Accelerated observers and Planck-scale kinematics

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$\Downarrow$

Upon introduction of a **"brick wall"** regulator obtain **entropy density** $\sim 1/L_P^2$

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The common view is that **quantum properties of spacetime** are responsible for a **finite horizon entropy density** in the same way quantization of the electromagnetic field leads to a finite **black-body entropy**

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Work in collaboration with Master’s student M. Laudonio
(Phys. Rev. D **97**, no. 8, 085004 (2018))

Accelerated observers

Accelerated observers

Four-velocity of observer with **acceleration** α

$$U^\mu = (\cosh \alpha\tau, \sinh \alpha\tau, 0, 0)$$

Lorentz boost by $\eta = \alpha\tau$ of four-velocity of static Minkowski observer $U^\mu = (1, 0, 0, 0)$

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Accelerated worldline = **Lorentz orbit** of the vector $(0, 1/\alpha)$

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Using a **dilation** generated by $D = -i\,x^{\mu}\partial_{\mu}$: a finite transformation of parameter δ

$$(t, x) \to (t', x') = e^{\delta}(t, x) \implies \boxed{\alpha \to \alpha' = e^{-\delta}\alpha}$$

Rindler space

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Define spatial **Rindler coordinate** ξ in terms of the **dilation parameter** $\delta = a\,\xi$ (with $1/a =$ [lenght])

$$\begin{cases} t = \frac{1}{a}\, e^{a\xi} \sinh a\eta \\ x = \frac{1}{a}\, e^{a\xi} \cosh a\eta \end{cases}$$

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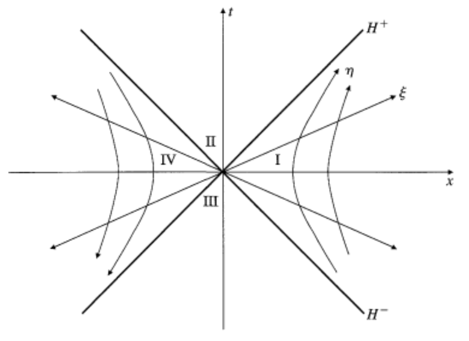

Accelerated observers and the Weyl-Poincaré group

Boosts and dilations in Minkowski space can be used to describe Rindler space

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Weyl-Poincaré algebra in 1+1 dimensions

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\begin{aligned}
\left[P_t, P_x\right] = 0\,, &\quad \left[D, N\right] = 0 \\
\left[N, P_t\right] = iP_x\,, &\quad \left[N, P_x\right] = iP_t \\
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This algebra contains *two abelian subalgebras* spanned by $\{P_t, P_x\}$ and $\{D, N\}$

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Besides usual reps in terms of $P_{t,x} = i\partial_{t,x}$ we have an **alternative reps** in terms of Rindler coordinates $\xi\,, \eta$

$$P_\xi = aD = i\partial_\xi\,, \quad P_\eta = aN = i\partial_\eta$$

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Note the role of acceleration scale a in order to get the right dimensions for P_ξ *and* P_η

Aside: the Unruh effect without space-time

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The simplest non-abelian Lie algebra

$$\big[D, P\big] = iP$$

relationship between **boundary** and **thermal effects** from representation theory
(work with Kowalski-Glikman: arXiv:1804.10550)

Rindler coordinates and reps of the Weyl-Poincaré algebra

Representation of the Weyl-Poincaré generators in terms of $(\partial_\xi, \partial_\eta)$

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P_\xi &= i\partial_\xi \\
P_\eta &= i\partial_\eta \\
P_t &= ie^{-a\xi}(\cosh a\eta\,\partial_\eta - \sinh a\eta\,\partial_\xi) \\
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Rindler translation generators are obtained from P_t and P_x in terms of a **boost** by $a\,\eta$ and a **dilation** by $a\,\xi$

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Rindler mass shell obtained from mass Casimir

$$\mathcal{C} = P_0^2 - P_x^2 = e^{-2a\xi}(P_\eta^2 - P_\xi^2)\,,$$

Doppler shift and Rindler horizon

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- At $\xi = -\infty$ i.e. on the light cone $x = |t|$ the photon's frequency will appear **infinitely blueshifted** (in analogy with Schwarzschild horizon)

The **Rindler horizon** is described by an *infinite contraction* generated by D

Mode counting in Minkowski space

Number of modes of a massless scalar field in a $3D$ box of size L

- **Nodes** on the wall of box

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by trivial integration leads to the known result for **density of states in Minkowski space**

$$n(E) = \frac{L^3 E^3}{6\pi^2}$$

used to derive all thermodynamical properties of a free massless field.

