Technische Universität München

The charm/bottom quark mass

from heavy quarkonium at N^3LO

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Based on work in collaboration with A. Pineda, M. Stahlhofen and J. Segovia
arxiv:1511.08210, arxiv:1806.05197

Outline

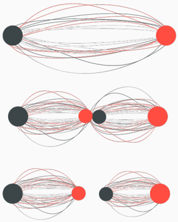
1. Heavy quarkonium at N^3LO
2. The quark masses
3. Other shifts and determination of α_s
4. Summary and conclusions

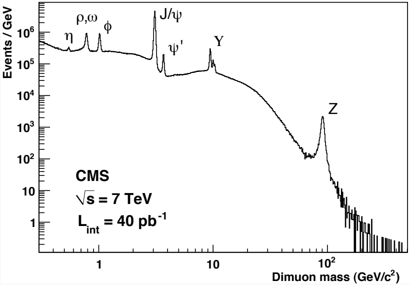
Heavy quarkonium at N^{3}LO

Motivation: The QCD parameters

QCD describes the interaction of quarks and gluons

- It is asymptotically free: predictability at high energies
- The strong interaction grows at large distances: confinement





Need for **precision** in QCD parameters: α_s and quark masses

- Tower of $q\bar{q}$ **bound states**: e.g. $J/\psi(c\bar{c})$, Υ $(b\bar{b})$, etc.

Bound states as NR systems

- q and $\bar{q}$ move with small relative velocity $v \ll 1$

Multi-scale problem

- Bound state scales:

  Hard: m_r, **Soft:** $|\mathbf{p}| \sim \frac{1}{r} \sim m_r v$, **Ultrasoft:** $E \sim \frac{\mathbf{p}^2}{2m_r} \sim m_r v^2$

- Confinement scale: **QCD:** Λ_{QCD}

- Non-relativistic systems fulfill the relation: $\boxed{m_r \gg |\mathbf{p}| \gg E}$

- **Heavy quark:** $\boxed{m_r \gg \Lambda_{\mathrm{QCD}}}$

  Strong coupling regime: $|\mathbf{p}| \sim \Lambda_{\mathrm{QCD}}$

  Weak coupling regime: $|\mathbf{p}| \gg \Lambda_{\mathrm{QCD}}$, Coulomb-like potential

Bound states as NR systems

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Multi-scale problem

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There is a hierarchy of scales $\Rightarrow$ Effective field theory

integrate out the hard and soft scales to obtain **pNRQCD**

pNRQCD at N^3LO

$$\left.\begin{array}{l}\left(i\partial_0 - \frac{\mathbf{p}^2}{2m_r} - V^{(0)}(r)\right)\phi(\mathbf{r}) = 0 \\ + \text{corrections to the potential} \\ + \text{interaction with other low-energy degrees of freedom}\end{array}\right\} \text{pNRQCD.}$$

Brambilla, Pineda, Soto, Vairo

pNRQCD at N^3LO

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The singlet potential

$$V_s(\mathbf{p},\mathbf{r}) = V^{(0)}(r) + \frac{V^{(1,0)}(r)}{m_1} + \frac{V^{(0,1)}(r)}{m_2} + \frac{V^{(2,0)}(\mathbf{p},\mathbf{r})}{m_1^2} + \frac{V^{(0,2)}(\mathbf{p},\mathbf{r})}{m_2^2} + \frac{V^{(1,1)}(\mathbf{p},\mathbf{r})}{m_1 m_2}$$

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3-loops 2-loops 1-loop

Wilson coefficients of the EFT: **matching** and **renormalization**

scheme dependent

pNRQCD at N^3LO

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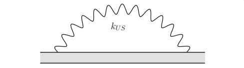
Brambilla, Pineda, Soto, Vairo

Contribution of US gluons:

perturbative for $mv^2 \gg \Lambda_{\rm QCD}$



pNRQCD at N^3LO

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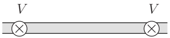
Brambilla, Pineda, Soto, Vairo

Energy levels

$$E(n,l,s,s^-,j) = E_n^C \left(1 + \frac{\alpha_s}{\pi} P_1(L_\nu) + \left(\frac{\alpha_s}{\pi}\right)^2 P_2(L_\nu) + \left(\frac{\alpha_s}{\pi}\right)^3 P_3(L_\nu)\right),$$

