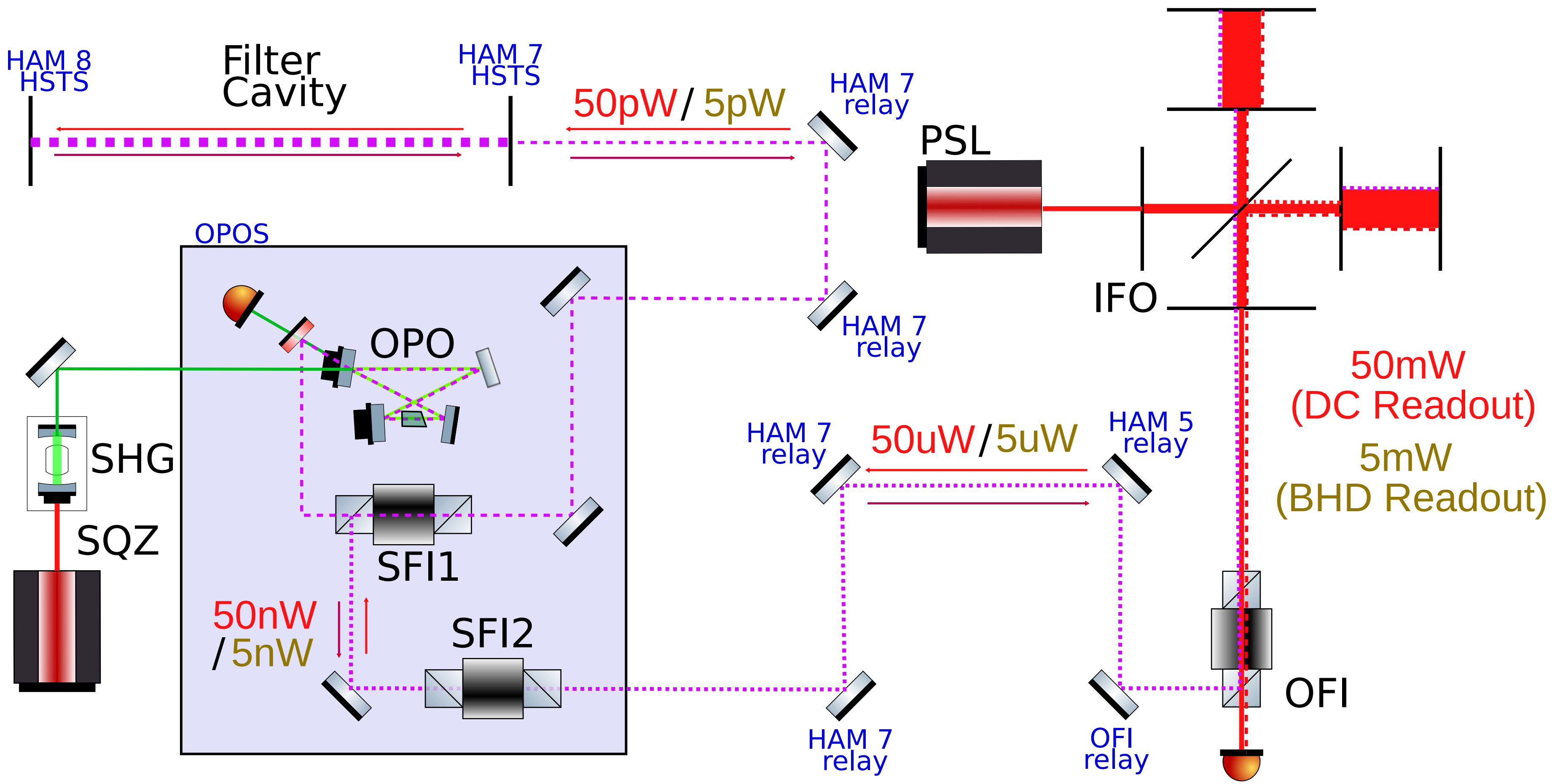
Scatter in Filter Cavities
(and some modeling thoughts)

