Scatter in Filter Cavities (and some modeling thoughts)

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The basic problem

- Faraday Isolators isolate only so well (30-40db)
- The filter cavity is high finesse (~5000 for 300m)
- IFO’s should be “quantum limited”
Requirements Drivers

To motivate simulation needs and “integrated approach” methods

- (Back)scatter is arbitrary in phase [unconstrained by control]
- Scatter field transforms same as vacuum – RPN curve does **NOT** relax requirements
  - Relay optics displacement noise
  - Filter Cavity length noise
    - Direct noise
    - Sensing/Witness noise injection!
  - Filter Cavity BRDF Scatter
- Three approaches
  - Analytic: “What are/How can I meet reqs”
  - Simulated/Integrated: “Am I meeting reqs”
  - Goal for today? Superintegrated: “where may I not be meeting reqs”
    - Unknown Unknowns → Known Unknowns

\[
|e| = \sqrt{\frac{10^8 E_\lambda}{2}}
\]

Amplitude

Phase

\[
\frac{2\pi}{\lambda} e \delta l
\]

Amplitude

Phase

Vacuum
(ideally SQZed)

AC noise
Requirements Drivers
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\[ |c| = \sqrt{\frac{10^8 E_\lambda}{2}} \]

Amplitude

Phase

Amplitude

20pW:

\[ 2\pi \frac{e\delta l}{\lambda} \]

Amplitude

Phase

Vacuum
(ideally SQZed)

Radiation Pressure

\[
\begin{bmatrix}
1 \\
-\kappa \\
0 \\
1
\end{bmatrix}
\]
Cavity Calculations

- Could use modeling software..
  - But analytic calcs good for documentation
- Ad-hoc
  - DC cavity field calculations easy
  - Derivatives are easy
  - AC cavity calculations more tedious (more parameters)
    - Must be simplified/decomposed afterwards
  - DC + Derivatives + synthesized AC
    - Start decomposed
    - Trust Kramers-Kronig for equivalence
    - Should do both and check with model

\[ \delta_L e = \frac{d}{dL} e_{\text{out}} = \frac{2ic}{\lambda f_{\text{pole}} L_{\text{cavity}}} e_{\text{in}} \]

\[ H_{\text{pole}}(f) = \frac{if_{\text{pole}}}{f_{\text{carrier}} + if_{\text{pole}} - f} \]

\[ |\delta_L^{\text{det}} e| = \left( |H_{\text{pole}}(f_{\text{det}} + f)|^2 + |H_{\text{pole}}(f_{\text{det}} - f)|^2 \right)^{1/2} |H_{\text{pole}}(f_{\text{det}})||\delta_L e| \]
Modeling Thoughts

- Analytic modeling is flexible
  - Many cases/classes of components can be reasoned about simultaneously
  - Requires cross checking, no test suite
- Simulators are combinatorially *complex* (like real instruments)
  - Single output projection for many D.O.F.s
  - But show most optical nonlinearity if parameters are scanned/modelled (this can be slow)
- Need more tools for combinatoric/incoherent tolerancing noise
  - Relay phase noise is actually a (frequency dependent) example of this
  - Implementing generic noise drives of incoherent nature (just like quantum noise) can model all linear tolerances and some quadratic (like SQZ phase noise)
    - Fast to compute, matrix implementation means budgets are possible
    - MCMC over tolerances
      - Corner plots are great, but need intelligent collection of parameters
Coherent Cavity Calculations

DC Source vectors
DC Transfer (optical cavity feedback)
- Quadrature vectors
- Or phasors

T_{DC}

Incident DC Fields (all optics)

AC Transfer (modulated optical cavity feedback, with drives)
- 2-photon q/p
- 2-photon +/-

T_{AC}

SVD of subspaces gives phase-indep. Gains in S, phases in UV, can be useful

\[ \vec{e}_{\text{out}} = T(f) \vec{e}_{\text{scatter}} \]

\[ USV = T(f) \]

AC transfers sourced by DC analysis

Incident DC Fields (all optics)

Incident AC Fields (all optics)
Incoh Cavity Calculations

$T_{DC}$

$T_{AC}$

$T\vec{e}$

Phase-space like representation of DC sources

This is essentially the same as QN calculations

$T_{AC}\{\cdot\}T_{AC}^\dagger$

(action on phase-space)

$(T\vec{e})(T\vec{e})^\dagger + (T'\vec{e})(T'\vec{e})^\dagger$

Primes are phase/parameter detunings
Length Noise Modelling

- Using actual measured SEI performance, rather than original design requirements (as our SEI outperforms them).
- SEI Spectra + SUS state-space → length noise budget
  - Need reference spectra
  - State space representations
    - quite concise,
    - easy to simulate,
    - probably good for MCMC, more advanced sim tools
  - Need reference output with safety factor (used GWINC)
Why the phase noise requirement?

