News from the FeynOnium project

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in collaboration with
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Outline

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**FeynCalc** [Mertig et al., 1991; VS et al., 2016] is a package for semi-automatic symbolic QFT calculations.

Available since 1990, my contributions started in 2014.

Popular among EFT practitioners, especially in the heavy quarkonium community.

- Familiar environment of Mathematica.
- Flat learning curve, usable documentation, many examples.
- Friendly mailing list for questions & bug reports.
- Easy to extend with own codes or interface with other tools.

Yet, in my view, applying **FeynCalc** to EFT calculations is still difficult.

- Only Lorentz tensors: What about Cartesian?
- Only standard \((p^2 - m^2 + i\eta)^{-1}\) propagators: Why not Euclidean or eikonal?
- Manipulation of unusual integrals (abundant in NREFTs)?
- Pauli algebra?
Common solution: elaborated private codes.
Often heavily tailored to a particular project.
High probability to forget how things work in < 1 year.
Been there, done that

- EM Van der Waals forces between hydrogen atoms in pNRQED: A lot of Cartesian integrals to deal with [Brambilla, VS, Tarrús Castellà, Vairo, 2017].
- Relativistic $O(v^2)$ corrections to $e^+e^- \rightarrow \chi_{cJ} \gamma$: QCD-NRQCD matching via the threshold expansion method [Braaten & Chen, 1996] to extract Wilson coefficients of choromelectric operators ⇒ Nonrelativistic expansions, tensor projections, Schouten’s identities [Brambilla, Chen, Jia, VS, Vairo, 2017].

Publication as the main goal, the codes are just by-products.
Idea at the end of my PhD: Recycle and generalize the codes.
Interesting side-project: (NR)EFTs made easy with FeynCalc.

Subdivided into 2 projects
- **FeynCalc 9.3**: improved package with broad support for various EFTs.
- **FeynOnium**: specific extras for NREFTs (mainly NRQCD/pNRQCD).

Reproduce key results from the literature to test the approach.

As usual, initial time estimates too optimistic …

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"I SPEND A LOT OF TIME ON THIS TASK. I SHOULD WRITE A PROGRAM AUTOMATING IT!"

Source: xkcd.com/1319
Many conceptual and technical issues to think over.

Account for relations between Lorentz and Cartesian tensors.

Every tensor that carries a Lorentz index can be decomposed into a temporal and a Cartesian piece

\[ p^\mu = p_\nu g^{\mu\nu} = p_0 g^{\mu 0} + p_i g^{\mu i} \]

Distinction between upper (contravariant) and lower (covariant) indices

\[ p^i = -p_i, \]

metric signature dependent!

Backward-compatibility: Extend the existing syntax but do not break it.

Prevent over-engineering: Add only minimal amount of new symbols.

Keep it simple: No new commands for standard operations (contractions, expansions etc.)
**Motivation**

- Tree-level calculations with NR quantities: Mostly completed during 2016-2017, presented at QWG 2017

- Loop-level calculations with nonstandard integrals: somewhat harder than expected, ongoing work since 2017.
Manipulations of Cartesian tensors (common in NREFTs) are easy. Calculating \( \left( \delta^{ij} + \frac{n^i k^j + k^i n^j}{k \cdot n} \right) q^i r^j (q \cdot r) \) amounts to

\[
\text{In}[2]:= \left( \text{KD}[i, j] + \frac{\text{CV}[k, j] \text{CV}[n, i] + \text{CV}[k, i] \text{CV}[n, j]}{\text{SP}[k, n]} \right) \text{CV}[q, i] \text{CV}[r, j] \text{CSP}[q, r] \quad // \quad \text{Contract}
\]

\[
\text{Out}[2]= (\vec{q} \cdot \vec{r})^2 + \frac{(k \cdot \vec{r})(\vec{n} \cdot \vec{q})(\vec{q} \cdot \vec{r})}{k \cdot n} + \frac{(k \cdot q)(\vec{n} \cdot \vec{r})(\vec{q} \cdot \vec{r})}{k \cdot n}
\]

\[
\text{In}[3]:= \% \quad // \quad \text{LorentzToCartesian}
\]

\[
\text{Out}[3]= (\vec{q} \cdot \vec{r})^2 + \frac{(k \cdot \vec{r})(\vec{n} \cdot \vec{q})(\vec{q} \cdot \vec{r})}{k^0 n^0 - k \cdot n} + \frac{(k \cdot q)(\vec{n} \cdot \vec{r})(\vec{q} \cdot \vec{r})}{k^0 n^0 - k \cdot n}
\]