- Expectation value + quantum mechanical perturbation theory



- Nonperturbative effects: parametrically $\sim \Lambda_{\mathrm{QCD}}^3 \langle r^2 \rangle$

History of spectrum computations

Pineda, Yndurain (1998)	$\mathcal{O}(m\alpha_s^4)$	$m_1 = m_2$
Brambilla et al. (2000)	$\mathcal{O}(m\alpha_s^4)$	$n = 1, m_1 \neq m_2$
Penin, Steinhauser (2002)	$\mathcal{O}(m\alpha_s^5)$	$n = 1, m_1 = m_2$
Beneke et al., Penin et al. (2005)	$\mathcal{O}(m\alpha_s^5)$	S-wave, $m_1 = m_2$
Kiyo, Sumino (2014)	$\mathcal{O}(m\alpha_s^5)$	$m_1 = m_2$
CP, Pineda, Stahlhofen (2015)	$\mathcal{O}(m\alpha_s^5)$	$m_1 \neq m_2$

Application to phenomenology

Physical systems:

$$\begin{cases} \text{Bottomonium (1S):} & |\mathbf{p}| \sim m_b v \sim 1.3 \text{ GeV} \\ B_c \text{ (1S):} & |\mathbf{p}| \sim 2 m_r v \sim 0.85 \text{ GeV} \\ \text{Charmonium (1S):} & |\mathbf{p}| \sim m_c v \sim 0.68 \text{ GeV} \end{cases}$$

How good are the results from weak coupling ($|\mathbf{p}| \gg \Lambda_{QCD}$)?
Is the US scale perturbative ($E \gg \Lambda_{QCD}$)?

Approach: obtain maximum improvement in weak coupling and **assess**

- Achieve high orders in PT: next-to-next-to-next-to
- Accelerate convergence
  - thershold masses
  - resumamtion of large logarithms
  - alternative computation schemes

Application to phenomenology

Physical systems:

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Approach: obtain maximum improvement in weak coupling and **assess**

- Achieve high orders in PT: next-to-next-to-next-to
- Accelerate convergence
  - thershold masses
  - resummation of large logarithms: see Dani Moreno's and my talk on Wed.
  - alternative computation schemes (see 1806.05197,1809.09124)

The quark masses

Convergence of the perturbative series

$$\mathcal{L} = \sum_i \frac{1}{m_q^i} C_i \mathcal{O}_i, \qquad C_i(\nu) = \tilde{C}_i + \sum_{n=0}^{\infty} C_{i,n} \alpha_s^{n+1}$$

- Wilson coefficients are **asymptotic** : $\boxed{C_{i,n} \sim n!}$

$\Rightarrow$ **BUT** comply the OPE: $m_q = m_{os} + \tilde{\Lambda}_{QCD}$ is renormalon free

$$m_{os} = m_{\overline{\rm MS}} \left(1 + B_1 \alpha_s + B_2 \alpha_s^2 + \dots \right), \qquad B_n \sim n!$$

- Redefine the mass such that C_i is **not asymptotic**: **threshold mass**

Renormalon subtracted schemes:

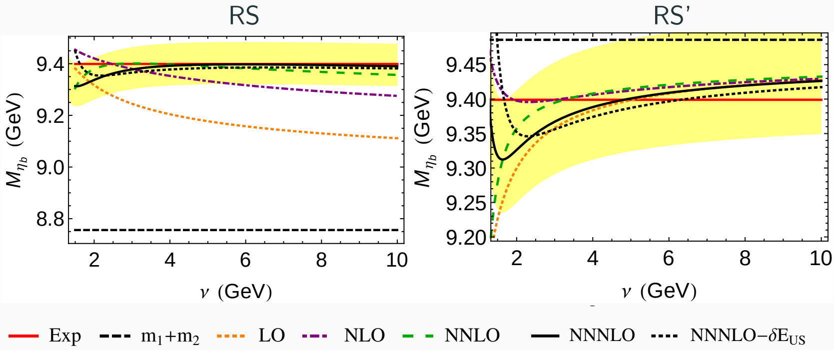
$$m_{\mathrm{RS}(')} = m_{OS} - N_m \pi \nu_f \sum_{N=0(1)}^{\infty} \left(\frac{\beta_0}{2} \right)^N \left(\frac{\alpha(n_l, \nu_f)}{\pi} \right)^{N+1} \sum_{n=0}^{\infty} \tilde{\chi}_n \frac{\Gamma(b+N+1-n)}{\Gamma(b+1-n)}$$