- Need end-to-end loop modeling!
  - Alignment sensing needs this far more desperately
- ALL measurements are differential, but how inertial is your reference?
  - In this case, the length-sensing field laser is not a freq. Reference
  - But the IFO filtered output is as stable as CARM motion
  - Must lock the two

(Need a simulator with noise budgets That are intelligible for J. Driggers realistic alignment-sensing-control (ASC) diagrams, full IFO complexity)

This is an example of a subtle req. hiding In the control system for just a single degree of freedom.

Highly shaped loop meets RMS reqs, but not with much margin for rolloff of sensing noise.

Assumes 1e-6/rtHz phase noise of FC sensing

300m FC Loop noises with 10Hz loop UGF [RMS 5.6e-12m]
Diffuse Scatter

- Forward/Reverse coupling follow an A-omega diffraction-limited collection area law (T940063 Flanagan, Thorne)
- Can ignore optical field strengths! (optical sensitivity is separable problem)

\[
S_{\text{diffuse}}^2 = \int_0^{2\pi} \int_0^{\pi/2} \frac{\lambda^2}{r^2(\theta)} B_{\text{optic}}(\theta) B_{\text{surface}}(\theta + X) \sin(\theta) \, d\theta \, d\phi
\]

Scattering “power”

Power-like Unitless Coupling for Amplitude Spectral Densities

\[
\delta L_{\text{cav}} = S_{\text{diffuse}} \delta L_{\text{surface}}
\]
Analytic Approach

• A: Filter cavity has enormously relaxed length sensitivity than the arms
  - Allows a worst-case analysis

• B: Usual approach worries that small angle scatter is large (from low-k mirror irregularities)

\[ B_{\text{optic}}(\theta) \propto \frac{1}{\theta^N} \]

• But! Assume/know total mirror scatter is small/bounded

\[ L_{\text{scatter}} = 2\pi \int_{0}^{\frac{\pi}{2}} B_{\text{optic}}(\theta) \sin(\theta) \, d\theta < 50 \text{ppm} \]

Must have some \( \theta_{\text{min}} \) cutoff scale, and be limited in scatter coefficient
Worst-Case Analysis

- Assume BRDF Monotonic
- Assume all scatter is in a disc at some cutoff
- Geometry mostly in $r(\theta, \phi)$

\[
B_{\text{optic}}(\theta) = \begin{cases} 
\alpha & \theta < \theta_{\text{min}} \\
\beta \cos(\theta) & \theta > \theta_{\text{min}} 
\end{cases}
\]

\[
\alpha \approx \frac{50 \text{ppm}}{2\pi \theta_{\text{min}}^2} \quad \beta \approx 50 \text{ppm}
\]

Now relatively tractable to evaluate for many geometries

\[
S_{\text{diffuse}}^2 = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{\lambda^2}{r^2(\theta, \phi)} B_{\text{optic}}^2(\theta) B_{\text{surface}}(\theta + X) \sin(\theta) \, d\theta \, d\phi
\]

Entirely Geometric - Scatter surface modeling can be separated from optical sensitivity.

Generally shows that near walls/baffles dominate from $\frac{1}{r^2}$ (contradicts arm-tube analysis?)
Conclusions

- Still useful to use analytic calculation to search parameter spaces, find solutions
- Useful to check all cases of chosen realization through simulation
  - Need tools to help here
- Diffuse scatter more a geometric problem, but plugs into optical sensitivities (determinable through incoherent simulation)
  - Is diffuse modeling fully separable?
  - Backscatter not separable, but also less geometric.
  - Specular scatter geometric, is it separably modellable

- (squeezed) shotnoise-limited field sensitivity sufficient for output backscatter calculations
  - Radiation Pressure effect “ignorable” (must use worst case)
  - (but does not relax reqs. W.R.T. SN.)
- Unmodelled sensing noise isn’t necessarily a scatter problem, but (more total) controls modeling may prevent design flaws.
  - Want to drive this point for future ASC design