- Lorentz (Cartesian) vectors in 4-dim (3-dim) have a bar.
- \( \text{KD}[i, j] \sim \delta^{ij} \): Cartesian metric (Kronecker Delta).
- \( \text{CV}[p, i] \sim p^i \): Cartesian 3-vector.
- \( \text{SP}[k, n] \sim k \cdot n \): Scalar product of two 4-vectors.
- \( \text{CSP}[q, r] \sim q \cdot r \): Scalar product of two 3-vectors.
- \( \text{TC}[k] \sim k^0 \): Temporal component of a 4-vector.
- Contract handles the contractions of Lorentz and Cartesian indices.
- LorentzToCartesian for breaking the manifest Lorentz covariance.
Differentiation w.r.t 3-vectors (projections, series expansions, differential equations) is straightforward.

Let us calculate \( \frac{\partial}{\partial q^i} \frac{1}{(q^2+m^2)(p+q)^2} \)

```math
In[4]:= 1 / (CSPD[q] + m^2) 1 / (CSPD[q + p])
Out[4]= \( \frac{1}{(m^2 + q^2)(p + q)^2} \)
```

```math
In[5]:= ThreeDivergence[f, CVD[q, i]]
Out[5]= \( -\frac{2 p^i}{(m^2 + q^2)(2(p \cdot q) + p^2 + q^2)^2} - \frac{2 q^i}{(m^2 + q^2)(2(p \cdot q) + p^2 + q^2)^2} - \frac{2 q^i}{(m^2 + q^2)(2(p \cdot q) + p^2 + q^2)^2} \)
```

\( \text{CSP} [q, r] \sim q \cdot r \): Scalar product of two 3-vectors in \( D - 1 \) dimensions.

\( \text{ThreeDivergence} [f, \text{CVD} [q, i]] \sim \frac{\partial}{\partial q^i} f. \)

In general, Cartesian tensors can live in 3, \( D - 1 \) or \( D - 4 \) dimensions.
Dirac matrices may carry Cartesian or temporal indices.

We can simplify chains of such matrices and compute traces as in

$$\text{Tr} \left( \not{p} \gamma^0 P_R \not{q} P_L \not{\epsilon}(n) \right),$$

with $\not{p} = \gamma^i p^i$.

$$\not{\epsilon} \bar{p} \bar{q} \not{\epsilon}(n)$$ stands for $\epsilon^{ijk} p^i q^j \epsilon^k(n)$.

CGS$[p] \sim \gamma^i p^i$: Dirac matrix contracted with a 3-vector.

GA$[6/7] \sim \frac{1}{2} (1 \pm \gamma_5)$: Chirality projectors.

TGA[] $\sim \gamma^0$.

DiracSimplify handles the Dirac algebra.
Tree-level calculations with NREFTs

- Pauli matrices are indispensable in NR calculations, can handle that too.

\[
\text{In}[10] := \text{PauliEta}[-i].\text{CSIS}[q].\text{CSIS}[\text{Polarization}[k1, -1]].\text{CSIS}[k1].\text{CSIS}[q]. \text{PauliXi}[i]
\]

\[
\text{Out}[10] = \eta^\dagger (\sigma \cdot q) (\sigma \cdot \bar{\epsilon}^*(k1)) (\sigma \cdot k1) (\sigma \cdot q) \xi
\]

\[
\text{In}[11] := \text{PauliTrick}[\%, \text{PauliReduce} \rightarrow \text{False}]
\]

\[
\text{Out}[11] = 2 (q \cdot \bar{\epsilon}^*(k1)) \eta^\dagger (\sigma \cdot k1)(\sigma \cdot q) \xi + 2 (k1 \cdot q) \eta^\dagger (\sigma \cdot \bar{\epsilon}^*(k1))(\sigma \cdot q) \xi + q^2 \eta^\dagger (\sigma \cdot \bar{\epsilon}^*(k1))(\sigma \cdot k1) \xi
\]