Pineda

- Introduction of a new scale $\nu_f \sim m_r \alpha_s$

The bottom quark mass

Fit $m_{b,RS(')}$: $M_{\eta_b(1S)} = 2m_{b,RS(')} + E(1,0)$



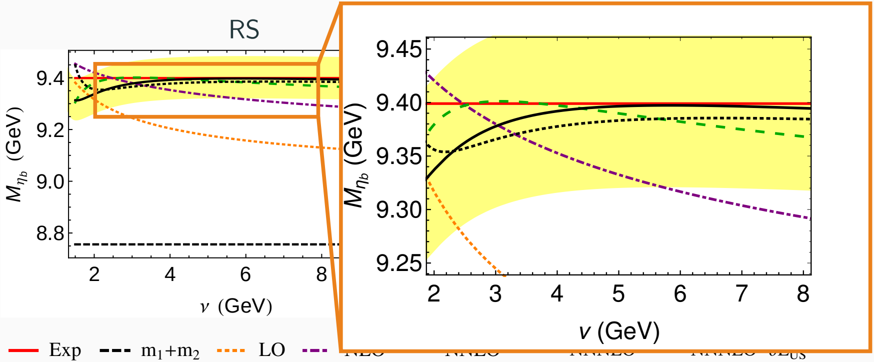
For $\nu = 5^{+3}_{-2}$ GeV, $\nu_f = 2^{+1}_{-1}$ GeV:

$$m_{b,\mathrm{RS}}(2\ \mathrm{GeV}) = 4\,379^{+1}_{+31}(\nu)^{-4}_{+5}(\nu_f)^{-5}_{+5}(\alpha_s)^{-32}_{+32}(N_m)$$

$$m_{b,\mathrm{RS}'}(2\ \mathrm{GeV}) = 4\,742^{-10}_{+39}(\nu)^{-2}_{+3}(\nu_f)^{+4}_{-4}(\alpha_s)^{-15}_{+15}(N_m)$$

The bottom quark mass

Fit $m_{b,RS(')}$: $M_{\eta_b(1S)} = 2m_{b,RS(')} + E(1,0)$



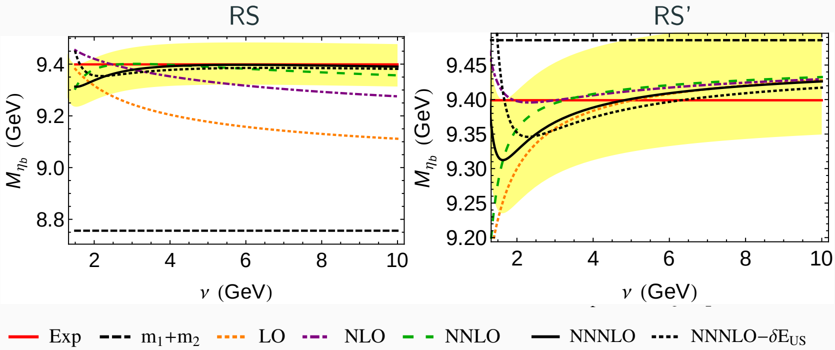
For $\nu = 5^{+3}_{-2}$ GeV, $\nu_f = 2^{+1}_{-1}$ GeV:

$$m_{b,\mathrm{RS}}(2\ \mathrm{GeV}) = \quad (4185 + 145 + 58 + 9 - 18)\ \mathrm{MeV}$$

$$m_{b,\mathrm{RS'}}(2\ \mathrm{GeV}) = \quad (4183 + 473 + 86 + 16 - 16)\ \mathrm{MeV}\ .$$

