# Quarkonium production in p+A collisions and nPDFs

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# COLD NUCLEAR MATTER EFFECTS

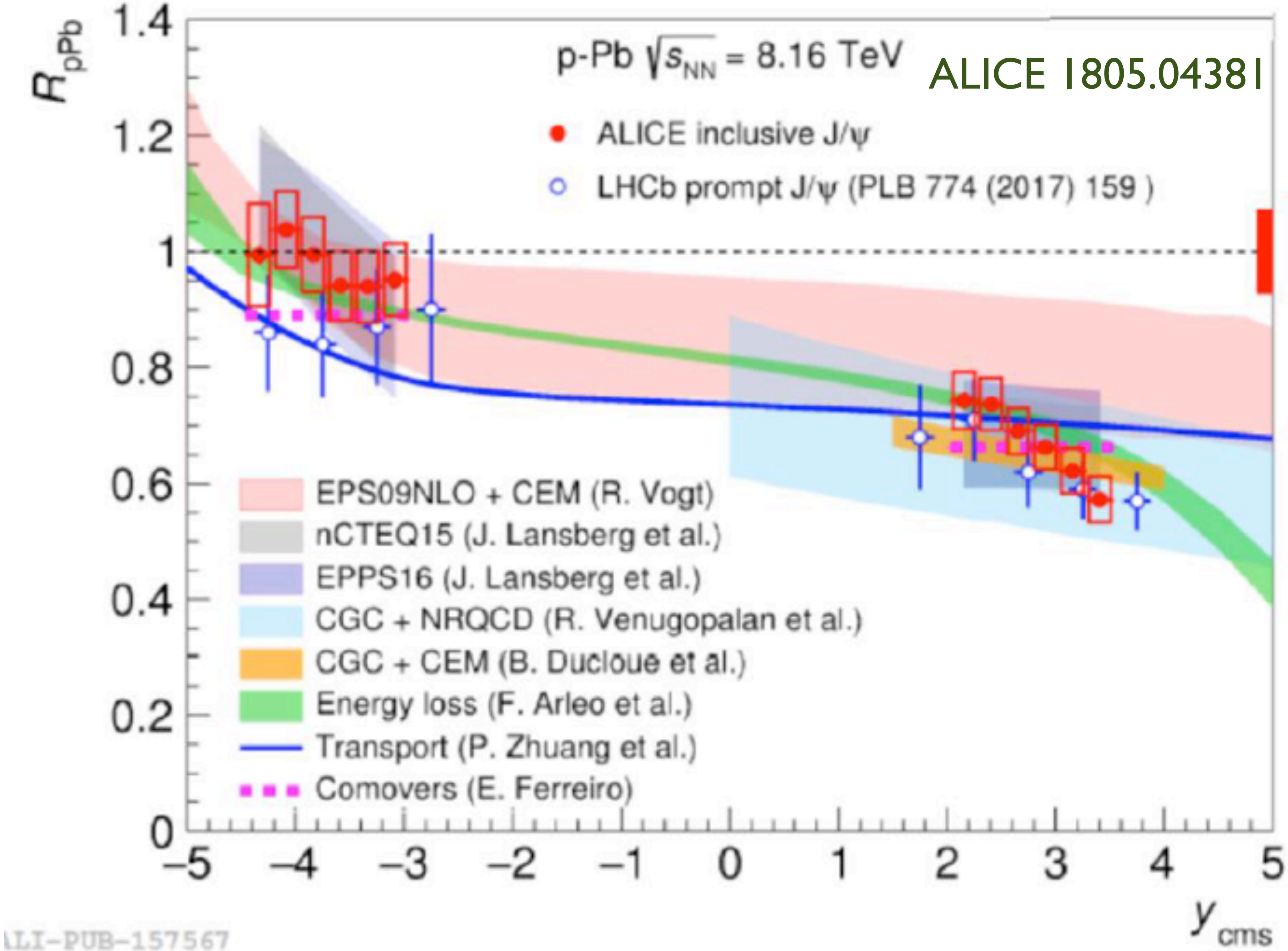
LPTHE LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

- **Initial state effects**
  - Modification of parton flux (e.g. shadowing) in nuclear PDF
  - Coherent or incoherent energy loss Arleo and Peigne '12; Sharma and Vitev '13
  - Colour filtering of intrinsic heavy-quark pair Brodsky and Hoyer '89
  - Saturation/small x/coherence effects Ducloué et al. '15; Ma et al.'15; Kharzeev et al. '09; ...
  - ...
- **Final state effects**
  - Coherent energy loss Arleo and Peigne '12
  - Break up in the nuclear matter: absorption effect Gerschel and Hufner '88; Vogt '99
  - Break up by comoving particles Ferreiro '15; Capella and Ferreiro '00'05; Gavin and Vogt '90
  - ...

*Cold nuclear matter effects are crucial to understand AA data*

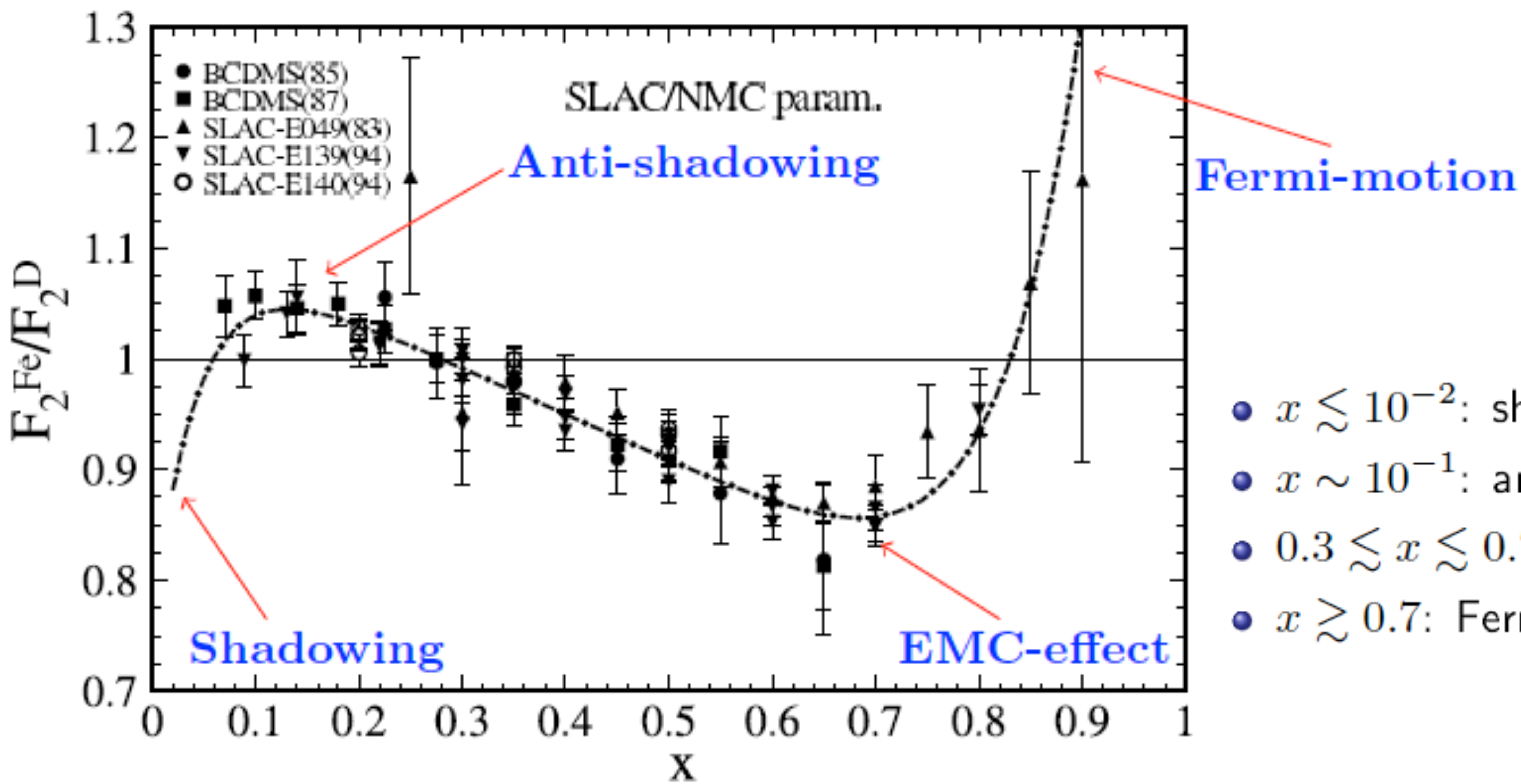
*Reference to disentangle genuine QGP effect in AA collisions*

# A MONEY PLOT




- **Challenging in differentiate them**
  - Importance of systematically improving the uncertainties

- **Cross-sections in nuclear collisions are modified**




- $x \lesssim 10^{-2}$: shadowing
- $x \sim 10^{-1}$: anti-shadowing
- $0.3 \lesssim x \lesssim 0.7$: EMC effect
- $x \gtrsim 0.7$: Fermi motion

*Such a modification can be translated into universal objects: nuclear PDFs (nPDFs)*

# A LESSON FOR PDF (20 YEARS AGO)

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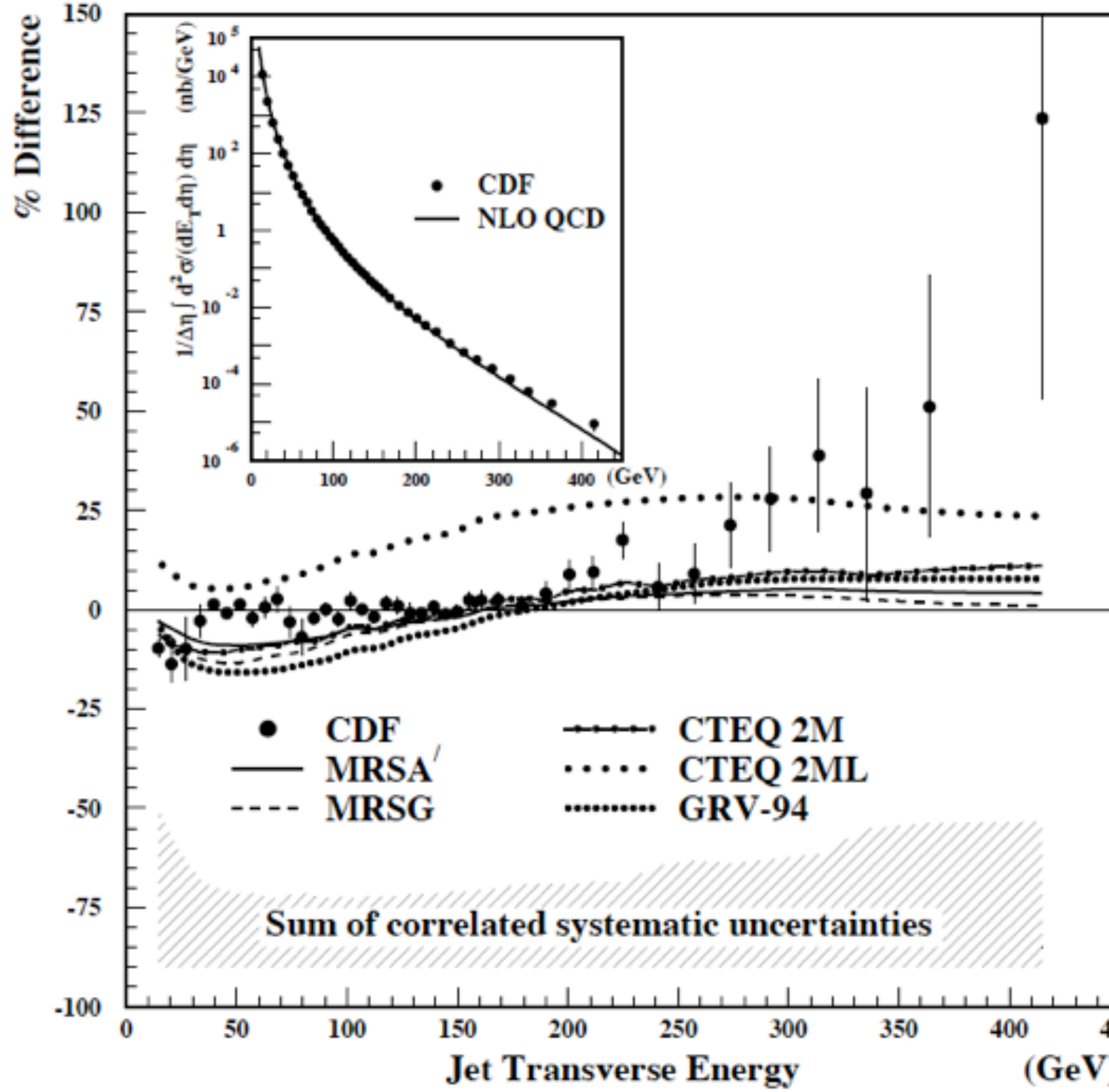
hep-ex/9601008

Inclusive jet cross section in $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV
(CDF Collaboration)

- High-pT excess in inclusive jet by CDF was initially triggering a lot of BSM studies, like quark compositeness.



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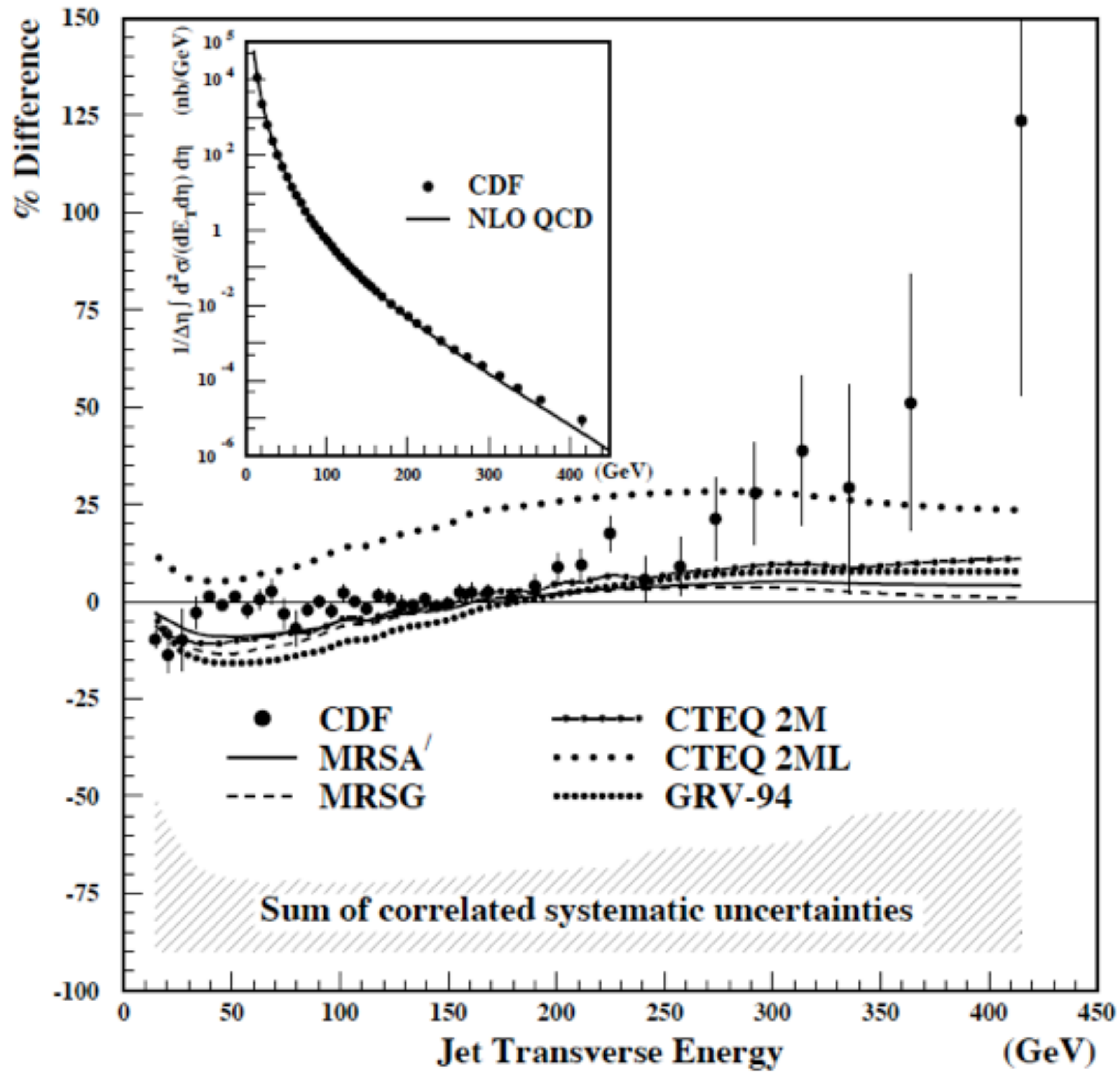