- \(\text{CSIS}[p] \sim \sigma^i p^i\): Pauli matrix contracted to a 3-vector.
- \text{PauliEta}, \text{PauliXi} denote the Pauli 2-spinors.
- \text{PauliTrick} simplifies a chain of Pauli matrices.
- \text{PauliReduce} governs the application of

\[
\sigma^i \sigma^j = \delta^{ij} + i \varepsilon^{ijk} \sigma^k.
\]

Important for DR calculations, \(\varepsilon^{ijk}\) is tricky in \(D - 1\) dimensions.
Validation is crucial for such software frameworks!

FeynCalc has a decent test-coverage with a large test-suite.

Still, unit-tests cannot replace real-life calculations.

Reproducing analytic results from the literature is very time-consuming.

Many publications are very concise regarding the technical details.

A selection of results reproduced using FeynCalc+FeynOnium

- \( Q\bar{Q}(\not{3}P_0^{[1]}) \rightarrow gg \) at \( \mathcal{O}(\alpha_s v^0) \) [Petrelli et al., 1998]
- \( \eta_c/\chi_{cJ} \rightarrow 2\gamma \) at \( \mathcal{O}(\alpha_s^0 v^4) \) [Brambilla et al., 2006]
- \( \eta_c \rightarrow 2\gamma \) at \( \mathcal{O}(\alpha_s v^2) \) [Jia et al., 2011]
- \( H \rightarrow J/\psi \gamma \) at \( \mathcal{O}(\alpha_s^0 v^2) \) [Bodwin et al., 2014]

See also today’s talk of Hee Sok Chung on \( \mathcal{O}(\alpha_s^0 v^4) \) corrections to 
\( H \rightarrow J/\psi \gamma \)!

No manipulations of nonstandard loop integrals were required here.
What is so special about loop integrals in (NR)EFTs?

For standard integrals at 1-loop we have the time-proven Passarino-Veltman (PaVe) [Passarino & Veltman, 1979] technique.

Old method, but still convenient and easy to apply.

Implemented in FeynCalc since version 1.0, subsequently improved in the newer versions.

The FeynCalc 1-loop toolbox includes

- FDS: Simplification using symmetries and loop momentum shifts.
- ApartFF: Partial fractioning, based on [Feng, 2012].
- ToPaVe: Conversion of scalar integrals to PaVe functions.
- PaVeUVPart (new in 9.3): extract the UV part of arbitrary PaVe functions, based on [Sulyok, 2017].
- $\text{KeepLogDivergentScalelessIntegrals}$: switch to prevent \( \left( \frac{1}{\varepsilon_{\text{UV}}} - \frac{1}{\varepsilon_{\text{IR}}} \right) \)-type integrals from being set to zero.
- TID: 1-loop tensor reduction (all-in-one).
Nonstandard integrals (not \((p^2 - m^2)^{-1}\)-type) are not that easy.

Not all tensor integrals reducable to scalars with unit numerators (even at 1-loop)

No universal basis like Passarino-Veltman.

Master integrals usually evaluated on case-by-case basis.

Need special care when simplifying the denominators, e.g. for eikonal

\[
\int \frac{d^D q}{[q^2 + i\eta][-2p \cdot q - m^2 + i\eta]} = - \int \frac{d^D q}{[q^2 + i\eta][2p \cdot q + m^2 - i\eta]}
\]
Loop-level calculations with NREFTs

- Most useful manipulations still applicable to almost all integrals.
- Partial fractioning (linear independence of propagators)

\[
\frac{4(p \cdot q)}{q^2(q + p)^2(q - p)^2} = \frac{1}{q^2(q-p)^2} + \frac{1}{q^2(q+p)^2}
\]

- Tensor reduction to remove loop momenta with uncontracted indices

\[
\int d^D q \frac{q^\mu q^\nu}{q^2 (q \cdot p - m^2)} = \frac{m^4}{(D-1)p^4} (Dp^\mu p^\nu - p^2 g^{\mu\nu}) \int \frac{d^D q}{q^2 (q \cdot p - m^2)}
\]

- Those are useful at 1-loop but also in multiloop calculations.
- Need a simple and consistent way to represent various (NR)EFT propagators in a symbolic code.
Surprisingly, in **FeynCalc** only 3 new shortcuts are needed.