Lee McCuller

GWADW 2019

The basic problem

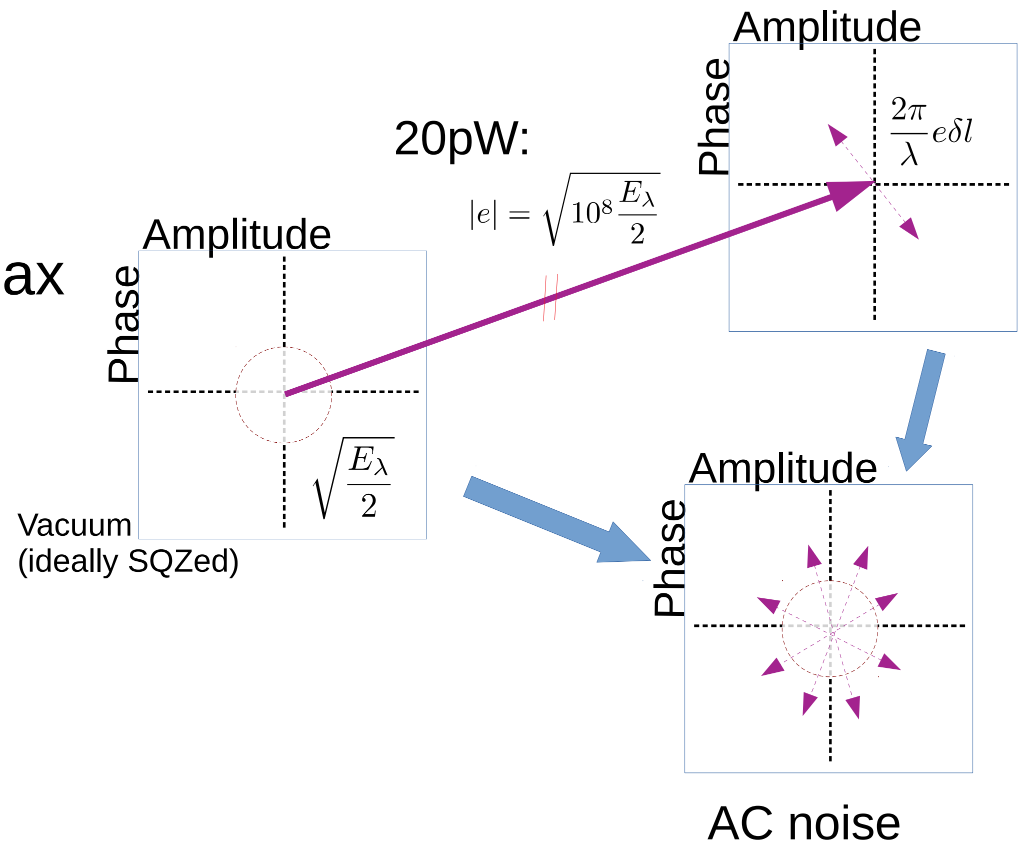
- Faraday Isolators isolate only so well (30-40db)
- The filter cavity is high finesse (~5000 for 300m)
- IFO’s should be “quantum limited”



Requirements Drivers

To motivate simulation needs and "integrated approach" methods

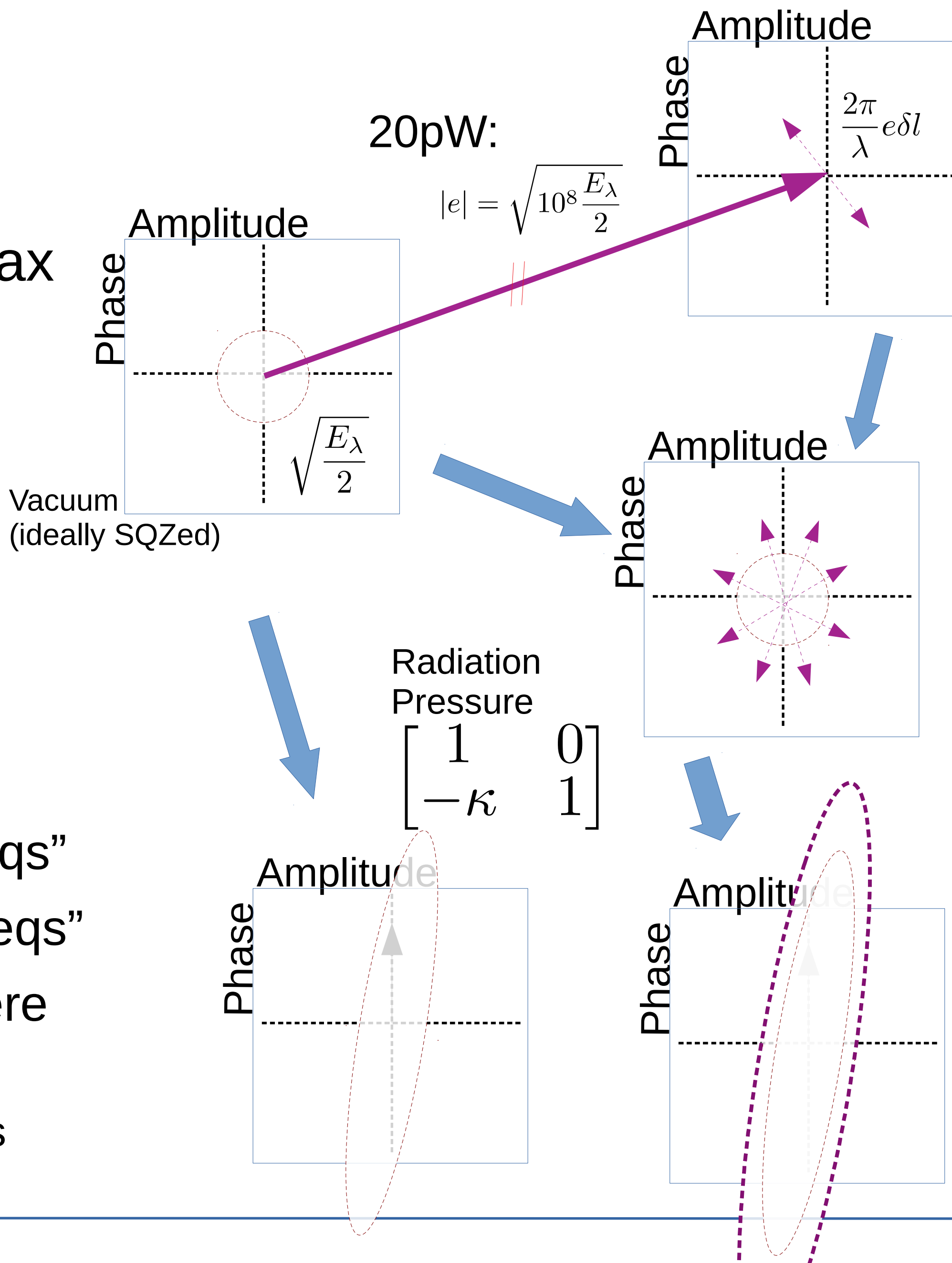
- (Back)scatter is arbitrary in phase [unconstrained by control]
- Scatter field transforms same as vacuum – RPN curve does **NOT** relax requirements
  - Relay optics displacement noise
  - Filter Cavity length noise
    - Direct noise
    - Sensing/Witness noise injection!
  - Filter Cavity BRDF Scatter
- Three approaches
  - Analytic: "What are/How can I meet reqs"
  - Simulated/Integrated: "Am I meeting reqs"
  - Goal for today? Superintegrated: "where may I not be meeting reqs"
    - Unknown Unknowns → Known Unknowns



