The dark matter distribution around the super-massive BH and its impact on the LIGO event rate

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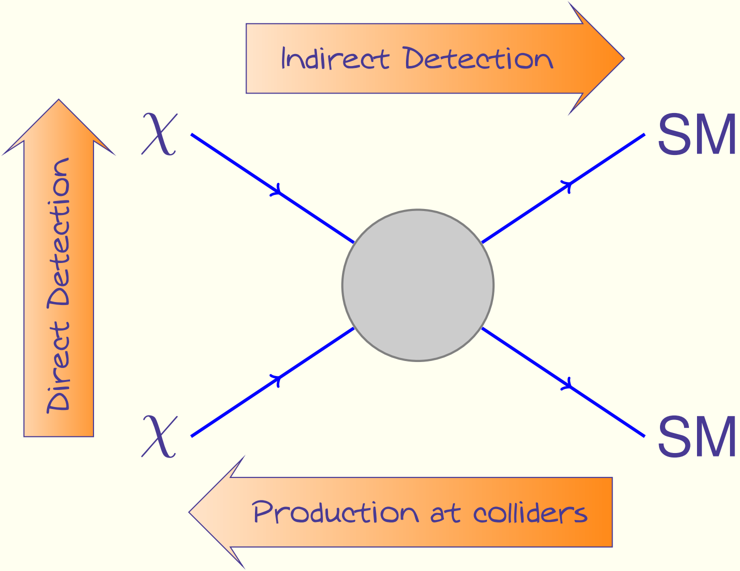
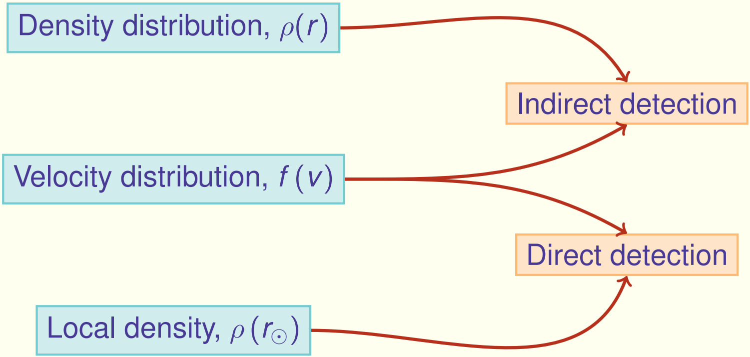
Università degli studi Roma Tre. May 28, 2018.



Indirect Detection
Direct Detection
Production at colliders
χ
χ
SM
SM

Astrophysical inputs



Outline

The role of the phase-space density on the IDMD

Recent results concerning dwarf spheroidals

IDMD and spikes around black holes

PBH as dark matter candidates

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- The role of the phase-space density on the IDMD
  - Recent results concerning dwarf spheroidals

- IDMD and spikes around black holes

- PBH as dark matter candidates

Velocity dependent cross-sections

If the amplitude for the process

$$\chi\,\chi \to SM\;SM$$

does not have singularities in momentum space, then the usual partial wave expansion gives:

$$\sigma v_{\rm rel} = a + b v_{\rm rel}^2 + \ldots .$$

If a is suppressed, the velocity dependence cannot be ignored.

Sommerfeld enhancement

In the Born approximation, the amplitude is proportional to the Fourier transform of the coordinate space matrix elements. As long as these fall off sufficiently rapidly at large separations, the amplitude will be analytic at zero momenta. However, if the scattering involves long-range forces the partial wave expansion no longer holds. Instead it typically gets multiplied by

$$S(v) = \frac{\pi \alpha_\chi}{v}.$$

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Additional sources of non-analyticity in generic particle physics models arise due to the presence of resonances, ...

The dark matter annihilation rate

FF & D. Hunter, JCAP 09 (2013) 005

The DM pair annihilation occurs with a probability per unit time

$$\mathrm{d}\Gamma_{2\chi} = \mathrm{d}\sigma \times \Phi_{2\chi},$$

where $\Phi_{2\chi}$ is the flux of either initial particle at the position of the other one

$$\Phi_{2\chi} = u_\chi \, n_\chi = u_\chi \frac{\rho_\chi}{m_\chi},$$

and the *relative velocity*

$$u_\chi = \frac{\sqrt{(p_1 \cdot p_2)^2 - m_\chi^4}}{E_1 E_2}$$

reduces to $u_\chi = 2\,|\boldsymbol{v}| = v_{\mathrm{rel}}$ in the center of mass frame when the two annihilating particles are identical.

The flux of photons that are generated in a volume element $\mathrm{d}V = l^2 \mathrm{d}l \mathrm{d}\Omega$ at the position $\boldsymbol{x}$ in the halo, containing $n_\chi(\boldsymbol{x})\mathrm{d}V$ dark matter particles, is:

$$\mathrm{d}\Phi_\gamma = \mathrm{d}l \, \frac{N_\gamma}{8\pi m_\chi^2} \langle \sigma v_{\mathrm{rel}} \rangle_{\boldsymbol{x}} \rho_\chi^2.$$

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If $\langle \sigma v_{\mathrm{rel}} \rangle_{\boldsymbol{x}}$ does not depend on the location in the halo, we obtain the usual expression:

$$\Phi_\gamma = \frac{N_\gamma \langle \sigma v \rangle}{8\pi m_\chi^2} \underbrace{\int_{los} \mathrm{d}l \rho_\chi^2}_{J} .$$

The Standard Halo

A. Drukier, K. Freese & D. Spergel, PRD 33 (1986) 3495

The first discussions of ID assumed that the DM distribution followed the profile:

$$\rho_{SIS}(r) = \frac{\sigma^2}{2\pi G r^2},$$

with a velocity distribution of the Maxwell-Boltzmann type

$$P_{\boldsymbol{x}}^{MB}(\boldsymbol{v}) = \frac{1}{\left(2\pi\sigma^2\right)^{3/2}} \exp\left(\frac{-v^2}{2\sigma^2}\right),$$

which does not depend on the location in the halo.
Furthermore, the *relative* velocity distribution is also MB, and

