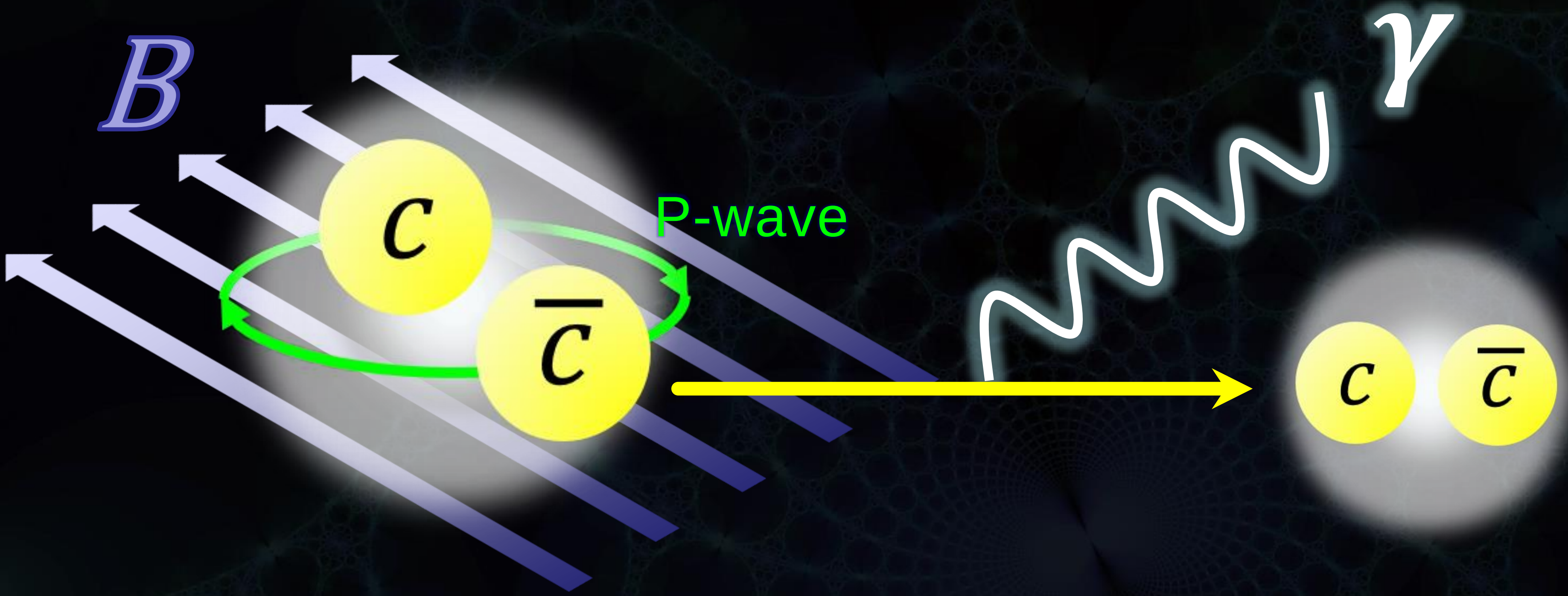
QWG 2019 - The 13th International Workshop on Heavy Quarkonium

Quarkonium radiative decays from the Hadronic Paschen-Back effect



Kei Suzuki (JAEA, Japan)

S. Iwasaki and K. Suzuki, Phys.Rev.D98, 054017 (2018)

S. Iwasaki, M. Oka, K. Suzuki, and T. Yoshida, Phys.Lett.B790, 71 (2019)

What's Paschen-Back effect?

= Strong B-field limit of Zeeman effect

$$|J; LS\rangle \longrightarrow |L_z S_z\rangle$$

No B-field:
(L_z and S_z are not good by LS coupling)

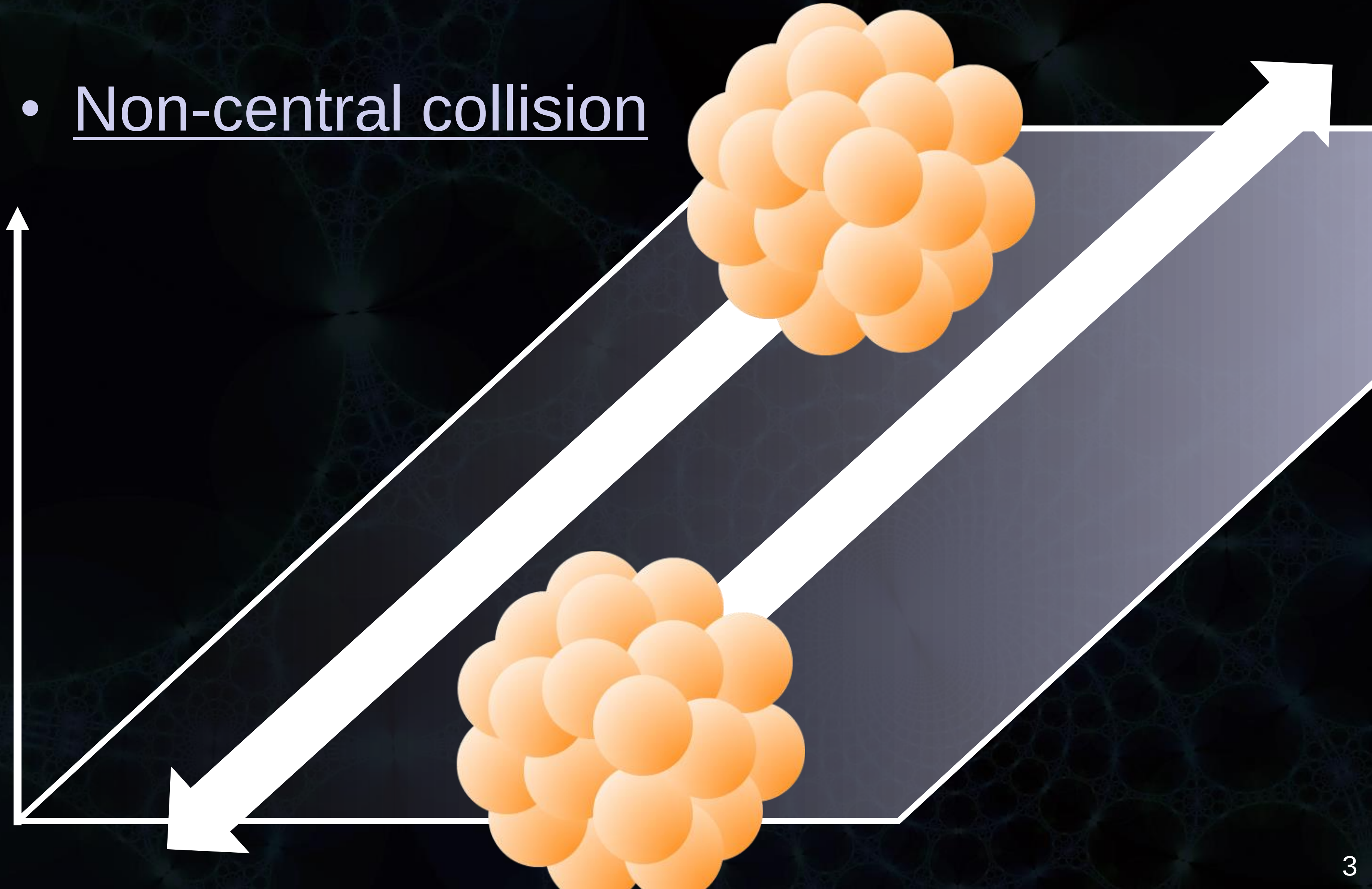
Paschen-Back limit
(J, L, S are not good by B-field)

Condition: Larger B-field than LS splitting

$$\sqrt{eB} \gg V_{LS}$$

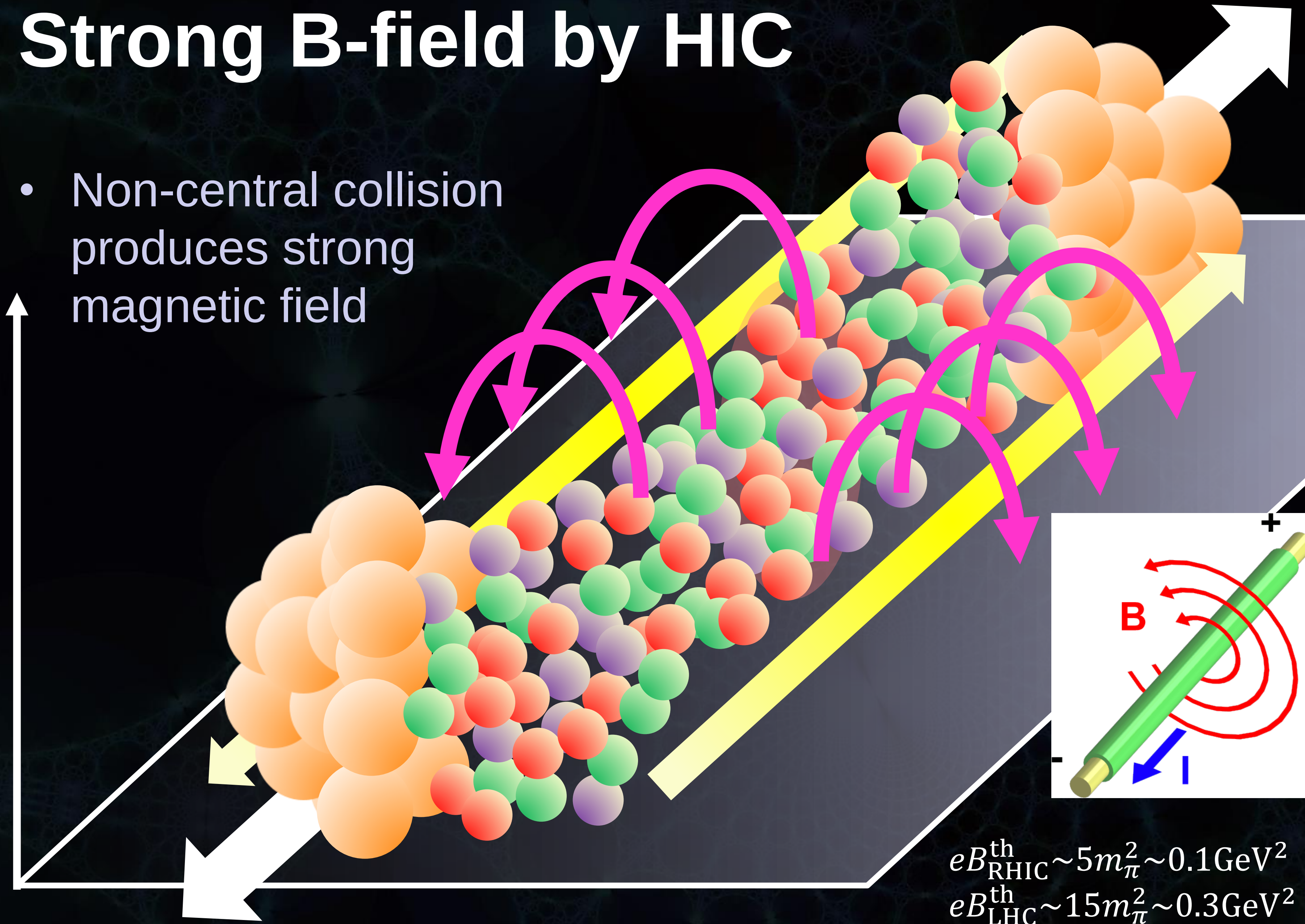
Strong B-field by HIC

- Non-central collision



Strong B-field by HIC

- Non-central collision produces strong magnetic field

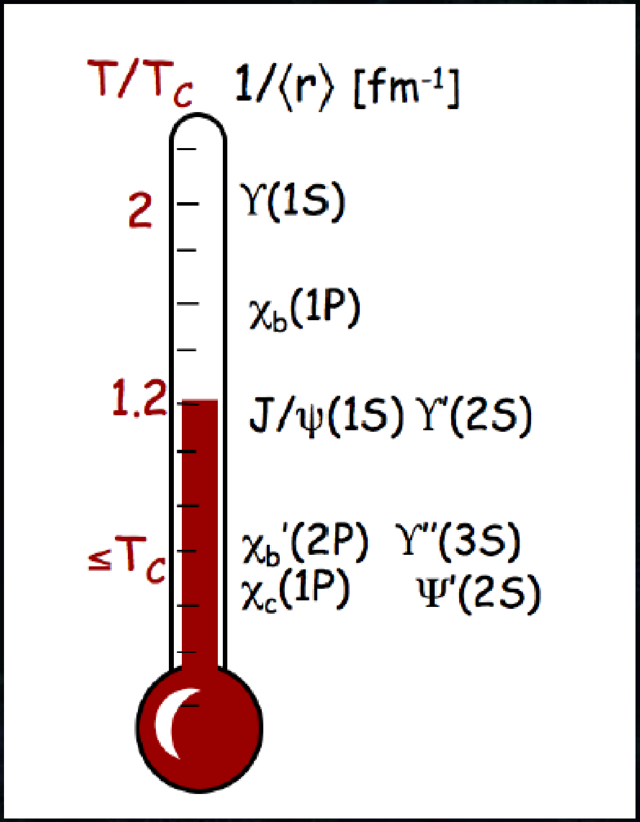


$eB_{\mathrm{RHIC}}^{\mathrm{th}} \sim 5 m_\pi^2 \sim 0.1\mathrm{GeV}^2$
$eB_{\mathrm{LHC}}^{\mathrm{th}} \sim 15 m_\pi^2 \sim 0.3\mathrm{GeV}^2$

One of the motivations:

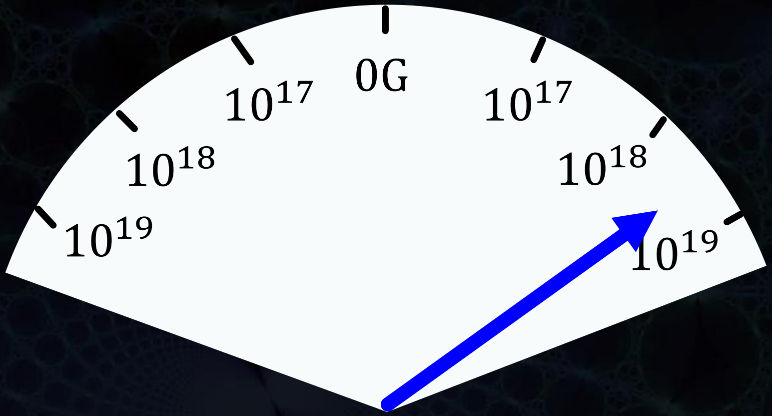
How to measure B-field by HIC?

