Two (plus one) α_s determinations from lattice QCD

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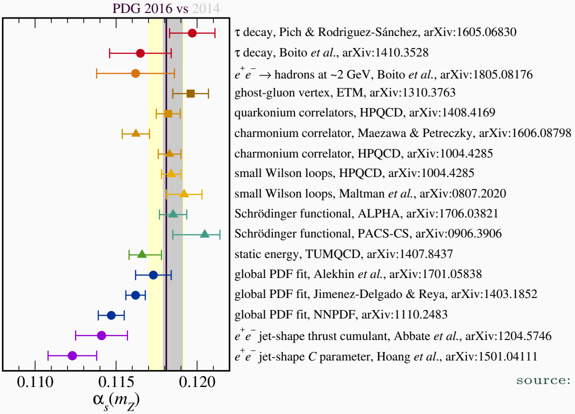
13th Quarkonium Working Group, Torino, 05/17/2019

YM, PP: PR D94 (2016) ⇒ PP, JHW: arXiv:1901.06424

TUMQCD: PR D90 (2014) ⇒ *in preparation*

Outline

Lattice determinations of α_s in context



source: A. S. Kronfeld

- PDG has increased the global error of α_s since 2014
- Lattice QCD (HPQCD) dominates the global average and error
- Spread hints at underestimated systematic uncertainties?

Conceptual idea of lattice determinations of α_s

- We compute hadronic observables on the lattice at sufficiently high scales for the weak-coupling approach to be applicable
- We compare continuum extrapolated lattice results to perturbative results in $\overline{\text{MS}}$ scheme to determine parameters

The time moments of (pseudoscalar) quarkonium correlators (2008-2019)

- The scale is set by the quark mass, $\nu = m_h$ where $m_h \gtrsim m_c$
- Conceptually similar to non-lattice methods
- Large quark masses cause large discretization errors $\sim (am_h)^n$

The QCD static energy of a (static) quark-antiquark pair (2010-2019)

- The scale is set by the (inverse) size of the system, $\nu = 1/r$
- Other scales are involved, i.e. the ultrasoft scale $\mu_{us} = \alpha_s/r$

Bibliography (I): Time moments of quarkonium correlators

- HPQCD collaboration[1] using 3 or 4 sea quark flavors (2008-2015)
- JLQCD collaboration[2] using 3 sea quark flavors (2016)
- BNL group[3] using 3 sea quark flavors (2016-now)
- Fermilab group[4] using 4 sea quark flavors (2016-now)

[1] Allison et al., Phys.Rev. D78 (2008) 054513
McNeile et al., Phys.Rev. D82 (2010) 034512
Chakraborty et al., Phys.Rev. D91 (2015) no.5, 054508

[2] Nakayama et al., PRD 94 (2016) 054507

[3] Maezawa, Petreczky, Phys.Rev. D94 (2016) no.3, 034507
Petreczky, JHW, arXiv:1901.06424

[4] Kronfeld et al., *in preparation*

Bibliography (II): QCD static energy of a quark-antiquark pair

- TUM group[5] using 3 sea quark flavors (2010-now)
- Frankfurt/Jena group[6] using 2 sea quark flavors (2012-2018)
- Kyushu group[7] using 3 sea quark flavors (2018)

Extension to 4 sea quark flavors is planned by the TUMQCD collaboration

[5] Brambilla et al., Phys. Rev. Lett. 105 (2010) 212001
Bazavov et al., Phys. Rev. D86 (2012) 114031
Bazavov et al., Phys. Rev. D90 (2014) 7, 074038
Bazavov et al. [TUMQCD], *in preparation*

[6] Jansen et al. [ETMC], JHEP 1201, 025 (2012)
Karbstein et al., JHEP 1409, 114 (2014)
Karbstein et al., Phys.Rev. D98 (2018) no.11, 114506

[7] Takaura et al., JHEP 1904, 155 (2019)
Takaura et al., Phys. Lett. B789, 598-602 (2019)

Gauge ensembles

- We use the (rooted) Highly Improved Staggered Quark (HISQ)[8] action for two degenerate light quarks and a phsyical strange quark
- We use the tree-level Symanzik-improved gauge action
- Discretization errors of HISQ action scale as $\alpha_s a^2$ and a^4
- We use high statistics ensembles generated by the HotQCD[9] collaboration for a study of EoS with a pion mass of $m_\pi \approx 160\,\mathrm{MeV}$ and a kaon mass of $m_K \approx 504\,\mathrm{MeV}$ in the continuum limit.
- We also use extra ensembles generated for another study of EoS at high T with a pion mass of $m_\pi \approx 320\,\mathrm{MeV}$ in the continuum limit[10]
- We use $\left(r^2 \frac{\partial V_S}{\partial r}\right)_{r=r_1} = 1$ to fix the lattice scale, $r_1 = 0.3106(14)(8)(4)\,\mathrm{fm}$.