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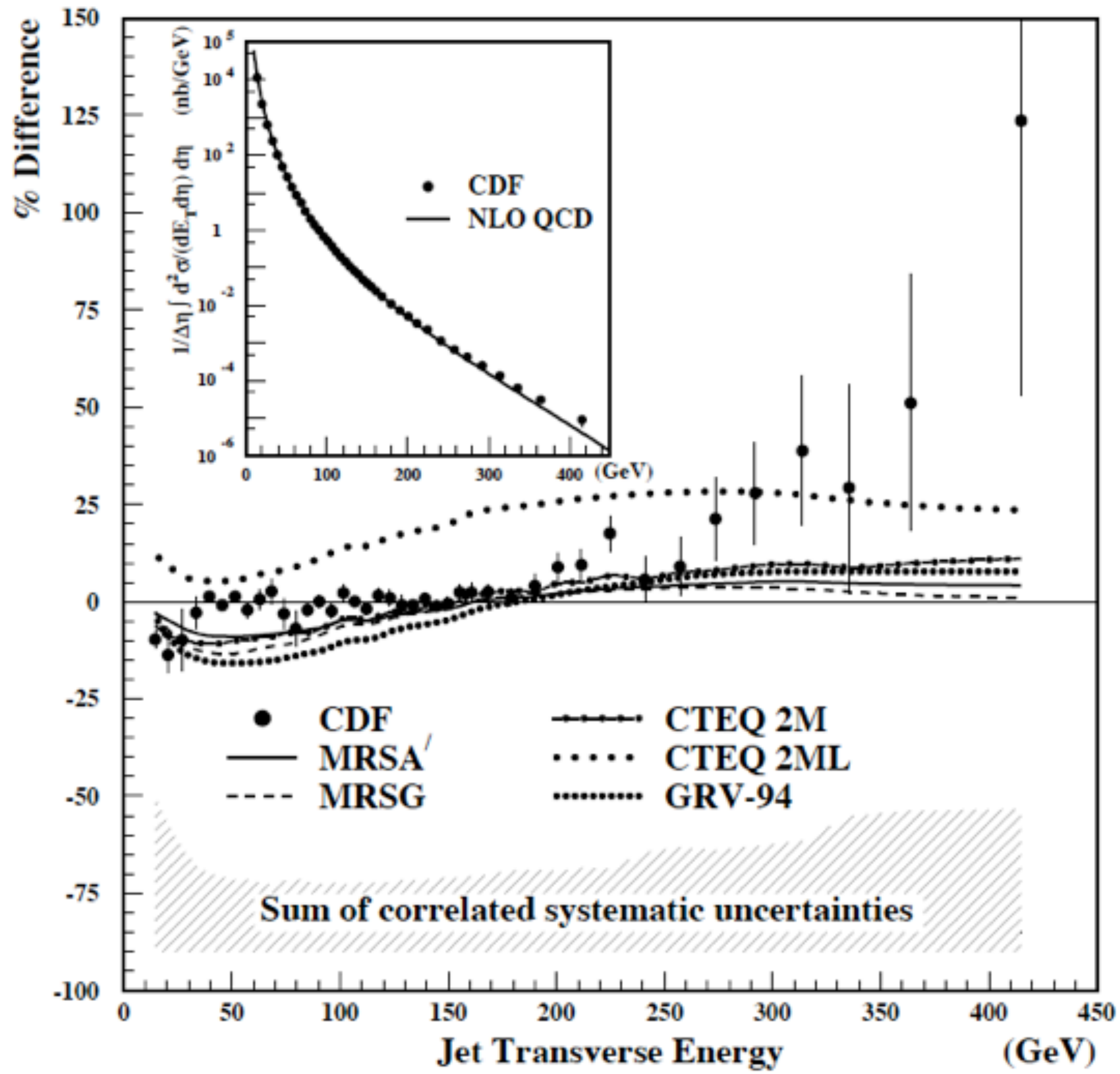

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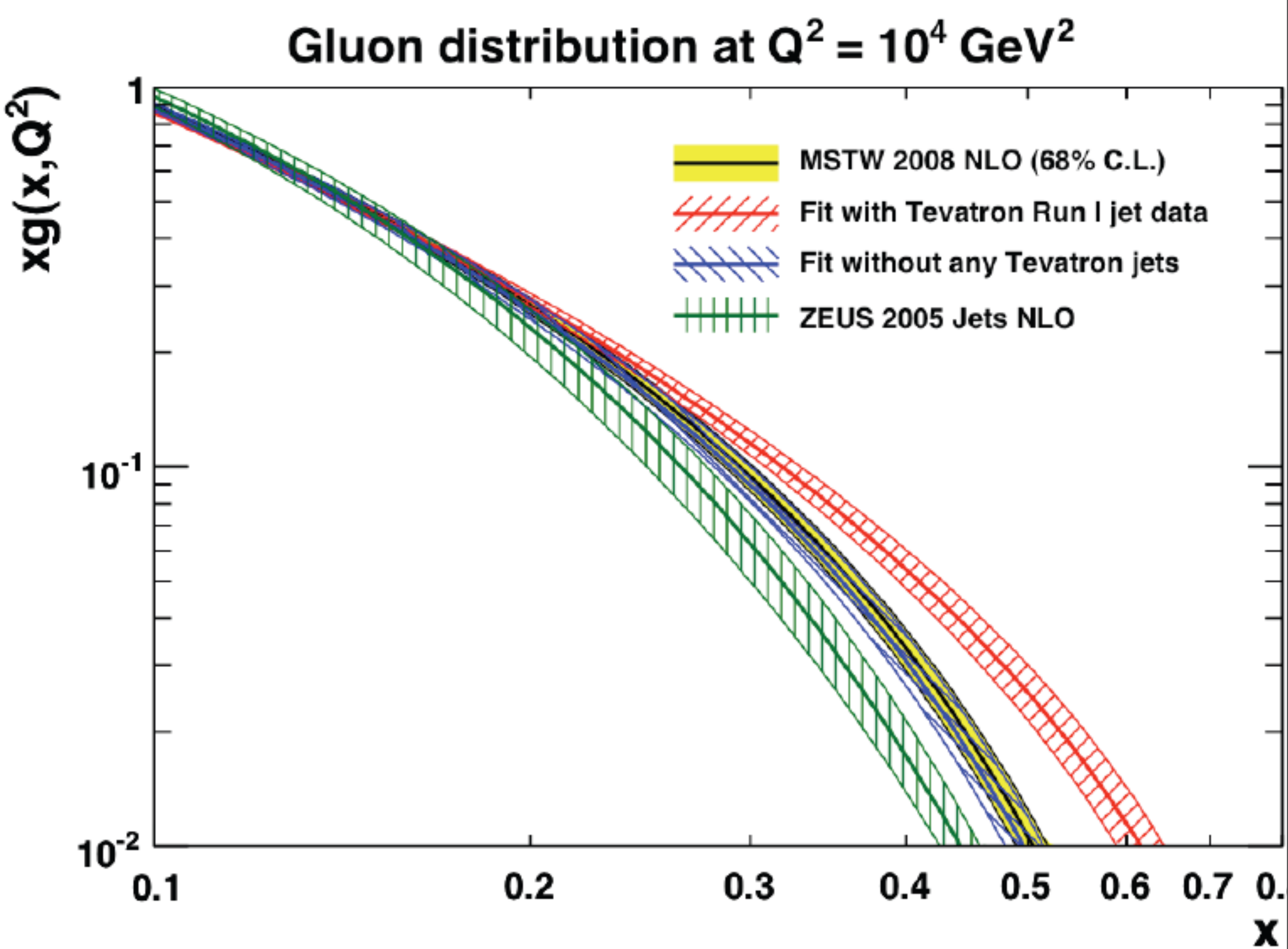

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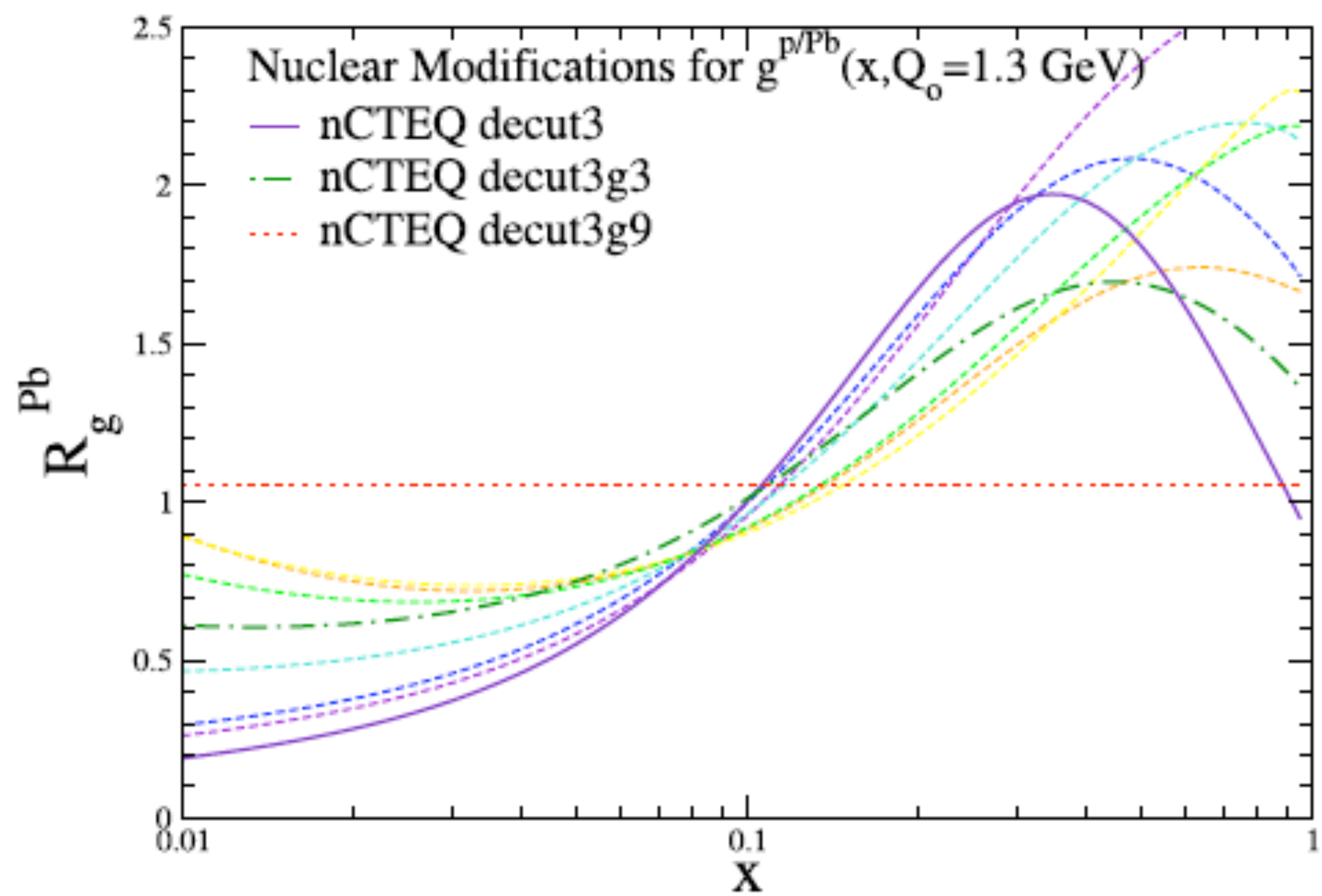
- It is finally known that due to our poor knowledge of gluon PDF in high x.
- Also important thing is no PDF uncertainty at that time.
- PDF uncertainty in the extrapolated region can be underestimated.
- Similarly, the conclusions based on nPDFs without taking into account the nPDF errors should be reexamined.