$$\langle \sigma v \rangle = a + 6b\sigma^2 + \dots$$

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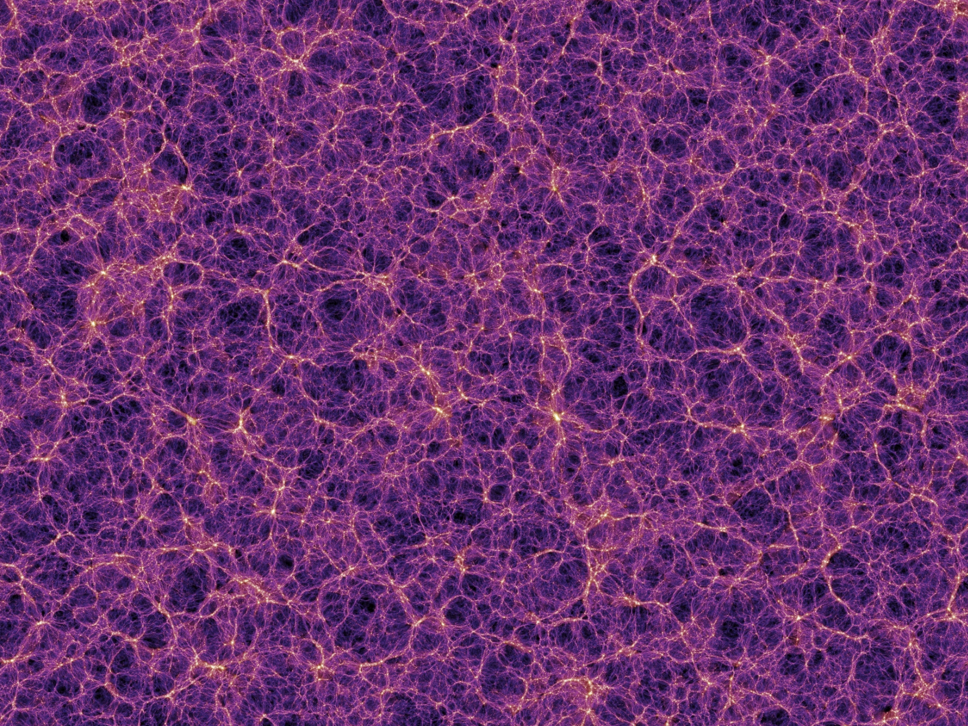
Over the years, the density profile was modified to better fit the results of N-body simulations (NFW, ...) and the J-factor was accordingly updated. *But, the MB velocity distribution continued to be used.*



The first N-body simulation

FIG. 4a FIG. 4b

FIG. 4a.—Tidal deformations corresponding to parabolic motions, clockwise rotations, and a distance of closest approach equal to the diameters of the nebulae. The spiral arms point in the direction of the rotation.

FIG. 4b.—Same as above, with the exception of counterclockwise rotations. The spiral arms point in the direction opposite to the rotation.

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Fig. 4a Fig. 4b

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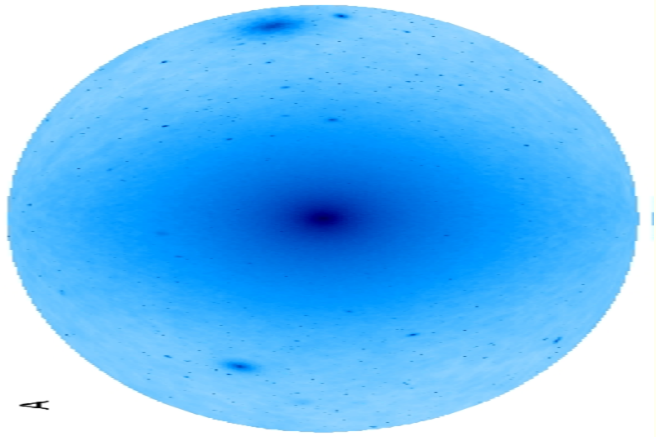
Fig. 4b.—Same as above, with the exception of counterclockwise rotations. The spiral arms point in the direction opposite to the rotation.

Holmberg in 1941 used light to mimic gravitational interactions. His analog computer calculation scales $\propto N$ (compare with $N \log N$ in modern hierarchical tree methods).

s-wave vs Sommerfeld in dwarfs



Lattanzi & Silk, PRD79 (2009) 083523

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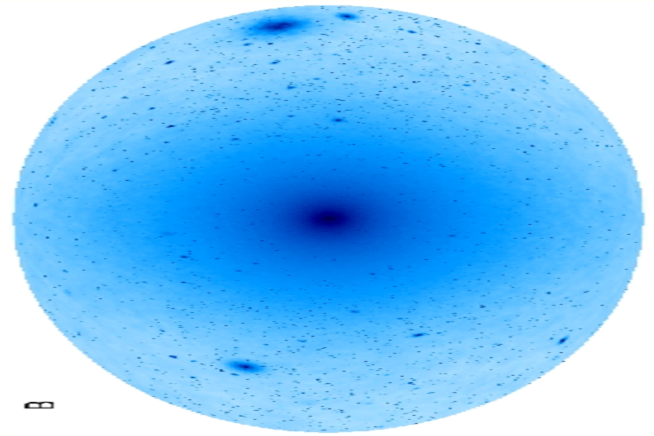
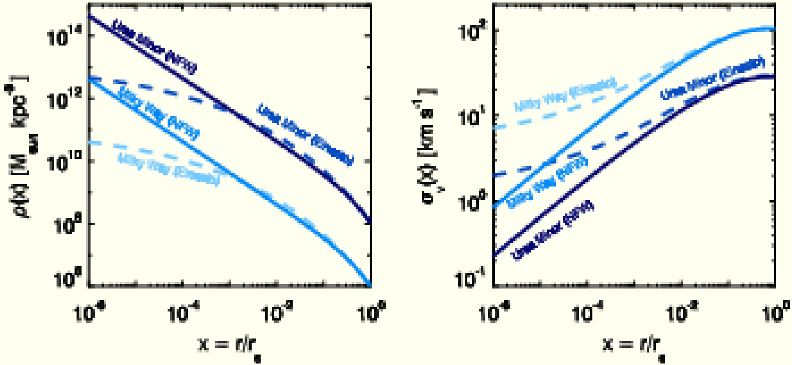


Lattanzi & Silk, PRD79 (2009) 083523

P-wave analysis

The position independent MB distribution is an *accident* of the SIS profile. For any other density profile we expect a position dependent velocity distribution, which is not of MB type.



Robertson & Zentner PRD 79 (2009) 083525

Consistent velocity distribution

For a spherical system confined by a known gravitational potential ψ we can find a unique isotropic distribution function using Eddington's result:

$$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2}\int_0^{\mathcal{E}} \frac{\mathrm{d}\Psi}{\sqrt{\mathcal{E}-\Psi}}\frac{\mathrm{d}^2\rho}{\mathrm{d}\Psi^2}.$$

- $\psi \equiv -\Phi$ is the relative gravitational potential.
- $\mathcal{E} = \psi - v^2/2$ is the relative energy.
- ρ is the known density distribution (e.g. from simulations).

Catena & Ullio, JCAP 05 (2012) 005

Example: NFW

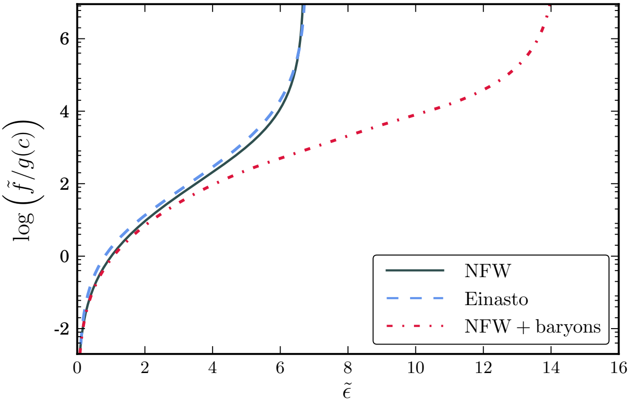
$$\rho_{\rm NFW} = \frac{\rho_0}{\frac{r}{a}\left(1+\frac{r}{a}\right)^2} = \frac{M_{\rm vir}}{4\pi r_{\rm vir}^3}\frac{1}{g_{\rm NFW}(c)} \times \underbrace{\frac{1}{cx(1+cx)^2}}_{\equiv\tilde{\rho}_{\rm NFW}},$$

where $x \equiv r/r_{vir}$. Solving Poison's equation one finds the associated gravitational potential:

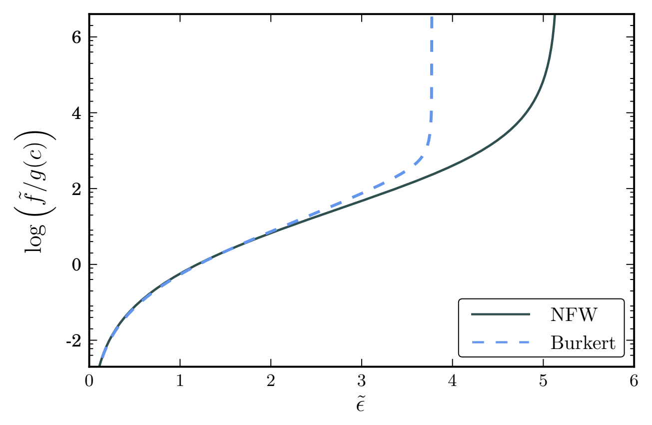
$$\tilde{\psi}_{\rm NFW} = \frac{\log(1+cx)}{c^3 g_{\rm NFW}(c)x}.$$

Since the potential has a maximum at $\tilde{\psi}(0)$, there is a maximum velocity of any bound particle.

- The contribution of the baryons in the disk and the bulge can be modelled adding $\tilde{\psi}_{\rm bulge}$ to the gravitational potential.

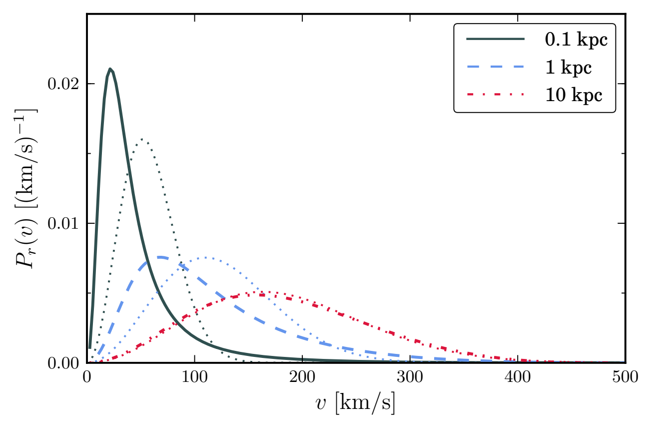


$\log\left(\tilde{f}/g(c)\right)$
6
4
2
0
-2
0
2
4
6
8
10
12
14
16
$\tilde{\epsilon}$
NFW
Einasto
NFW + baryons



$\log\left(\tilde{f}/g(c)\right)$
6
4
2
0
-2
0
1
2
3
4
5
6
$\tilde{\epsilon}$
NFW
Burkert

The galactic velocity distribution



At large distances it can be approximated by a MB shape. However, as we move towards the center, the distribution shifts to lower values of the velocity and is highly non-gaussian.

The relative velocity distribution

The yield from DM annihilations depends on the *relative* velocity distribution:

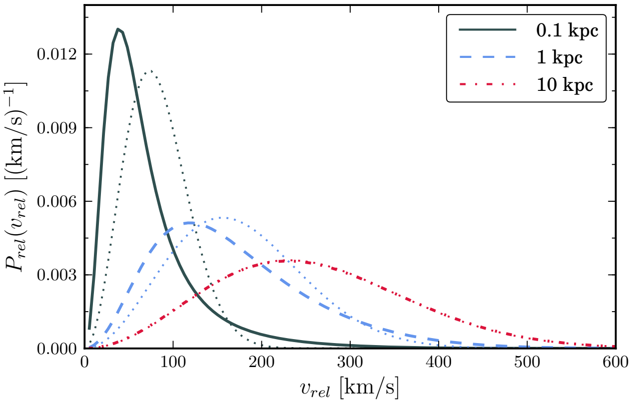
$$P_{\boldsymbol{x}}\left(\boldsymbol{v}_1\right) P_{\boldsymbol{x}}\left(\boldsymbol{v}_2\right) \mathrm{d}^3\boldsymbol{v}_1 \mathrm{d}^3\boldsymbol{v}_2 = P_{\boldsymbol{x}}\left(\boldsymbol{v}_{\mathrm{cm}} + \boldsymbol{v}_{\mathrm{rel}}/2\right) P_{\boldsymbol{x}}\left(\boldsymbol{v}_{\mathrm{cm}} - \boldsymbol{v}_{\mathrm{rel}}/2\right) \mathrm{d}^3\boldsymbol{v}_{\mathrm{cm}} \mathrm{d}^3\boldsymbol{v}$$

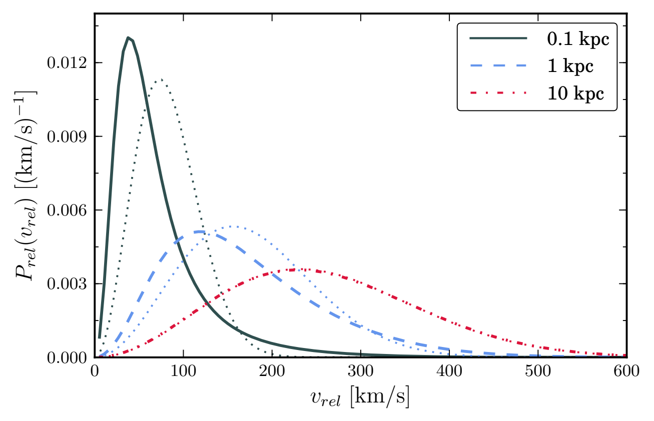
integrated over the center of mass velocity.

Moreover, since $\tilde{f} = \tilde{f}\left(v^2\right)$ at any point in the halo, the individual velocity distributions only depend on the combinations:

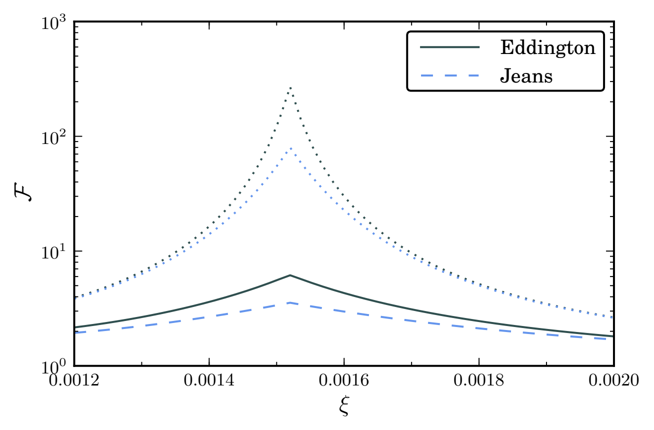
$$P_{\boldsymbol{x}}\left(v_{1,2}^2\right) = P_{\boldsymbol{x}}\left(v_{\mathrm{cm}}^2 + v_{\mathrm{rel}}^2/4 \pm v_{\mathrm{cm}} v_{\mathrm{rel}} z\right),$$

where z is the cosine of the angle between the relative and the center of mass velocities.


