1)^{L+S}|\bar{f}f;J,L,S\rangle$$

This implies that π^0, being a $^1S_0$ state of $d\bar{d}$ and $u\bar{u}$ must have C-parity of 1.

Tests of C-invariance

C_γ can be inferred from classical field theory: $\vec{A}(\vec{x},t) \rightarrow C_\gamma \vec{A}(\vec{x},t)$

and since all electric charges swap, electric field and scalar potential also change sign:

$$\vec{E}(\vec{x},t) \rightarrow -\vec{E}(\vec{x},t), \qquad \varphi(\vec{x},t) \rightarrow -\varphi(x,t)$$

Which upon substitution into: $\vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{\partial t}$ gives $C_\gamma = -1$

- Another confirmation of C-invariance comes from observation of η-meson decays:

$$\eta \rightarrow \gamma + \gamma$$
$$\eta \rightarrow \pi^0 + \pi^0 + \pi^0$$
$$\eta \rightarrow \pi^+ + \pi^- + \pi^0$$

These are em decays, and first two clearly indicate that C =1.
Identical charged pions momenta distribution in third, confirms C-invariance.

Charge conjugation (C) simply means to change each particle into its anti-particle. This changes the sign of each of the charge-like numbers. The neutron is neutral, nonetheless it has charge-like quantum numbers. It is made of three quarks, and charge conjugation change them into three anti-quarks. Charge conjugation leaves spin and momentum unchanged.

The interesting question is, does a world composed completely of anti-matter have the same behavior. For example, in neutron decay, there is a correlation between the spin of the neutron and the direction of the electron that is emitted when the neutron decays. The electron spin is also directed opposite to its direction of motion.

	$\leftarrow$		$\rightarrow$	momentum direction
n	$\bar{\nu}$	p	e	
$\Leftarrow$	$\Leftarrow$	$\Rightarrow$	$\Leftarrow$	spin direction

Charge conjugated:

	$\leftarrow$		$\rightarrow$	momentum direction
$\bar{n}$	ν	$\bar{p}$	$\bar{e}$	
$\Leftarrow$	$\Leftarrow$	$\Rightarrow$	$\Leftarrow$	spin direction

This is not what an anti-neutron decay looks like! The laws of physics responsible for neutron decay are not invariant with respect to charge conjugation.

In weak interactions C invariance is broken:

LH neutrino n → LH antineutrino

(which dos not exists)

However under combined CP:

LH neutrino n → RH antineutrino

$$
\begin{aligned}
a + b &\rightarrow c + d \\
a + \bar{c} &\rightarrow \bar{b} + d \\
a + \bar{d} &\rightarrow c + \bar{b} \\
a &\rightarrow \bar{b} + c + d \\
c + d &\rightarrow a + b
\end{aligned}
$$

