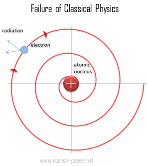
Giancarlo Panizzo
INFN di Trieste
Gruppo collegato di Udine

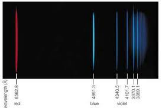
Quantum Mechanics

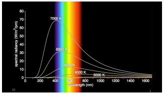
Introduction

In late XIX century, classical mechanics proved to be unable to explain some well known phenomena:

1. Black body radiation
2. Stability of the atom
3. Spectral series of Hydrogen







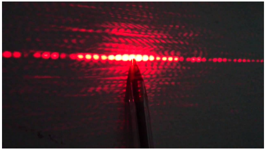
"On the theory of the Energy distribution law of the Normal Spectrum"

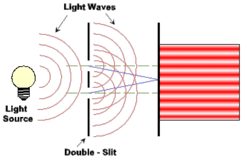
1900, Max Planck

This was the start of a big revolution in Physics, i.e. of Quantum Mechanics.

Black Body Spectrum

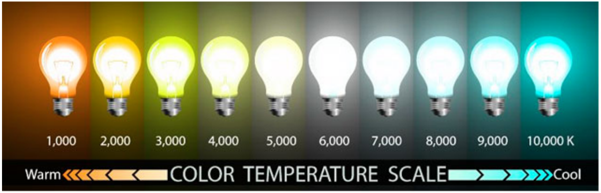
Light behaves as a wave ...





... but ...

Black Body Spectrum



- < 1900 K: Candle light
- 2700–3300 K: Warm white
- 2200–3400 K: Incandescent light bulbs
- > 3900 K: Fluorescent lamps
- 4000–5000 K: Neutral white
- 5100–5400 K: Midday sun in summer
- 6500 K: Standard light C (xenon test lamp)
- 5000–6800 K: Daylight white
- > 9000 K: Midday blue sky in December

$$B \overset{?}{\longleftrightarrow} T$$

$$B = \left(\text{Output power/unit wavelenght} \atop \approx \text{Intensity spectrum}\right)$$

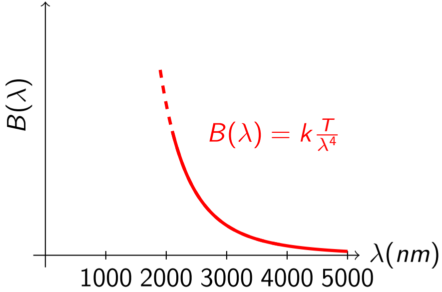
$$T = \text{Temperature}$$

Black Body Spectrum

Assuming light is a wave, classical mechanics predicts:

$$B(\lambda) = \frac{2ck_BT}{\lambda^4} \propto \frac{T}{\lambda^4}$$

"Ultra-violet catastrophe"



UNIVERSITÀ DEGLI STUDI DI UDINE
hic sunt futura

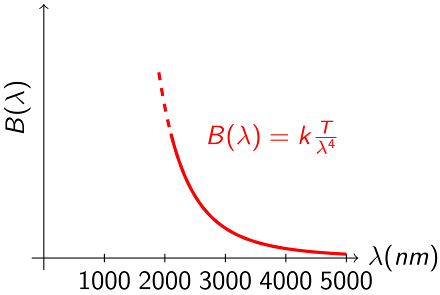
Planck's proposal

Light is emitted in quanta of energy

$$E = h\,\nu$$

- ν (Greek letter: "nu") is the light's fequency ($\nu = \frac{c}{\lambda}$)
- h is the Planck's constant

$$h = 6.62606957 \times 10^{-34}\ Js$$



UNIVERSITÀ DEGLI STUDI DI UDINE
hic sunt futura

Planck's proposal

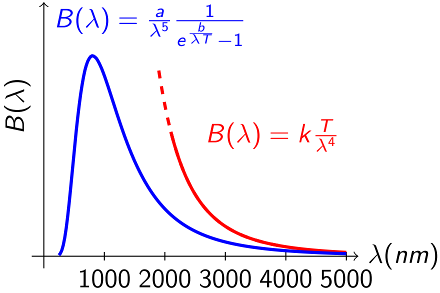
Light is emitted in quanta of energy

$$E = h\,\nu$$

Predicted spectrum becomes:

$$B(\lambda) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

- Same behaviour at high λ
- Dumping factor
  - heals $\lambda \to 0$



UNIVERSITÀ DEGLI STUDI DI UDINE
hic sunt futura

Planck's proposal

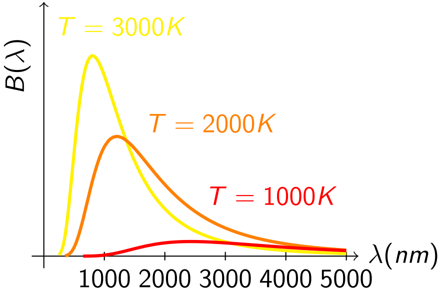
Light is emitted in quanta of energy

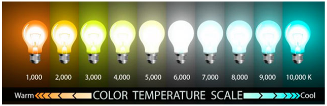
$$E = h\,\nu$$

Predicted spectrum becomes:

$$B(\lambda) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

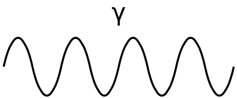
- Same behaviour at high λ
- Dumping factor
  - heals $\lambda \to 0$
  - agrees with observations



