Anisotropies of the astrophysical background of gravitational waves

Giulia Cusin

UNIVERSITY OF OXFORD

Phys.Rev. D96 (2017) 10, 103019
arXiv: 1711.11345, accepted by PRD
arXiv: 1803.03236, PRL

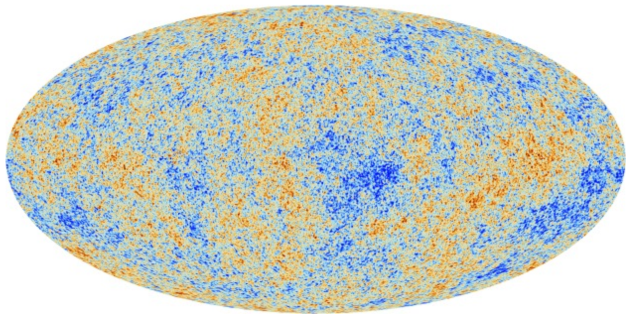
in collaboration with I. Dvorkin, C. Pitrou, J.P. Uzan

Outline

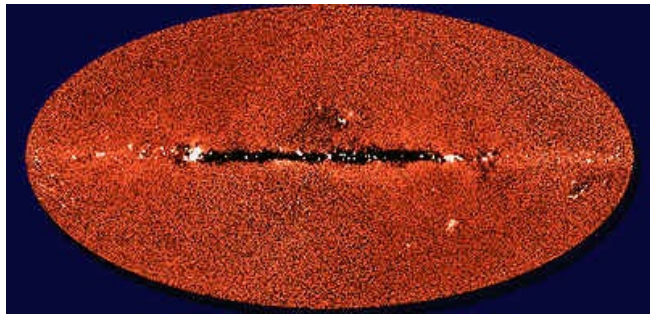
- GW background: what it is and how to characterize it
- Status of the art of observations
- Framework to study anisotropies
- First numerical predictions: what can we learn?
- Work in progress

Stochastic backgrounds of radiation

	cosmological origin	astrophysical origin
EM radiation	CMB	IR extragalactic background
GW radiation	cosmological background	**astrophysical background**



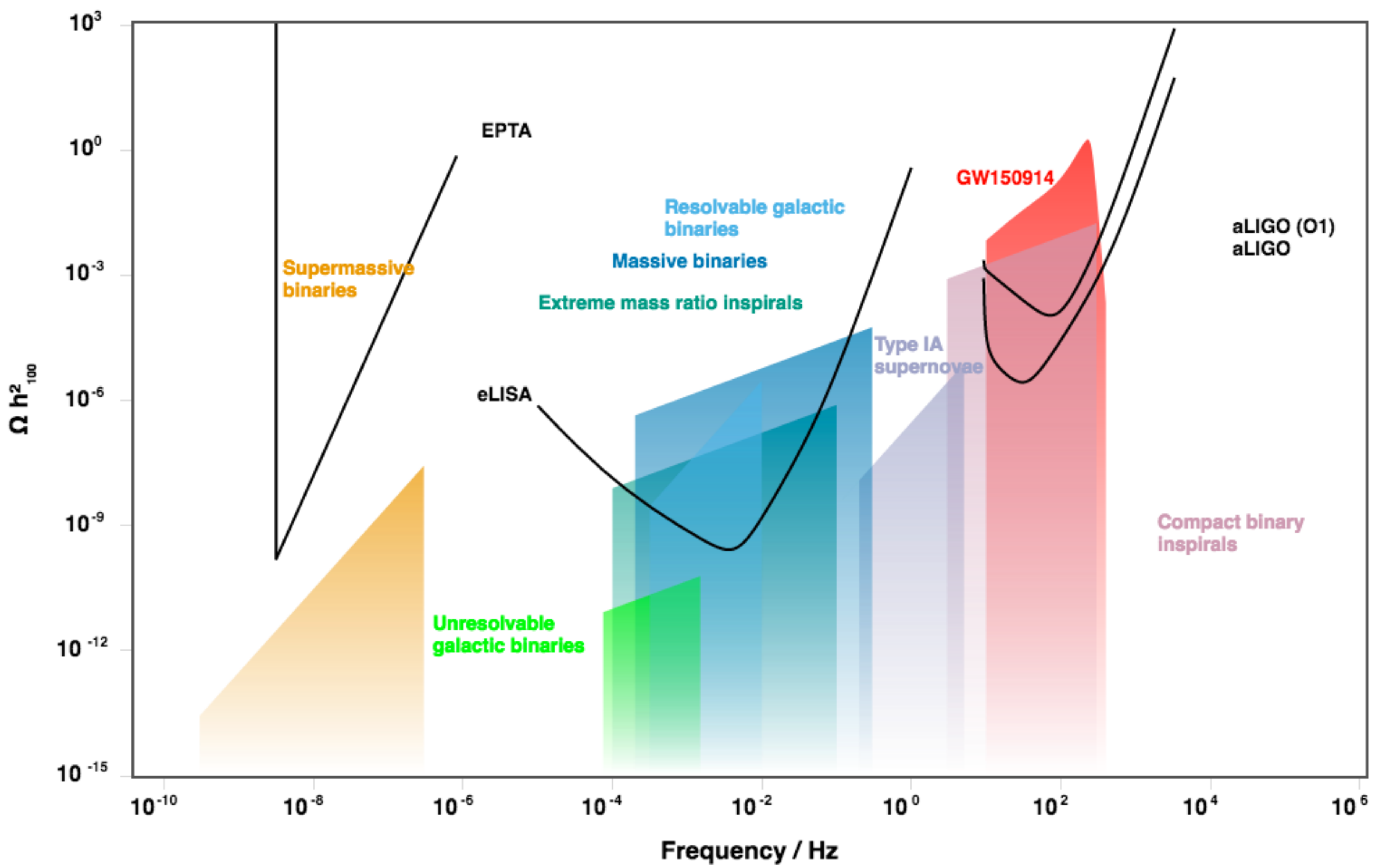
Plank CMB map



Plank IR map

Different contributions to the astro background

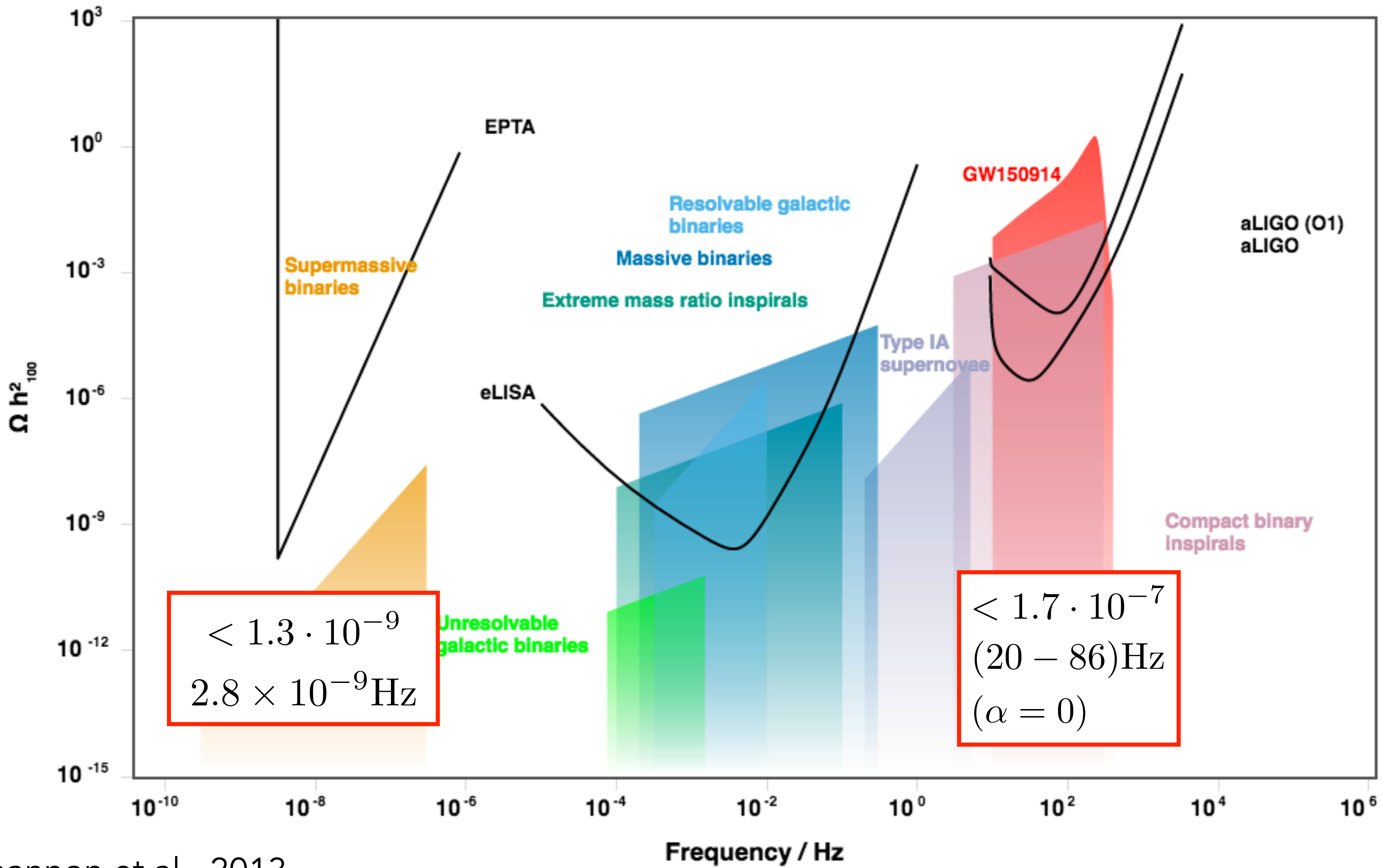
$$\Omega_{GW}(\nu_O) = \frac{\nu_O}{\rho_c}\frac{d\rho_{GW}(\nu_O)}{d\nu_O}$$



http://rhcole.com/apps/GWplotter/

Different contributions to the astro background

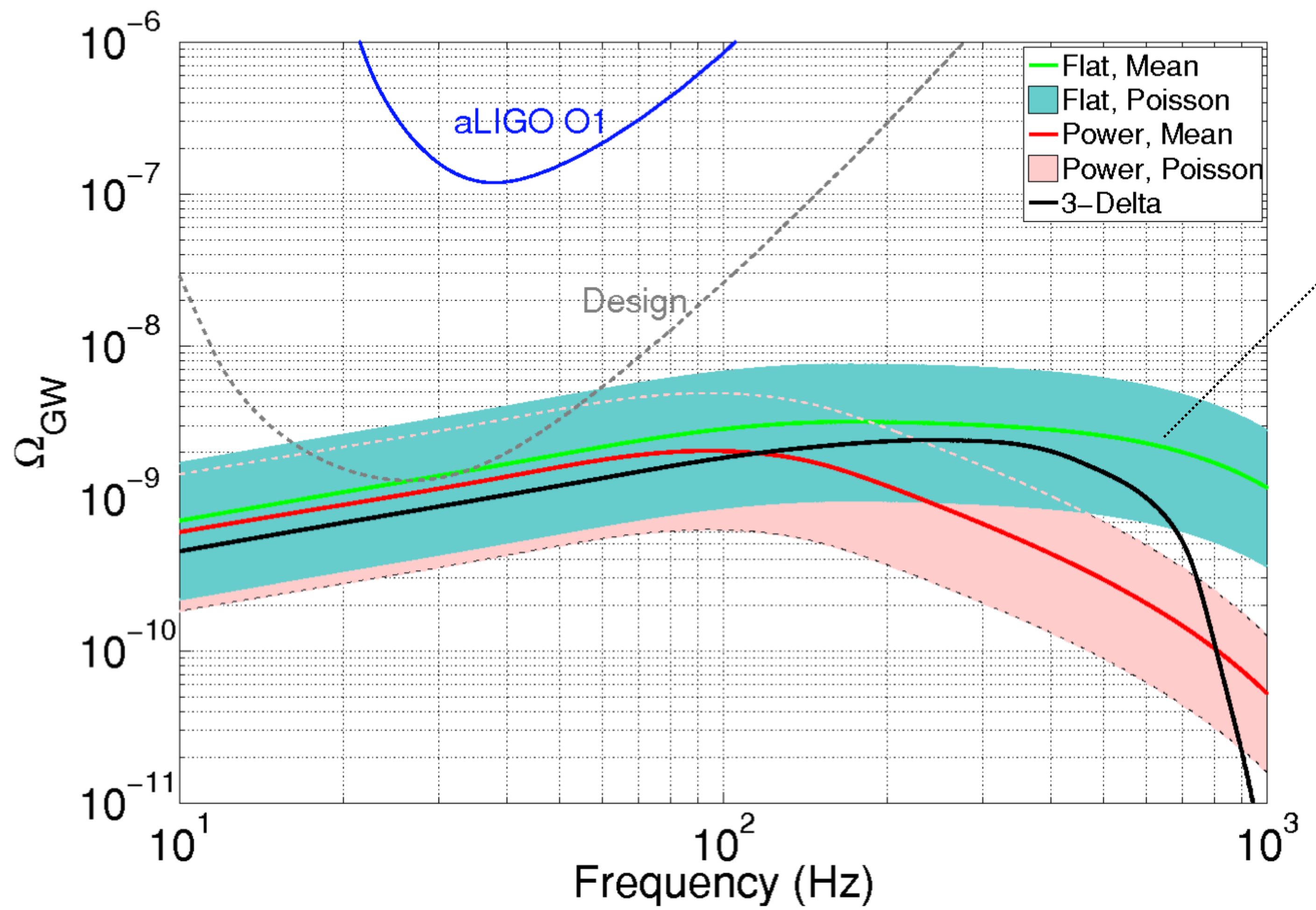
$$\Omega_{GW}(\nu_O) = \frac{\nu_O}{\rho_c}\frac{d\rho_{GW}(\nu_O)}{d\nu_O}$$



LIGO-VIRGO
PRL 118, 121101, 2017

Shannon et al., 2013

LIGO: what is expected from next run



uncertainty due to uncertainty on local rate measurements

LIGO-VIRGO
PRL 118, 121101, 2017

Detection of BBHs background highly probable as the designed sensitivity is reached!