Quarkonia as a thermometer



arXiv:0811.0337

Quarkonia as a **gaussmeter!**



Other motivations:

Compact stars, lattice simulations…

Cf.) J. Alford and M. Strickland, PRD88 (2013)

Nonrelativistic two-body Hamiltonian in B-field

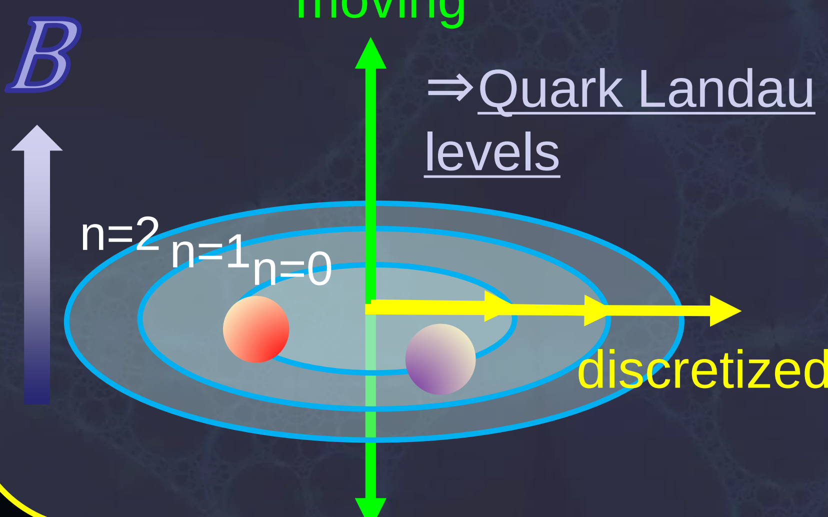
$$A = \frac{1}{2} B \times \boldsymbol{r}_i$$

Linear + Coulomb + Spin-spin + LS + Tensor

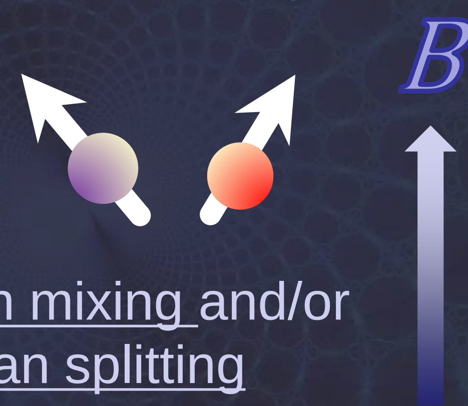
$$H = \sum_{i=1,2} \frac{1}{2m_i}(\boldsymbol{p}_i - q_i A)^2 - \boldsymbol{\mu}_i \cdot B + m_i + V(r)$$

(1) Modification of kinetic energy perpendicular to B

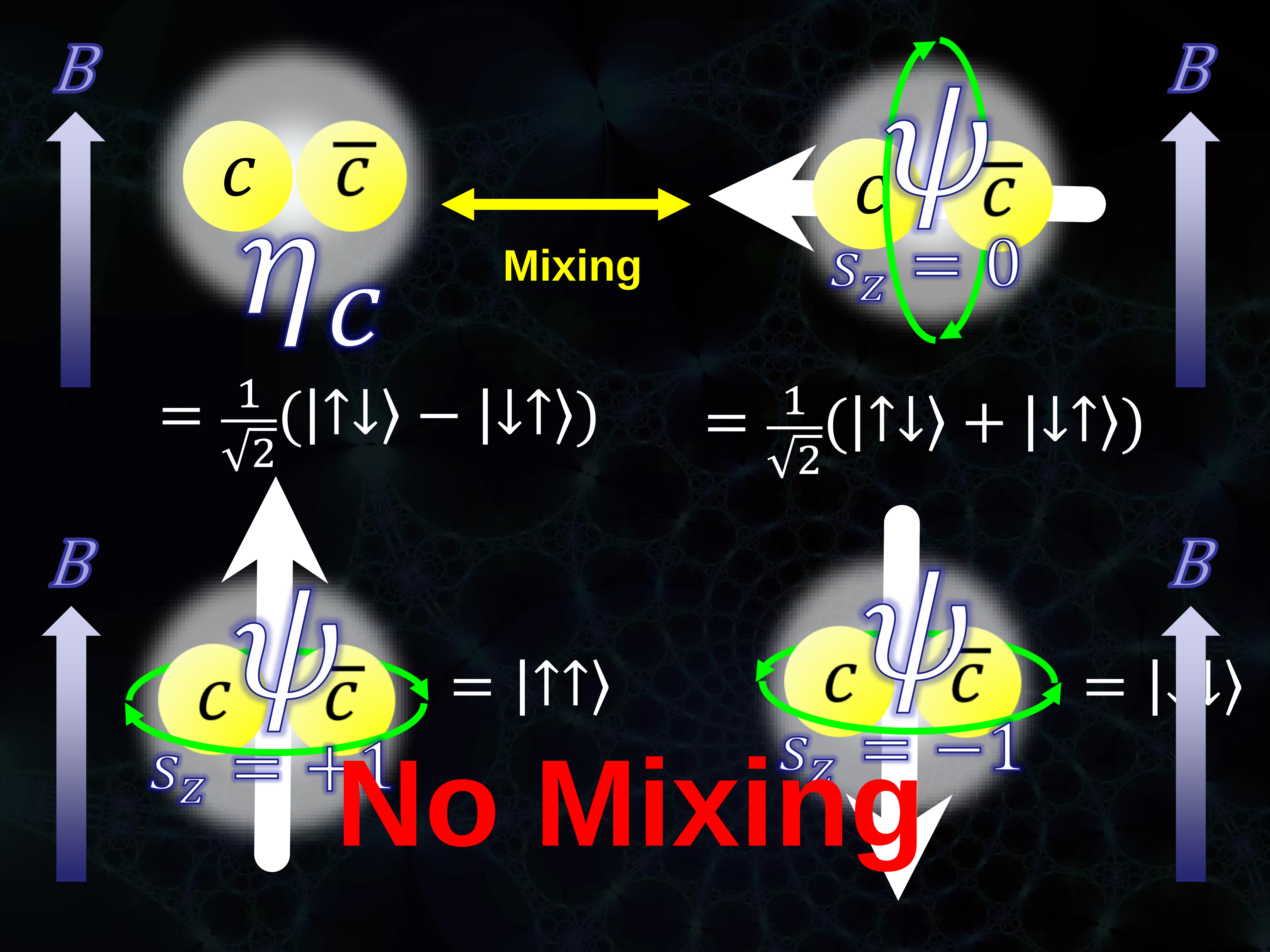
⇒Quark Landau levels



(2) Alignment of magnetic moment



⇒Spin mixing and/or Zeeman splitting



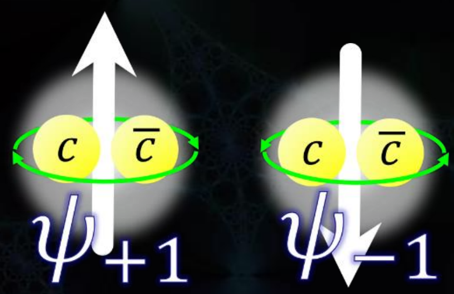
B
c $\bar{c}$
η_c
Mixing
ψ
c $\bar{c}$
$S_z = 0$
B
$= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
$= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
B
ψ
c $\bar{c}$
$= |\uparrow\uparrow\rangle$
$S_z = +1$
ψ
c $\bar{c}$
$= |\downarrow\downarrow\rangle$
$S_z = -1$
B
No Mixing

Strictly speaking…

$$\hat{H} = \boldsymbol{\mu} \cdot \boldsymbol{B} = (\boldsymbol{\mu}_q + \boldsymbol{\mu}_{\bar{q}}) \cdot \boldsymbol{B}, \text{ where } \boldsymbol{\mu}_i = \frac{gq}{2m_i}\boldsymbol{S}_i$$

$$\hat{H}|\uparrow\uparrow\rangle = -\frac{gqB}{4m_1}|\uparrow\uparrow\rangle + \frac{gqB}{4m_2}|\uparrow\uparrow\rangle$$

$$\hat{H}|\downarrow\downarrow\rangle = \frac{gqB}{4m_1}|\downarrow\downarrow\rangle - \frac{gqB}{4m_2}|\downarrow\downarrow\rangle$$



⇒If $m_1 \neq m_2$, diagonal component is finite = Hadron Zeeman splitting

⇒If $m_1 = m_2$, diagonal component is zero = No Zeeman splitting

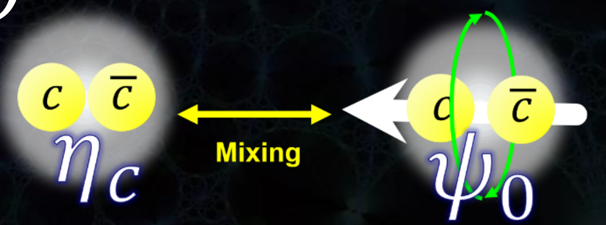
$$\hat{H}\,(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$= -\frac{gqB}{4m_1}|\uparrow\downarrow\rangle - \frac{gqB}{4m_1}|\downarrow\uparrow\rangle - \frac{gqB}{4m_2}|\uparrow\downarrow\rangle - \frac{gqB}{4m_2}|\downarrow\uparrow\rangle$$

$$= -\left(\frac{gqB}{4m_1} + \frac{gqB}{4m_2}\right)(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