${N_\sigma}^3 \times N_\tau$	a^{-1} [GeV]	# TU
48^4	$\lesssim 2.4$	8-16K
$48^3 \times 64$	$\lesssim 3.2$	8-9K
64^4	$\lesssim 4.9$	9K

${N_\sigma}^3 \times N_\tau$	a^{-1} [Gev]	# TU
48^4	2.4	3K
64^4	$\lesssim 7.9$	8K

[8] Follana et al. [HPQCD], Phys.Rev. D75, 054502 (2007)
[9] Bazavov et al. [HotQCD], Phys.Rev. D90, 094503 (2014)
[10] Bazavov et al., Phys.Rev. D97, no. 1, 014510 (2018))

Lattice setup and heavy quark parameters

β	$\frac{m_\ell}{m_s}$	$N_\sigma^3 \times N_\tau$	a^{-1} GeV	L_σ fm	am_{c0}	am_{b0}
6.740	0.05	48^4	1.81	5.2	0.5633(10)	
6.880	0.05	48^4	2.07	4.6	0.4800(10)	
7.030	0.05	48^4	2.39	4.0	0.4047(9)	
7.150	0.05	$48^3 \times 64$	2.67	3.5	0.3547(9)	
7.280	0.05	$48^3 \times 64$	3.01	3.1	0.3086(13)	
7.373	0.05	$48^3 \times 64$	3.28	2.9	0.2793(5)	
7.596	0.05	64^4	4.00	3.2	0.2220(2)	1.019(8)
7.825	0.05	64^4	4.89	2.6	0.1775(3)	0.7985(5)
7.030	0.20	48^4	2.39	4.0	0.4047(9)	
7.825	0.20	64^4	4.89	2.6	0.1775(3)	0.7985(5)
8.000	0.20	64^4	5.58	2.3	0.1495(6)	0.6710(6)
8.200	0.20	64^4	6.62	1.9	0.1227(3)	0.5519(6)
8.400	0.20	64^4	7.85	1.6	0.1019(27)	0.4578(6)

- Pseudoscalar meson operator $j_5(x) = \bar{\psi}(x)\gamma_5\psi(x)$
- RGI pseudoscalar meson correlator

$$G(\tau) = a^8 m_{h0}^2 \sum_{\boldsymbol{x}} \langle j_5(\boldsymbol{x},\tau) j_5(0,0)\rangle_U \quad \lim_{\tau\to 0} \quad \left(\frac{a}{\tau}\right)^4$$

- HISQ valence quarks, m_c and m_b tuned using η_c and η_b masses
- Meson correlators with $am_{h0} = 1, 1.5, 2, 3, 4\ am_{c0}$, and am_{b0}

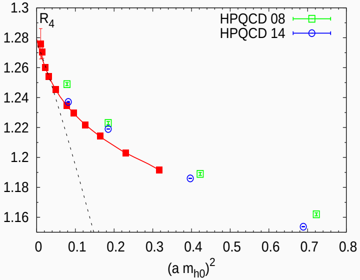
Quarkonium moments with HISQ action

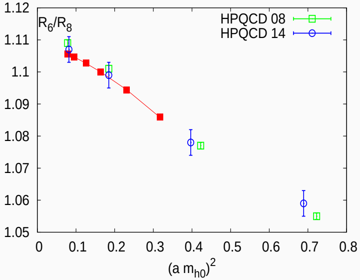
- Time moments are finite for $n \geq 4$ defined on the lattice as

$$G_n = \sum_{\tau/a=1}^{N_\tau/2} \left(\frac{\tau}{a}\right)^n [G(\tau) + G(aN_\tau - \tau)]$$

- Use random color wall sources – statistical errors become irrelevant
- Fluctuations and mass dependence reduced in ratios $G_n^{\frac{1}{n-4}} / G_{n+2}^{\frac{1}{n-2}}$
- Artifacts $\sim \alpha_s^0 (am_{h0})^n$ cancel in reduced moments $R_n = \left(\frac{G_n^{QCD}}{G_n^0}\right)^{\frac{1}{n-4}}$
- Artifacts $\sim \alpha_s^m (am_{h0})^n$ persist in R_n, no artifacts $\sim (a\Lambda_{\mathrm{QCD}})^n$ relevant
- Artifacts are worse in lower moments ($\tau \sim a$) and for larger masses
- Finite size effects are worse in higher moments ($\tau \sim aN_\tau$) and for free theory moments G_n^0 ("quark-antiquark" scattering states, not hadrons)