The bottom quark mass

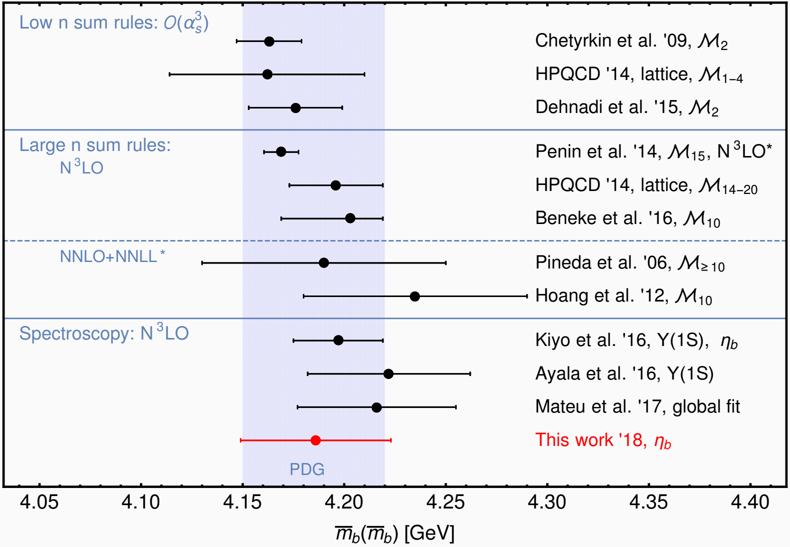
Fit $m_{b,RS(')}$: $M_{\eta_b(1S)} = 2m_{b,RS(')} + E(1,0)$



At $\nu = 2.5$ GeV:

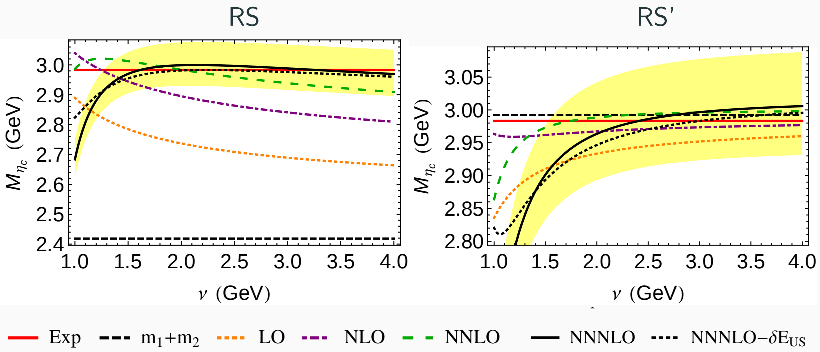
$$\overline{m}_b(\overline{m}_b) = 4186(37)\mathrm{MeV}$$

Other determinations of the bottom mass



The charm quark mass

Fit $m_{c,RS(')}$: $M_{\eta_c(1S)} = 2m_{c,RS(')} + E(1,0)$



For $\nu = 2.5^{+1.5}_{-1}$ GeV, $\nu_f = 1^{+0.5}_{-0.3}$ GeV:

$$m_{c,\mathrm{RS}}(1\mathrm{GeV}) = 1\,202^{+15}_{+16}(\nu)^{-15}_{+11}(\nu_f)^{-10}_{+10}(\alpha_s)^{-34}_{+34}(N_m)$$

$$m_{c,\mathrm{RS'}}(1\ \mathrm{GeV}) = 1\,495^{-11}_{+50}(\nu)^{-9}_{+20}(\nu_f)^{+4}_{-4}(\alpha_s)^{-20}_{+20}(N_m)$$

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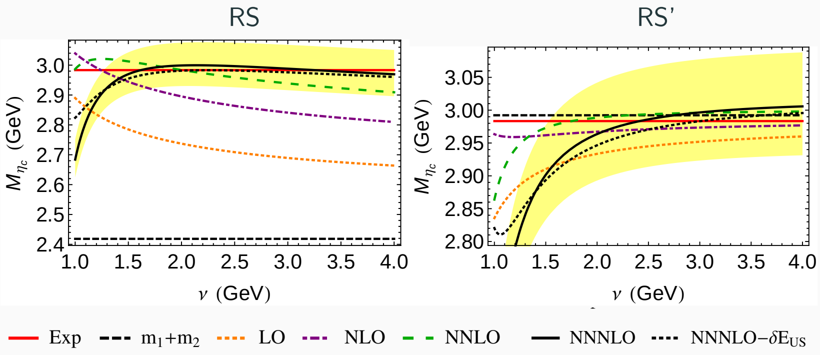
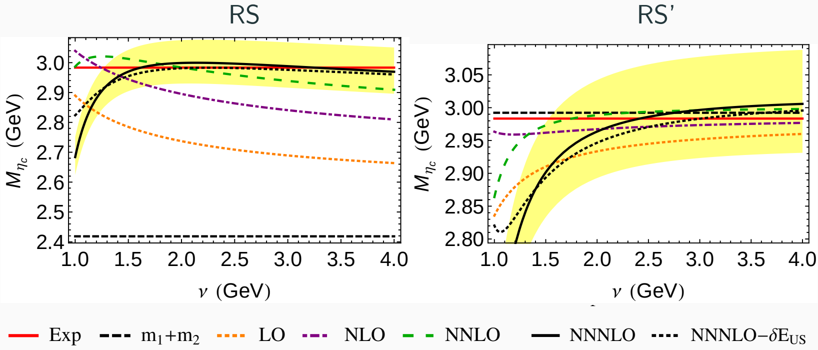