⇒Diagonal component is zero

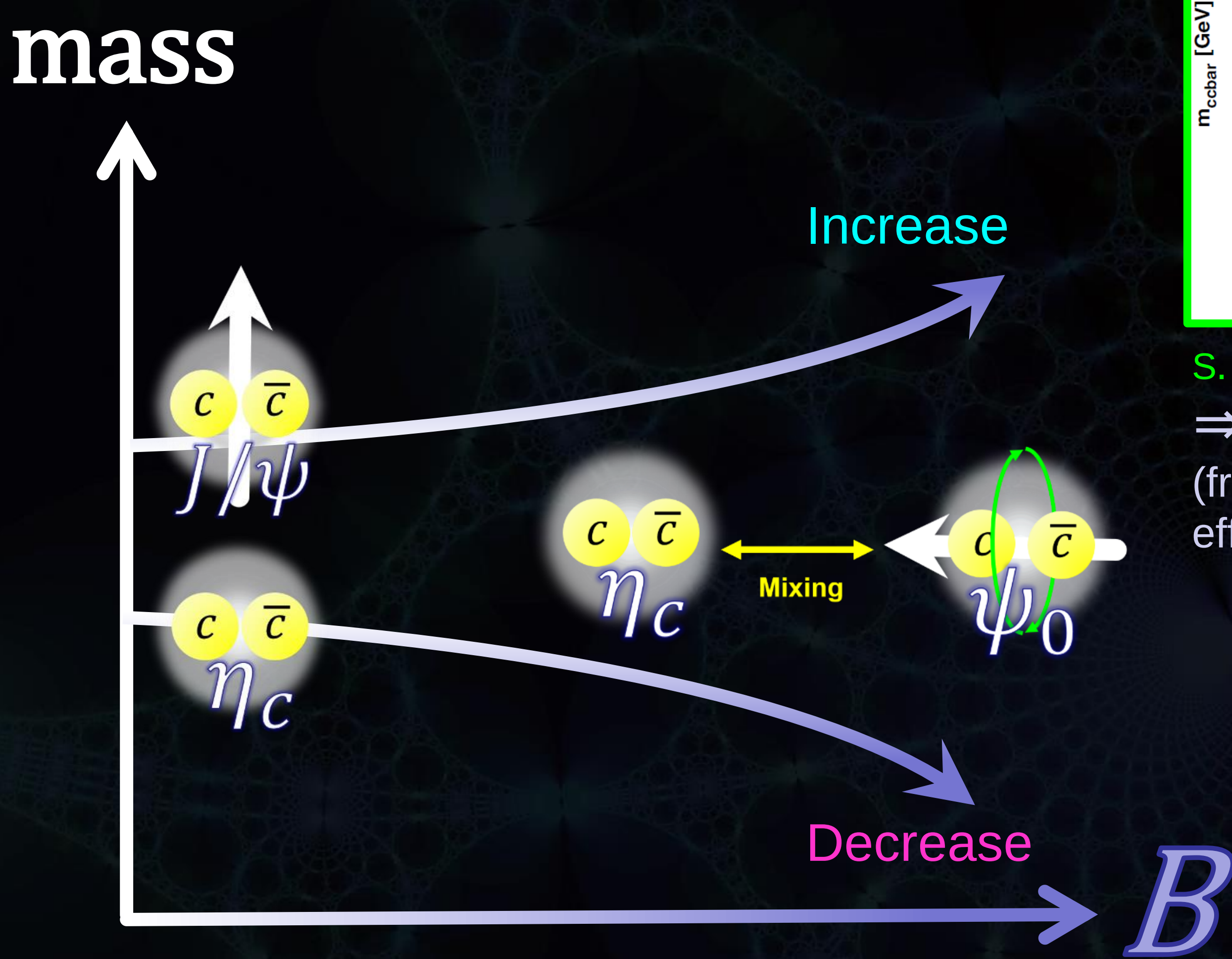
⇒Finite off-diagonal component = Spin mixing

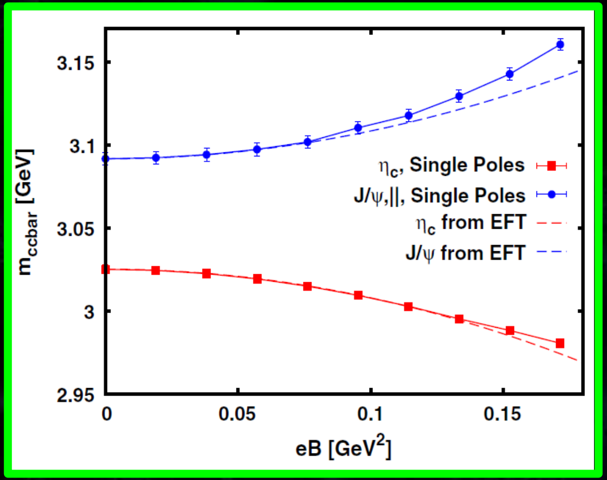


One of observables:

C.S. Machado et al. PRD88 (2013), J. Alford and M. Strickland, PRD88 (2013), S. Cho et al. PRL113 (2014)

Mass shift by spin mixing



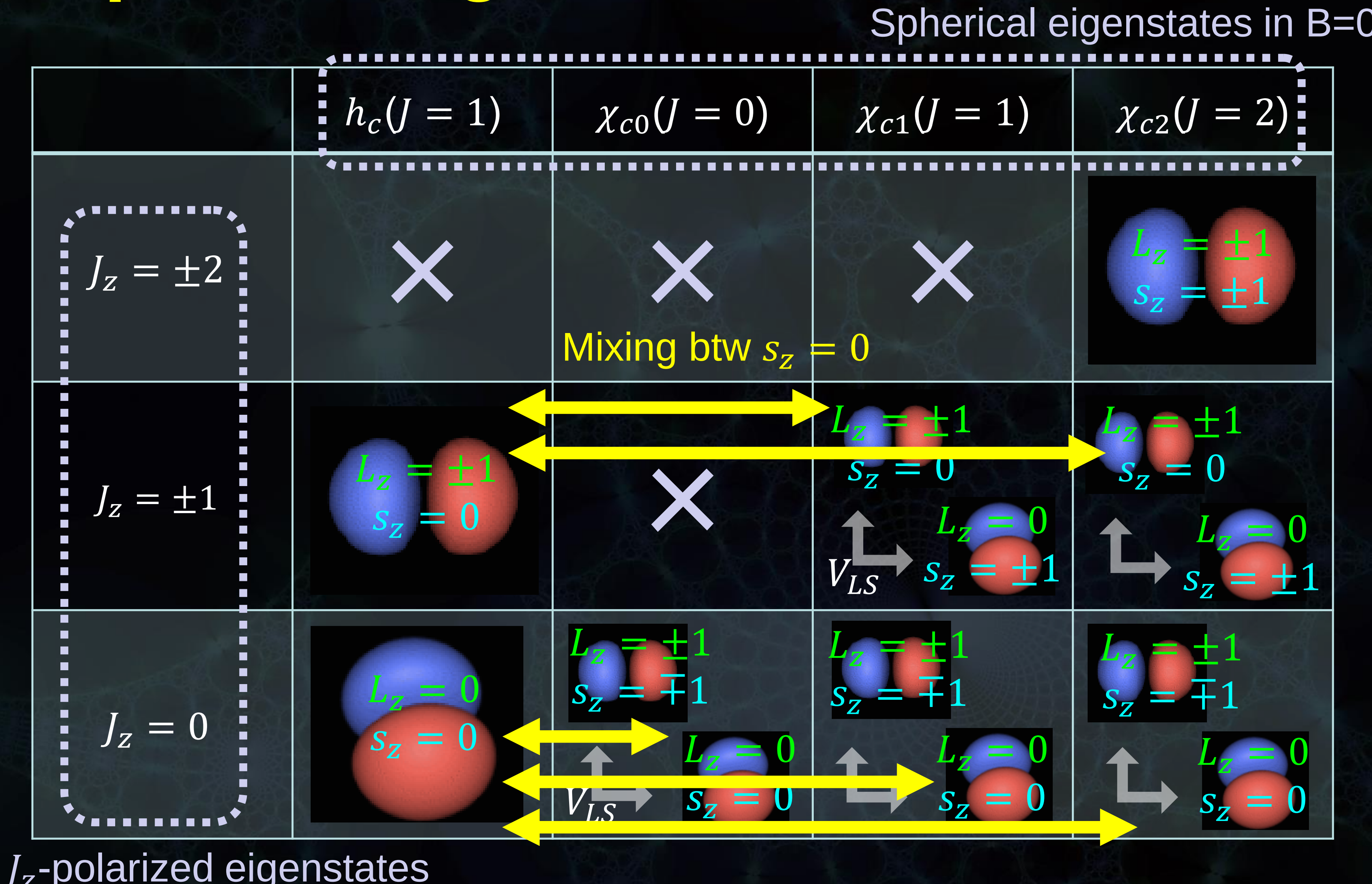


S. Cho et al. PRL113 (2014)

⇒ Results for S-waves

(from QCD sum rules and effective models)

Spin mixing for P-waves



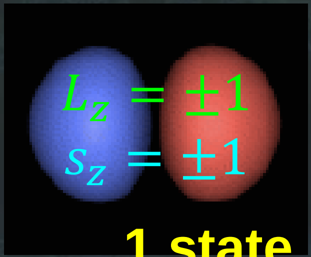
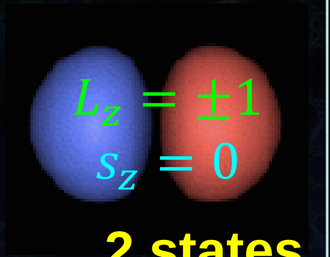
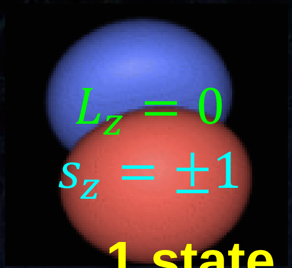
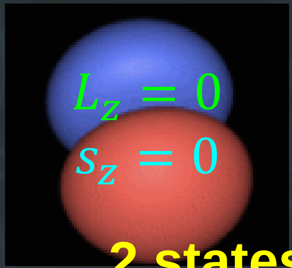
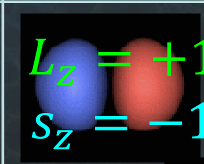
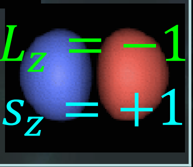
Let's remember Paschen-Back effect

Condition: Larger B-field than LS splitting

$$\sqrt{eB} \gg V_{LS}$$

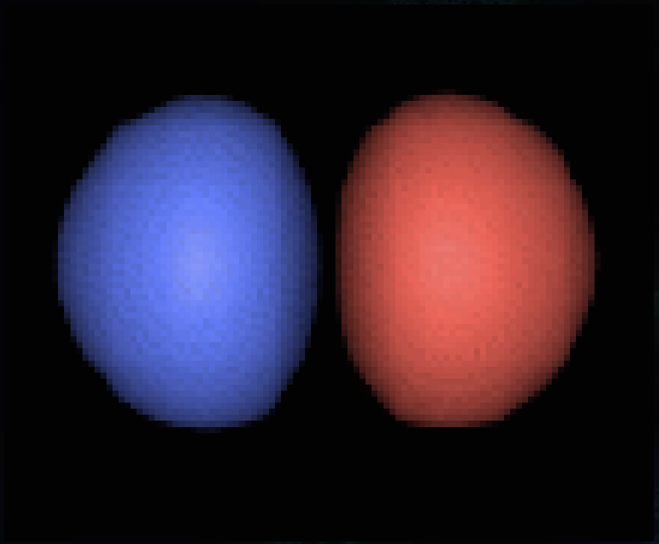
- LS mixing is (approximately) broken
- and, Spin mixing becomes dominant !

P-wave eigenstates **in PB limit**

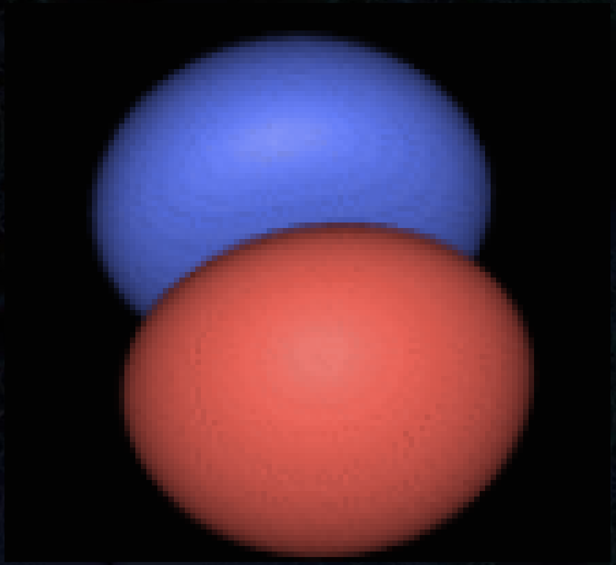
	$L_z = 0$ $s_z = 0$	$L_z = \pm 1$ $s_z = 0$	$L_z = 0$ $s_z = \pm 1$	$L_z = \pm 1$ $s_z = \pm 1$
$J_z = \pm 2$	×	×	×	 **1 state**
$J_z = \pm 1$	×	 **2 states**	 **1 state**	×
$J_z = 0$	 **2 states**	×	×	 **2 states** 

Conclusion:

Wave functions <u>in PB limit</u>



or

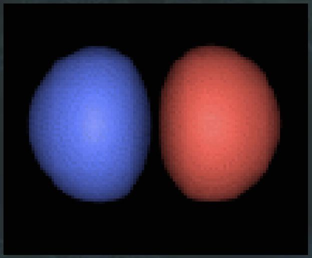
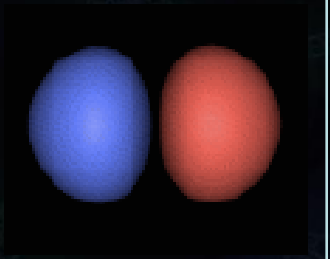
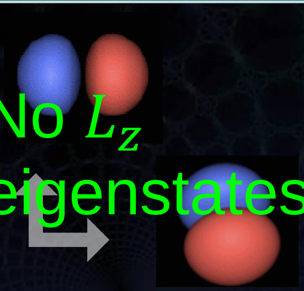
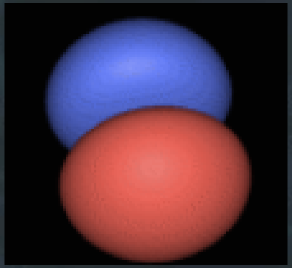
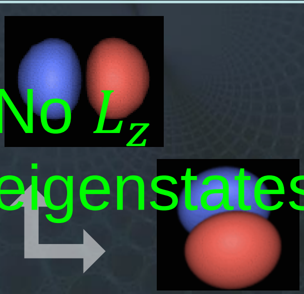
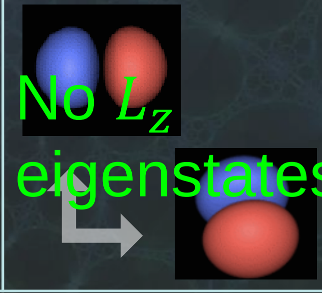


$L_z = \pm 1$
(Transverse)

$L_z = 0$
(Longitudinal)

Notice:

J_z-polarization is NOT the PB limit

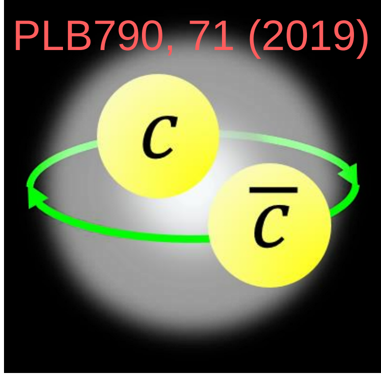
	$h_c(J=1)$	$\chi_{c0}(J=0)$	$\chi_{c1}(J=1)$	$\chi_{c2}(J=2)$
$J_z = \pm 2$	×	×	×	
$J_z = \pm 1$		×	 No L_z eigenstates	 No L_z eigenstates
$J_z = 0$		 No L_z eigenstates	 No L_z eigenstates	 No L_z eigenstates

Let's numerically check it!