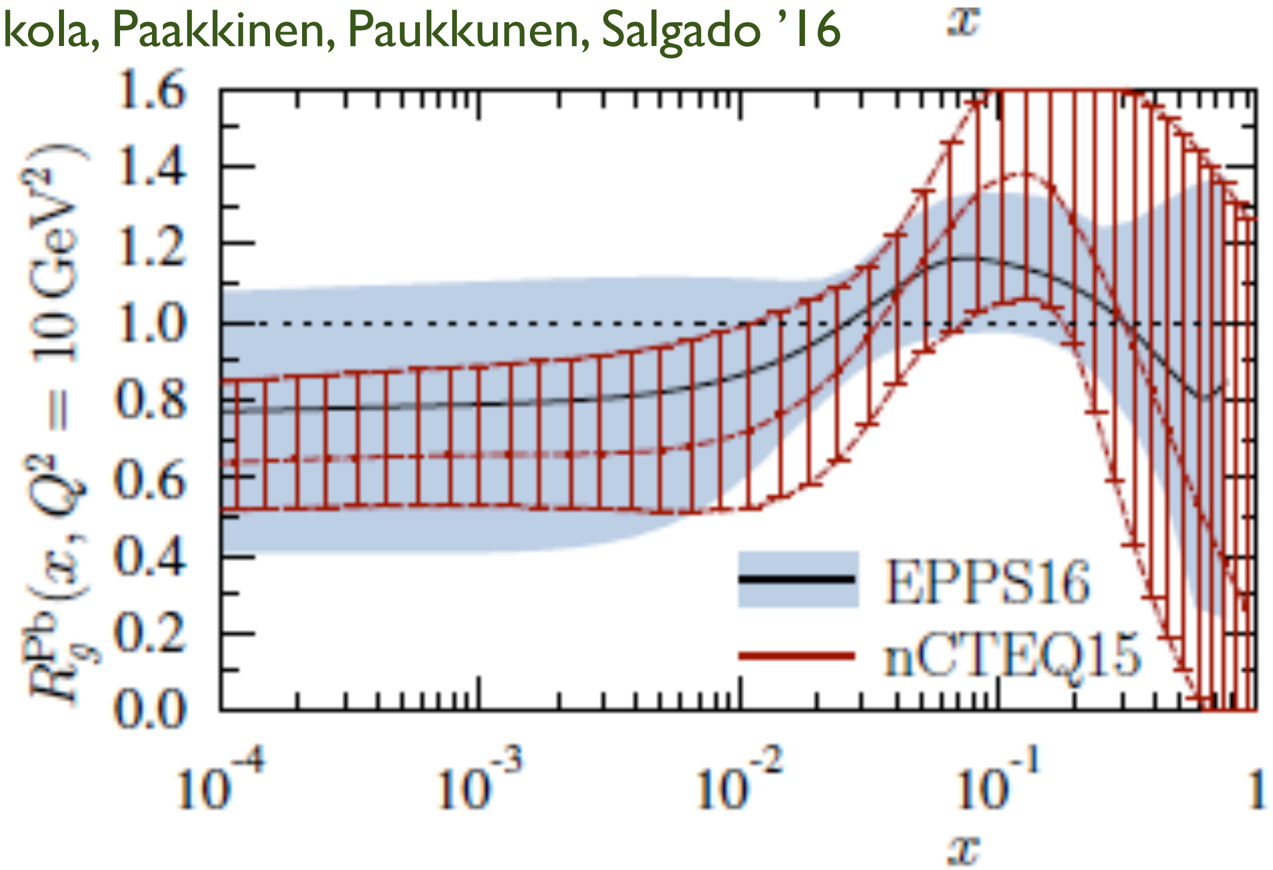
# THE SUMMARY OF NPDFS

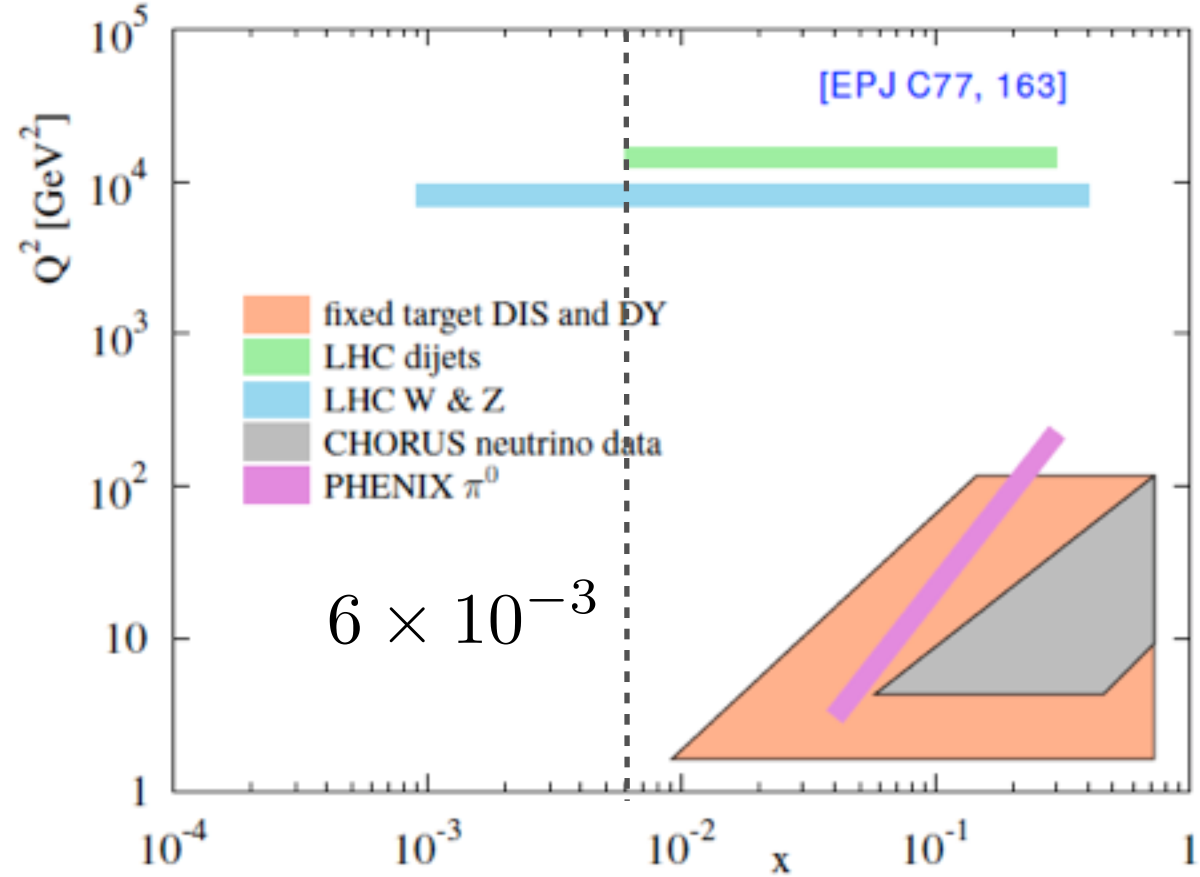
From Armesto's DIS2019 talk

| SET | | EPS09 JHEP 0904 (2009) 065 | DSSZ PRD85 (2012) 074028 | nCTEQ15 PRD93 (2016) 085037 | KA15 PRD93 (2016) 014036 | EPPS16 EPJC C77 (2017)163 | nNNPDF1.0 1904.00018 |
|---|---|---|---|---|---|---|---|
| data | eDIS | ✔ | ✔ | ✔ | ✔ | ✔ | ✔ |
| data | DY | ✔ | ✔ | ✔ | ✔ | ✔ | ✘ |
| data | $\pi^0$ | ✔ | ✔ | ✔ | ✘ | ✔ | ✘ |
| data | νDIS | ✘ | ✔ | ✘ | ✘ | ✔ | ✘ |
| data | pPb | ✘ | ✘ | ✘ | ✘ | ✔ | ✘ |
| # data | | 929 | 1579 | 740 | 1479 | 1811 | 451 |
| order | | NLO | NLO | NLO | NNLO | NLO | NNLO |
| proton PDF | | CTEQ6.1 | MSTW2008 | ~CTEQ6.1 | JR09 | CT14NLO | NNPDF3.1 |
| mass scheme | | ZM-VFNS | GM-VFNS | GM-VFNS | ZM-VFNS | GM-VFNS | FONLL-B |
| comments | | $\Delta\chi^2$=50, ratios, huge shadowing-antishadowing | $\Delta\chi^2$=30, ratios, medium-modified FFs for $\pi^0$ | $\Delta\chi^2$=35, PDFs, valence flavour sep., not enough sensitivity | PDFs, deuteron data included | $\Delta\chi^2$=52, flavour sep., ratios, LHC pPb data | NNPDF methodology, isoscalarity assumed |

# TYPICAL GLUON NUCLEAR PDFS

Eskola, Paakkinen, Paukkunen, Salgado '16




- For the gluons, only the shadowing depletion is established although its magnitude is still discussed.
- The gluon antishadowing not yet observed although used in many studies; hence, absent in some nPDF fit.
- The gluon EMC effect is even less known, hence the uncertainty there.
- The heavy-quark production at the LHC may help to understand better the gluon density in nuclei.

# AN AUTOMATED CODE TO EVALUATE NPDF EFFECTS

Lansberg, HSS '17

- Partonic scattering cross section fit from pp data with a Crystal Ball function parametrizing $|\mathcal{A}_{gg\to\mathcal{H}X}|^2$

Kom, Kulesza, Stirling '11

$$\overline{|\mathcal{A}(k_1k_2\to\mathcal{H}+k_3)|^2}=\frac{\lambda^2\kappa s x_1 x_2}{M_{\mathcal{H}}^2}\exp\left(-\kappa\frac{\min(P_T^2,\langle P_T\rangle^2)}{M_{\mathcal{H}}^2}\right)\left(1+\theta(P_T^2-\langle P_T\rangle^2)\frac{\kappa}{n}\frac{P_T^2-\langle P_T\rangle^2}{M_Q^2}\right)^{-n}$$

- It is in principle can be applied to any single-inclusive particle production as long as knowing the fraction of initial partonic luminosity in priori (e.g. gluon-gluon dominance for heavy-flavour production at high-energy collisions).

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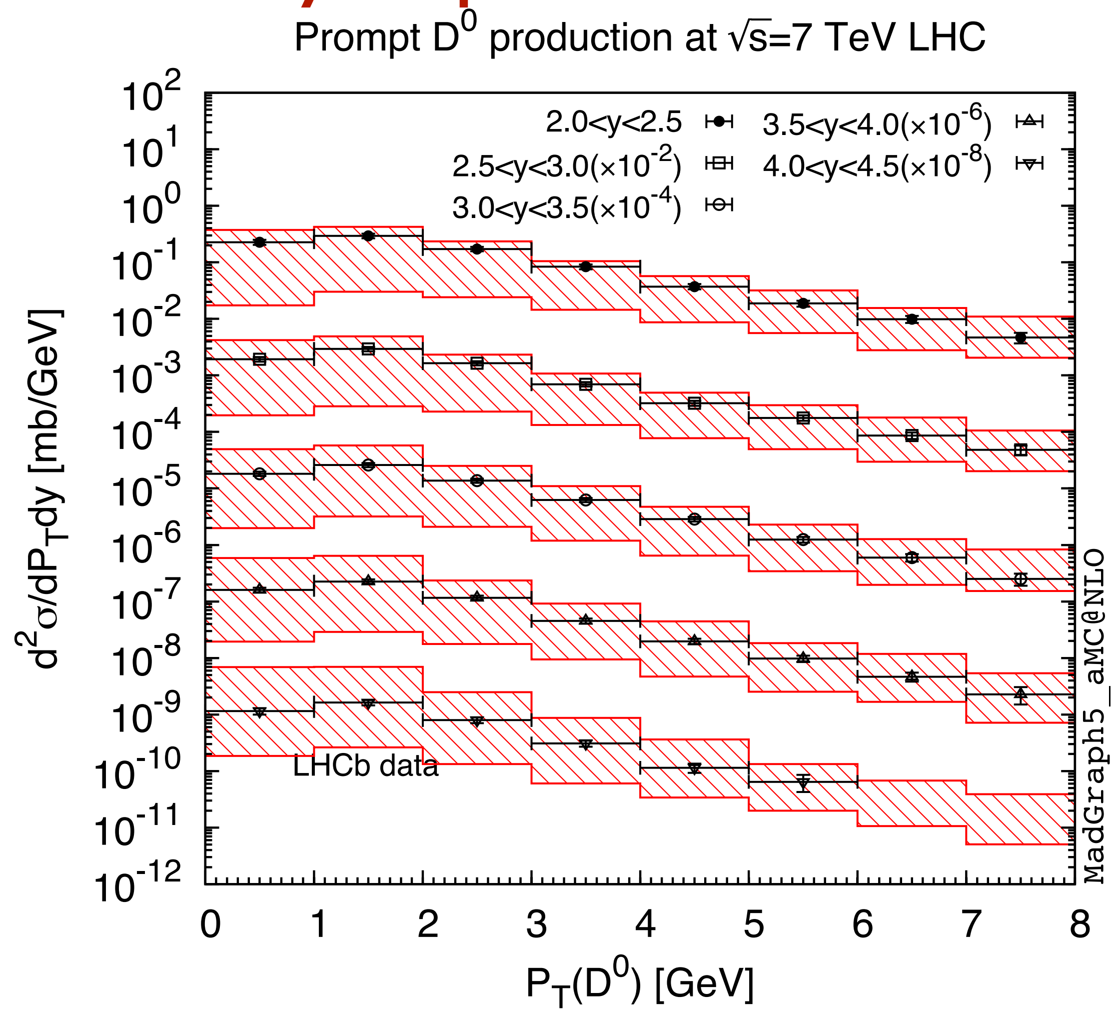


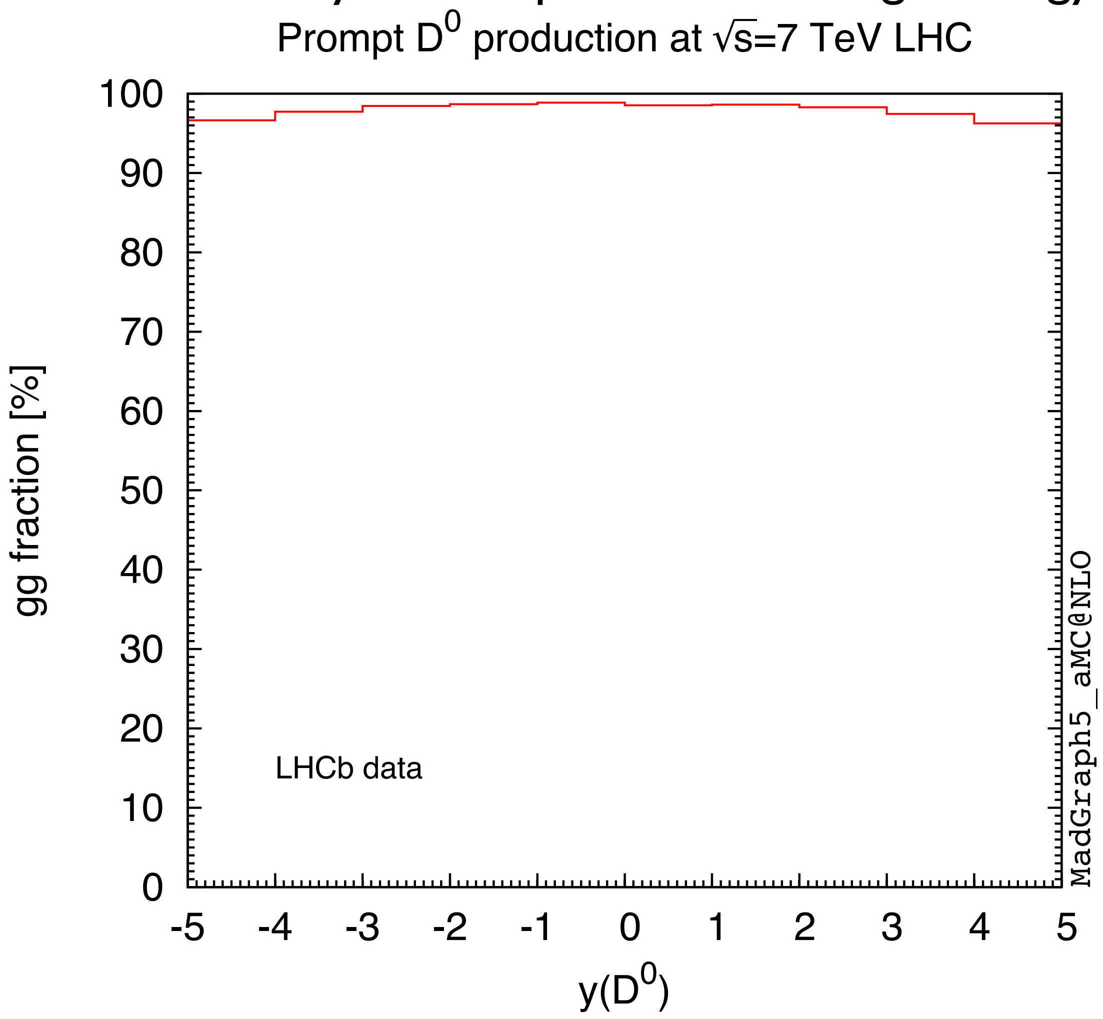

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- It is in principle can be applied to any single-inclusive particle production as long as knowing the fraction of initial partonic luminosity in priori (e.g. gluon-gluon dominance for heavy-flavour production at high-energy collisions).

- Applied to open/hidden charm/beauty hadrons (J/psi, Y, D and B)
- It is a way to evade the quarkonium-production-mechanism controversy (at least to some extent).

- The key point to compute nPDF effects is to have a partonic XS
- It can be validated with state-of-the-art pQCD computations (e.g. FONLL, GM-VFNS)
- Any nPDF set available in LHAPDF 5 or 6 can be used

# AN AUTOMATED CODE TO EVALUATE NPDF EFFECTS

Lansberg, HSS '17

- Extensive comparisons directly with data

  makes sense only when nPDF are the dominant CNM

- One can test this hypothesis by comparing our curves with data

  Global agreement $\overset{?}{\Rightarrow}$ only nPDFs matter

- One can go further in the theory-data comparison with reweighting
- Bonus: since the pp yields are fit, the procedure sometimes hints a normalisation issues (bar $R_{FB}$) which could otherwise be misinterpreted as nuclear suppressions or enhancements.
- It allows one to study different nPDF sets AND the scale uncertainties as well as a better control of the theory uncertainties
- Last but not least: it allows one to study different nPDF sets AND the scale uncertainties as well as a better control of the theory uncertainties
- Disclaimer: it does not provide any insight on the production mechanisms but provides us efficient and controlled (inter/extra)polations of the differential XS in the space ($x_1$,$x_2$,y,$p_T$).