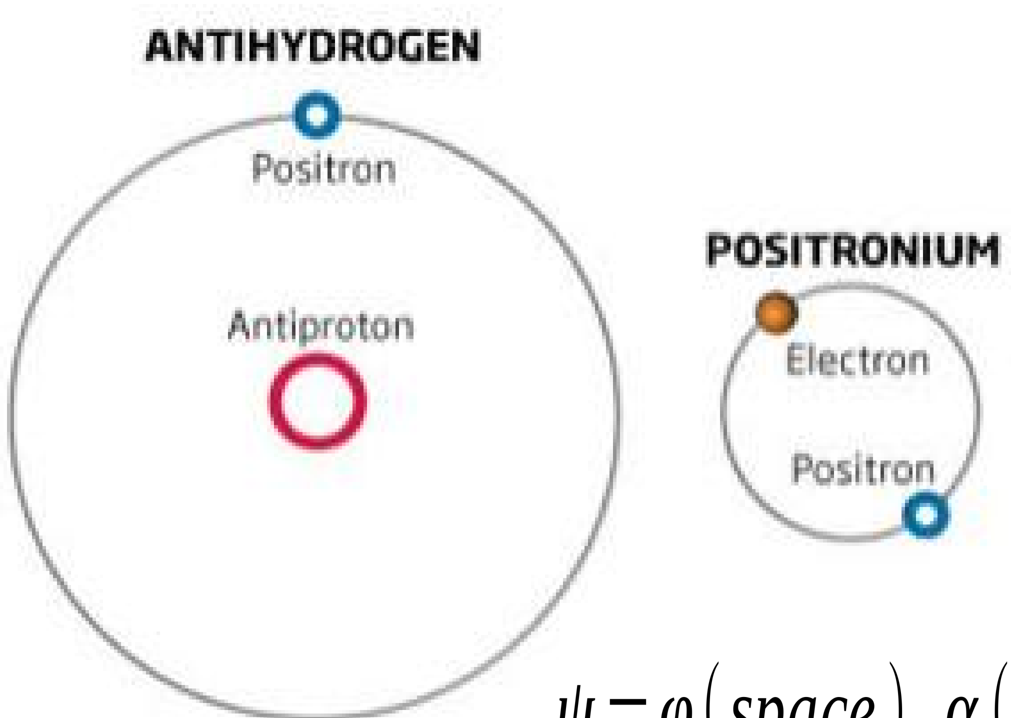
Weak interactions are eigenstates of CP

This statement is not completely true:

CP violation in weak interactions does occur at the 10^{-4} level

Positronium decays

Similar to the H atom. Actually, the «true» atom.
We require the total wavefunction to be antysimmetric, considering the electron and the positron as different C-states of the same particle



$$\psi = \varphi(space)\ \alpha(spin)\ \chi(C)$$

The lack of symmetry under the parity operation was discovered in the fifties following the suggestion of Lee and Yang that this symmetry was not well tested experimentally. It is now known that parity is violated in the weak interaction, but not in strong and electromagnetic interactions.

The situation with charge conjugation symmetry is similar; the lack of symmetry under charge conjugation exists only in the weak interaction.

The Standard Model incorporates parity violation and charge conjugation symmetry violation in the structure of the weak interaction properties of the quarks and leptons and in the form of the weak interaction itself.

P /C for a fermion-antifermion system

$P=(-1)^{l+1}$ $C=(-1)^{l+s}$

Trovare i valori di J^{PC} di particella-antiparticella di spin 1/2 in l=0, 1 ($p \neq p$, e^+e^-, $q \neq q$)

Se l=0 (onda S) P=–, se l=1 (onda P), P=+

Notazione spettroscopica: $^{2\hat{}s\hat{}+1}L_J$

$^1S_0 \Leftrightarrow J^{PC}=0^{-+}$

$^3S_1 \Leftrightarrow J^{PC}=1^{--}$

$^1P_1 \Leftrightarrow J^{PC}=1^{+-}$

$^3P_0 \Leftrightarrow J^{PC}=0^{++}$

$^3P_1 \Leftrightarrow J^{PC}=1^{++}$

$^3P_2 \Leftrightarrow J^{PC}=2^{++}$

$J^{PC}= 0^{+-}, 0^{--}, 1^{-+}$,…..non possono essere fatti da quark e antiquark se i quark hanno spin 1/2

Time Inversion

It changes the time arrow T

Classical dynamical equations are invariant because of second order in time

$$\frac{d\omega\left(q_i, p_i\right)}{dt} = \left\{ \omega, H \right\}$$

Classical microscopic systems : T invariance

Classical macroscopic systems: time arrow selected statistically (non decrease of entropy)

In the quantum case :

$$i\hbar \frac{\partial}{\partial t}\psi = H\psi$$ Is not invariant for $$\psi\left(\vec{r}, t\right) \xrightarrow{T} \psi\left(\vec{r}, -t\right)$$

Let us now start from the Conjugate Schroedinger Equation :

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = H\,\psi(\vec{r},t) \qquad\qquad -i\hbar \frac{\partial}{\partial t}\psi^{¿}(\vec{r},t) = H\,\psi^{¿}(\vec{r},t)$$

Let us define the T-inversion operator : T $\qquad \psi(\vec{r},t) \xrightarrow{T} \psi^{¿}(\vec{r},-t)$

$$-i\hbar \frac{\partial}{\partial t}\psi^{¿}(\vec{r},t) = H\,\psi^{¿}(\vec{r},t) \qquad\qquad i\hbar \frac{\partial}{\partial t}\psi^{¿}(\vec{r},-t) = H\,\psi^{¿}(\vec{r},-t)$$

So, with this definition of T, we have: $\quad i\hbar \frac{\partial}{\partial t}\left[T\psi(\vec{r},t)\right] = H\left[T\,\psi(\vec{r},t)\right]$

The operator representing T is an antilinear operator.

