The search for gravitational waves from white dwarf binaries in gravimetric data using the Earth's normal modes response in the mHz frequency band

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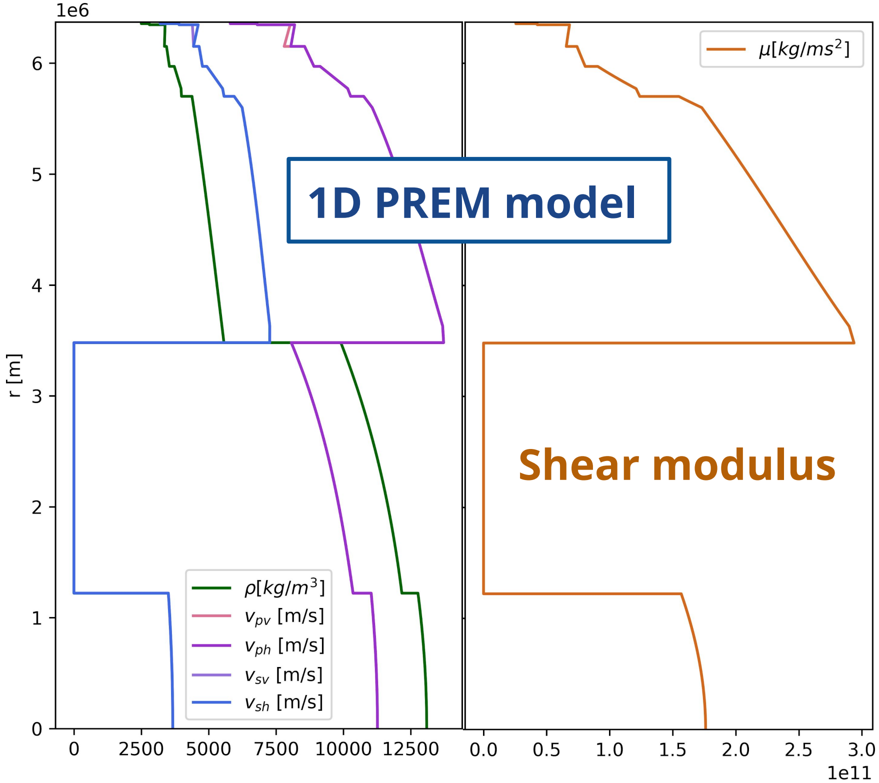
Earth response model to GWs

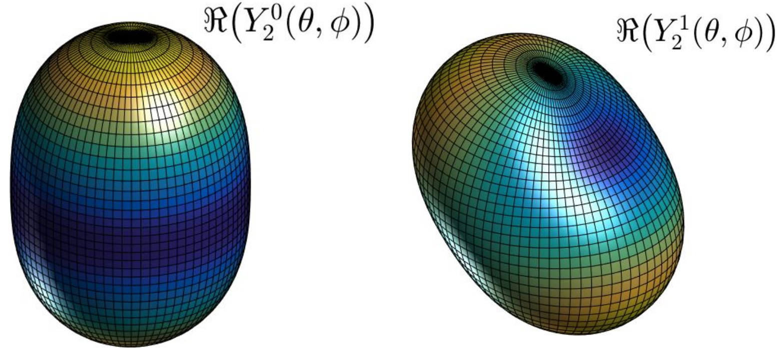
Flat-Earth Model

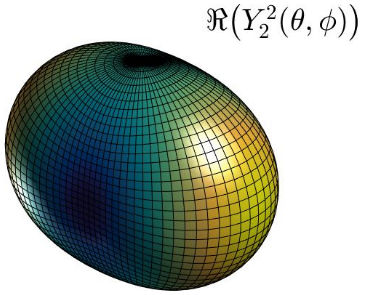
Dyson, 1969



Non-rotating 1D Earth Model

Ben-Menahem, 1983





Quadrupole modes $l = 2$

Modelling Earth response to GWs from the binary systems

Induced response using Green tensor formalism (Ben-Menahem)

$$\mathbf{s}(\mathbf{r},t) = \int_{-\infty}^{t}\int_{V} \mathbf{G}(\mathbf{r},\mathbf{r}';t-t') \cdot \mathbf{f}(\mathbf{r}',t')\, dV'\,dt' + \int_{-\infty}^{t}\int_{S} \mathbf{G}(\mathbf{r},\mathbf{r}';t-t') \cdot \mathbf{t}(\mathbf{r}',t')\, d\Sigma'\,dt'$$

Rotating 3D Earth model

Splitting and coupling of normal modes

Force term (Dyson)

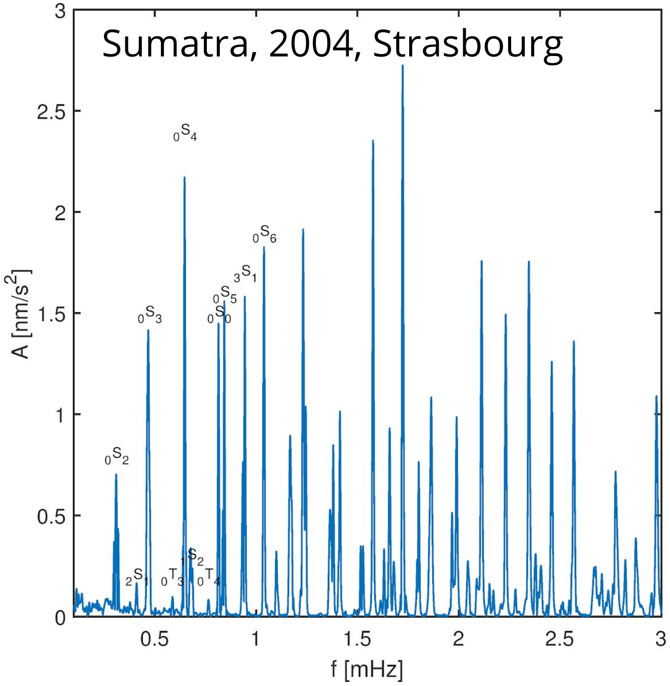
$$\mathbf{f}(\mathbf{r},t) = -\frac{\partial \mu}{\partial r}\mathbf{e}_r \cdot \mathbf{h}(\mathbf{r},t)$$

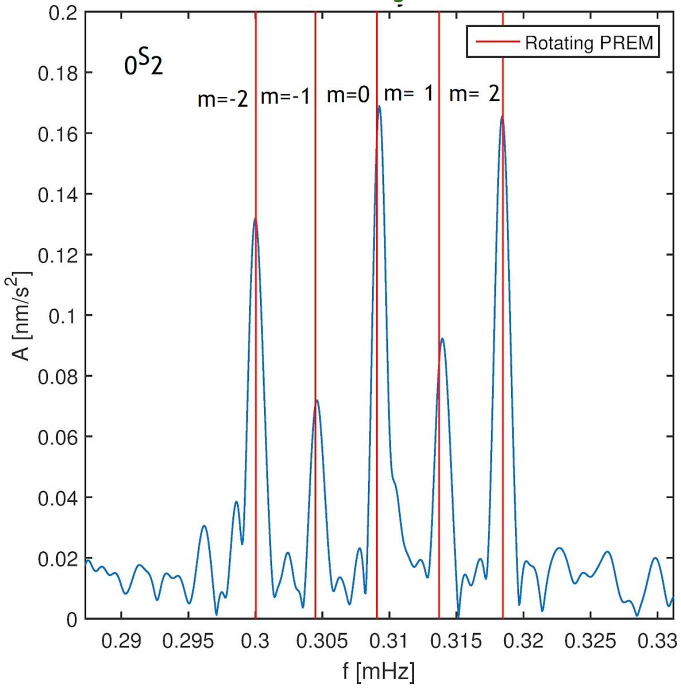
Binary stars

Terrestrial Reference System

The normal modes summation and the perturbation theory

${}_nS_l^m$





GROUP-COUPLING APPROXIMATION

$$\mathbf{G}(\mathbf{r},\mathbf{r}';t) = \Re \sum_k (i\nu_k)^{-1}\mathbf{s}_k(\mathbf{r})\mathbf{s}_k^*(\mathbf{r}')e^{i\nu_k t}$$

$$\mathbf{H} = \mathbf{N} - \nu_0\mathbf{I} + \mathbf{W} + (2\omega_0)^{-1}\left[\mathbf{V}^{\mathrm{ell+cen}} + \mathbf{V}^{\mathrm{lat}} + i\mathbf{A} - \omega_0^2(\mathbf{T}^{\mathrm{ell}} + \mathbf{T}^{\mathrm{lat}})\right]$$

The metric perturbation - binary star system

$$\mathbf{h} = h_+ \mathbf{e}_+ + h_\times \mathbf{e}_\times$$

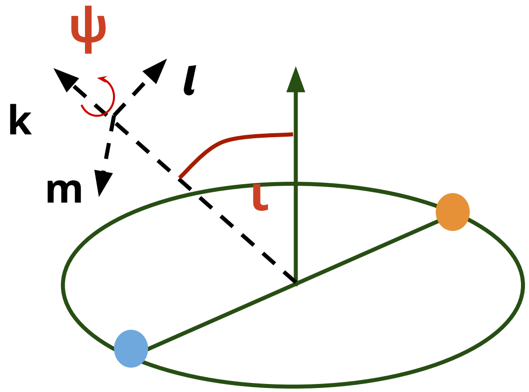
$$h_+ = -2(1+\cos^2\iota)\frac{\mu(M\Omega)^{2/3}}{r}\cos(2\Omega t)$$

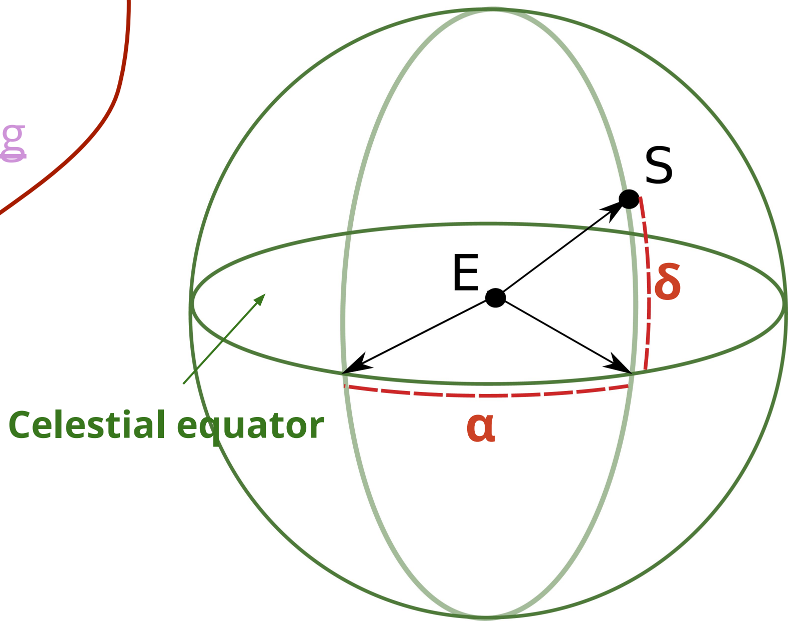
$$h_\times = -4\cos\iota\frac{\mu(M\Omega)^{2/3}}{r}\sin(2\Omega t)$$