0.1 kpc
1 kpc
10 kpc
0.012
0.009
0.006
0.003
0.000
0
100
200
300
400
500
600
$P_{rel}(v_{rel})$ [(km/s)$^{-1}$]
v_{rel} [km/s]



After all this work we have all the ingredients to calculate fluxes with a consistent velocity distribution.



Annihilation fluxes can be significantly enhanced in comparison to the usual MB expectation. The presence of baryons, however, reduces the boost.

Dwarf spheroidals have large M/L ratios with little baryon content, which makes them optimal targets for indirect detection.

- The density profile can be constrained by observations of l.o.s. velocities of stars in the dwarf.
- One of the most stringent limits on the annihilation cross-section using Fermi data comes from combining information from the dwarfs with largest J-factor.
- Since there are almost no baryons, any boost will be preserved.

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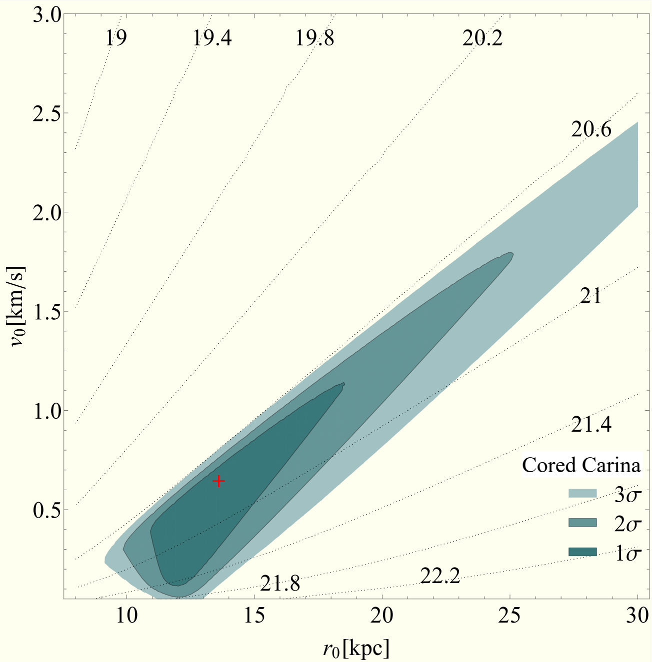
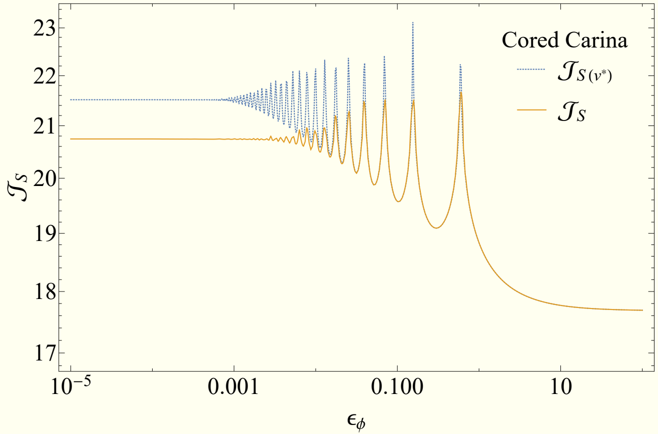
Recent studies have found large boost factors in several dwarfs if DM annihilation is Sommerfeld enhanced. The ordering in terms of J is not preserved.

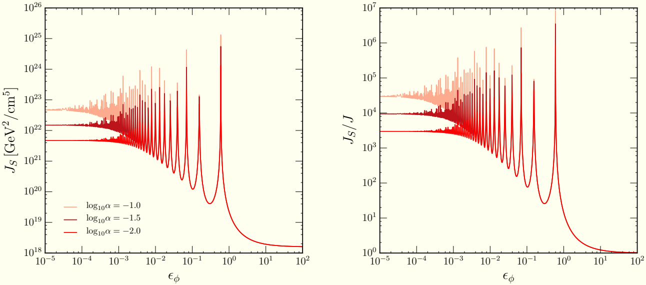
Body et al., PRD 95 (2017); Bergstrom, Catena, et al 1712.03188



3.0
2.5
2.0
1.5
1.0
0.5
v_0[km/s]
10
15
20
25
30
r_0[kpc]
19
19.4
19.8
20.2
20.6
21
21.4
21.8
22.2
Cored Carina
3σ
2σ
1σ



Cored Carina
$\mathcal{J}_{S(v^*)}$
$\mathcal{J}_S$
23
22
21
20
19
18
17
$\mathcal{J}_S$
10^{-5}
0.001
0.100
10
ϵ_ϕ



$J_S\,[\mathrm{GeV}^2/\mathrm{cm}^5]$
ϵ_ϕ
$\log_{10}\alpha = -1.0$
$\log_{10}\alpha = -1.5$
$\log_{10}\alpha = -2.0$
J_S/J

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- The role of the phase-space density on the IDMD
    - Recent results concerning dwarf spheroidals
- IDMD and spikes around black holes
- PBH as dark matter candidates

Super-massive black holes

Sadeghian, Will & FF, PRD 2013; FF, Medeiros & Will, PRD 2017

- Will focus on the super-massive BH at the center of the Galaxy.
- Similar effects will occur in the cores of AGNs, or in IMBHs.

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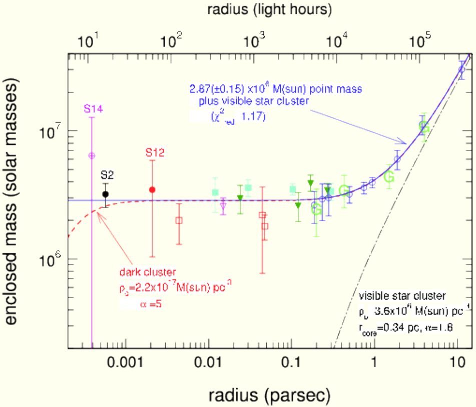

Is the growth adiabatic?

We will assume that a black hole of mass $4 \times 10^6 M_\odot$ grows adiabatically over $\sim 10^{10}$yr.

- Growth time

$$\frac{r_h}{\sigma} \leq \frac{m}{\dot{m}_{\rm Edd}}$$

$$r_h \approx \frac{Gm}{\sigma^2} \rightarrow t_{\rm dyn} \approx 10^4 {\rm yr} \leq t_{\rm Salpeter} \approx 5 \times 10^7 {\rm yr}$$

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- Dynamical time

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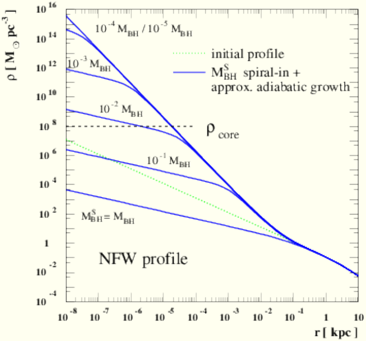
The central supermassive black hole

Caveats: Hierarchical mergers, initial BH seed off-center, kinetic heating of DM by stars, . . .