Approach to continuum



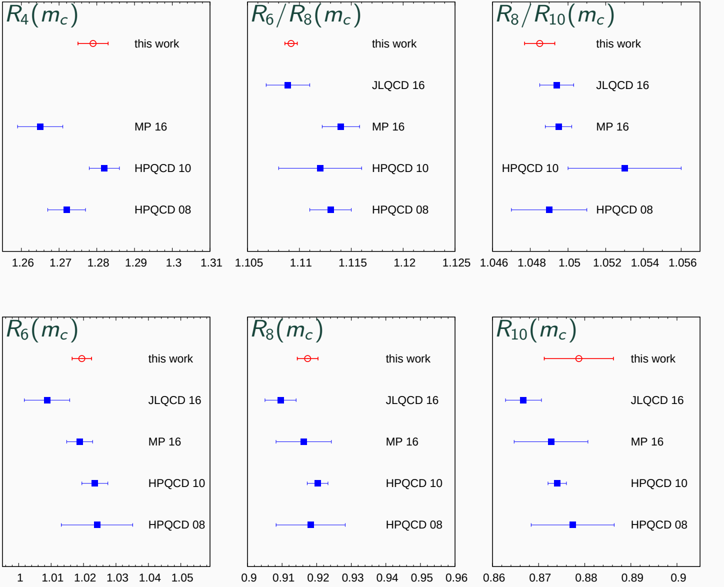


- Unresolved logs $\Rightarrow$ R_4 under- and R_6/R_8 or R_8/R_{10} overestimated
- **Boosted coupling** $\alpha_s^{\rm lat} = 10/(4\pi\beta u_0^4)$, where u_0 is the tadpole factor, i.e., an average link U defined via the plaquette, $u_0^4 = \langle {\rm Tr}\, U_\square \rangle /3$
- We extrapolate the reduced moments and ratios to the continuum using

$$R(\alpha_s^{\rm lat}, am_h) = \sum_{n=1}^{N} \sum_{j=1}^{J} c_{nj}\, (\alpha_s^{\rm lat})^n (am_h)^{2j}, \quad N \leq 3,\; J \leq 5$$

- Similar for larger m_h; control of continuum limit up to $m_h = 3m_c$

Continuum limit at the charm scale



Reduced quarkonium moments in perturbation theory

- We compare to the known weak-coupling result[11] at order α_s^3

$$R_n = \begin{cases} r_4 & (n=4) \\ r_n \cdot \frac{m_{h0}}{m_h} & (n \geq 6) \end{cases} , \qquad r_n = 1 + \sum_{j=1}^{3} r_{nj}\left(m_h, \frac{\mu}{m_h}\right) \alpha_s^j(\mu)$$

- We estimate the uncertainty due to the truncation of the perturbative series with an α_s^4 term, whose coefficient is varied in the range $\pm 5 r_{n3}$
- **Nonperturbative physics enters only via QCD condensates**

$\Rightarrow$ Leading nonperturbative contribution due to the gluon condensate[12]

- We determine $\alpha_s(m_h)$ from the nonlinear equations

$$R_4(\alpha_s(m_h)) = 1 + \sum_{j=1}^{3} r_{4,j}\left(m_h, 1\right) \alpha_s^j(m_h) + \frac{1}{m_h^4}\frac{11}{4}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \quad \text{etc.} \ ,$$

using the gluon condensate $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = -0.006(12)\,\text{GeV}$ from τ decays[13]

[11] Sturm, JHEP 0809 (2008) 075
Kiyo et al., Nulc. Phys. B 823, 269 (2009)
Maier et al., Nucl. Phys. B 824, 1 (2010)

[12] Broadhurst et al., Phys. Lett. B **329**, 103 (1994)

[13] Geshkenbein et al., Phys. Rev. D **64**, 093009 (2001)

α_s at the heavy quark scale m_h

$\frac{m_h}{m_c}$	R_4	R_6/R_8	R_8/R_{10}	av.	$\Lambda^{N_f=3}_{\mathrm{QCD}}$ MeV
1.0	0.3815(55)(30)(22)	0.3837(25)(180)(40)	0.3550(63)(140)(88)	0.3788(65)	315(9)
1.5	0.3119(28)(4)(4)	0.3073(42)(63)(7)	0.2954(75)(60)(17)	0.3099(48)	311(10)
2.0	0.2651(28)(7)(1)	0.2689(26)(35)(2)	0.2587(37)(34)(6)	0.2649(29)	285(8)
3.0	0.2155(83)(3)(1)	0.2338(35)(19)(1)	0.2215(367)(17)(1)	0.2303(150)	284(48)

