kHz gravitational waves from numerical relativity

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Outline

0. Introduction
1. Binary neutron star merger
2. Black hole-neutron star tidal disruption
3. Stellar core collapse of massive stars

I will not talk
- Low-mass BH+BH merger
- Higher QNM modes ($l \geq 3$) for BH-BH
- Collapse of exotic objects to BH (e.g., cosmic string loop, axion stars) & merger of exotics
Introduction: What are high-freq sources?

Targets should be low-mass: < 20 solar mass

e.g., GW150914

NS-NS

adv LIGO

ET-B

$h_{\text{eff}} (D=400\text{Mpc})$

$10^{-21}$

$10^{-22}$

$10^{-23}$

$10^{-24}$

$f (\text{Hz})$

$10$  $100$  $1000$  $10000$
Orbital & oscillation timescale of stars

\[ f_{\text{orb}} \approx \frac{1}{\pi} \sqrt{\frac{GM}{r^3}} = 1.16 \text{ kHz} \left( \frac{r}{30 \text{ km}} \right)^{-3/2} \left( \frac{M}{2.7 M_{\text{sun}}} \right)^{1/2} \]

\[ \tau_{\text{osc}} \approx \frac{1}{\sqrt{G\rho}} = 0.387 \text{ ms} \left( \frac{\rho}{10^{14} \text{ g/cm}^3} \right)^{-1/2} \]

\[ \Rightarrow f_{\text{osc}} \approx \sqrt{G\rho} = 2.58 \text{ kHz} \left( \frac{\rho}{10^{14} \text{ g/cm}^3} \right)^{1/2} \]

• **Neutron stars** are only (non-exotic) matter sources of kHz-gravitational waves: other (normal) stars are not
1 Binary neutron stars: Their fates

Likely for $M < \sim 2.8M_{\text{sun}}$

I.e., irrespective of EOS, threshold mass $>\sim 2.8M_{\text{sun}}$
## Compact NS-NS system in our galaxy

<table>
<thead>
<tr>
<th>PSR</th>
<th>$P$(day)</th>
<th>$e$</th>
<th>$m(M_{\odot})$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$T_{\text{GW}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1913+16</td>
<td>0.323</td>
<td>0.617</td>
<td>2.828</td>
<td>1.441</td>
<td>1.387</td>
<td>3.0</td>
</tr>
<tr>
<td>B1534+12</td>
<td>0.421</td>
<td>0.274</td>
<td>2.678</td>
<td>1.333</td>
<td>1.345</td>
<td>27</td>
</tr>
<tr>
<td>B2127+11C</td>
<td>0.335</td>
<td>0.681</td>
<td>2.71</td>
<td>1.35</td>
<td>1.36</td>
<td>2.2</td>
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<tr>
<td>J0737-3039</td>
<td>0.102</td>
<td>0.088</td>
<td>2.58</td>
<td>1.34</td>
<td>1.25</td>
<td>0.86</td>
</tr>
<tr>
<td>J1756-2251</td>
<td>0.32</td>
<td>0.181</td>
<td>2.57</td>
<td>1.34</td>
<td>1.23</td>
<td>17</td>
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<tr>
<td>J1906+746</td>
<td>0.166</td>
<td>0.085</td>
<td>2.61</td>
<td>1.29</td>
<td>1.32</td>
<td>3.1</td>
</tr>
<tr>
<td>J1913+1102</td>
<td>0.206</td>
<td>0.090</td>
<td>2.875</td>
<td>1.65</td>
<td>1.24</td>
<td>5.0</td>
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<td>J1757-1854</td>
<td>0.184</td>
<td>0.606</td>
<td>2.74</td>
<td>1.35</td>
<td>1.39</td>
<td>0.77</td>
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<td>J1946+2052</td>
<td>0.078</td>
<td>0.064</td>
<td>$\sim$2.50</td>
<td>$\sim$1.2</td>
<td>$\sim$1.3</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Total Mass of NS in compact NS-NS is in a narrow range, $m \approx 2.7 \pm 0.2 \ M_{\odot}$ and for many, $m < 2.8 \ M_{\odot}$
**NS-NS typical case**

- **Early Inspiral**
  \[ r_{\text{orb}} >> R_{\text{NS}} \]