# FITTING THE PP DATA

Lansberg, HSS '17

- Starting with the J/psi

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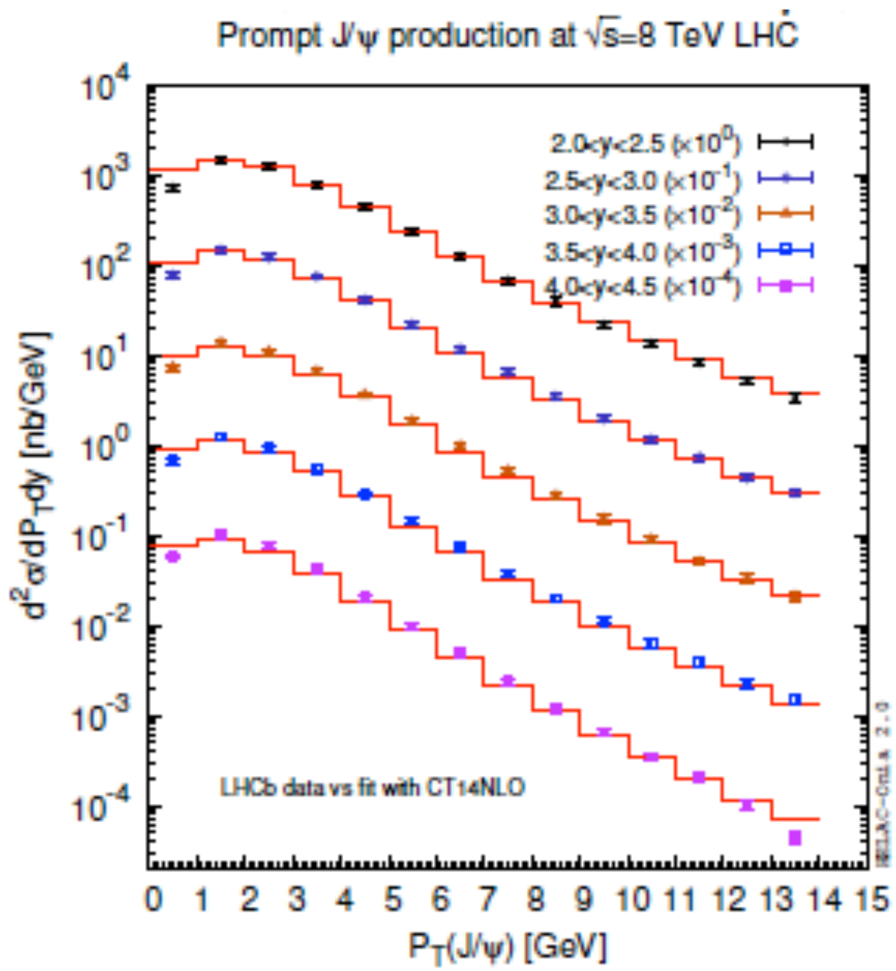
- Extremely good fit of the LHCb data (bar may be the 1st bin)



# FITTING THE PP DATA

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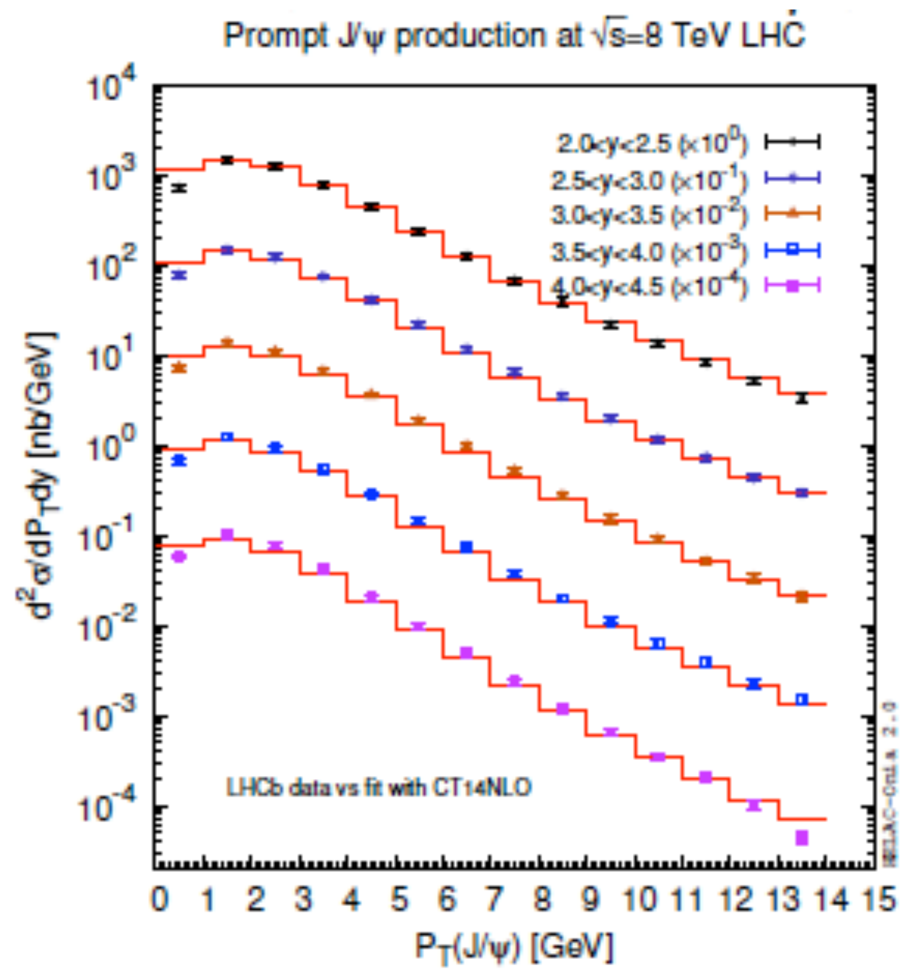
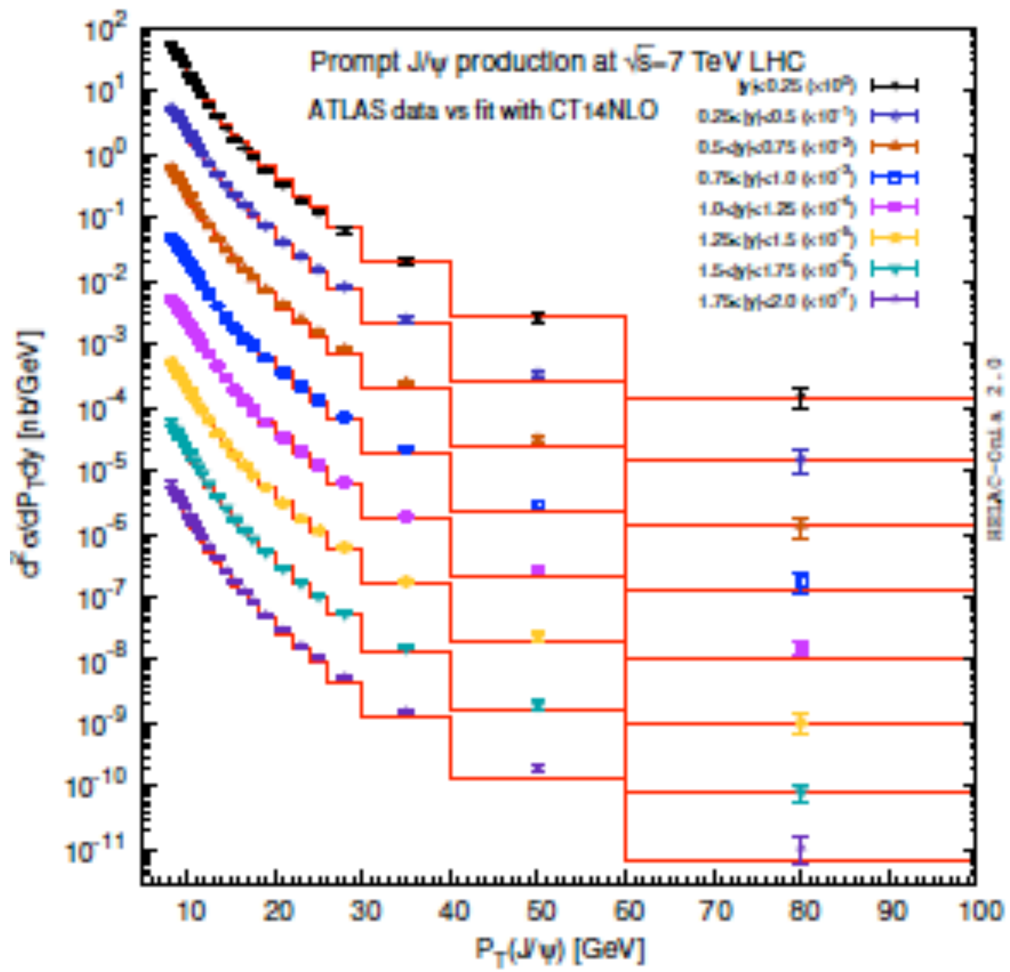
- Starting with the J/psi
- Extremely good fit of the LHCb data (bar may be the 1st bin)
- Also good at high $p_T$ with ATLAS ...

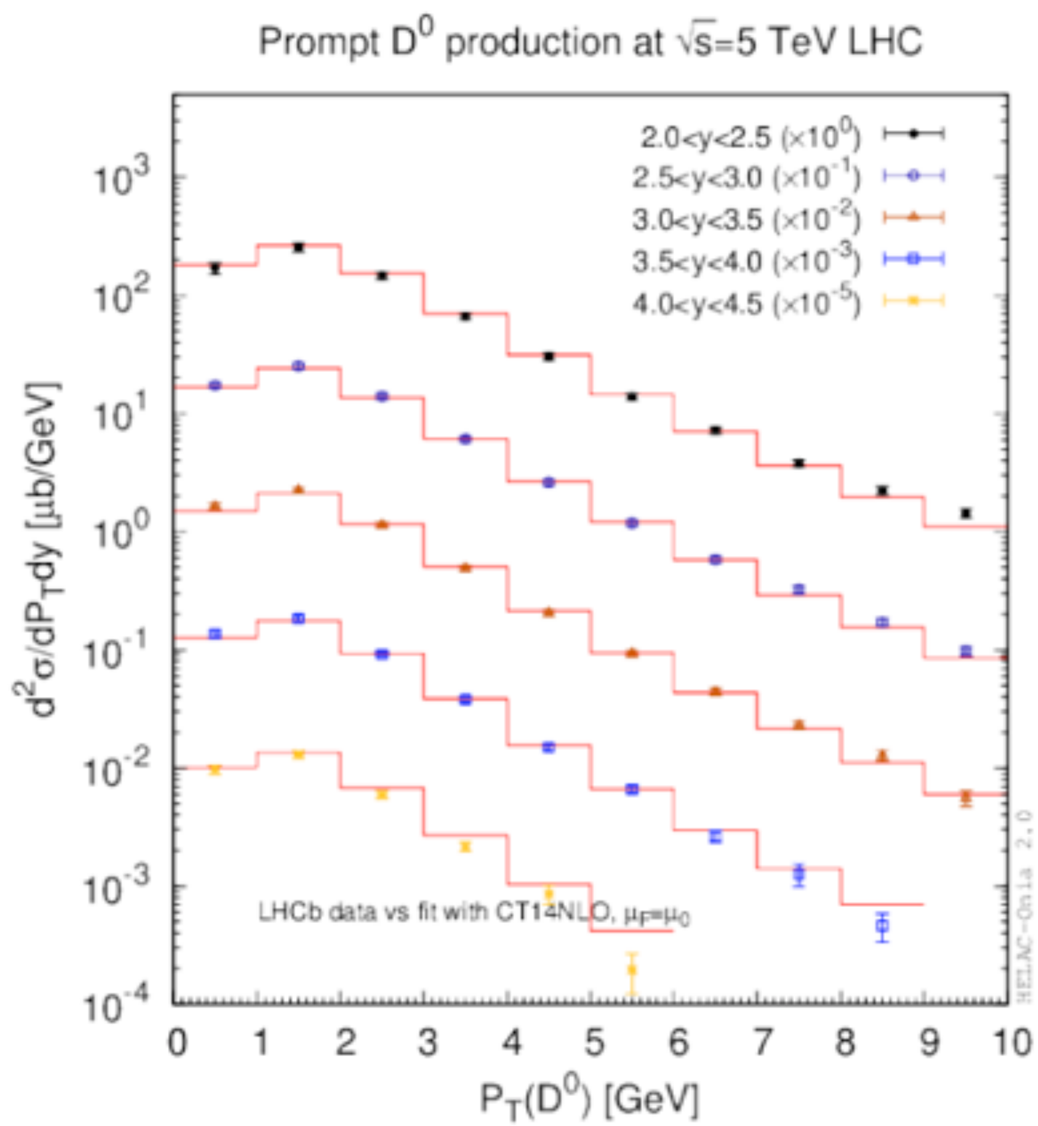


# FITTING THE PP DATA

Lansberg, HSS '17

- Above exercises can be used also for Y, eta_c, D, B etc
- Especially, one can compare with relatively well-understood pQCD computations for open charm/beauty
- For example, extremely good fit for $D^0$ measured by LHCb



# USED DATA SETS

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Kusina, Lansberg, Schienbein, HSS '17

| | $D^0$ | $J/\psi$ | $B \to J/\psi$ | $\Upsilon(1S)$ |
|---|---|---|---|---|
| $\mu_0$ | $\sqrt{4M_{D^0}^2 + P_{T,D^0}^2}$ | $\sqrt{M_{J/\psi}^2 + P_{T,J/\psi}^2}$ | $\sqrt{4M_B^2 + \left(\frac{M_B}{M_{J/\psi}} P_{T,J/\psi}\right)^2}$ | $\sqrt{M_{\Upsilon(1S)}^2 + P_{T,\Upsilon(1S)}^2}$ |
| $p$+$p$ data | LHCb (1) | LHCb (2; 3) | LHCb (2; 3) | ALICE (4), ATLAS (5), CMS (6), LHCb (7; 8) |
| $R_{pPb}$ data | ALICE (9), LHCb (15) | ALICE (10; 11), LHCb (16; 12) | LHCb (12) | ALICE (13), ATLAS (14), LHCb (17) |


[1] LHCb, R. Aaij et al., JHEP **06**, 147 (2017), 1610.02230.
[2] LHCb, R. Aaij et al., Eur. Phys. J. **C71**, 1645 (2011), 1103.0423.
[3] LHCb, R. Aaij et al., JHEP **06**, 064 (2013), 1304.6977.
[4] ALICE, B. B. Abelev et al., Eur. Phys. J. **C74**, 2974 (2014), 1403.3648.
[5] ATLAS, G. Aad et al., Phys. Rev. **D87**, 052004 (2013), 1211.7255.
[6] CMS, S. Chatrchyan et al., Phys. Lett. **B727**, 101 (2013), 1303.5900.
[7] LHCb, R. Aaij et al., Eur. Phys. J. **C72**, 2025 (2012), 1202.6579.
[8] LHCb, R. Aaij et al., JHEP **11**, 103 (2015), 1509.02372.
[9] ALICE, B. B. Abelev et al., Phys. Rev. Lett. **113**, 232301 (2014), 1405.3452.
[10] ALICE, J. Adam et al., JHEP **06**, 055 (2015), 1503.07179.
[11] ALICE, B. B. Abelev et al., JHEP **02**, 073 (2014), 1308.6726.
[12] LHCb, R. Aaij et al., (2017), 1706.07122.
[13] ALICE, B. B. Abelev et al., Phys. Lett. **B740**, 105 (2015), 1410.2234.
[14] The ATLAS collaboration, (2015), ATLAS-CONF-2015-050.
[15] LHCb, R. Aaij et al., (2017), 1707.02750.
[16] LHCb, R. Aaij et al., JHEP **02**, 072 (2014), 1308.6729.
[17] LHCb, R. Aaij et al., JHEP **07**, 094 (2014), 1405.5152.