LISA verification double white dwarf catalog

Contain **rotation matrix** from celestial to terrestrial reference frame:

- declination **δ**
- right ascension **α**
- Greenwich Sidereal Time **γ(t)**
- polarization angle **ψ**





Induced forced spheroidal motion for one GW source and *l* = 2

$$s(a,t) = \sum_k \sum_m U_k(a) Y_l^m(\theta,\phi) \left[h_{+,c} \bar{g}_+^m(t,\Omega,\nu_m) f_+^m(\gamma(t),\alpha,\delta,\psi) + h_{\times,c} \bar{g}_\times^m(t,\Omega,\nu_m) f_\times^m(\gamma(t),\alpha,\delta,\psi) \right] \alpha_k(a)$$

Model dependent (PREM)
- 24 multiplet groups

Position of the station
- Latitude and longitude

Binary parameters (catalog)
- **m1, m2, d, ɿ (inclination)**

Source-time function
- **Ω (GW frequency)**
- **ν_m (split eigenfrequency)**

f-function
- **γ(t) (GST), α (right ascension), δ (declination), ψ (polarization angle)**

${}_0S_2$ - ${}_0T_2$ - ${}_2S_1$ - ${}_0S_3$

${}_0T_3$ - ${}_0S_4$ - ${}_1S_2$

${}_0T_5$ - ${}_2S_2$ - ${}_1S_3$ - ${}_3S_1$

${}_3S_2$

${}_5S_1$ - ${}_4S_2$ - ${}_0S_{10}$ - ${}_0T_{11}$ - ${}_1T_5$

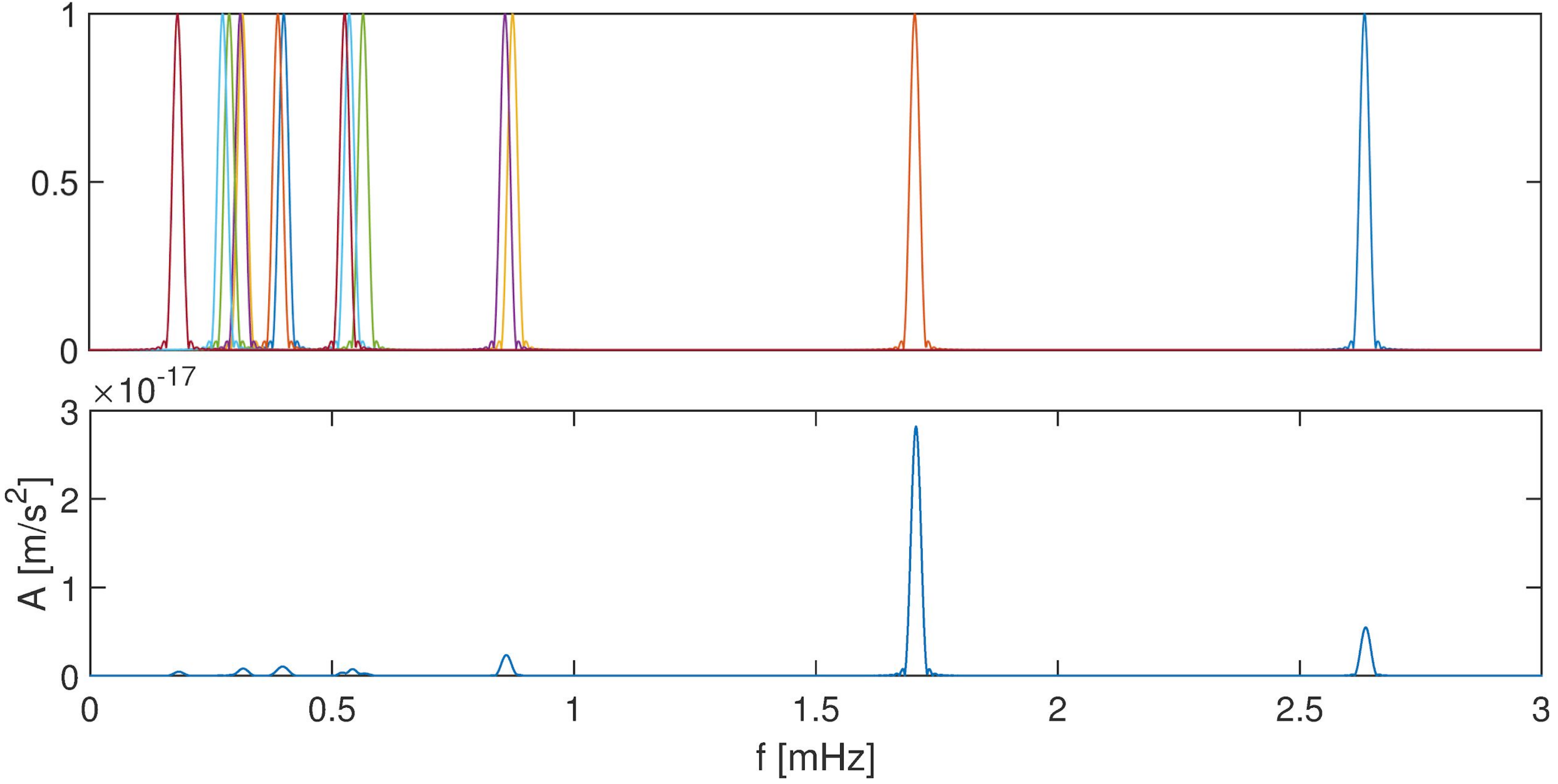
${}_5S_2$ - ${}_0T_{14}$ - ${}_1T_7$ - ${}_0S_{13}$

