R measurement at KEDR

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KEDR collaboration

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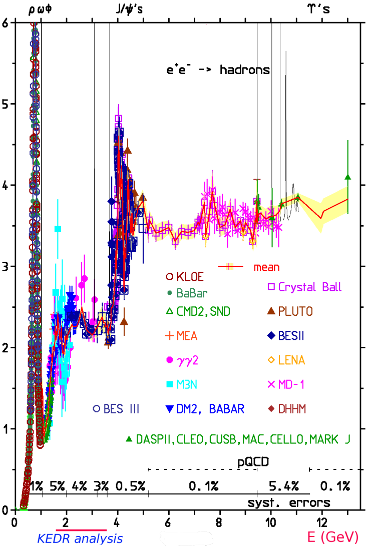
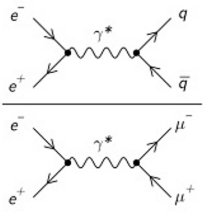
The 13th International Workshop on Jeavy Quarconium

$R(s)$ measurement. Motivation.



F. Jegerlehner arXiv:1511.0447

$$R = \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^-\mu^+)} \approx$$



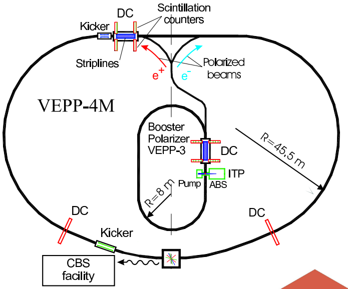
In first approximation:

$$R(s) \simeq 3 \sum e_q^2$$

$R(s)$ is used to determine:

- $\alpha_s(s)$
- $(g_\mu - 2)/2$
- $\alpha(M_Z^2)$
- m_Q

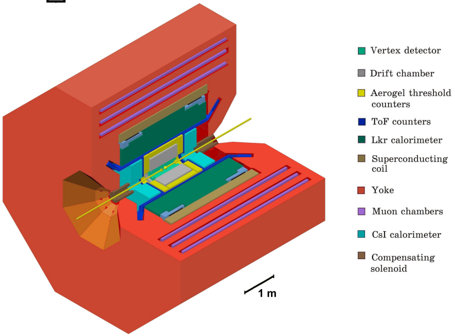
VEPP-4M and KEDR



Beam energy	$1 \div 5$ GeV
Number of bunches	2×2
Luminosity 1.8 GeV	1.5×10^{30} cm^{-2} s^{-1}

Energy measurement:

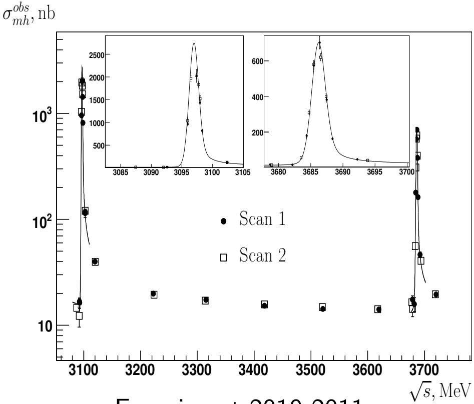
- Resonant depolarization method:
  Instant measurement accuracy ~ 1 keV
  Energy interpolation accuracy $10 \div 30$ keV
- Compton backscattering method ~ 100 keV



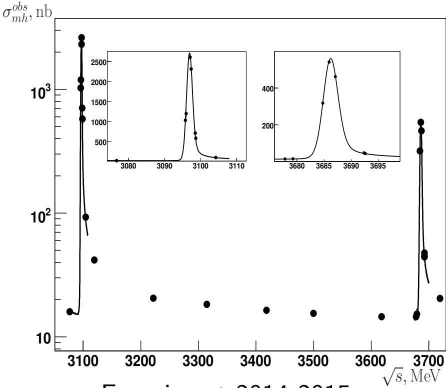
Kedr is a siberian pine somewhat similar to lebanon cedar

R measurement between J/ψ and $\psi(2S)$

The observed multihadron cross section as a function of the c.m. energy



Experiment 2010-2011



Experiment 2014-2015

- The c.m. energy range between 3.076 and 3.72 GeV studied
- An integrated luminosity of 2.7 pb^{-1} collected at 9 energies 3.077, 3.120,3.223, 3.315, 3.418, 3.500, 3.521, 3.618, 3.719 GeV
- $\sim(2-6)\times10^3$ m.h. events per point, $\sim 38\times10^3$ in total

Analysis

The way that we are measuring R:

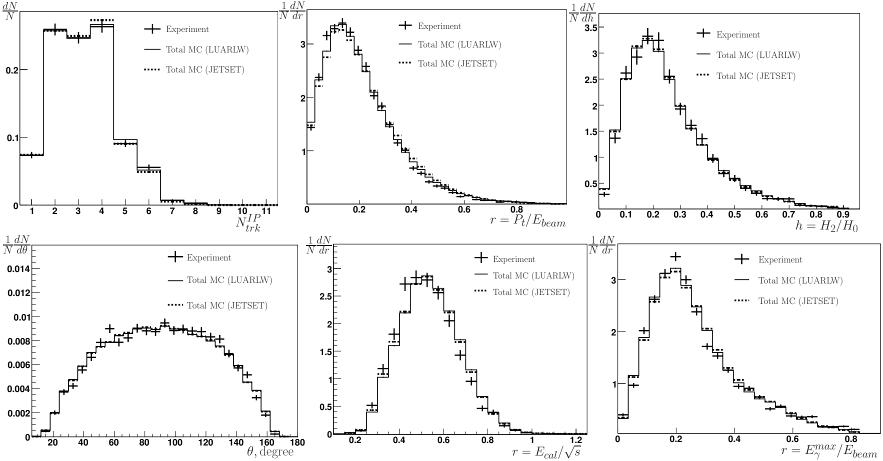
$$R = \frac{\sigma_{obs}(s) - \sum \varepsilon_{\psi}^{tail}(s)\sigma_{\psi}^{tail}(s) - \sum \varepsilon_{bg}^{i}(s)\sigma_{bg}^{i}(s)}{\varepsilon(s)(1+\delta(s))\sigma_{\mu\mu}^{0}}$$

with $\sigma_{obs}(s) = \frac{N_{mh} - N_{res.bg.}}{\int \mathcal{L}dt}$ where N_{mh} represent all events pass hadronic selection criteria, $N_{res.bg.}$ – residual machine background
$\sum \varepsilon_{\psi}^{tail}(s)\sigma_{\psi}^{tail}(s)$ is contribution from J/ψ and $\psi(2S)$ resonances
$\sum \varepsilon_{bg}^{i}(s)\sigma_{bg}^{i}(s)$ is contribution from physical processes: $e^+e^- \to l^+l^-$, $\gamma\gamma$-processes.
$\varepsilon(s)$ – multihadron efficiency.