# USED DATA SETS

Kusina, Lansberg, Schienbein, HSS '17

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|---|---|---|---|---|
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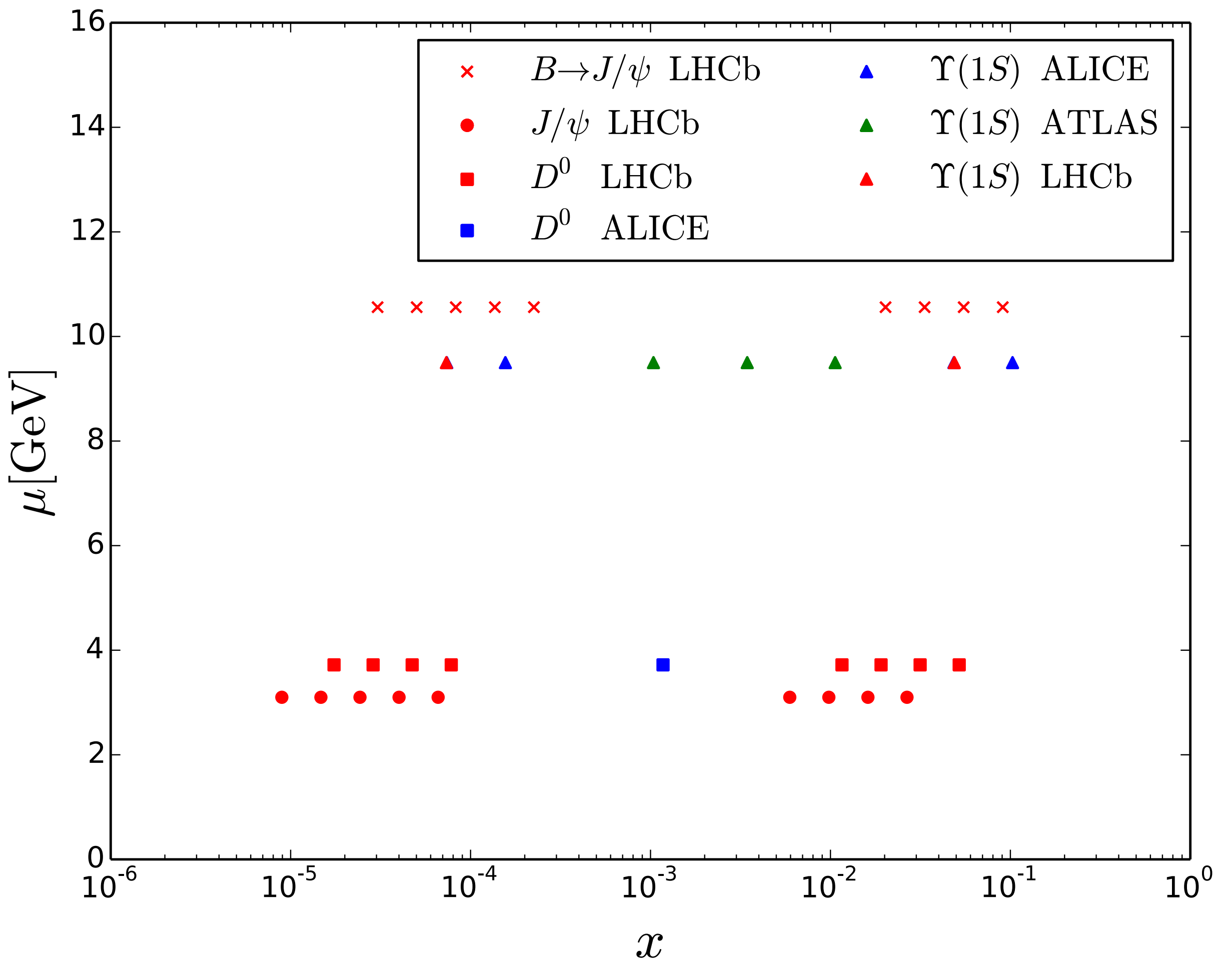
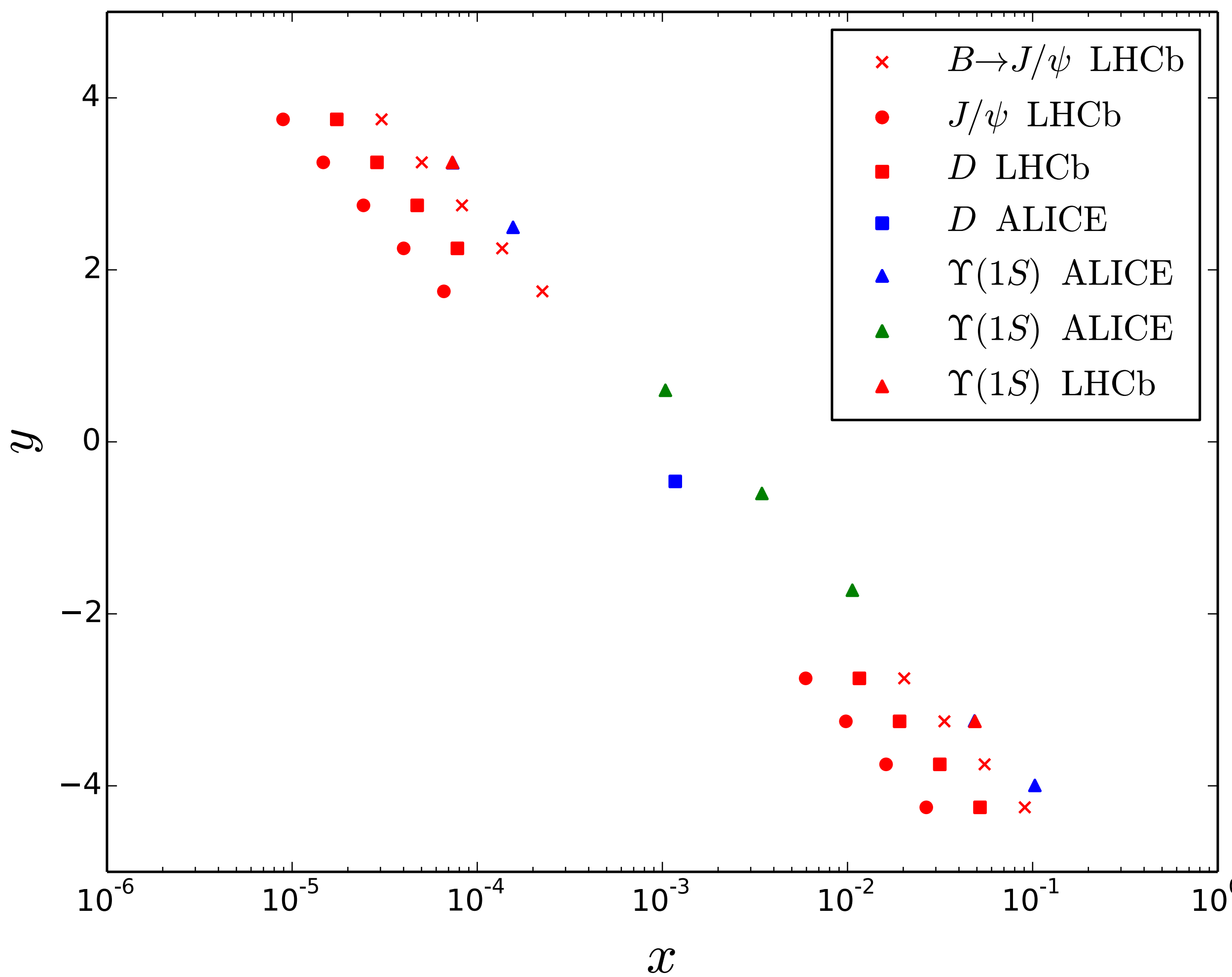
| p+p data | LHCb (1) | LHCb (2; 3) | LHCb (2; 3) | ALICE (4), ATLAS (5), |





[13] ALICE, B. B. Abelev et al., Phys. Lett. **B740**, 105 (2015), 1410.2234.
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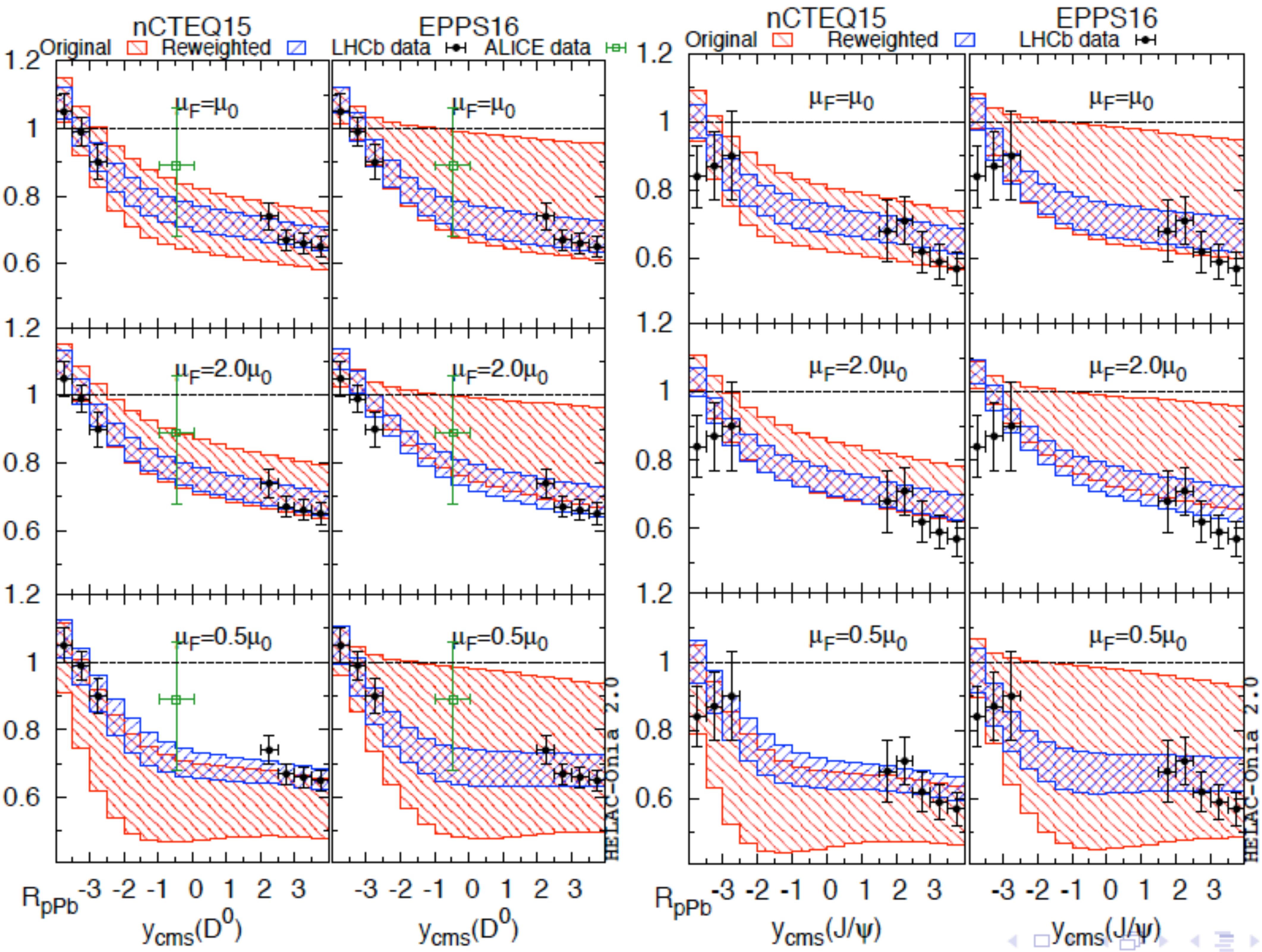
# REWEIGHTING RESULTS: $D^0$ AND J/PSI

68% CL

Kusina, Lansberg, Schienbein, HSS '17

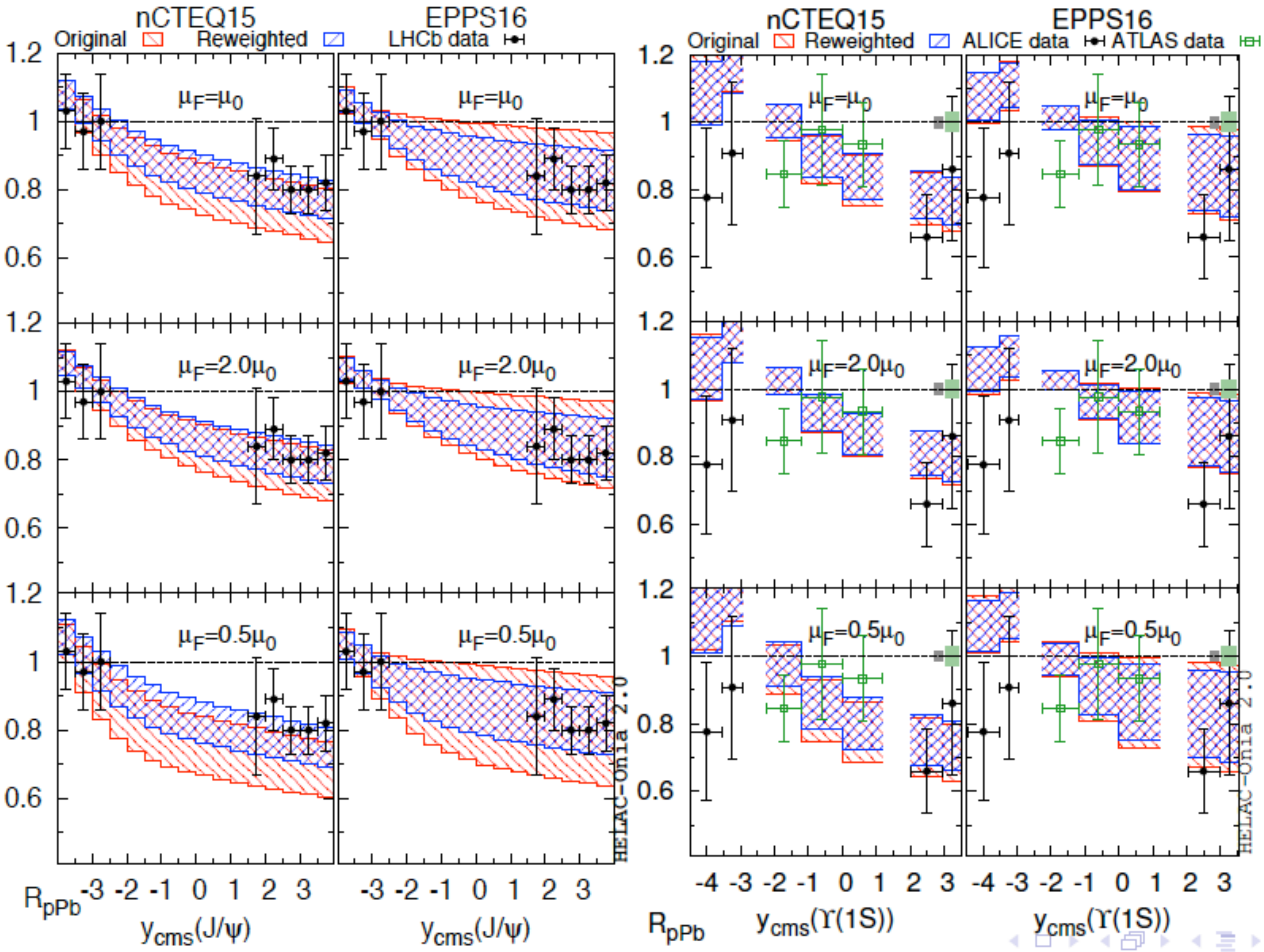



Changing the scale has two effects:
1) the uncertainty tends to increase at low $\mu_F$
2) since the shadowing suppression (in/de)creases for (de/in)creasing $\mu_F$, the reweighted nPDF from data shifts within the original uncertaities