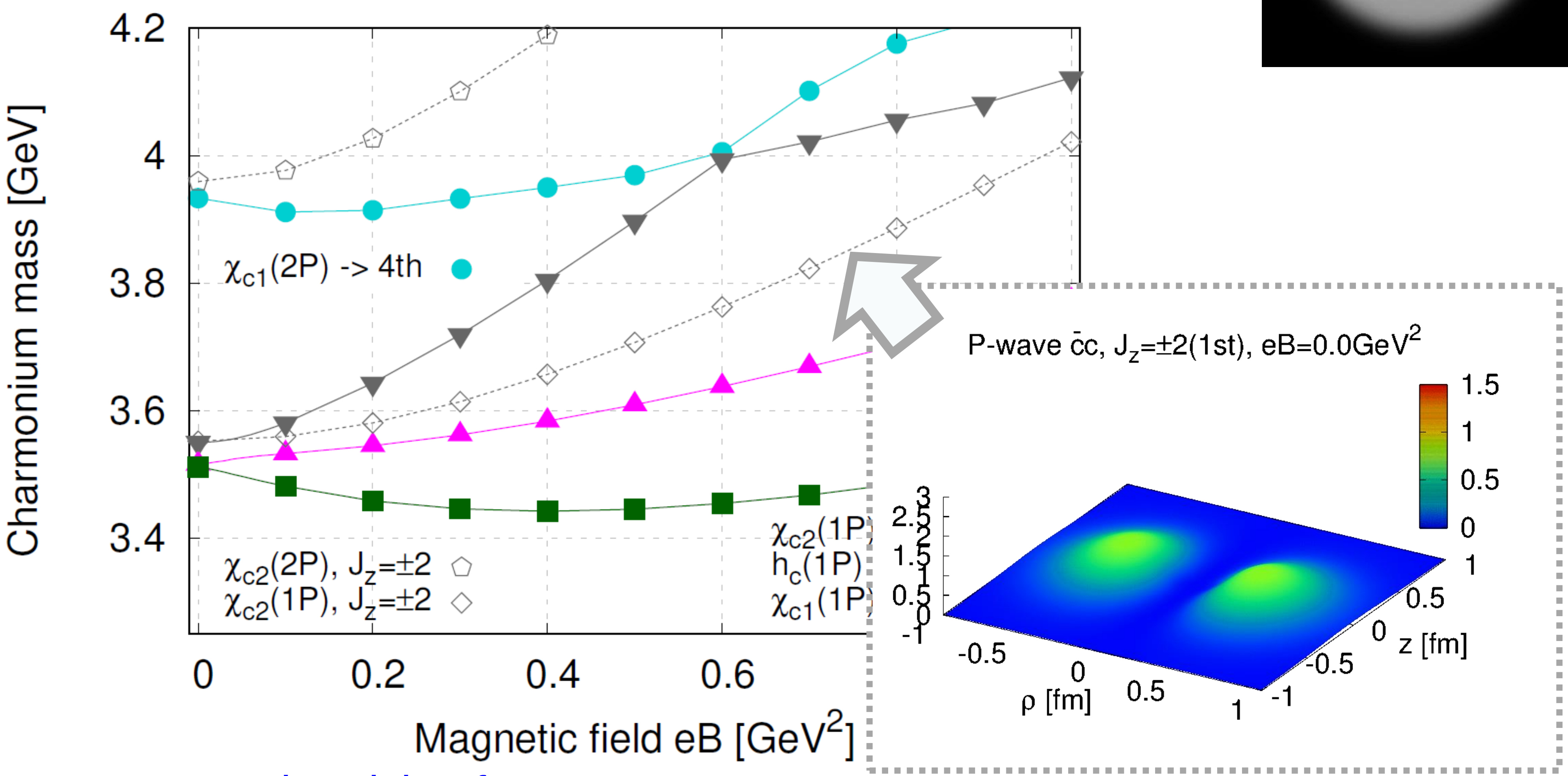
(from quark model)

S. Iwasaki, M. Oka, K. Suzuki, and T. Yoshida, Phys.Lett.B790, 71 (2019)

S. Iwasaki, M. Oka, KS, T. Yoshida, PLB790, 71 (2019)



Mass and WF ($J_z = \pm 2$)



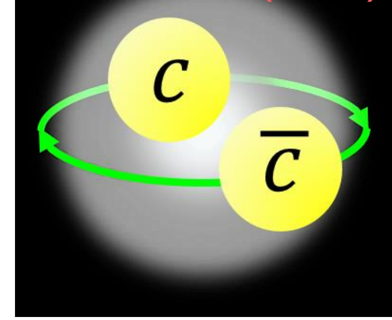
- No spin-mixing for $J_z = \pm 2$

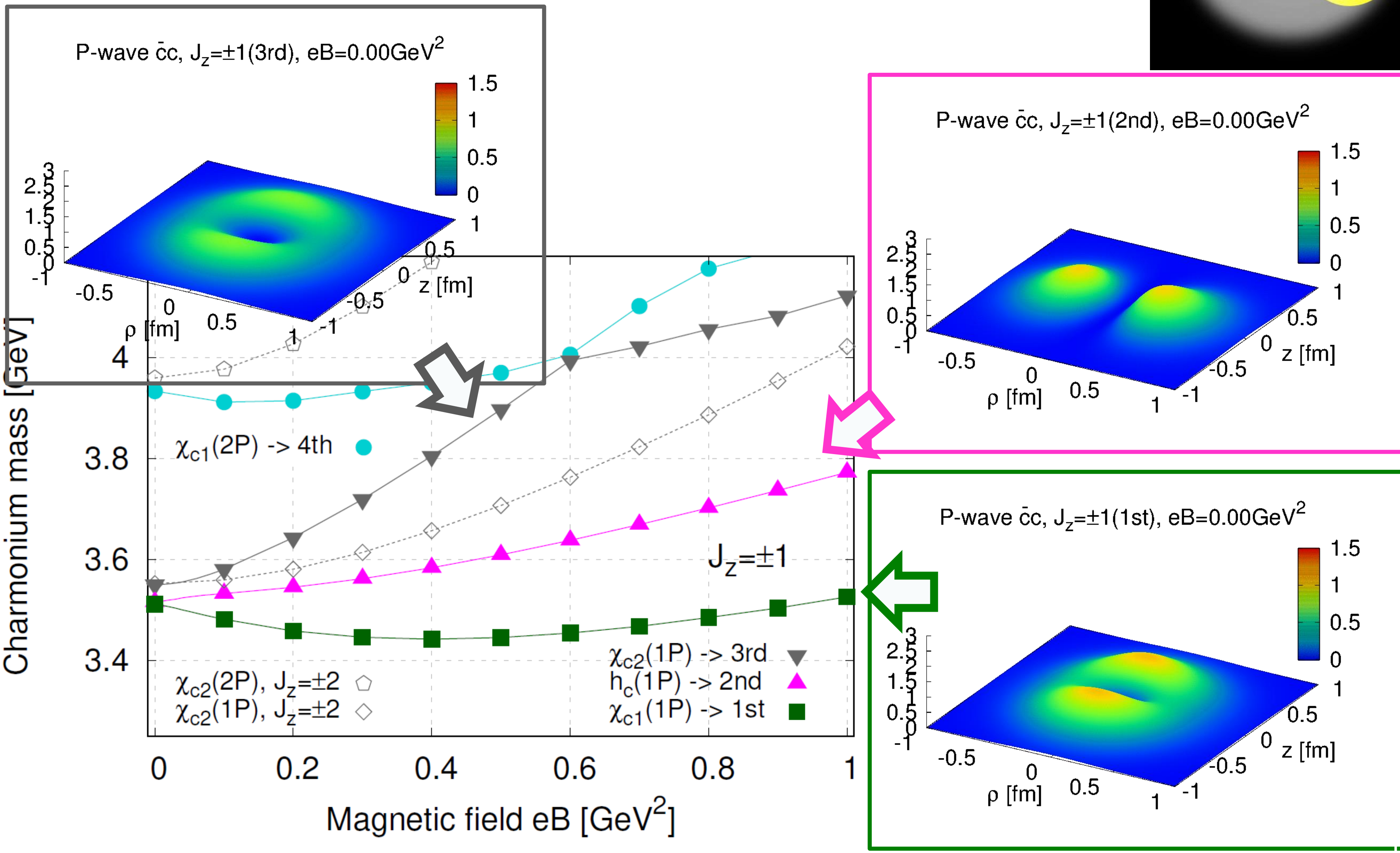
⇒ ρ-squeezing of wave function (=charm quark Landau levels, <u>not PB effect</u>)

⇒ Increase of charmonium mass

S. Iwasaki, M. Oka, KS, T. Yoshida, PLB790, 71 (2019)

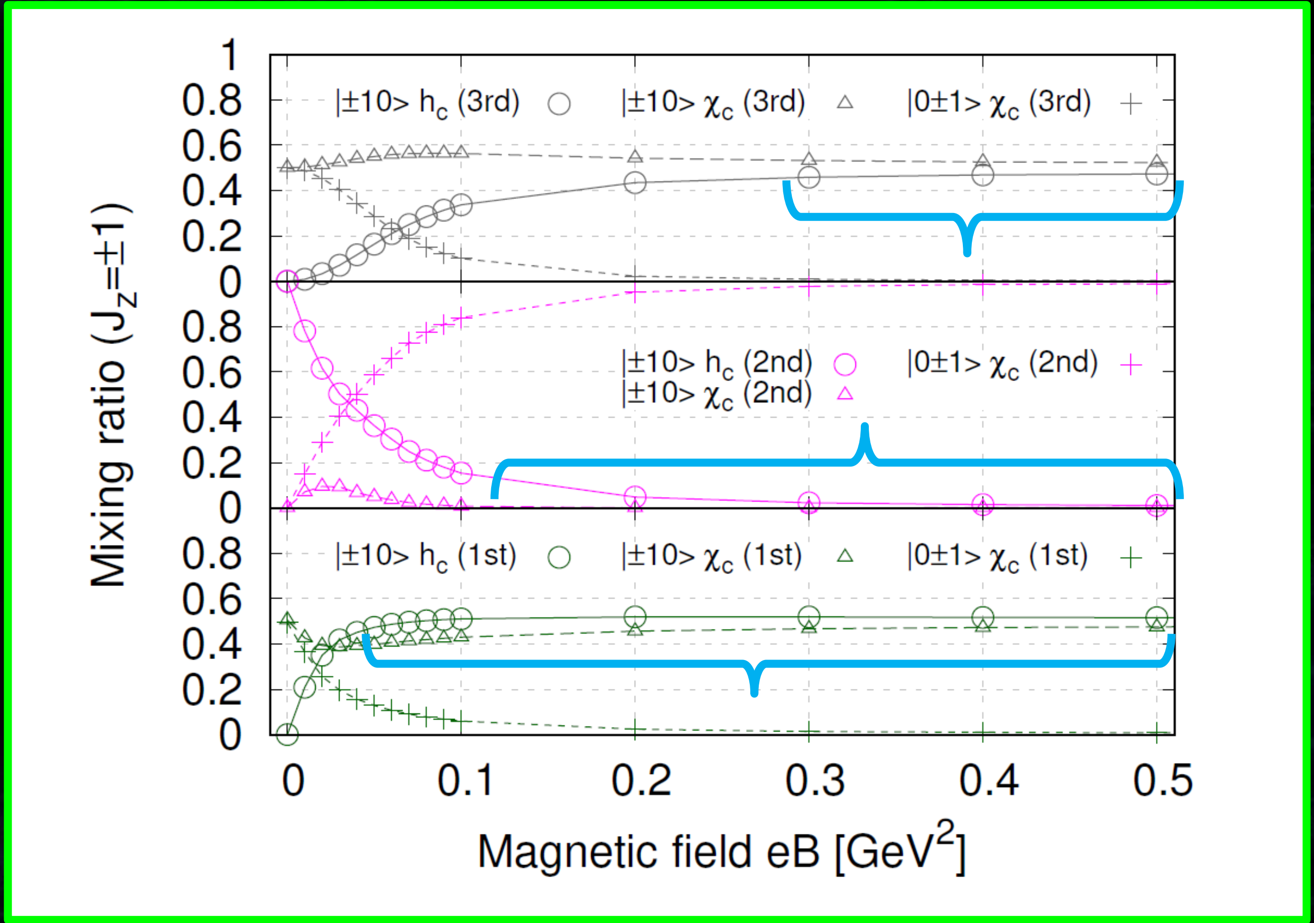
Mass and WF ($J_z = \pm 1$)





