News from the FeynOnium project

Vladyslav Shtabovenko
Zhejiang University

in collaboration with
N. Brambilla, H. S. Chung and A. Vairo

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ZHEJIANG UNIVERSITY

Technische Universität München

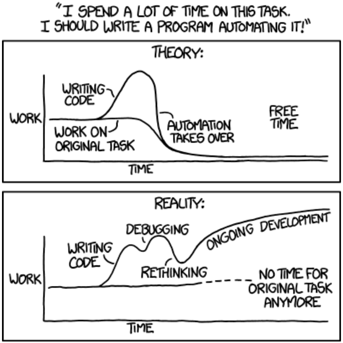
Outline

1. Motivation
2. Tree-level calculations with NREFTs
3. Loop-level calculations with NREFTs
4. Summary and Outlook

- FEYNCALC [Mertig et al., 1991; VS et al., 2016] is a package for semi-automatic symbolic QFT calculations.
- Available since 1990, my contributions started in 2014.
- Popular among EFT practitioners, especially in the heavy quarkonium community.
  - Familiar environment of MATHEMATICA.
  - Flat learning curve, usable documentation, many examples.
  - Friendly mailing list for questions & bug reports.
  - Easy to extend with own codes or interface with other tools.
- Yet, in my view, applying FEYNCALC to EFT calculations is still difficult.
  - Only Lorentz tensors: What about Cartesian?
  - Only standard $(p^2 - m^2 + i\eta)^{-1}$ propagators: Why not Euclidean or eikonal?
  - Manipulation of unusual integrals (abundant in NREFTs)?
  - Pauli algebra?

- Common solution: elaborated private codes.
- Often heavily tailored to a particular project.
- High probability to forget how things work in < 1 year.
- Been there, done that
  - EM Van der Waals forces between hydrogen atoms in pNRQED: A lot of Cartesian integrals to deal with [Brambilla, VS, Tarrús Castellà, Vairo, 2017].
  - Relativistic $\mathcal{O}(v^2)$ corrections to $e^+e^- \to \chi_{cJ}\,\gamma$: QCD-NRQCD matching via the threshold expansion method [Braaten & Chen, 1996] to extract Wilson coefficients of choromelectric operators $\Rightarrow$ Nonrelativistic expansions, tensor projections, Schouten's identities [Brambilla, Chen, Jia, VS, Vairo, 2017].
- Publication as the main goal, the codes are just by-products.
- Idea at the end of my PhD: Recycle and generalize the codes.
- Interesting side-project: (NR)EFTs made easy with FEYNCALC.

- Main focus on the strong sector: χPT [Weinberg, 1979; Gasser & Leutwyler, 1984], SCET [Bauer et al., 2000-2002; Beneke et al, 2002], NRQCD [Caswell & Lepage, 1986; Bodwin et al., 1995], pNRQCD [Pineda & Soto, 1998; Brambilla et al., 2000].
- Subdivided into 2 projects
  - FeynCalc 9.3: improved package with broad support for various EFTs.
  - FeynOnium: speicific extras for NREFTs (mainly NRQCD/pNRQCD).
- Reproduce key results from the literature to test the approach.
- As usual, initial time estimates too optimistic …



Source: xkcd.com/1319

- Many conceptual and technical issues to think over.
- Account for relations between Lorentz and Cartesian tensors.
- Every tensor that carries a Lorentz index can be decomposed into a temporal and a Cartesian piece

$$p^\mu = p_\nu g^{\mu\nu} = p_0 g^{\mu 0} + p_i g^{\mu i}.$$

- Distinction between upper (contravariant) and lower (covariant) indices

$$\boldsymbol{p}^i = -\boldsymbol{p}_i,$$

  metric signature dependent!
- Backward-compatibility: Extend the existing syntax but do not break it.
- Prevent over-engineering: Add only minimal amount of new symbols.
- Keep it simple: No new commands for standard operations (contractions, expansions etc.)

- Tree-level calculations with NR quantities: Mostly completed during 2016-2017, presented at QWG 2017

- Loop-level calculations with nonstandard integrals: somewhat harder than expected, ongoing work since 2017.