$$1+\delta(s) = \int dx \frac{1}{1-x} \frac{\mathcal{F}(s,x)}{\left|1-\tilde{\Pi}(s(1-x))\right|^2} \frac{\tilde{R}(s(1-x))\varepsilon(s(1-x))}{R(s)\varepsilon(s)}$$

$\mathcal{F}(s,x)$ – radiative correction kernel (**E.A.Kuraev, V.S.Fadin Sov.J.Nucl.Phys.41(466-472)1985**) Here $\tilde{\Pi}$ and $\tilde{R}$ does not includes J/ψ and $\psi(2S)$ resonances. **To determine the contributions of the J/ψ and $\psi(2S)$ without external data, the additional data samples of about 0.4 pb^{-1}(2010-2011) and 0.34 pb^{-1}(2014-2015) were collected in the vicinity of peak regions.**

Simulation: JETSET and LUARLW



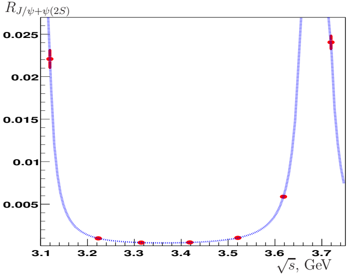
Properties of hadronic events produced in the uds continuum at 3.119 GeV (2014-2015).

Here N is the number of events, N_{trk}^{IP} is the number of tracks originated from IP, P_t is a transverse momentum of the track, H_2 and H_0 are Fox-Wolfram moments, θ is a polar angle of the track, E_{cal} is energy deposited in the calorimeter, E_γ^{max} is energy of the most energetic photon.

Systematic uncertainties

Source	Syst. uncertainty, %		
	Scan 1 and 2 (2010-2011)	Scan 2014-2015	Correlated
Luminosity	1.1	0.9	0.4
Rad. corr.	$0.4 \div 0.6$	$0.5 \div 0.8$	$0.2 \div 0.4$
uds simulation	$1.3 \div 2.0$	1.1	0.9
Track reconstruction	0.5	0.4	–
J/ψ	$0.1 \div 2.7$	$0.1 \div 1.8$	–
$\psi(2S)$ (at 3.72 GeV)	1.4	1.1	–
l^+l^-	$0.1 \div 0.2$	$0.3 \div 0.4$	$0.1 \div 0.2$
e^+e^-X	$0.1 \div 0.2$	0.1	0.1
Trigger	0.2	0.2	0.2
Nuclear interaction	0.2	0.2	0.2
Machine background	$0.5 \div 1.1$	$0.4 \div 0.8$	–
Cuts	0.6	0.6	–
Total	$2.1 \div 3.6$ (correlated $1.8 \div 2.5$)	$1.9 \div 2.7$	1.1

R for $\sqrt{s}$= 3.12 – 3.72 GeV



Using J/ψ and $\psi(2S)$ parameters, we obtain $R_{uds}(s) + R_{J/\psi+\psi(2S)} \Longrightarrow R(s)$

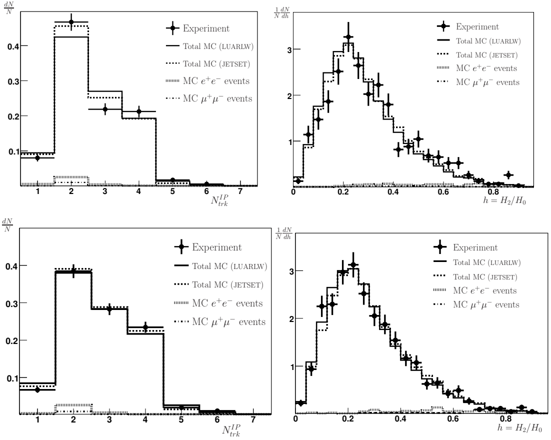
Data 2010-2011		Data 2014-2015		Combination	
$\sqrt{s}$, MeV	$R_{\mathbf{uds}}(s)$	$\sqrt{s}$, MeV	$R_{\mathbf{uds}}(s)$	$\sqrt{s}$, MeV	$R_{\mathbf{uds}}(s)\{R(s)\}$
-	-	3076.7 ± 0.2	2.188 ± 0.056 ± 0.042	3076.7 ± 0.2	2.188 ± 0.056 ± 0.042
3119.9 ± 0.2	2.215 ± 0.089 ± 0.066	3119.2 ± 0.2	2.211 ± 0.046 ± 0.060	3119.6 ± 0.4	2.212{2.235} ± 0.042 ± 0.049
3223.0 ± 0.6	2.172 ± 0.057 ± 0.045	3221.8 ± 0.2	2.214 ± 0.055 ± 0.042	3222.5 ± 0.8	2.194{2.195} ± 0.040 ± 0.035
3314.7 ± 0.7	2.200 ± 0.056 ± 0.043	3314.7 ± 0.4	2.233 ± 0.044 ± 0.042	3314.7 ± 0.6	2.219{2.219} ± 0.035 ± 0.035
3418.2 ± 0.2	2.168 ± 0.050 ± 0.042	3418.3 ± 0.4	2.197 ± 0.047 ± 0.040	3418.3 ± 0.3	2.185{2.185} ± 0.032 ± 0.035
-	-	3499.6 ± 0.4	2.224 ± 0.054 ± 0.040	3499.6 ± 0.4	2.224{2.224} ± 0.054 ± 0.040
3520.8 ± 0.4	2.200 ± 0.050 ± 0.044	-	-	3520.8 ± 0.4	2.200{2.201} ± 0.050 ± 0.044
3618.2 ± 1.0	2.201 ± 0.059 ± 0.044	3618.1 ± 0.4	2.220 ± 0.049 ± 0.042	3618.2 ± 0.7	2.212{2.218} ± 0.038 ± 0.035
3719.4 ± 0.7	2.187 ± 0.068 ± 0.060	3719.6 ± 0.2	2.213 ± 0.047 ± 0.049	3719.5 ± 0.5	2.204{2.228} ± 0.039 ± 0.042

V.V.Anashin et al., Phys.Lett. B 753, 533-541 (2016).[arXiv:1510.02667]
V.V.Anashin et al., Phys.Lett. B 788, 42-51 (2019).[arXiv:1805.06235]

R for $\sqrt{s}$= 1.84 – 3.05 GeV

- An integrated luminosity 0.66 pb^{-1} collected at 13 equidistant points with a step $\sim$ 0.1 GeV: 1.841, 1.937 ... 3.048 GeV
- $\sim 10^3$ hadronic events per point, 14.8×10^3 events in total
- Simulation of the *uds* continuum based on the LUARLW generator, tuned JETSET alternatively used at 6 points for a cross-check.