S. Iwasaki, M. Oka, KS, T. Yoshida, PLB790, 71 (2019)

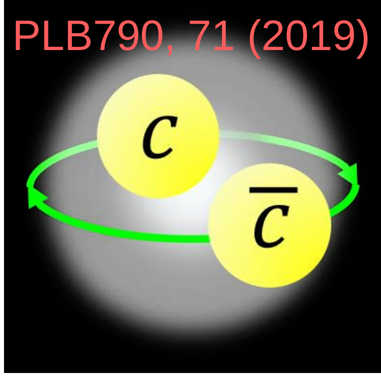
Mixing ratios for $J_z = \pm 1$

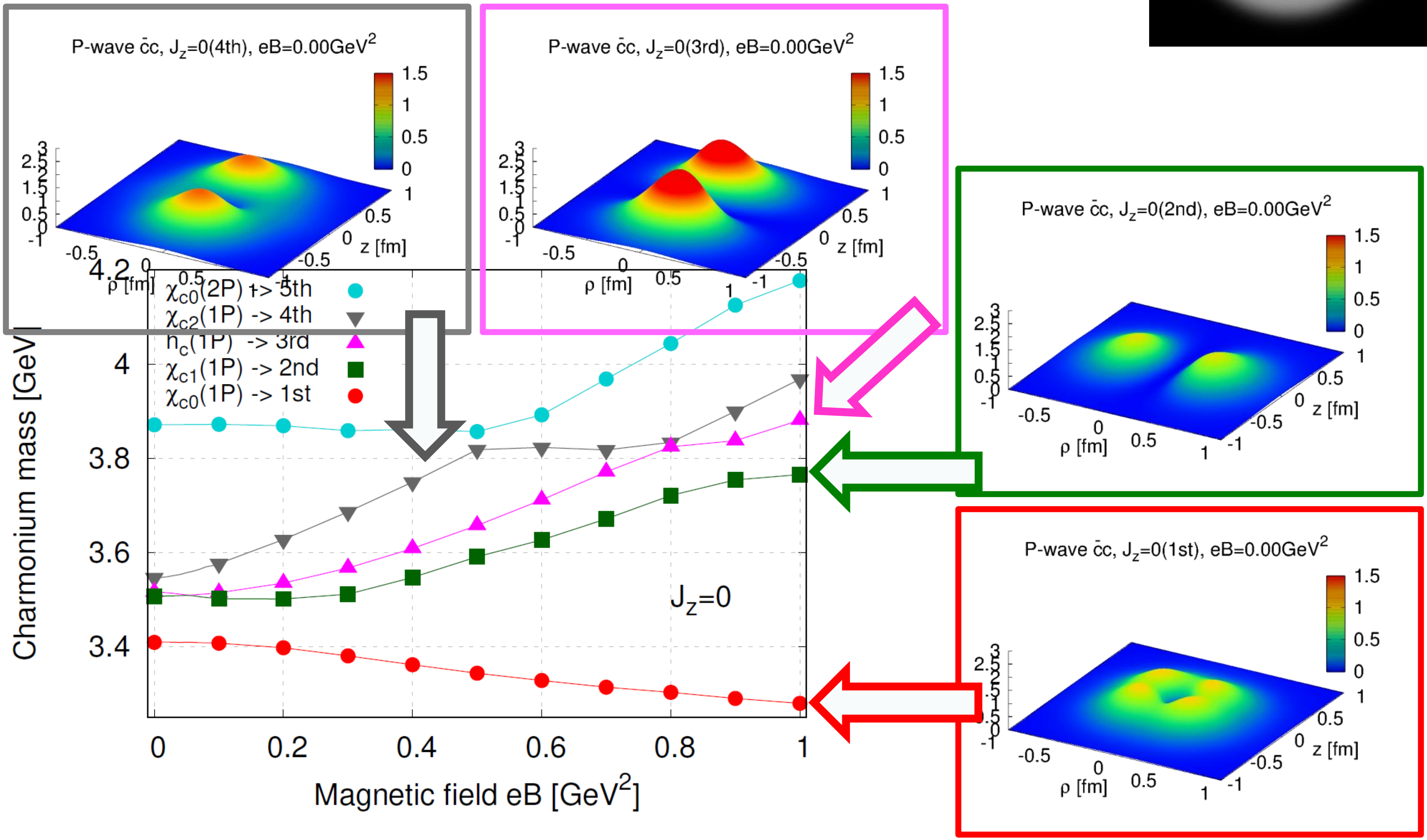


- Saturation of mixing ratios (=Paschen-Back region)
- It can be realized even for $\mathrm{eB} = 0.01 - 0.1\ \mathrm{GeV}^2$ due to $\sqrt{eB} \gg V_{LS}$

S. Iwasaki, M. Oka, KS, T. Yoshida, PLB790, 71 (2019)

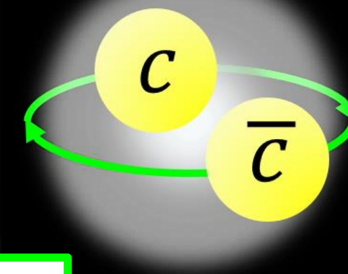
Mass and WF ($J_z = 0$)

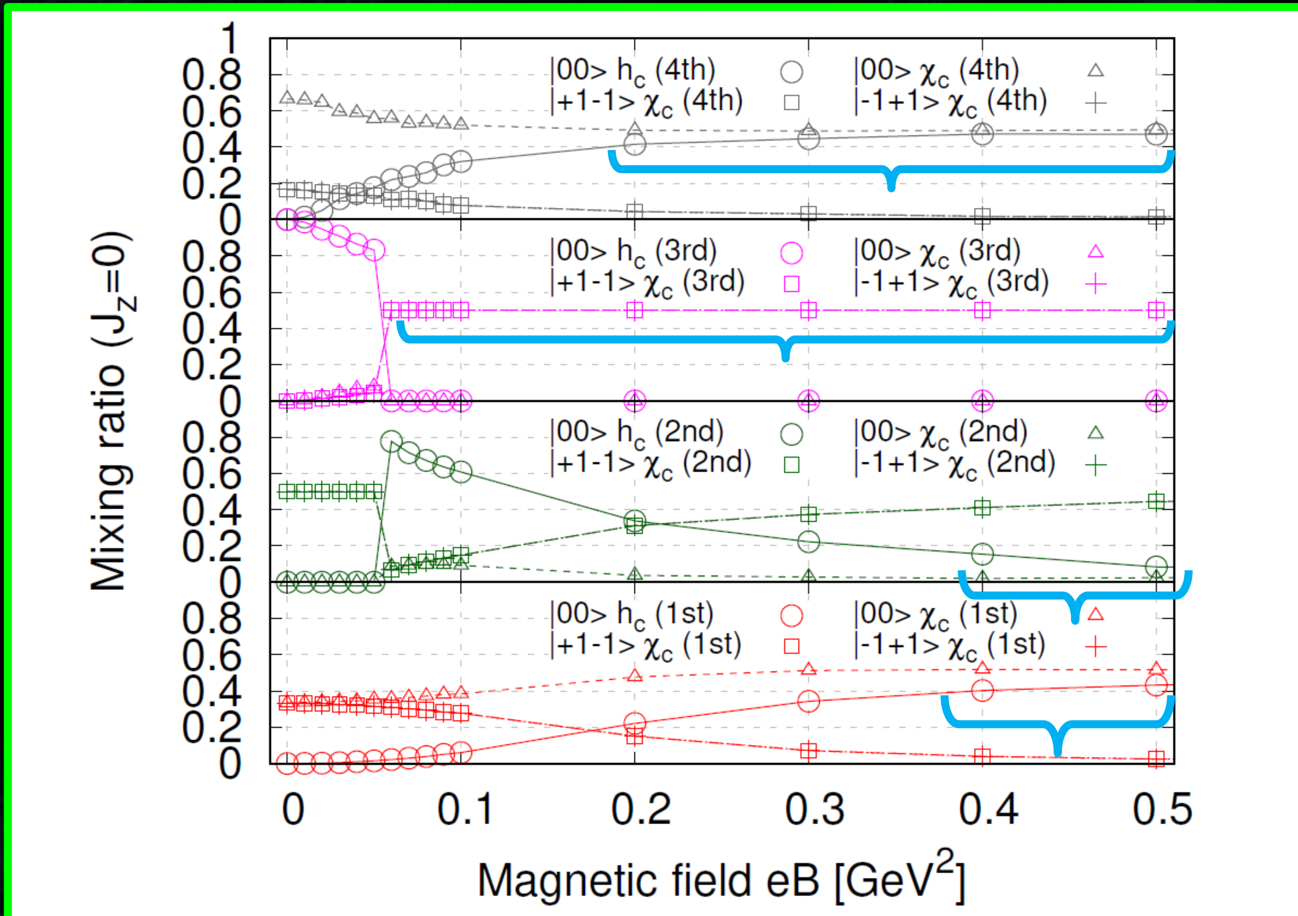




S. Iwasaki, M. Oka, KS, T. Yoshida, PLB790, 71 (2019)

Mixing ratios for $J_z = 0$



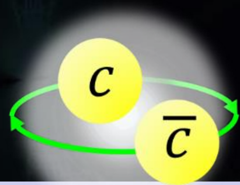


- Saturation of mixing ratios (=Paschen-Back region)
- Note: tensor mixing survives ($L_z = +1 \Leftrightarrow L_z = -1$)

Radiative decays from pNRQCD

[E1] N. Brambilla, Y. Jia, and A. Vairo, PRD73, 054005 (2006)
[M1] N. Brambilla, P. Pietrulewicz, and A. Vairo, PRD85, 094005 (2012)
[E1] A. Pineda and J. Segovia, PRD87, 074024 (2013)
[M1] J. Segovia, S. Steinbeißer, and A. Vairo, PRD99, 074011 (2019)

P-wave WF

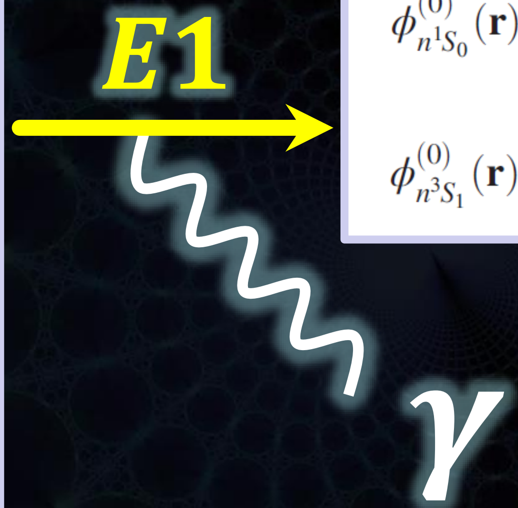


$$\phi^{(0)}_{n^1P_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\,\mathbf{e}_{n^1P_1}(\lambda)\cdot\hat{\mathbf{r}},$$

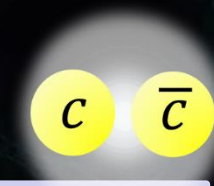
$$\phi^{(0)}_{n^3P_0}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n1}(r)\,\boldsymbol{\sigma}\cdot\hat{\mathbf{r}},$$

$$\phi^{(0)}_{n^3P_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{16\pi}} R_{n1}(r)\,\boldsymbol{\sigma}\cdot(\hat{\mathbf{r}}\times\mathbf{e}_{n^3P_1}(\lambda)),$$

$$\phi^{(0)}_{n^3P_2(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\,\boldsymbol{\sigma}^i h^{ij}_{n^3P_2}\hat{\mathbf{r}}^j,$$