Requirements Drivers

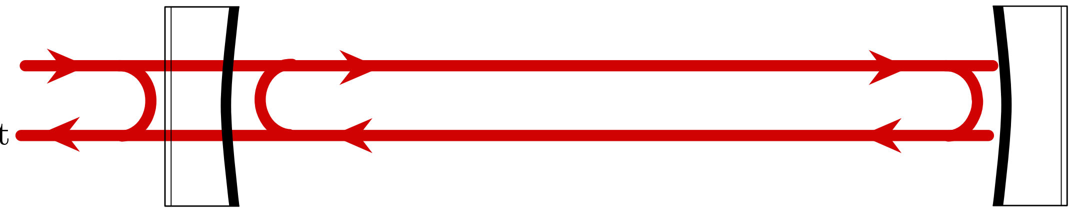
To motivate simulation needs and “integrated approach” methods

- (Back)scatter is arbitrary in phase [unconstrained by control]
- Scatter field transforms same as vacuum – RPN curve does **NOT** relax requirements
  - Relay optics displacement noise
  - Filter Cavity length noise
    - Direct noise
    - Sensing/Witness noise injection!
  - Filter Cavity BRDF Scatter
- Three approaches
  - Analytic: “What are/How can I meet reqs”
  - Simulated/Integrated: “Am I meeting reqs”
  - Goal for today? Superintegrated: “where may I not be meeting reqs”
    - Unknown Unknowns → Known Unknowns



Cavity Calculations

- Could use modeling software..
  - But analytic calcs good for documentation
- Ad-hoc
  - DC cavity field calculations easy
  - Derivatives are easy
  - AC cavity calculations more tedious (more parameters)
    - Must be simplified/decomposed afterwards
  - DC + Derivatives + synthesized AC
    - Start decomposed
    - Trust Kramers-Kronig for equivalence
    - Should do both and check with model