In addition to the standard FAD (FeynAmpDenominator) introduce

- SFAD (StandardFeynAmpDenominator): Covariant integrals.
- CFAD (CartesianFeynAmpDenominator): Cartesian integrals.
- GFAD (GenericFeynAmpDenominator): anything else.

<table>
<thead>
<tr>
<th>Shortcut in <strong>FeynCalc</strong></th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAD ([l - p_1 - \ldots, m])</td>
<td>(\frac{1}{(l-p_1-\ldots)^2-m^2})</td>
</tr>
<tr>
<td>SFAD ([l - p_1 - \ldots, \pm l.(q_1 + \ldots), \pm m^2, \pm 1, n])</td>
<td>(\left[\frac{1}{(l-p_1-\ldots)^2 \pm l.(q_1 + \ldots) \mp m^2 \pm i\eta}\right]^n)</td>
</tr>
<tr>
<td>CFAD ([l - p_1 - \ldots, \pm l.(q_1 + \ldots), \pm m^2, \pm 1, n])</td>
<td>(\left[\frac{1}{(l-p_1-\ldots)^2 \pm l.(q_1 + \ldots) \mp m^2 \pm i\eta}\right]^n)</td>
</tr>
<tr>
<td>GFAD ([x, \pm 1, n])</td>
<td>(\left[\frac{1}{x \pm i\eta}\right]^n)</td>
</tr>
</tbody>
</table>

Notice that the mass term and \(i\eta\) can be positive or negative.
Simplify the 1-loop heavy nucleon propagator in baryonic $\chi$PT [Ecker & Mojzis, 1996] (example from S. Scherer’s lectures [Scherer, 2003])

$$\int \frac{d^D k}{(2\pi)^D} (-S_v \cdot k) \sigma^i \frac{1}{v \cdot (r - k) + i\eta} \frac{1}{k^2 - M^2 + i\eta} (S_v \cdot k) \sigma^i,$$

with $S^\mu_v = -\frac{1}{2} \gamma_5 (\gamma^\mu \not v - v^\mu)$.

Painful in FeynCalc 9.2, trivial with version 9.3
Loop-level calculations with NREFTs

- Tensor reduction of Cartesian integrals e.g.

\[ \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \frac{k^i k^j}{[k^2 + m^2][(k - q)^2 + m^2]} \]

- Very common in pNRQCD calculations if you prefer to handle spatial and temporal integrations separately.

- Sort of possible in \textsc{FeynCalc} 9.2, but not straightforward.

- Now it just works

In[44]:= \text{exp} = \text{CVD}[k, i] \text{CVD}[k, j] \text{CFAD}[[k, m^2], [k - q, m^2]]

Out[44]= \frac{k^i k^j}{(k^2 + m^2 - i \eta)((k - q)^2 + m^2 - i \eta)}

In[47]:= \text{TID}[\text{exp}, k]

Out[47]= \frac{(D - 1) q^i q^j - q^2 \delta^i j + 2 q^i q^j}{2(2 - D) q^2 (k^2 + m^2 - i \eta)} - \frac{(D - 1) q^2 q^i q^j - 4 m^2 q^2 \delta^i j + 4 m^2 q^i q^j - q^4 \delta^i j}{4(2 - D) q^2 (k^2 + m^2 - i \eta)((k - q)^2 + m^2 - i \eta)}
Loop-level calculations with NREFTs

- My favorite new propagator type: GFAD, the generic propagator
- GFAD can be anything, so it works with *highly* nonstandard propagators
- The idea for this came from my work on the Energy-Energy-Correlation (EEC) event shape variable in QCD.

EEC [Basham et al., 1978] is a classical (IR safe) hadronic observable in $e^+e^-$ annihilation: $e^+e^- \rightarrow \gamma^* / Z^0 \rightarrow a + b + X$.

- Formal definition ($\cos \theta_{ab} = \hat{p}_a \cdot \hat{p}_b$)

$$\frac{1}{\sigma_{tot}} \frac{d\Sigma(\chi)}{d\cos \chi} = \sum_{a,b} \int \frac{E_a E_b}{Q^2} \delta(\cos \theta_{ab} - \cos \chi) \times \: d\sigma_{a+b+X}.$$  

- 2 calorimeters at relative angle $\chi$ measure the energies of the hadrons $a$ and $b$.

