Application of Feldman-Cousins upper limits to gravitational wave data analysis

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Upper limits

- Confidence intervals are the traditional way in which experimentalists report results of experiments.
- A special case is upper limits which play a very important role in gravitational wave physics.
- Currently Bayesian approach is very popular.
- However frequentist ULs are also commonly used.

Problems with frequentist upper limits

- Sometimes frequentist upper limits give unphysical results: typically an upper limit turns out to be negative for a quantity that by definition is positive (e.g. mass or amplitude).
- Feldman-Cousins provided a definition of confidence intervals that overcomes this difficulty.

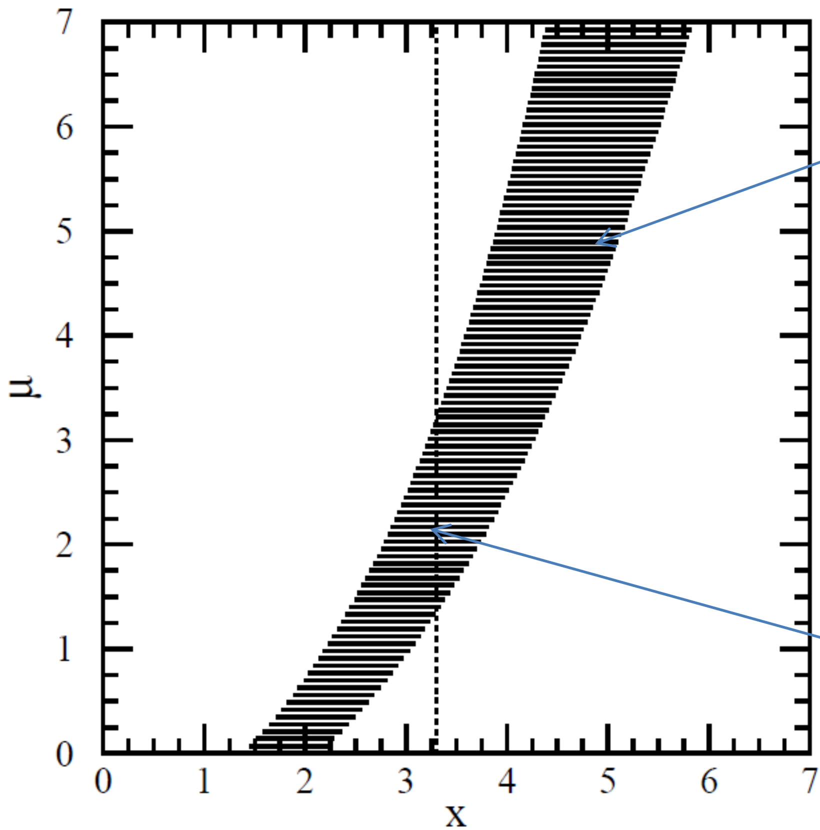
Feldman, G. J., & Cousin, R. D. 1998, A unified approach to the classical statistical analysis of small signals, Phys. Rev. D **57**, 3873

Cassical confidence intervals

J. **Neyman**, Phil. Trans. Royal Soc. London, Series A, 236 333-80 (**1937**).

Construct horizontally

$$P(x \in [x_1, x_2] \,|\mu) = \alpha.$$



Acceptance region

Read vertically

$$P(\mu \in [\mu_1, \mu_2]) = \alpha.$$

Confidence interval

A special case - upper limit: $P(x < x_1|\mu) = 1 - \alpha$

FC confidence intervals

Gaussian distribution: $$P(x|\mu) = \frac{1}{\sqrt{2\pi}} \exp(-(x-\mu)^2/2).$$

FC construction of acceptance region:

1. Let $\mu_{\rm best}$ be physically allowed value for which P is maximum

$$\mu_{\rm best} = \max(0, x) \quad P(x|\mu_{\rm best}) = \begin{cases} 1/\sqrt{2\pi}, & x \geq 0 \\ \exp(-x^2/2)/\sqrt{2\pi}, & x < 0. \end{cases}$$

