



# Stability of relativistic magnetized jets

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## Abstract

Astrophysical jets are collimated outflows extending in length several times their radii. Even though current – driven and Kelvin – Helmholtz instabilities form along the flow, jets are observed to retain their shape almost intact. Using the ideal MHD equations we describe the jet dynamics and we study their linear stability. The growth rates of the perturbations are found through numerical integration of the perturbed system. Finally, we elaborate on the results and outline the main conclusions focusing on their dependence on the bulk Lorentz factor and the current distribution.

## 1. Theoretical framework

- Two main categories of instabilities:  
Kelvin–Helmholtz → shear velocity profile or fluids in contact with different velocities.  
Current–driven → helical magnetic field.  
Stability properties of outflows through a linear stability analysis.
- Jet dynamics described by ideal relativistic magnetohydrodynamics (MHD).
- Jets have to be in dynamic equilibrium along the radial direction, i.e., the radial component of the momentum equation must be satisfied:

$$\gamma \rho_0 (\mathbf{v} \cdot \nabla) (\xi \gamma \mathbf{v}) = \nabla P + \frac{J^0 \mathbf{E} + \mathbf{J} \times \mathbf{B}}{c}$$

where,  $\gamma$  is the Lorentz factor,  $\mathbf{v}$  the bulk flow velocity,  $\rho_0$  is the proper density and  $\xi = 1 + \frac{\Gamma P}{\Gamma - 1 \rho_0 c^2}$  is the specific enthalpy. Also,  $P$  is the thermal pressure,  $J^0$ ,  $\mathbf{J}$  are the electric charge and current densities, and finally  $\mathbf{E}$ ,  $\mathbf{B}$  the electric and the magnetic fields.

- Jets are in equilibrium with their environment, which is assumed to be a stationary unmagnetized medium.
- Small perturbations are of the form:  
 $\delta Q = Q_1(\varpi) \exp[i(m\phi + kz - \omega t)]$ , with  $|Q_1| \ll |Q_0|$ .
- $\omega = \Re\omega + i\Im\omega \in \mathbb{C}$ . This leads to a time–dependent amplitude for the perturbations. Whenever  $\Im\omega > 0$  we have an unstable mode with growth timescale  $\frac{1}{\Im\omega}$ .
- The final linearized set of equations is a 2x2 first order complex differential equation system.
- Our main goal is to find the dispersion relation of the system,  $\omega = \omega(k)$ .

## 2. Jet models

We analyse two configurations.

- First: Relativistic jets without current sheets [1].

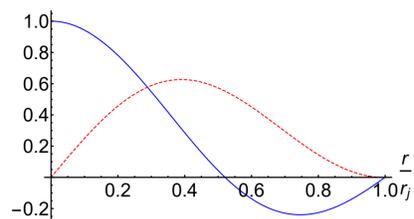


Figure 1: Magnetic field configuration from [1]: Azimuthal magnetic field (red line) and the z–component (blue line).

- Second: Relativistic jet [2] with two distinct areas, “spine–sheath” model.

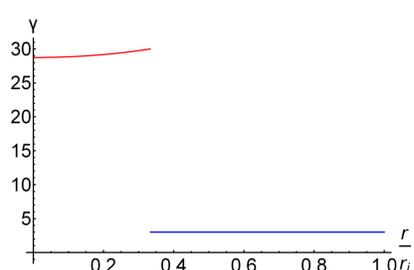


Figure 2: Plot of Lorentz factor from [2].

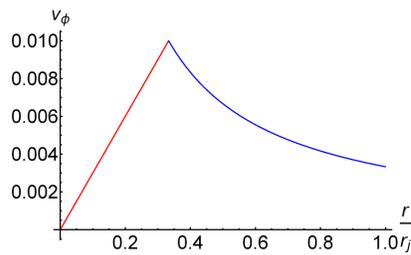


Figure 3: Azimuthal bulk velocity (in units  $c$ ). The  $\phi$ –component of the magnetic field has a similar profile.

Both models are cylindrical, steady–state and axisymmetric, so the unperturbed physical quantities depend only on the cylindrical radius  $r$ .

## 3. Methodology

We solve numerically the linearized differential equations using a shooting method algorithm. We provide the algorithm with:

- differential equations
- boundary conditions defined on the axis, at the jet–environment interface and at great distances ( $r \rightarrow \infty$ )
- The solutions in the environment are Bessel functions.