Experimental distribution and two variants of MC simulation based on LUARLW and tuned JETSET are plotted ($\sqrt{s} = 1.94$ GeV and $\sqrt{s} = 2.14$ GeV).



R for $\sqrt{s} = 1.84 - 3.05$ GeV

Measured value of $R = \frac{\sigma_{obs}(s) - \sum \varepsilon^i_{bg}(s)\sigma^i_{bg}(s)}{\varepsilon(s)(1+\delta(s))\sigma^0_{\mu\mu}}$

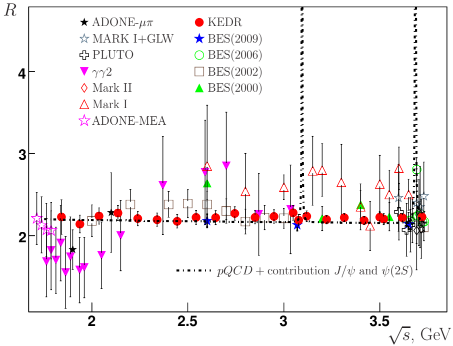
$\sqrt{s}$, MeV	$R(s)$
1841.0	$2.226 \pm 0.139 \pm 0.158$
1937.0	$2.141 \pm 0.081 \pm 0.073$
2037.3	$2.238 \pm 0.068 \pm 0.072$
2135.7	$2.275 \pm 0.072 \pm 0.055$
2239.2	$2.208 \pm 0.069 \pm 0.053$
2339.5	$2.194 \pm 0.064 \pm 0.048$
2444.1	$2.175 \pm 0.067 \pm 0.048$
2542.6	$2.222 \pm 0.070 \pm 0.047$
2644.8	$2.220 \pm 0.069 \pm 0.049$
2744.6	$2.269 \pm 0.065 \pm 0.050$
2849.7	$2.223 \pm 0.065 \pm 0.047$
2948.9	$2.234 \pm 0.064 \pm 0.051$
3048.1	$2.278 \pm 0.075 \pm 0.048$

The main systematic uncertainties in the R:

Source	Error,%
Luminosity	1.2
Rad. corr.	$0.5 \div 2.0$
uds simulation	$1.2 \div 6.6$
l^+l^-	$0.3 \div 0.6$
e^+e^-X	0.2
Trigger	0.3
Nuclear interaction	0.4
Machine background	$0.4 \div 0.9$
Cuts	0.7
Total	$2.1 \div 7.1$

V.V. Anashin et al., Phys.Lett. B 770C, 174 (2017)

Comparison with others experiments



The quantity R versus the c.m. energy and the sum of the prediction of perturbative QCD and a contribution of narrow resonances.

In the c.m.energy range 3.08-3.72 GeV the weighted average $\overline{R}_{\mathrm{uds}} = 2.204 \pm 0.014 \pm 0.026$ is approximately one sigma higher than that theoretically expected, $R_{\mathrm{uds}}^{\mathrm{pQCD}} = 2.16 \pm 0.01$ calculated according to the pQCD In the lower c.m.energy range 1.84-3.05 GeV the weighted average is $2.225 \pm 0.020 \pm 0.047$ (the pQCD prediction of 2.18 ± 0.02).

An application of the $R(s)$

Correlated uncertainties of R_{uds} in %

Source	Uncertainty in %	
	Data 2010	Data 2010 / 2011,2014
Luminosity		
Cross section calc.	0.5	0.4
Calorimeter response	0.7	-
Calorimeter alignment	0.2	0.2
Rad. correction		
Π approx.	0.3	0.1
$\delta R_{\text{uds}}(s)$	0.2	0.2
$\delta\epsilon(s)$	0.3	0.2
Continuum simulation	1.2	$0.4 \div 0.8$
Track reconstr.	0.5	0.4
e^+e^-X contribution	0.2	0.1
l^+l^- contribution	0.3	0.2
Trigger efficiency	0.3	0.2
Nuclear interaction	0.4	0.2
Sum in quadrature	1.8	$0.8 \div 1.1$

$$R_{\text{uds}}(s) \simeq 2\times\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s^2}{\pi^2}\times\left(\frac{365}{24}-9\zeta_3-\frac{11}{4}\right)\right)$$

where ζ is the Euler-Riemann zeta function,

$$\alpha_s(s) = \frac{1}{b_0 t}\left(1 - \frac{b_1 l}{b_0^2 t} + \frac{b_1(l^2 - l - 1) + b_0 b_2}{b_0^4 t^2} + \frac{b_1^3(-2l^3 + 5l^2 + 4l - 1) - 6 b_0 b_2 b_1 l + b_0^2 b^3}{2 b_0^6 t^3}\right)$$

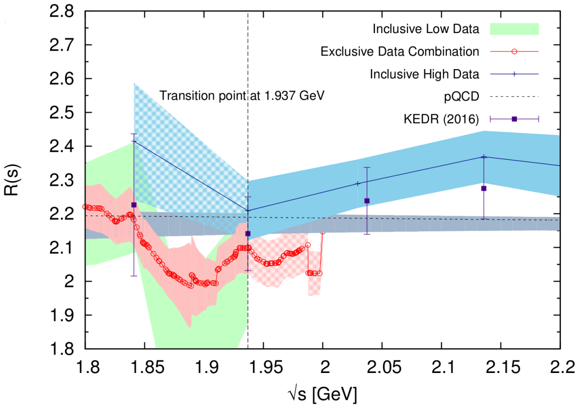
with $t = \ln\frac{s}{\Lambda^2}$, $l = \ln t$ parametrized in terms of the QCD scale parameter Λ and coefficients b_0, b_1, b_3 (can be found in PDG). To determine Λ, we minimise the χ^2 function

$$\chi^2 = \sum_i\sum_j\left(R^{\text{meas}}_{\text{uds}}(s_i) - R^{\text{calc}}_{\text{uds}}(s_i)\right)C^{-1}_{ij}\left(R^{\text{meas}}_{uds}(s_j) - R^{\text{calc}}_{\text{uds}}(s_j)\right),$$

The obtained value of $\Lambda = 0.361^{+0.155}_{-0.174}$ GeV corresponds to $\alpha_s(m_\tau) = 0.332^{+0.100}_{-0.092}$. If the next order of pQCD is included in the expansion of R_{uds}, the fitting results are as follows: $\Lambda = 0.437^{+0.210}_{-0.215}$ GeV and $\alpha_s(m_\tau) = 0.378^{+0.173}_{-0.120}$.