- Manipulations of Cartesian tensors (common in NREFTs) are easy.
- Calculating $\left(\delta^{ij} + \frac{\boldsymbol{n}^i \boldsymbol{k}^j + \boldsymbol{k}^i \boldsymbol{n}^j}{k \cdot n}\right) \boldsymbol{q}^i \boldsymbol{r}^j (\boldsymbol{q} \cdot \boldsymbol{r})$ amounts to

```
In[2]:= (KD[i, j] + (CV[k, j] CV[n, i] + CV[k, i] CV[n, j])/SP[k, n]) CV[q, i] CV[r, j] CSP[q, r] //
          Contract
```

$$\text{Out[2]=}\ (\overline{\boldsymbol{q}} \cdot \overline{\boldsymbol{r}})^2 + \frac{(\overline{\boldsymbol{k}} \cdot \overline{\boldsymbol{r}})(\overline{\boldsymbol{n}} \cdot \overline{\boldsymbol{q}})(\overline{\boldsymbol{q}} \cdot \overline{\boldsymbol{r}})}{\overline{k} \cdot \overline{n}} + \frac{(\overline{\boldsymbol{k}} \cdot \overline{\boldsymbol{q}})(\overline{\boldsymbol{n}} \cdot \overline{\boldsymbol{r}})(\overline{\boldsymbol{q}} \cdot \overline{\boldsymbol{r}})}{\overline{k} \cdot \overline{n}}$$

```
In[3]:= % // LorentzToCartesian
```

$$\text{Out[3]=}\ (\overline{\boldsymbol{q}} \cdot \overline{\boldsymbol{r}})^2 + \frac{(\overline{\boldsymbol{k}} \cdot \overline{\boldsymbol{r}})(\overline{\boldsymbol{n}} \cdot \overline{\boldsymbol{q}})(\overline{\boldsymbol{q}} \cdot \overline{\boldsymbol{r}})}{k^0 n^0 - \overline{\boldsymbol{k}} \cdot \overline{\boldsymbol{n}}} + \frac{(\overline{\boldsymbol{k}} \cdot \overline{\boldsymbol{q}})(\overline{\boldsymbol{n}} \cdot \overline{\boldsymbol{r}})(\overline{\boldsymbol{q}} \cdot \overline{\boldsymbol{r}})}{k^0 n^0 - \overline{\boldsymbol{k}} \cdot \overline{\boldsymbol{n}}}$$

- Lorentz (Cartesian) vectors in 4-dim (3-dim) have a bar.
- `KD[i,j]` $\sim \delta^{ij}$: Cartesian metric (Kronecker Delta).
- `CV[p,i]` $\sim \boldsymbol{p}^i$: Cartesian 3-vector.
- `SP[k,n]` $\sim k \cdot n$: Scalar product of two 4-vectors.
- `CSP[q,r]` $\sim \boldsymbol{q} \cdot \boldsymbol{r}$: Scalar product of two 3-vectors.
- `TC[k]` $\sim k^0$: Temporal component of a 4-vector.
- `Contract` handles the contractions of Lorentz and Cartesian indices.
- `LorentzToCartesian` for breaking the manifest Lorentz covariance.

- Differentiation w.r.t 3-vectors (projections, series expansions, differential equations) is straightforward.
- Let us calculate $\frac{\partial}{\partial \boldsymbol{q}^i} \frac{1}{(\boldsymbol{q}^2+m^2)(\boldsymbol{p}+\boldsymbol{q})^2}$

```
In[4]:= 1 / (CSPD[q] + m ^ 2)   1 / (CSPD[q + p])
```

Out[4]= $\frac{1}{(m^2 + \boldsymbol{q}^2)(\boldsymbol{p} + \boldsymbol{q})^2}$

```
In[5]:= ThreeDivergence[%, CVD[q, i]]
```

Out[5]= $-\frac{2\,\boldsymbol{p}^i}{(m^2 + \boldsymbol{q}^2)(2(\boldsymbol{p}\cdot\boldsymbol{q}) + \boldsymbol{p}^2 + \boldsymbol{q}^2)^2} - \frac{2\,\boldsymbol{q}^i}{(m^2 + \boldsymbol{q}^2)^2(2(\boldsymbol{p}\cdot\boldsymbol{q}) + \boldsymbol{p}^2 + \boldsymbol{q}^2)} - \frac{2\,\boldsymbol{q}^i}{(m^2 + \boldsymbol{q}^2)(2(\boldsymbol{p}\cdot\boldsymbol{q}) + \boldsymbol{p}^2 + \boldsymbol{q}^2)^2}$

- `CSP[q,r]` $\sim \boldsymbol{q}\cdot\boldsymbol{r}$: Scalar product of two 3-vectors in $D-1$ dimensions.
- `ThreeDivergence[f, CVD[q,i]]` $\sim \frac{\partial}{\partial \boldsymbol{q}^i} f$.
- In general, Cartesian tensors can live in 3, $D-1$ or $D-4$ dimensions.