- Three errors of $\alpha_s(m_h)$ due to the continuum-extrapolated lattice data, the truncation of the perturbative series, and the gluon condensate
- The latter two shrink at the expense of the lattice error for $m_h > m_c$
- All three errors generally increase for the ratios, and α_s from R_8/R_{10} is usually lower than α_s from R_4 or R_6/R_8 for no apparent reason
- Weighted average of the three observables at each scale, and determine the minimal uncertainty such that it has overlapping errors with each
- Consistency of three $\alpha_s(m_h)$ is powerful check for the continuum limit

At $\mu = m_c$: $\alpha_s(M_Z, N_f = 5) = 0.1166(7)$ vs $\alpha_s(M_Z, N_f = 5) = 0.1183(7)$[14]

[14] McNeile et al., Phys.Rev. D82 (2010) 034512

Heavy quark masses m_h from higher moments

$\frac{m_h}{m_c}$	R_6	R_8	R_{10}
1.0	1.2740(25)(17)(11)(61)	1.2783(28)(23)(00)(43)	1.2700(72)(46)(13)(33)
1.5	1.7147(83)(11)(03)(60)	1.7204(42)(14)(00)(40)	1.7192(35)(29)(04)(30)
2.0	2.1412(134)(07)(01)(44)	2.1512(71)(10)(00)(29)	2.1531(74)(19)(02)(21)
3.0	2.9788(175)(06)(00)(319)	2.9940(156)(08)(00)(201)	3.0016(170)(16)(00)(143)
4.0	3.7770(284)(06)(00)(109)	3.7934(159)(08)(00)(68)	3.8025(152)(15)(00)(47)
$\frac{m_b}{m_c}$	4.1888(260)(05)(00)(111)	4.2045(280)(07)(00)(69)	4.2023(270)(14)(00)(47)

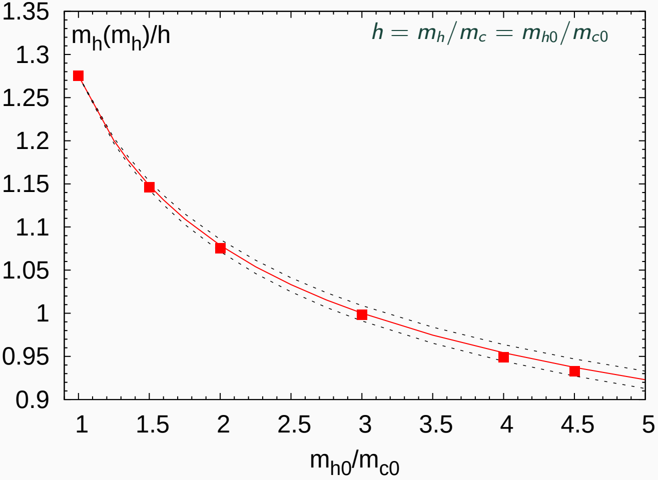
- Four errors of m_h due to the continuum-extrapolated lattice data, truncation of the perturbative series, the gluon condensate, and $\alpha_s(m_h)$
- The error due to the lattice scale r_1 is not included in the table
- Continuum extrapolation of R_6, R_8, and R_{10} is unproblematic for all m_h
- At each $m_h \leq 3m_c$ we obtain Λ_{QCD} from m_h and $\alpha_s(m_h)$, and take the unweighted average of $\Lambda_{\text{QCD}}^{N_f=3}$, and use the spread as systematic error

$$\Lambda_{\text{QCD}}^{N_f=3} = 301 \pm 16\,\text{MeV}, \qquad \alpha_s(M_Z, F_f = 5) = 0.1161(12),$$

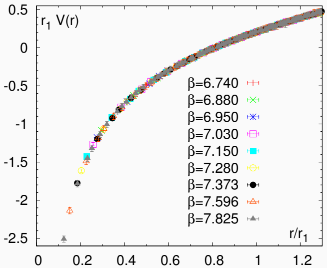
- For $m_h > 3m_c$: unweighted average of Λ_{QCD}, then use 4-loop running to obtain $\alpha_s(4m_c)$ and $\alpha_s(m_b)$, matching to 4 or 5 flavors at 1.5 or 4.7 GeV

$$m_c(m_c, N_f = 4) = 1.2672(84)\,\text{GeV}, \quad m_b(m_b, N_f = 5) = 4.188(29)\,\text{GeV}$$