AGWB observation: angular searches

$$\Omega_{GW}(\nu_O) = \int d^2\mathbf{e}_O \, \boxed{\Omega_{GW}(\nu_O, \mathbf{e}_O)}$$

$$\Omega_{GW}(\nu_O, \mathbf{e}_O) = \frac{\nu_O}{\rho_c} \frac{d^3\rho_{GW}(\nu_O, \mathbf{e}_O)}{d^2\mathbf{e}_O d\nu_O}$$

- **LIGO**: directional searches for persistent GW background

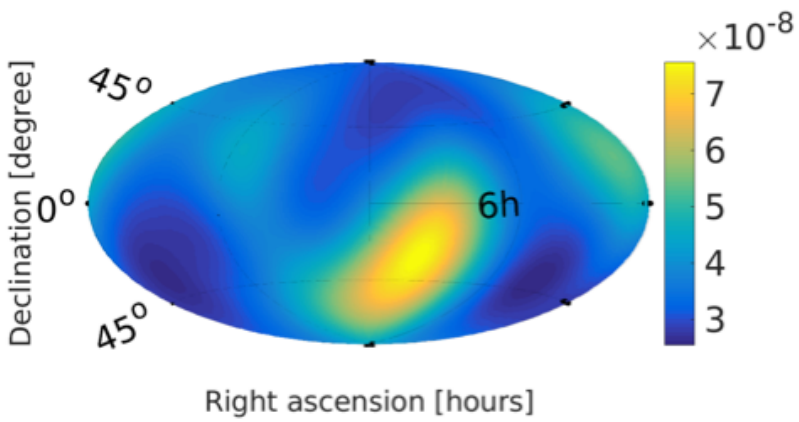
(LIGO-VIRGO PRL 118, 121102, 2017)

- **PTA,** see e.g. Taylor, Gair 2013

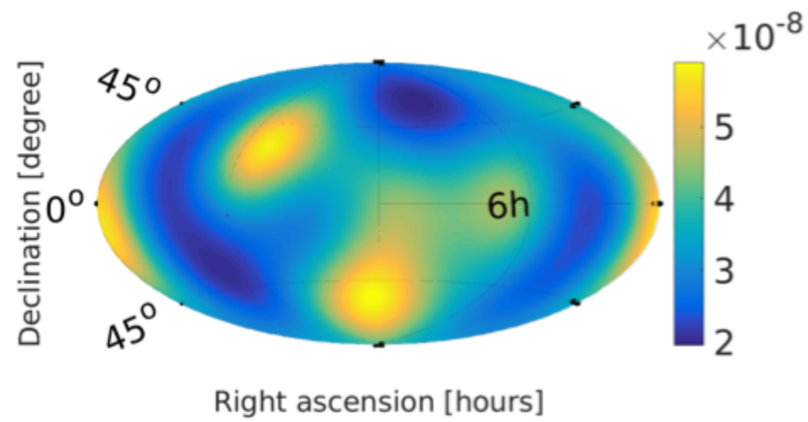
LIGO directional searches

(LIGO-VIRGO PRL 118, 121102, 2017)

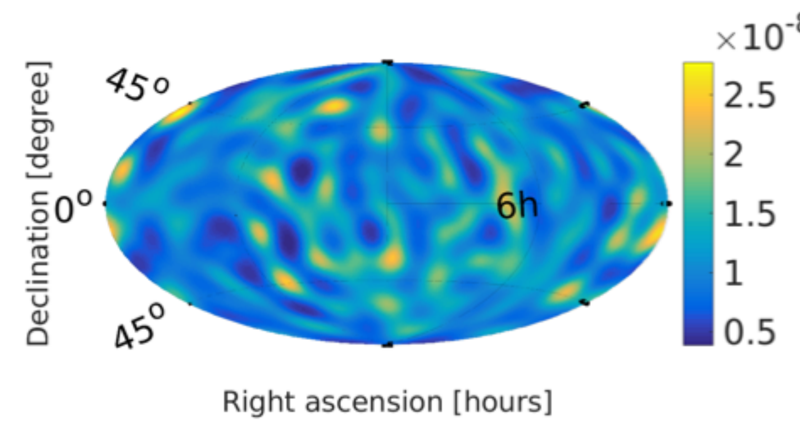
upper limits at 90% confidence level



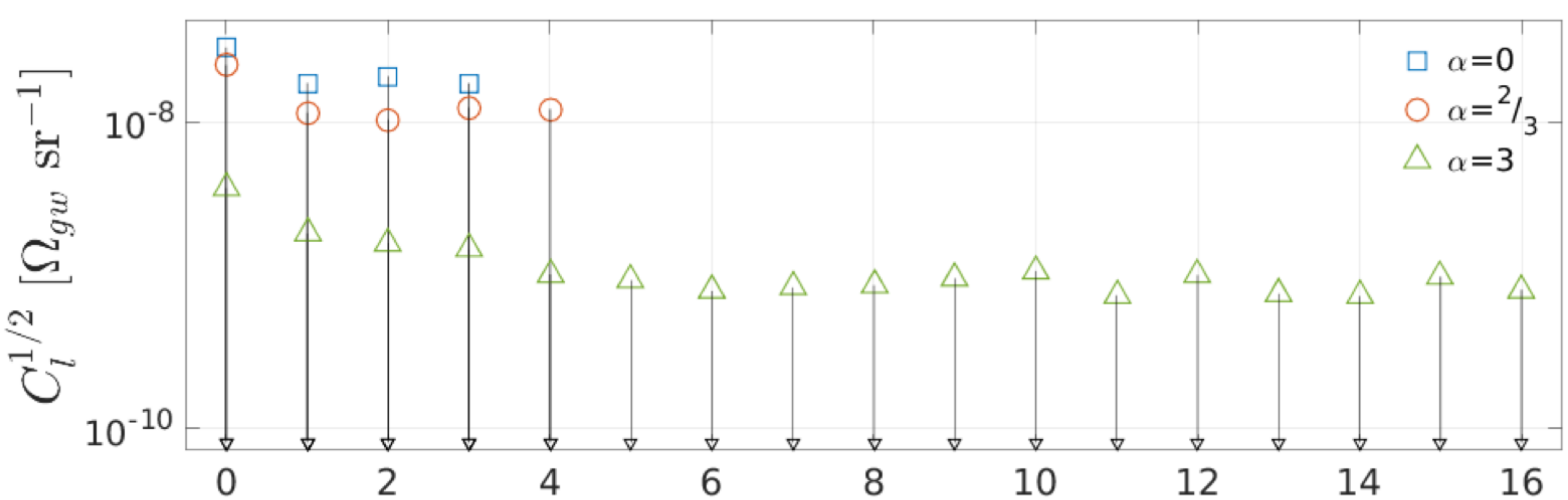
$\alpha = 0$



$\alpha = 2/3$



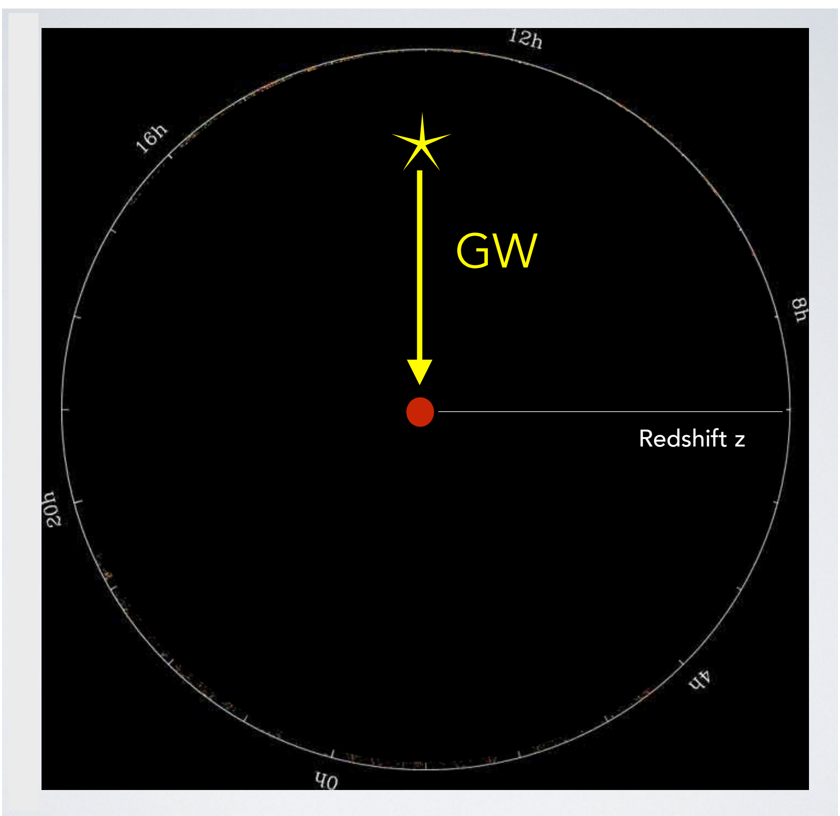
$\alpha = 3$



- SNR consistent with gaussian noise
- limits on monopole consistent with the ones from isotropic searches

Astrophysical background: theory

Ω_{GW} computed assuming FLRW Universe:

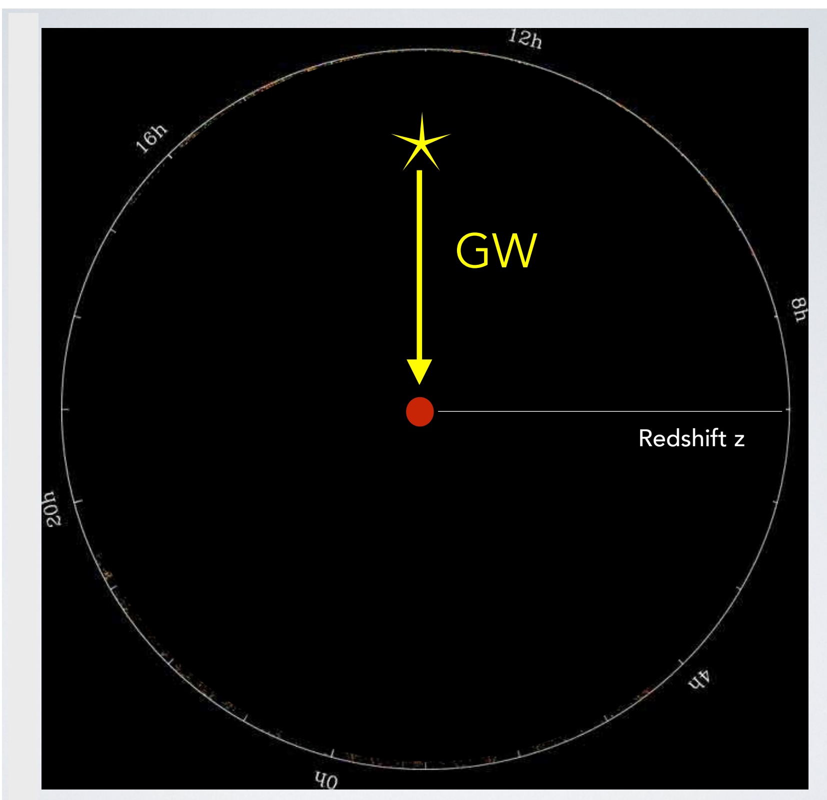


- Isotropic distribution of sources (**no structures**)
- Propagation of GW in **homogeneous** medium

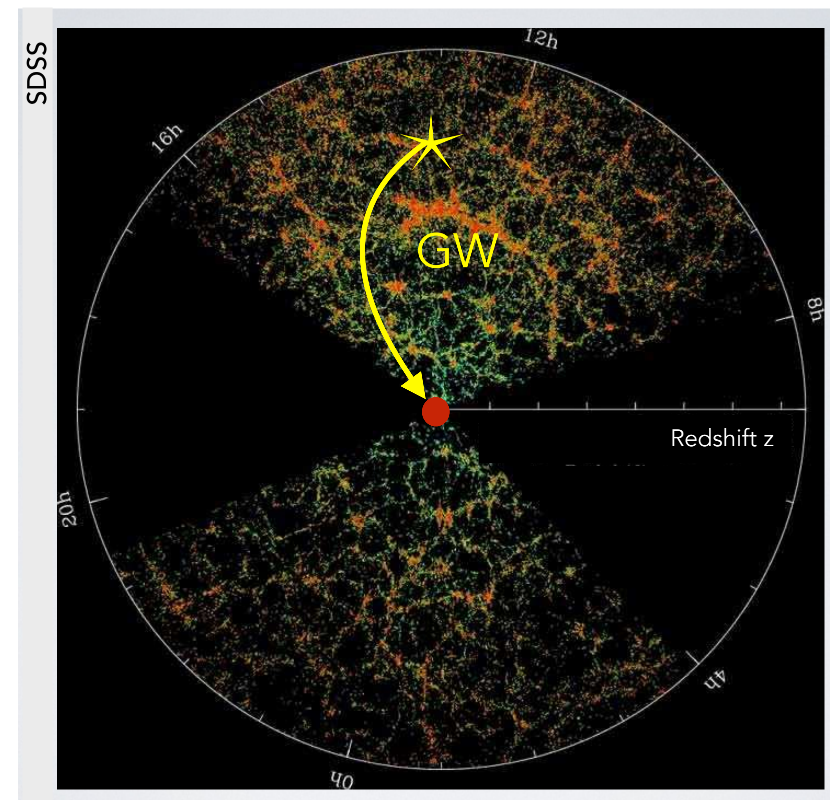
Not sufficient to compute anisotropies….

Astrophysical background: theory

FLRW



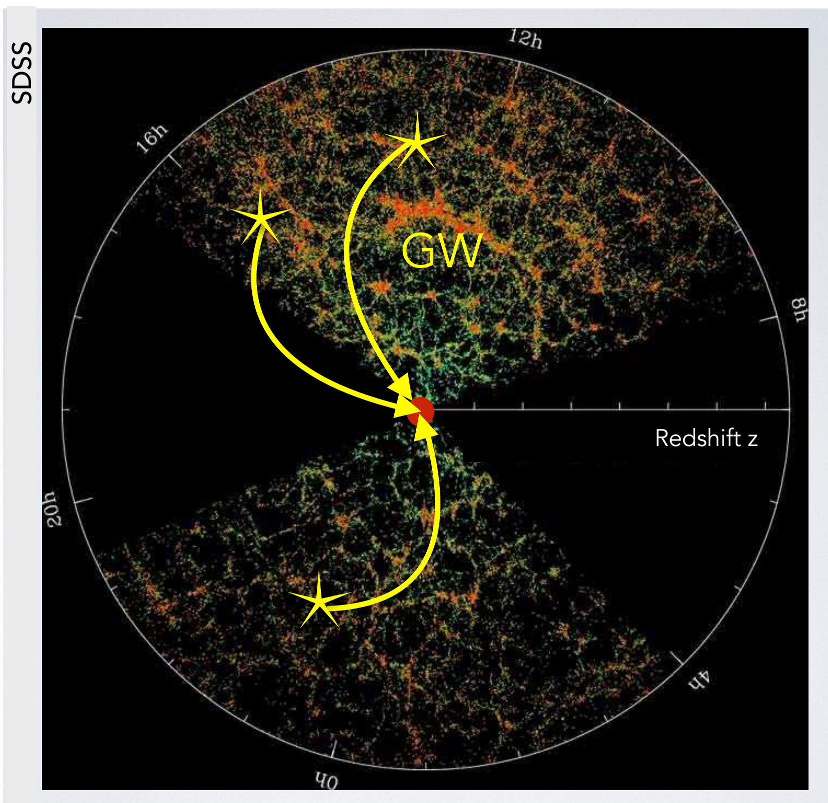
Structures & lensing effect



Our goal: compute the total flux of GW from astrophysical sources, received per units of frequency and solid angle around the direction of observation