- Dirac matrices may carry Cartesian or temporal indices.
- We can simplify chains of such matrices and compute traces as in

$$\mathrm{Tr}\left(\not{\boldsymbol{p}}\,\gamma^0 P_R\,\not{\boldsymbol{q}}\,P_L\not{\boldsymbol{\varepsilon}}(n)\right),$$

with $\not{\boldsymbol{p}} = \gamma^i \boldsymbol{p}^i$.

```
In[9]:= DiracTrace[CGS[p].TGA[].GA[6].CGS[q].GA[7].CGS[Polarization[n, I]]]
```

Out[9]= $\mathrm{tr}((\overline{\gamma}\cdot\overline{p}).\overline{\gamma}^0.\overline{\gamma}^6.(\overline{\gamma}\cdot\overline{q}).\overline{\gamma}^7.(\overline{\gamma}\cdot\overline{\varepsilon}(n)))$

```
In[10]:= % // DiracSimplify
```

Out[10]= $2\,i\,\bar{\epsilon}^{\overline{p}\,\overline{q}\,\overline{\varepsilon}(n)}$

- $\bar{\epsilon}^{\bar{\boldsymbol{p}}\bar{\boldsymbol{q}}\bar{\boldsymbol{\varepsilon}}(n)}$ stands for $\epsilon^{ijk}\boldsymbol{p}^i\boldsymbol{q}^j\boldsymbol{\varepsilon}^k(n)$.
- `CGS[p]` $\sim \gamma^i\boldsymbol{p}^i$: Dirac matrix contracted with a 3-vector.
- `GA[6/7]` $\sim \frac{1}{2}(1\pm\gamma_5)$: Chirality projectors.
- `TGA[]` $\sim \gamma^0$.
- `DiracSimplify` handles the Dirac algebra.

- Pauli matrices are indispensable in NR calculations, can handle that too.

```
In[10]:= PauliEta[-i].CSIS[q].CSIS[Polarization[k1, -I]].CSIS[k1].CSIS[q].
          PauliXi[i]
```

Out[10]=

$$\eta^{\dagger}.(\overline{\sigma}\cdot\overline{\boldsymbol{q}}).(\overline{\sigma}\cdot\overline{\varepsilon}^{*}(\mathrm{k1})).(\overline{\sigma}\cdot\overline{\mathbf{k1}}).(\overline{\sigma}\cdot\overline{\boldsymbol{q}}).\xi$$

```
In[11]:= PauliTrick[%, PauliReduce → False]
```

Out[11]=

$$2\,(\overline{\boldsymbol{q}}\cdot\overline{\varepsilon}^{*}(\mathrm{k1}))\,\eta^{\dagger}.(\overline{\sigma}\cdot\overline{\mathbf{k1}}).(\overline{\sigma}\cdot\overline{\boldsymbol{q}}).\xi-2\,(\overline{\mathbf{k1}}\cdot\overline{\boldsymbol{q}})\,\eta^{\dagger}.(\overline{\sigma}\cdot\overline{\varepsilon}^{*}(\mathrm{k1})).(\overline{\sigma}\cdot\overline{\boldsymbol{q}}).\xi+\overline{\boldsymbol{q}}^{2}\,\eta^{\dagger}.(\overline{\sigma}\cdot\overline{\varepsilon}^{*}(\mathrm{k1})).(\overline{\sigma}\cdot\overline{\mathbf{k1}}).\xi$$

- `CSIS[p]` $\sim \sigma^i \boldsymbol{p}^i$: Pauli matrix contracted to a 3-vector.
- `PauliEta`, `PauliXi` denote the Pauli 2-spinors.
- `PauliTrick` simplifies a chain of Pauli matrices.
- `PauliReduce` governs the application of

$$\sigma^i \sigma^j = \delta^{ij} + i\varepsilon^{ijk}\sigma^k.$$

Important for DR calculations, ε^{ijk} is tricky in $D-1$ dimensions.