${}_5S_4$ - ${}_4S_5$ - ${}_2S_{10}$ - ${}_2T_4$ - ${}_6S_2$

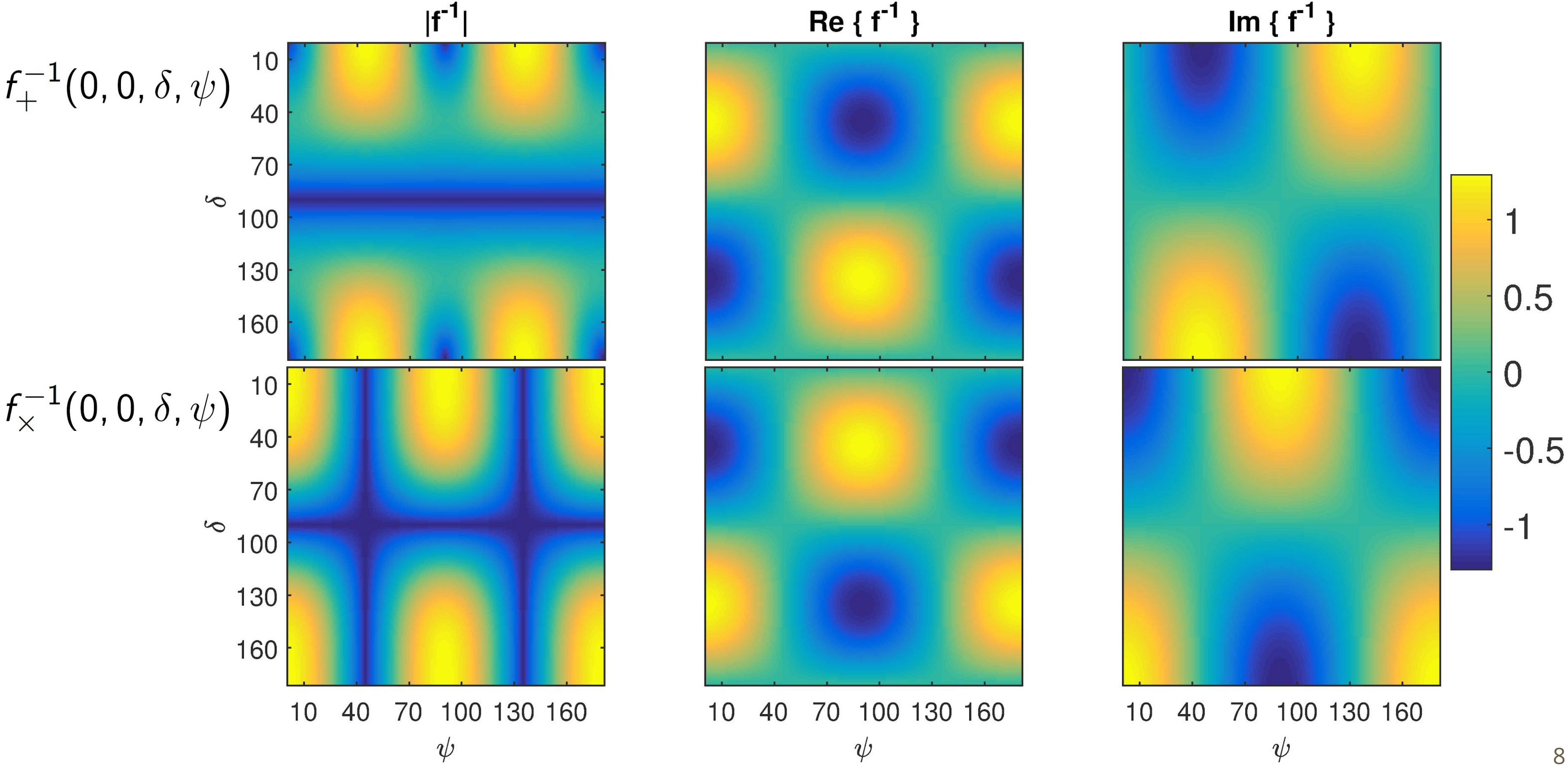
${}_7S_2$ - ${}_2S_0$

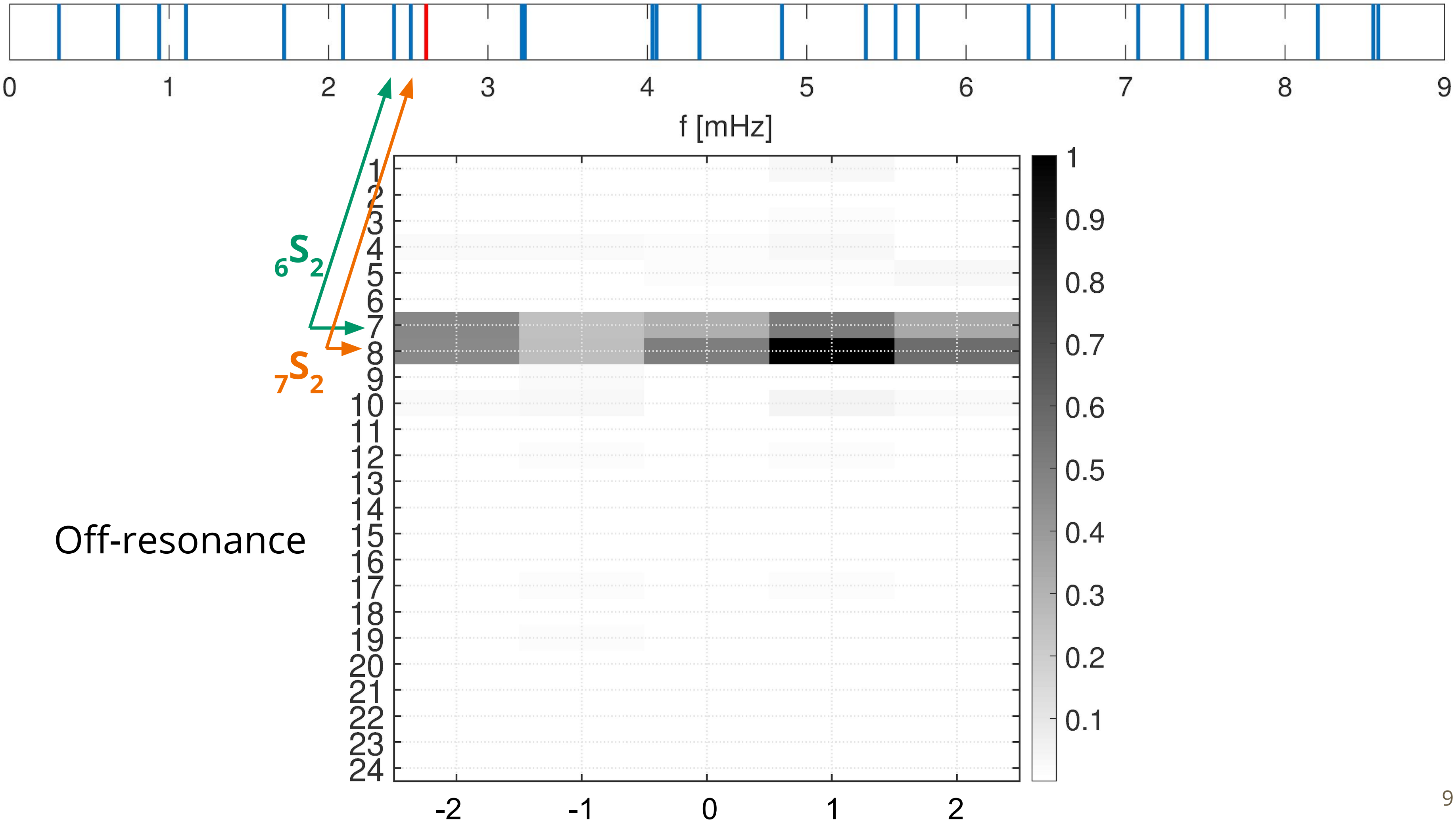
${}_8S_2$ … ${}_{23}S_2$

Induced forced spheroidal motion for 14 GW sources and 24 normal modes

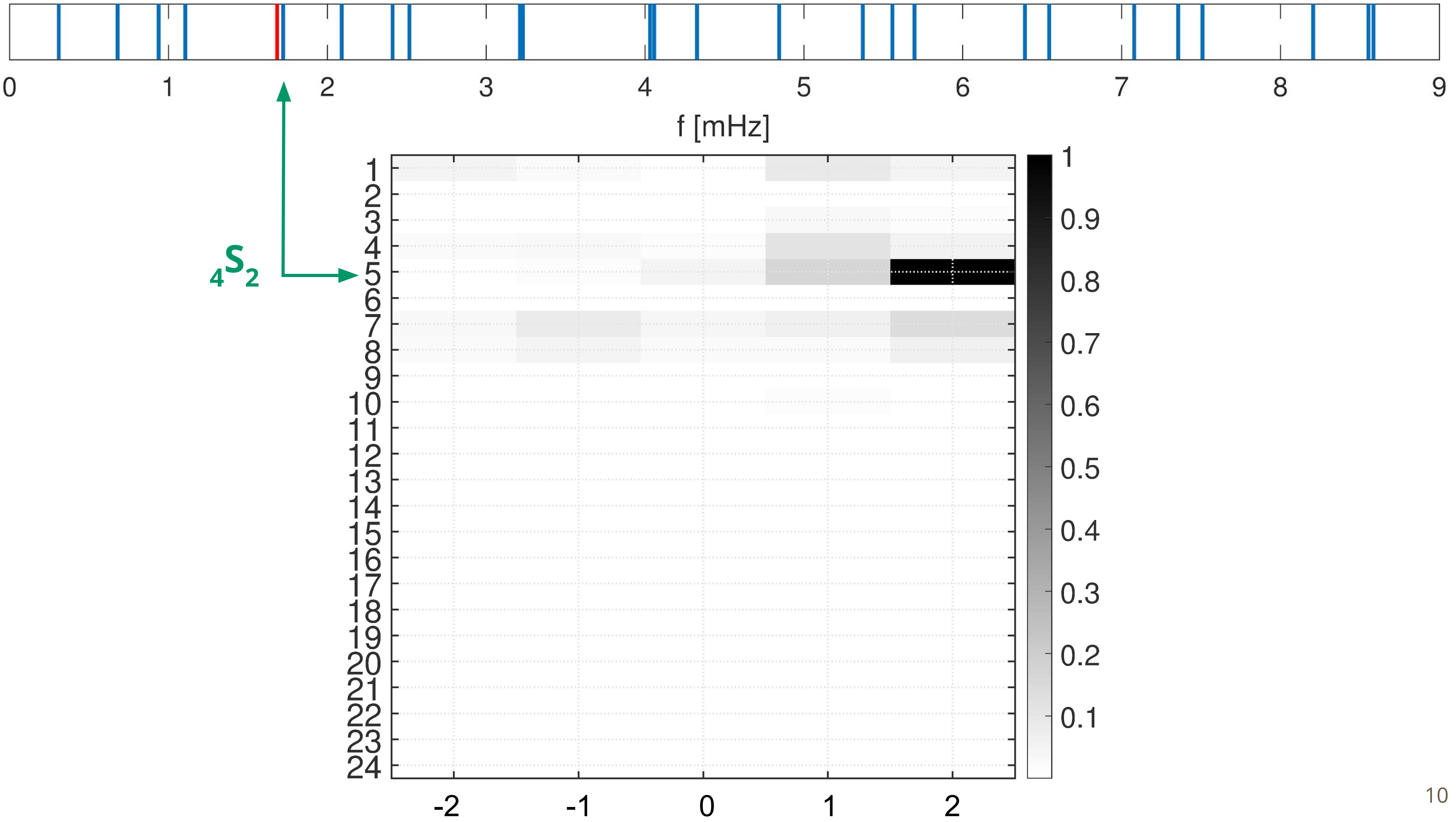


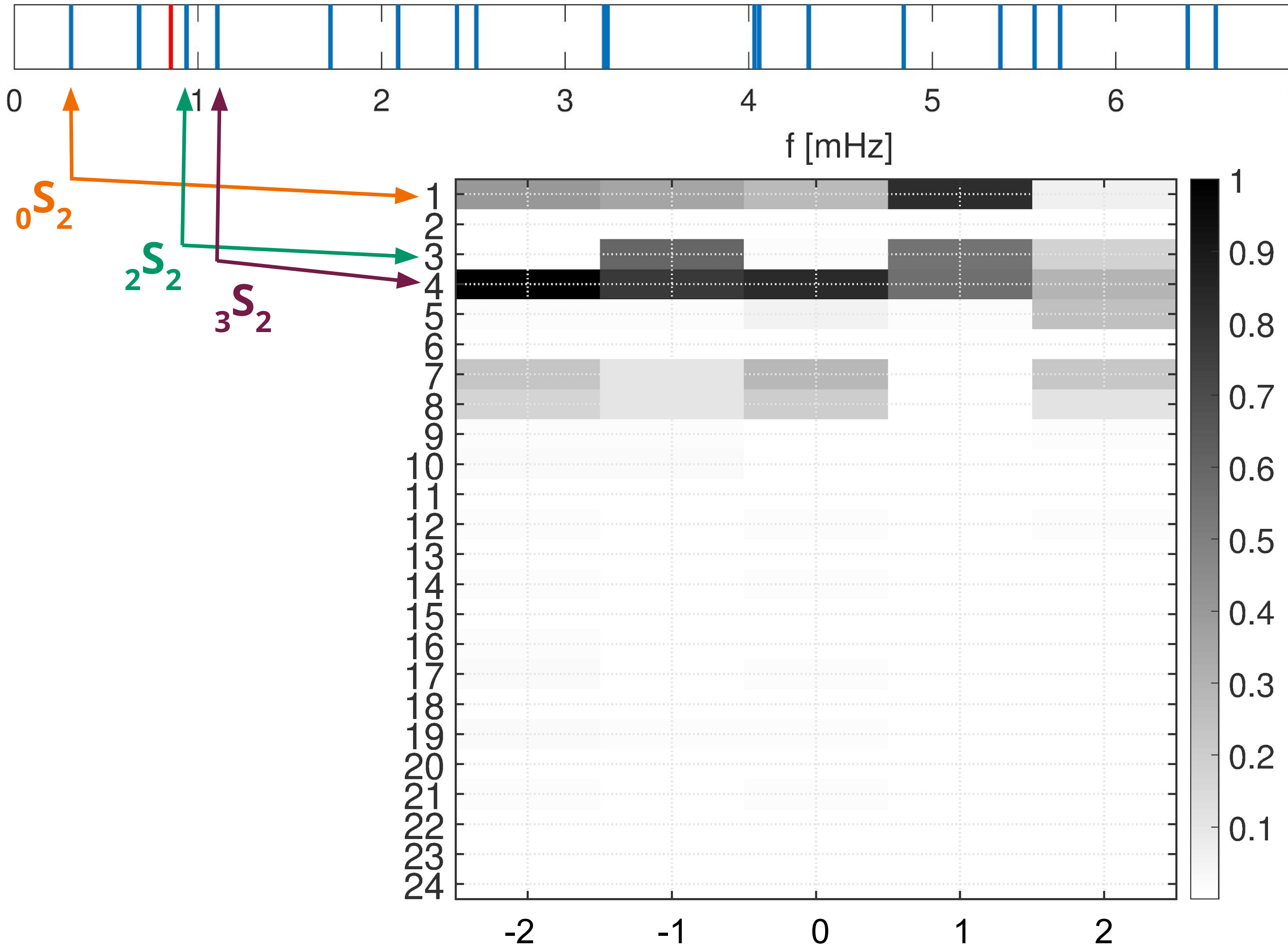
Declination and polarization angles for t=0,α=0