S-wave WF



$$\phi^{(0)}_{n^1S_0}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n0}(r),$$

$$\phi^{(0)}_{n^3S_1}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n0}(r)\,\boldsymbol{\sigma}\cdot\mathbf{e}_{n^3S_1}(\lambda),$$

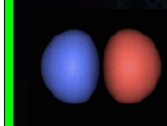
S. Iwasaki and KS, PRD98, 054017 (2018)

Radiative decays in the PB limit

- Wave functions are “polarized” only by $L_z S_z$

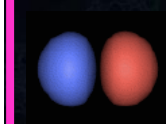
$J_z = \pm 2$

$$\phi^{(0)}_{L_z=\pm1;(S,S_z)=(1,\pm1)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\hat{r}^{\pm}\sigma^{\pm}.$$

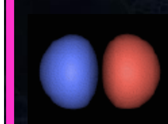


$J_z = \pm 1$

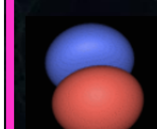
$$\phi^{(0)}_{L_z=\pm1;(S,S_z)=(0,0)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\hat{r}^{\pm},$$



$$\phi^{(0)}_{L_z=\pm1;(S,S_z)=(1,0)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\hat{r}^{\pm}\sigma^{z},$$

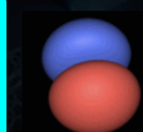


$$\phi^{(0)}_{L_z=0;(S,S_z)=(1,\pm1)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\hat{z}\sigma^{\pm}.$$

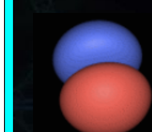


$J_z = 0$

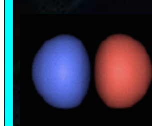
$$\phi^{(0)}_{L_z=0;(S,S_z)=(0,0)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\hat{z},$$



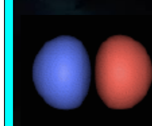
$$\phi^{(0)}_{L_z=0;(S,S_z)=(1,0)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\hat{z}\sigma^{z},$$



$$\phi^{(0)}_{L_z=+1;(S,S_z)=(1,-1)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\hat{r}^{+}\sigma^{-},$$



$$\phi^{(0)}_{L_z=-1;(S,S_z)=(1,+1)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\hat{r}^{-}\sigma^{+},$$

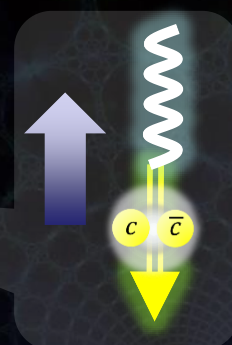


Anisotropic radiative decays in the PB limit

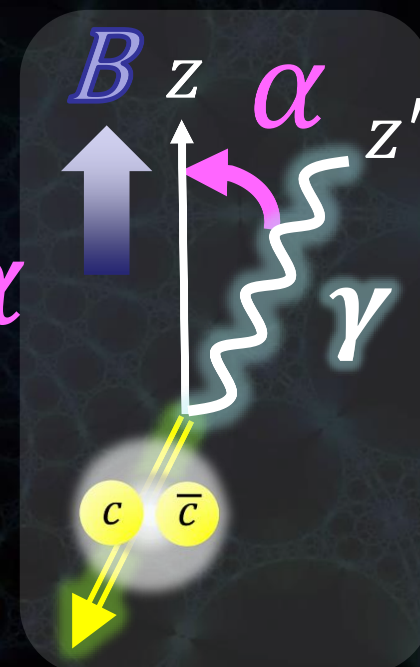
- L_z -Polarized wave functions leads to…

$$\left\langle L_z = +1 \middle| \vec{r}\cdot\vec{\epsilon}^{\pm} \middle| c\,\bar{c}\ \text{S-wave} \right\rangle \propto \cos\alpha \pm 1$$

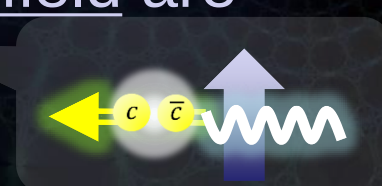
⇒Decays parallel to B-field are enhanced



$$\left\langle L_z = 0 \middle| \vec{r}\cdot\vec{\epsilon}^{\pm} \middle| c\,\bar{c}\ \text{S-wave} \right\rangle \propto \sin\alpha$$



⇒Decays perpendicular to B-field are enhanced



ΔJ_z	E1 Decay process	Decay factor	$\alpha = 0, \pi$	$\alpha = \pi/2$
$0 \to 0$	$\frac{1}{\sqrt{2}}[Y_{10}\chi_{00} \pm Y_{10}\chi_{10}] \to \frac{1}{\sqrt{2}}[Y_{00}\chi_{00} \pm Y_{00}\chi_{10}]$	$\frac{3}{4}\sin^2\alpha$	0	$\frac{3}{4}$
$0 \to -1$	$\frac{1}{\sqrt{2}}[Y_{11}\chi_{1-1} \pm Y_{1-1}\chi_{11}] \to Y_{00}\chi_{1-1}$	$\frac{3}{32}(\cos^2\alpha + 1)$	$\frac{3}{16}$	$\frac{3}{32}$
$0 \to +1$	$\frac{1}{\sqrt{2}}[Y_{11}\chi_{1-1} \pm Y_{1-1}\chi_{11}] \to Y_{00}\chi_{11}$	$\frac{3}{32}(\cos^2\alpha + 1)$	$\frac{3}{16}$	$\frac{3}{32}$
$+1 \to 0$	$\frac{1}{\sqrt{2}}[Y_{11}\chi_{00} \pm Y_{11}\chi_{10}] \to \frac{1}{\sqrt{2}}[Y_{00}\chi_{00} \pm Y_{00}\chi_{10}]$	$\frac{3}{32}(\cos^2\alpha + 1)$	$\frac{3}{16}$	$\frac{3}{32}$
$\pm 1 \to \pm 1$	$Y_{10}\chi_{1\pm 1} \to Y_{00}\chi_{1\pm 1}$	$\frac{3}{2}\sin^2\alpha$	0	$\frac{3}{2}$
$-1 \to 0$	$\frac{1}{\sqrt{2}}[Y_{1-1}\chi_{00} \pm Y_{1-1}\chi_{10}] \to \frac{1}{\sqrt{2}}[Y_{00}\chi_{00} \pm Y_{00}\chi_{10}]$	$\frac{3}{32}(\cos^2\alpha + 1)$	$\frac{3}{16}$	$\frac{3}{32}$
$+2 \to +1$	$Y_{11}\chi_{11} \to Y_{00}\chi_{11}$	$\frac{3}{16}(\cos^2\alpha + 1)$	$\frac{3}{8}$	$\frac{3}{16}$
$-2 \to -1$	$Y_{1-1}\chi_{1-1} \to Y_{00}\chi_{1-1}$	$\frac{3}{16}(\cos^2\alpha + 1)$	$\frac{3}{8}$	$\frac{3}{16}$

ΔJ_z	M1 Decay process	Decay factor	$\alpha = 0, \pi$	$\alpha = \pi/2$
$0 \to +1$	$\frac{1}{\sqrt{2}}[Y_{10}\chi_{00} \pm Y_{10}\chi_{10}] \to Y_{10}\chi_{11}$	$\frac{1}{2}(\cos^2\alpha + 1)$	1	$\frac{1}{2}$
$0 \to 0$	$\frac{1}{\sqrt{2}}[Y_{10}\chi_{00} + Y_{10}\chi_{10}] \to \frac{1}{\sqrt{2}}[Y_{10}\chi_{00} - Y_{10}\chi_{10}]$	$\sin^2\alpha$	0	1
$0 \to 0$	$\frac{1}{\sqrt{2}}[Y_{10}\chi_{00} - Y_{10}\chi_{10}] \to \frac{1}{\sqrt{2}}[Y_{10}\chi_{00} + Y_{10}\chi_{10}]$	$\sin^2\alpha$	0	1
$0 \to -1$	$\frac{1}{\sqrt{2}}[Y_{10}\chi_{00} \pm Y_{10}\chi_{10}] \to Y_{10}\chi_{1-1}$	$\frac{1}{2}(\cos^2\alpha + 1)$	1	$\frac{1}{2}$
$0 \to +1$	$\frac{1}{\sqrt{2}}[Y_{11}\chi_{1-1} \pm Y_{1-1}\chi_{11}] \to \frac{1}{\sqrt{2}}[Y_{11}\chi_{00} \pm Y_{11}\chi_{10}]$	$\frac{1}{4}(\cos^2\alpha + 1)$	$\frac{1}{2}$	$\frac{1}{4}$
$0 \to -1$	$\frac{1}{\sqrt{2}}[Y_{11}\chi_{1-1} \pm Y_{1-1}\chi_{11}] \to \frac{1}{\sqrt{2}}[Y_{1-1}\chi_{00} \pm Y_{1-1}\chi_{10}]$	$\frac{1}{4}(\cos^2\alpha + 1)$	$\frac{1}{2}$	$\frac{1}{4}$
$+1 \to +2$	$\frac{1}{\sqrt{2}}[Y_{11}\chi_{00} \pm Y_{11}\chi_{10}] \to Y_{11}\chi_{11}$	$\frac{1}{2}(\cos^2\alpha + 1)$	1	$\frac{1}{2}$
$+1 \to 0$	$\frac{1}{\sqrt{2}}[Y_{11}\chi_{00} \pm Y_{11}\chi_{10}] \to \frac{1}{\sqrt{2}}[Y_{11}\chi_{1-1} \pm Y_{1-1}\chi_{11}]$	$\frac{1}{4}(\cos^2\alpha + 1)$	$\frac{1}{2}$	$\frac{1}{4}$
$\pm 1 \to \pm 1$	$\frac{1}{\sqrt{2}}[Y_{1\pm 1}\chi_{00} + Y_{1\pm 1}\chi_{10}] \to \frac{1}{\sqrt{2}}[Y_{1\pm 1}\chi_{00} - Y_{1\pm 1}\chi_{10}]$	$\sin^2\alpha$	0	1
$\pm 1 \to \pm 1$	$\frac{1}{\sqrt{2}}[Y_{1\pm 1}\chi_{00} - Y_{1\pm 1}\chi_{10}] \to \frac{1}{\sqrt{2}}[Y_{1\pm 1}\chi_{00} + Y_{1\pm 1}\chi_{10}]$	$\sin^2\alpha$	0	1
$-1 \to 0$	$\frac{1}{\sqrt{2}}[Y_{1-1}\chi_{00} \pm Y_{1-1}\chi_{10}] \to \frac{1}{\sqrt{2}}[Y_{11}\chi_{1-1} \pm Y_{1-1}\chi_{11}]$	$\frac{1}{4}(\cos^2\alpha + 1)$	$\frac{1}{2}$	$\frac{1}{4}$
$-1 \to -2$	$\frac{1}{\sqrt{2}}[Y_{1-1}\chi_{00} \pm Y_{1-1}\chi_{10}] \to Y_{1-1}\chi_{1-1}$	$\frac{1}{2}(\cos^2\alpha + 1)$	1	$\frac{1}{2}$
$+1 \to 0$	$Y_{10}\chi_{11} \to \frac{1}{\sqrt{2}}[Y_{10}\chi_{00} \pm Y_{10}\chi_{10}]$	$\frac{1}{2}(\cos^2\alpha + 1)$	1	$\frac{1}{2}$
$-1 \to 0$	$Y_{10}\chi_{1-1} \to \frac{1}{\sqrt{2}}[Y_{10}\chi_{00} \pm Y_{10}\chi_{10}]$	$\frac{1}{2}(\cos^2\alpha + 1)$	1	$\frac{1}{2}$
$+2 \to +1$	$Y_{11}\chi_{11} \to \frac{1}{\sqrt{2}}[Y_{11}\chi_{00} \pm Y_{11}\chi_{10}]$	$\frac{1}{2}(\cos^2\alpha + 1)$	1	$\frac{1}{2}$
$-2 \to -1$	$Y_{1-1}\chi_{1-1} \to \frac{1}{\sqrt{2}}[Y_{1-1}\chi_{00} \pm Y_{1-1}\chi_{10}]$	$\frac{1}{2}(\cos^2\alpha + 1)$	1	$\frac{1}{2}$

Summary and Outlook

Today's message:

<u>Paschen-Back limit</u> is a new frontier of quarkonium physics

- This limit is realized in <u>relatively weak</u> magnetic fields (even for $\mathrm{eB} = 0.01 - 0.1\ \mathrm{GeV}^2$)
- <u>Deformed ("polarized") wave functions</u>
- <u>Anisotropic decays</u>