# REWEIGHTING RESULTS: B->J/PSI AND Y

68% CL

Kusina, Lansberg, Schienbein, HSS '17

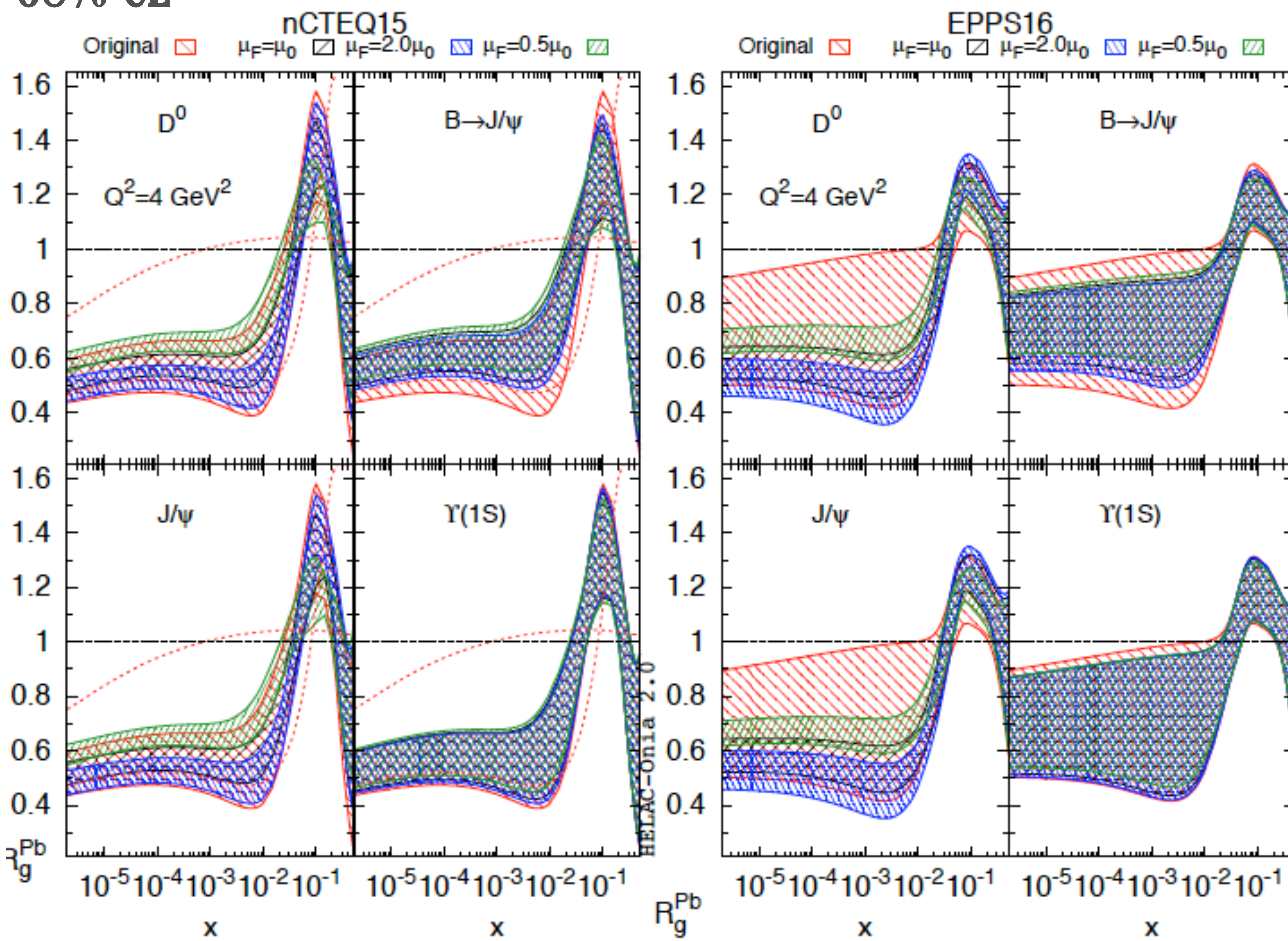



Compared to the $D$ and $J/\psi$ cases,
1) the scales uncertainties are smaller, but
2) the data are not yet as precise

# RESULTS OF REWEIGHTED NPDFS

68% CL

Kusina, Lansberg, Schienbein, HSS '17




- Global coherence of the data constrains: necessary condition to assume a shadowing-only approach
- First clear exp. obser. on gluon shadowing at low $x_{bj}$: visible reduction of EPPS16 uncertainties; confirmation of nCTEQ15 extrapolation (reduction after including two similar-good extreme cases)

- The scale ambiguity for D and J/psi production is now the dominant uncertainty
- B or non-prompt J/psi are promising if precision of the data can be improved

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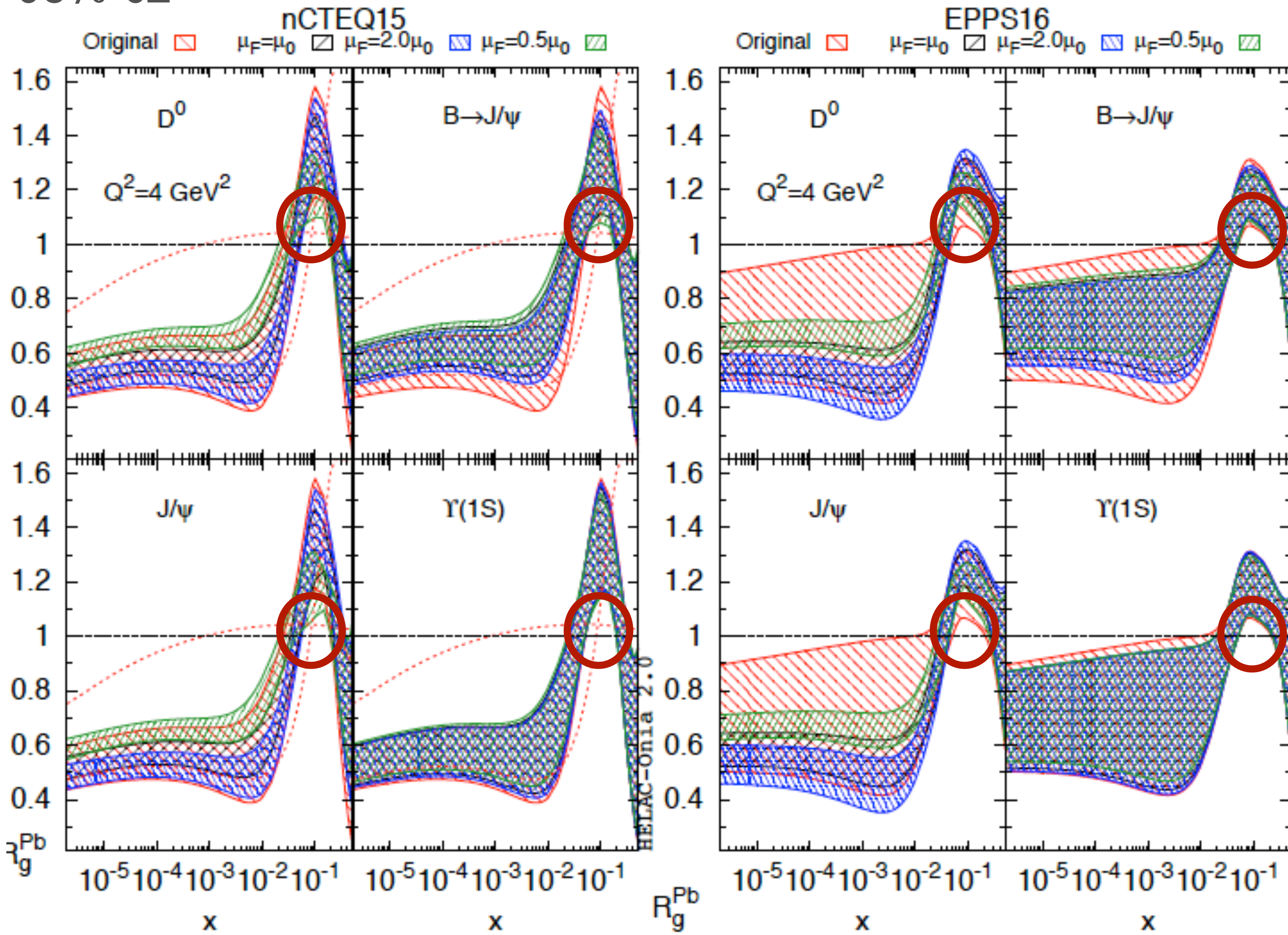


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- The scale ambiguity for D and J/psi production is now the dominant uncertainty
- B or non-prompt J/psi are promising if precision of the data can be improved
- Confirmation of the existence of a gluon anti-shadowing: $R_g(0.05 \lesssim x < \lesssim 0.1) > 1$

# RESULTS OF REWEIGHTED NPDFS

Kusina, Lansberg, Schienbein, HSS '17

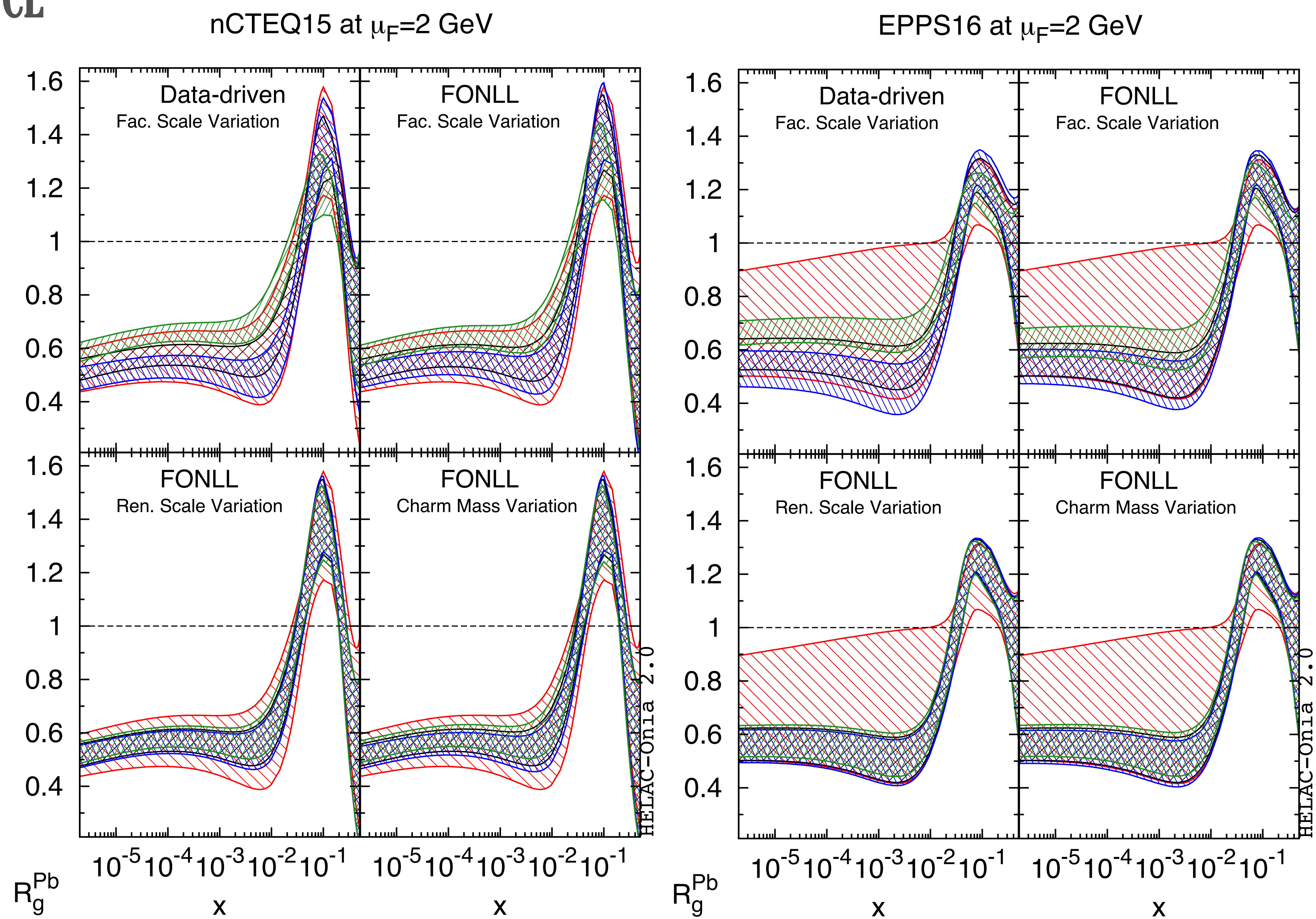
| | | $D^0$ | $J/\psi$ | $B \to J/\psi$ | $\Upsilon(1S)$ |
|---|---|---|---|---|---|
| | $N_{\text{data}}$ | 38 | 71 | 37 | 12 |
| Original nCTEQ15 | $\xi = 0.5$ | 142 | 131 | 39 | 14 |
| | $\xi = 1.0$ | 39 | 63 | 23 | 11 |
| | $\xi = 2.0$ | 63 | 90 | 15 | 11 |
| Reweighted nCTEQ15 | $\xi = 0.5$ | 56 | 46 | 14 | 13 |
| | $\xi = 1.0$ | 56 | 53 | 11 | 11 |
| | $\xi = 2.0$ | 56 | 46 | 9 | 11 |
| Original EPPS16 | $\xi = 0.5$ | 53 | 62 | 9 | 10 |
| | $\xi = 1.0$ | 140 | 150 | 7 | 10 |
| | $\xi = 2.0$ | 218 | 220 | 8 | 11 |
| Reweighted EPPS16 | $\xi = 0.5$ | 37 | 59 | 7 | 10 |
| | $\xi = 1.0$ | 37 | 59 | 7 | 10 |
| | $\xi = 2.0$ | 37 | 59 | 7 | 11 |

- The chi2 are improved in general !

# VALIDATE WITH FONLL FOR OPEN CHARM/BEAUTY

Kusina, Lansberg, Schienbein, HSS '17

68% CL

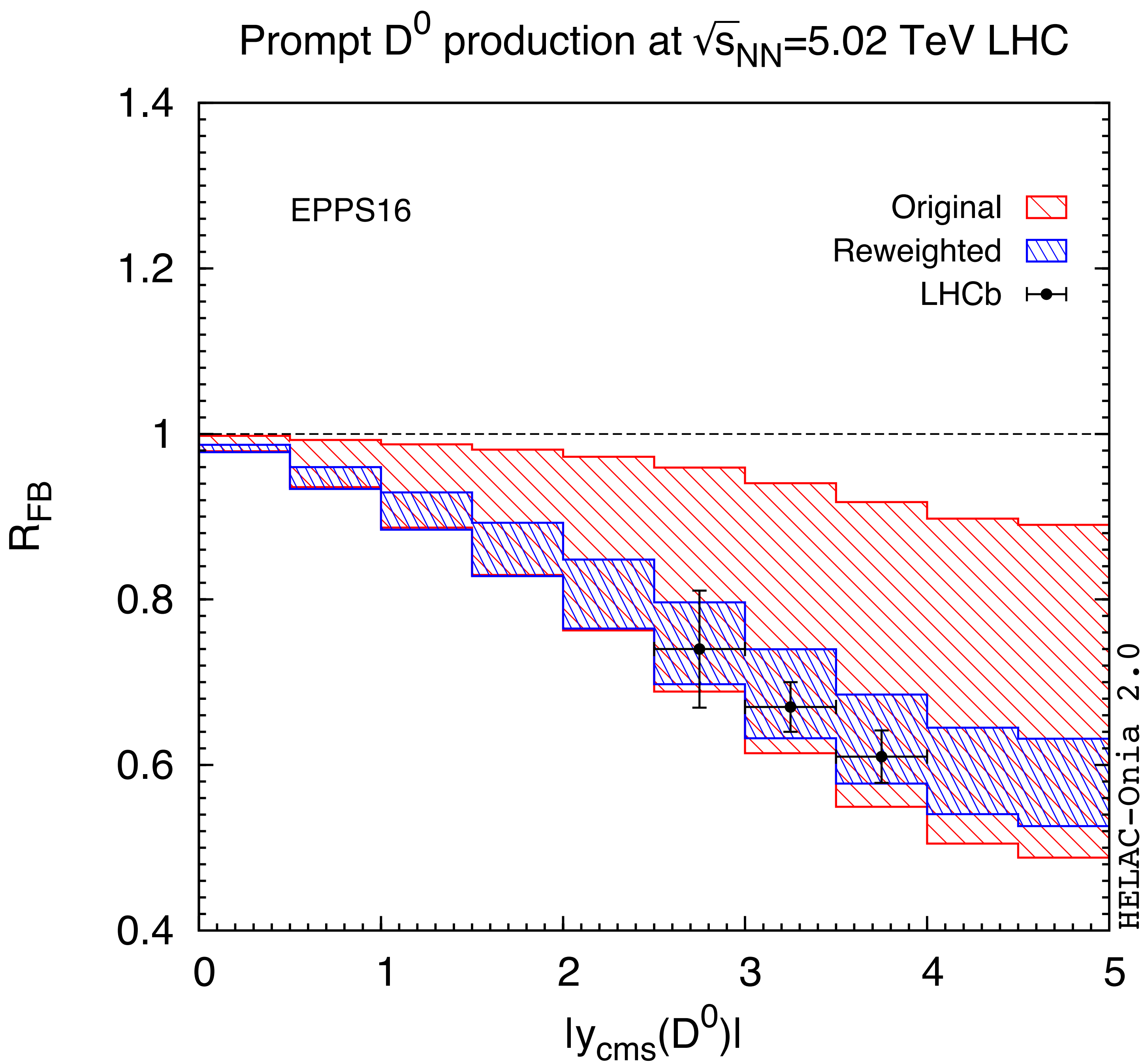



- Validated with the state-of-the-art pQCD calculations !