Bertone, Hooper & Silk, Phys. Rep. 2004

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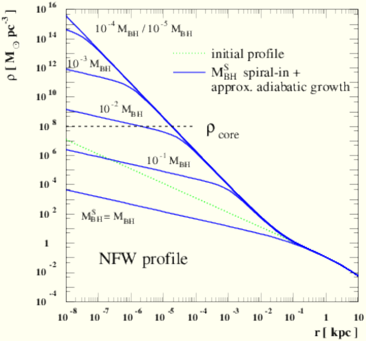
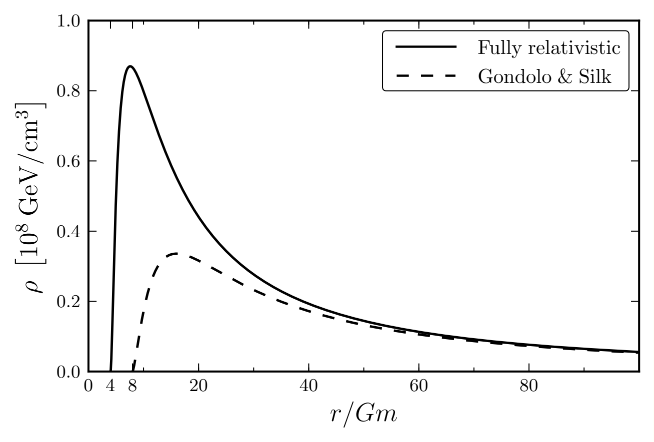


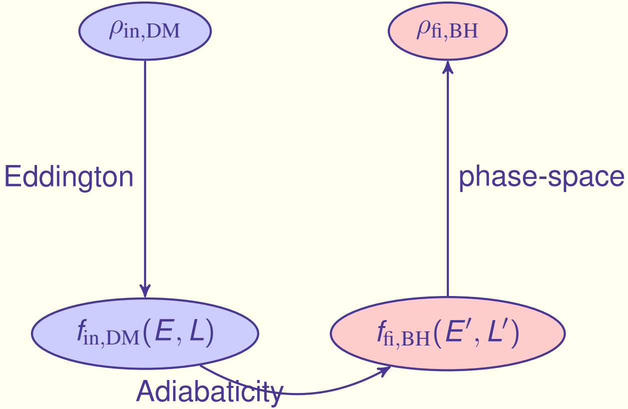
Bertone, Hooper & Silk, Phys. Rep. 2004

These processes are not universal!

The DM spike


1.0
0.8
0.6
0.4
0.2
0.0
ρ [10^8 GeV/cm^3]
0
4
8
20
40
60
80
r/Gm
Fully relativistic
Gondolo & Silk

Adiabatic growth of a BH



Adiabatic invariants

- Each particle in an initial DM distribution $f(E, L)$, will react to the change by the growth of the BH. However, *the adiabatic invariants remain fixed*:

$$I_r(E, L) = I_r(E', L') \qquad L = L' \qquad L_z = L'_z$$

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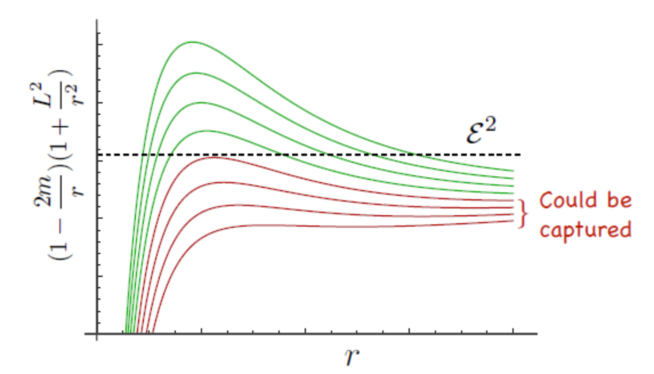
- The limits of integration are set by the requirements that the actions should be real and that the DM particle should be bound to the halo.
- NR: Take into account particles trapped inside the event horizon by modifying boundary conditions in an *ad hoc* manner: $L \geq 2cR_S$.

Relativistic analysis

The positivity of the radial action determines the boundary conditions, *including the effects of the horizon*.



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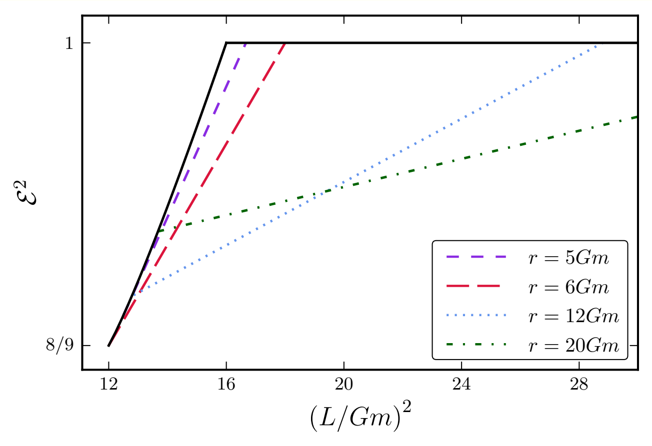
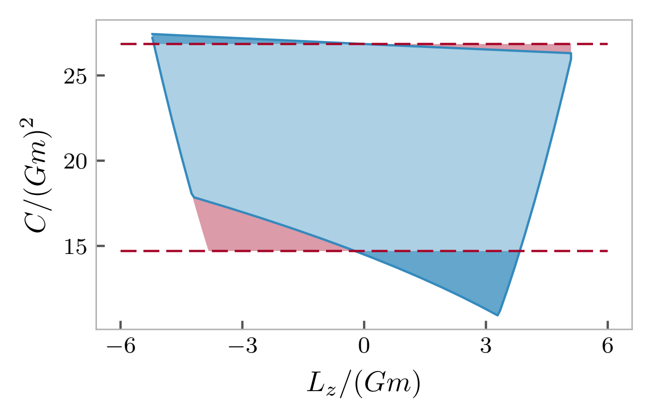


Adding spin



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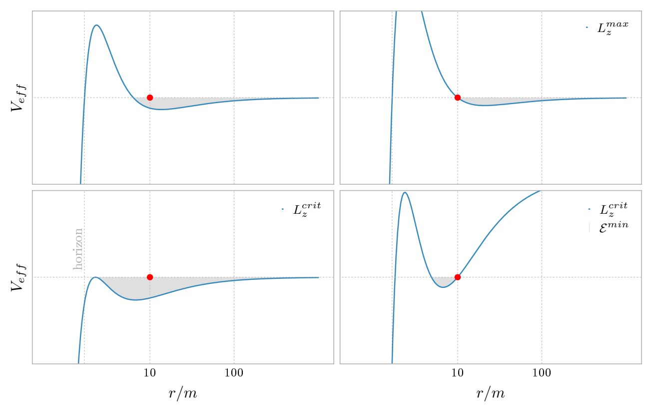
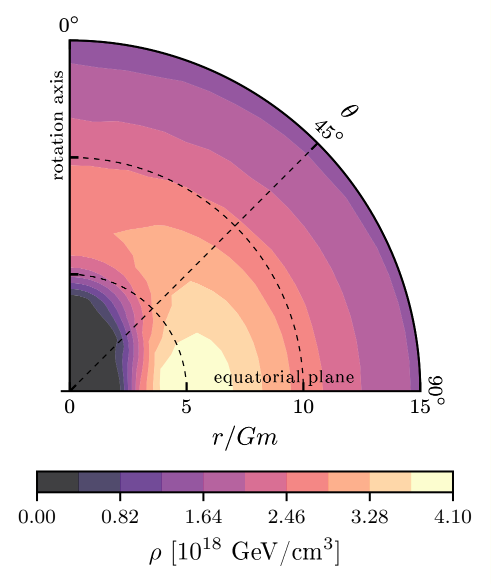