GW background: anisotropies

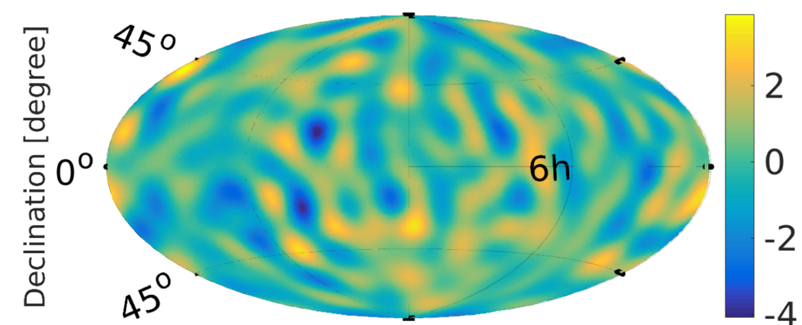
Structures & lensing effect

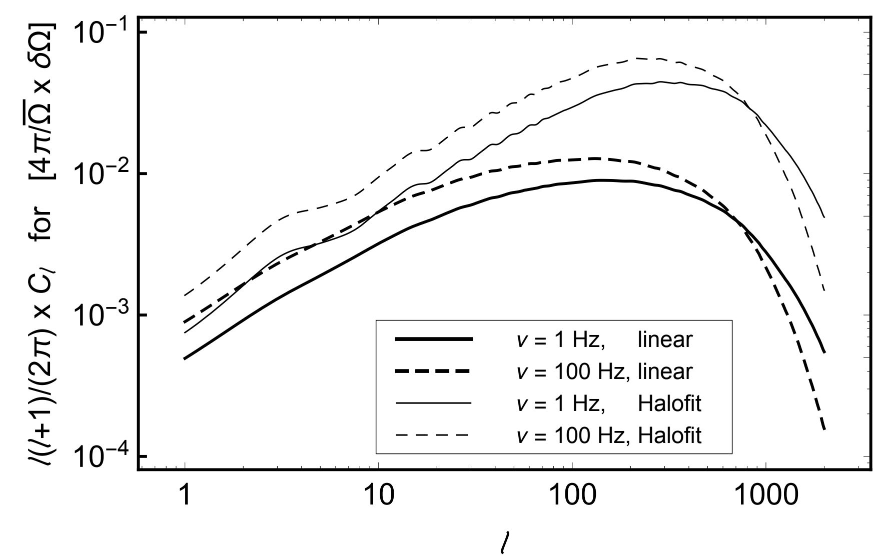


$$\Omega_{GW}(\nu_O, \mathbf{e}_O)$$

[Cusin, Pitrou, Uzan 2017]

sky map (LIGO-VIRGO PRL 118, 121102, 2017)





[Cusin, Dvorkin, Uzan, Pitrou 2018]

New observable: at the crossroad astro-cosmology

Final results contain information about

(1) cosmology: cosmic structure at large scales, spacetime geometry…

(2) local physics: GW production, star formation rate, distribution sources…

New observable: at the crossroad astro-cosmology

Final results contain information about

(1) cosmology: cosmic structure at large scales, spacetime geometry…

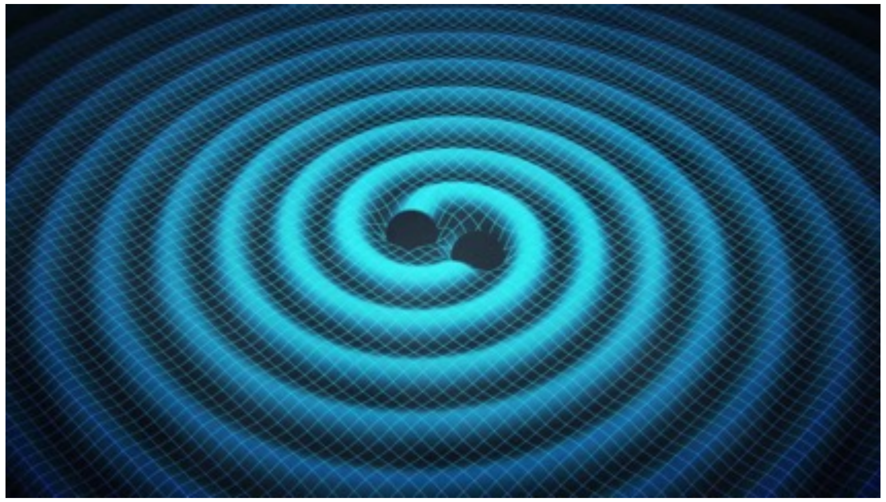
(2) local physics: GW production, star formation rate, distribution sources…

by comparison with observations we can put constraints on astrophysics

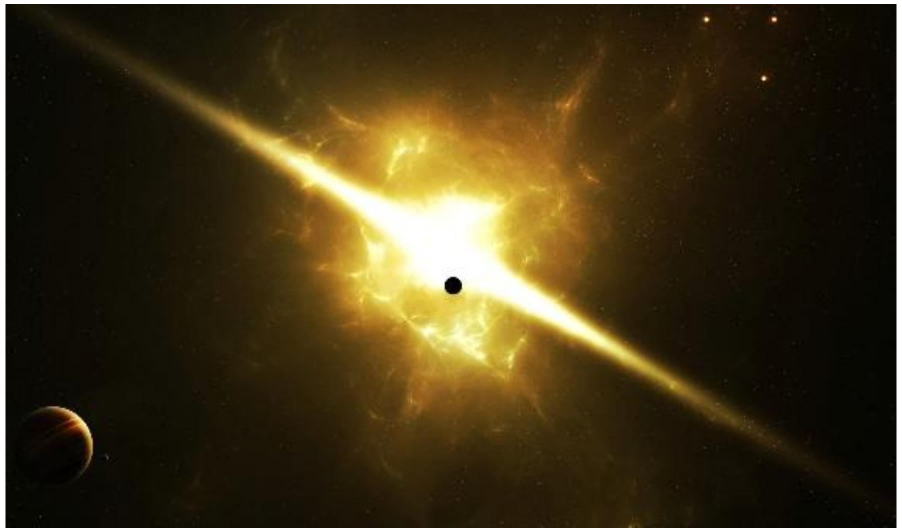
Sources of GW considered

We distinguish the following **sources of GW** (inside a galaxy)

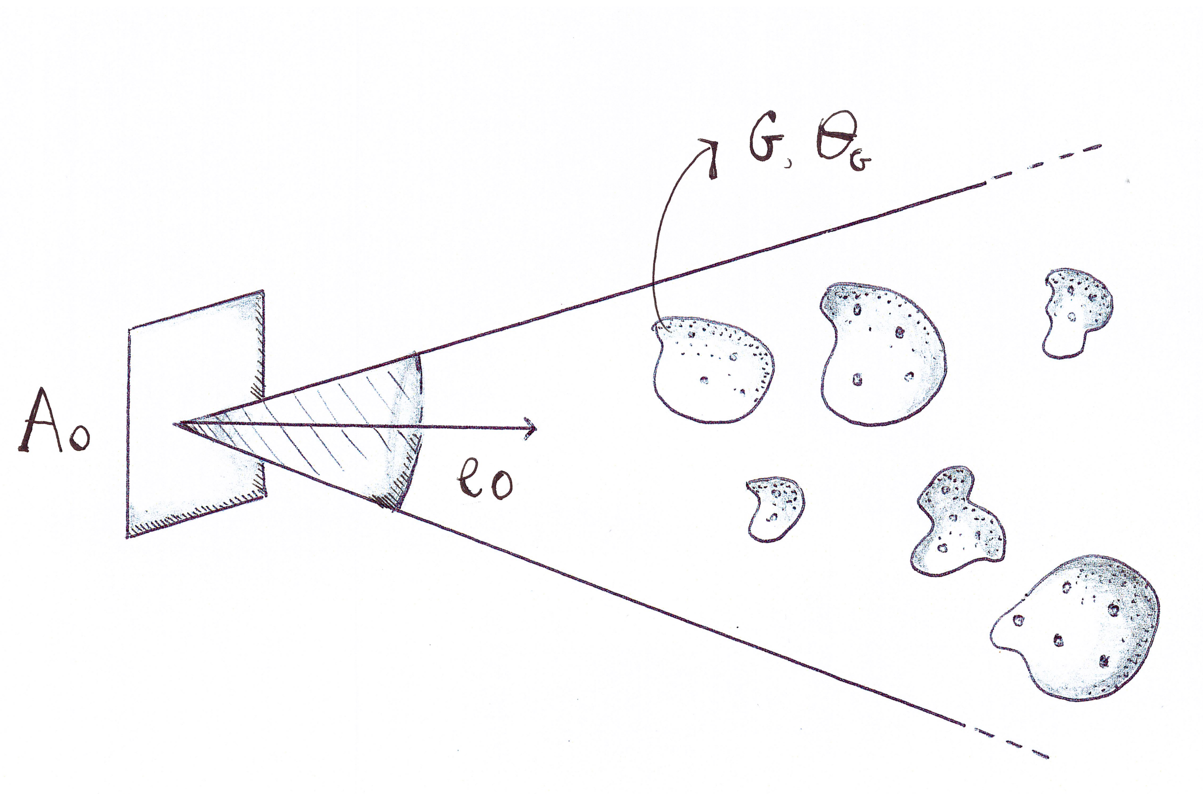
(1) mergers of compact objects



(2) exploding supernovae



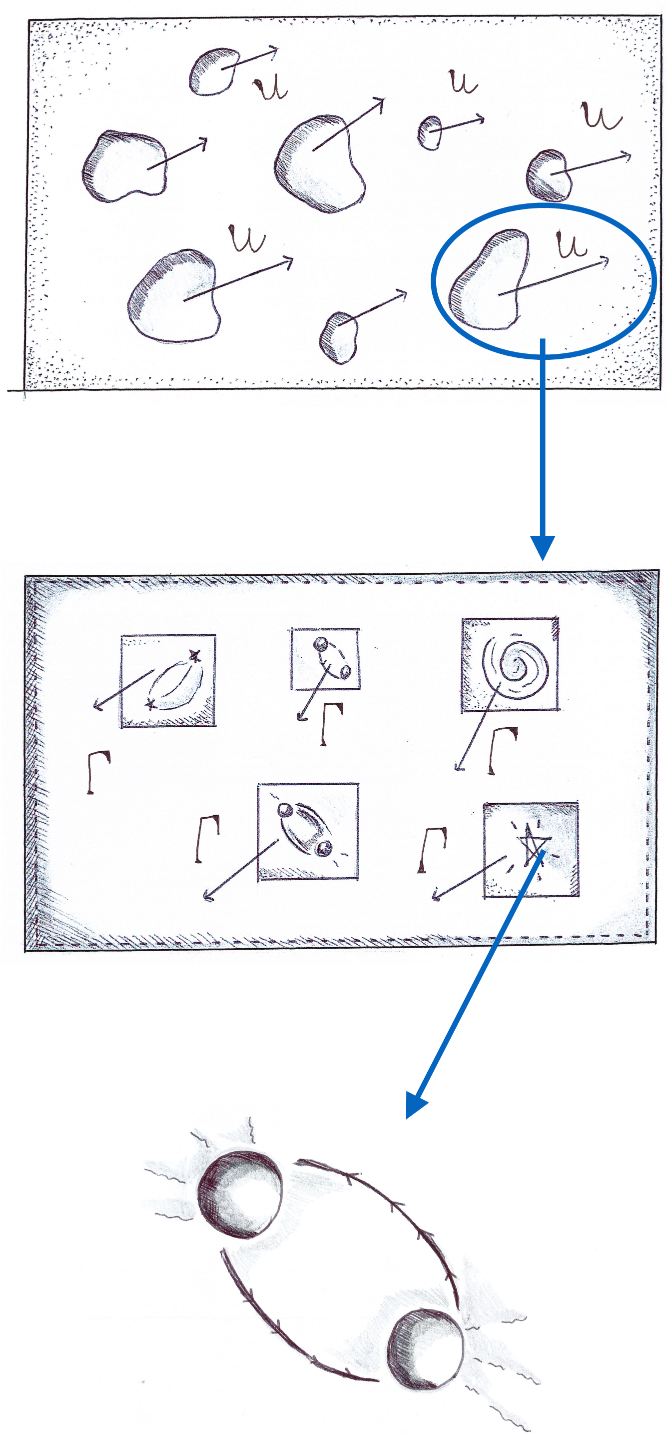
Scheme of our approach



$$\Phi(e_O, z_G, \theta_G) \equiv \frac{\text{Energy}}{A_O \Delta t_O}$$

Total flux received: sum the contributions from all the galaxies in the solid angle of observation

From cosmological to local scale



cosmological scale

local scale

$$\Phi = \frac{(1+z_G)}{D_L^2}\boxed{\mathcal{L}_G}$$

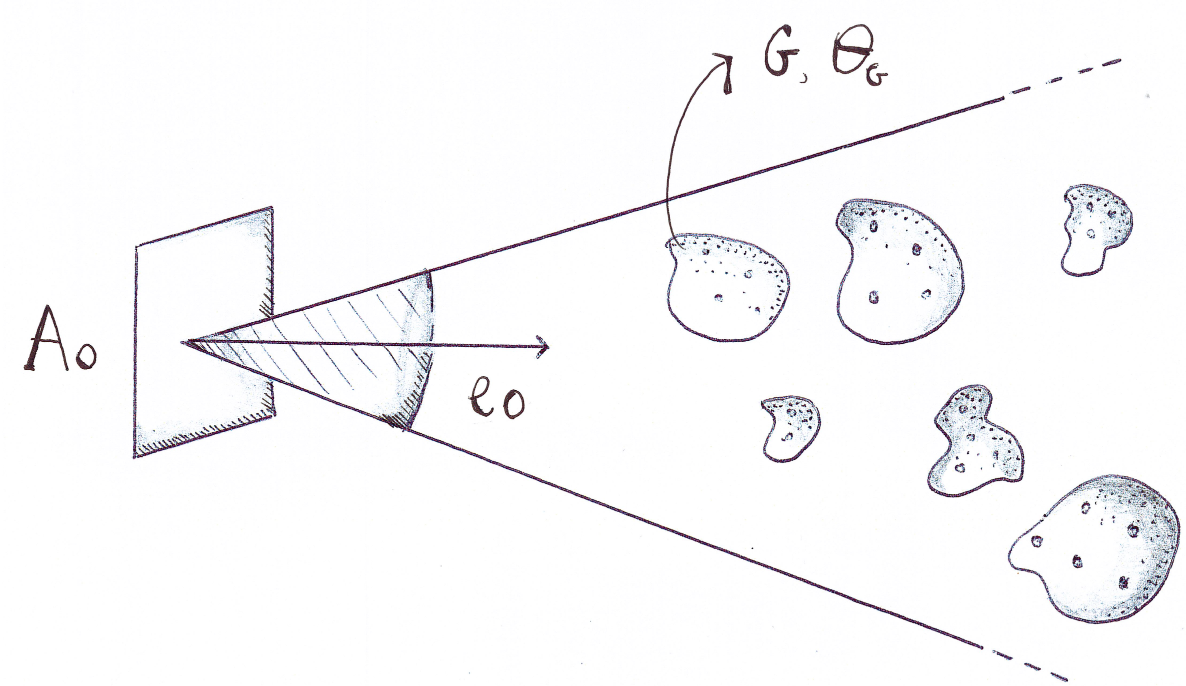
function local
quantities at sources

Final parametrization

$$\Omega_{GW}(\nu_O, \mathbf{e}_O) = \frac{\nu_O}{\rho_c} \int dz_G \int d\theta_G \, \boxed{\Phi[z_G, \nu_O, \theta_G]} \, \boxed{\frac{d^3\mathcal{N}_G}{dz_G d^2\mathbf{e}_O}(z_G, \theta_G)}$$