- Validation is crucial for such software frameworks!
- FEYNCALC has a decent test-coverage with a large test-suite.
- Still, unit-tests cannot replace real-life calculations.
- Reproducing analytic results from the literature is very time-consuming.
- Many publications are very concise regarding the technical details.
- A selection of results reproduced using FEYNCALC+FEYNONIUM
  - $Q\bar{Q}({}^3P_0^{[1]}) \to gg$ at $\mathcal{O}(\alpha_s v^0)$ [Petrelli et al., 1998]
  - $\eta_c/\chi_{cJ} \to 2\gamma$ at $\mathcal{O}(\alpha_s^0 v^4)$ [Brambilla et al., 2006]
  - $\eta_c \to 2\gamma$ at $\mathcal{O}(\alpha_s v^2)$ [Jia et al., 2011]
  - $H \to J/\psi\,\gamma$ at $\mathcal{O}(\alpha_s^0 v^2)$ [Bodwin et al., 2014]
  - See also today's talk of Hee Sok Chung on $\mathcal{O}(\alpha_s^0 v^4)$ corrections to $H \to J/\psi\,\gamma$!
- No manipulations of nonstandard loop integrals were required here.

- What is so special about loop integrals in (NR)EFTs?
- For standard integrals at 1-loop we have the time-proven Passarino-Veltman (PaVe) [Passarino & Veltman, 1979] technique.
- Old method, but still convenient and easy to apply.
- Implemented in FeynCalc since version 1.0, subsequently improved in the newer versions.
- The FeynCalc 1-loop toolbox includes
  - `FDS`: Simplification using symmetries and loop momentum shifts.
  - `ApartFF`: Partial fractioning, based on [Feng, 2012].
  - `ToPaVe`: Conversion of scalar integrals to PaVe functions.
  - `PaVeUVPart` (new in 9.3): extract the UV part of arbitrary PaVe functions, based on [Sulyok, 2017].
  - `$KeepLogDivergentScalelessIntegrals`: switch to prevent $\left(\frac{1}{\varepsilon_{\rm UV}} - \frac{1}{\varepsilon_{\rm IR}}\right)$-type integrals from being set to zero.
  - `TID`: 1-loop tensor reduction (all-in-one).

- Nonstandard integrals (not $(p^2 - m^2)^{-1}$-type) are not that easy.
- Not all tensor integrals reducable to scalars with unit numerators (even at 1-loop)
- No universal basis like Passarino-Veltman.
- Master integrals usually evaluated on case-by-case basis.
- Need special care when simplifying the denominators, e. g. for eikonal

$$\int \frac{d^D q}{[q^2 + i\eta]\,[-2p \cdot q - m^2 + i\eta]} = -\int \frac{d^D q}{[q^2 + i\eta]\,[2p \cdot q + m^2 - i\eta]}$$

- Most useful manipulations still applicable to almost all integrals.
- Partial fractioning (linear independence of propagators)

$$\frac{4(\boldsymbol{p}\cdot\boldsymbol{q})}{\boldsymbol{q}^2(\boldsymbol{q}+p)^2(\boldsymbol{q}-p)^2} = \frac{1}{\boldsymbol{q}^2(\boldsymbol{q}-\boldsymbol{p})^2} + \frac{1}{\boldsymbol{q}^2(\boldsymbol{q}+\boldsymbol{p})^2}$$

- Tensor reduction to remove loop momenta with uncontracted indices

$$\int d^D q \frac{q^\mu q^\nu}{q^2\,(q\cdot p - m^2)} = \frac{m^4}{(D-1)p^4}(D p^\mu p^\nu - p^2 g^{\mu\nu}) \int \frac{d^D q}{q^2\,(q\cdot p - m^2)}$$

- Those are useful at 1-loop but also in multiloop calculations.
- Need a simple and consistent way to represent various (NR)EFT propagators in a symbolic code.

- Surprisingly, in FeynCalc only 3 new shortcuts are needed.
- In addition to the standard `FAD` (`FeynAmpDenominator`) introduce
  - `SFAD` (`StandardFeynAmpDenominator`): Covariant integrals.
  - `CFAD` (`CartesianFeynAmpDenominator`): Cartesian integrals.
  - `GFAD` (`GenericFeynAmpDenominator`): anything else.

Shortcut in FeynCalc	Meaning
$\texttt{FAD}[l - p_1 - \ldots,\ m]$	$\frac{1}{(l-p_1-\ldots)^2-m^2}$
$\texttt{SFAD}[l - p_1 - \ldots,\ \pm l.(q_1 + \ldots),\ \pm m^2,\ \pm 1,\ n]$	$\left[\frac{1}{(l-p_1-\ldots)^2 \pm l.(q_1+\ldots) \mp m^2 \pm i\eta}\right]^n$
$\texttt{CFAD}[l - p_1 - \ldots,\ \pm l.(q_1 + \ldots),\ \pm m^2,\ \pm 1,\ n]$	$\left[\frac{1}{(\boldsymbol{l}-\boldsymbol{p}_1-\ldots)^2 \pm \boldsymbol{l}.(\boldsymbol{q}_1+\ldots) \pm m^2 \pm i\eta}\right]^n$
$\texttt{GFAD}[x,\ \pm 1,\ n]$	$\left[\frac{1}{x \pm i\eta}\right]^n$

- Notice that the mass term and $i\eta$ can be positive or negative.