- Diff. angular distribution from measuring the energies $(E_a, E_b)$ of and the angle $\theta_{ab}$ between all hadrons pairs produced in each event.

Source: [Tulipánt et al., 2017].
Numerically, EEC is currently known at NNLO \cite{DelDuca:2016wmj, Tulipant:2017hto}.

Analytically (fixed-order), the LO result was state of the art for 40 years \cite{Basham:1978is}

\[
\frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma(\chi)}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi} \frac{C_F}{4(1 - z)z^5} \left[ 3z(2 - 3z) + 2(2z^2 - 6z + 3) \log(1 - z) \right] + O(\alpha_s^2), \quad \text{with } z = \frac{(1 - \cos\chi)}{2}.
\]

With my collaborators we obtained fixed-order analytic NLO results for
- $e^+ e^- \rightarrow \gamma^* \rightarrow \text{partons}$ in pQCD \cite{Dixon:2018sos}
- $H \rightarrow \text{gluons} \rightarrow \text{partons}$ in HEFT \cite{Luo:2019hjv}

"NLO" in this context: by calculating 3-loop phase space integrals with special propagators from the measurement function.

Done using familiar multiloop tools (\textbf{FORM} \cite{Vermaseren:2000nd}, \textbf{COLOR} \cite{vanRitbergen:1998pn}, \textbf{FIRE 5} \cite{Smirnov:2015mct}, \textbf{LiteRed} \cite{Lee:2012cn}, \textbf{Fuchsia} \cite{Gituliar:2016xpv})

In the HEFT calculation \textbf{FeynCalc} was employed at some intermediate stages.
Loop-level calculations with NREFTs

- **EEC:**
  \[
  \frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma(\chi)}{d\cos \chi} = \sum_{a,b} \int \frac{E_a E_b}{Q^2} \delta(\cos \theta_{ab} - \cos \chi) d\sigma_{a+b+x}.
  \]

- Measurement function in the rest frame, setting \(Q^2 = 1\)
  \[
  E_a E_b \delta(\cos \theta_{ab} - \cos \chi) = (p_a \cdot Q)^2 (p_b \cdot Q)^2 \delta(2z p_a \cdot Q p_b \cdot Q - p_a \cdot p_b).
  \]

- Analytic calculation of phase-space integrals using the reverse unitarity method [Anastasiou & Melnikov, 2002; Anastasiou et al., 2003]
  \[
  \delta(2z p_a \cdot Q p_b \cdot Q - p_a \cdot p_b) \sim \frac{1}{2z p_a \cdot Q p_b \cdot Q - p_a \cdot p_b} \bigg|_{\text{cut}}
  \]
  \([2z p_a \cdot Q p_b \cdot Q - p_a \cdot p_b]^{-1}\) is nonlinear in the scalar products.

- Nonstandard propagator that is not supported by public codes.

- Initially we used "hacked" **LiteRed** to derive IBPs [Chetyrkin & Tkachov, 1981] and differential equations [Kotikov, 1991a, 1991b, 1991c; Bern et al., 1994; Remiddi, 1997; Gehrmann & Remiddi, 2000].

- Later we discovered interesting tricks to optimize the IBP equations and simplify the integrands.

- For "experiments" I needed a more convenient software framework.
Loop-level calculations with NREFTs

- Using GFADs it is easy to put the nonlinear propagators into FeynCalc

Example: calculate

\[
\int d^D p_1 d^D p_2 \frac{\partial}{\partial p_1^\mu} \frac{p_1^\mu (p_2 \cdot Q)}{p_1^2 p_2^2 (Q - p_1 - p_2)^2 (Q - p_1)^2 (Q - p_2)^2} \frac{1}{2z p_a \cdot Q p_b \cdot Q - p_a \cdot p_b}
\]

\[\text{In}[72]:= \text{SFAD}[p_1, p_2, Q - p_1 - p_2, Q - p_1, Q - p_2] \text{GFAD}[2 z \text{SPD}[p_1, Q] \text{SPD}[p_2, q] - \text{SPD}[p_1, p_2]]\]