flux from one galaxy

galaxies in comoving volume



Final parametrization

$$\Omega_{GW}(\nu_O, \mathbf{e}_O) = \frac{\nu_O}{\rho_c} \int dz_G \int d\theta_G \boxed{\Phi[z_G, \nu_O, \theta_G]} \boxed{\frac{d^3\mathcal{N}_G}{dz_G d^2\mathbf{e}_O}(z_G, \theta_G)}$$

rewritten in terms of comoving density and comoving volume

rewritten in terms of luminosity

predictions in a generic cosmology, G.C. et al Phys.Rev. D96 (2017) 10, 103019

Results in cosmological context

We consider a Friedman universe with scalar perturbations

$$\Omega_{GW} = \boxed{\frac{\bar{\Omega}_{GW}}{4\pi}} + \boxed{\delta\Omega_{GW}(\mathbf{e}, \nu_O)}$$

new

we recover standard results in the literature

Results in cosmological context

cosmological part

$$\delta\Omega_{GW}(\boldsymbol{e},\nu_{\rm O}) = \frac{\nu_{\rm O}}{4\pi\rho_c}\int_{\eta_*}^{\eta_{\rm O}} \mathrm{d}\eta\, \mathcal{A}(\eta,\nu_{\rm O}) \left[\delta_{\rm G} + 4\Psi - 2\boldsymbol{e}\cdot\nabla v + 6\int_{\eta}^{\eta_{\rm O}} \mathrm{d}\eta'\dot{\Psi}\right]$$

$$+ \frac{\nu_{\rm O}}{4\pi\rho_c}\int_{\eta_*}^{\eta_{\rm O}} \mathrm{d}\eta\, \mathcal{B}(\eta,\nu_{\rm O}) \left[\boldsymbol{e}\cdot\nabla v - \Psi - 2\int_{\eta}^{\eta_{\rm O}} d\eta'\dot{\Psi}\right]$$

astrophysical part

$$\mathcal{A}(\eta,\nu_{\rm O}) \equiv a^4 \bar{n}_{\rm G}(\eta)\int d\theta_{\rm G}\, \mathcal{L}_{\rm G}(\eta,\bar{\nu}_{\rm G},\theta_{\rm G})$$

$$\mathcal{B}(\eta,\nu_{\rm O}) \equiv a^3\,\nu_{\rm O}\bar{n}_{\rm G}(\eta)\int d\theta_{\rm G}\,\frac{\partial\mathcal{L}_{\rm G}}{\partial\nu_{\rm G}}\bigg|_{\bar{\nu}_{\rm G}}(\eta,\bar{\nu}_{\rm G},\theta_{\rm G})$$

Angular power spectrum

Non-vanishing auto-correlation linked to **correlation of large scale structures** (with modulation from local physics)

$$C(\nu_{\rm O}, \theta) = \langle \delta\Omega_{GW}(\nu_{\rm O}, \boldsymbol{e}_1)\delta\Omega_{GW}(\nu_{\rm O}, \boldsymbol{e}_2)\rangle$$

$$\sum_\ell \frac{2\ell+1}{2\pi} C_\ell(\nu_{\rm O}) P_\ell(\boldsymbol{e}_1 \cdot \boldsymbol{e}_2)$$

depends on frequency

$$\boxed{C_\ell(\nu_{\rm O}) = \frac{2}{\pi}\int \mathrm{d}k\, k^2 |\delta\hat{\Omega}_\ell(k, \nu_{\rm O})|^2}$$

Correlation with other cosmological probes

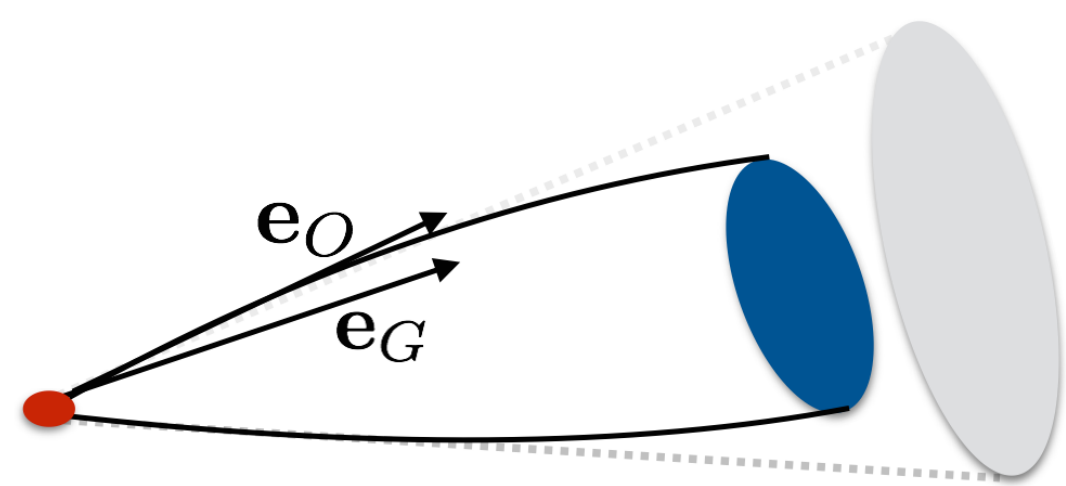
Number counts: number of galaxies as a function of direction and redshift

(see e.g. Bonvin & Durrer 2011)

Weak lensing describes the deformation of the shape of a given galaxy by the gravitational potential of the large scale structures

$$\mathbf{e}_G = \mathcal{A} \cdot \mathbf{e}_O$$

amplification matrix



Summary of correlations

	Ω_{GW}	lensing κ	galaxy number counts Δ
Ω_{GW}	$C_\ell(\nu_O)$	$B_\ell(\nu_O)$	$D_\ell(\nu_O)$
κ	$B_\ell(\nu_O)$	κ_ℓ	γ_ℓ
Δ	$D_\ell(\nu_O)$	γ_ℓ	Δ_ℓ

new!

known

Numerical results: BH merges

cosmological part

$$\delta\Omega_{GW}(\boldsymbol{e},\nu_{\rm O}) = \frac{\nu_{\rm O}}{4\pi\rho_c}\int_{\eta_*}^{\eta_{\rm O}} \mathrm{d}\eta\, \mathcal{A}(\eta,\nu_{\rm O}) \left[\delta_{\rm G} + 4\Psi - 2\boldsymbol{e}\cdot\nabla v + 6\int_{\eta}^{\eta_{\rm O}} \mathrm{d}\eta'\dot{\Psi}\right] + \frac{\nu_{\rm O}}{4\pi\rho_c}\int_{\eta_*}^{\eta_{\rm O}} \mathrm{d}\eta\, \mathcal{B}(\eta,\nu_{\rm O}) \left[\boldsymbol{e}\cdot\nabla v - \Psi - 2\int_{\eta}^{\eta_{\rm O}} d\eta'\dot{\Psi}\right]$$

cosmological evolution

astrophysical part

$$\mathcal{A}(\eta,\nu_{\rm O}) \equiv a^4 \bar{n}_{\rm G}(\eta)\int d\theta_{\rm G}\mathcal{L}_{\rm G}(\eta,\bar{\nu}_{\rm G},\theta_{\rm G})$$

$$\mathcal{B}(\eta,\nu_{\rm O}) \equiv a^3\,\nu_{\rm O}\bar{n}_{\rm G}(\eta)\int d\theta_{\rm G}\left.\frac{\partial\mathcal{L}_{\rm G}}{\partial\nu_{\rm G}}\right|_{\bar{\nu}_{\rm G}}(\eta,\bar{\nu}_{\rm G}$$

astrophysical model

Astrophysical model: ingredients

Luminosity of a galaxy

$$\mathcal{L}_{\mathrm{G}} = \int \mathrm{d}m\,\mathrm{d}m'\,\mathrm{d}a_{\mathrm{f}}\,\underbrace{\boxed{\frac{\mathrm{d}E}{\mathrm{d}\nu}}}_{\text{spectrum}} \times \underbrace{\boxed{\mathcal{R}_{\mathrm{m}}[m, m', a_{\mathrm{f}}, t]}}_{\text{merger rate}}$$

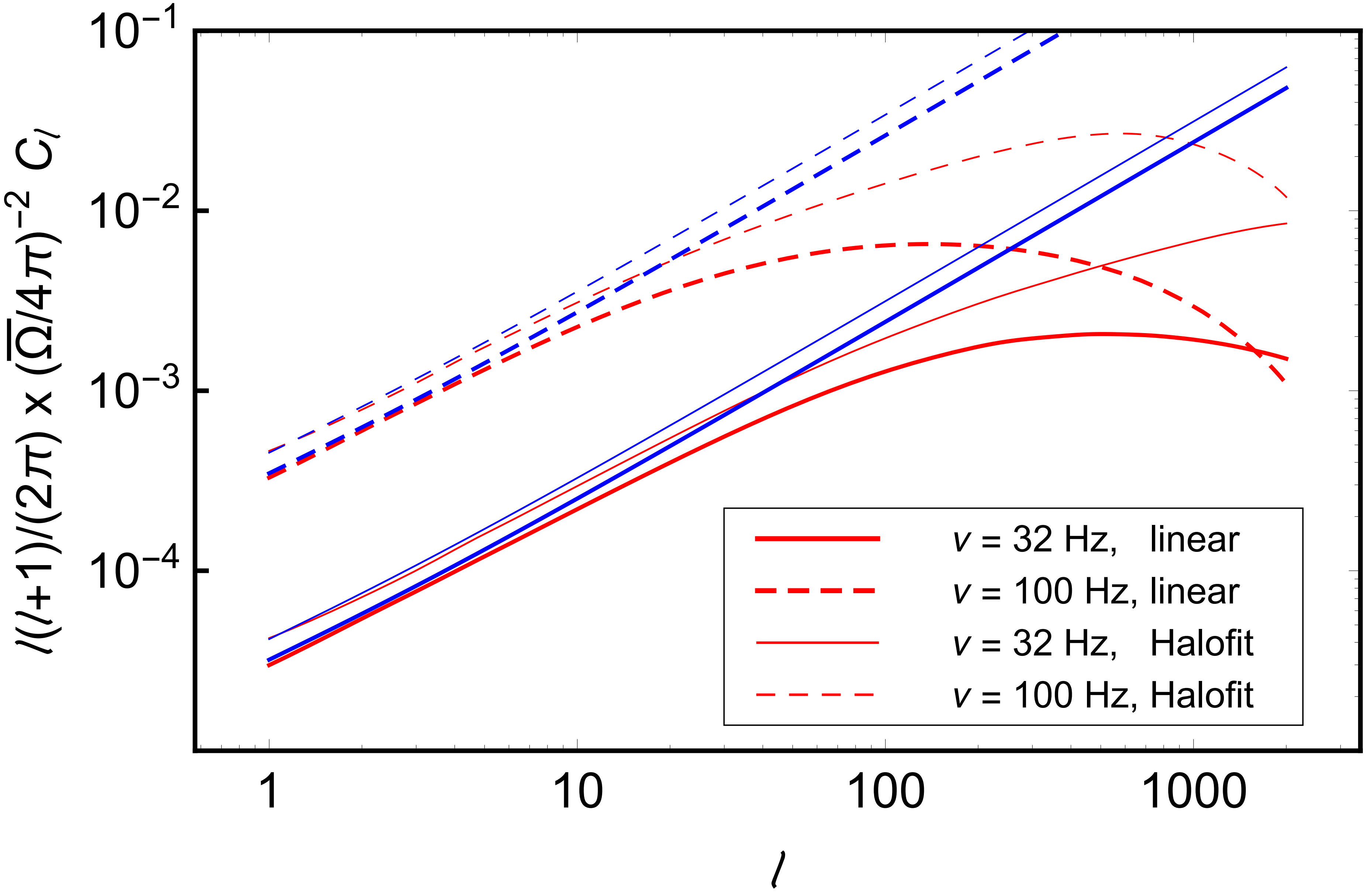
- star formation rate
- stellar evolution model (Fryer et al.)
- rate of BH formation
- fraction BH in binary systems
- birth rate of binaries

see Cusin, Dvorkin, Pitrou, Uzan, arXiv: 1803.03236, PRL

Cosmological evolution

- Perturbations evolved from primordial power spectrum after inflation
- Linear evolution of perturbations with CMBquick
- Inclusion of non-linearities with HALO fit
- Cosmological parameters from Planck 2015 data