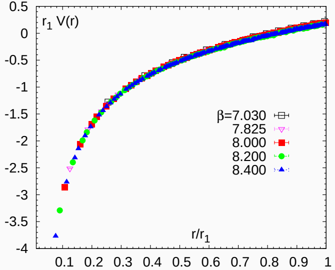
Running of the $\overline{MS}$ mass renormalization factor $m_h(m_h)/(m_h/m_c)$



Static energy on the lattice: 2014 vs 2019



2014 edition[15], $a^{-1} \leq 4.9\,\mathrm{Gev}$



2019 edition[16], $a^{-1} \leq 7.9\,\mathrm{Gev}$

- Smallest distance $r = 0.04\,\mathrm{fm}$
- Perturbative errors dominant
- Very light pion $m_\pi = 160\,\mathrm{MeV}$
- Consistent with 2012 edition[17]

- Three extra fine lattice spacings at $T = 0$
- Include for shortest distances free energies at $T > 0$ $\Rightarrow a^{-1} \lesssim 22\,\mathrm{GeV}$

[15] Bazavov et al., Phys. Rev. D90 (2014) 7, 074038
[16] Bazavov et al. [TUMQCD], *in preparation*
[17] Bazavov et al., Phys. Rev. D86 (2012) 114031

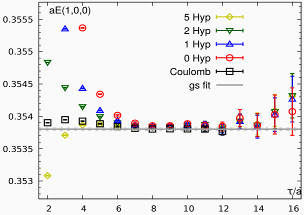
Wilson loops vs Wilson line correlators in Coulomb gauge

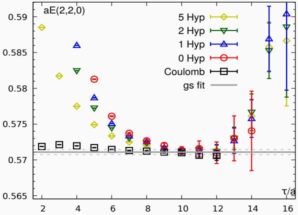
Wilson loops on the lattice

+ Explicit gauge invariance
– Cusp divergences due to corners
– Extra cusp divergences for off-axis separation
– Self-energy divergences due to spatial Wilson lines

Wilson line correlator on the lattice

– Must fix some gauge, i.e. Coulomb gauge
+ No corners, no cusps
+ On- and off-axis separation have same divergence
+ No spatial Wilson lines





- Same ground state for both, but Wilson lines technically favorable
- Distortions at small distance and time for both operators

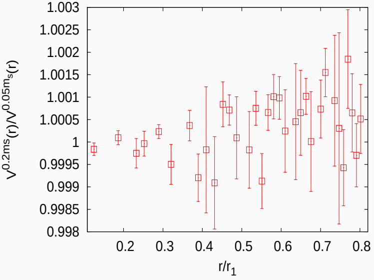
Quark mass dependence and topology

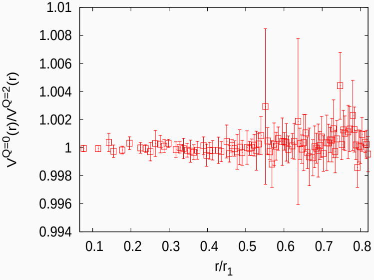
- Combine gauge ensembles with different light sea quark mass

⇒ No statistically significant quark mass effects up to $r \approx 0.5 r_1$

- Fine gauge ensembles with fully suppressed topological tunneling

⇒ No statistically significant difference between static energy in different topological sectors up to $r \approx 0.5 r_1$ observed[18]

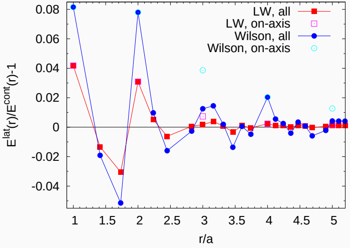


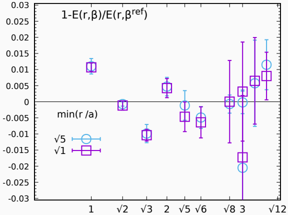


[18] Bazavov et al., arXiv:1811.12902

Lattice artifacts in the static quark-antiquark energy

- The static energy at short distances has percent-level lattice artifacts





- Improved gauge action (Lüscher–Weisz) – reduced symmetry breaking
- Tree-level improvement: $\frac{E^{\mathrm{lat}}(r)}{E^{\mathrm{cont}}(r)}$ for OGE without running coupling
- After tree-level correction – smaller cutoff effects with similar pattern[19]
- E on fine lattices as continuum estimate, correct for cutoff effects
- Alternatively use only data with $r/a \geq \sqrt{8}$ omitting $r/a = \sqrt{12}$