\[\text{Out}[72]= \frac{1}{(p_1^2 + i \eta).(p_2^2 + i \eta).((-p_1 - p_2 + Q)^2 + i \eta).((Q - p_1)^2 + i \eta).((Q - p_2)^2 + i \eta).((2 z (p_1 \cdot Q) (p_2 \cdot q) - p_1 \cdot p_2 + i \eta).(p_1^2 + i \eta).(p_2^2 + i \eta).((Q - p_1)^2 + i \eta))}
\]

\[\text{In}[73]:= \text{FourDivergence}[\text{FVD}[p_1, \mu] \text{SPD}[p_2, Q], \text{FVD}[p_1, \mu]]\]

\[\text{Out}[73]= \frac{D (p_2 \cdot Q)}{(Q - p_2)^2 + i \eta).((-p_1 - p_2 + Q)^2 + i \eta).(2 z (p_1 \cdot Q) (p_2 \cdot q) - p_1 \cdot p_2 + i \eta).(p_1^2 + i \eta).(p_2^2 + i \eta).((Q - p_1)^2 + i \eta) - 2 p_1^2 (p_2 \cdot Q)}
\]

\[\text{Out}[73]= \frac{2 (p_1 \cdot Q) (p_2 \cdot Q) - 2 p_1^2 (p_2 \cdot Q)}{(Q - p_2)^2 + i \eta).((-p_1 - p_2 + Q)^2 + i \eta).(2 z (p_1 \cdot Q) (p_2 \cdot q) - p_1 \cdot p_2 + i \eta).(p_1^2 + i \eta).(p_2^2 + i \eta).((Q - p_1)^2 + i \eta) - 2 p_1^2 (p_2 \cdot Q) - 2 (p_1 \cdot p_2) (p_2 \cdot Q) + 2 (p_1 \cdot Q) (p_2 \cdot Q)}
\]

\[\text{Out}[73]= \frac{(Q - p_2)^2 + i \eta).((Q - p_1)^2 + i \eta).((Q - p_2)^2 + i \eta).((Q - p_1)^2 + i \eta) - 2 p_1^2 (p_2 \cdot Q) - 2 (p_1 \cdot p_2) (p_2 \cdot Q) + 2 (p_1 \cdot Q) (p_2 \cdot Q)}{(Q - p_2)^2 + i \eta).((-p_1 - p_2 + Q)^2 + i \eta).(2 z (p_1 \cdot Q) (p_2 \cdot q) - p_1 \cdot p_2 + i \eta).(p_1^2 + i \eta).(p_2^2 + i \eta).((Q - p_1)^2 + i \eta) - 2 p_1^2 (p_2 \cdot Q) - 2 (p_1 \cdot p_2) (p_2 \cdot Q) + 2 (p_1 \cdot Q) (p_2 \cdot Q)}
\]
In **FeynCalc** 9.3 propagators can *in principle* have symbolic powers. With little effort one can write a custom function for deriving IBPs for the given topology à la **LiteRed**.