$\alpha_s(m_\tau)$ determined from our $R(s)$ results is consistent with obtained in semileptonic τ decays ($\alpha_s(m_\tau) = 0.331 \pm 0.013$)

Comparison with exclusive data

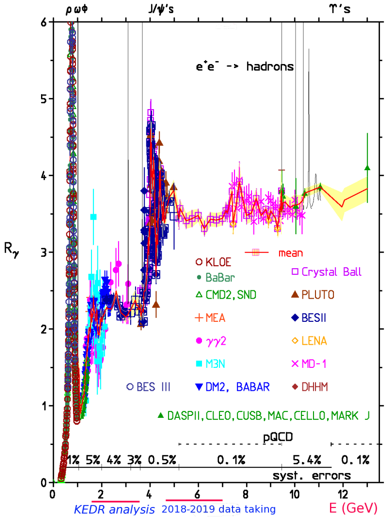


A. Keshavarzi,D. Nomura and T. Teubner. The muon $g-2$ and $\alpha(M_Z^2)$: a new data-based analysis. Phys. Rev. D **97**, 114025 (2018).[arXiv:1802.02995].

Outlook

R measurement in the energy range
4.56-6.96 GeV.

- First scan finished in 2018. An integrated luminosity $\sim$ 4 pb^{-1} collected at 8 equidistant points with a step $\sim$ 0.3 GeV from 4.71 to 6.81 GeV
- In April 2019 we have started the second scan (10 equidistant points in the energy range 4.56 $\div$ 6.96 GeV)



F. Jegerlehner arXiv:1511.0447

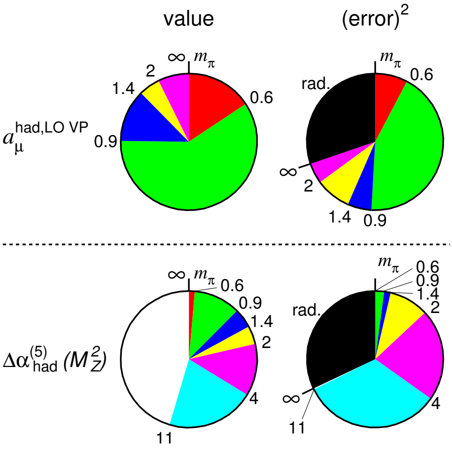
Summary

- KEDR has measured the R values at 22 center-of-mass energies between 1.84 and 3.72 GeV.
- In the energy range between 1.84 and 3.05 GeV the achieved accuracy is about or better than 3.9% at most of the energy points with a systematic uncertainty less than 2.4%.
- For the energies above J/ψ resonance the total error is about or better than 2.6% and a systematic uncertainty of about 1.9%.
- We are taking data in the energy range from 4.56 to 6.96 GeV

Thank you for your time and attention

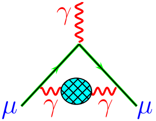
BACKUP SLIDES

R contribution in a_μ and $\alpha(M_Z^2)$



K.Hagiwara et al. arxiv:1105.3149

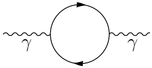
$$a_\mu^{exp} = (g_\mu - 2)/2$$



$$a_\mu^{LO\ \ VP} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{K(s)R(s)}{s}ds$$

Low energy contributions dominate

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}$$

$\Delta\alpha = \sum_f$  $= \Delta\alpha_{\mathbf{lep}}(s) + \Delta\alpha_{\mathbf{had}}(s)$

$$\Delta\alpha^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\,\mathrm{Re}\int_{m_\pi^2}^{\infty}\frac{R(s)ds}{s(s-M_Z^2-i\epsilon)}$$

$\sigma^{e^+e^-\to \text{hadrons}}$ and $\sigma^{e^+e^-\to e^+e^-}$ nearby a narrow resonance

In the soft photon approximation analytical expression for the annihilation cross section nearby a narrow resonance.
Ya.I. Azimov *et al.* JETP Lett. 21 (1975) 172. With up-today modifications one has

$$\sigma^{e^+e^-\to \text{hadr}}(s) = \sigma^{e^+e^-\to \text{hadr}}_{\text{continuum}} + \frac{12\pi}{s}\,(1+\delta_{sf})\left[\frac{\Gamma_{ee}\tilde{\Gamma}_h}{\Gamma M}\,\text{Im}\,f(s) - \frac{2\alpha\sqrt{R\,\Gamma_{ee}\tilde{\Gamma}_h}}{3\sqrt{s}}\,\lambda\,\text{Re}\,\frac{f^*(s)}{1-\Pi_0}\right],$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{ee\to ee} = \left(\frac{d\sigma}{d\Omega}\right)^{ee\to ee}_{\text{QED}} + \frac{1}{s}\,(1+\delta_{sf})\left\{\frac{9}{4}\frac{\Gamma_{ee}^2}{\Gamma M}(1+\cos^2\theta)\,\text{Im}\,f - \right.$$
$$\left.\frac{3\alpha}{2}\frac{\Gamma_{ee}}{M}\left[(1+\cos^2\theta)\,\text{Re}\,\frac{f^*}{1-\Pi_0(s)} - \frac{(1+\cos\theta)^2}{(1-\cos\theta)}\,\text{Re}\,\frac{f^*}{1-\Pi_0(t)}\right]\right\},$$

Recently it was verified in the work X. Y. Zhou, Y. D. Wang and L. G. Xia, Chin. Phys. C **41** (2017) no.8,083001

$$\delta = \frac{3}{4}\beta + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + \beta^2\left(\frac{37}{96} - \frac{\pi^2}{12} - \frac{L}{72}\right), \quad L = \ln\left(s/m_e^2\right), \beta = \frac{2\alpha}{\pi}(L-1),$$

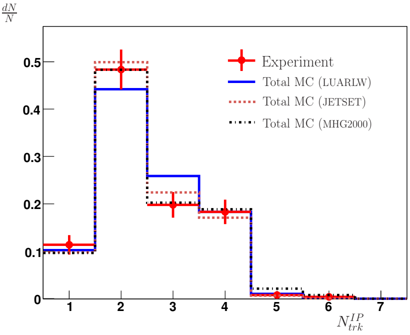
$$f(s) = \frac{\pi\beta}{\sin\pi\beta}\left(\frac{s}{M^2 - s - iM\Gamma}\right)^{1-\beta}$$

Γ_{ee}, Γ, M – 'dressed' parameters including corrections to the vacuum polarization,
$\Gamma_{ee} = \Gamma_{ee}^{(0)}/|1-\Pi_0|^2$, λ–parameter controls the resonance–continuum interference, $\tilde{\Gamma}_h \neq \Gamma_h$

Numerical convolution with the collision energy distribution is used to fit resonance.