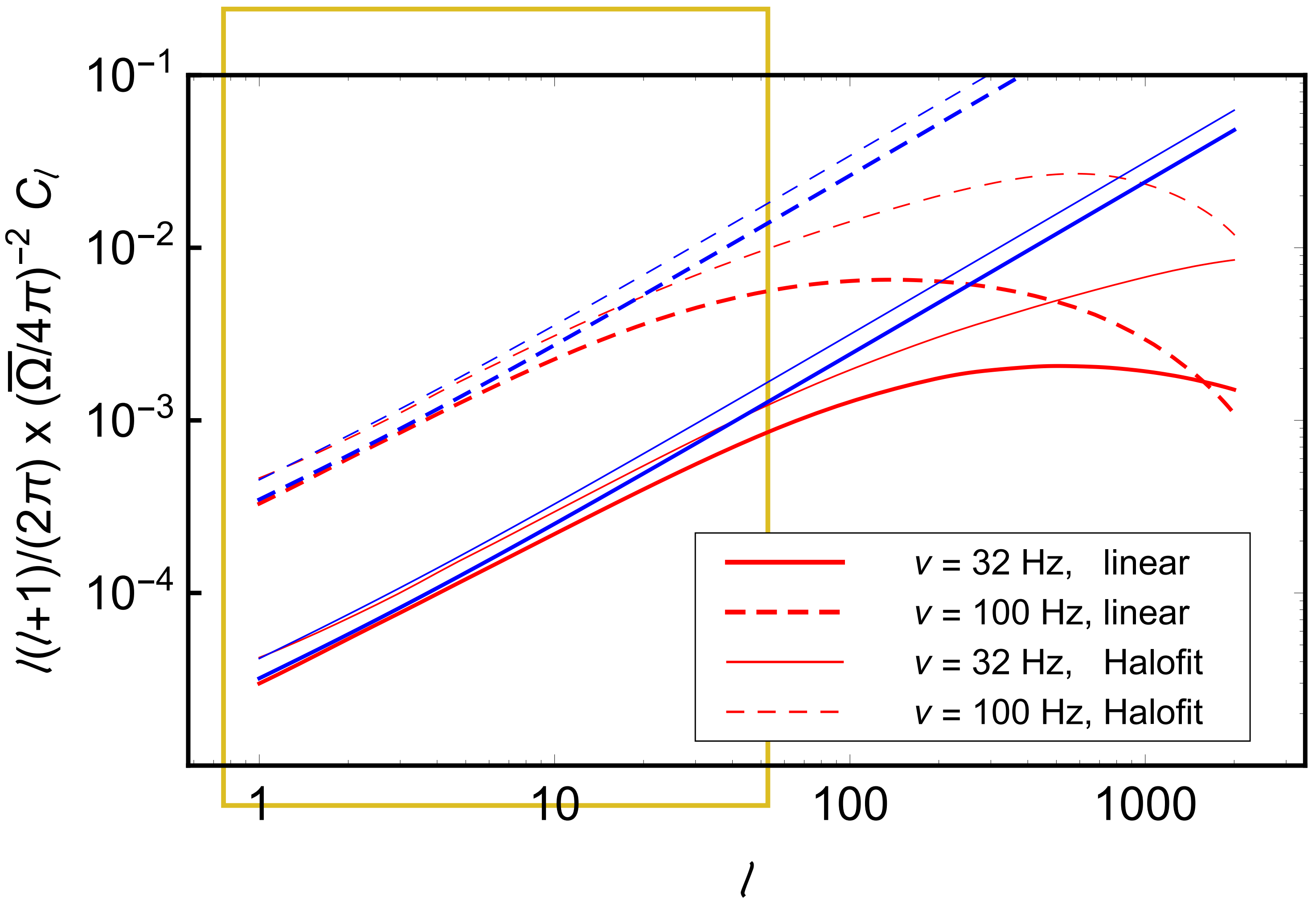
Angular power spectrum



GC, Dvorkin, Pitrou, Uzan, arXiv: 1803.03236

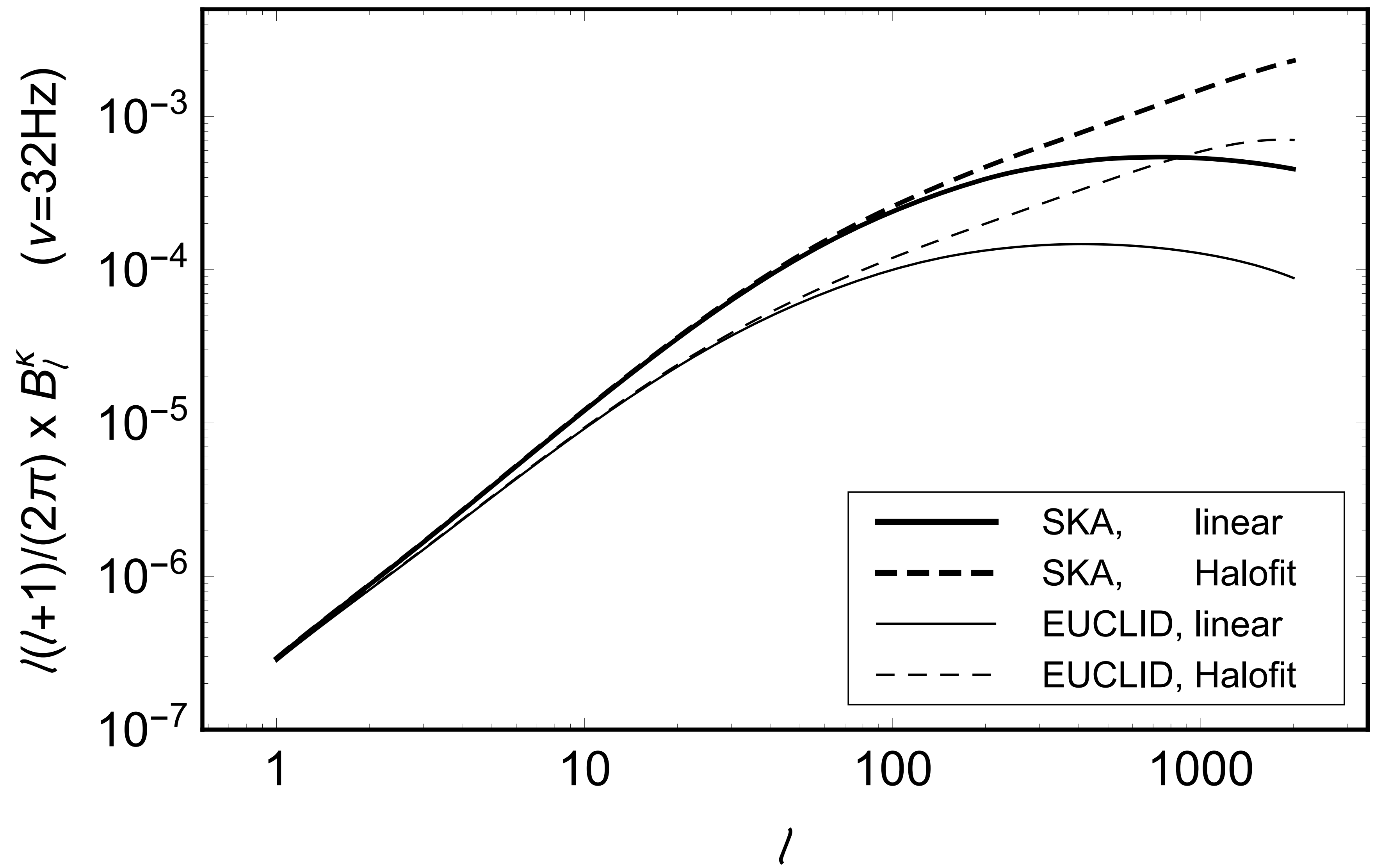
Angular power spectrum

variations of order 30%



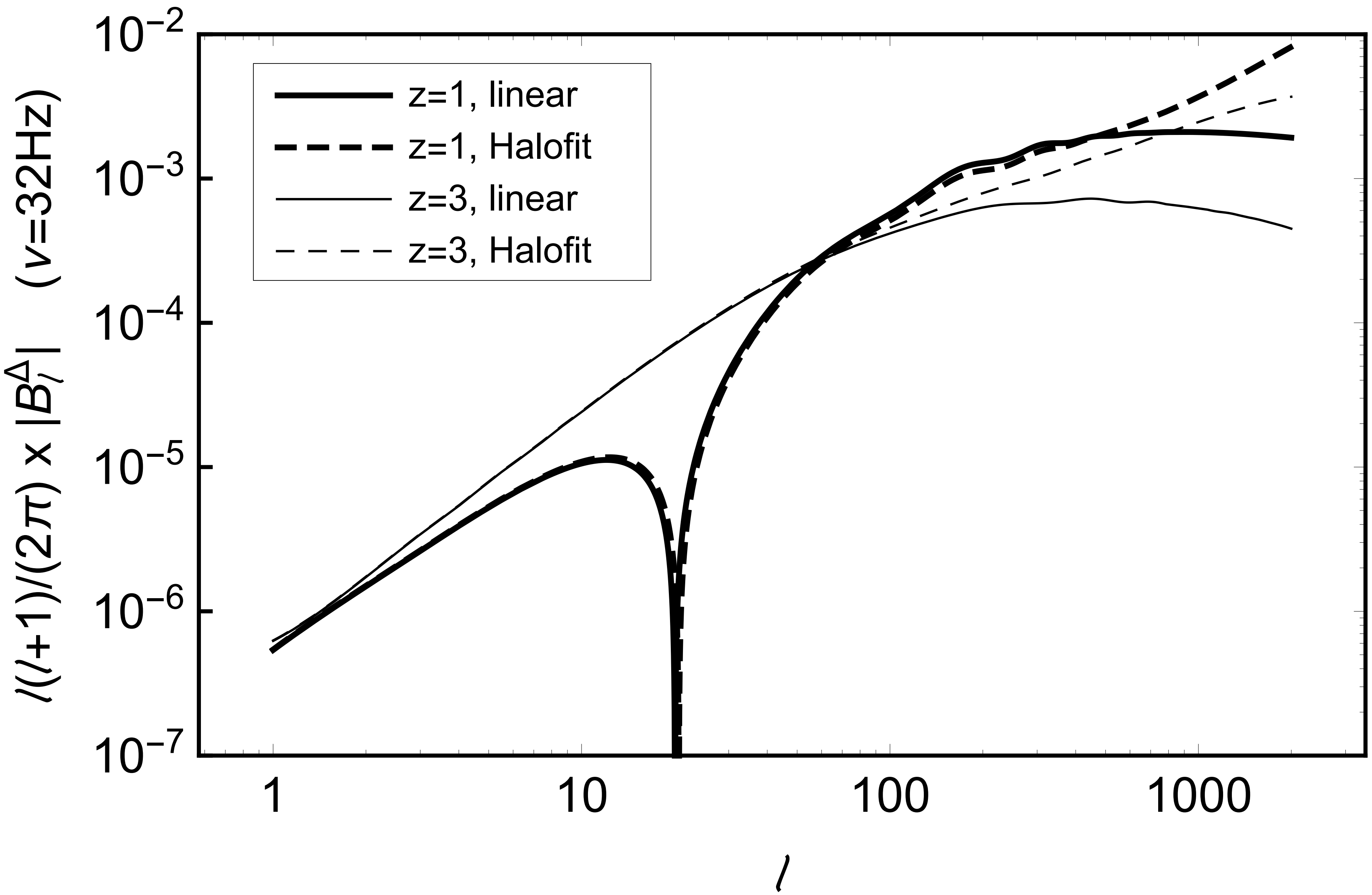
$$(\ell + \tfrac{1}{2})C_\ell(\nu_{\rm o}) \simeq \left[\frac{\nu_{\rm o}\mathcal{A}(\eta_{\rm o},\nu_{\rm o})b(\eta_{\rm o})}{4\pi\rho_{\rm c}}\right]^2 \int_{k_{\rm min}} P_\delta(k)\mathrm{d}k$$

Cross correlation: lensing



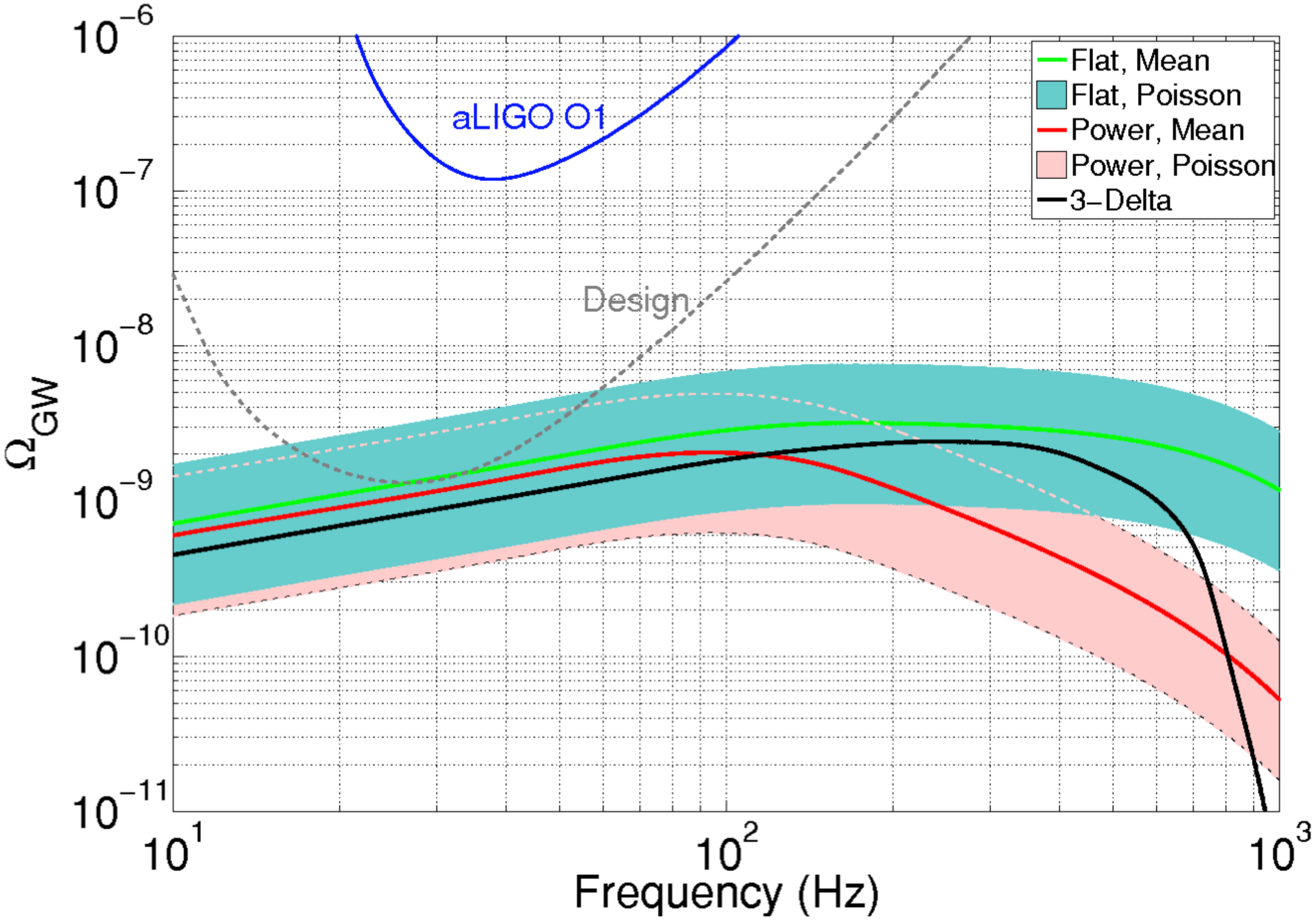
GC, Dvorkin, Pitrou, Uzan, arXiv: 1803.03236

Cross correlation: number counts



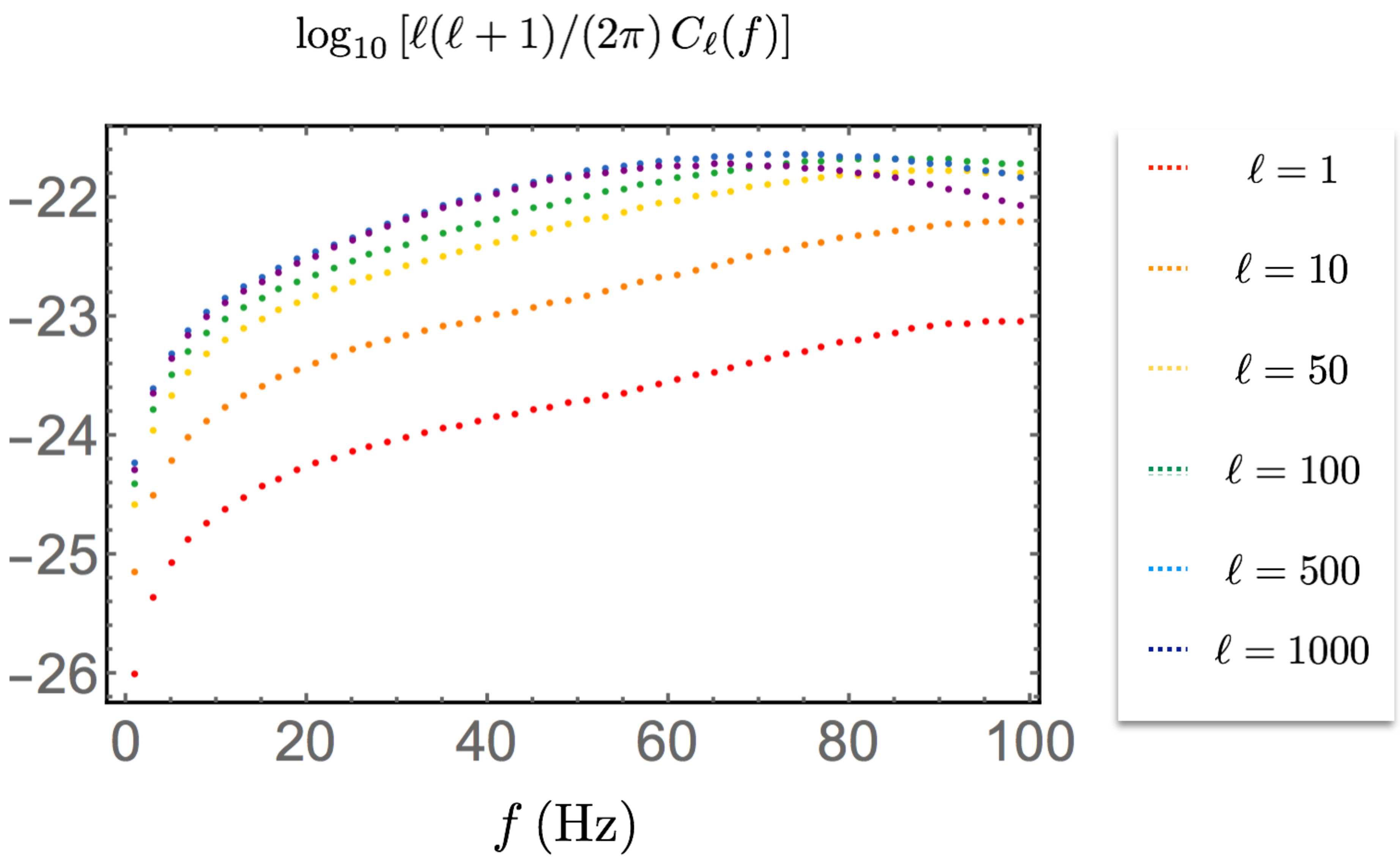
GC, Dvorkin, Pitrou, Uzan, arXiv: 1803.03236