- Simplify the 1-loop heavy nucleon propagator in baryonic χPT [Ecker & Mojzis, 1996] (example from S. Scherer's lectures [Scherer, 2003])

$$\int \frac{d^D k}{(2\pi)^D} (-S_v \cdot k)\sigma^i \frac{1}{v \cdot (r-k) + i\eta} \frac{1}{k^2 - M^2 + i\eta} (S_v \cdot k)\sigma^i,$$

with $S_v^\mu = -\frac{1}{2}\gamma_5(\gamma^\mu \not{v} - v^\mu)$.

- Painful in FEYNCALC 9.2, trivial with version 9.3

```
In[40]:= exp = (-(-1/2 GA[5].(GAD[μ].GSD[v] - FVD[v, μ]).CSI[i] FVD[k, μ])).
    SFAD[{{0, v.(r - k)}}, {k, M^2}].(-1/2 GA[5].
      (GAD[ν].GSD[v] - FVD[v, ν]).CSI[i]) FVD[k, ν]
```

Out[40]=

$$k^\nu \left(\frac{1}{2} k^\mu \bar{\gamma}^5.(\gamma^\mu.(\gamma \cdot v) - v^\mu).\bar{\sigma}^i\right).\frac{1}{((r-k)\cdot v + i\eta).(k^2 - M^2 + i\eta)}.\left(-\frac{1}{2}\bar{\gamma}^5.(\gamma^\nu.(\gamma \cdot v) - v^\nu).\bar{\sigma}^i\right)$$

```
In[41]:= TID[PauliTrick[exp], k, FCE → True]
```

Out[41]=

$$\frac{3(r \cdot v)}{4(k^2 - M^2 + i\eta)} - \frac{3((r \cdot v)^2 - M^2 v^2)}{4(k^2 - M^2 + i\eta).((r-k)\cdot v + i\eta)}$$

- Tensor reduction of Cartesian integrals e. g.

$$\int \frac{d^{D-1}k}{(2\pi)^{D-1}} \frac{\boldsymbol{k}^i \boldsymbol{k}^j}{[\boldsymbol{k}^2 + m^2][(\boldsymbol{k}-\boldsymbol{q})^2 + m^2]}$$

- Very common in pNRQCD calculations if you prefer to handle spatial and temporal integrations separately.
- *Sort of* possible in FeynCalc 9.2, but not straightforward.
- Now it *just works*

```
In[44]:= exp = CVD[k, i] CVD[k, j] CFAD[{k, m^2}, {k - q, m^2}]
```

Out[44]=

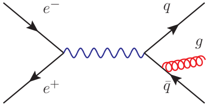
$$\frac{\boldsymbol{k}^i \boldsymbol{k}^j}{(\boldsymbol{k}^2+m^2-i\eta).((\boldsymbol{k}-\boldsymbol{q})^2+m^2-i\eta)}$$

```
In[47]:= TID[exp, k]
```

Out[47]=

$$\frac{-(D-1)\,\boldsymbol{q}^i\,\boldsymbol{q}^j - \boldsymbol{q}^2\,\delta^{ij} + 2\,\boldsymbol{q}^i\,\boldsymbol{q}^j}{2\,(2-D)\,\boldsymbol{q}^2\,(\boldsymbol{k}^2+m^2-i\eta)} - \frac{(D-1)\,\boldsymbol{q}^2\,\boldsymbol{q}^i\,\boldsymbol{q}^j - 4\,m^2\,\boldsymbol{q}^2\,\delta^{ij} + 4\,m^2\,\boldsymbol{q}^i\,\boldsymbol{q}^j - \boldsymbol{q}^4\,\delta^{ij}}{4\,(2-D)\,\boldsymbol{q}^2\,(\boldsymbol{k}^2+m^2-i\eta).((\boldsymbol{k}-\boldsymbol{q})^2+m^2-i\eta)}$$

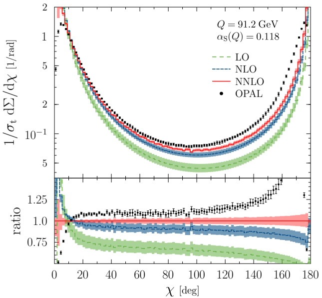
- My favorite new propagator type: `GFAD`, the generic propagator
- `GFAD` can be anything, so it works with *highly* nonstandard propagators
- The idea for this came from my work on the Energy-Energy-Correlation (EEC) event shape variable in QCD.