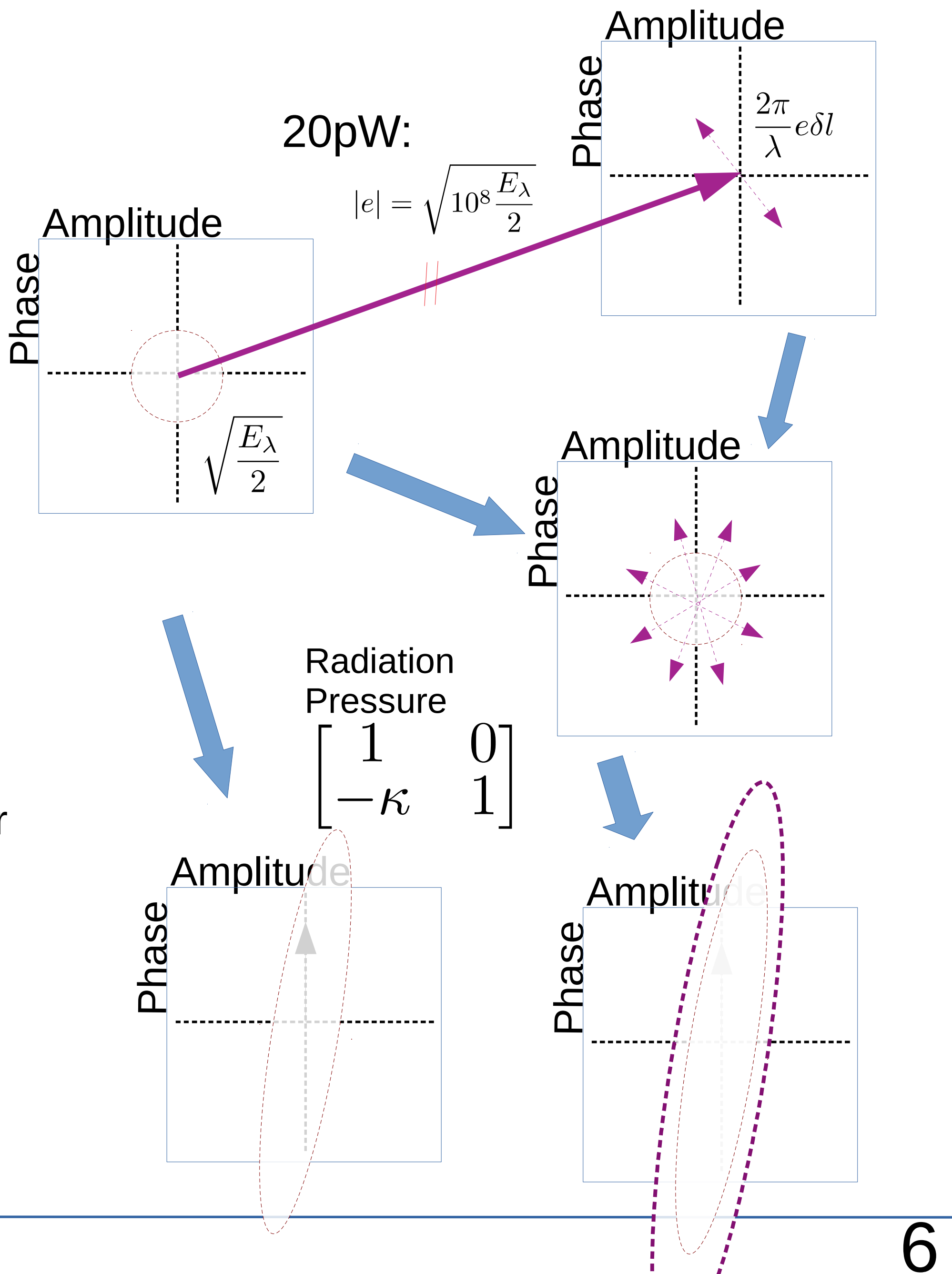
$$\delta_L e = \frac{\mathrm{d}}{\mathrm{d}L} e_{\mathrm{out}} = \frac{2ic}{\lambda f_{\mathrm{pole}} L_{\mathrm{cavity}}} e_{\mathrm{in}}$$

$$H_{\mathrm{pole}}(f) = \frac{i f_{\mathrm{pole}}}{f_{\mathrm{carrier}} + i f_{\mathrm{pole}} - f}$$

$$|\delta_L^{\mathrm{det}} e| = \left(|H_{\mathrm{pole}}(f_{\mathrm{det}} + f)|^2 + |H^*_{\mathrm{pole}}(f_{\mathrm{det}} - f)|^2 \right)^{1/2} |H_{\mathrm{pole}}(f_{\mathrm{det}})||\delta_L e|$$

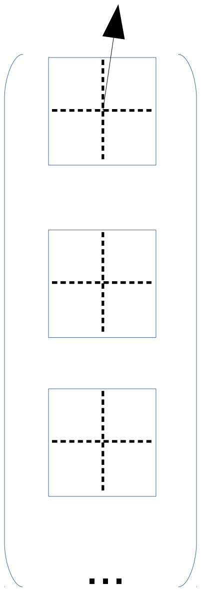
Modeling Thoughts

- Analytic modeling is flexible
  - Many cases/classes of components can be reasoned about simultaneously
  - Requires cross checking, no test suite
- Simulators are combinatorially *complex* (like real instruments)
  - Single output projection for many D.O.F.s
  - But show most optical nonlinearity if parameters are scanned/modelled (this can be slow)
- Need more tools for combinatoric/incoherent tolerancing noise
  - Relay phase noise is actually a (frequency dependent) example of this
  - Implementing generic noise drives of incoherent nature (just like quantum noise) can model all linear tolerances and some quadratic (like SQZ phase noise)
    - Fast to compute, matrix implementation means budgets are possible
  - MCMC over tolerances
    - Corner plots are great, but need intelligent collection of parameters