[19] Bazavov et al, Phys.Rev. D98 (2018) no.5, 054511

Static quark-antiquark energy in perturbation theory

- Static energy determined from large-time behavior of Wilson loops

$$E_0(r) = \Lambda_s - \frac{C_F \alpha_s}{r}\left(1 + \#\alpha_s + \#\alpha_s^2 + \#\alpha_s^3 + \#\alpha_s^3 \ln\alpha_s + \#\alpha_s^4 \ln^2\alpha_s + \#\alpha_s^4 \ln\alpha_s + \ldots\right) \quad @\ 3\text{loop}$$

- Contributions to the static energy can be understood in pNRQCD

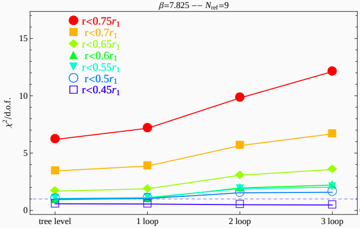
$$E_0(r) = \Lambda_s - V_s(r,\mu) - i\frac{g^2}{N_c} V_A^2 \int_0^\infty dt e^{-it(V_o - V_s)} \langle \mathrm{Tr}\, \boldsymbol{r}\cdot\boldsymbol{E}(t)\boldsymbol{r}\cdot\boldsymbol{E}(0)\rangle(\mu) + \ldots$$

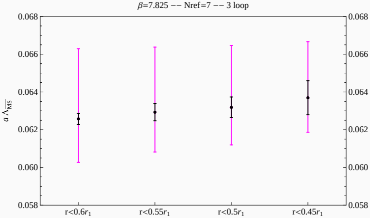
  as to include the singlet potential and an ultrasoft contribution[20]

- The factorization of the ultrasoft contribution gives rise to the ultrasoft scale μ_{us}, the scale of transitions between singlet and octet
- Cancellation of intermediate scale[21]: $\ln\alpha_s = \ln\left(\frac{\mu}{1/r}\right) + \ln\left(\frac{\alpha_s/r}{\mu}\right)$

[20] Brambilla et al., Nucl. Phys. B566 (2000) 275
[21] Brambilla et al., Phys. Rev. D60 (1999) 091502

Fitting lattice results of the static energy (2014)





Different perturbative orders

- χ^2/dof reduces for higher orders at shorter distances

$\Rightarrow$ Weak-coupling suitable for static energy for $r \lesssim 0.15$ fm

- At shortest distances little sensititivy to perturbative order

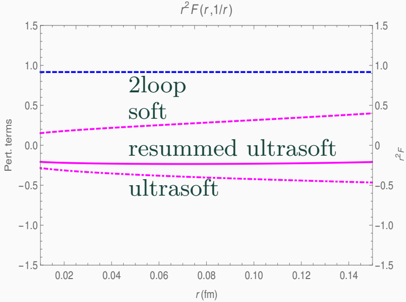
When going to shorter distances

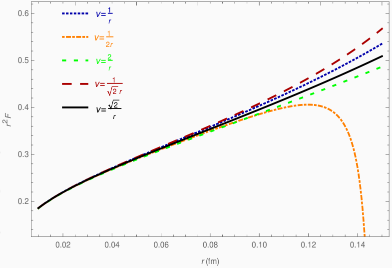
- Statistical errors increase
- Perturbative errors decrease

Perturbative errors estimated from

- scale variation $\nu = \frac{1}{\sqrt{2}r}$ to $\frac{\sqrt{2}}{r}$
- generic higher order term $\pm\frac{\alpha_s^4}{r}$

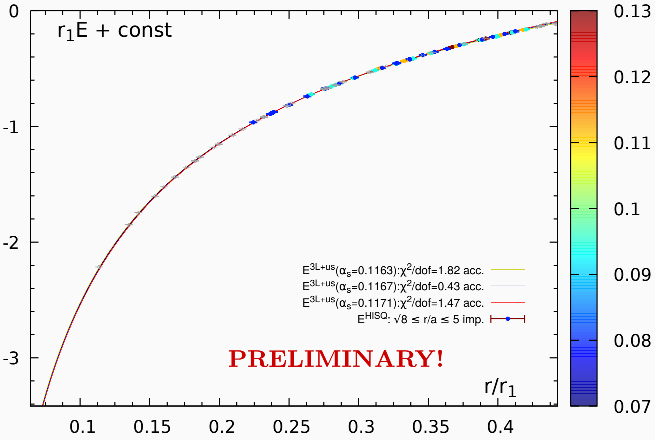
Perturbative uncertainty in the 2019 edition





- Ultrasoft resummation not required – use 3loop + unresummed US
- Soft scale variation generates the dominant uncertainty at 3loop
- More conservative soft scale variation in 2019 edition: $\nu = \frac{1}{2r}$ to $\frac{2}{r}$
- Nonmonotonic soft scale dependence is minimal for $\nu \approx 1/(\sqrt{2}r)$
- Soft scale $\nu \approx 1/(2r)$ not suitable for $r \gtrsim 0.1\,\text{fm}$