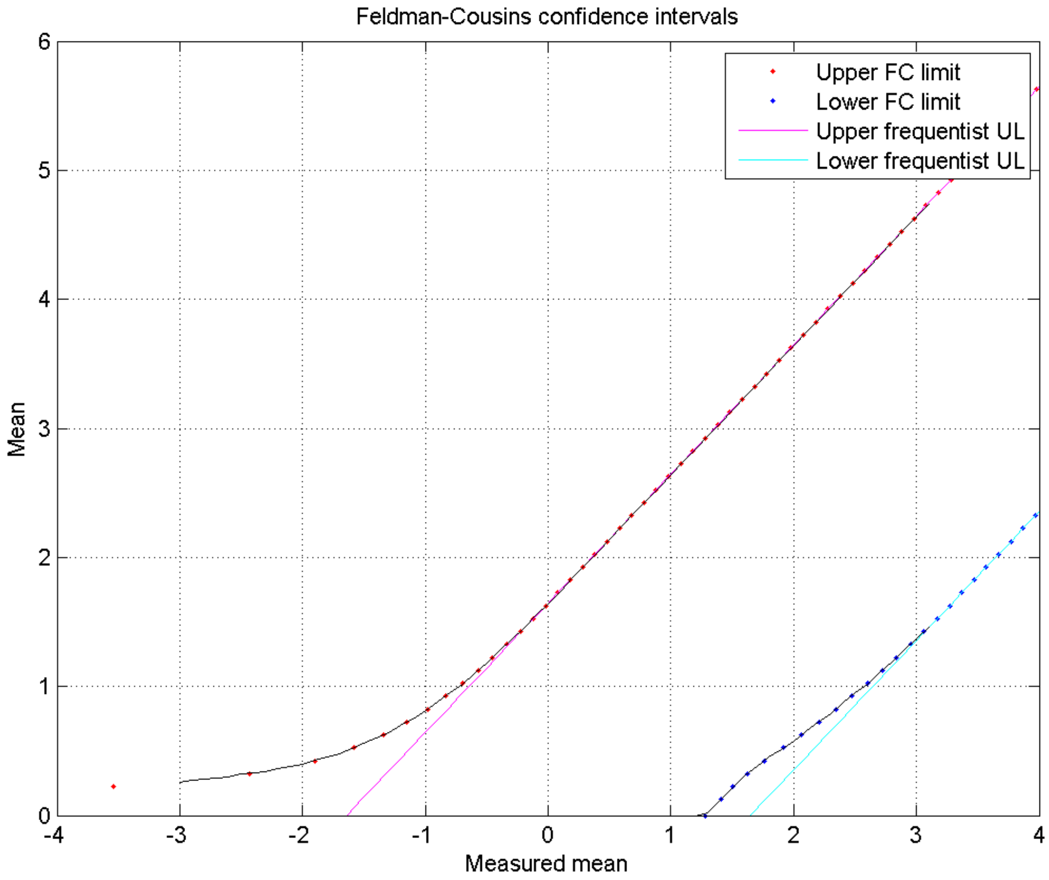
2. Define ordering funstion R as

$$R(x) = \frac{P(x|\mu)}{P(x|\mu_{\rm best})} = \begin{cases} \exp(-(x-\mu)^2/2), & x \geq 0 \\ \exp(x\mu - \mu^2/2), & x < 0. \end{cases}$$

3. Acceptance region $[x_1, x_2]$ defined as

$$\int_{x_1}^{x_2} P(x|\mu)dx = \alpha.$$ such that $$R(x_1) = R(x_2)$$

FC intervals for Gaussian case



Application of FC upper limit to GW searches for known pulsars using the F-statistic

- A popular way to do CW searches is to use the F-statistic:

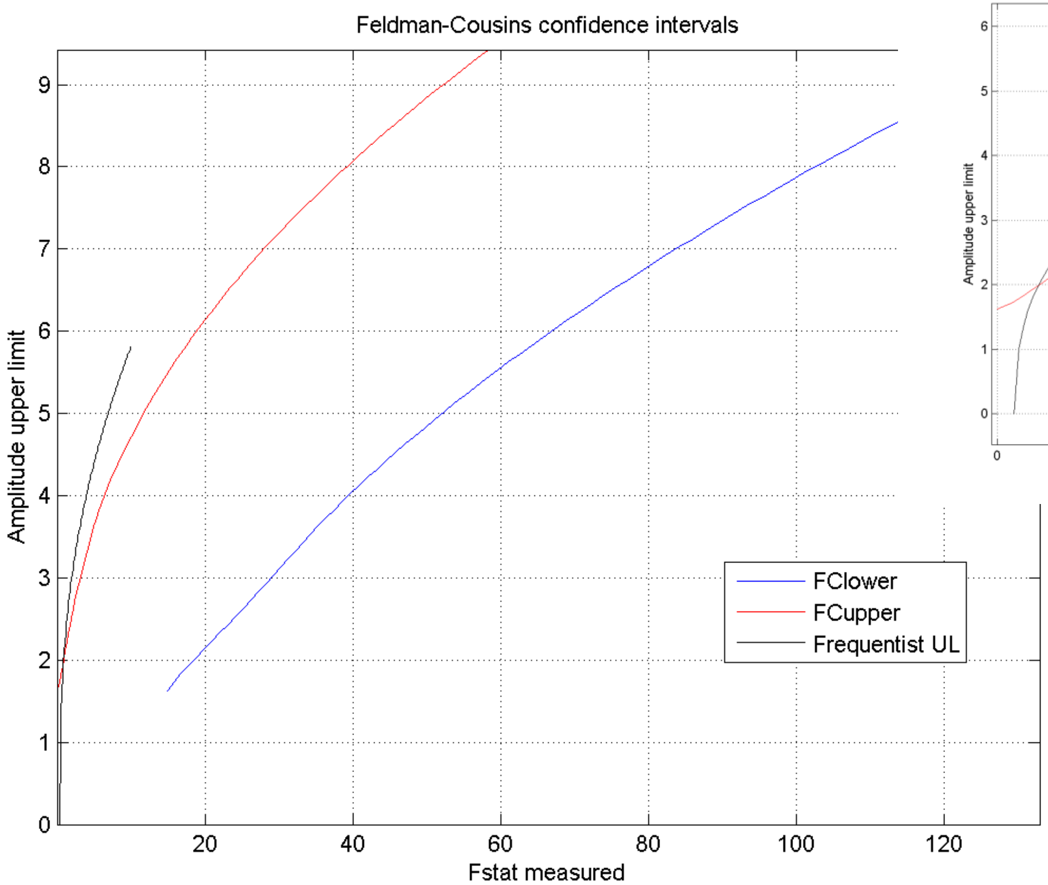
Jaranowski, P., Królak, A., & Schutz, B. F. 1998, Data analysis of gravitational-wave signals from pulsars: The signal and its detection, Phys. Rev. D **58**, 063001

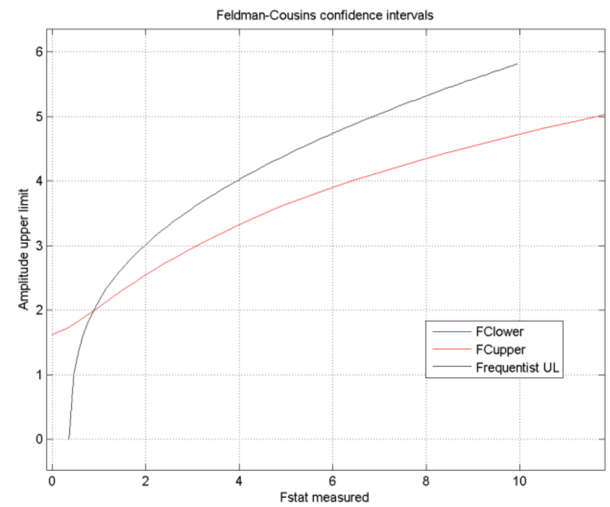
F-statistic is particularly suitable for searches for known pulsars as it depends only on **frequency, frequency derivatives and sky position** and not on unknown parameters: **amplitude, polarization, phase.**

Thus search consists of evaluating one number – the value of F.

For Gaussian noise F-statistic has χ^2 distribution with 4 degrees of freedom (central χ^2 for noise and non-central χ^2 parameter SNR^2 when signal is present).

FC intervals for χ^2 case





FC intervals for searches of real data

Real data are usually not normally distributed and consequently the F-statistic does not have the theoretical χ^2 distribution.