Coherent Cavity Calculations

DC Source vectors

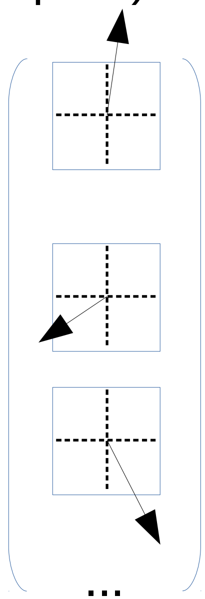


DC Transfer (optical cavity feedback)

- Quadrature vectors
- Or phasors

$$\mathbb{T}_{\mathrm{DC}}$$

Incident DC Fields (all optics)

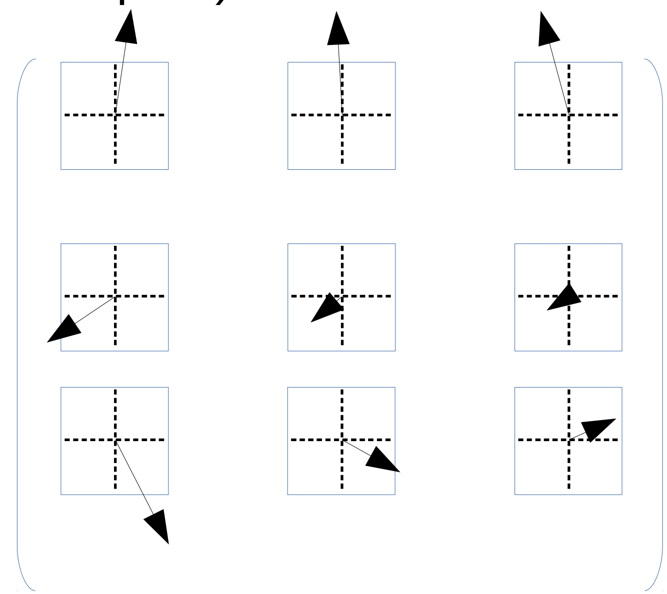


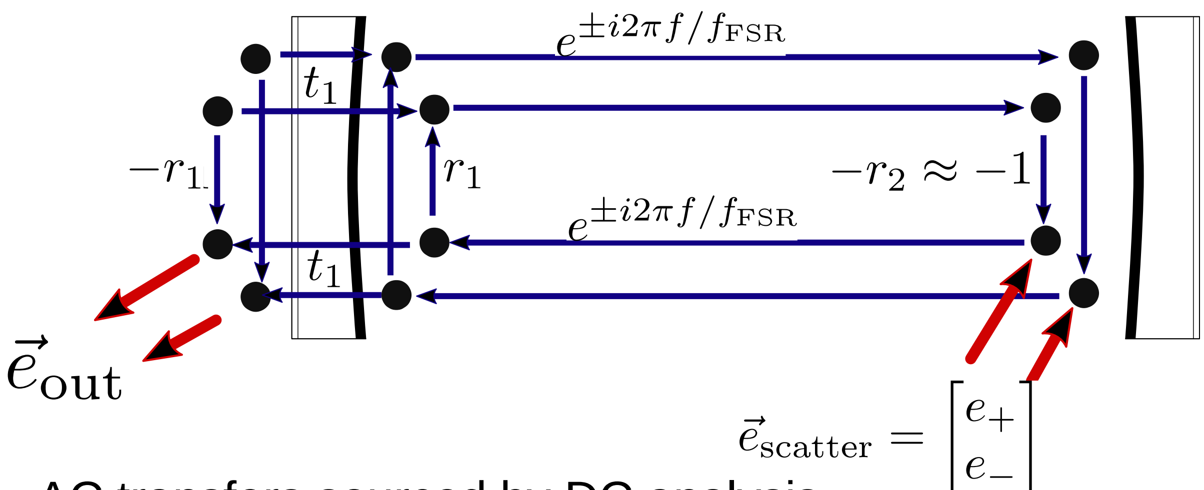
AC Transfer (modulated optical cavity feedback, with drives)

- 2-photon q/p
- 2-photon +/-

$$\mathbb{T}_{\mathrm{AC}}$$

Incident AC Fields (all optics)