- **Late inspiral**
  \[ r_{\text{orb}} \leq 5R_{\text{NS}} \]

- **Merger**
  \[ \text{Massive NS + torus } \rightarrow \text{GRB?} \]

**Post-Newtonian**

- **Point mass** + adiabatic phase
- **Tidally deformed phase**
- **Dynamical & GR phase**

**Post merger**

- **Massive NS / BH**

**Tidal deformability**

- Target of 2\(^{\text{nd}}\) G

**Frequency bands**

- \( f < \sim 1000 \text{ Hz} \)
- \( f \sim 2 - 4 \text{ kHz} \)
- \( f \sim 6.5 \text{ kHz} \)

**Mass**

- \( M \sim 2.5 - 2.8 M_{\odot} \)
Gravitational waveform from NS-NS
(1.35-1.35 solar mass)

Align the phase here

Tidal deformability (Hiderler-Flanagan)

Late inspiral

Post merger

Black: BH (by SXS)
Red: SFHo (11.9 km)
Green: DD2 (13.2 km)
Blue: TM1 (14.5 km)

Hotokezaka et al. 2016 (Many efforts by Bernuzzi, Dietrich, etc)
Spectrum

Peak frequency depends on EOS

SNR = 0.9—1.6 (for $f > 1$ kHz)
@total SNR = 30 event: small for 2nd generation detectors

Possible source of 3rd generation detectors

Advanced LIGO: Zero-detuned high-P

Hotokezaka et al. PRD 93 2016

f (kHz)
Clear correlation between peak and radius

\[ f \propto \sqrt{\frac{GM}{R^3}} \]

NS radius will be constrained within 1km error

At one good event

See also Rezzolla et al, …
Popular field ? in Germany
and APR4-130160, this gain of the angular momentum gets the escape velocity. For models APR4-140150 object just before the formation of a black hole and star material gains a sufficient torque from the merged merger, and a fraction of the tidally elongated neutron-neutron star is tidally elongated during the tidal torque. In the presence of mass asymmetry, the less-ejection occurs for the case that the mass asymmetry is primary role in the mass ejection. A significant mass black hole, and thus, the shock heating does not play a collision of two neutron stars is soon swallowed by the hole is promptly formed, a region shock-heated at the instance of the merger, i.e., during a short duration before the formation of a black hole. Because a black formation of a black hole occurs only for APR4 with the total mass.

FIG. 18 (color online). The same as Fig.

FIG. 19 (color online). The same as Fig.

Remnant: $M=2.80$ solar mass & $a=0.77$ $f \sim 6.5$ kHz: too high even for 3G
Caution:
Physical state of remnant NS is very uncertain, similar to supernova case

✧ Magnetic fields are surely amplified
   (Price-Rosswog 2006, Kiuchi et al. 2014, 15, 18)
→ It is natural to consider that turbulence will be excited
   (cf. the first talk in the today’s morning on Tokamak)
→ Turbulence viscosity is likely to be excited
→ Typically $\nu=\alpha_v \, c_s \, H$ with $\alpha_v=0.01$: but uncertain
   ($c_s$: sound velocity, $H$: scale height)
\( \alpha_v = 0.01: \) With viscosity

\( \alpha_v = 0: \) No viscosity

Non-axisymmetry disappears in short timescale

Shibata & Kiuchi, PRD 2017
Gravitational waveforms in viscous hydro

\[ D m_0 \Re[\Psi_4] \]

\[ t_{\text{ret}} [\text{ms}] \]

\( \alpha_v = 0.00 \) \( \alpha_v = 0.01 \) \( \alpha_v = 0.02 \)

Shibata & Kiuchi, PRD 2017
May be challenging but deserves to challenge.