α_s from $T = 0$ in the 2019 edition



- Restrict lattice data to $r < 0.14\,\mathrm{fm} \approx 0.45 r_1$
- Combined analysis of lattice data with $a \leq 0.06\,\mathrm{fm}$, i.e., $a/r_1 \leq 0.2$
- Analysis for $r/a \geq \sqrt{8} \Rightarrow$ lattice artifacts are statistically irrelevant

Systematic errors in the 2019 edition

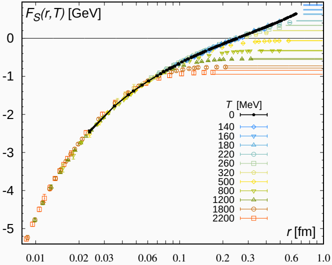
$\min(r/a)$	$\max(r)$ fm	α_s	δ^{stat}	$\delta^{\text{pert}}_{2014}$	$\delta^{\text{pert}}_{2019}$
$\sqrt{8}$	0.097	0.1168	0.0005	$^{+0.0006}_{-0.0003}$	$^{+0.0015}_{-0.0003}$
$\sqrt{8}$	0.131	0.1167	0.0004	$^{+0.0008}_{-0.0003}$	$^{+0.0019}_{-0.0005}$
1	0.055	0.1158	0.0007	$^{+0.0003}_{-0.0001}$	$^{+0.0007}_{-0.0002}$
1	0.073	0.1163	0.0006	$^{+0.0004}_{-0.0001}$	$^{+0.0010}_{-0.0003}$
1	0.098	0.1165	0.0005	$^{+0.0005}_{-0.0002}$	$^{+0.0012}_{-0.0003}$
1	0.131	0.1166	0.0003	$^{+0.0007}_{-0.0004}$	$^{+0.0016}_{-0.0004}$

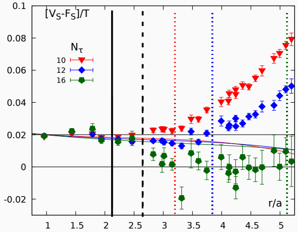
- Must keep $r \lesssim 0.1$ fm to enable the full soft scale variation
- Central value α_s for soft scale $1/(\sqrt{2}r) \le \nu \le \sqrt{2}/r$ is very stable against variation of $\max(r)$
- Include $r/a < \sqrt{8}$ to reduce the impact of scale variation

PRELIMINARY!

$$\Lambda^{N_f=3}_{\text{QCD}} = 313^{+18}_{-9} \pm 2(\text{scale})\,\text{MeV}, \qquad \alpha_s(M_Z, N_f=5) = 0.1165^{+13}_{-6}$$

$T > 0$ data in the 2019 edition

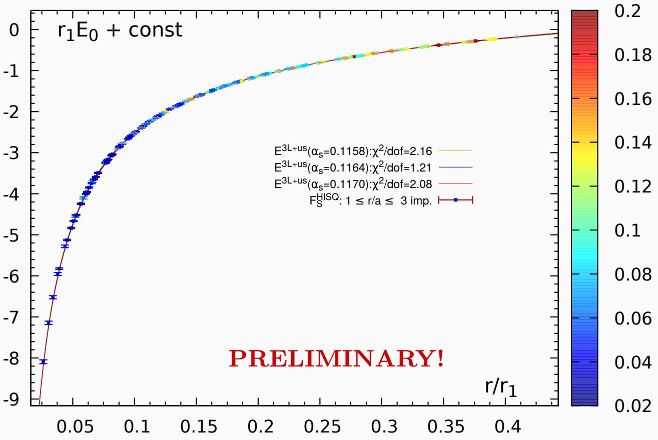




- Singlet free energy for $T > 0$ with much finer lattice spacing[22]
- $T > 0$ effects exponentially suppressed for $\alpha_s/r \gg T$, i.e., $r/a \ll \alpha_s N_\tau$
- Nonconstant $T > 0$ effects are numerically small for $r/a \lesssim 0.30 N_\tau$

[22] Bazavov et al, Phys.Rev. D98 (2018) no.5, 054511

α_s from $T > 0$



- Restrict $T > 0$ lattice data to $r/a \leq 3$, i.e., $r \leq 0.25/T$
- Cannot avoid having to correct for the lattice artifacts