For $\nu = 2.5^{+1.5}_{-1}$ GeV, $\nu_f = 1^{+0.5}_{-0.3}$ GeV:

$$m_{c,\mathrm{RS}}(1\ \mathrm{GeV}) = (1217 - 35 + 18 + 11 - 9)\ \mathrm{MeV}$$

$$m_{c,\mathrm{RS}'}(1\ \mathrm{GeV}) = (1222 + 182 + 62 + 29 - 1)\ \mathrm{MeV}$$

Fit $m_{c,RS(')}$: $M_{\eta_c(1S)} = 2m_{c,RS(')} + E(1,0)$

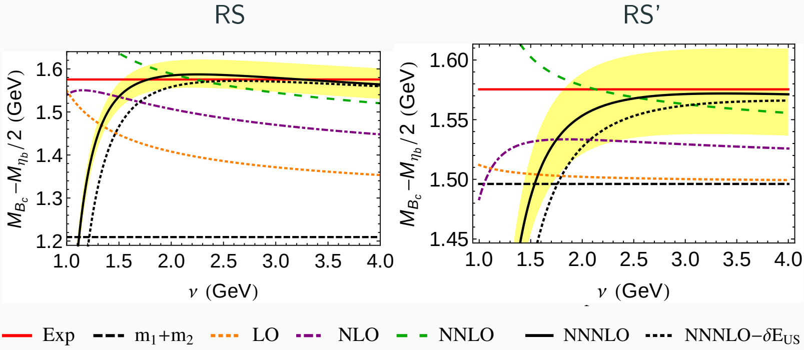


At $\nu = 1.5$ GeV:

$$\overline{m}_c(\overline{m}_c) = 1220(45)\mathrm{MeV}$$

The charm quark mass

Fit $m_{c,RS(')}$: $M_{B_c} - M_{\eta_b}/2 = m_{c,RS(')} + E(1,0)$



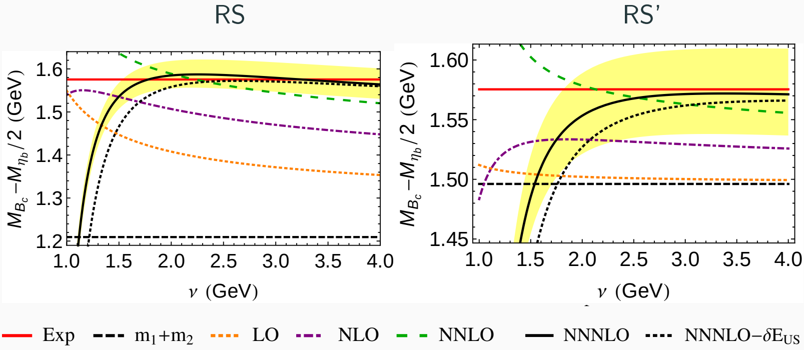
For $\nu = 3^{+1.5}_{-1}$ GeV, $\nu_f = 1^{+0.5}_{-0.3}$ GeV:

$$m_{c,\mathrm{RS}}(1\mathrm{GeV}) = 1\,204^{+27}_{-8}(\nu)^{-26}_{+18}(\nu_f)^{-17}_{+16}(\alpha_s)^{-33}_{+33}(N_m)^{-1}_{+1}(\overline{m}_b)$$

$$m_{c,\mathrm{RS'}}(1\ \mathrm{GeV}) = 1\,501^{+1}_{+23}(\nu)^{-14}_{-27}(\nu_f)^{-2}_{+2}(\alpha_s)^{-18}_{+18}(N_m)^{-1}_{+1}(\overline{m}_b)$$