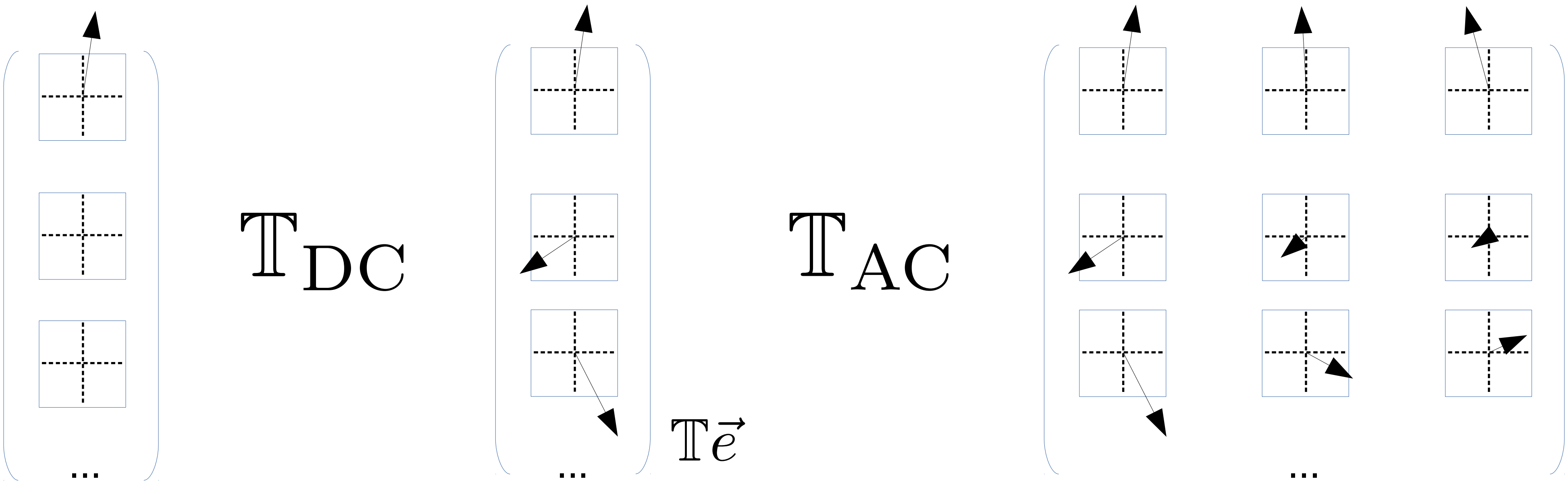
$$\vec{e}_{\mathrm{out}} = \mathbb{T}(f)\vec{e}_{\mathrm{scatter}}$$

$$\mathbf{U}S\mathbf{V} = \mathbb{T}(f)$$

AC transfers sourced by DC analysis

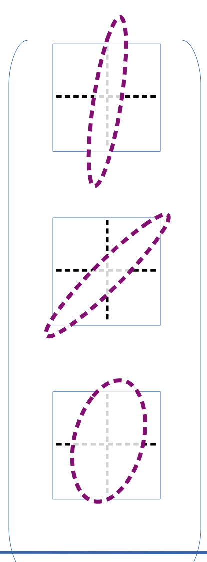
SVD of subspaces gives phase-indep. Gains in S, phases in UV, can be useful

Incoh Cavity Calculations



Phase-space like representation of DC sources

This is essentially the same as QN calculations

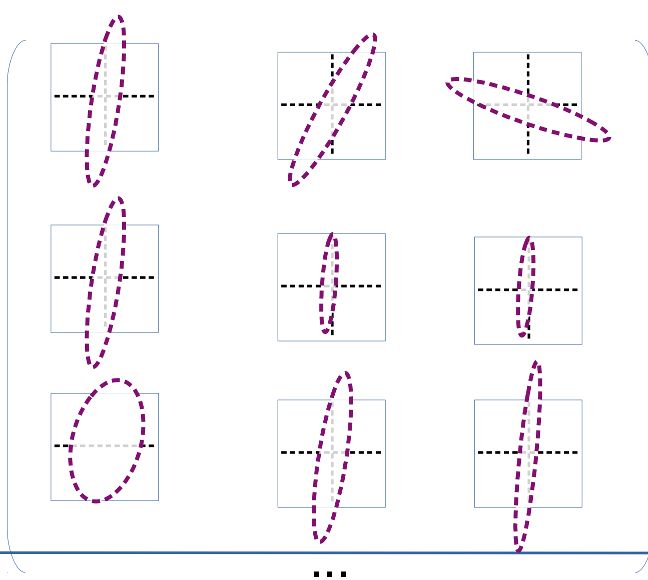


$$\mathbb{T}_{\mathrm{AC}}\{\cdot\}\mathbb{T}_{\mathrm{AC}}^{\dagger}$$

(action on phase-space)