- EEC [Basham et al., 1978] is a classical (IR safe) hadronic observable in e^+e^- annihilation: $e^+e^- \to \gamma^*/Z^0 \to a + b + X$.
- Formal definition ($\cos\theta_{ab} = \hat{\boldsymbol{p}}_a \cdot \hat{\boldsymbol{p}}_b$)

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma(\chi)}{d\cos\chi} = \sum_{a,b} \int \underbrace{\frac{E_a E_b}{Q^2} \delta(\cos\theta_{ab} - \cos\chi)}_{\text{measurement function}} \times d\sigma_{a+b+X}.$$

- 2 calorimeters at relative angle χ measure the energies of the hadrons a and b.



Source: [Tulipánt et al., 2017].

- Diff. angular distribution from measuring the energies (E_a, E_b) of and the angle θ_{ab} between all hadrons pairs produced in each event.

- Numerically, EEC is currently known at NNLO [Del Duca et al., 2016; Tulipánt et al., 2017].
- Analytically (fixed-order), the LO result was state of the art for 40 years [Basham et al., 1978]

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\Sigma(\chi)}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi} C_F \frac{3-2z}{4(1-z)z^5}\left[3z(2-3z)+2(2z^2-6z+3)\log(1-z)\right] + \mathcal{O}(\alpha_s^2), \quad \text{with } z=(1-\cos\chi)/2.$$

- With my collaborators we obtained fixed-order analytic NLO results for
  - $e^+e^- \to \gamma^* \to \text{partons}$ in pQCD [Dixon, Luo, VS, Yang, Zhu, 2018]
  - $H \to \text{gluons} \to \text{partons}$ in HEFT [Luo, VS, Yang, Zhu, 2019]
- "NLO" in this context: by calculating 3-loop phase space integrals with special propagators from the measurement function.
- Done using familiar multiloop tools (FORM [Vermaseren, 2000], Color [van Ritbergen et al., 1999], FIRE 5 [Smirnov, 2015], LiteRed [Lee, 2012], Fuchsia [Gituliar & Magerya, 2017])
- In the HEFT calculation FeynCalc was employed at some intermediate stages.

- EEC: $\frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma(\chi)}{d \cos \chi} = \sum_{a,b} \int \frac{E_a E_b}{Q^2} \delta(\cos \theta_{ab} - \cos \chi) d\sigma_{a+b+X}$.
- Measurement function in the rest frame, setting $Q^2 = 1$

$$E_a E_b \, \delta(\cos \theta_{ab} - \cos \chi) = (p_a \cdot Q)^2 (p_b \cdot Q)^2 \delta \left(2z \, p_a \cdot Q \, p_b \cdot Q - p_a \cdot p_b\right).$$

- Analytic calculation of phase-space integrals using the reverse unitarity method [Anastasiou & Melnikov, 2002; Anastasiou et al., 2003]

$$\delta \left(2z \, p_a \cdot Q \, p_b \cdot Q - p_a \cdot p_b\right) \sim \frac{1}{2z \, p_a \cdot Q \, p_b \cdot Q - p_a \cdot p_b}\bigg|_{\text{cut}}$$

- $[2z \, p_a \cdot Q \, p_b \cdot Q - p_a \cdot p_b]^{-1}$ is nonlinear in the scalar products.
- Nonstandard propagator that is not supported by public codes.
- Initially we used “hacked” LiteRed to derive IBPs [Chetyrkin & Tkachov, 1981] and differential equations [Kotikov, 1991a, 1991b, 1991c; Bern et al., 1994; Remiddi, 1997; Gehrmann & Remiddi, 2000].
- Later we discovered interesting tricks to optimize the IBP equations and simplify the integrands.
- For “experiments” I needed a more convenient software framework.