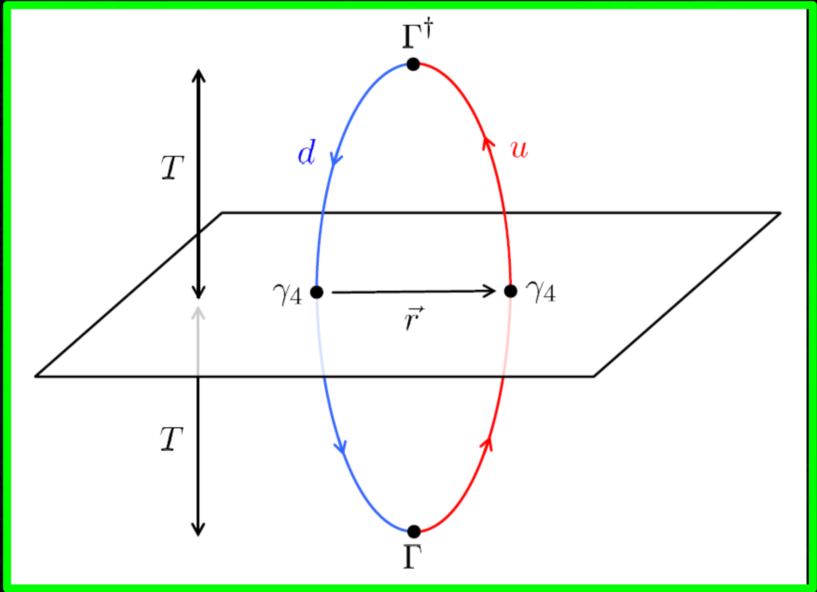
- Color-octet states in pNRQCD
- Other hadrons with $L \neq 0$ (D mesons, baryons, light hadrons…)
- Other anisotropic processes (strong decays, production…)
- Beyond the PB limit (EFT or QFT with B-field)
- Application to Heavy-ion collisions, Compact stars, Lattice QCD…

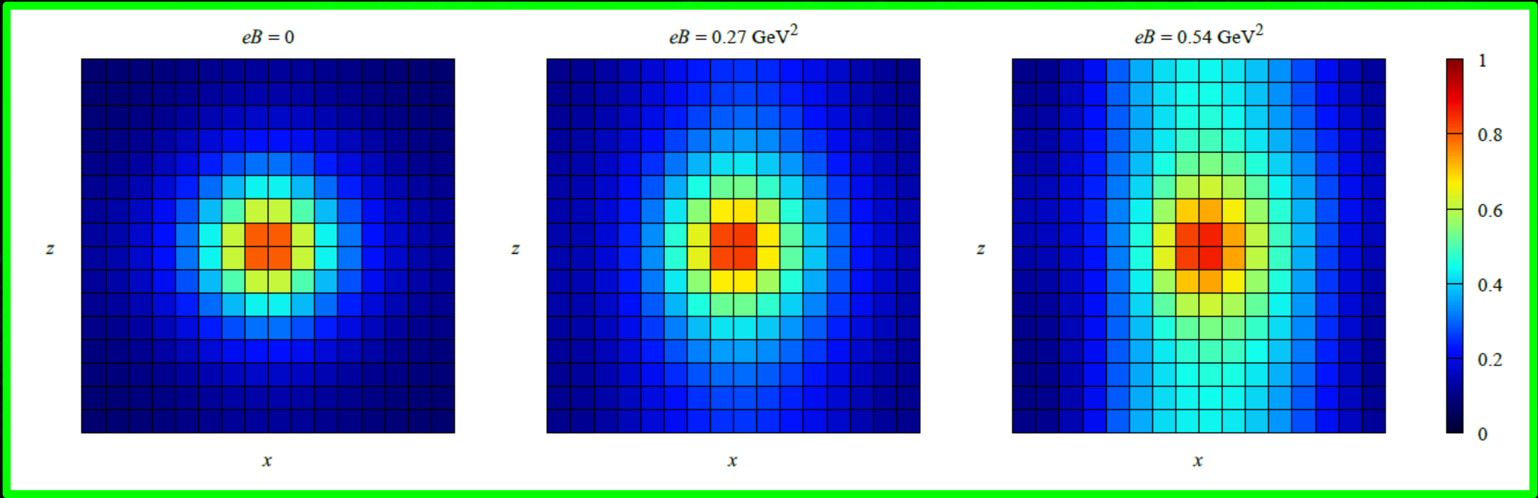
Backup

States	J_z	Bases $(Y_{LL_z}\chi_{SS_z})$
h_c $(^1P_1)$	0	$Y_{10}\chi_{00}$
	±1	$Y_{1\pm1}\chi_{00}$
χ_{c0} $(^3P_0)$	0	$\frac{1}{\sqrt{3}}[Y_{11}\chi_{1-1} - Y_{10}\chi_{10} + Y_{1-1}\chi_{11}]$
χ_{c1} $(^3P_1)$	0	$\frac{1}{\sqrt{2}}[Y_{1-1}\chi_{11} - Y_{11}\chi_{1-1}]$
	±1	$\pm\frac{1}{\sqrt{2}}[Y_{10}\chi_{1\pm1} - Y_{1\pm1}\chi_{10}]$
χ_{c2} $(^3P_2)$	0	$\frac{1}{\sqrt{6}}[Y_{11}\chi_{1-1} + 2Y_{10}\chi_{10} + Y_{1-1}\chi_{11}]$
	±1	$\frac{1}{\sqrt{2}}[Y_{1\pm1}\chi_{10} + Y_{10}\chi_{1\pm1}]$
	±2	$Y_{1\pm1}\chi_{1\pm1}$

J_z	Bases $(Y_{LL_z}\chi_{SS_z})$
0	$\frac{1}{\sqrt{2}}[Y_{10}\chi_{00} + Y_{10}\chi_{10}]$
0	$\frac{1}{\sqrt{2}}[Y_{11}\chi_{1-1} + Y_{1-1}\chi_{11}]$
0	$\frac{1}{\sqrt{2}}[Y_{11}\chi_{1-1} - Y_{1-1}\chi_{11}]$
0	$\frac{1}{\sqrt{2}}[Y_{10}\chi_{00} - Y_{10}\chi_{10}]$
±1	$\frac{1}{\sqrt{2}}[Y_{1\pm1}\chi_{00} + Y_{1\pm1}\chi_{10}]$
±1	$Y_{10}\chi_{1\pm1}$
±1	$\frac{1}{\sqrt{2}}[Y_{1\pm1}\chi_{00} - Y_{1\pm1}\chi_{10}]$
±2	$Y_{1\pm1}\chi_{1\pm1}$

WF deformation from Lattice QCD



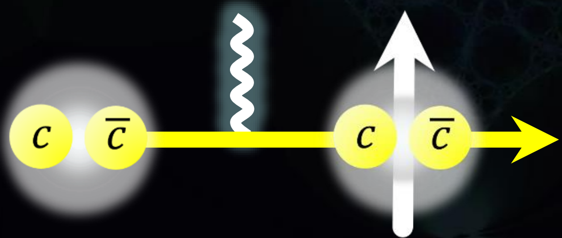


S-wave charmonia

Approaches for quarkonia in B-field

(1) Hadron EFT

$$\mathcal{L}_{EFT} = g\tilde{F}_{\mu\nu}\,(\partial^{\mu}P)\,V^{\nu}$$



$\Rightarrow \eta_c$-J/ψ-γ vertex

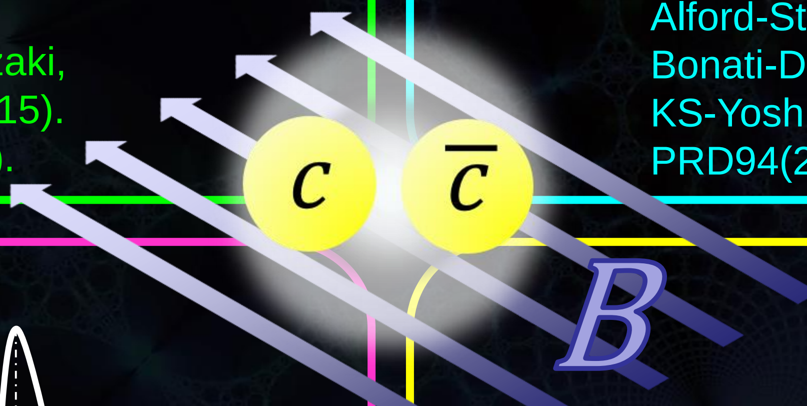
Cho-Hattori-Lee-Morita-Ozaki, PRL113(2014); PRD91(2015).
Yoshida-KS, PRD94(2016).

(2) Potential models

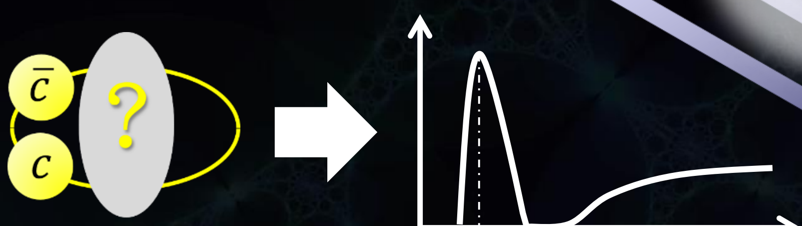
$$H = \sum_{i=1,2} \frac{1}{2m_i}(\boldsymbol{p}_i - q_i\boldsymbol{A})^2 - \boldsymbol{\mu}_i \cdot \boldsymbol{B} + m_i + V(r)$$

$\Rightarrow$Constituent quark LL and magnetic moment effect

Alford-Strickland, PRD88(2013).
Bonati-D'Elia-Rucci, PRD92(2015).
KS-Yoshida, PRD93(2016); PRD94(2016).



(3) QCD sum rules



OPE in B-field $\Rightarrow$ Spectral function

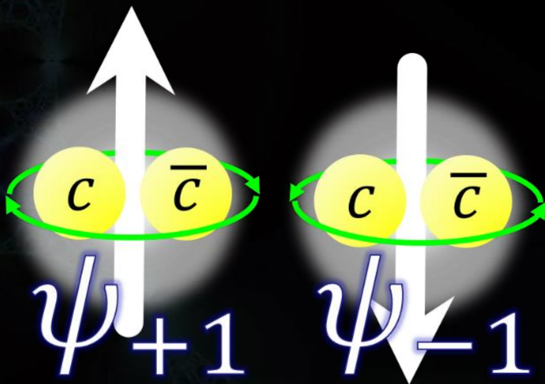
Cho-Hattori-Lee-Morita-Ozaki, PRL113(2014); PRD91(2015).

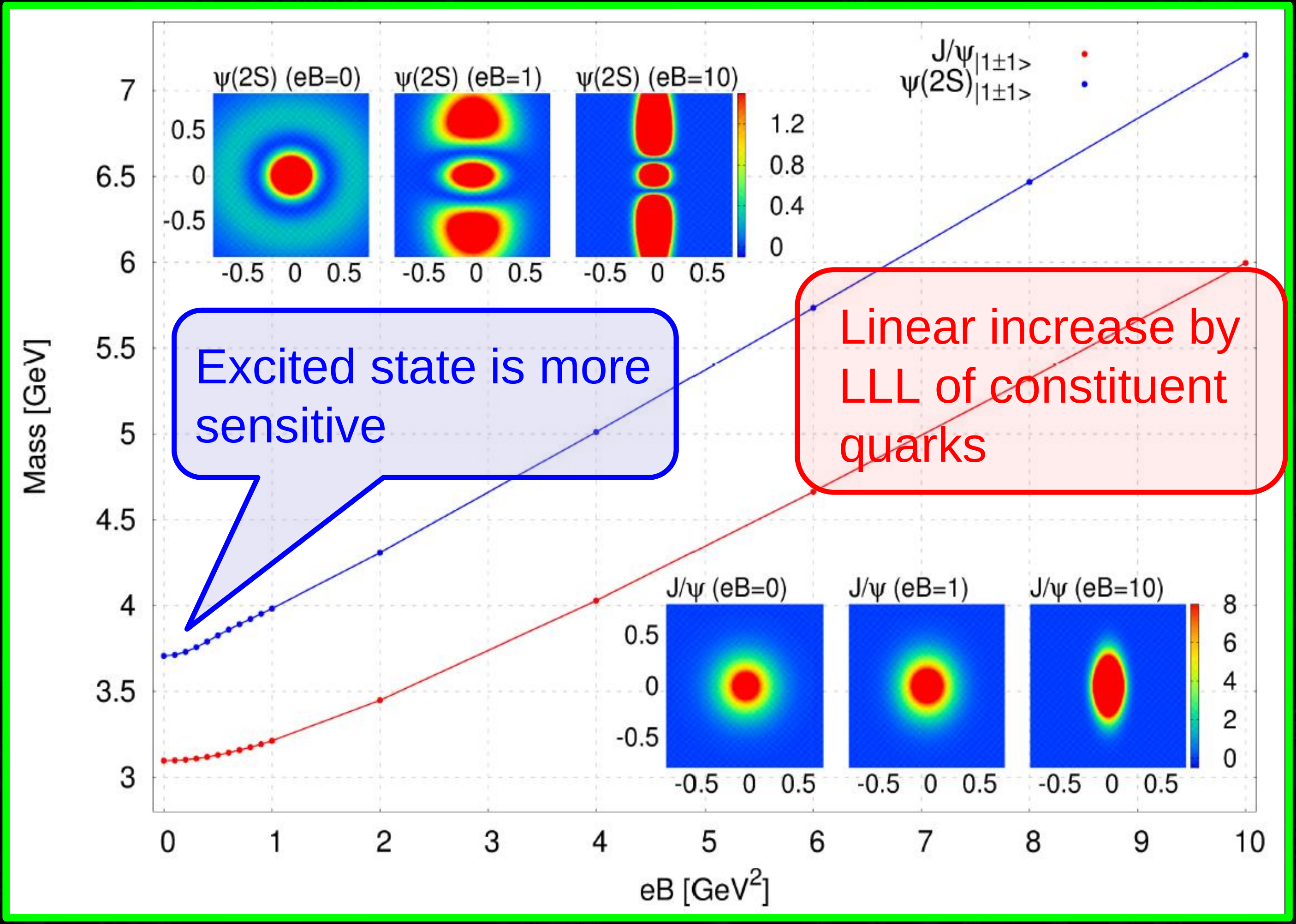
(4) Lattice QCD

There are no results!
(for heavy hadrons)