Complementarity isotropic-anisotropic info



LIGO-VIRGO
PRL 118, 121101, 2017

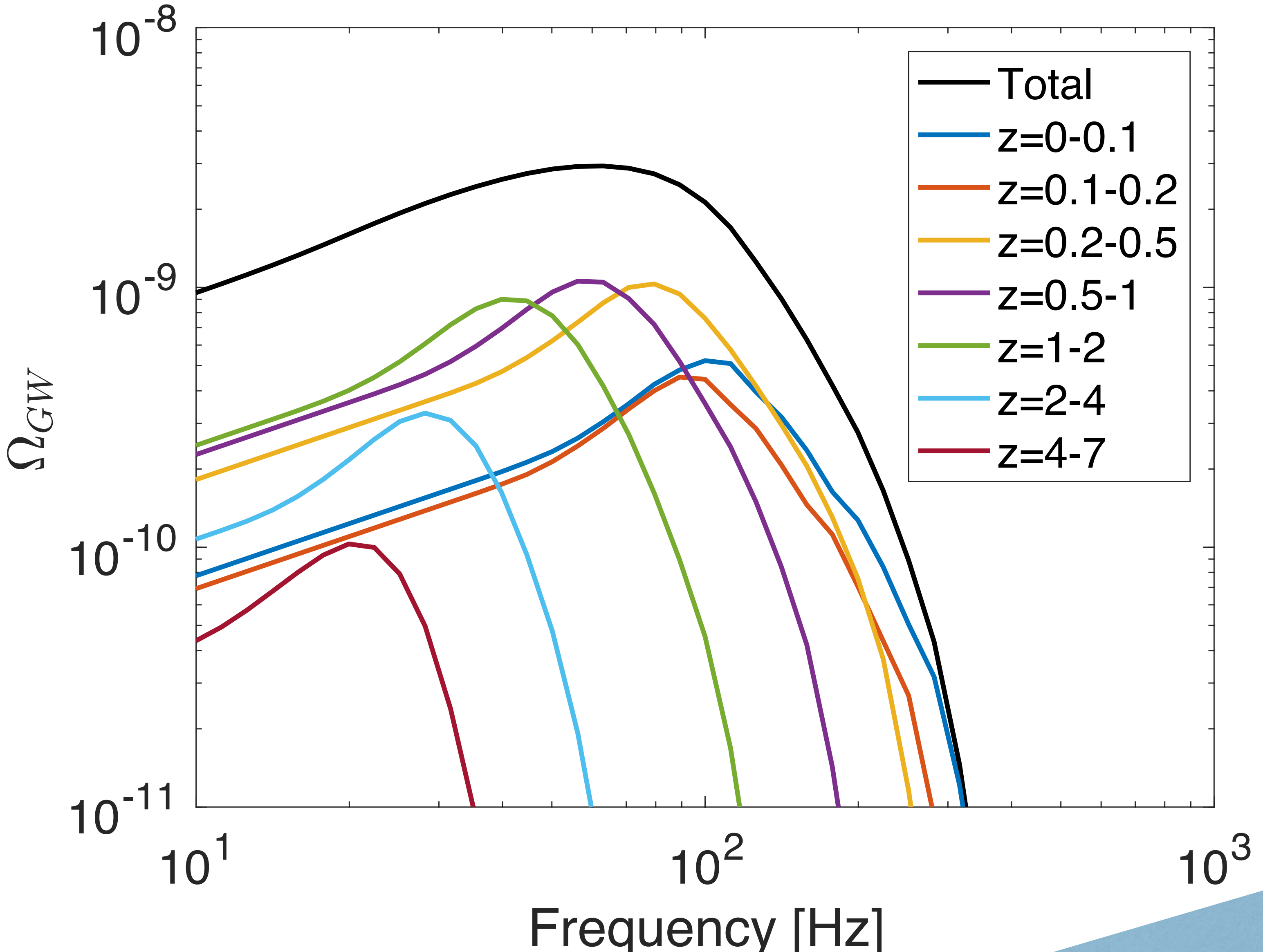
Complementarity isotropic-anisotropic info



In progress…

- Test the dependence on the astro model/parameters - star formation rate, binary mass distribution, role of eccentricity, model stellar evolution…
- Inclusion of other sources - neutron star inspiralling and merging and supernovae exploding
- Building estimators- S/N ratio and matching with current searches

Which info can be extracted...

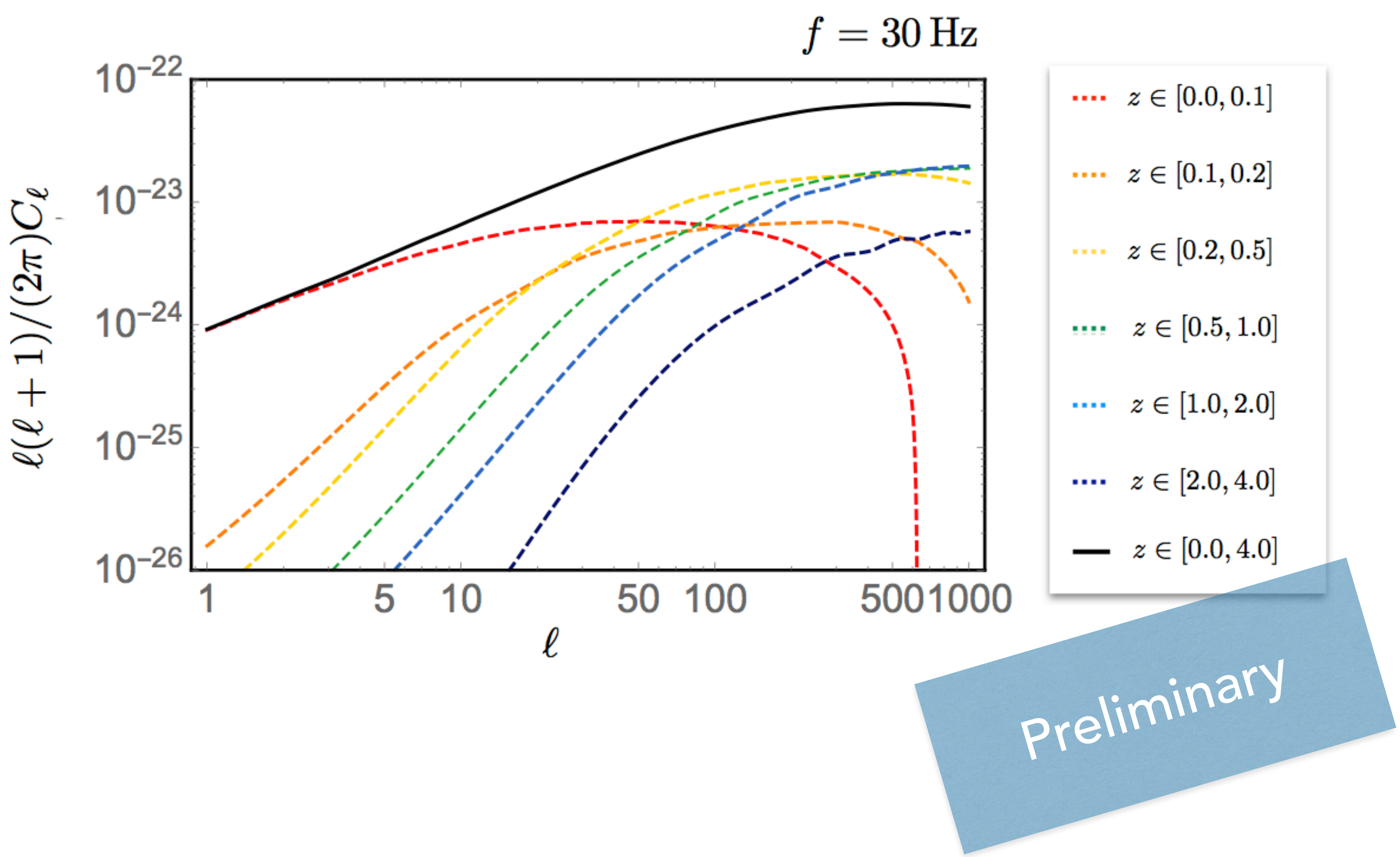


Cumulative plot: monopole

Preliminary

Which info can be extracted…

Cumulative plot: multipoles



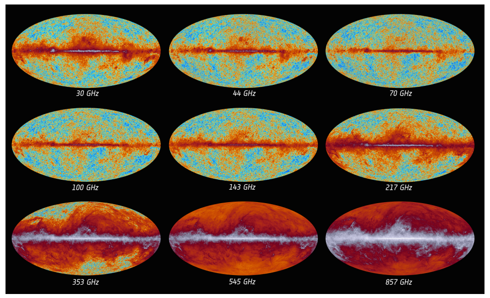
CMB

$\ell = 0$ ℓ

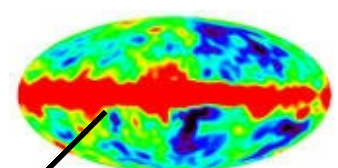
Penzias & Wilson '65

Sachs-Wolfe formula '67

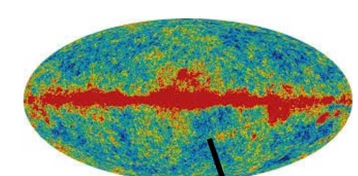
Planck '13

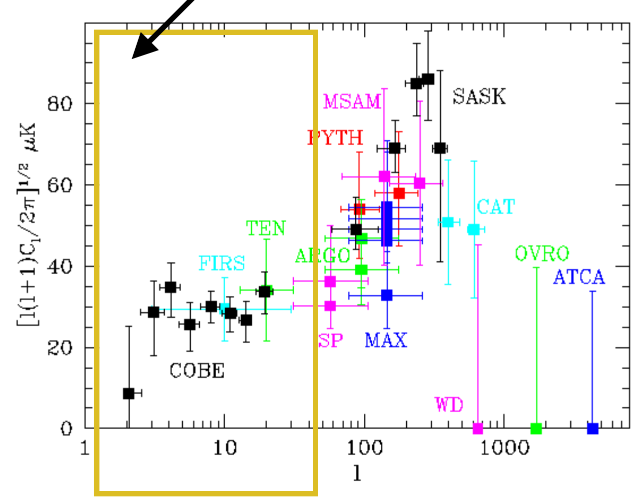


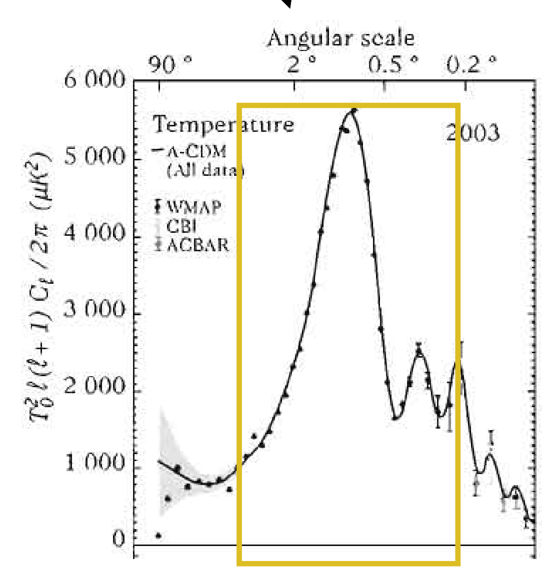
COBE '89



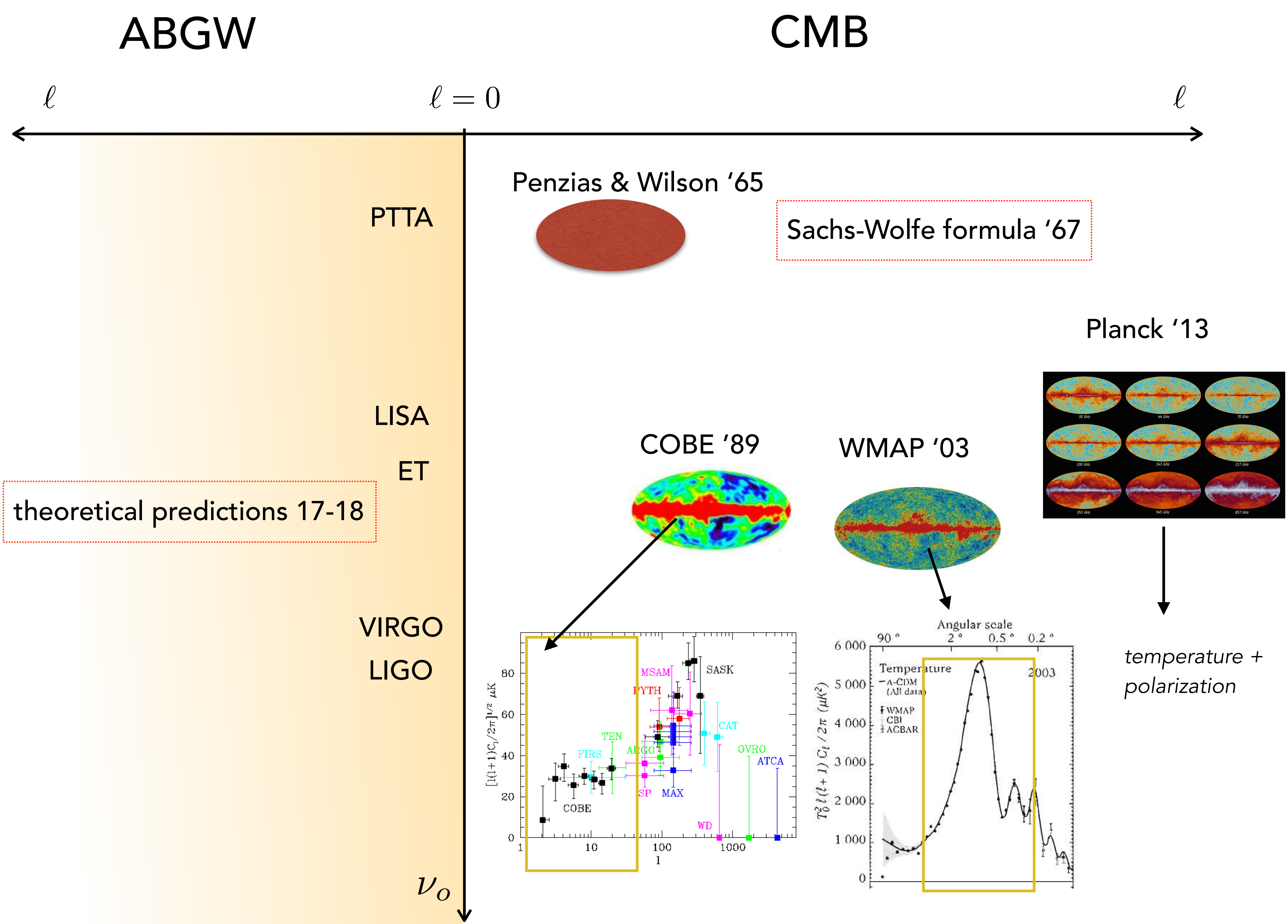
WMAP '03







temperature + polarization


ABGW
CMB
ℓ
$\ell = 0$
ℓ
PTTA
LISA
ET
theoretical predictions 17-18
VIRGO
LIGO
ν_o
Penzias & Wilson '65
Sachs-Wolfe formula '67
Planck '13
COBE '89
WMAP '03
temperature + polarization
COBE
FIRS
TEN
ARGO
SP
MAX
MSAM
PYTH
SASK
CAT
OVRO
ATCA
WD
Angular scale
Temperature
2003
WMAP
CBI
ACBAR