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Density of states: 3+1 Minkowski vs. Rindler

$$dn_M = \frac{L^3}{2\pi^2} k^2 dk \qquad \text{vs} \qquad dn_R = \frac{L^2\, k_\perp\, k_\xi}{2\pi^2}\, dk_\perp d\xi$$

Area law from flat space

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Using **Rindler dispersion relation** $E^2 = k^2 + e^{2a\xi} k_\perp^2$ we integrate dn_R over $k_\perp$

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With a **"brick wall"** at ξ_{min} we obtain **density of states** as function of energy

$$n_R(E) = \frac{E^3 L^2}{6\pi^2} \int_{\xi_{min}}^{\log(aR)/a} d\xi\, e^{-2a\xi} = \frac{E^3 L^2}{12\pi^2} \frac{1}{a} \left[e^{-2a\xi_{min}} - \frac{1}{(aR)^2} \right].$$

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From $\log Q = \beta \int_0^\infty dE \frac{n(E)}{e^{\beta E} - 1}$ calculate **entropy**, which scales as L^2!

$$S = \frac{\pi^2}{45} \frac{L^2}{a\beta^3} \left[e^{-2a\xi_{min}} - \frac{1}{(aR)^2} \right] = S_{wall} + \mathrm{IR\ \ box\ \ contribution}\,,$$

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With a **"brick wall"** at ξ_{min} we obtain **density of states** as function of energy

$$n_R(E) = \frac{E^3\, L^2}{6\pi^2} \int_{\xi_{min}}^{\log(aR)/a} d\xi\, e^{-2a\xi} = \frac{E^3\, L^2}{12\pi^2}\frac{1}{a}\left[e^{-2a\xi_{min}} - \frac{1}{(aR)^2}\right] .$$

From $\log Q = \beta \int_0^\infty dE \frac{n(E)}{e^{\beta E}-1}$ calculate **entropy**, which scales as L^2!

$$S = \frac{\pi^2}{45}\frac{L^2}{a\beta^3}\left[e^{-2a\xi_{min}} - \frac{1}{(aR)^2}\right] = S_{wall} + \text{IR box contribution}\, ,$$

For $\beta \sim 1/T_U \sim 2\pi/a$, ξ_{min} can be fixed $\Rightarrow$ **Bekenstein-Hawking entropy density**

$$\boxed{\sigma_{wall} = S_{wall}/L^2 = \frac{1}{4L_p^2}}$$

The covariant recipe for counting states

State counting can be recast in a covariant framework by considering **state space of a massless relativistic field**

$$dn \sim \underbrace{2E\, dt\, dx^3\, \delta(t)}_{\textit{covariant space volume}} \quad \times \quad \overbrace{d^4p\, \delta(\mathcal{C})\, \theta(E)}^{\textit{covariant momentum-space volume}}$$

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In Minkwoski space, integrating over a spatial volume $V = L^3$

$$n_M(E) = \frac{V}{(2\pi)^3} \int_E d^4p\, 2p_0\, \delta(p^2)\, \theta(p_0) = \frac{L^3 E^3}{6\pi^2}$$

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For a **Rindler field**

$$n_R(E) = \frac{V_\perp}{(2\pi)^3} \int_{\mathbb{R}} d\xi \int dp_\eta dp_\xi dp_\perp^2\, 2p_\eta e^{-2a\xi}\, \delta(\mathcal{C})\theta(p_\eta) = \frac{V_\perp}{(2\pi)^3} \frac{4\pi}{3} E^3 \int_{-\infty}^{\infty} d\xi\, e^{-2a\xi}$$

The twisted Weyl-Poincaré algebra

The Weyl-Poincaré algebra $\mathfrak{pw}(3,1)$ in $3+1$ dimensions

$$[P_\mu, P_\nu] = 0\ , \quad [P_\mu, M_{\rho\nu}] = i\left(\eta_{\mu\rho} P_\nu - \eta_{\mu\nu} P_\rho\right)$$

$$\left[M_{\mu\nu}, M_{\rho\sigma}\right] = i\left(\eta_{\mu\sigma} M_{\nu\rho} - \eta_{\mu\sigma} M_{\nu\rho} + \eta_{\nu\rho} M_{\mu\rho} - \eta_{\nu\rho} M_{\mu\sigma}\right)$$

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Consider deformation by **Jordanian twist** (Aschieri, Borowiec and Pachol, JHEP **1710**, 152 (2017))

$$W^{\mathcal{F}} = \bar{f}^\alpha(W)\bar{f}_\alpha \,, \quad W \in \mathfrak{pw}(3,1)$$

where $\mathcal{F} = f^\alpha \otimes f_\alpha = exp(-iD \otimes \sigma)\,,\ \sigma = \log\left(1 + \ell P_0\right)$ and $\mathcal{F}^{-1} = \bar{f}^\alpha \otimes \bar{f}_\alpha$