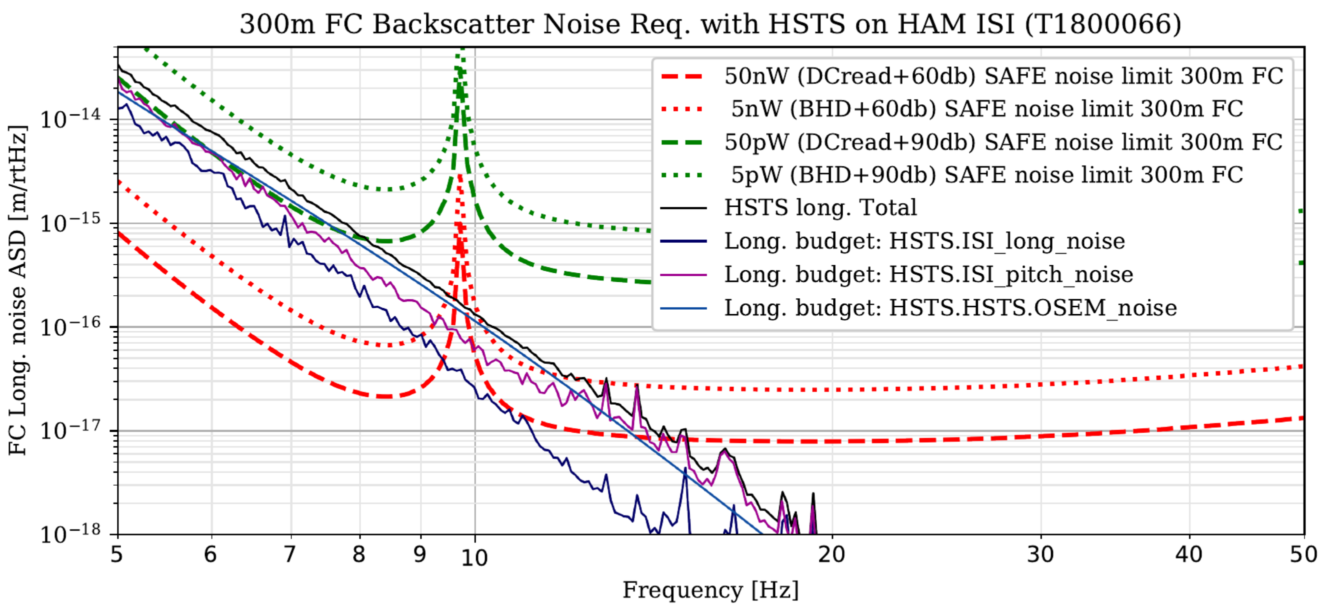
$$(\mathbb{T}\vec{e})(\mathbb{T}\vec{e})^{\dagger} + (\mathbb{T}'\vec{e})(\mathbb{T}'\vec{e})^{\dagger}$$

Primes are phase/parameter detunings



Length Noise Modelling

- Using actual measured SEI performance, rather than original design requirements (as our SEI outperforms them).
- SEI Spectra + SUS state-space → length noise budget
  - Need reference spectra
  - State space representations
    - quite concise,
    - easy to simulate,
    - probably good for MCMC, more advanced sim tools
  - Need reference output with safety factor (used GWINC)



Why the phase noise requirement?

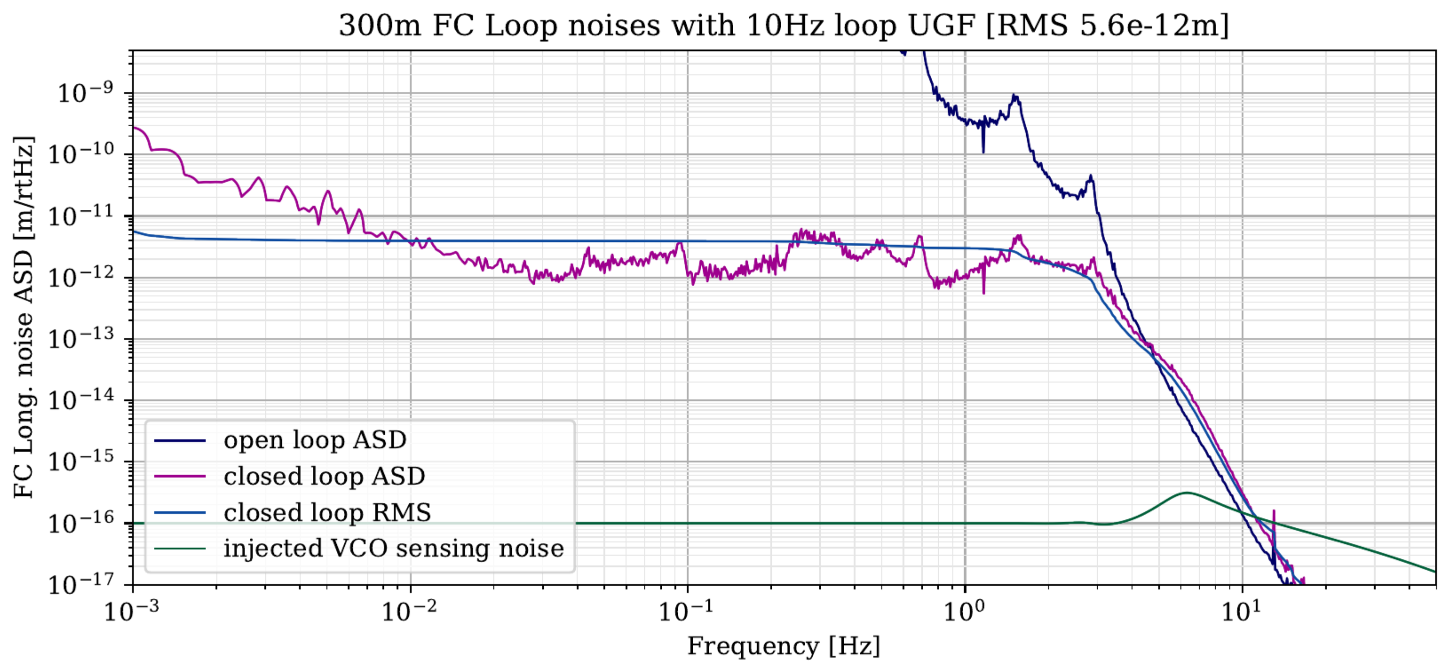
- Need end-to-end loop modeling!
  - Alignment sensing needs this far more desperately
- ALL measurements are differential, but how inertial is your reference?
  - In this case, the length-sensing field laser is not a freq. Reference
  - But the IFO filtered output is as stable as CARM motion
  - Must lock the two