The square modulus of transition amplitudes is conserved

Time Inversion

Time reversal means to reverse the direction of time. There are a number of ways in which we can consider time reversal. For example, if we look at collisions on a billiard table it would clearly violate our sense of how things work if time were reversed. It is very unlikely that we would have a set of billiard balls moving in just the directions and speeds necessary for them to collect and form a perfect triangle at rest, with the cue ball moving away.

However, if we look at any individual collision, reversing time results in a perfectly normal looking collision (if we ignore the small loss in kinetic energy due to inelasticity in the collision). The former lack of time reversal invariance has to do with the laws of thermodynamics; we here are interested in individual processes for which the laws of thermodynamics are not important.

Time reversal reverses momenta and also spin, since the latter is the cross product of a momentum (which changes sign) and a coordinate, which does not.

Fermi Golden Rule

$$\frac{d^2N}{dAdt} = \sigma \cdot n_b \cdot n_a v_i \qquad W = \sigma \cdot n_a \cdot v_i$$

W = numero di interazioni per unità di tempo per particella bersaglio

La sezione d'urto contiene informazioni sulle proprietà delle particelle interagenti e sulla dinamica dell'interazione. Scrivendo l'hamiltoniana come:

$$H = H_0 + H'$$

si può dimostrare che al primo ordine perturbativo:

$$W = \frac{2\pi}{\hbar}\left|M_{if}\right|^2 \frac{dn}{dE_0}$$

Fermi Golden Rule

$$M_{if} = \int \psi_f^* H' \psi_i d\tau$$

contiene la dinamica dell'interazione

$\frac{dn}{dE_0}$ densità degli stati finali (o spazio delle fasi)

$$\sigma(a+b \to c+d) = \frac{1}{\pi\hbar^4}\left|M_{if}\right|^2 \frac{(2s_c+1)(2s_d+1)}{v_i v_f} p_f^2$$

Detailed Balance principle

$$a + b \rightleftarrows c + d$$

Dall'invarianza per T e P si ottiene:

$$\left|M_{if}\right|^2 = \left|M_{fi}\right|^2$$

$$\langle f(\vec{p}_c,\vec{p}_d,s_c,s_d)|H'|i(\vec{p}_a,\vec{p}_b,s_a,s_b)\rangle$$

$$\downarrow T$$

$$\langle i(-\vec{p}_a,-\vec{p}_b,-s_a,-s_b)|H'|f(-\vec{p}_c,-\vec{p}_d,-s_c,-s_d)\rangle$$

$$\downarrow P$$

$$\langle i(\vec{p}_a,\vec{p}_b,-s_a,-s_b)|H'|f(\vec{p}_c,\vec{p}_d,-s_c,-s_d)\rangle$$

Se non c'è invarianza per *T* e *P* (interazioni deboli) si applica la teoria delle perturbazioni al primo ordine e dalla hermiticità di H′ si ottiene lo stesso risultato.

Sommando su tutte le *(2s+1)* proiezioni di spin si ottiene per l'appunto $|M_{if}|^2=|M_{fi}|^2$.