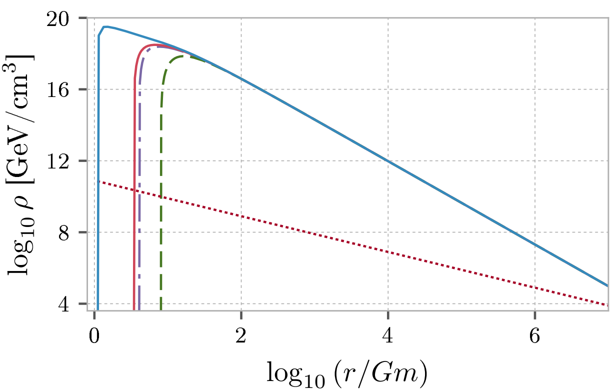
V_{eff}
L_z^{max}
L_z^{crit}
horizon
V_{eff}
L_z^{crit}
$\mathcal{E}^{min}$
10
100
r/m
10
100
r/m

DM density

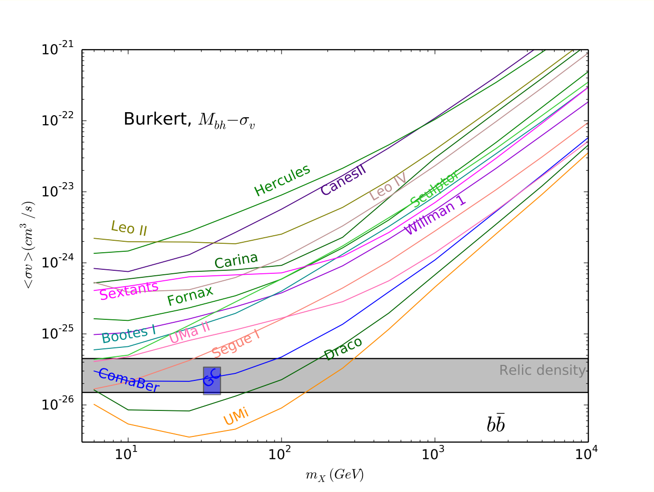


FF, A Medeiros & C Will, 1707.06302

Spike comparison

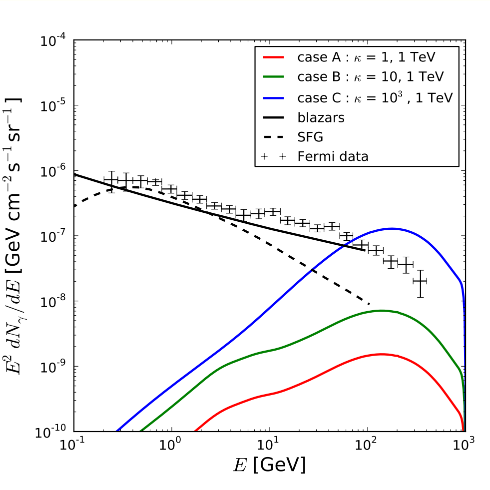


Limits from dwarf spheroidals



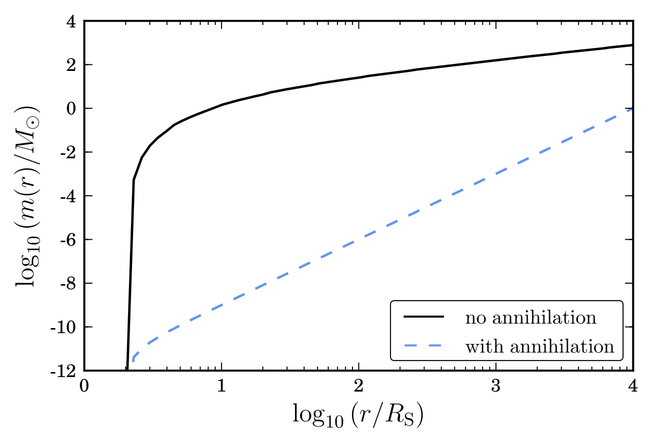
Gonzalez-Morales, Profumo & Queiroz, 1406.2424

The diffuse γ-ray background

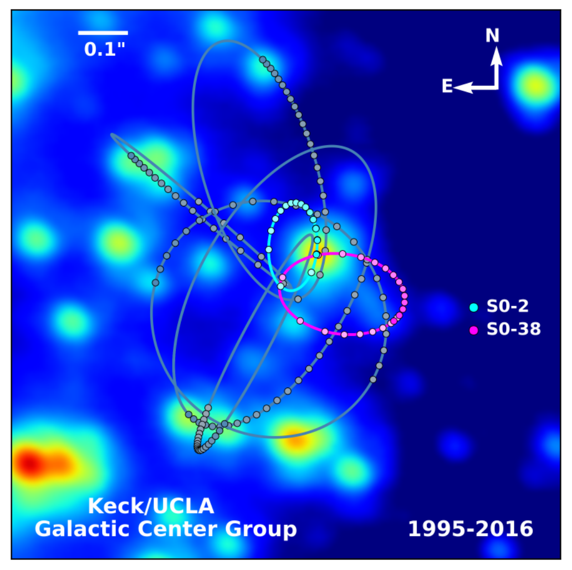


Belikov & Silk, 1312.0007

The gravitational potential

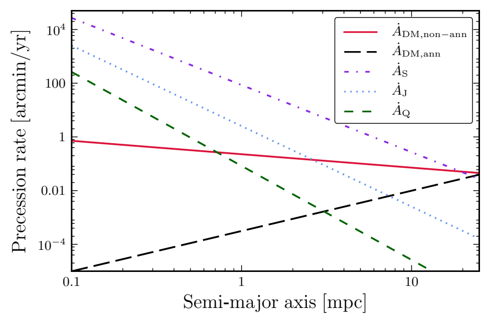


S stars



Genzel, Eisenhauer & Gillessen, RMP 82, 3221 (2013)

Tests of the *no-hair* theorem



$$Q_2 = -J^2/m$$

Future directions

- Improved observations with Einstein Telescope.
- Spikes in PBH
- Implications for LIGO