Detection efficiency uncertainty in the energy range $\sqrt{s} = 1.84 \div 3.05$ GeV

- Used two essentially different MC generators (LUARLW and tuned JETSET)
- We validated our estimate of the systematic uncertainty related to simulation of the *uds* continuum using an unfolding method (Chinise Physics C Vol. 37, No. 6 (2013) 063001).
- The estimate at the most problematic energy point 1.84 GeV was additionally verified using the exclusive generator MHG2000.



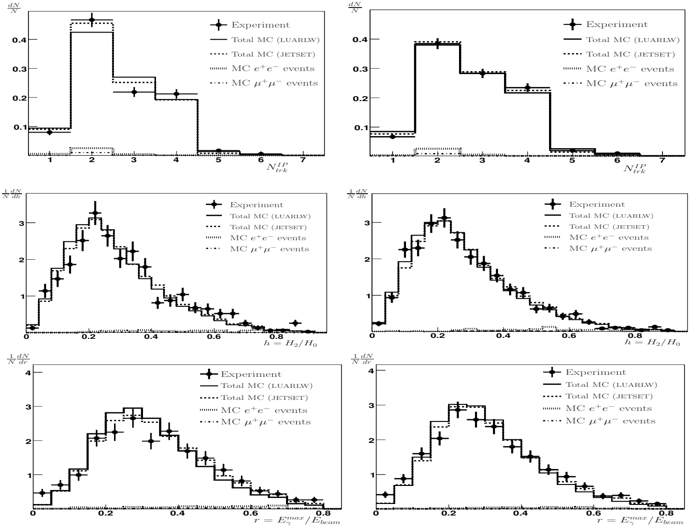
Detection efficiency uncertainties obtained by different methods

Energy, MeV	$\delta\epsilon/\epsilon$		
	LUARLW JETSET	Unfolding method	LUARLW MHG2000
1841.0	6.6%	3.6%	3.8%
1937.0 ÷ 2135.7	2.5%	1.9%	–
2135.7 ÷ 3048.1	1.2%	0.5%	–

Selection criteria

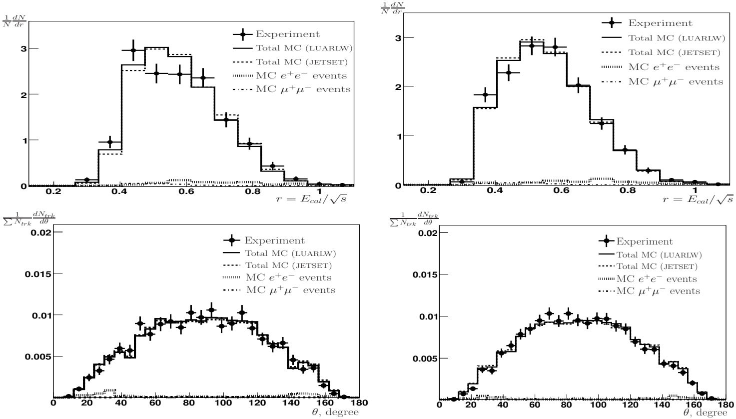
Selection criteria for hadronic events which were used by AND.

Variable	Allowed range	
	3.12-3.72 GeV	1.84 - 3.05 GeV
$N_{\text{track}}^{\text{IP}}$	≥ 1	≥ 1
E_{obs}	> 1.6 GeV	> 1.4 GeV ($> \mathbf{1.3}$ **GeV if** $E_{beam} < \mathbf{1.05}$ **GeV**)
$E_{\gamma}^{\text{max}}/E_{\text{beam}}$	< 0.8	< 0.8
$E_{\text{obs}} - E_{\gamma}^{\text{max}}$		> 1.2 GeV ($> \mathbf{1.1}$ **GeV if** $E_{beam} < \mathbf{1.05}$ **GeV**)
E_{cal}	> 0.75 GeV	> 0.55 GeV
H_2/H_0	< 0.85	< 0.9
$\lvert P_{\text{z}}^{\text{miss}}/E_{\text{obs}}\rvert$	< 0.6	< 0.7
$E_{\text{LKr}}/E_{\text{cal}}^{\text{tot}}$	> 0.15	> 0.15
$\lvert Z_{\text{vertex}}\rvert$	< 20.0 cm	< 15.0 cm
	$N_{\text{particles}} \geq 4$ or $\tilde{N}_{\text{track}}^{\text{IP}} \geq 2$	$N_{\text{particles}} \geq 3$ or $\tilde{N}_{\text{track}}^{\text{IP}} \geq 2$

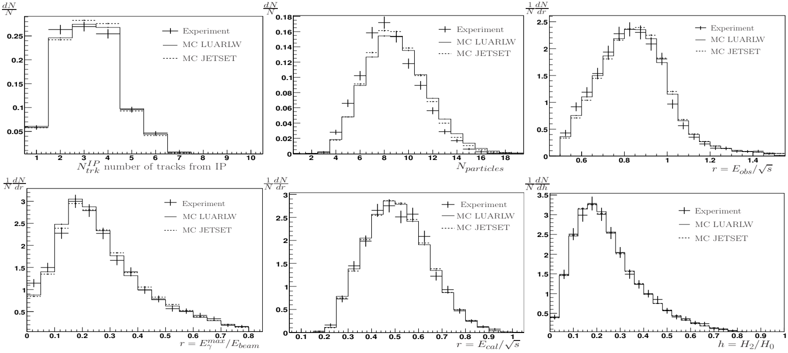


Properties of hadronic events produced in uds continuum at 1.94 GeV (left) and 2.14 GeV (right). Here, N is the number of events, H_2 and H_0 are Fox-Wolfram moments, $E_\gamma^{\max}$ is energy of the most energetic photon, N_{trk} is the number of tracks in event.