The charm quark mass

Fit $m_{c,RS(')}$: $M_{B_c} - M_{\eta_b}/2 = m_{c,RS(')} + E(1,0)$



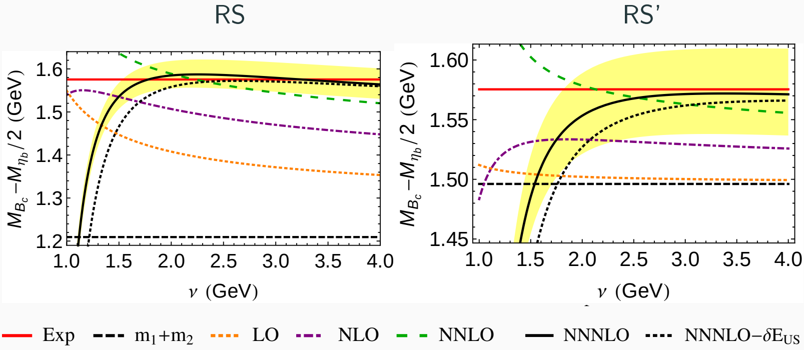
For $\nu = 3^{+1.5}_{-1}$ GeV, $\nu_f = 1^{+0.5}_{-0.3}$ GeV:

$$m_{c,\mathrm{RS}}(1\ \mathrm{GeV}) = (1220 - 35 + 18 + 11 - 10)\ \mathrm{MeV}$$

$$m_{c,\mathrm{RS'}}(1\ \mathrm{GeV}) = (1227 + 183 + 62 + 30 - 1)\ \mathrm{MeV}$$

The charm quark mass

Fit $m_{c,RS(')}$: $M_{B_c} - M_{\eta_b}/2 = m_{c,RS(')} + E(1,0)$



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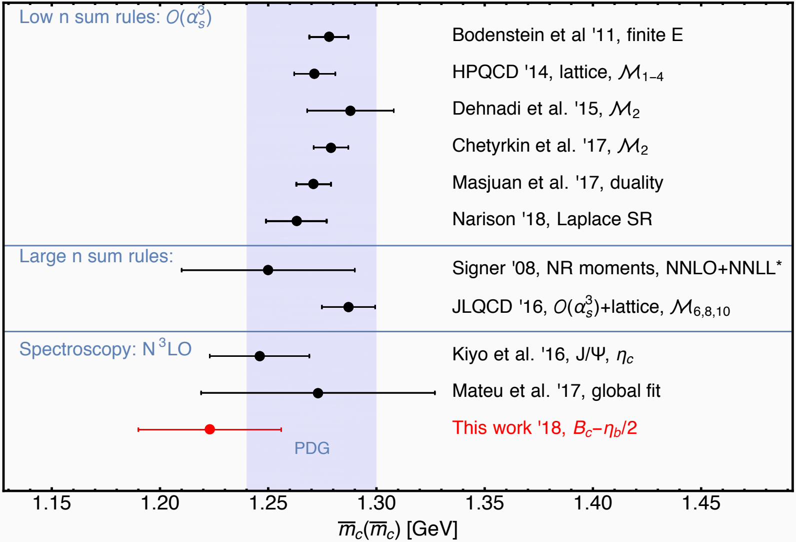
Other determinations of the charm mass



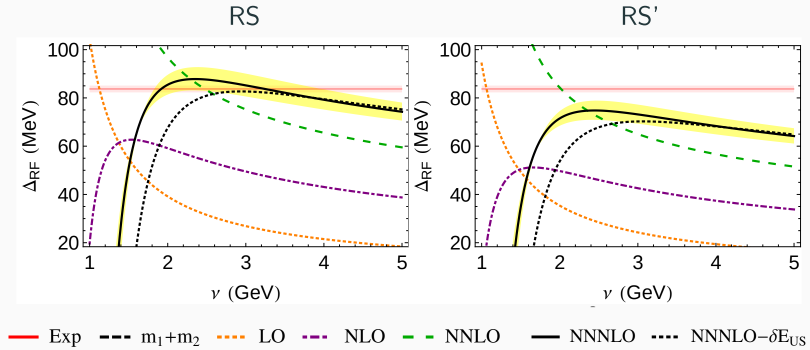
Other shifts and determination of α_s

The renormalon free shift for B_c and α_s

Consider:

$$\Delta_{\rm RF} = M_{B_c} - M_{\eta_b}/2 - M_{\eta_c}/2 = E(1,0)|_{B_c} - E(1,0)/2|_{\eta_b} - E(1,0)/2|_{\eta_c}$$

with $\nu_f = 1$ GeV



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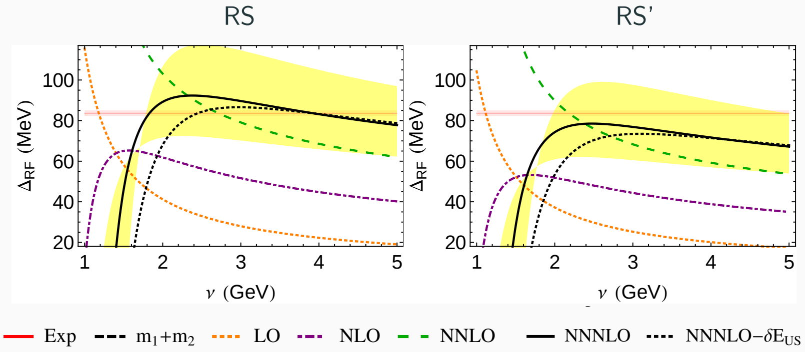
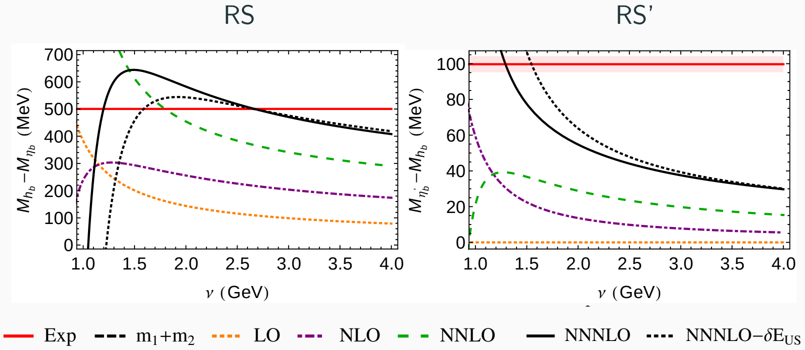


$$\alpha_s(M_z) = 0.1195(53)$$

Other renormalon free shifts

Consider also $n = 2$: $M_{h_b} - M_{\eta_b(1S)}$, $\quad M_{\eta_b(2S)} - M_{h_b}$

with $\nu_f = 1$ GeV



Summary and conclusions

Summary and conclusions

- We compute the **N^3LO spectrum** in pNRQCD for **different masses**
  - The potentials obtained are valid for $\mathbf{mv} \gg \Lambda_{\rm QCD}$
  - The US contribution is valid for $\mathbf{mv^2} \gg \Lambda_{\rm QCD}$

From M_{η_b} and $M_{B_c} - M_{\eta_b}/2$ @N^{3}LO we obtain

$$\overline{m}_b(\overline{m}_b) = 4186(37)\mathrm{MeV} \qquad \overline{m}_c(\overline{m}_c) = 1223(33)\mathrm{MeV}$$

From $\Delta_{\rm RF} = M_{B_c} - M_{\eta_b}/2 - M_{\eta_c}/2$ @N^{3}LO

$$\alpha_s(M_z) = 0.1195(53)$$

- The ultrasoft effects look small

Improvements in the near future:

- Resummation of large logs
- Promising new threshold masses: TUMQCD '17, Ayala et al. '19

Thank you!

Alternative computational scheme

- Exact solution of the Schrödinger equation:

$$\left[\frac{\mathbf{p}^2}{2m_r} + V_N^{(0)}(r;\nu)\right]\phi_{nl}^{(0)}(\mathbf{r}) = E_{nl}^{(0)}\phi_{nl}^{(0)}(\mathbf{r})$$

where

$$V_N^{(0)}(r;\nu) = -\frac{C_f\,\alpha_s(\nu)}{r}\left\{1 + \sum_{n=1}^{N}\left(\frac{\alpha_s(\nu)}{4\pi}\right)^n a_n(\nu;r)\right\}$$