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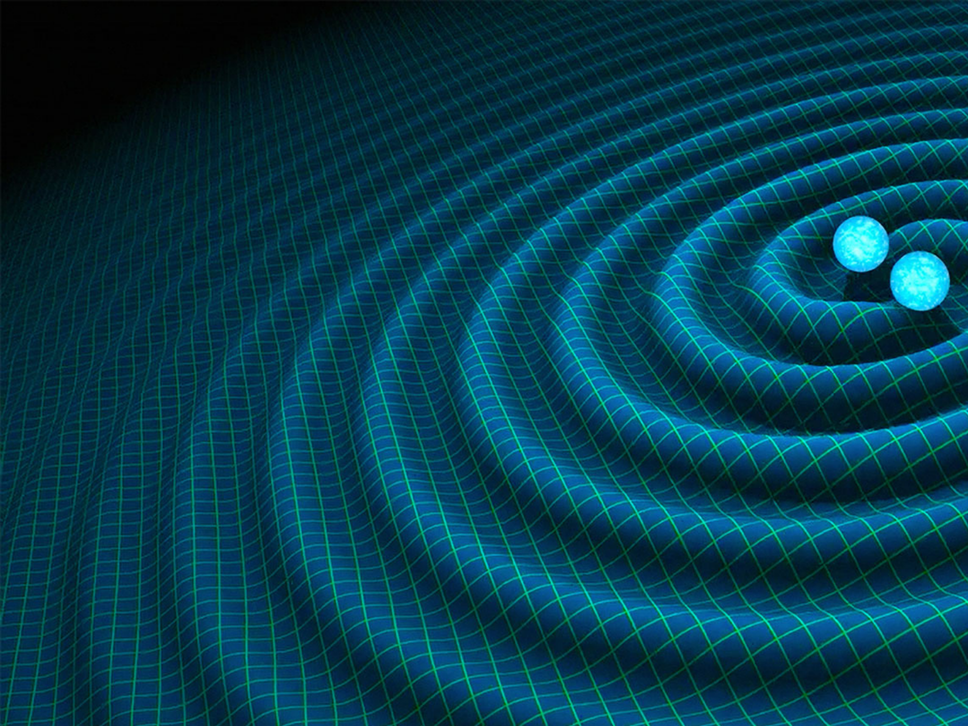
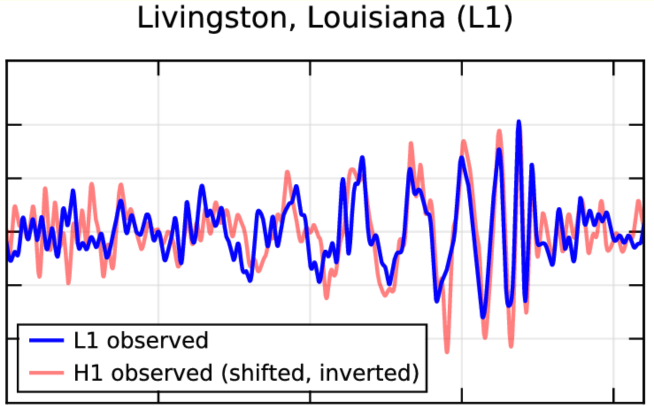
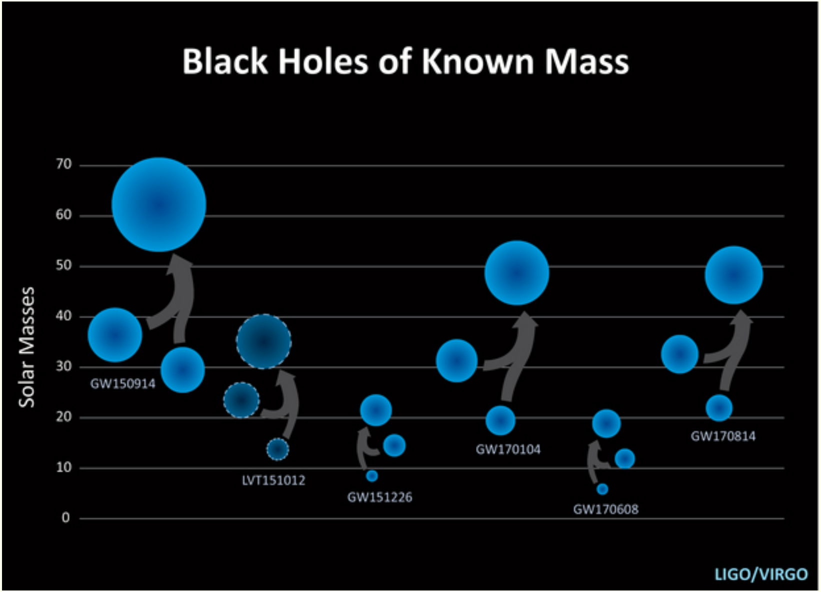
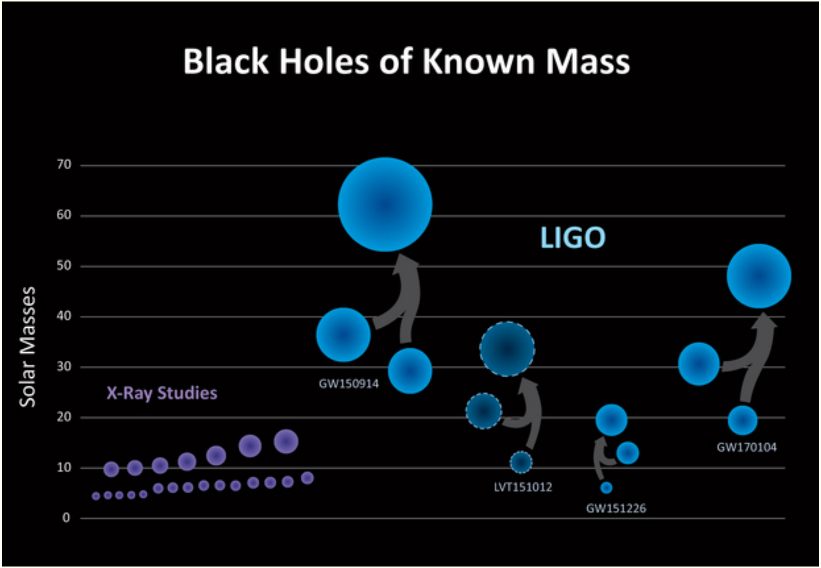
PBH as dark matter candidates




Livingston, Louisiana (L1)
L1 observed
H1 observed (shifted, inverted)



Black Holes of Known Mass
Solar Masses
70
60
50
40
30
20
10
0
GW150914
LVT151012
GW151226
GW170104
GW170608
GW170814
LIGO/VIRGO



Black Holes of Known Mass
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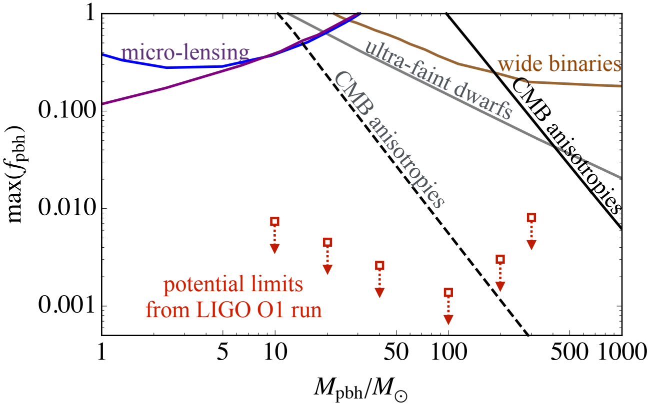
X-Ray Studies
GW150914
LVT151012
GW151226
GW170104

Could they be be primordial? Is DM made (partly) of PBHs?