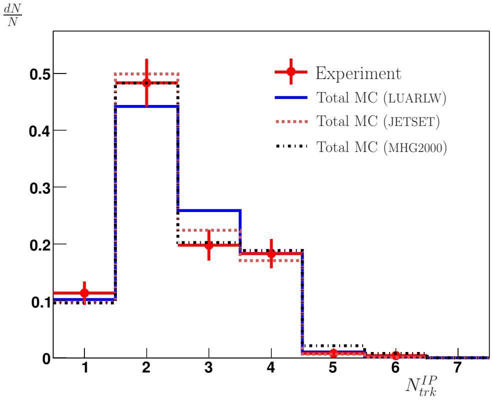
Simulation at 1.94 and 2.14 GeV: JETSET and LUARLW



Properties of hadronic events produced in uds continuum at 1.94 GeV (left) and 2.14 GeV (right). Here, N is the number of events, E_{cal} is energy deposited in the calorimeter, θ is polar angle, N_{trk} is the number of tracks in event. Integrals of all distributions are normalized to unity.



Properties of hadronic events produced in uds continuum at 3.12 GeV. Here N is the number of events, H_2 and H_0 are Fox-Wolfram moments. Integrals of all distributions are normalized to unity.



To obtain the detection efficiency required for calculation of the radiative correction, we performed simulation of the hadronic events using LUARLW and the event generator MHG2000 developed by the CMD-3 collaboration. MGH2000 generates about 30 exclusive channels accounting for the resonance production below 1.9 GeV.

Detection efficiency: JETSET and LUARLW

Detection efficiency for the uds continuum in % (statistical errors only).

$\sqrt{s}$, MeV	ϵ_{LUARLW}	ϵ_{JETSET}	$\delta\epsilon/\epsilon$
1841.0	42.2 ± 0.1	45.0 ± 0.1	-6.6 ± 0.3
1937.0	47.2 ± 0.1	46.0 ± 0.1	-2.5 ± 0.3
2037.3	53.4 ± 0.1		
2135.7	52.5 ± 0.1	51.3 ± 0.1	-1.2 ± 0.3
2239.2	57.0 ± 0.1		
2339.5	61.6 ± 0.1		
2444.1	64.3 ± 0.1		
2542.6	66.7 ± 0.1		
2644.8	68.2 ± 0.1	68.0 ± 0.1	-0.2 ± 0.2
2744.6	70.3 ± 0.1	70.6 ± 0.1	$+0.4 \pm 0.2$
2849.7	71.6 ± 0.1		
2948.9	73.0 ± 0.1		
3048.1	72.4 ± 0.1	73.2 ± 0.1	$+1.1 \pm 0.2$

Detection efficiency: JETSET and LUARLW

$\sqrt{s}$, MeV	ϵ_{JETSET}	ϵ_{LUARLW}	$\delta\epsilon/\epsilon$
Scan 1			
3119.9	75.5 ± 0.1	75.0 ± 0.1	-0.7 ± 0.2
3222.4	76.9 ± 0.1	76.2 ± 0.1	-0.9 ± 0.2
3315.2	77.0 ± 0.1	77.0 ± 0.1	0.0 ± 0.2
3418.1	78.1 ± 0.1	77.4 ± 0.1	-0.9 ± 0.2
3521.0	78.3 ± 0.1	78.2 ± 0.1	-0.1 ± 0.2
3619.7	79.6 ± 0.1	78.6 ± 0.1	-1.3 ± 0.2
3720.4	80.8 ± 0.1	79.2 ± 0.1	-2.0 ± 0.2
Scan 2			
3120.1	75.3 ± 0.1	74.9 ± 0.1	-0.5 ± 0.2
3223.6	75.9 ± 0.1	75.1 ± 0.1	-1.1 ± 0.2
3313.9	77.5 ± 0.1	77.3 ± 0.1	-0.3 ± 0.2
3418.4	78.7 ± 0.1	78.0 ± 0.1	-0.9 ± 0.2
3520.3	78.8 ± 0.1	78.7 ± 0.1	-0.1 ± 0.2
3617.6	80.0 ± 0.1	79.0 ± 0.1	-1.3 ± 0.2
3718.9	80.9 ± 0.1	79.4 ± 0.1	-1.9 ± 0.2

Luminosity determination: 3.12-3.72 GeV

$e^+e- \to e^+e^-(\gamma)$ events detected by the LKr calorimeter $41^\circ < \theta < 159^\circ$ and CsI calorimeter $20^\circ < \theta < 32^\circ$ and $148^\circ < \theta < 160^\circ$

Systematic uncertainties of the luminosity determination in %.

Source	Uncertainty, %
Calorimeter response	0.7
Calorimeter alignment	0.2
Polar angle resolution	0.2
Cross section calculation	0.5
Background	0.1
MC statistics	0.1
Variation of cuts	0.6
Sum in quadrature	1.1

Differences of an integrated luminosities obtained using the LKr and CsI calorimeters in two scans are $0.5 \pm 0.5\%$ and $0.0 \pm 0.5\%$, respectively.

Correction to residual machine background: 3.12-3.72 GeV

- The contribution of residual machine background was estimated using runs with separated e^+ and e^- bunches.
- The residual background was evaluated and subtracted using the number of events which passed selection criteria in the background runs in the assumption that the background rate is proportional to the beam current and the measured vacuum pressure.
- As alternative we assumed that background rate is proportional to the current only. The difference between the numbers of background events obtained with the two assumption was considered as the uncertainty estimate for given energy point.

The residual machine background in % of observed cross section

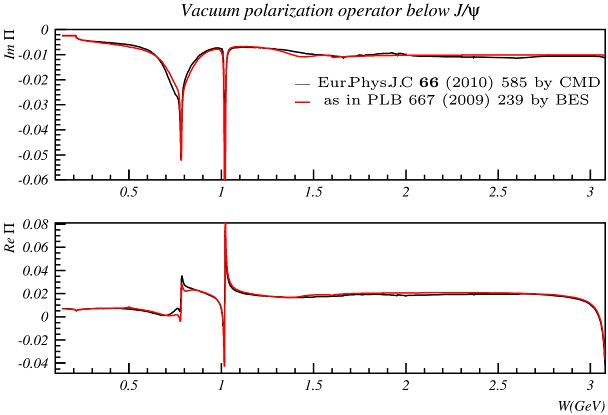
Point	Scan 1	Scan 2
1	$1.3 \pm 0.2 \pm 0.4$	$1.3 \pm 0.2 \pm 0.4$
2	$2.4 \pm 0.4 \pm 0.5$	$2.7 \pm 0.4 \pm 0.5$
3	$2.7 \pm 0.5 \pm 0.4$	$3.0 \pm 0.5 \pm 0.4$
4	$2.9 \pm 0.5 \pm 0.4$	$3.6 \pm 0.6 \pm 0.4$
5	$3.1 \pm 0.6 \pm 0.5$	$3.3 \pm 0.5 \pm 0.5$
6	$2.7 \pm 0.5 \pm 0.4$	$3.7 \pm 0.6 \pm 0.4$
7	$2.1 \pm 0.4 \pm 0.2$	$2.2 \pm 0.3 \pm 0.2$