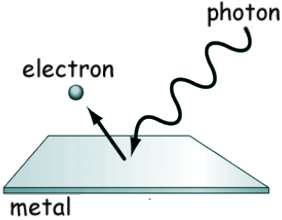


Plank & Einstein

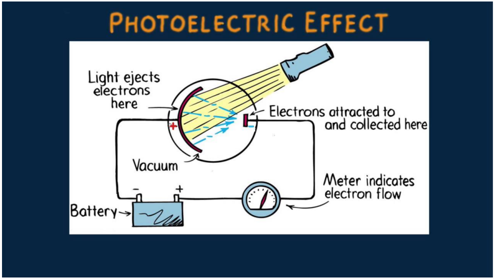
- Plank's proposed $E = h\nu$ as a *mathematical* assumption
- Einstein even further: light *is composed* by quanta, later called "photons"



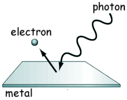
  - Photoelectric effect, Nobel Prize 1921



Photoelectric Effect



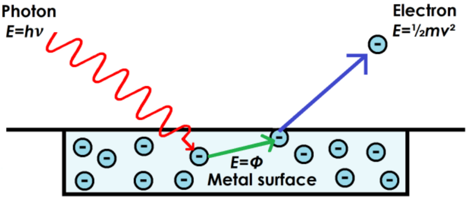
- Shining ultraviolet light on the metal plate
  - gives flow of negative charge *(Hertz, 1887)*



- Brightness $\propto$ current, but ...
- Flow can be stopped with a specific voltage V_0
  - **independent** of the **brightness**
  - depends only on the frequency *(Lenard, 1902)*

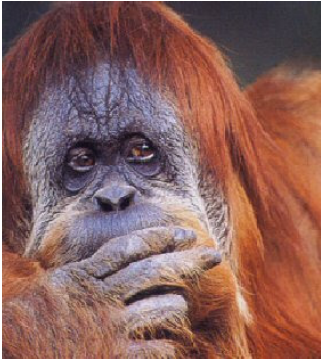
Photoelectric Effect

- Light is actually **made up** out of particles "photons" *(Einstein, 1905)*
  - of energy $E = h\nu$
- Kinetic energy of the emitted electrons is the energy left over after the electron has been "lifted" over the work function barrier



UNIVERSITÀ DEGLI STUDI DI UDINE
hic sunt futura

Wave or particle?



So we have seen that light behaves

- as a **wave** (interference, diffraction ...)
- and **also** is made of **particles** (photoelectric effect, black body radiation, ...)

"Wave-particle duality"

...not yet the end of the story!

UNIVERSITÀ DEGLI STUDI DI UDINE
hic sunt futura

Atom structure

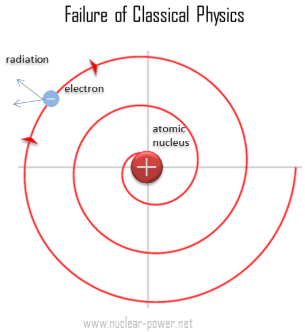
early 20th century

Rutherford, 1911, **atom** seen as

- diffuse cloud of e^-
- dense positively charges nucleus

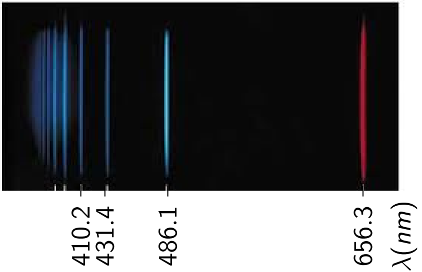
How can be an atom stable?

- Electrons on circular (accelerated!) orbit
  - Radiates photons (Larmor formula)
- loose energy. Catastrophe!



Atom structure

Bohr



Hot hydrogen emits light in a set of spectral lines (*"Balmer series"*)

⇒ set of lines in the visible spectrum

Solution (Bohr, 1913)

⇒ Electron's **angular momentum** L **quantized** in units of $\hbar = \frac{h}{2\pi}$

Bohr's radius

in one slide!