Spin of $\pi^{\pm}$

Lo spin dei pioni carichi è stato determinato applicando il principio del bilancio dettagliato alla reazione:

$$p + p \rightleftarrows \pi^+ + d$$

$$\sigma_{pp\to\pi^+d} = |M_{if}|^2 \frac{(2s_\pi+1)(2s_d+1)}{v_i v_f} p_\pi^2$$

$$\sigma_{\pi^+d\to pp} = \frac{1}{2}|M_{fi}|^2 \frac{(2s_p+1)^2}{v_f v_i} p_p^2$$

(il fattore ½ deriva dall'integrazione su mezzo angolo solido, dovuta al fatto che ci sono due fermioni identici nello stato finale).

$$\frac{\sigma_{pp\to\pi^+d}}{\sigma_{\pi^+d\to pp}} = 2\frac{(2s_\pi+1)(2s_d+1)}{(2s_p+1)^2}\frac{p_\pi^2}{p_p^2}$$

Dalla misura delle sezioni d'urto per la reazione diretta e quella inversa si è ottenuto:

$$s_\pi = 0$$

An important consequence of T-invariance at the microscopic level concerns the transition amplitudes :

$$\left|M_{i \to f}\right| = \left|M_{f \to i}\right| \qquad \text{(detailed balance)}$$

Note: the detailed balance DOES NOT imply the equality of the reaction rates:

$$\left|M_{i \to f}\right| = \left|M_{f \to i}\right| \quad \not\Longrightarrow \quad P: Y_l^m(\theta, \phi) \to (-1)^l \, Y_l^m(\theta, \phi)$$

A “classical” test, the study of the reaction $p + {}^{27}Al \Leftrightarrow \alpha + {}^{24}Mg$

T is violated at the microscopic level in the Weak Nuclear Interactions

Physical Review Letters 109 (2012) 211801. BaBar experiment at SLAC

Comparing the reactions: $B' / \Leftrightarrow / B^0$

T violation

Now let's consider what happens when we apply time reversal (T) to the case of the neutron decay.

	$\leftarrow$		$\rightarrow$	momentum direction
n	$\bar{\nu}$	p	e	
$\Leftarrow$	$\Leftarrow$	$\Rightarrow$	$\Leftarrow$	spin direction

Time reversal:

	$\rightarrow$		$\leftarrow$	momentum direction
n	$\bar{\nu}$	p	e	
$\Rightarrow$	$\Rightarrow$	$\Leftarrow$	$\Rightarrow$	spin direction

This looks just fine, the electron spin is opposite to its momentum and the electron direction is opposite to the neutron's spin.

So, at least for neutron decay, the laws of physics appear to be symmetric under time reversal invariance.

Quantity	Under C	Under T	Under P
position	r	r	-r
momentum	p	-p	-p
Spin	σ	$-\sigma$	σ
E field	E	E	-E
B field	B	-B	B
Magnetic dipole momentum	$\sigma \cdot B$	$\sigma \cdot B$	$\sigma \cdot B$
Electric dipole momentum	$\sigma \cdot E$	$-\sigma \cdot E$	$-\sigma \cdot E$
Long. polarisation	$\sigma \cdot p$	$\sigma \cdot p$	$-\sigma \cdot p$
Transverse polarisation	$\sigma \cdot (p_1 x p_2)$	$-\sigma \cdot (p_1 x p_2)$	$\sigma \cdot (p_1 x p_2)$

The CPT Theorem

In a local, Lorentz-invariant quantum field theory, the interaction (Hamiltonian) is invariant with respect to the combined action of C,P,T (Pauli, Luders, Villars, 1957)

A few consequences :

1) Mass of the particle = Mass of the antiparticle
2) (Magnetic moment of the particle) = -- (Magnetic moment of antiparticle)
3) Lifetime of particle = Lifetime of antiparticle

Protons, electrons

	Proton	Antiproton	Electron	Positron
Q	**+e**	**-e**	-e	+e
B o L(e)	**+1**	**-1**	**+1**	**-1**
μ	$+2.79\left(e\hbar/2Mc\right)$	$-2.79\left(e\hbar/2Mc\right)$	$-e\hbar/2mc$	$+e\hbar/2mc$
σ	$\hbar/2$	$\hbar/2$	$\hbar/2$	$\hbar/2$

The CPT Theorem

If the charge of all particles in universe were changed to the opposite charge (so that all particles change to their antiparticles); & at the same time, all were reflected in a mirror; & at the same time, time started to run backwards:

This new world would be indistinguishable from the old world.