\[
\begin{align*}
\text{In[29]} &: \quad \text{SPD} [\mathbf{Q}] = 1;
\text{In[30]} &: \quad \text{topo} = \text{FCTopology}[1,
\{\text{SFAD}[\mathbf{p}_1], \text{SFAD}[\mathbf{p}_2], \text{SFAD}[\mathbf{-p}_1 - \mathbf{p}_2 + \mathbf{Q}], \text{SFAD}[-\mathbf{p}_2 + \mathbf{Q}], \text{SFAD}[-\mathbf{p}_1 + \mathbf{Q}],
\text{GFAD}[-\text{SPD}[\mathbf{p}_1, \mathbf{p}_2] + 2 \, \text{SPD}[\mathbf{p}_1, \mathbf{Q}] \, \text{SPD}[\mathbf{p}_2, \mathbf{Q}])\}]
\text{Out[30]} &= \quad \text{FCTopology}\left(1,\left\{\frac{1}{(p1^2 + i \eta)}, \frac{1}{(p2^2 + i \eta)}, \frac{1}{((-p1 - p2 + Q)^2 + i \eta)}, \frac{1}{((Q - p1)^2 + i \eta)}, \frac{1}{((Q - p2)^2 + i \eta)}, \frac{1}{(2 \, z \, (p1 \cdot Q) (p2 \cdot Q) - p1 \cdot p2 + i \eta)}\right\}\right)
\text{In[33]} &: \quad \text{FCCreateIBPEquationsEEC[topo, \{\mathbf{p}_1, \mathbf{p}_2\}]\{1 ;; 3\}] \\
\text{Out[33]} &= \quad \left\{-a(3) \, Y(3) \, Ym(1) - a(5) \, Y(5) \, Ym(1) + a(3) \, Y(3) \, Ym(4) + a(5) \, Y(5) - 2 \, a(1) - a(3) - a(5) - a(6) + D, \right.
\frac{1}{2} \, a(6) \, Y(6) \, Ym(1)^2 \, z - a(6) \, Y(6) \, Ym(1) \, (z - 1) + a(6) \, Y(6) \, Ym(5) \, Ym(1) \, z - \frac{1}{2} \, a(6) \, Y(6) \, Ym(5)^2 \, z + \\
a(6) \, Y(6) \, Ym(5) \, z - a(3) \, Y(3) \, Ym(1) + a(4) \, Y(4) \, Ym(1) - a(2) \, Y(2) \, Ym(3) - a(4) \, Y(4) \, Ym(3) + \\
a(2) \, Y(2) \, Ym(4) + a(3) \, Y(3) \, Ym(4) + a(2) \, Y(2) \, Ym(5) - \frac{1}{2} \, a(6) \, Y(6) \, z - a(2) \, Y(2) - a(3) + a(4), \\
\left. - \frac{1}{2} \, a(6) \, Y(6) \, Ym(2)^2 \, z - a(6) \, Y(6) \, Ym(2) \, (z - 1) + a(6) \, Y(6) \, Ym(4) \, Ym(2) \, z - \frac{1}{2} \, a(6) \, Y(6) \, Ym(4)^2 \, z + \\
a(6) \, Y(6) \, Ym(4) \, z - a(3) \, Y(3) \, Ym(2) + a(5) \, Y(5) \, Ym(2) - a(1) \, Y(1) \, Ym(3) - a(5) \, Y(5) \, Ym(3) + \\
a(1) \, Y(1) \, Ym(4) + a(1) \, Y(1) \, Ym(5) + a(3) \, Y(3) \, Ym(5) - \frac{1}{2} \, a(6) \, Y(6) \, z - a(1) \, Y(1) - a(3) + a(5)\right\} \\
\end{align*}
\]

- \( Y[i] \) (\( Ym[i] \)) increase (decrease) the power of the \( i^{th} \) propagator
- \( a[i] \) denotes the original power of the \( i^{th} \) propagator.
- The output can be directly put into **FIRE** for the IBP reduction.
The packages are *publicly* (GNU GPLv3) available on GitHub ([https://github.com/FeynCalc](https://github.com/FeynCalc))

- Works on *Mathematica* starting with version 8.0.
- If you are brave, you can try out the development version [https://github.com/FeynCalc/feyncalc/wiki/Installation](https://github.com/FeynCalc/feyncalc/wiki/Installation)
- ...or just wait until the preprints appear on the arXiv.

There will be 2 publications related to this project very soon

- **FeynCalc** 9.3 (for the broad audience): coding finished, documentation updated, writing everything up. Huge changelog: [https://github.com/FeynCalc/feyncalc/blob/master/FeynCalc/Changelog.md](https://github.com/FeynCalc/feyncalc/blob/master/FeynCalc/Changelog.md)
- **FeynOnium** 1.0 (for the NREFT community): employed in several ongoing projects, already usable but needs a bit of polishing.
Summary

Our long-term (2016-2019) project to develop a toolbox for (NR)EFT calculations is now in its final stage.

Publicly available code to handle nonrelativistic expressions and a wide range of nonstandard integrals.

Not a fully automatic all-in-one solution, but a handy tool for knowledgeable people.

The focus is on tree and 1-loop level.

Validation by reproducing existing results from the literature and obtaining new predictions.

Good progress, given the side-project nature of this work.

Outlook

Currently no user feedback, expect a sharp rise after the publications.

Many things that could be implemented or improved, unfortunately only finite amount of time.