Shibata & Kiuchi, PRD 2017

Decrease by $\frac{\tau_v}{\tau_{v=0}}$
**2 BH-NS: Tidal disruption or not**

For tidal disruption, \( \frac{M_{NS}}{(\alpha R_{NS})^2} < \frac{M_{BH}(\alpha R_{NS})}{r^3} \) \( (\alpha > 1) \)

\( \Rightarrow \)

\[ \frac{M_{BH}}{r_{ISCO}} \left( \frac{M_{NS}}{M_{BH}} \right)^2 \left( \frac{\alpha R_{NS}}{M_{NS}} \right)^3 \leq 1 \]

- For tidal disruption
  - Large NS Radius or
  - Small BH mass or
  - High corotation spin is necessary
BH-NS with aligned BH spin

$M_{\text{BH}} = 6.75 M_{\text{sun}}$
$a = 0.75$
$M_{\text{NS}} = 1.35 M_{\text{sun}}$
$R = 11.1 \text{ km}$

$M_{\text{BH}} = 4.05 M_{\text{sun}}$
$a = 0$
$M_{\text{NS}} = 1.35 M_{\text{sun}}$
$R = 11.0 \text{ km}$
BH-NS with aligned BH spin

\[ M_{\text{BH}} = 6.75 M_{\text{sun}} \]
\[ a = 0.75 \]
\[ M_{\text{NS}} = 1.35 M_{\text{sun}} \]
\[ R = 11.1 \text{ km} \]

\[ M_{\text{BH}} = 4.05 M_{\text{sun}} \]
\[ a = 0 \]
\[ M_{\text{NS}} = 1.35 M_{\text{sun}} \]
\[ R = 11.0 \text{ km} \]

High BH spin is key for TD

Kyutoku et al. 2011, 2015; See also great effort by Foucart+
GW frequency at tidal force

\[ \Omega = \sqrt{\frac{GM}{r_{TD}^3}} = \sqrt{\frac{GM_{NS} (1+Q)}{r_{TD}^3}} = \sqrt{\frac{GM_{NS} (1+Q)}{(\alpha R_{NS})^{\frac{1}{3}}}} = \sqrt{G\rho} \sqrt{\frac{4\pi (1+Q)}{3\alpha^3 Q}} \]

Mass ratio \( Q = \frac{M_{BH}}{M_{NS}} \)

At the onset of tidal disruption, \( r_{TD} = \alpha R_{NS} Q^{1/3} \)

Here, \( \frac{4\pi}{3\alpha^3} = 1.02 \left( \frac{\alpha}{1.6} \right)^3 \& \frac{1+Q}{Q} = 1.05 \sim 1.2 \) for \( Q = 5 - 20 \)

\[ \rightarrow f = \frac{\Omega}{\pi} \propto \sqrt{G\rho} \]

This frequency reflects neutron-star average density!
BH-NS: Signal of tidal disruption

Tidal disruption

Weak tidal disruption

Green=Tayloy T4
BH-NS Fourier spectrum

For all, \( \chi = 0.5 \) & NS:BH=1.35:5.40\( M_{\odot} \)

\[
f_{\text{tidal dist}} \sim \frac{1}{2\pi} \sqrt{\frac{G M_{\text{NS}}}{R_{\text{NS}}^3}} \sim 1 - 2 \text{ kHz}
\]

Lackey et al. 14; Pannarale et al. 14
Realistic case: $9.45-1.35 \, M_{\odot}$ & $a=0.75$ cases

Most optimistic direction

Black hole ringdown frequency

Larger NS radius

$|f h(\theta)| \, (D=100 \, \text{Mpc})$

Larger NS radius

Excluded by GW170817

$|f h(\theta)| \asymp 10^{-21}$

$|f h(\theta)| \asymp 10^{-22}$

$\frac{1}{2\pi} \sqrt{\frac{GM_{\odot}}{R_{NS}^3}} \sim 1-2 \, \text{kHz}$

Kyutoku+ 2015: See also work by Foucart+  $f (\text{Hz})$
3 supernovae

$\rho_c \sim 4 \times 10^9 \text{g/cm}^3$  

$\rho_c \sim 10^{12} \text{g/cm}^3$ 

$\rho_c \sim 3 \times 10^{14} \text{g/cm}^3$

Neutral star

Explosion

Neutrino sphere

Shock

Standing shock

Shock revival

Oscillate $\rightarrow$ GW
further developed, we are limited in our efforts If BH formation takes place in a rapidly spinning progenitor, it will be accompanied by an intense spike-like burst of GW emission at the point of relativistic collapse, followed by a fast ringdown as the newly formed BH settles down to a Kerr spacetime. By contrast, BH formation during the first seconds after collapse in non-rotating or slowly rotating progenitors is likely to manifest itself only as an abrupt cutoff of GW emission after a long period of moderate-amplitude GW emission. Prior to BH formation, the characteristic frequencies of PNS oscillation modes in the spectrum will increase to several kHz.