So if we start from the classical relation for electron's motion:

$$F_{em} = m_e a, \quad \left(a = \frac{v^2}{r} \right)$$

(circular orbit), and remembering Coulomb's law

$$F_{em} = k_C \frac{Ze^2}{r^2}$$

we get

$$F_{em} = k_C \frac{Ze^2}{r^2} = m_e \frac{v^2}{r} = m_e a$$

And finally using Bohr's assumption $L = m_e vr = n\hbar$:

Quantized Bohr's radii

$$r_n = \frac{n^2 \hbar^2}{Z k_C e^2 m_e}$$

De Broglie, 1924

Put together:

1. Photons are quanta of light, with $E = pc = h\nu = \frac{hc}{\lambda}$
   $\Rightarrow$ light has also a particle behaviour
2. Electrons have quantized angular momenta $L = n\hbar$

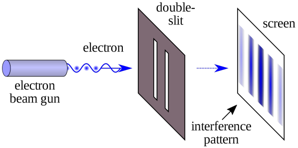
But then also **particles** can behave as waves: "**matter-waves**" with wavelength

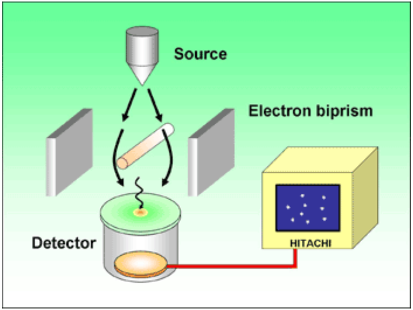
De Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Davisson and Germer **measured** $\lambda_e = \frac{h}{m_e v}$ using diffraction.

Electrons as waves





UNIVERSITÀ DEGLI STUDI DI UDINE
hic sunt futura

Wave function

Wave-like nature of light/particles described by

$$\psi(t, \mathbf{x}) \neq \psi^*(t, \mathbf{x})$$

Probability:

$$P \propto |\psi(t, \mathbf{x})|^2, \quad \int dx\ dt\ |\psi(t, \mathbf{x})|^2 = 1$$

Uncertainty principle

Heisenberg

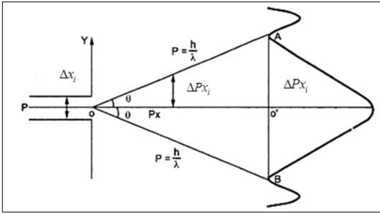
Only one of the "position" or "momentum" can be measured accurately at a single moment within the instrumental limit.

..or

It is impossible to measure both the position and momentum simultaneously with unlimited accuracy.

$\Delta x \rightarrow$ uncertainty in position

$\Delta p_x \rightarrow$ **uncertainty in momentum**



then

$$\Delta x \; \Delta p_x \geq \frac{\hbar}{2} \qquad \left(\hbar = \frac{h}{2\pi}\right)$$

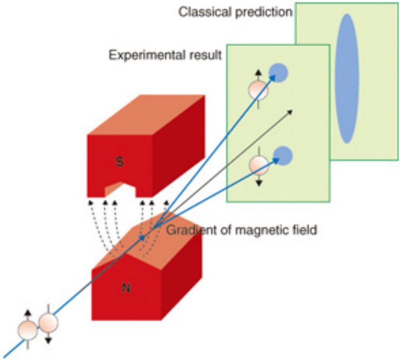
The product $\Delta x \; \Delta p_x$ of an object is greater than or equal to 2

(*equivalent to state that interference is intrinsic*)

Spin

Stern & Gerlach, 1922:

- Electrons are deviated by a magnetic field, in different directions
- Two "types" of electrons ⇔ **spin** $s = \pm\frac{1}{2}$



- Fermions: half-integer spin (electrons, ...)
- Bosons: integer spin (photons, ...)

Scalars have zero spin (Higgs)

Schroedinger equation

Wave-particles wave function governed by

$$i\hbar\frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \hat{H}\psi(t,\mathbf{x})$$

where $\hat{H}$ is the "Hamiltonian" $(E+V)$ and

$$\begin{aligned} \vec{p} &\rightarrow -\imath\hbar\left(\tfrac{\partial}{\partial x},\tfrac{\partial}{\partial y},\tfrac{\partial}{\partial z}\right) = -\imath\hbar\vec{\nabla} \\ E &\rightarrow \imath\hbar\tfrac{\partial}{\partial t} \end{aligned}$$

Example of a free particle possible solution (1D):

$$\begin{aligned} \hat{H} &= \tfrac{p^2}{2m} \\ \psi(t,x) &= \exp\left(\tfrac{\imath}{\hbar}\left(px - \tfrac{p^2}{2m}t\right)\right) \end{aligned}$$

(verify as an exercise)