Off-resonance


0
1
2
3
4
5
6
7
8
9
f [mHz]
$_4S_2$
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
-2
-1
0
1
2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1



f [mHz]
$_0S_2$
$_2S_2$
$_3S_2$

The matched filtering (MF)

$$x(t) = 4\mathrm{Re}\int_0^{\infty} \frac{\tilde{d}(f)\tilde{s}^*_{template}(f)}{S_n(f)} e^{i2\pi ft} df$$

$$\sigma_h^2 = 4\int_0^{\infty} \frac{|\tilde{s}_{template}(f)\tilde{s}^*_{template}(f)|}{S_n(f)} df$$

$$\rho(t) = \frac{|x(t)|}{\sigma_h}$$

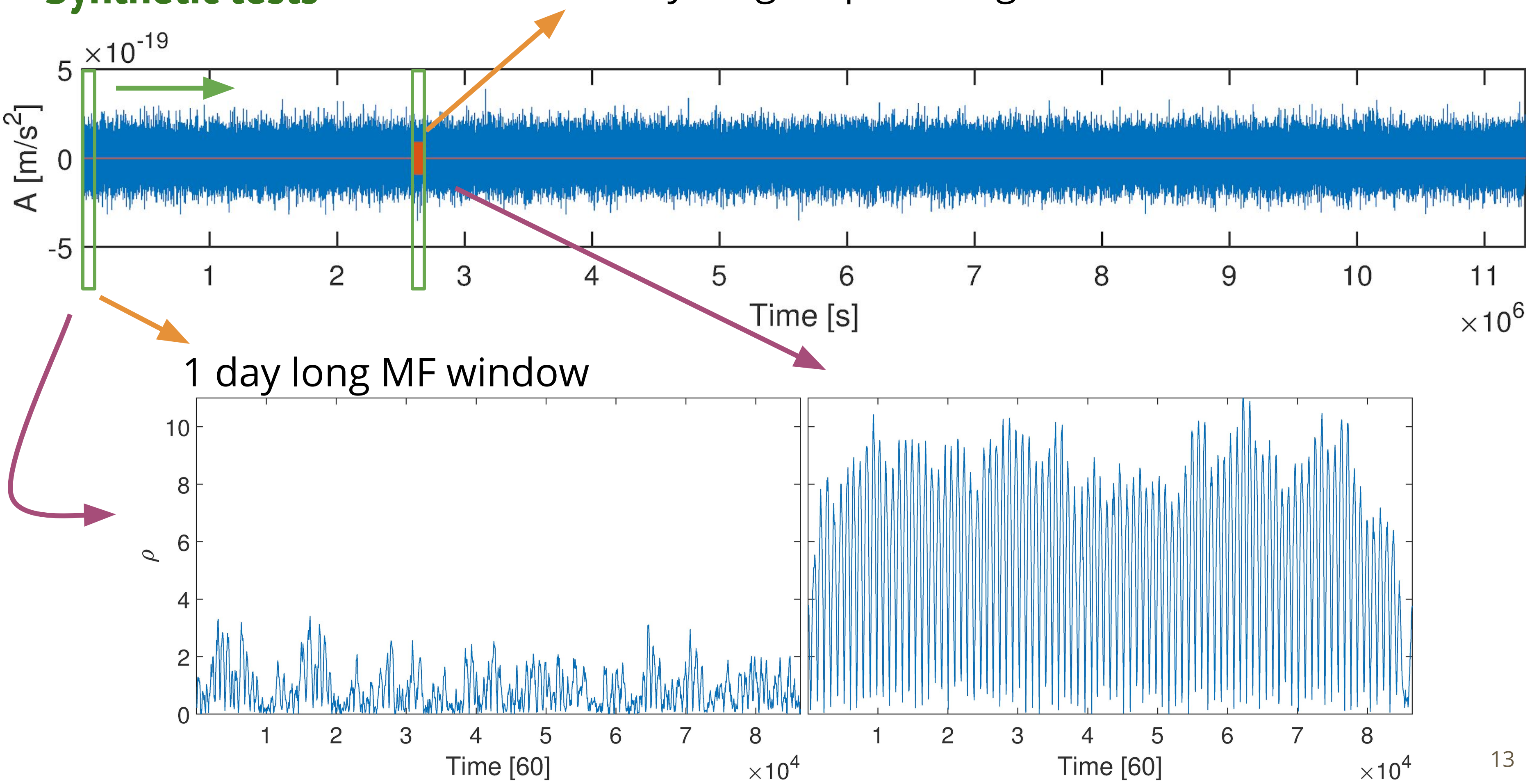
Synthetic tests

$$s_{template}(t; p_i)$$
$$i = 1, \cdots, 7$$

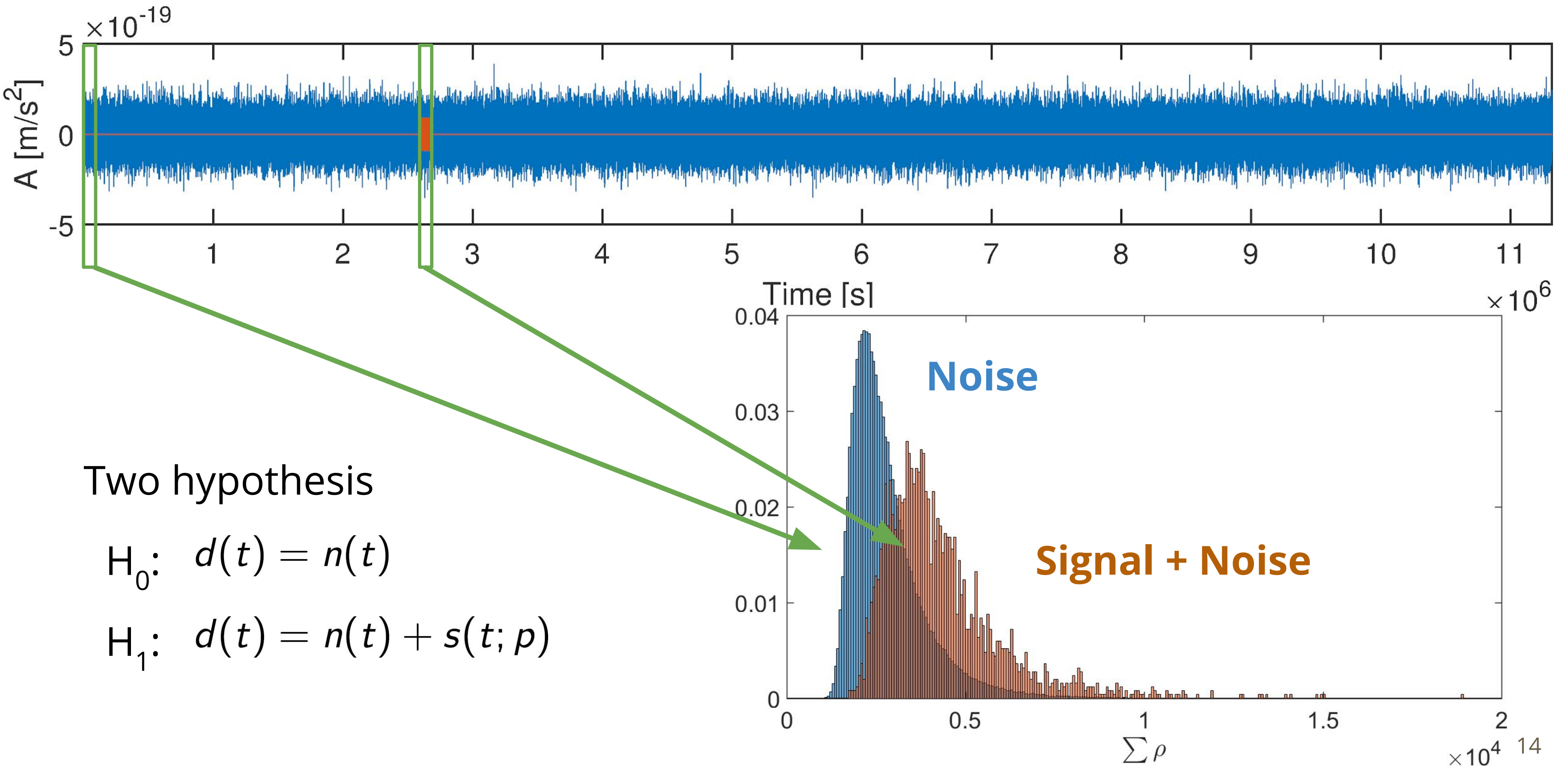
GW frequency
Mass
Distance
Inclination
Right ascension
Declination
Polarization angle

Template bank for each p_i defined by catalog uncertainties.