We estimate ULs by injecting artificial signals to the data with frequency offset from the frequency of the pulsar we are searching for.

We chose a set of signal amplitudes and for each amplitude we add signals for randomly chosen polarizations.

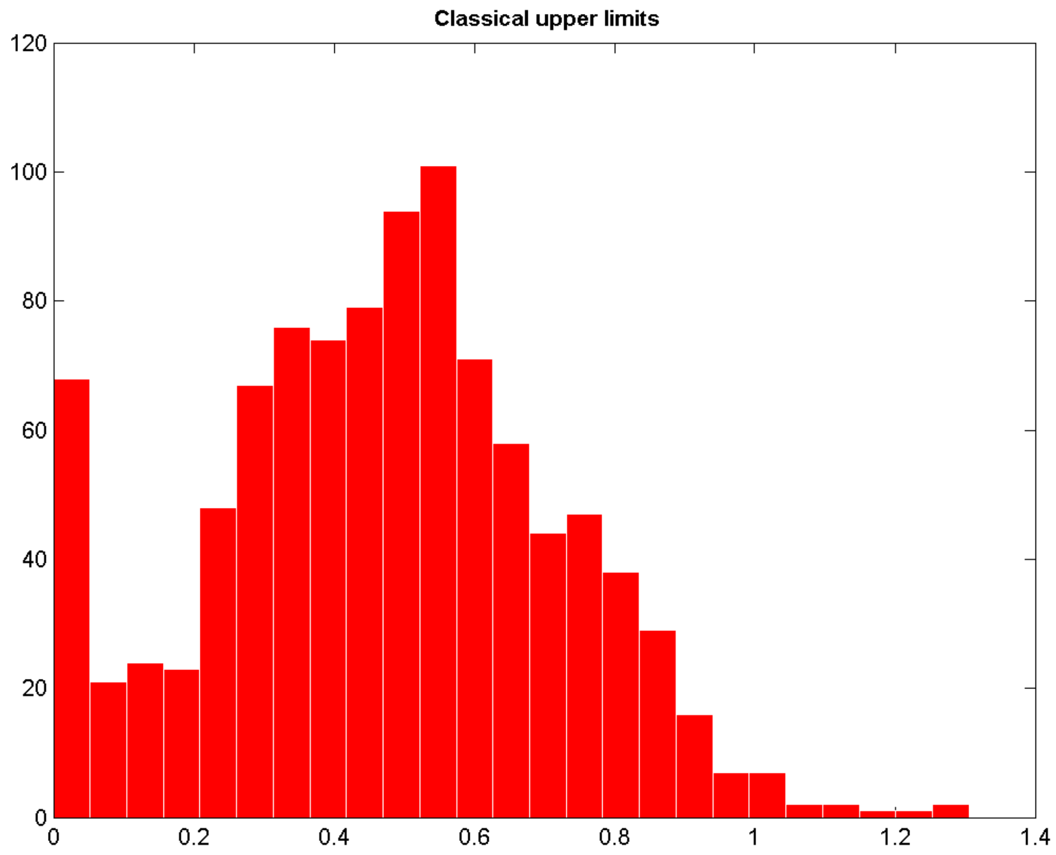
MC simulation

- 1000 fake pulsar data were generated.
- Around 10 amplitudes were chosen.
- For each amplitude 2500 pulsar signals were added with randomly offset frequency and randomly chosen polarization.
- FC acceptance region was determined by estimating pdf from the simulations and by interpolation.
- FC upper limits were determined by interpolation.
- Frequentist upper limits were obtained for comparison.

FC order function

- function   r = orderfcFstatho(x,hov,k,inpd)
- % ORDERFCFSTATHO Order function for simulated trageted searches
- % using F-statistic
- %
- % Physically allowed value of ho
- % for which pdf is maximum
- for l = 1:length(hov)
-     load(['uplimsim' inpd num2str(l)],'F')
-     [pdfFvec,Fvec] = ksdensity(F,'support','positive');
-     Px(l) = interp1(Fvec,pdfFvec,x,'cubic');
- end
- Pm = max(Px);
- r = Px(k)/Pm;

Classical ULs



Feldman-Cousins ULs