ABGW

CMB

ℓ $\ell = 0$ ℓ

PTTA

LISA

ET

New era for astrophysics

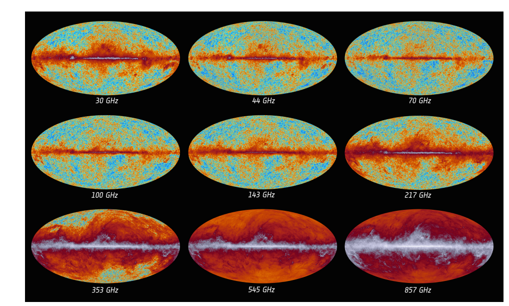
VIRGO

LIGO

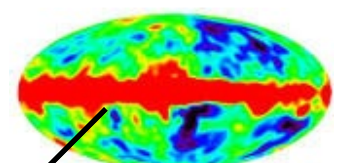
Penzias & Wilson '65

Sachs-Wolfe formula '67

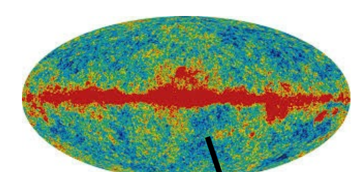
Planck '13

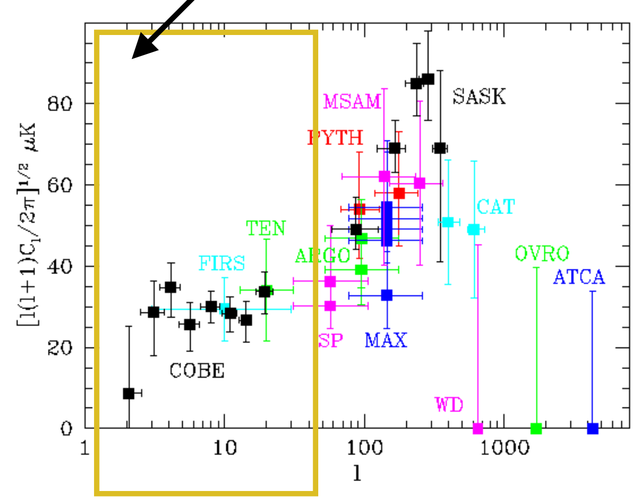


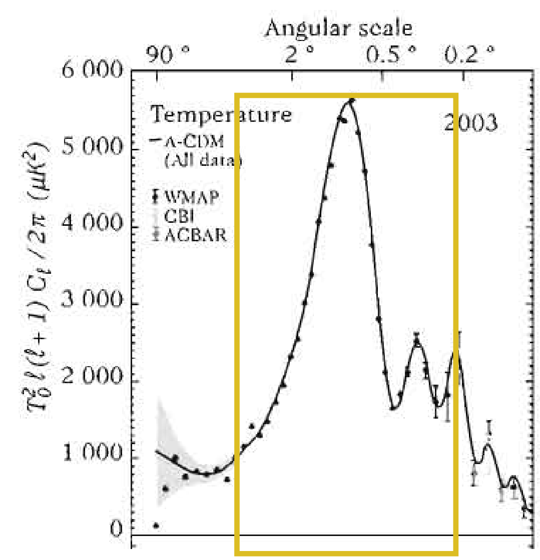
COBE '89



WMAP '03







temperature + polarization

Thank you

Astrophysical model

Astrophysical model

The computation of the astrophysical part of the result requires

(1) Luminosity of a galaxy

$$\mathcal{L}_{\mathrm{G}} = \int \mathrm{d}m\, \mathrm{d}m'\, \mathrm{d}a_{\mathrm{f}} \underbrace{\boxed{\frac{\mathrm{d}E}{\mathrm{d}\nu}}}_{\text{spectrum}} \times \underbrace{\boxed{\mathcal{R}_{\mathrm{m}}[m, m', a_{\mathrm{f}}, t]}}_{\text{merger rate}}$$

(2) Sum over the entire galaxy population using the halo mass function calibrated with simulations

Ingredients needed

- star formation rate $\psi(M_G, t)$

- stellar evolution model Fryer et al. to compute mass of BH formed for a star with given initial mass and metallicity $m = g_s(M_*, Z_*)$ and lifetime $\tau(M_*)$

- rate of BH formation $\mathcal{R}_1(m, t) = \psi[M_G, t - \tau(M_*)]\boxed{\phi(M_*) \times dM_*/dm}$

  # black holes formed from given initial mass

- fraction BH in binary systems $\mathcal{R}_2(m, t) = \beta \mathcal{R}_1(m, t)$

- birth rate of binaries $\mathcal{R}_{\rm bin}(m, m') = \mathcal{R}_2(m)\mathcal{R}_2(m')\boxed{P(m, m')}$

  2-mass distribution

Merger rate

- birth rate of binaries $\rightarrow$ $$\boxed{\mathcal{R}_f[m, m', a_f, t] = \mathcal{R}_{bin}(m, m') f(a_f)}$$

initial distribution orbital parameters

- merger time of a system $\tau_m(m, m', a_f)$

$$\downarrow$$

$$\boxed{\mathcal{R}_m[m, m', a_f, t] = \mathcal{R}_f[m, m', a_f, t - \tau_m(m, m', a_f)]}$$

Final result

The computation of the astrophysical part of the result requires

(1) Luminosity of a galaxy

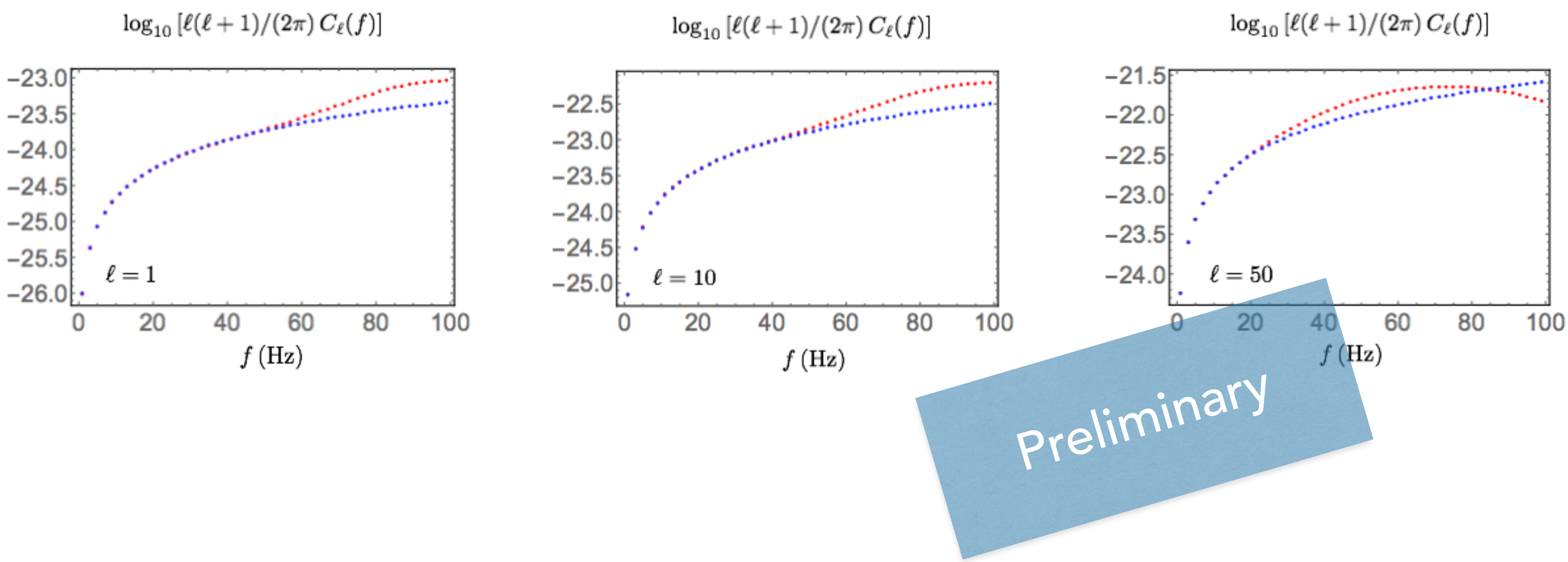
$$\mathcal{L}_{\mathrm{G}} = \int \mathrm{d}m\,\mathrm{d}m'\,\mathrm{d}a_{\mathrm{f}}\,\underbrace{\frac{\mathrm{d}E}{\mathrm{d}\nu}}_{\text{spectrum}} \times \underbrace{\mathcal{R}_{\mathrm{m}}[m, m', a_{\mathrm{f}}, t]}_{\text{merger rate}}$$

(2) Sum over the entire galaxy population using the halo mass function calibrated with simulations

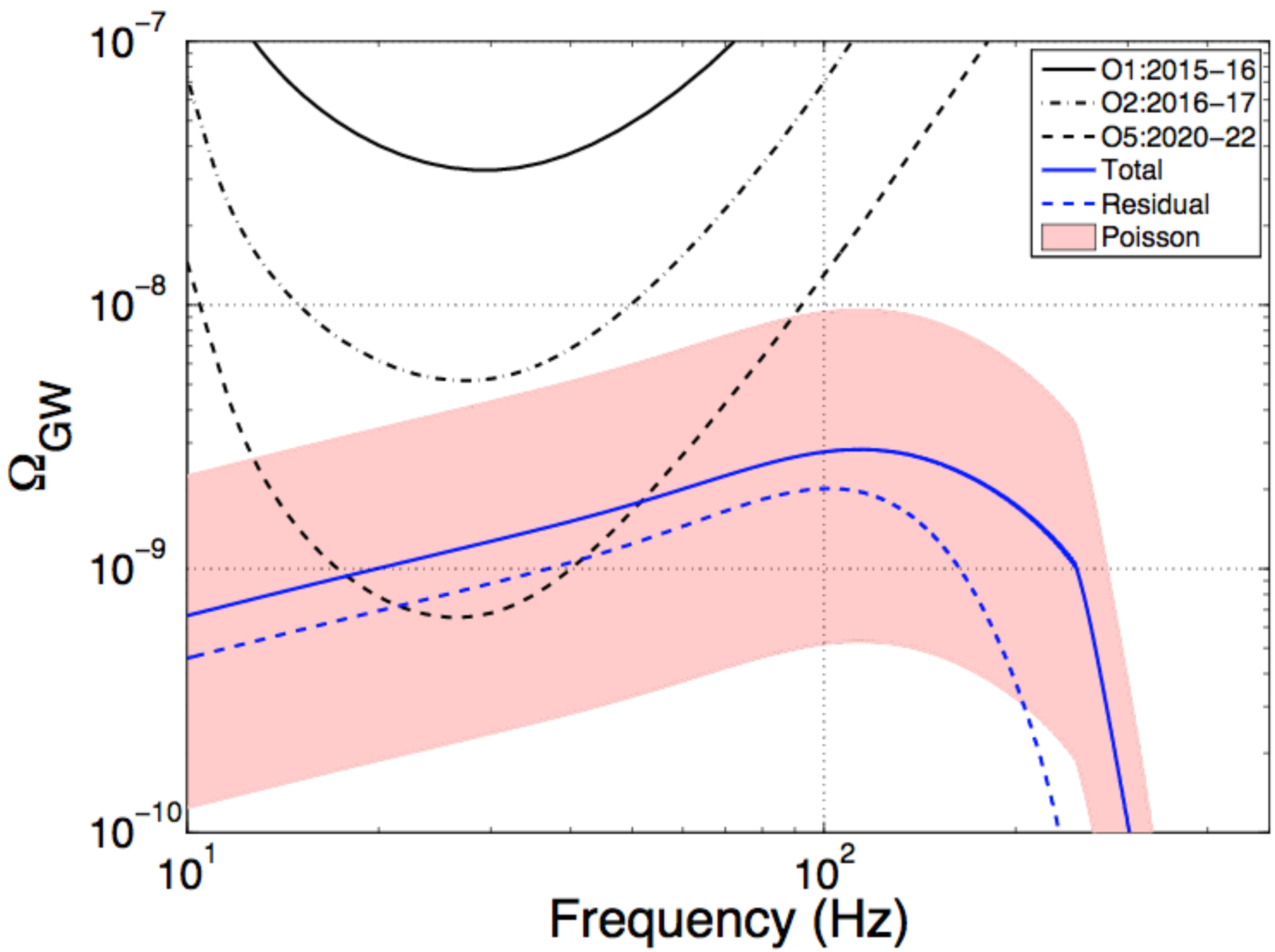
Test hypothesis used in current searches