Synthetic tests



Synthetic tests



Two hypothesis

H_0: $d(t) = n(t)$

H_1: $d(t) = n(t) + s(t; p)$

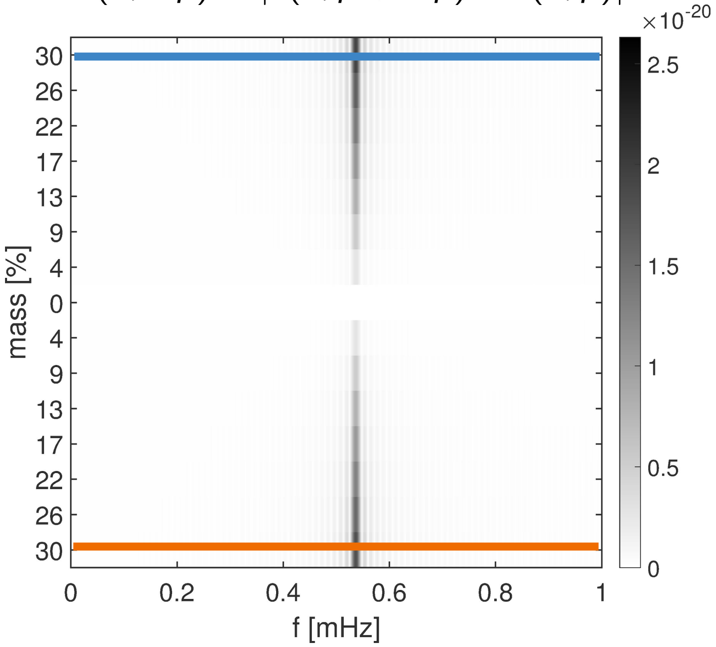
Synthetic tests

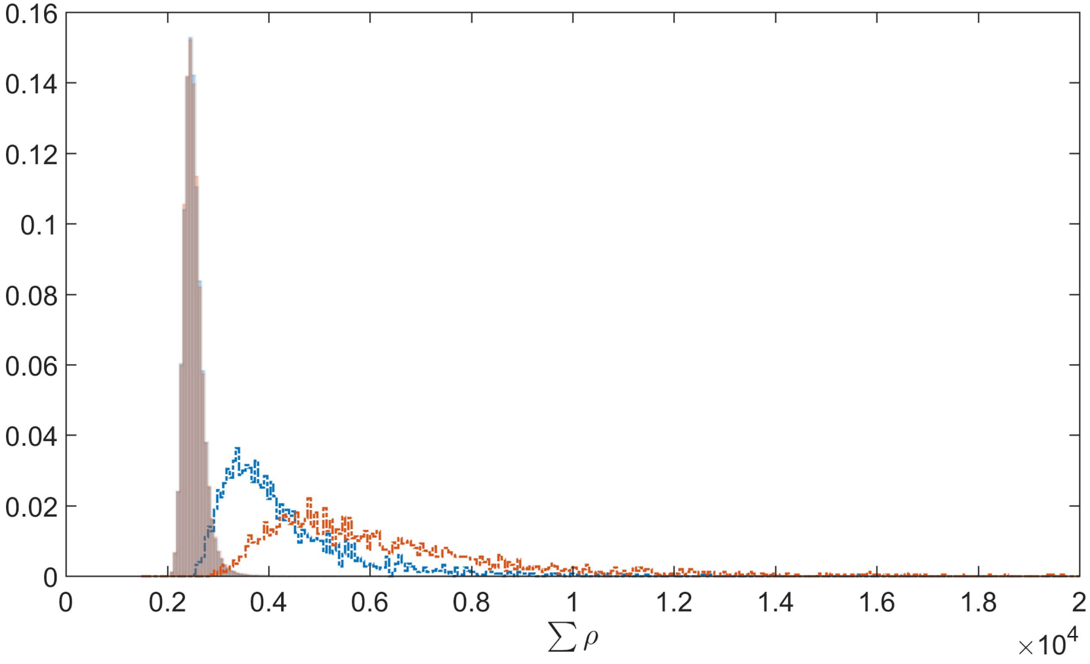
- T1 - testing the MF performance for a set of templates of one parameter where input signal and template match

- T2 - testing the MF performance for a set of templates when input signal and used template **slightly mismatch**