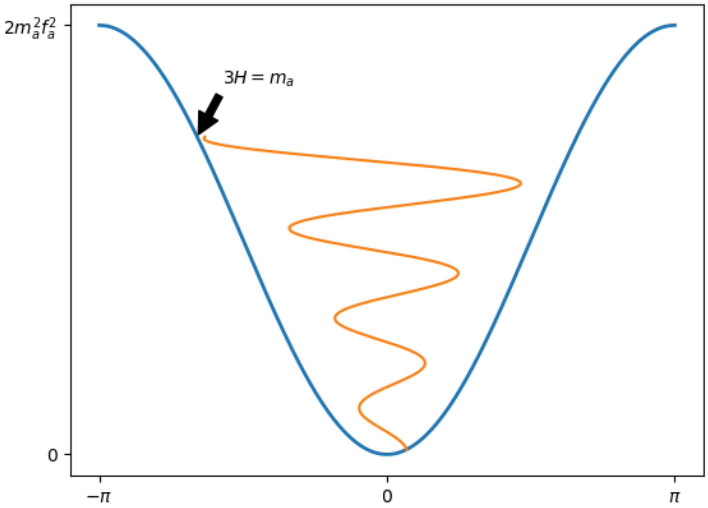
Ali-Haïmoud, Kovetz & Kamionkowski, 1709.06576

PBHs from axions?

- Most inflationary mechanisms for PBH formation require an ad hoc feature in the scalar potential.
- PBHs could also be formed during phase transitions in the early universe.

Could PBHs be related to axions?

Axion DM from misalignment



Axion DM from topological defects

In the post-inflationary scenario, two separate phase-transitions after reheating enable the creation of a system of hybrid string-wall defects. The decay of this network gives an additional contribution of mildly relativistic axions.

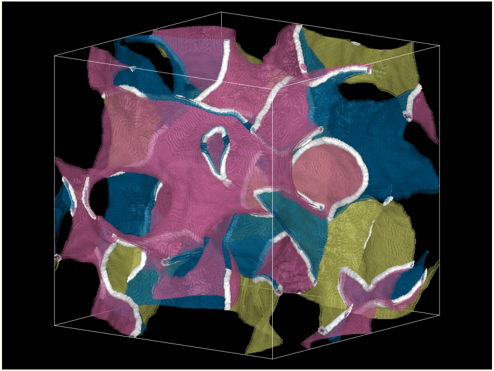
Axion DM from topological defects

In the post-inflationary scenario, two separate phase-transitions after reheating enable the creation of a system of hybrid string-wall defects. The decay of this network gives an additional contribution of mildly relativistic axions.

- Axionic strings form first at the PQ phase transition $T_{\mathrm{PQ}} \sim f_a$.
- Strings evolve entering the scaling regime, $\rho_s \sim \mu_s H^2$, and keep radiating axions.
- As the universe cools down and approaches T_{QCD}, gauge instantons generate a periodic potential for the axion. This potential leads to the formation of domain walls.

$N_{DW} > 1$

There are N_{DW} domain walls attached to every string, each one pulling in a different direction. The network can actually be stable, and dominate the universe.



T. Hiramatsu, *et al.*, JCAP 1301 (2013) 001

A fourth DM component?

The collapse of closed domain walls, which belong to the hybrid string-wall network can lead to the formation of PBHs.

T. Vachaspati, 1706.03868

- This mechanism does not rely on (nor complicate) the physics of inflation.
- GW astronomy can potentially probe the physics of axions.

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T. Vachaspati, 1706.03868

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It is crucial that the annihilation of the network proceeds slowly. The result is a universe with DM made of a mixture of BHs and a smooth component.

FF, Massó, Panico, Pujolàs & Rompineve, in progress

Pair formation

Most of the BH pairs that merge today form in the early universe, deep in the radiation era. Pairs form due to the chance proximity of PHB pairs and merge on a time-scale:

$$t_{merge} = \frac{3c^5}{170 G_N^3} \frac{a^4 (1 - e^2)^{7/2}}{M_{pbh}^3}$$

Several processes (torques due to other BHs, encounters with other BHs, DM spikes around PBHs, ...) influence the merger rate that is measured by LIGO.

Ali-Haïmoud, Kovetz & Kamionkowski, 1709.06576

Kavanagh, Gaggero & Bertone, 1805.09034

Pair formation in present day halos

Binary BHs can also form in present day halos from GW emission

$$\sigma = 1.37 \times 10^{-14} \left(\frac{M_{\rm PBH}}{30 M_\odot} \right)^2 \left(\frac{v_{\rm rel}}{200 {\rm km/s}} \right)^{-18/7} {\rm pc}^2.$$

These binaries are very tight and highly eccentric so that they coalesce within a very short timescale.

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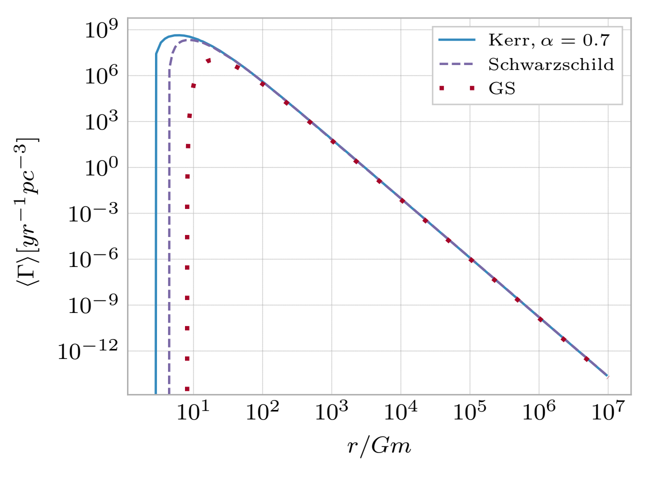
These binaries are very tight and highly eccentric so that they coalesce within a very short timescale. In principle this population gives a subdominant contribution to the LIGO observed events, but:

- The SMBH could greatly enhance the event rate.

  Nishikawa et al. 1708.08449

- The formation cross-section has a steep velocity dependence and phase-space details could matter.

  FF, Augusto Medeiros, in progress



Binaries formed in the early universe still provide the dominant contribution, but dynamical friction effects could alter the balance.

FF, Augusto Medeiros, in progress

Conclusions

- Taking into account the full phase-space structure of the DM distribution is crucial to obtain realistic indirect detection fluxes.
- The distribution close to the galactic center is dramatically altered by the presence of a SMBH that creates a spike. General relativistic calculations confirm early Newtonian estimates.
- The spike enhances the contribution of binary mergers formed in virialized structures to the observed gravitational wave events.
- Scenarios that predict the formation of PBHs generally feature mixed BH-smooth DM distributions with a rich phenomenology that we are just starting to survey.