# PREDICTIONS & CONSISTENCES

## 68% CL

Kusina, Lansberg, Schienbein, HSS '17




- Other observables: e.g. $R_{FB}$

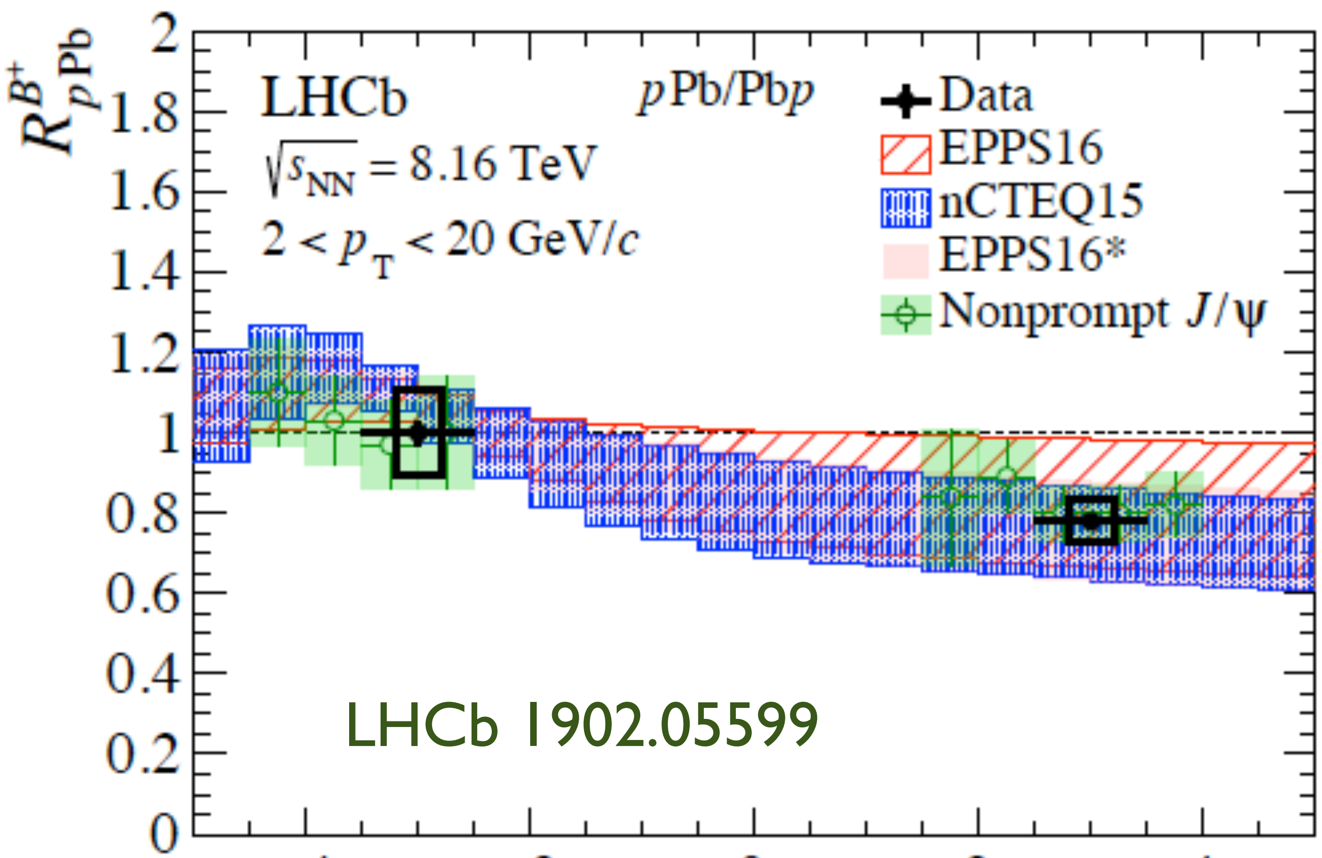
# PREDICTIONS & CONSISTENCES

Kusina, Lansberg, Schienbein, HSS '17

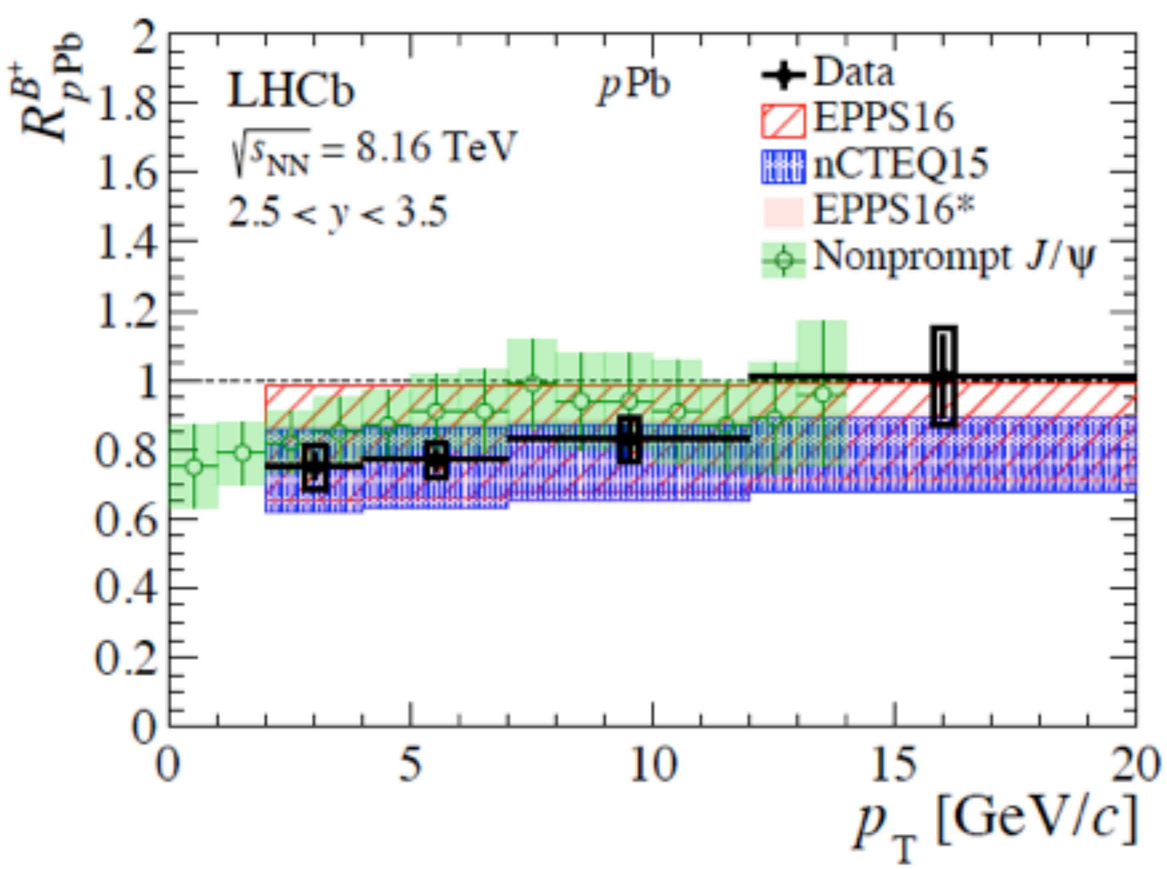
- Other observables: e.g. $R_{FB}$
- Consistent with other measurements, e.g. at RHIC

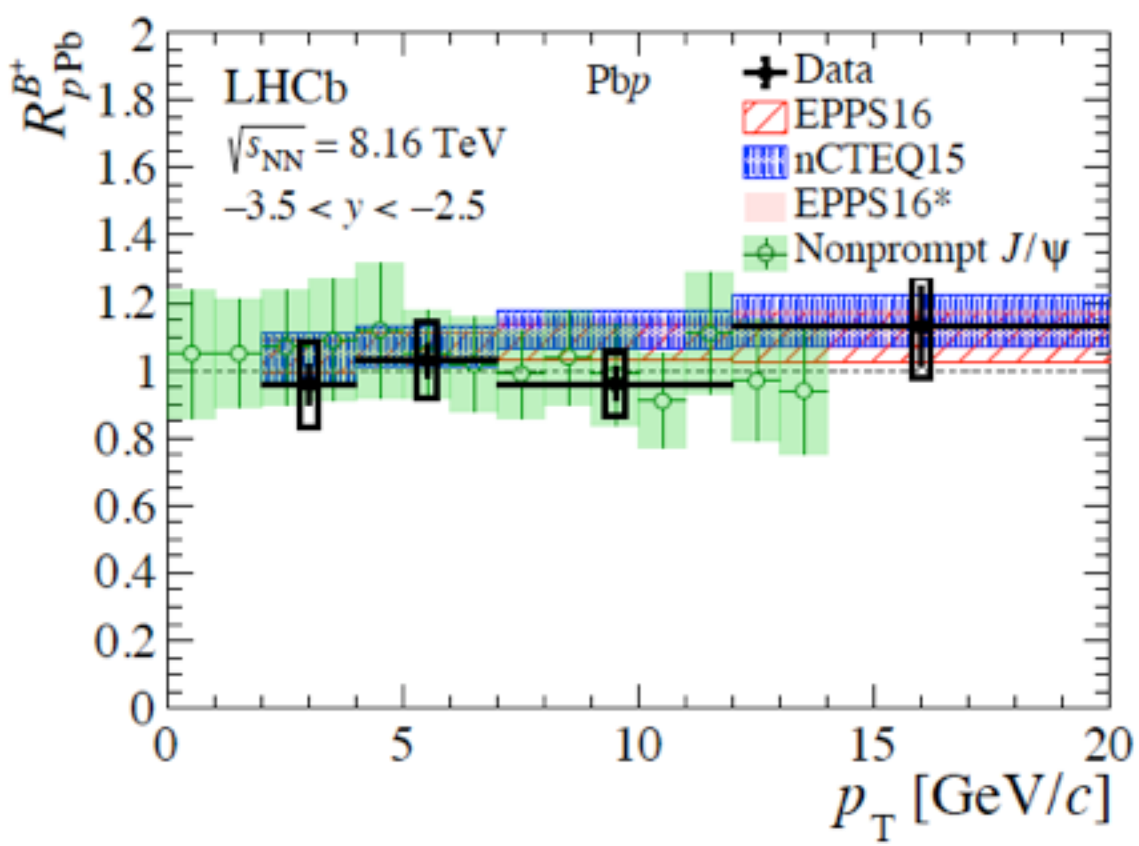
| | | | Original | Reweighted | | | |
|---|---|---|---|---|---|---|---|
| | | | | $D^0$ | $J/\psi$ | $B \to J/\psi$ | $\Upsilon(1S)$ |
| PHENIX $J/\psi$ ($N_{data} = 74$) | nCTEQ15 | $\xi = 0.5$ | 265 | – | 134 | – | – |
| | | $\xi = 1.0$ | 189 | – | 176 | – | – |
| | | $\xi = 2.0$ | 231 | – | 205 | – | – |
| | EPPS16 | $\xi = 0.5$ | 133 | – | 138 | – | – |
| | | $\xi = 1.0$ | 207 | – | 167 | – | – |
| | | $\xi = 2.0$ | 263 | – | 209 | – | – |
| LHC W/Z ($N_{data} = 102$) | nCTEQ15 | $\xi = 0.5$ | 248 | 218 | 230 | 212 | 229 |
| | | $\xi = 1.0$ | | 254 | 271 | 214 | 238 |
| | | $\xi = 2.0$ | | 317 | 332 | 219 | 243 |
| NMC $F_2^{Sn}/F_2^C$ ($N_{data} = 111$) | nCTEQ15 | $\xi = 0.5$ | 65 | 93 | 98 | 86 | 70 |
| | | $\xi = 1.0$ | | 65 | 66 | 78 | 67 |
| | | $\xi = 2.0$ | | 62 | 62 | 71 | 65 |
| NMC $F_2^{Pb}/F_2^C$ ($N_{data} = 14$) | nCTEQ15 | $\xi = 0.5$ | 8 | 8 | 8 | 8 | 7 |
| | | $\xi = 1.0$ | | 7 | 6 | 7 | 7 |
| | | $\xi = 2.0$ | | 9 | 8 | 7 | 8 |

# PREDICTIONS & CONSISTENCES




- Other observables: e.g. $R_{FB}$
- Consistent with other measurements, e.g. at RHIC
- New LHC measurements versus our new predictions start to appear.