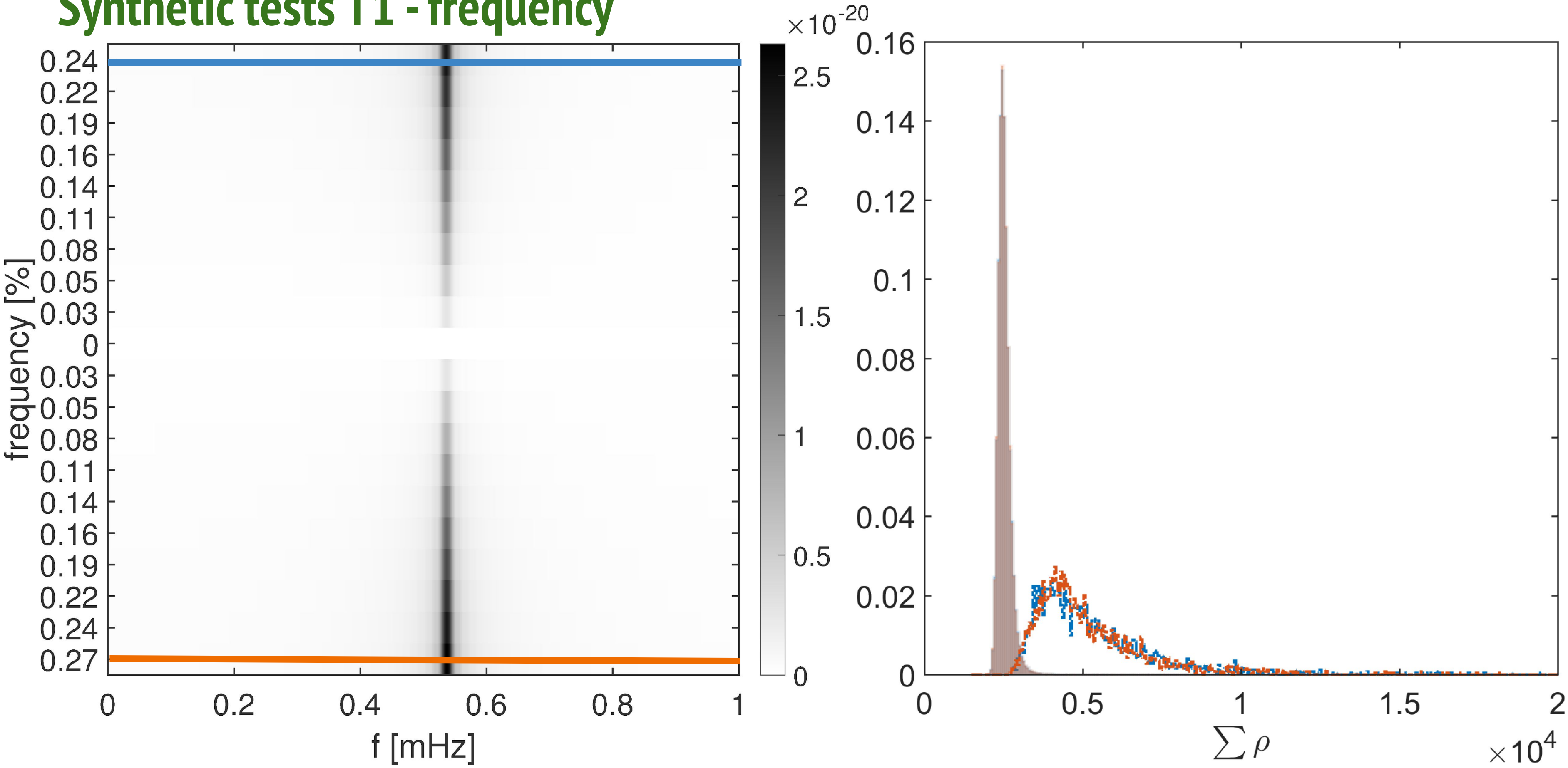
Synthetic tests T1 - Mass

$$S(f;\Delta p) = |s(f;p+\Delta p) - s(f;p)|$$

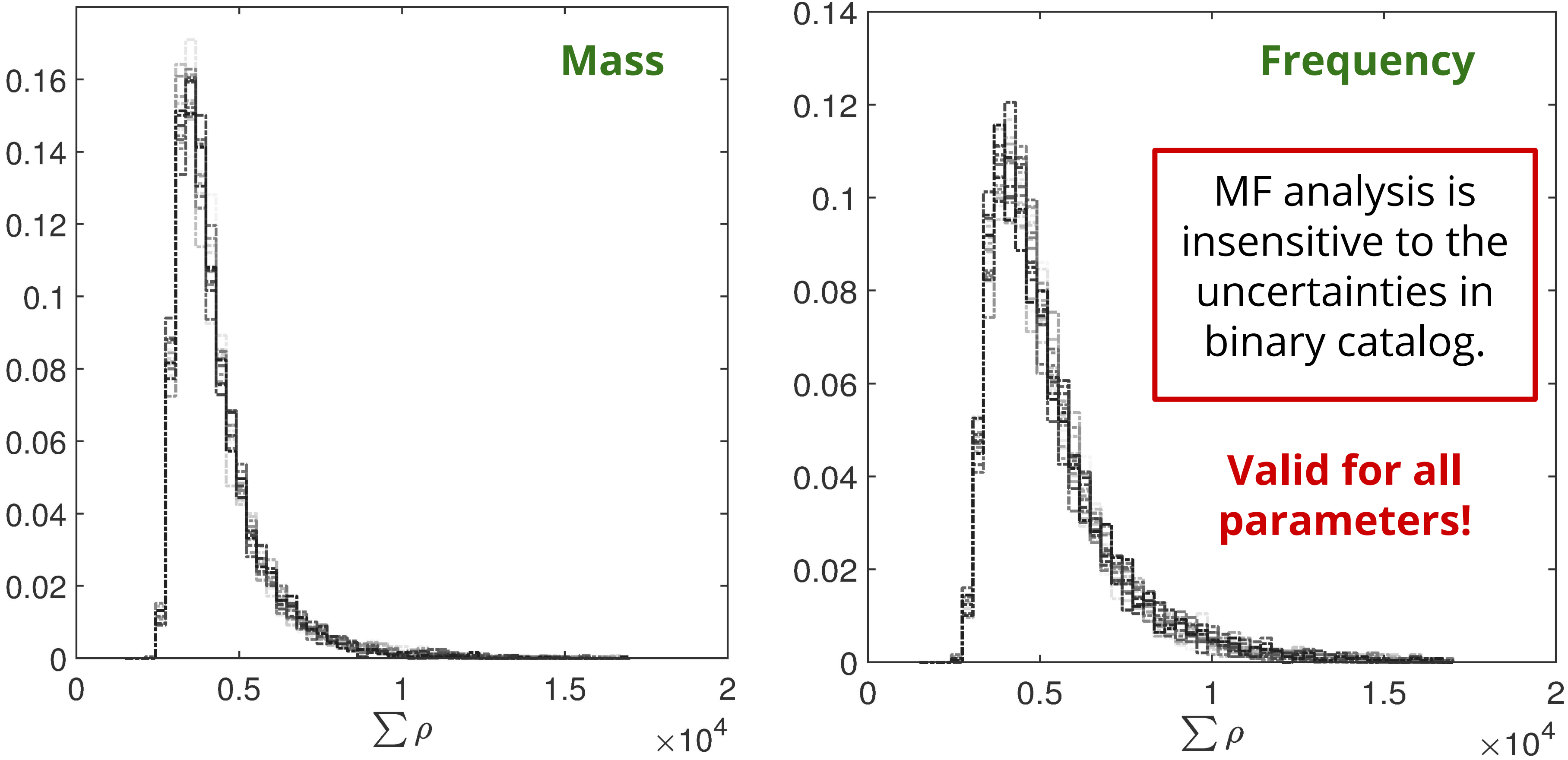




Synthetic tests T1 - frequency



Synthetic tests T2 - noise + signal histograms

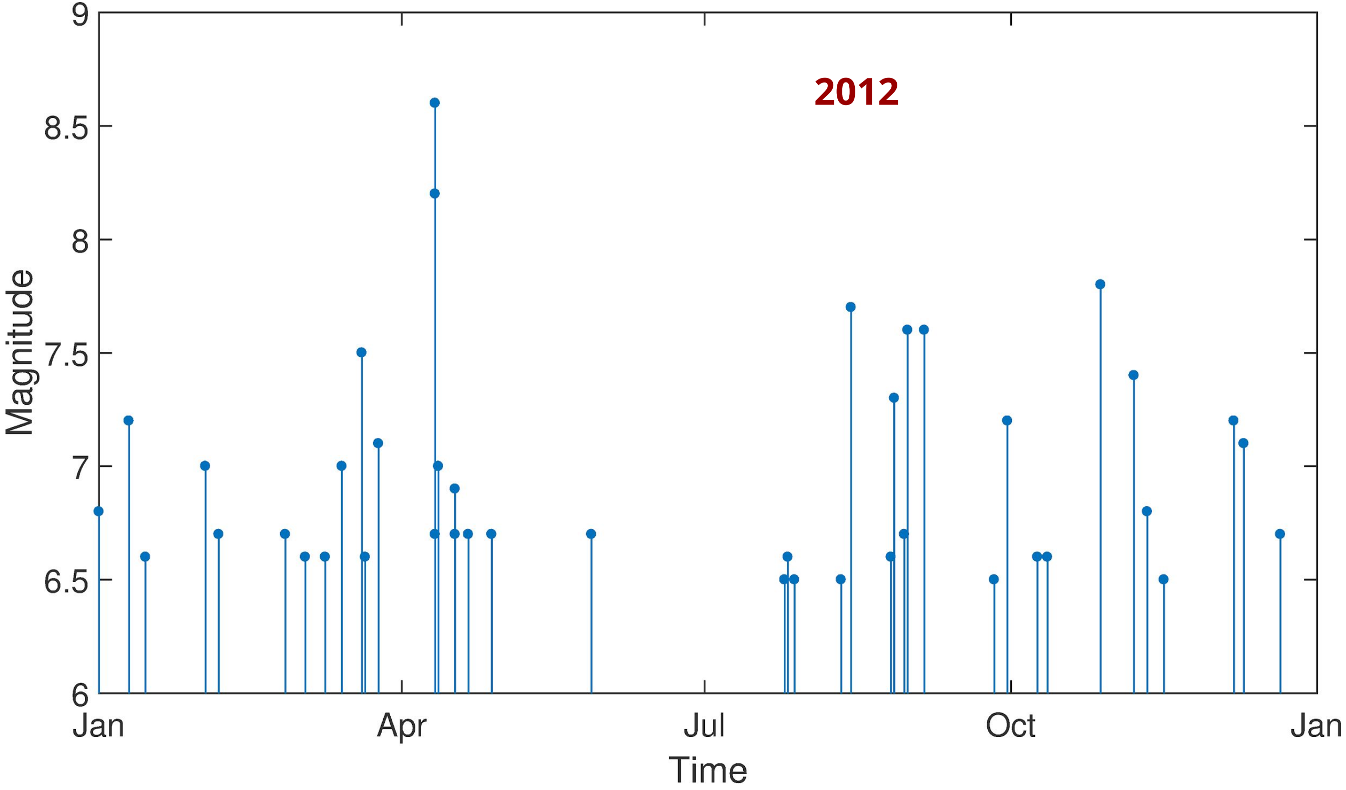


MF analysis is insensitive to the uncertainties in binary catalog.

Valid for all parameters!

Observations - an example study

- Gravimeter data from the superconducting gravimeter
- Earthquakes, tides, atmospheric pressure effects removed-coda remains



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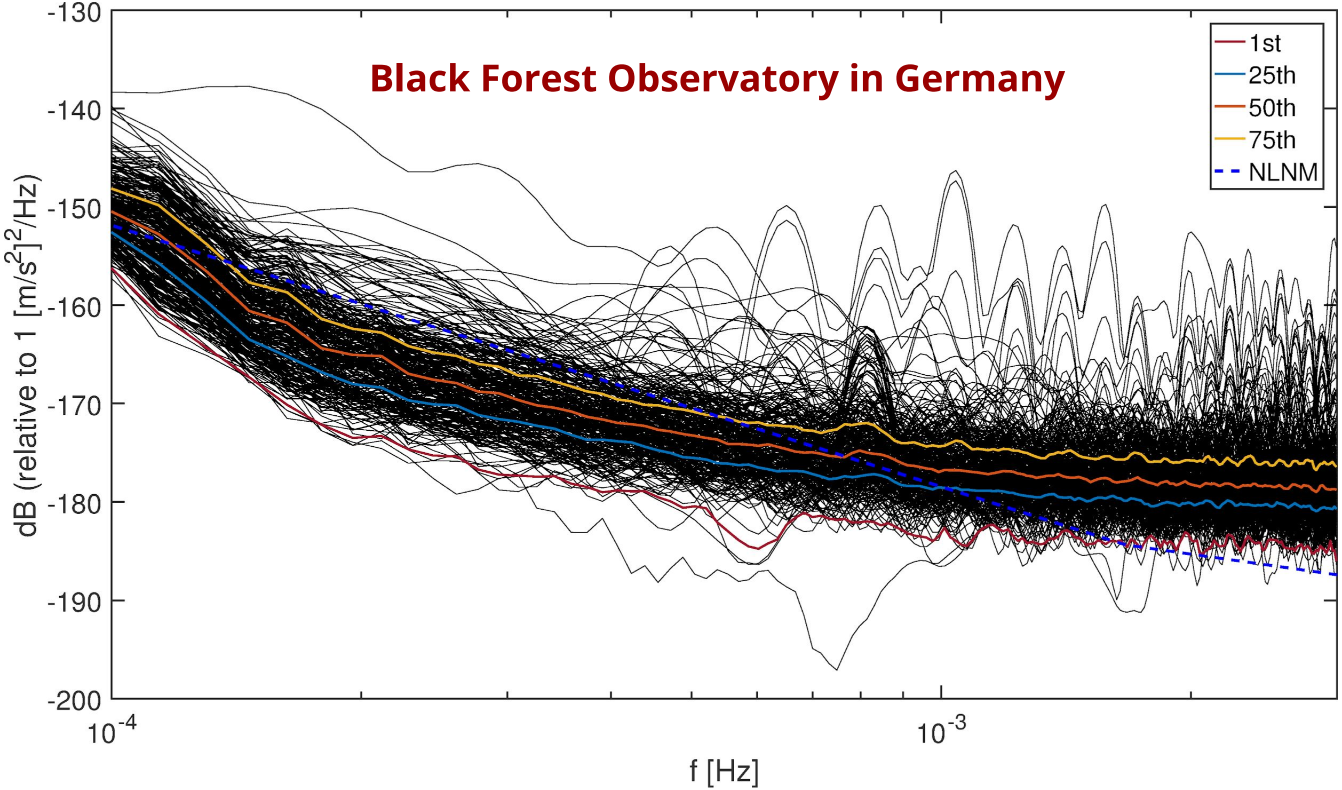
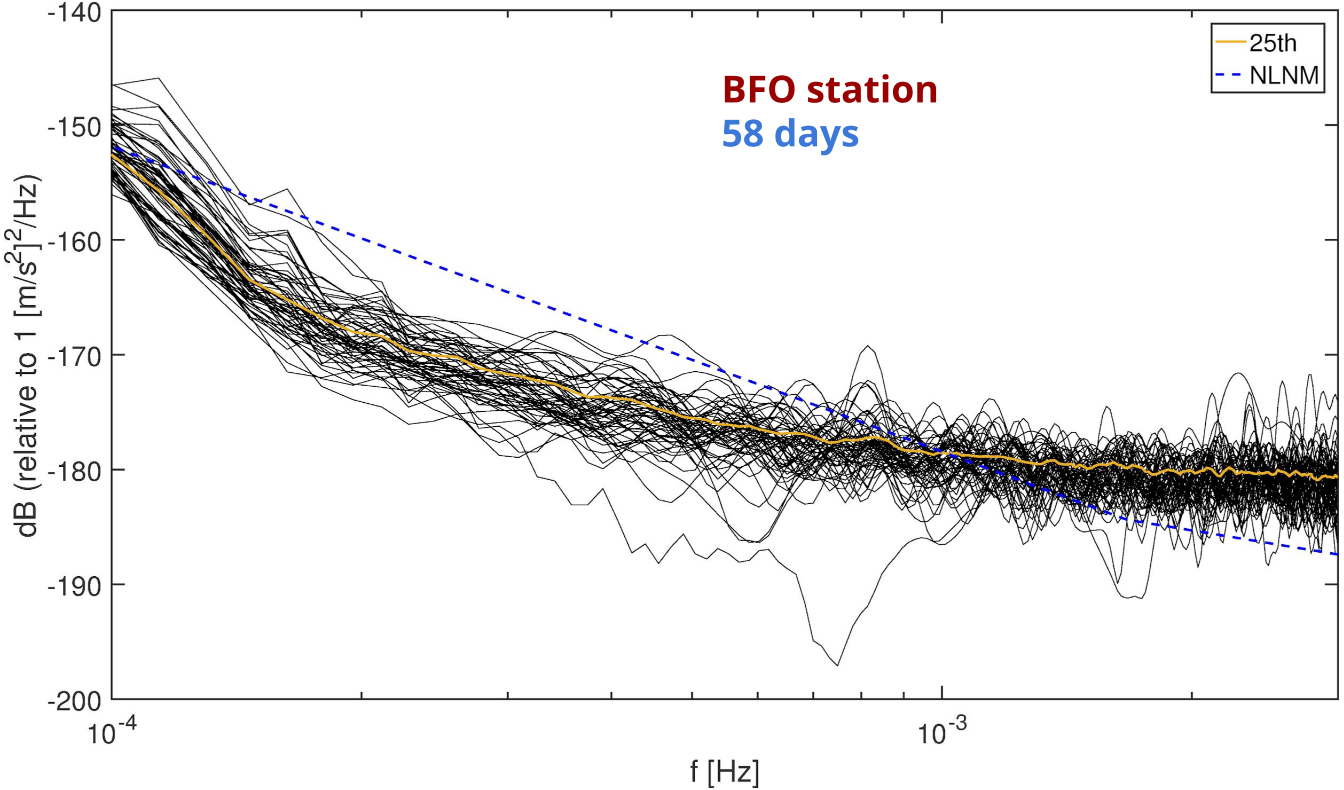