Definition of the RS scheme:

$$V_{N,\mathrm{RS}'}^{(0)}(r) = \begin{cases} (V_N^{(0)} + 2\delta m_{\mathrm{RS}'}^{(N)})|_{\nu=\nu} \equiv \sum_{n=0}^{N} V_{\mathrm{RS}',n}\alpha_s^{n+1}(\nu) & \text{if} \quad r > \nu_r^{-1} \\ (V_N^{(0)} + 2\delta m_{\mathrm{RS}'}^{(N)})|_{\nu=1/r} \equiv \sum_{n=0}^{N} V_{\mathrm{RS}',n}\alpha_s^{n+1}(1/r) & \text{if} \quad r < \nu_r^{-1}. \end{cases}$$

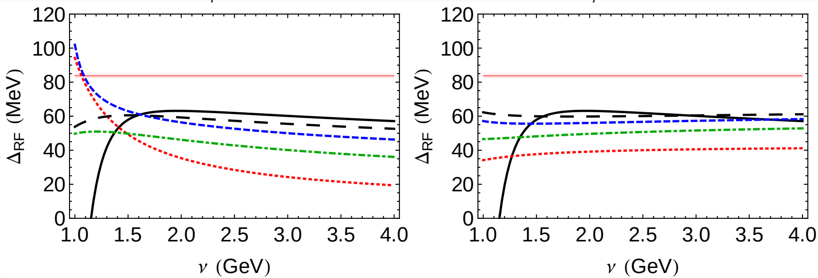
Relativistic potentials:

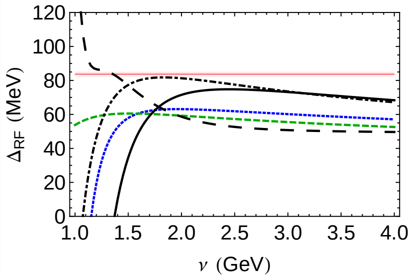
$$\delta E^{\mathrm{nc}} = \langle\phi^{(0)}|\delta V|\phi^{(0)}\rangle$$

where $\delta V = V_s - V^{(0)}$

Alternative scheme for Δ_{RF}

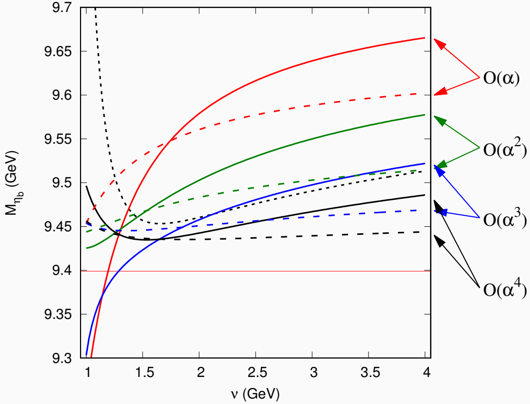
$\nu_r = \infty$ $\qquad$ $\nu_r = 1$ GeV





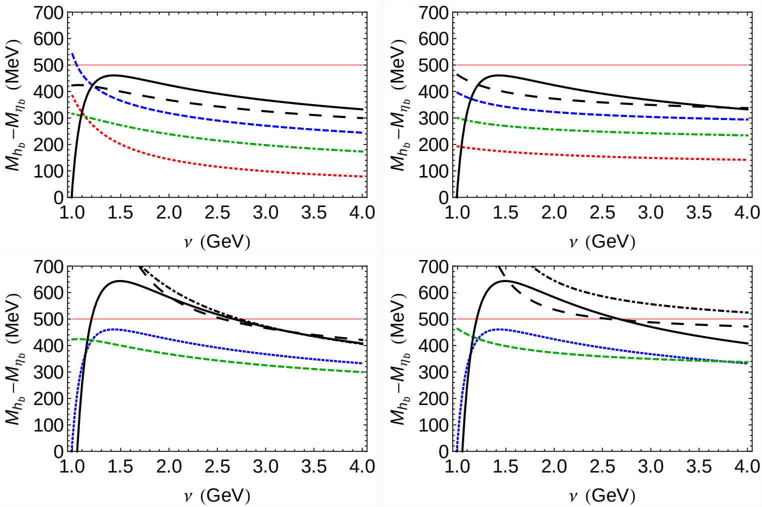
Higher order resummation for $\eta_b(1S)$

Effect of the factorization scale:



- $\nu_r = \infty$: solid
- $\nu_r = 1$ GeV: dashed

Renormalon free shifts



Renormalon free shifts