Checking Hp of frequency factorization

$$\delta\Omega^{\rm obs}_{\rm GW}(\mathbf{n}, f) = H(f)P(\mathbf{n})$$



Total vs unresolved contributions



LIGO collaboration 1602.03847

Filtering out resolved sources

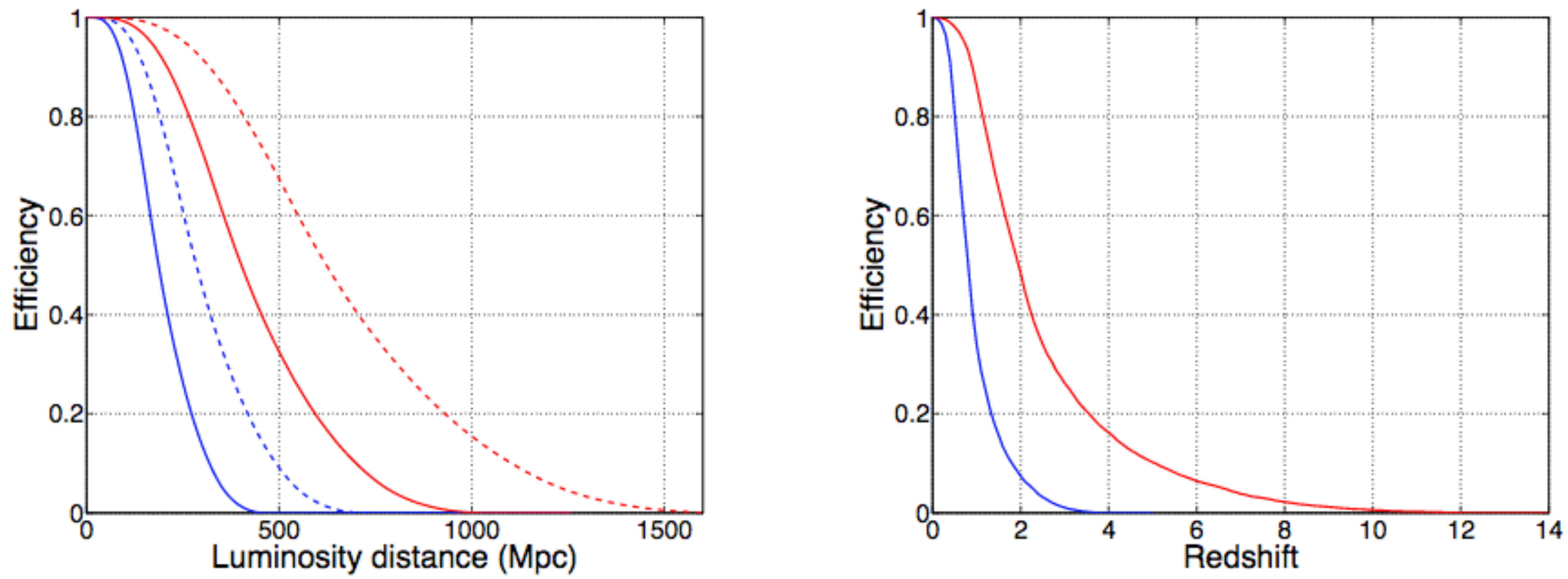
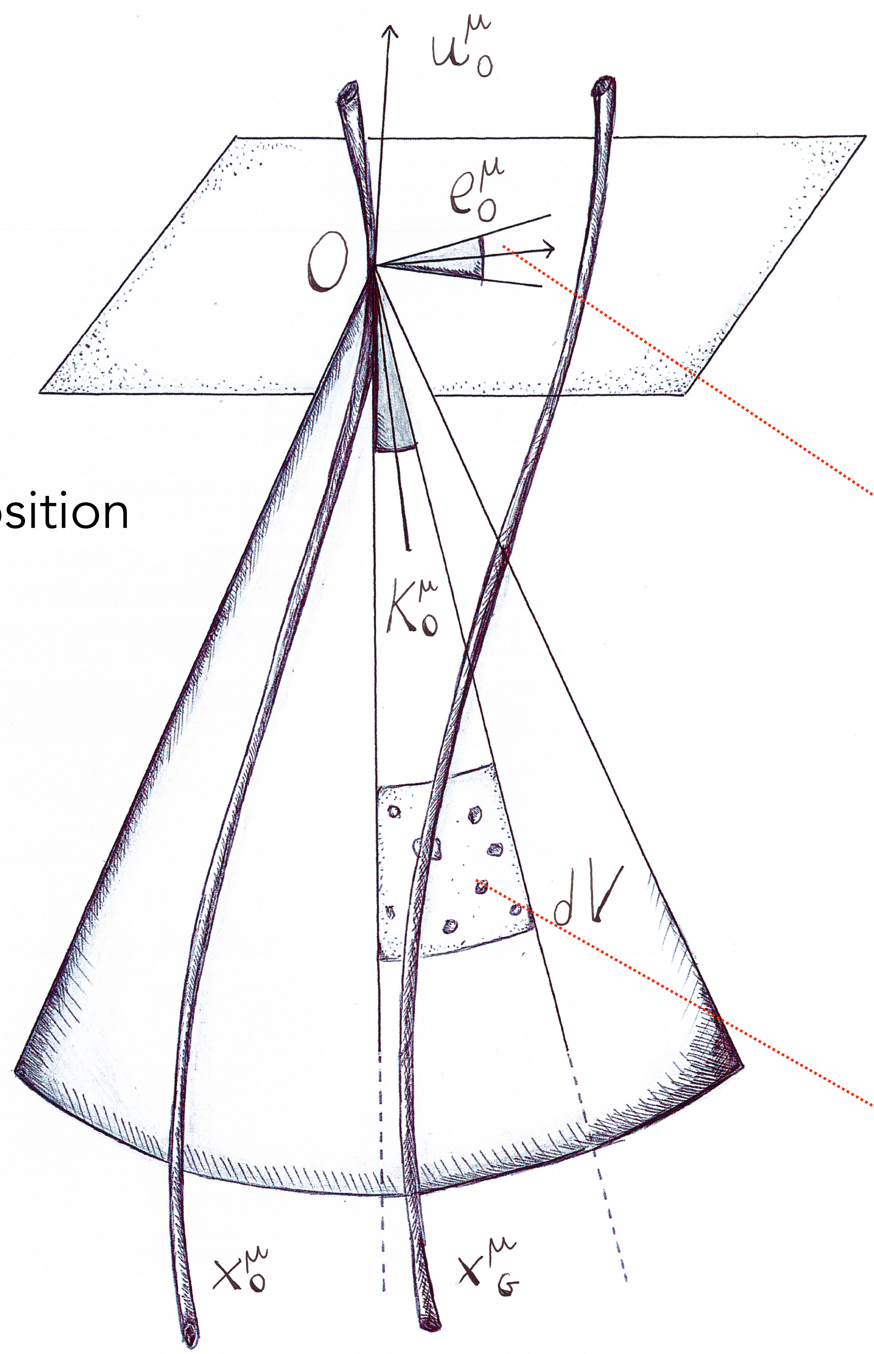


Fig. 5.— Left: GW detection efficiency as a function of luminosity distance of BNS (blue) and NS-BH (red) for the ALV network. The continuous and dashed lines correspond to signal-to-noise ratio threshold of 12 and 8 respectively. Right: GW detection efficiency as a function of redshift of BNS (blue) and NS-BH (red) for ET and a signal-to-noise ratio threshold of 8. We assumed masses of 1.4 $M_\odot$ for neutron stars and 10 $M_\odot$ for black holes.

Regimbau et al. 2014

Line of sight approach

3+1 decomposition



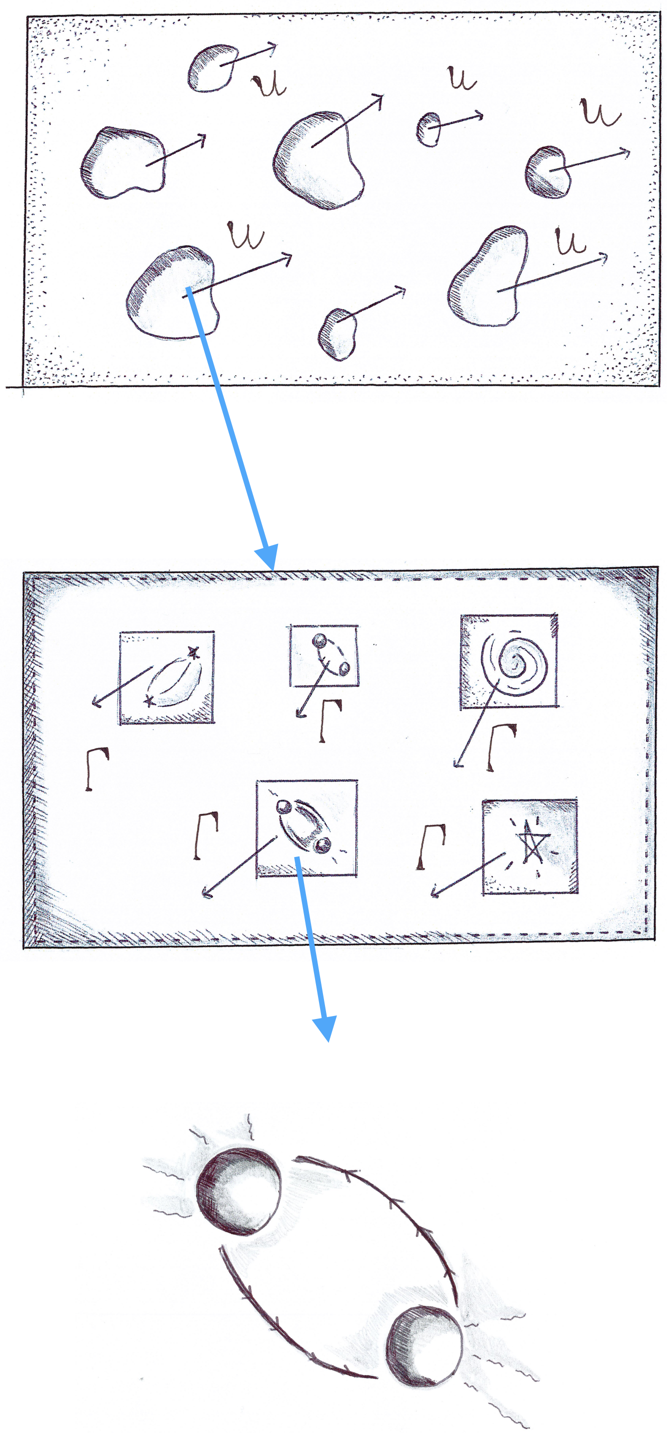
wave vector graviton

$$\frac{dx^\mu}{d\lambda} \equiv k^\mu = E(u^\mu - e^\mu)$$

e_O^μ in solid angle defines a null bundle along the past light cone

intersection of 4-volume with the observer past light cone

Three scales in our problem



cosmological scale. Galaxies: point-like sources moving with the cosmic flow

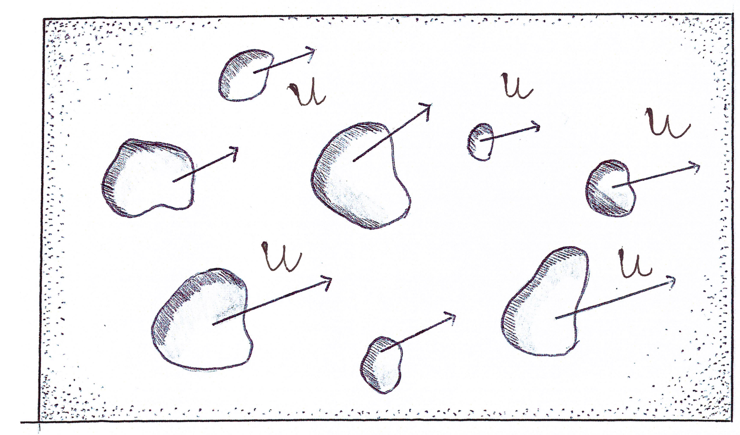
galactic scale. Effective luminosity of a galaxy defined taking into account the various contributions of the sources

local scale: single GW sources inside a galaxy

Cosmological scale

$$\int_0^\infty \mathcal{L}_G(\nu_G, \theta_G) d\nu_G = L_G(\theta_G)$$ effective luminosity

$$\nu_G = (1 + z_G)\nu_O$$ effective frequency



Relation flux-luminosity:

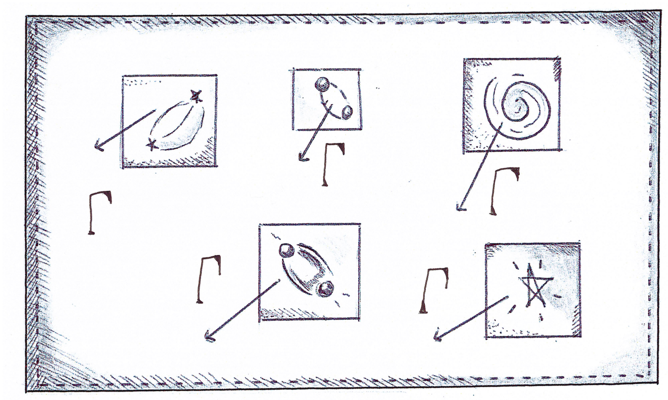
$$\Phi(z_G, e_O, \theta_G) \equiv \frac{1}{4\pi D_L^2(z_G, e_O)} L_G(\theta_G)$$

$$\Phi_\nu(z_G, e_O, \nu_O, \theta_G) d\nu_O \equiv \frac{(1 + z_G)}{4\pi D_L^2(z_G, e_O)} \mathcal{L}_G(\nu_G, \theta_G) d\nu_O$$

Galactic scale

$\mathbf{\Gamma} = \mathbf{\Gamma}(\theta^{(i)}, \theta_G)$ with distribution function $f(\mathbf{\Gamma}, \theta_G)$

with $\int d^3\Gamma f(\mathbf{\Gamma}, \theta_G) = 1$



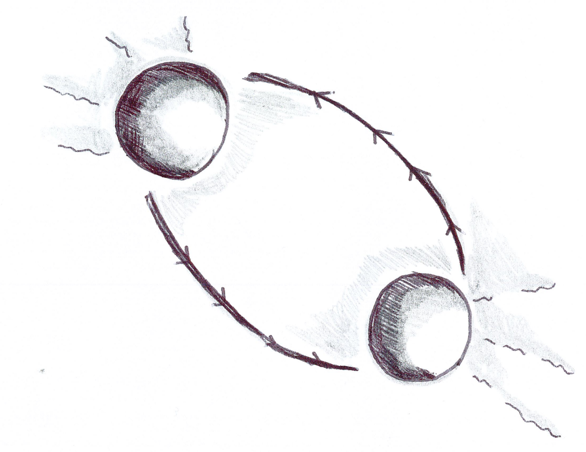
Effective luminosity

$$\mathcal{L}_G(\theta_G, \nu_G) = \mathcal{L}_G^I(\theta_G, \nu_G) + \mathcal{L}_G^M(\theta_G, \nu_G) + \mathcal{L}_G^{SN}(\theta_G, \nu_G)$$

$$\mathcal{L}_G^{M,SN}(\theta_G, \nu_G) = \sum_{(i)}^{M,SN} \int d\theta^{(i)} \underbrace{\boxed{\frac{d\mathcal{N}^{(i)}}{dt_G}(\theta^{(i)}, \theta_G)}}_{\text{rate}} \int d^3\mathbf{\Gamma}\, f(\mathbf{\Gamma}, \theta_G) \underbrace{\boxed{\frac{dE_G^{(i)}}{d\nu_G}(\nu_G, \mathbf{\Gamma}, \theta_G)}}_{\text{spectrum}}$$

Local scale

Results in the literature computed in the source local frame: **change of reference frame** needed



$$e.g. \quad \left.\frac{d^2E}{dt\,d\nu}\right|_G = \mathrm{boost}\left(\left.\frac{d^2E}{dt\,d\nu}\right|_S, \mathbf{\Gamma}\right)$$

galaxy frame

source frame