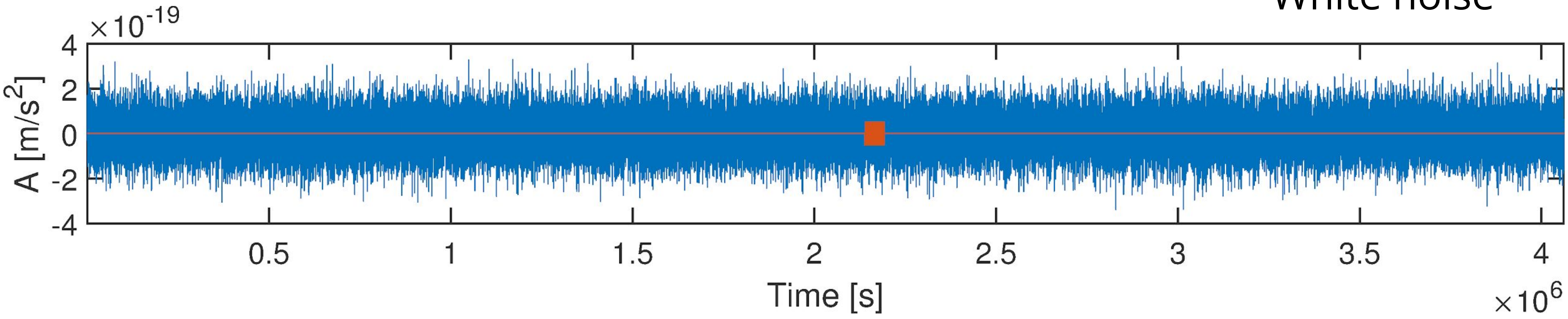
Observations - an example study

- Gravimeter data from SG instrument
- Removed earthquakes, tides, atmospheric oscillations - <u>coda remains</u>