Unfolding method

- An efficiency matrix ϵ_{ij} describes the efficiency of an event generated with j charged tracks to be reconstructed with i charged tracks.
- The distribution of the number of observed charged track events in data, N_i^{obs} , is known. The true multiplicity distribution in data can be estimated from the observed multiplicity distribution in data and the efficiency matrix by minimizing the χ^2.
- $$\chi^2 = \sum_{i=1}^{i=8} \frac{N_i^{obs} - \sum_{i=1}^{i=8} \epsilon_{ij} \times N_j}{N_i^{obs}}$$

  where the N_j $(j = 0, 2, 4, 6, 8)$ describe the true multiplicity distribution in data and are taken as floating parameters in the fit.
- The total «true» number of events in data can be obtained by summing all fitted N_j.

$\Pi(s)$ calculation



Luminosity determination

$e^+e- \to e^+e^-(\gamma)$ events detected by the LKr calorimeter $41^\circ < \theta < 159^\circ$ and CsI calorimeter $20^\circ < \theta < 32^\circ$ and $148^\circ < \theta < 160^\circ$

Systematic uncertainties of the luminosity determination in %.

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Calorimeter response	0.7
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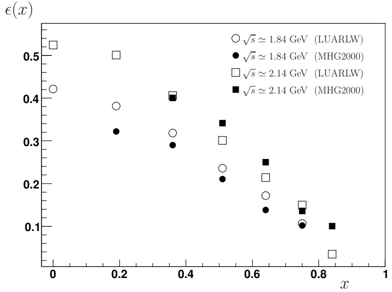
Cross section calculation	0.5
Background	0.1
MC statistics	0.1
Variation of cuts	0.6
Sum in quadrature	1.1

Differences of an integrated luminosities obtained using the LKr and CsI calorimeters in two scans are $0.5 \pm 0.5\%$ and $0.0 \pm 0.5\%$, respectively.

Radiation correction calculation in the energy range 1.84 – 3.05 GeV



Detection efficiency vs variable x at 1.84 and 2.14 GeV.

$$1+\delta(s) = \int \frac{dx}{1-x} \frac{\mathcal{F}(s,x)}{\left|1-\Pi((1-x)s)\right|^2} \frac{R((1-x)s)\varepsilon((1-x)s)}{R(s)\varepsilon(s)}$$

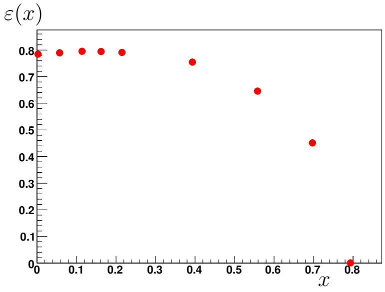
$$R(s) = -\frac{3}{\alpha} \operatorname{Im} \Pi_{\text{hadr}}(s)$$

Vacuum polarization according to CMD-2 data compilation: Eur. Phys. J. C66 (2010) 585

Radiative correction factor $1+\delta$

$\sqrt{s}$, MeV	$1+\delta$	$\sqrt{s}$, MeV	$1+\delta$
1841.0	1.0423 ± 0.0208	2542.6	1.0739 ± 0.0054
1937.0	1.0429 ± 0.0156	2644.8	1.0796 ± 0.0054
2037.3	1.0515 ± 0.0126	2744.6	1.0809 ± 0.0054
2135.7	1.0634 ± 0.0106	2849.7	1.0823 ± 0.0054
2239.2	1.0645 ± 0.0096	2948.9	1.0774 ± 0.0054
2339.5	1.0664 ± 0.0075	3048.1	1.0584 ± 0.0053
2444.1	1.0684 ± 0.0064		

Radiation correction calculation in the energy range 3.12 – 3.72 GeV



Detection efficiency vs variable x (scan 1, $\sqrt{s} = 3.52$ GeV).

$$1+\delta(s) = \int \frac{dx}{1-x} \frac{\mathcal{F}(s,x)}{\left|1-\tilde{\Pi}((1-x)s)\right|^2} \frac{\tilde{R}((1-x)s)\varepsilon((1-x)s)}{R(s)\varepsilon(s)}$$

$$R(s) = -\frac{3}{\alpha}\,\mathrm{Im}\,\Pi_{\mathrm{hadr}}(s)$$

Vacuum polarization according to CMD-2 data compilation:
Eur. Phys. J. C66 (2010) 585

$\sqrt{s}$, MeV	Scan 1	Scan 2	Uncertainty,%				Total
	$1+\delta$		$\Pi(s)$	δR	$\delta\varepsilon$	$\delta_{calc.}$	
3119.9	1.0941 ± 0.0066	1.1074 ± 0.0066	0.3	0.5	0.2	0.2	0.6
3223.0	1.0949 ± 0.0055	1.1049 ± 0.0055	0.1	0.4	0.2	0.2	0.5
3314.7	1.0959 ± 0.0055	1.1100 ± 0.0056	0.1	0.4	0.2	0.2	0.5
3418.2	1.0982 ± 0.0044	1.1094 ± 0.0044	0.1	0.3	0.2	0.2	0.4
3520.8	1.1032 ± 0.0044	1.1102 ± 0.0044	0.1	0.3	0.2	0.2	0.4
3618.2	1.1021 ± 0.0044	1.1098 ± 0.0044	0.1	0.3	0.2	0.2	0.4
3719.4	1.1049 ± 0.0055	1.1067 ± 0.0055	0.4	0.3	0.2	0.2	0.5

List of systematic uncertainties in the energy range 1.84-3.05 GeV

R systematic uncertainties (in %) assigned to each energy point.