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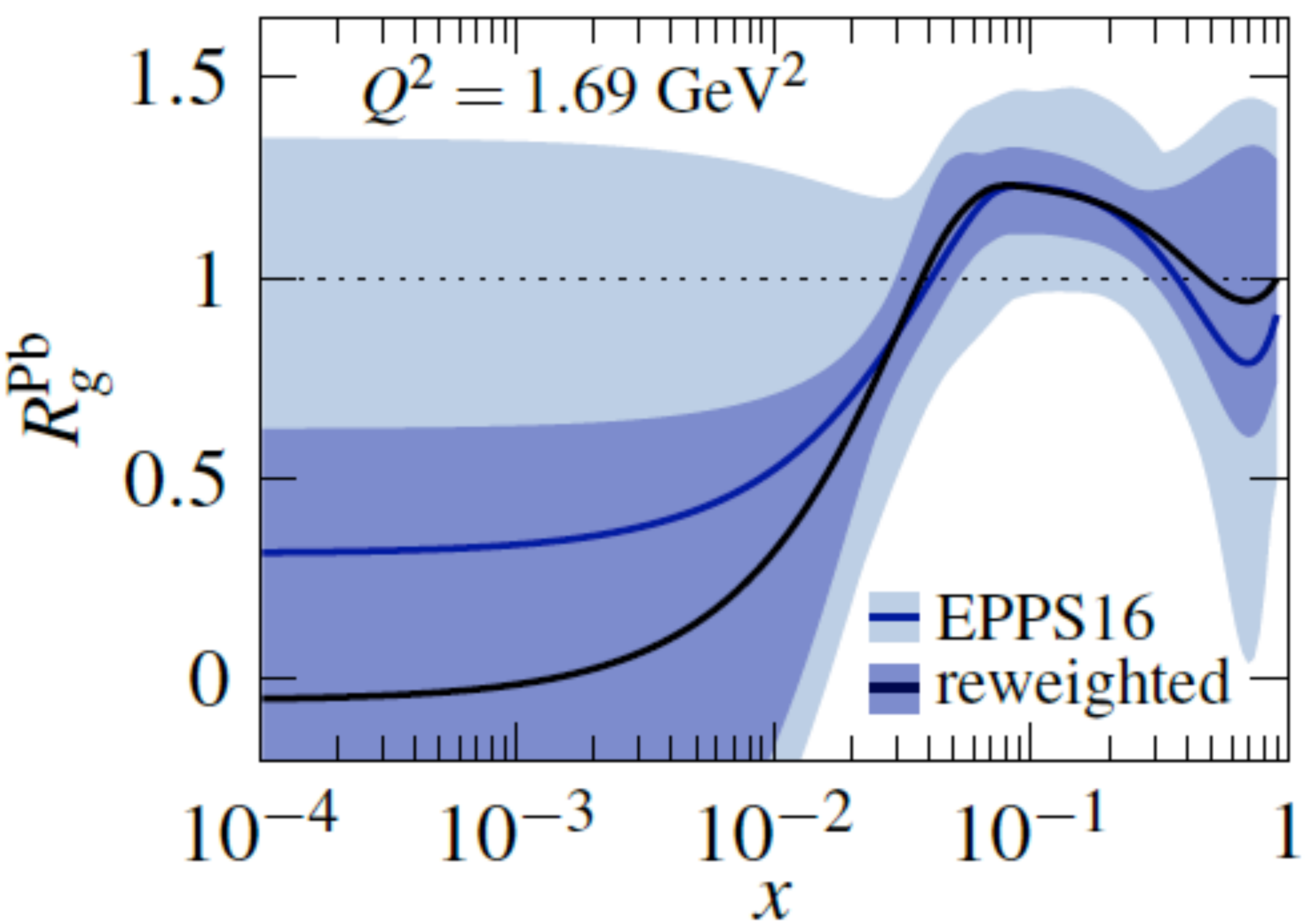


Eskola, Paakkinen, Paukkunen '19

- Other observables: e.g. $R_{FB}$
- Consistent with other measurements, e.g. at RHIC
- New LHC measurements versus our new predictions start to appear.
- Large gluon shadowing is confirmed by dijet in p+Pb

# CONCLUSIONS

- Gluon nPDFs at low x are extrapolated: no low x data used in fits
  $\rightarrow$ need for new constraints at $x \leq 10^{-3}$
- We have proposed a quick and robust method to evaluate nPDF effects, which is complementary to full but time consuming pQCD computations
- With standard theory-data comparisons, and with (n)PDF Bayesian reweighting technique, we tested and validated a shadowing-only hypothesis with HF (D, J/psi, B->J/psi, Y) LHC data
- Under this hypothesis, we call for an experimental observation of shadowing and anti-shadowing
- We thoroughly considered the scale uncertainty in pA for the 1st time
- For charm, it induces uncertainties as large as the reweighted nPDF err
- Our extraction is consistent with other observables. Stay tuned for the more comparisons.



Wednesday, May 8, 19

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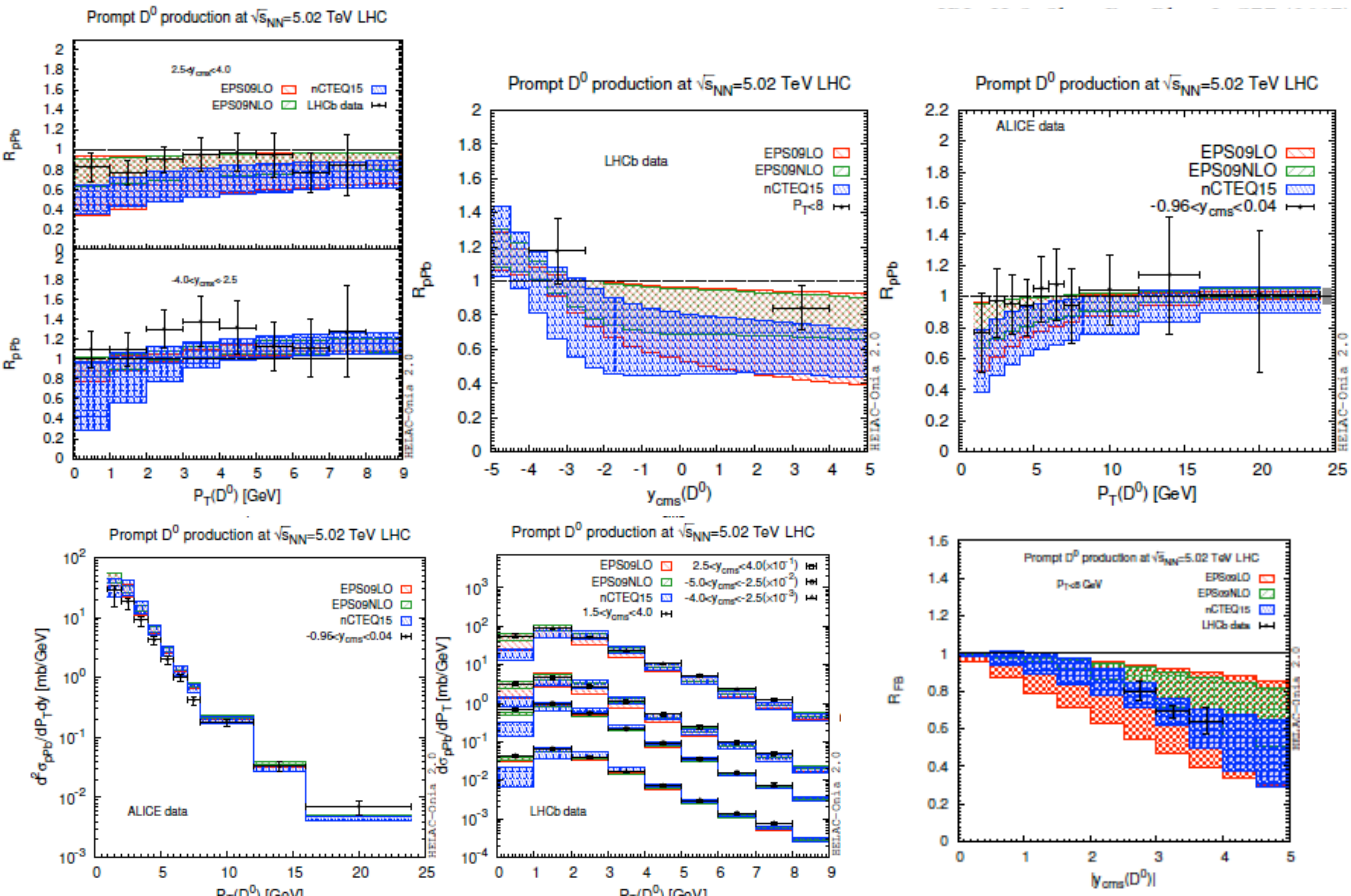
**Enjoy the dinner !**

# Backup

# RESULTS FOR PA: $D^0$

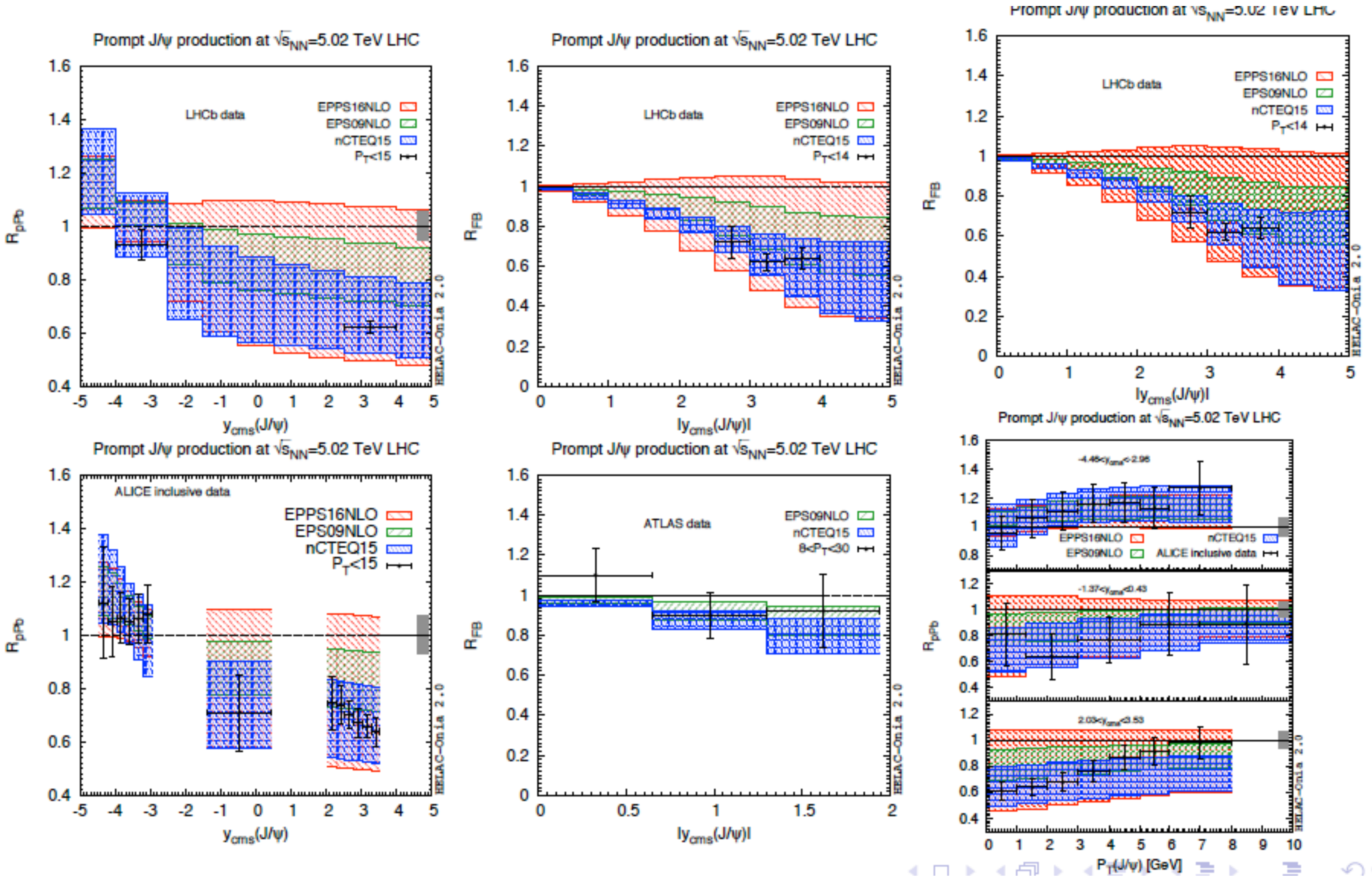
Lansberg, HSS '17



# RESULTS FOR PA: J/PSI

Lansberg, HSS '17

- nCTEQ15, EPPS16, EPS09 etc



# REWEIGHTING FOR HESSIAN PDFS

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Giele, Keller ’98; Ball et al. ’11; Sato, Owens, Prosper ’14; Paukkunen, Zurita ’14;

1. Convert Hessian error PDFs into replicas

$$f_k = f_0 + \sum_i^N \frac{f_i^{(+)} - f_i^{(-)}}{2} R_{ki},$$

2. Calculate weights for each replica

$$w_k = \frac{e^{-\frac{1}{2}\chi_k^2/T}}{\frac{1}{N_{\rm rep}} \sum_i^{N_{\rm rep}} e^{-\frac{1}{2}\chi_k^2/T}}, \qquad \chi_k^2 = \sum_j^{N_{\rm data}} \frac{(D_j - T_j^k)^2}{\sigma_j^2}$$

3. Calculate observables with new (reweighted) PDFs

$$\langle \mathcal{O} \rangle_{\rm new} = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} w_k \mathcal{O}(f_k),$$

$$\delta \langle \mathcal{O} \rangle_{\rm new} = \sqrt{\frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} w_k \left(\mathcal{O}(f_k) - \langle \mathcal{O} \rangle\right)^2}.$$

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Giele, Keller '98; Ball et al. '11; Sato, Owens, Prosper '14; Paukkunen, Zurita '14;

1. Convert Hessian error PDFs into replicas

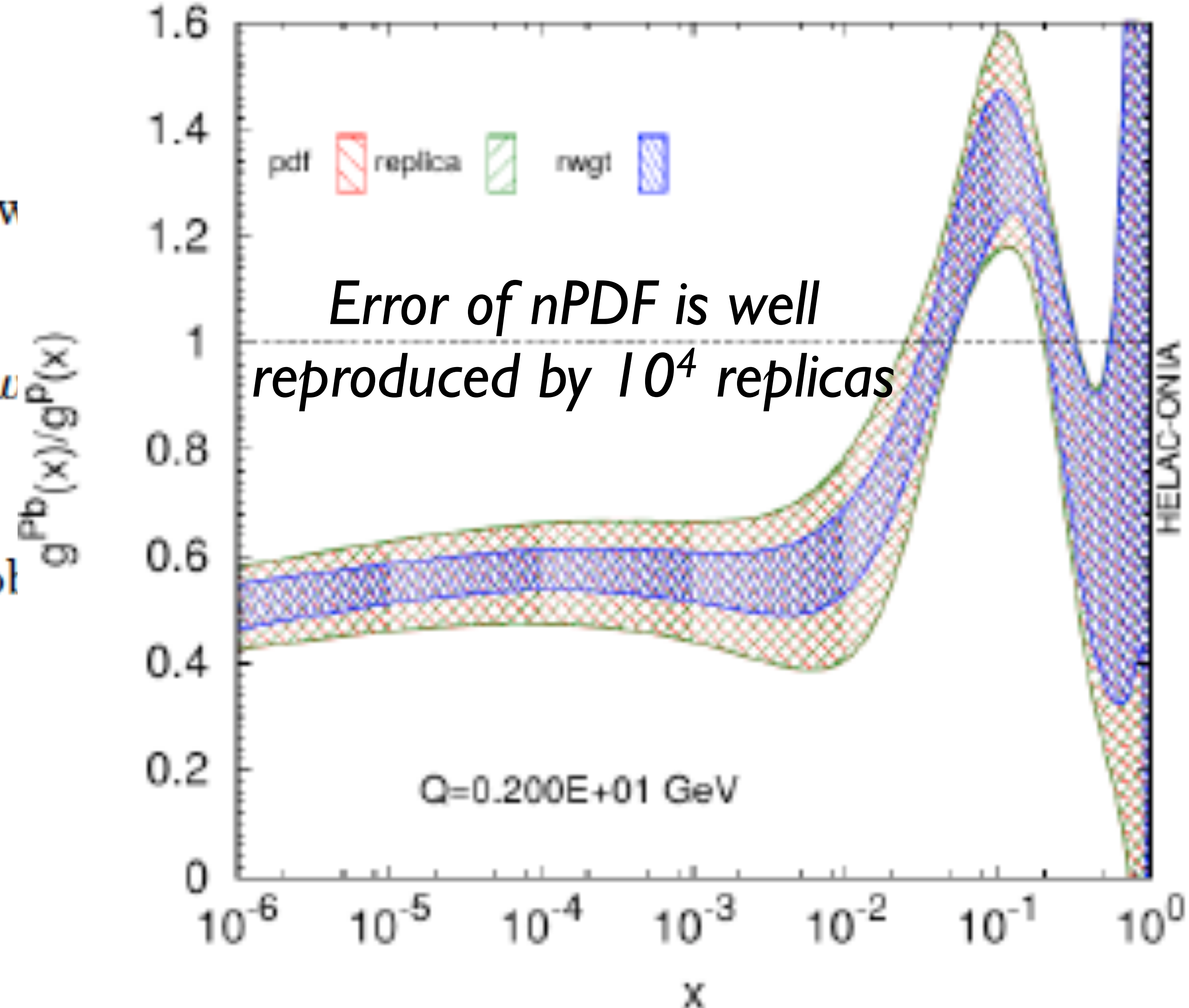
$N \quad (+) \quad (-)$

2. Calculate w

$w$

$i - T_j^k)^2$ / $\sigma_j^2$

3. Calculate ol




$\sqrt{\phantom{x}} \; N_{\rm rep} \quad k=1$



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