$T = 0$ vs $T > 0$

N_τ	max(r/a)	max(r) fm	α_s	$\delta^{\rm stat}$	$\delta^{\rm pert}_{2014}$	$\delta^{\rm pert}_{2019}$
64	2	0.057	0.1157	0.0009	$^{+0.0003}_{-0.0001}$	$^{+0.0007}_{-0.0002}$
64	2	0.078	0.1161	0.0009	$^{+0.0004}_{-0.0001}$	$^{+0.0009}_{-0.0003}$
64	2	0.096	0.1163	0.0008	$^{+0.0004}_{-0.0002}$	$^{+0.0011}_{-0.0003}$
12	2	0.057	0.1152	0.0012	$^{+0.0002}_{-0.0001}$	$^{+0.0005}_{-0.0002}$
12	2	0.078	0.1157	0.0011	$^{+0.0002}_{-0.0001}$	$^{+0.0007}_{-0.0002}$
12	2	0.091	0.1159	0.0011	$^{+0.0003}_{-0.0001}$	$^{+0.0008}_{-0.0002}$
64	3	0.055	0.1158	0.0007	$^{+0.0003}_{-0.0001}$	$^{+0.0007}_{-0.0002}$
64	3	0.073	0.1163	0.0006	$^{+0.0004}_{-0.0001}$	$^{+0.0010}_{-0.0003}$
64	3	0.096	0.1165	0.0005	$^{+0.0005}_{-0.0002}$	$^{+0.0012}_{-0.0003}$
64	3	0.134	0.1166	0.0004	$^{+0.0007}_{-0.0004}$	$^{+0.0016}_{-0.0004}$
12	3	0.055	0.1161	0.0008	$^{+0.0002}_{-0.0001}$	$^{+0.0005}_{-0.0002}$
12	3	0.073	0.1163	0.0007	$^{+0.0003}_{-0.0001}$	$^{+0.0007}_{-0.0002}$
12	3	0.096	0.1164	0.0006	$^{+0.0003}_{-0.0001}$	$^{+0.0009}_{-0.0002}$
12	3	0.133	0.1166	0.0005	$^{+0.0005}_{-0.0004}$	$^{+0.0011}_{-0.0004}$

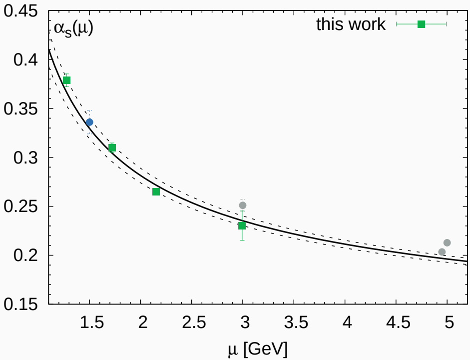
Complete agreement between α_s from $T = 0$ or $T > 0$

Summary

- We determine the strong coupling constant α_s and the charm and bottom quark masses using moments of PS quarkonium correlators, with 6 heavy quark masses, 11 lattice spacings and 2 sea quark masses
- We determine the strong coupling constant α_s from the static energy using 6 lattice spacings with more conservative perturbative errors
- We determine the strong coupling constant α_s from the singlet free energy using 15 lattice spacings (and two N_τ, resp., temperatures)

Quarkonium	2016	2019
$\alpha_s(m_Z, N_f = 5)$	0.11622(84)	0.1161(12)
$\Lambda_{\text{QCD}}(N_f = 3)$	308(12) MeV	301(16) MeV
$m_c(m_c, N_f = 4)$	1.267(12) GeV	1.2672(84) GeV
$m_b(m_b, N_f = 5)$	4.184(89) GeV	4.188(29) GeV
Static energy	**2014**	**2019 (PRELIMINARY!)**
$\alpha_s(m_Z, N_f = 5)$	0.1166^{+12}_{-8}	0.1165^{+13}_{-6}
$\Lambda_{\text{QCD}}(N_f = 3)$	315^{+18}_{-12} MeV	313^{+19}_{-8} MeV
Singlet free energy	**past**	**2019 (PRELIMINARY!)**
$\alpha_s(m_Z, N_f = 5)$	NA	0.1164^{+11}_{-7}
$\Lambda_{\text{QCD}}(N_f = 3)$	NA	311^{+16}_{-10} MeV

Running of α_s at low scales



From left to right

- TUMQCD static energy[23]
- HPQCD quarkonium correlators[24]

[23] Bazavov et al., Phys. Rev. D90 (2014) 7, 074038

[24] Chakraborty et al., Phys.Rev. D91 (2015) no.5, 054508
McNeile et al., Phys.Rev. D82 (2010) 034512
Allison et al., Phys.Rev. D78 (2008) 054513

Thank you!