(Need a simulator with noise budgets
That are intelligible for J. Driggers realistic
alignment-sensing-control (ASC)
diagrams, full IFO complexity)

This is an example of a subtle req. hiding
In the control system for just a single degree
of freedom.

Highly shaped loop meets RMS reqs, but not with much margin for rolloff of sensing noise

Assumes 1e-6/rtHz phase noise of FC sensing



Diffuse Scatter

- Forward/Reverse coupling follow an A-omega diffraction-limited collection area law (T940063 Flanagan, Thorne)

- Can ignore optical field strengths! (optical sensitivity is separable problem)

BRDF of optic

BRDF of scatter surface

Scattering “power”

$$S^2_{\mathrm{diffuse}} = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{\lambda^2}{r^2(\theta)} B^2_{\mathrm{optic}}(\theta) B_{\mathrm{surface}}(\theta + X) \sin(\theta) \mathrm{d}\theta \mathrm{d}\phi$$

Power-like Unitless Coupling for Amplitude Spectral Densities

$$\delta L_{\mathrm{cav}} = S_{\mathrm{diffuse}} \delta L_{\mathrm{surface}}$$

Analytic Approach

- A: Filter cavity has enormously relaxed length sensitivity than the arms
  - Allows a worst-case analysis
- B: Usual approach worries that small angle scatter is large (from low-k mirror irregularities)

$$B_{\mathrm{optic}}(\theta) \propto \frac{1}{\theta^N}$$

- But! Assume/know total mirror scatter is small/bounded

$$L_{\mathrm{scatter}} = 2\pi \int_0^{\frac{\pi}{2}} B_{\mathrm{optic}}(\theta) \sin(\theta) \mathrm{d}\theta < 50\mathrm{ppm}$$

Must have some θ_{min} cutoff scale, and be limited in scatter coefficient

Worst-Case Analysis

- Assume BRDF Monotonic
- Assume all scatter is in a disc at some cutoff
- Geometry mostly in $r(\theta, \phi)$

$$B_{\mathrm{optic}}(\theta) = \begin{cases} \alpha & \theta < \theta_{\mathrm{min}} \\ \beta \cos(\theta) & \theta > \theta_{\mathrm{min}} \end{cases}$$

$$\alpha \approx \frac{50\mathrm{ppm}}{2\pi\theta_{\mathrm{min}}^2} \qquad \beta \approx 50\mathrm{ppm}$$

Now relatively tractable to evaluate for many geometries

$$S_{\mathrm{diffuse}}^2 = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{\lambda^2}{r^2(\theta,\phi)} B_{\mathrm{optic}}^2(\theta) B_{\mathrm{surface}}(\theta + X) \sin(\theta) \mathrm{d}\theta \mathrm{d}\phi$$

Entirely Geometric - Scatter surface modeling can be separated from optical sensitivity.

Generally shows that near walls/baffles dominate from $\frac{1}{r^2}$
(contradicts arm-tube analysis?)

Conclusions

- Still useful to use analytic calculation to search parameter spaces, find solutions
- Useful to check all cases of chosen realization through simulation
  - Need tools to help here
- Diffuse scatter more a geometric problem, but plugs into optical sensitivities (determinable through incoherent simulation)
  - Is diffuse modeling fully separable?
  - Backscatter not separable, but also less geometric.
  - Specular scatter geometric, is it separably modellable
- (squeezed) shotnoise-limited field sensitivity sufficient for output backscatter calculations
  - Radiation Pressure effect “ignorable” (must use worst case)
  - (but does not relax reqs. W.R.T. SN.)
- Unmodelled sensing noise isn’t necessarily a scatter problem, but (more total) controls modeling may prevent design flaws.
  - Want to drive this point for future ASC design