Figure 1: Characteristic strain vs. frequency of three typical 3D ccSN simulations: C15, W15-4, and TM1. The Einstein Telescope in xylophone D (ET-D) configuration, the Cosmic Explorer (CE), and the Advanced LIGO design also shown.

Sources of Continuous Gravitational Waves

The emission of continuous high-frequency GWs at detectable amplitudes requires a time-varying mass quadrupole in a fast rotating compact object. They are expected whenever there is a sustained non-axisymmetric distribution of matter in a rotating compact object. This can happen due to a variety of mechanisms. Most prominent examples include elastic stresses building up in the crust and giving rise to local deformations, deformations due to magnetic fields, which can occur in isolated NSs, and the growth of r-modes in accreting NSs (a fluid mode of oscillation for which the restoring force is the Coriolis force). Whereas the amplitude of a GW signal depends on the details of the emission mechanism and on the source, the possible signal morphologies do not differ much. Typical continuous GWs are sinusoidal signals with a small spin-down or spin-up (|\dot{f}| no larger than $10^{-7}$ Hz/s and most often smaller than $10^{-9}$ Hz/s) and a duration of at least a few weeks and most typically years. As the loss rate of rotational energy caused by GW radiation is proportional to the sixth power of the spin frequency, the most powerful sources must possess rapid spin. Such large amounts of spin angular momentum can be a birth property of a newborn NS, or it may result from the recycling of an old NS via accretion of matter and angular momentum from a companion star.
Black hole formation from direct collapse of very massive star

Black hole mass at formation $\sim 20$ solar mass
dimensionless spin $\sim 0.8$

Uchida et al., PRD 99, 041402, 2019
Spectrum: spin $> \sim 0.5$ is promising

$10^{-24}$

$10^{-23}$

$10^{-22}$

$10^{-21}$

$2h_{\text{eff, ave}} (D=50\text{Mpc})$

$f \sim 630\text{Hz (}30M_{\text{sun}}/M\text{)}$

Uchida et al., PRD 99, 041402, 2019
Summary

• Promising high-freq sources for 3G detectors
  1. Merger remnant of NS-NS \(\rightarrow\) NS radius
  2. Tidal disruption of BH-NS \(\rightarrow\) NS average density
  3. Supernovae, protoNS \(\rightarrow\) Mechanism of SN
  4. Supernovae, BH formation \(\rightarrow\) Finding BH formation from collapse for the first time

• BH-BH \(\rightarrow\) ringdown (relatively low mass)
• Other exotic possibilities: Axion star collapse, cosmic string collapse, primordial BH-BH mergers, axion stars merger ……
Sensitivity of LIGO & VIRGO O2

\[ \text{Strain} \left( \frac{1}{\sqrt{\text{Hz}}} \right) \]

- Virgo
- Hanford
- Livingston

\[ D_{\text{eff}} = 100 \text{Mpc} \]
\[ 1.35 - 1.35 \text{ solar mass} \]

\[ f^{-2/3} \rightarrow f > 1 \text{kHz is not reachable now} \]

PRL 119, 141101
High-resolution GRMHD for NS-NS
Kiuchi et al. 2015

Kelvin-Helmholtz instability:
- Magnetic field should be amplified by winding
- Quick angular momentum transport? (not yet seen)
Magnetic energy: Resolution dependence

B field would be amplified in $\Delta t \ll 1$ ms $\rightarrow$ turbulence?

Still NOT convergent...

Higher resolution

$E_B$ [erg] vs. $t - t_{\text{mrg}}$ [ms]

$B_{\text{max}} = 10^{13}$ G

$\tau_{\text{KH}} \sim \frac{\lambda}{v} \propto \Delta x$

Kiuchi et al. 2015
Evolution of angular velocity $\alpha_v = 0.01$

Inside of remnant neutron star

$\rightarrow$ Rigid rotation
Frequency evolution

![Graph showing frequency evolution with labels for PNS Oscillations and SASI, and a black box labeled with 25 $M_\odot$.]