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The resulting *twisted generators*

$$P^{\mathcal{F}}_\mu = \frac{P_\mu}{1 + \ell P_0} \, , \quad M^{\mathcal{F}}_{\mu\nu} = M_{\mu\nu} \, , \quad D^{\mathcal{F}} = D.$$

very similar to the “non-linear” redefinition of translation generators used by Magueijo and Smolin in their early DSR model (Phys. Rev. Lett. **88**, 190403 (2002))

The twisted Weyl-Poincaré algebra (continued)

In terms of the *twisted commutator*

$$[W^{\mathcal{F}}, V^{\mathcal{F}}]_{\mathcal{F}} = W^{\mathcal{F}} V^{\mathcal{F}} - (\bar{R}^{\alpha}(V))^{\mathcal{F}} (\bar{R}_{\alpha}(W))^{\mathcal{F}}.$$

the twisted generators obey an **undeformed** Weyl-Poincaré algebra.

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This translates in the following **deformed commutators**

$$[M^{\mathcal{F}}_{\mu\nu}, P^{\mathcal{F}}_{\rho}] = i\big(\eta_{\rho\nu} P^{\mathcal{F}}_{\mu} - \eta_{\rho\mu} P^{\mathcal{F}}_{\nu}\big) - i\ell \delta_{\mu 0}\delta_{\nu i} P^{\mathcal{F}}_{\rho} P^{\mathcal{F}}_{i}$$
$$[D^{\mathcal{F}}, P^{\mathcal{F}}_{\mu}] = i P^{\mathcal{F}}_{\mu} - i\ell P^{\mathcal{F}}_{\mu} P^{\mathcal{F}}_{0}$$

while all other commutators remain undeformed.

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The mass Casimir $\mathcal{C} = P_{\mu} P^{\mu}$ in terms of the twisted translation generators $P^{\mathcal{F}}_{\mu}$ becomes

$$\mathcal{C}^{\mathcal{F}} = \frac{(P_{\mu} P^{\mu})^{\mathcal{F}}}{(1 - \ell P^{\mathcal{F}}_{0})^2}.$$

At the algebraic level this is all we need to go and play the "DSR game"

DSR finite boosts

Twisted DSR finite boosts in the 1-direction

From the deformed algebra we have

$$\frac{d\omega}{d\phi} = -i[N_1, \omega] = k_1(1 - \ell\omega) \qquad \omega(\phi) = \frac{\omega^0 \cosh\phi + k_1^0 \sinh\phi}{A}$$

$$\frac{dk_1}{d\phi} = -i[N_1, k_1] = (\omega - \ell k_1 k^1) \qquad \Longrightarrow \qquad k_1(\phi) = \frac{\omega^0 \sinh\phi + k_1^0 \cosh\phi}{A}$$

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where $\boxed{A = 1 - \ell\omega^0 + \ell\omega^0 \cosh\phi + \ell k_1^0 \sinh\phi}$

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Boosts **saturate at the Planck scale!**

$$\lim_{\phi\to\infty} \omega(\phi) = \frac{1}{\ell}\,, \quad \lim_{\phi\to\infty} k_1 = \frac{1}{\ell}\,, \quad \lim_{\phi\to\infty} k_i = 0$$

Twisted dilations

The same procedure can be used to derive the **twisted dilation transformation**

$$\frac{d\omega}{d\delta} = -i[D,\omega] = \omega(1-\ell\omega) \qquad \Longrightarrow \qquad \omega(\delta) = \frac{\omega^0}{\omega^0\ell + (1-\omega^0\ell)e^{-\delta}}$$

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For $\delta \to -\infty$ both energy and momentum vanish, as in the undeformed case

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For $\delta \to \infty$ **dilation transformations saturate at the Planck scale!**

$$\lim_{\delta\to\infty} \omega(\delta) = \frac{1}{\ell} \qquad \lim_{\delta\to\infty} k_i(\delta) = \frac{k_i^0}{\ell\omega^0}$$

Deformed Rindler modes and finite blueshift

Rindler translation generators associated to accelerated observers in the 1-direction

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$$P^{\mathcal{F}}_{i} = \frac{P^{\mathcal{F}}_i}{1 - \ell P^{\mathcal{F}}_0 + \ell P^{\mathcal{F}}_0 \cosh a\eta + \ell P^{\mathcal{F}}_1 \sinh a\eta}\,, \; i = 2, 3.$$

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Finite blueshift at the accelerated horizon $\xi \to -\infty$!

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Finite blueshift at the accelerated horizon $\xi \to -\infty$!
Could this act as a "covariant brick wall"?