The last piece of information is an initial “guess” for  $k$ ,  $\Re(\omega)$ , and  $\Im(\omega)$  in order to begin the shooting method.

## 4. Results

For the first model we studied jet’s stability properties for various values of Lorentz factor. The main results are:

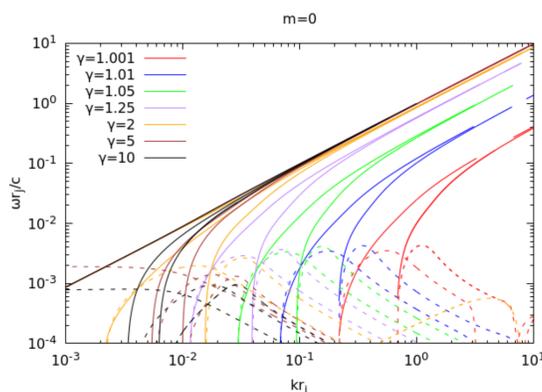


Figure 4: Plot of the dispersion relation for  $m = 0$  for various Lorentz factors from [3].

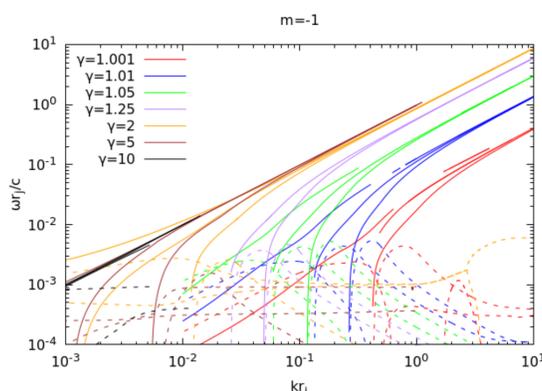


Figure 5: Plot of the dispersion relation for  $m = -1$  for various Lorentz factors from [3]. The  $m=1$  case has very similar plot.

- The non–relativistic case is in accordance with the results presented in [4].
- Also the results for higher Lorentz factors are in agreement with [5].

- For high Lorentz factors we have more stable jet. We have a decrease in the  $\Im\omega$  by a factor of 10.
- The main reason for this behaviour is the increased inertia of the matter, as the flow reaches greater velocities.
- Except from the current–sheet–free configuration, the enhanced stability of this model could be due to the low magnetization  $\sigma \left( \frac{B_\phi^2}{4\pi\gamma^2\rho_0 c^2} \right) \sim 10^{-8}$ .
- Kelvin–Helmholtz instabilities don’t seem to affect jet’s stability. Plots for cases  $m=1$  and  $m=-1$  are similar indicating the above statement [6].

For the second model we investigated the dependence of  $\Im\omega$  with magnetization parameter  $\sigma$ .

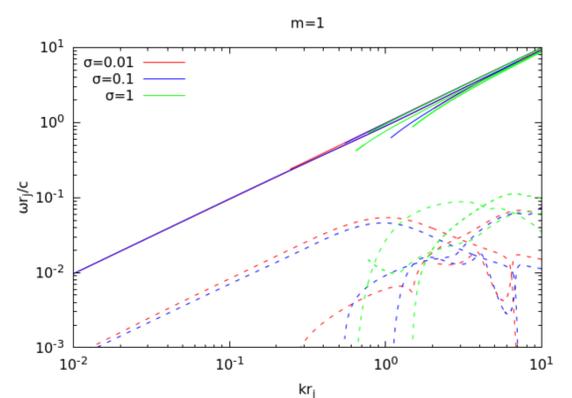


Figure 6: Dispersion relations for the second jet model. Both, the other two cases,  $m = 0$  and  $m = -1$  exhibit similar trends.

- Jet becomes more unstable when we increase  $\sigma$ .
- Current–driven instabilities may become more important when magnetization increases.
- Timescales in [2] in agreement with the above results.

## 5. Conclusions

The stability properties of the outflows, in general, depend on the magnetization, the bulk Lorentz factor and the ratio  $\frac{B_\phi}{B_z}$ . Rough agreement with the result

$$\Im\omega \sim \frac{1}{\gamma} \left[ 0.133 \frac{v_a B_\phi}{r_j B_z} \right]_{\text{comoving}} \quad \text{from [7].}$$

## References

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