- Using GFADs it is easy to put the nonlinear propagators into FEYNCALC
- Example: calculate

$$\int d^D p_1 d^D p_2 \frac{\partial}{\partial p_1^\mu} \frac{p_1^\mu (p_2 \cdot Q)}{p_1^2 p_2^2 (Q - p_1 - p_2)^2 (Q - p_1)^2 (Q - p_2)^2} \frac{1}{2z\, p_a \cdot Q\, p_b \cdot Q - p_a \cdot p_b}$$

```
In[72]:= SFAD[p1, p2, Q - p1 - p2, Q - p1, Q - p2] GFAD[2 z SPD[p1, Q] SPD[p2, q] - SPD[p1, p2]]
```

Out[72]=

$$\frac{1}{(\mathrm{p1}^2+i\eta).(\mathrm{p2}^2+i\eta).((-\mathrm{p1}-\mathrm{p2}+Q)^2+i\eta).((Q-\mathrm{p1})^2+i\eta).((Q-\mathrm{p2})^2+i\eta)\,(2z(\mathrm{p1}\cdot Q)(\mathrm{p2}\cdot q)-\mathrm{p1}\cdot\mathrm{p2}+i\eta)}$$

```
In[73]:= FourDivergence[FVD[p1, μ] SPD[p2, Q] %, FVD[p1, μ]]
```

Out[73]=

$$\frac{D\,(\mathrm{p2}\cdot Q)}{((Q-\mathrm{p2})^2+i\eta).((-\mathrm{p1}-\mathrm{p2}+Q)^2+i\eta).(2z(\mathrm{p1}\cdot Q)(\mathrm{p2}\cdot q)-\mathrm{p1}\cdot\mathrm{p2}+i\eta).(\mathrm{p1}^2+i\eta).(\mathrm{p2}^2+i\eta).((Q-\mathrm{p1})^2+i\eta)} -$$

$$\frac{2\,\mathrm{p1}^2\,(\mathrm{p2}\cdot Q)}{(\mathrm{p1}^2+i\eta)^2.((Q-\mathrm{p1})^2+i\eta).((Q-\mathrm{p2})^2+i\eta).((-\mathrm{p1}-\mathrm{p2}+Q)^2+i\eta).(2z(\mathrm{p1}\cdot Q)(\mathrm{p2}\cdot q)-\mathrm{p1}\cdot\mathrm{p2}+i\eta).(\mathrm{p2}^2+i\eta)} +$$

$$\frac{2(\mathrm{p1}\cdot Q)(\mathrm{p2}\cdot Q)-2\,\mathrm{p1}^2(\mathrm{p2}\cdot Q)}{((-\mathrm{p1}-\mathrm{p2}+Q)^2+i\eta).(2z(\mathrm{p1}\cdot Q)(\mathrm{p2}\cdot q)-\mathrm{p1}\cdot\mathrm{p2}+i\eta).(\mathrm{p1}^2+i\eta).(\mathrm{p2}^2+i\eta).((Q-\mathrm{p1})^2+i\eta)^2.((Q-\mathrm{p2})^2+i\eta)} +$$

$$\frac{-2\,\mathrm{p1}^2(\mathrm{p2}\cdot Q)-2(\mathrm{p1}\cdot\mathrm{p2})(\mathrm{p2}\cdot Q)+2(\mathrm{p1}\cdot Q)(\mathrm{p2}\cdot Q)}{((Q-\mathrm{p2})^2+i\eta).(2z(\mathrm{p1}\cdot Q)(\mathrm{p2}\cdot q)-\mathrm{p1}\cdot\mathrm{p2}+i\eta).(\mathrm{p1}^2+i\eta).(\mathrm{p2}^2+i\eta).((Q-\mathrm{p1})^2+i\eta).((-\mathrm{p1}-\mathrm{p2}+Q)^2+i\eta)^2} +$$

$$\frac{(\mathrm{p1}\cdot\mathrm{p2})(\mathrm{p2}\cdot Q)-2z(\mathrm{p1}\cdot Q)(\mathrm{p2}\cdot q)(\mathrm{p2}\cdot Q)}{((-\mathrm{p1}-\mathrm{p2}+Q)^2+i\eta).(2z(\mathrm{p1}\cdot Q)(\mathrm{p2}\cdot q)-\mathrm{p1}\cdot\mathrm{p2}+i\eta)^2.(\mathrm{p1}^2+i\eta).(\mathrm{p2}^2+i\eta).((Q-\mathrm{p1})^2+i\eta).((Q-\mathrm{p2})^2+i\eta)}$$

- In FeynCalc 9.3 propagators can *in principle* have symbolic powers.
- With little effort one can write a custom function for deriving IBPs for the given topology à la LiteRed

```
In[29]:= SPD[Q] = 1;

In[30]:= topo = FCTopology[1,
  {SFAD[p1], SFAD[p2], SFAD[-p1 - p2 + Q], SFAD[-p2 + Q], SFAD[-p1 + Q],
    GFAD[-SPD[p1, p2] + 2 z SPD[p1, Q] SPD[p2, Q]]}]
```