	1841.0	1937.0	2037.3	2135.7	2239.2	2339.5	2444.1
Luminosity	1.2	1.2	1.2	1.2	1.2	1.2	1.2
Radiative correction	2.0	1.5	1.2	1.0	0.9	0.7	0.6
Continuum simulation	6.6	2.5	2.5	1.2	1.2	1.2	1.2
Track reconstruction	0.5	0.5	0.5	0.5	0.5	0.5	0.5
l^+l^- contribution	0.6	0.5	0.4	0.4	0.4	0.4	0.3
e^+e^-X contribution	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Trigger efficiency	0.3	0.3	0.3	0.3	0.3	0.3	0.3
Nuclear interaction	0.4	0.4	0.4	0.4	0.4	0.4	0.4
Neutral events	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Cuts variation	0.7	0.7	0.7	0.7	0.7	0.7	0.7
Machine background	0.6	0.5	0.4	0.7	0.8	0.6	0.8
Energy determination	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Sum in quadrature	7.1	3.4	3.2	2.4	2.4	2.2	2.2
	2542.6	2644.8	2744.6	2849.7	2948.9	3048.1	
Luminosity	1.2	1.2	1.2	1.2	1.2	1.2	
Radiative correction	0.5	0.5	0.5	0.5	0.5	0.5	
Continuum simulation	1.2	1.2	1.2	1.2	1.2	1.2	
Track reconstruction	0.5	0.5	0.5	0.5	0.5	0.5	
l^+l^- contribution	0.4	0.4	0.4	0.4	0.4	0.4	
e^+e^-X contribution	0.2	0.2	0.2	0.2	0.2	0.2	
Trigger efficiency	0.3	0.3	0.3	0.3	0.3	0.3	
Nuclear interaction	0.4	0.4	0.4	0.4	0.4	0.4	
Neutral events	0.2	0.2	0.2	0.2	0.2	0.2	
Cuts variation	0.7	0.7	0.7	0.7	0.7	0.7	
Machine background	0.4	0.6	0.8	0.4	0.9	0.5	
Energy determination	0.1	0.1	0.1	0.1	0.1	0.1	
Sum in quadrature	2.1	2.2	2.2	2.1	2.3	2.1	

List of systematic uncertainties in the energy range 3.12-3.72 GeV

R_{uds} systematic uncertainties (in %) assigned to each energy point.

	3119.9	3223.0	3314.7	3418.2	3520.8	3618.2	3719.4
			Scan 1				
Luminosity	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Radiative correction	0.6	0.5	0.5	0.4	0.4	0.4	0.5
Continuum simulation	1.4	1.4	1.4	1.4	1.4	1.4	2.1
J/ψ contribution	2.7	0.5	0.3	0.2	0.2	0.1	0.1
$\psi(2S)$ contribution							1.4
e^+e^-X contribution	0.1	0.1	0.1	0.2	0.2	0.2	0.2
l^+l^- contribution	0.1	0.1	0.1	0.1	0.1	0.2	0.2
Trigger efficiency	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Nuclear interaction	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Cuts variation	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Machine background	1.1	0.8	0.7	0.7	0.9	0.7	0.7
Sum in quadrature	3.5	2.2	2.1	2.1	2.2	2.1	3.0
			Scan 2				
Luminosity	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Radiative correction	0.6	0.5	0.5	0.4	0.4	0.4	0.5
Continuum simulation	1.4	1.4	1.4	1.4	1.4	1.4	2.1
J/ψ contribution	2.8	0.6	0.3	0.2	0.2	0.1	0.1
$\psi(2S)$ contribution							1.3
e^+e^-X contribution	0.1	0.1	0.1	0.2	0.2	0.2	0.2
l^+l^- contribution	0.1	0.1	0.1	0.1	0.1	0.2	0.2
Trigger efficiency	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Nuclear interaction	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Cuts variation	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Machine background	1.1	0.8	0.7	0.8	0.8	0.7	0.5
Sum in quadrature	3.6	2.2	2.1	2.1	2.1	2.1	2.9

pQCD calculation

$R(s)$, obtained in:
P.A.Baikov *et al.* **Nucl. and Part. Phys. Proceed. 261-262(2015)**:

$$R^{n_f=3}(s) = 2\left[1 + \frac{\alpha_s}{\pi} + 1.6398\left(\frac{\alpha_s}{\pi}\right)^2 - 10.2839\left(\frac{\alpha_s}{\pi}\right)^3 - 106.8798\left(\frac{\alpha_s}{\pi}\right)^4\right].$$

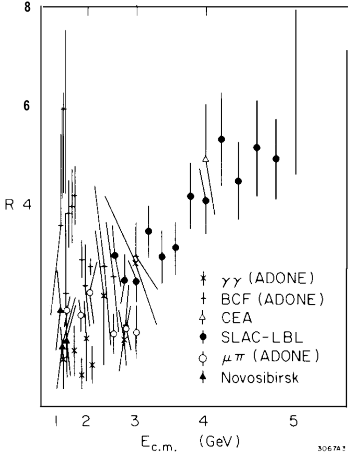
α_s obtained in **K.G.Chetyrkin, B.A.Kniehl, M.Steinhauser PRL 79 (1997)**

$$\alpha_s = \frac{1}{\beta_0 L} - \frac{1}{(\beta_0 L)^2}\frac{\beta_1}{\beta_0}\ln L + \frac{1}{(\beta_0 L)^3}\left[\left(\frac{\beta_1}{\beta_0}\right)^2(\ln^2 L - \ln L - 1) + \frac{\beta_2}{\beta_0}\right]$$
$$+ \frac{1}{(\beta_0 L)^4}\left[\left(\frac{\beta_1}{\beta_0}\right)^3\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3\frac{\beta_1\beta_2}{\beta_0^2}\ln L + \frac{\beta_3}{2\beta_0}\right]$$

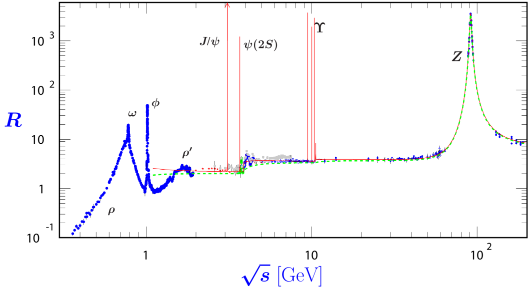
for $n_f = 3$ $\beta_0 = \frac{9}{4}, \beta_1 = 4, \beta_2 = \frac{3863}{384}, \beta_3 = \frac{445}{32}\zeta(3) + \frac{140599}{4608}, L = \ln^2\frac{Q^2}{\Lambda^2_{\overline{MS}}}$

$\alpha_s(m_\tau^2) = 0.331 \pm 0.013$ (**A.Pich Nucl. and Part. Phys. Proceed. 260 (2015) 61-69**) allow to get $R_{uds}^{pQCD} = 2.16 \pm 0.01$ in energy range $3.1 \div 3.7$ GeV.

$R(s)$ measurement. Motivation.



"The ratio R as of July 1974"
Presented by Richter at the
London Conference in July 1974.



PDG at the present time.