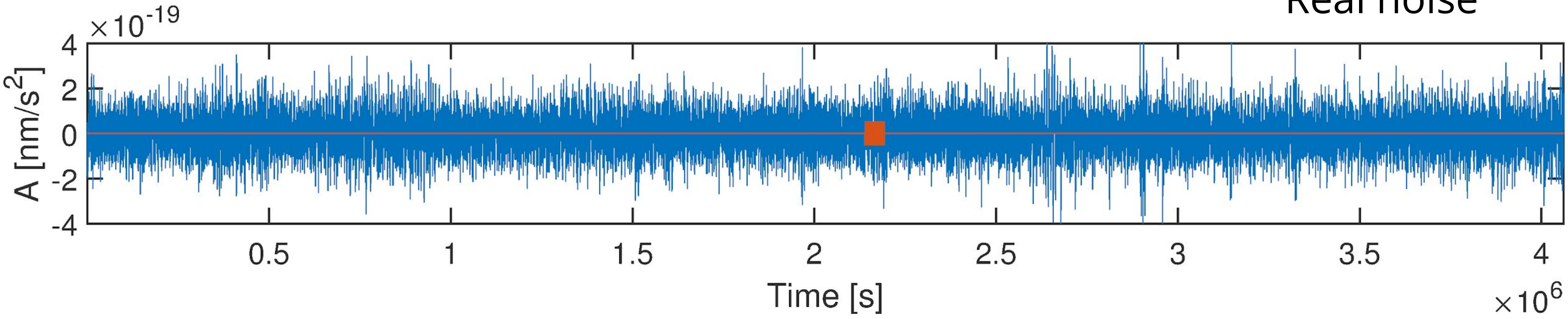


White vs real noise

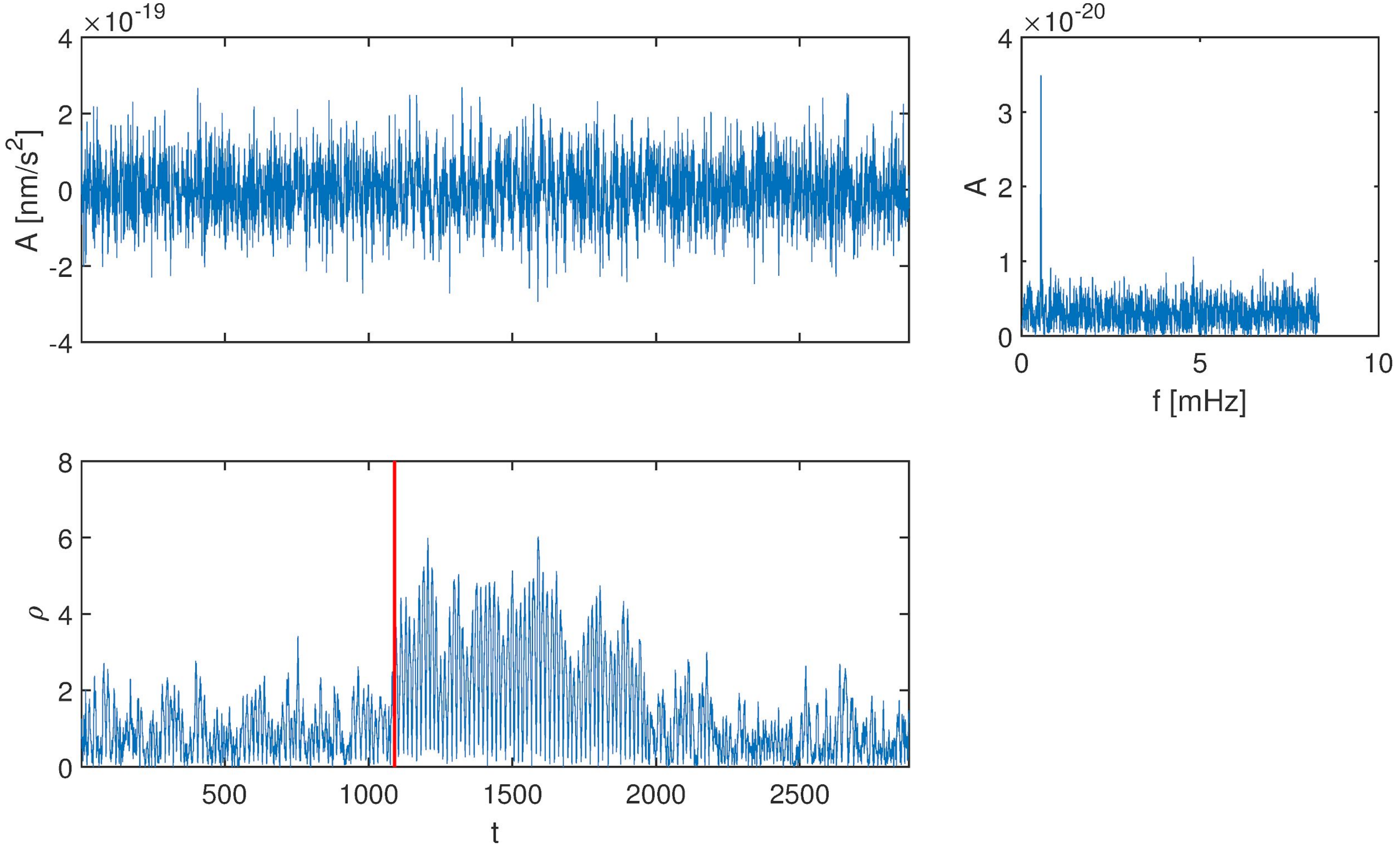
White noise



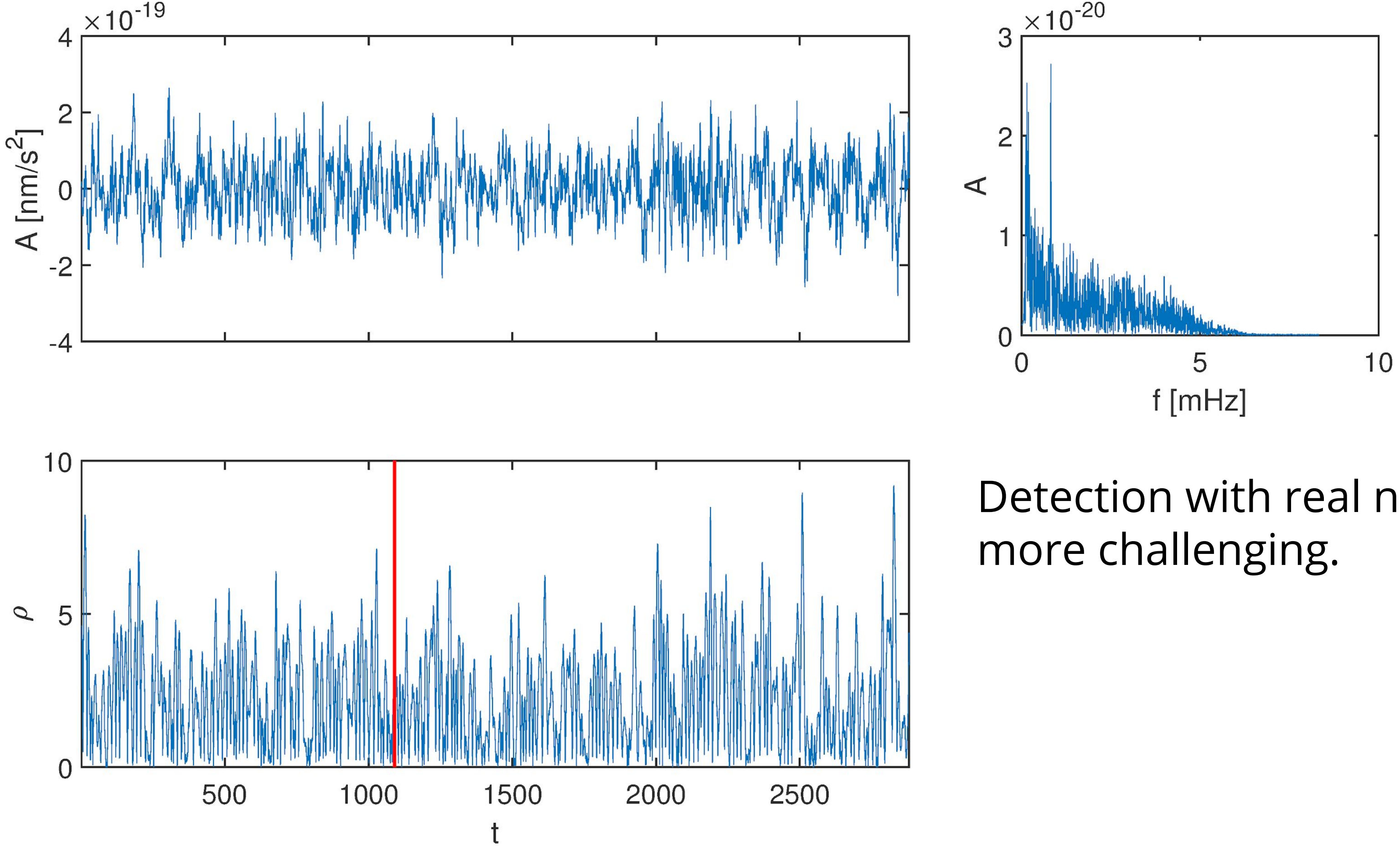
Real noise



White noise MF output



Real noise MF output



Detection with real noise more challenging.

Summary

- We derive the Earth response to GW from the binary star for a 3D rotating Earth model;
- We include the effects of splitting and coupling within the normal modes by considering group coupling approximation;
- The metric perturbation is transformed from celestial to terrestrial reference system;
- Induced spheroidal motion is quadrupole (l=2);
- Pattern f-functions define which singlets are going to be excited;
- The modes with frequencies close to GW source frequencies, but not necessarily the closest, contribute the most to building the induced response of this particular source;
- The MF analysis is insensitive to the uncertainties in binary catalog;
- Detection with real noise is more challenging.

Summary

- We derive the Earth response to GW from the binary star for a 3D rotating Earth model;
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Thank you for your attention!

Induced quadrupole modes

$$\mathbf{s}_k(\mathbf{r},t) = \mathbf{s}_k(\mathbf{r})\left(h_{+,c}\bar{g}_+(t)\mathbf{e}_+ + h_{\times,c}\bar{g}_\times(t)\mathbf{e}_\times\right) : \Bigg[\left(\mu(a)U_k(a)a^2 - \int_r \frac{\partial\mu}{\partial r}U_k(r)r^2 dr\right)\int_\Omega \mathbf{e}_r\mathbf{e}_r Y_l^{m*}(\theta,\phi)d\Omega$$
$$+\left(\mu(a)\kappa^{-1}V_k(a)a^2 - \int_r \frac{\partial\mu}{\partial r}\kappa^{-1}V_k(r)r^2 dr\right)\int_\Omega \mathbf{e}_r\nabla_1 Y_l^{m*}(\theta,\phi)d\Omega\Bigg]$$

$$\mathbf{e}_r\mathbf{e}_r = \begin{bmatrix} \sin^2\theta\cos^2\phi & \sin^2\sin\phi\cos\phi & \sin\theta\cos\theta\cos\phi \\ \sin^2\theta\sin\phi\cos\phi & \sin^2\theta\sin^2\phi & \sin\theta\cos\theta\sin\phi \\ \sin\theta\cos\theta\cos\phi & \sin\theta\cos\theta\sin\phi & \cos^2\theta \end{bmatrix}$$

$$I_1 = \frac{2\sqrt{\pi}}{3}\delta_{l,0}\delta_{m,0}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{2}{3}\sqrt{\frac{\pi}{5}}\delta_{l,2}\delta_{m,0}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$
$$+\sqrt{\frac{2\pi}{15}}\delta_{l,2}\begin{bmatrix} \delta_{m,2}+\delta_{m,-2} & -i\delta_{m,2}+i\delta_{m,-2} & -\delta_{m,1}+\delta_{m,-1} \\ -i\delta_{m,2}+i\delta_{m,-2} & -\delta_{m,2}-\delta_{m,-2} & i\delta_{m,1}+i\delta_{m,-1} \\ -\delta_{m,1}+\delta_{m,-1} & i\delta_{m,1}+i\delta_{m,-1} & 0 \end{bmatrix}$$

Source-time function

$$\bar{g}_+(t) = \frac{1}{4\pi}(i\nu_k)^{-1}\frac{1}{\gamma'_k + i(2\Omega - \omega'_k)}\mathrm{e}^{i2\Omega t} + \frac{1}{4\pi}(i\nu_k)^{-1}\frac{1}{\gamma'_k + i(-2\Omega - \omega'_k)}\mathrm{e}^{-i2\Omega t}$$

$$\bar{g}_\times(t) = -\frac{i}{4\pi}(i\nu_k)^{-1}\frac{1}{\gamma'_k + i(2\Omega - \omega'_k)}\mathrm{e}^{i2\Omega t} + \frac{i}{4\pi}(i\nu_k)^{-1}\frac{1}{\gamma'_k + i(-2\Omega - \omega'_k)}\mathrm{e}^{-i2\Omega t}$$

F-function

$$
\begin{aligned}
f_+^m(\gamma(t), \alpha, \delta, \psi) = & -2\sqrt{\frac{\pi}{5}}\delta_{m,0}\cos^2\delta\cos 2\psi \\
& + \frac{1}{2}\sqrt{\frac{2\pi}{15}}e^{-2i(\alpha-\gamma(t))}\delta_{m,2}[-4i\sin 2\psi\sin\delta + (-3+\cos 2\delta)\cos 2\psi] \\
& + \frac{1}{2}\sqrt{\frac{2\pi}{15}}e^{2i(\alpha-\gamma(t))}\delta_{m,-2}[4i\sin 2\psi\sin\delta + (-3+\cos 2\delta)\cos 2\psi] \\
& - \sqrt{\frac{2\pi}{15}}e^{-i(\alpha-\gamma(t))}\delta_{m,1}[2i\sin 2\psi\cos\delta + \sin 2\delta\cos 2\psi] \\
& + \sqrt{\frac{2\pi}{15}}e^{i(\alpha-\gamma(t))}\delta_{m,-1}[-2i\sin 2\psi\cos\delta + \sin 2\delta\cos 2\psi]
\end{aligned}
$$

LISA verification double white dwarf binary catalog

Name	Period	Pdot	d	M2	q	M1	i	Mv	V	RA	DEC	l	b
	[s]	[s/s]	[pc]	Msun		Msun	[deg]			[h:m:s]	[d:m:s]	[deg]	[deg]
SDSS J0651+2844	765.4+/-7.9	?	~1000	0.50	0.5	0.25	86.9+1.6-1	?	g=19.1	06 51 33.338	+28 44 23.37	186.93	12.69
SDSS J0935+4411	1188+/-44	?	~660	>0.14	?	0.32	?	?	g=17.7	09 35 XX	+44 11 YY		
SDSS J0106-1000	2346+/-2	?	~2400	0.43	0.4	0.17	67+/-13	?	g=19.8	01 06 57.39	-10 00 03.3	135.72	-72.47
SDSS J1630+4233	2390+/-4	?	~830	>0.52	?	0.31	?	?	g=	16 30 XX	+42 33 YY		
SDSS J1053+5200	3680+/-10	?	~1100	>0.26	?	0.20	?	?	g=18.87	10 53 53.89	+52 00 31.0	156.40	+56.79
SDSS J0923+3028	3884	?	270	>0.34	?	0.23	?	?	g=	09 23 45.59	+30 28 05.0	195.82	44.78
SDSS J1436+5010	3957 +/-10	?	~800	>0.46	?	0.24	?	?	g=18.16	14 36 33.29	+50 10 26.8	089.01	+59.46
WD 0957-666	5269.81080+/-0.00007	?	135 +/- 20	0.32 +/- 0.03	1.15 +/- 0.10	0.37 +/- 0.02	50 – 86	8.94	14.60	09 58 54.96	–66 53 10.2	287.14	-9.46
SDSS J0755+4906	5445	?	2620	>0.81	?	0.17	?	?		07 55 52.40	+49 06 27.9	169.76	30.42
SDSS J0849+0445	6800	?	930	>0.64	?	0.17	?	?		08 49 10.13	+04 45 28.7	222.70	28.27
SDSS J0022-1014	6902	?	790	>0.19	?	0.33	?	?		00 22 07.65	-10 14 23.5	99.30	-71.75
SDSS J2119-0018	7497	?	2500	>0.75	?	0.17	?	?		21 19 21.96	-00 18 25.8	51.58	-32.54
SDSS J1234-0228	7900	?	780	>0.09	?	0.23	?	?		12 34 10.36	-02 28 02.8	294.25	60.11
WD 1101+364	12503 +/- 5	?	97 +/- 15	0.36	0.87 +/- 0.03	0.31	25	9.55	14.49	11 04 32.61	+36 10 49.5	184.48	+65.62
WD 1704+4807BC	12511 +/- 2	?		0.56 +/- 0.07	0.70 +/- 0.03	0.39 +/- 0.05	61		14.5	17 05 30.1	+48 03 17	74.25	+37.19

Amplitudes and Q-factors of normal modes