Out[30]=

$$\text{FCTopology}\left(1, \left\{\frac{1}{(\text{p1}^2+i\eta)}, \frac{1}{(\text{p2}^2+i\eta)}, \frac{1}{((-\text{p1}-\text{p2}+Q)^2+i\eta)}, \frac{1}{((Q-\text{p2})^2+i\eta)}, \frac{1}{((Q-\text{p1})^2+i\eta)}, \frac{1}{(2z(\text{p1}\cdot Q)(\text{p2}\cdot Q)-\text{p1}\cdot\text{p2}+i\eta)}\right\}\right)$$

```
In[33]:= FCCreateIBPEquationsEEC[topo, {p1, p2}][[1 ;; 3]]
```

Out[33]=

$$\{-a(3)\,Y(3)\,\text{Ym}(1)-a(5)\,Y(5)\,\text{Ym}(1)+a(3)\,Y(3)\,\text{Ym}(4)+a(5)\,Y(5)-2\,a(1)-a(3)-a(5)-a(6)+D,$$

$$-\frac{1}{2}a(6)\,Y(6)\,\text{Ym}(1)^2 z-a(6)\,Y(6)\,\text{Ym}(1)(z-1)+a(6)\,Y(6)\,\text{Ym}(5)\,\text{Ym}(1)\,z-\frac{1}{2}a(6)\,Y(6)\,\text{Ym}(5)^2 z+ a(6)\,Y(6)\,\text{Ym}(5)\,z-a(3)\,Y(3)\,\text{Ym}(1)+a(4)\,Y(4)\,\text{Ym}(1)-a(2)\,Y(2)\,\text{Ym}(3)-a(4)\,Y(4)\,\text{Ym}(3)+ a(2)\,Y(2)\,\text{Ym}(4)+a(3)\,Y(3)\,\text{Ym}(4)+a(2)\,Y(2)\,\text{Ym}(5)-\frac{1}{2}a(6)\,Y(6)\,z-a(2)\,Y(2)-a(3)+a(4),$$

$$-\frac{1}{2}a(6)\,Y(6)\,\text{Ym}(2)^2 z-a(6)\,Y(6)\,\text{Ym}(2)(z-1)+a(6)\,Y(6)\,\text{Ym}(4)\,\text{Ym}(2)\,z-\frac{1}{2}a(6)\,Y(6)\,\text{Ym}(4)^2 z+ a(6)\,Y(6)\,\text{Ym}(4)\,z-a(3)\,Y(3)\,\text{Ym}(2)+a(5)\,Y(5)\,\text{Ym}(2)-a(1)\,Y(1)\,\text{Ym}(3)-a(5)\,Y(5)\,\text{Ym}(3)+ a(1)\,Y(1)\,\text{Ym}(4)+a(1)\,Y(1)\,\text{Ym}(5)+a(3)\,Y(3)\,\text{Ym}(5)-\frac{1}{2}a(6)\,Y(6)\,z-a(1)\,Y(1)-a(3)+a(5)\}$$

- `Y[i]` (`Ym[i]`) increase (decrease) the power of the i^{th} propagator
- `a[i]` denotes the original power of the i^{th} propagator.
- The output can be directly put into FIRE for the IBP reduction.

- The packages are *publicly* (GNU GPLv3) available on GitHub (https://github.com/FeynCalc)
- Works on Mathematica starting with version 8.0.
- If you are brave, you can try out the development version https://github.com/FeynCalc/feyncalc/wiki/Installation
- ...or just wait until the preprints appear on the arXiv.
- There will be 2 publications related to this project very soon
  - FeynCalc 9.3 (for the broad audience): coding finished, documentation updated, writing everything up. Huge changelog: https://github.com/FeynCalc/feyncalc/blob/master/FeynCalc/Changelog.md
  - FeynOnium 1.0 (for the NREFT community): employed in several ongoing projects, already usable but needs a bit of polishing.

Summary

- Our long-term (2016-2019) project to develop a toolbox for (NR)EFT calculations is now in its final stage.
- Publicly available code to handle nonrelativistic expressions and a wide range of nonstandard integrals.
- Not a fully automatic all-in-one solution, but a handy tool for knowledgeable people.
- The focus is on tree and 1-loop level.
- Validation by reproducing existing results from the literature and obtaining new predictions.
- Good progress, given the side-project nature of this work.

Outlook

- Currently no user feedback, expect a sharp rise after the publications.
- Many things that could be implemented or improved, unfortunately only finite amount of time.