Counting modes in twisted Poincaré

Warm up: mode counting for a field in twisted Poincaré

$$n(E) = \frac{V}{(2\pi)^3} \int_E d\mu(p)\, 2p_0\, \delta(\mathcal{C})\, \theta(p_0)\,,$$

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To LIV or not to LIV? (Gubitosi and Magueijo, Class. Quant. Grav. 33, no. 11, 115021 (2016))

Breaks Lorentz

$$d\mu(p)_{\mathrm{LIV}} = d^4 p$$

Covariant under deformed boosts

$$d\mu(p)_{\mathrm{C}} = \frac{d^4 p}{(1-\ell p_0)^5}$$

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Breaks Lorentz ← $d\mu(p)_{\mathrm{LIV}} = d^4p$

Covariant under deformed boosts ← $d\mu(p)_{\mathrm{C}} = \dfrac{d^4p}{(1-\ell p_0)^5}$

The resulting density of states are

$$n(E)_{\mathrm{LIV}} = \frac{2V}{(2\pi)^2}\left[\frac{E^3}{3} - \frac{\ell E^4}{2} + \frac{\ell^2 E^5}{5}\right]$$

$$n(E)_{\mathrm{C}} = \frac{V}{(2\pi)^2}\frac{1}{\ell^3}\left[\frac{\ell E(3\ell E - 2)}{(1-\ell E)^2} - 2\log(1-\ell E)\right].$$

Finte density of states fo LIV measure

Boosts saturate at $1/\ell$, **maximal energy**, what about density of states?

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$$\lim_{E \to 1/\ell} n(E)_{\mathrm{C}} = \infty$$

However using the LIV measure

$$\lim_{E \to 1/\ell} n(E)_{\mathrm{LIV}} = \frac{V}{(2\pi)^2} \frac{1}{15\ell^3}\,,$$

we have a **finite number of states all the way up to the Planck scale**

Deformed Rindler: brick-wall from twist?

Look at **twisted generalization** of

$$n(E) = \frac{V_\perp}{(2\pi)^3} \int_{\mathbb{R}} d\xi \int dp_\eta dp_\xi dp_\perp^2 \, 2p_\eta e^{-2a\xi} \, \delta(\mathcal{C})\theta(p_\eta) \, .$$

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Sparing you the gruesome details the **final result** is

$$n(E) = \frac{V_\perp}{(2\pi)^3} \int_{\mathbb{R}} d\xi \int d\mu(p_\eta, p_\xi, p_\perp) \, \frac{p_\eta (1-\ell p_\eta)^2 e^{2a\xi} \delta(p_\eta - \omega_p)}{(\ell p_\eta + (1-\ell p_\eta) e^{a\xi})(p_\eta e^{a\xi} + \ell p_\xi^2 (1 - e^{a\xi}))} \, \theta(p_\eta)$$

where $\omega_p =$ on-shell energy obtained from **deformed Rindler dispersion relation**

$$\mathcal{C}^{\mathcal{F}} = \frac{e^{-2a\xi}}{(1-\ell P_\eta^{\mathcal{F}})^2} \left[-(P_\eta^{\mathcal{F}})^2 + (P_\xi^{\mathcal{F}})^2 + (P_\perp^{\mathcal{F}})^2 (P_\eta^{\mathcal{F}} \ell + (1-\ell P_\eta^{\mathcal{F}}) e^{a\xi})^2 \right]$$

Brick-wall from twist? Only if LIV!

Calculate density of states in fully **covariant picture**

$$n(E)_{\mathrm{C}} = \frac{V_\perp}{(2\pi)^2} \frac{e^{-2a\xi_{min}}}{a} \left[\frac{E^3}{3} - \frac{\ell E^4}{2} + \frac{\ell^2 E^5}{5} \right],$$

still need a "brick-wall" regulator ξ_{min} ...

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still need a "brick-wall" regulator ξ_{min} ...

If we LIV we get a **finite density of states!**

$$n(E)_{\mathrm{LIV}} = -\frac{V_\perp}{(2\pi)^2} \frac{1}{6a} \frac{1}{\ell^3} \log(1 - \ell E),$$

a bitter win...

Bekenstein-Hawking entropy from "twisted brick wall"

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Evaluate the entropy associated to $n(E)_{\mathrm{LIV}}$

Consider the logarithm of the **deformed partition function**

$$\log Q = \beta \int_0^{1/\ell} dE \, \frac{n(E)}{e^{\frac{\beta E}{1-\ell E}} - 1}$$

(deformed B-E distribution natural choice determined by the twist map $P_\mu^{\mathcal{F}} = \frac{P_\mu}{1+\ell P_0}$)

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Setting $\beta = \frac{1}{T_U} = \frac{2\pi}{a}$ we obtain

$$S_{\mathrm{LIV}} = \frac{V_\perp}{144\pi\ell^2} - \frac{V_\perp a}{\ell} \frac{\zeta(3)}{32\pi^4} + \frac{V_\perp a^2}{2160\pi} + \mathcal{O}(\ell)$$

at leading order, with $\ell = \frac{L_P}{6\sqrt{\pi}}$ leads to the **Bekenstein-Hawking entropy**...

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