KS and T. Yoshida, PRD93, 051502 (2016)

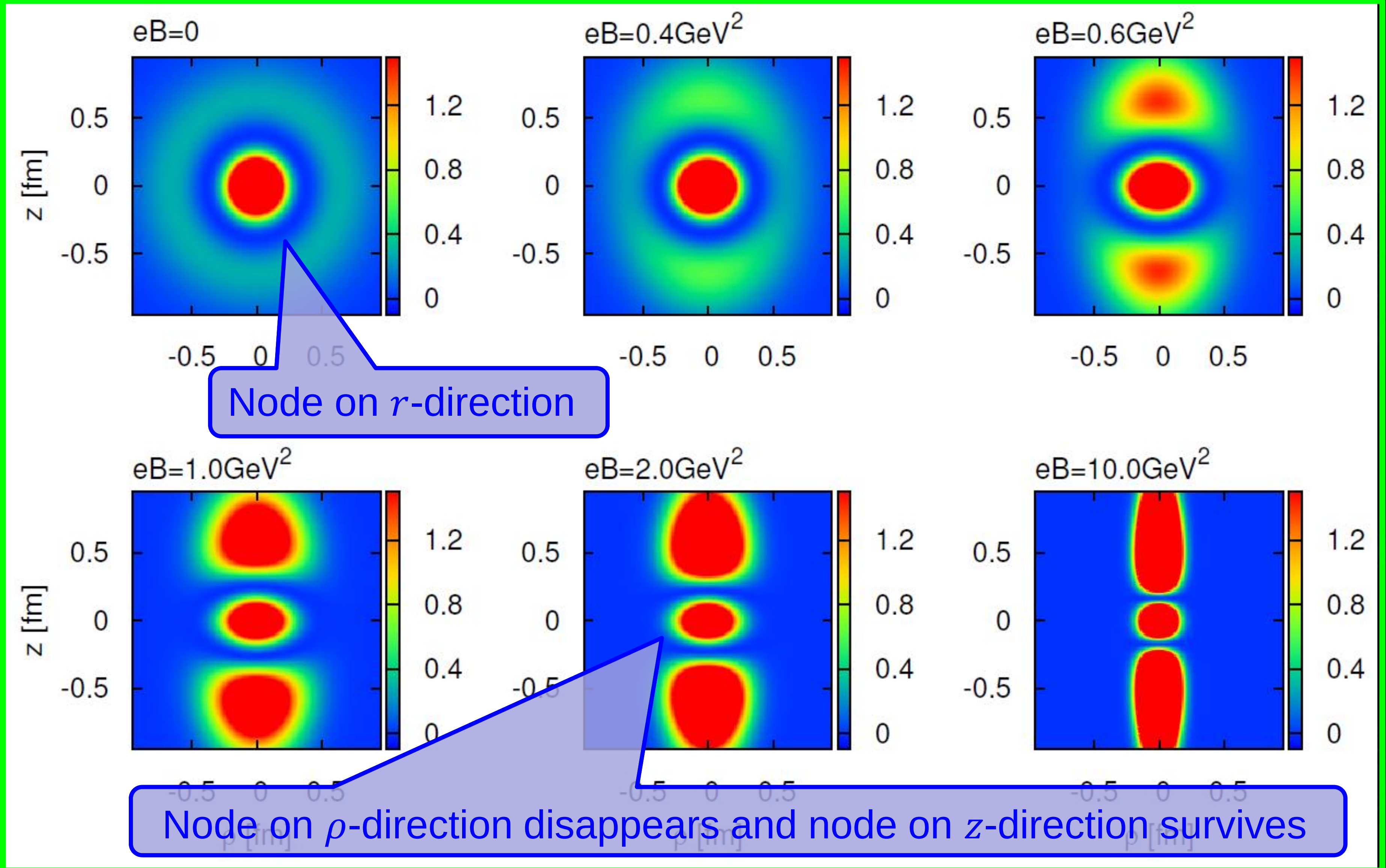
$\mathrm{J}/\psi\ (J_z = \pm 1)$ (no mixing)





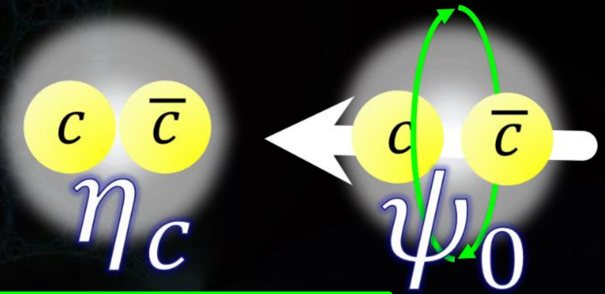
KS and T. Yoshida, PRD93, 051502 (2016)

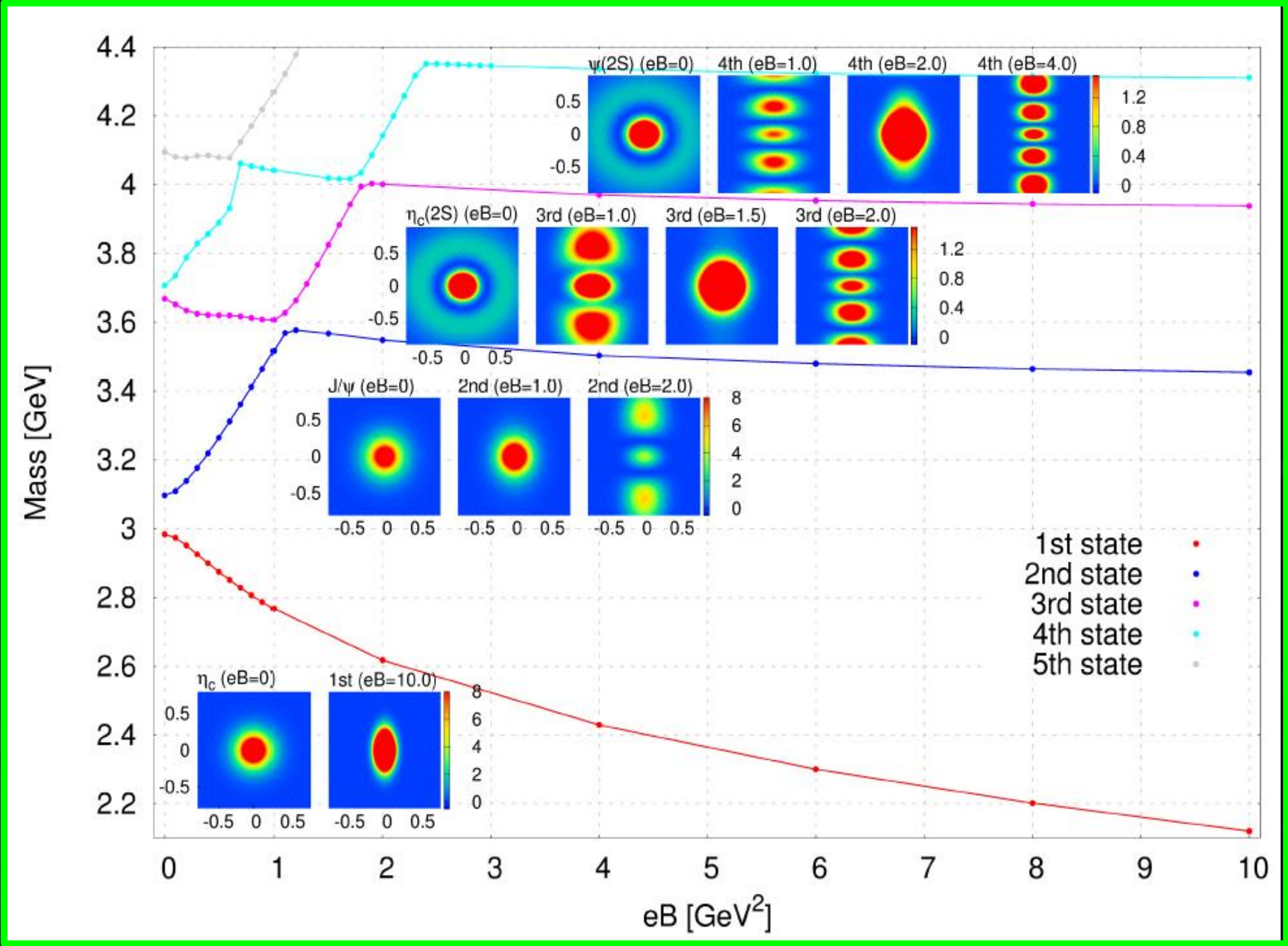
Excited state (2S) deformation



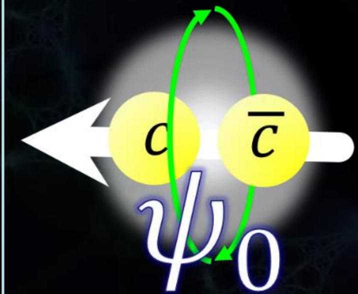
KS and T. Yoshida, PRD93, 051502 (2016)

η_c and J/ψ ($J_z = 0$)





Summary of mass shift

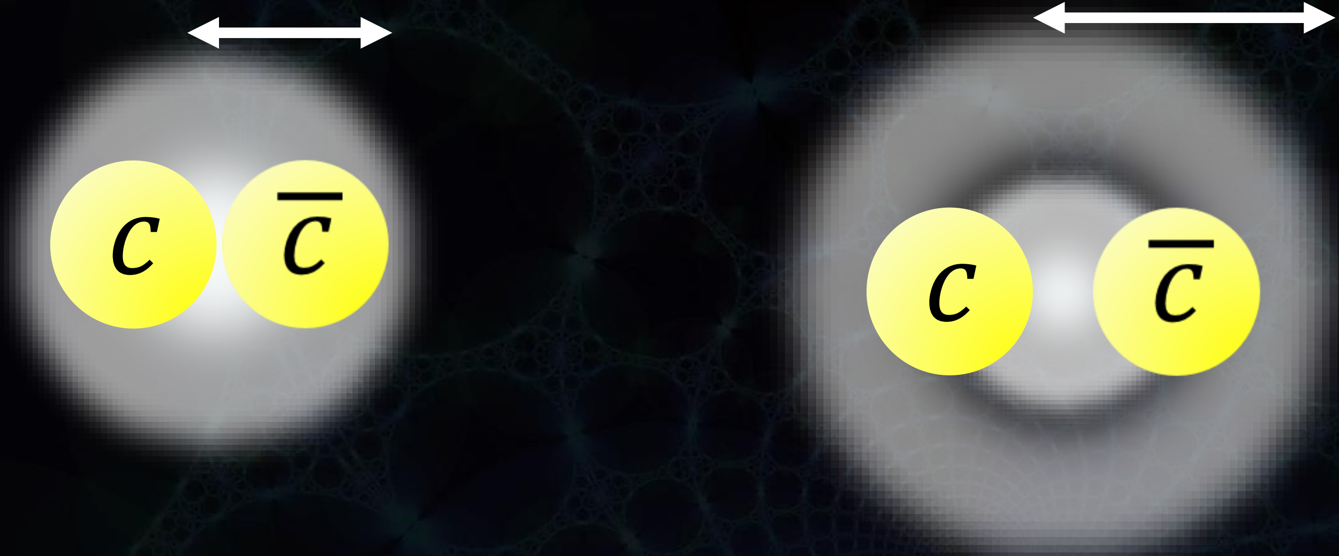
				
Spin mixing	 Decrease ↘	Increase ↗	No	No
Quark Landau levels	Increase ↗	Increase ↗	Increase ↗	Increase ↗

Excited states (ψ'-η_c') are more sensitive than ground states

Excited states

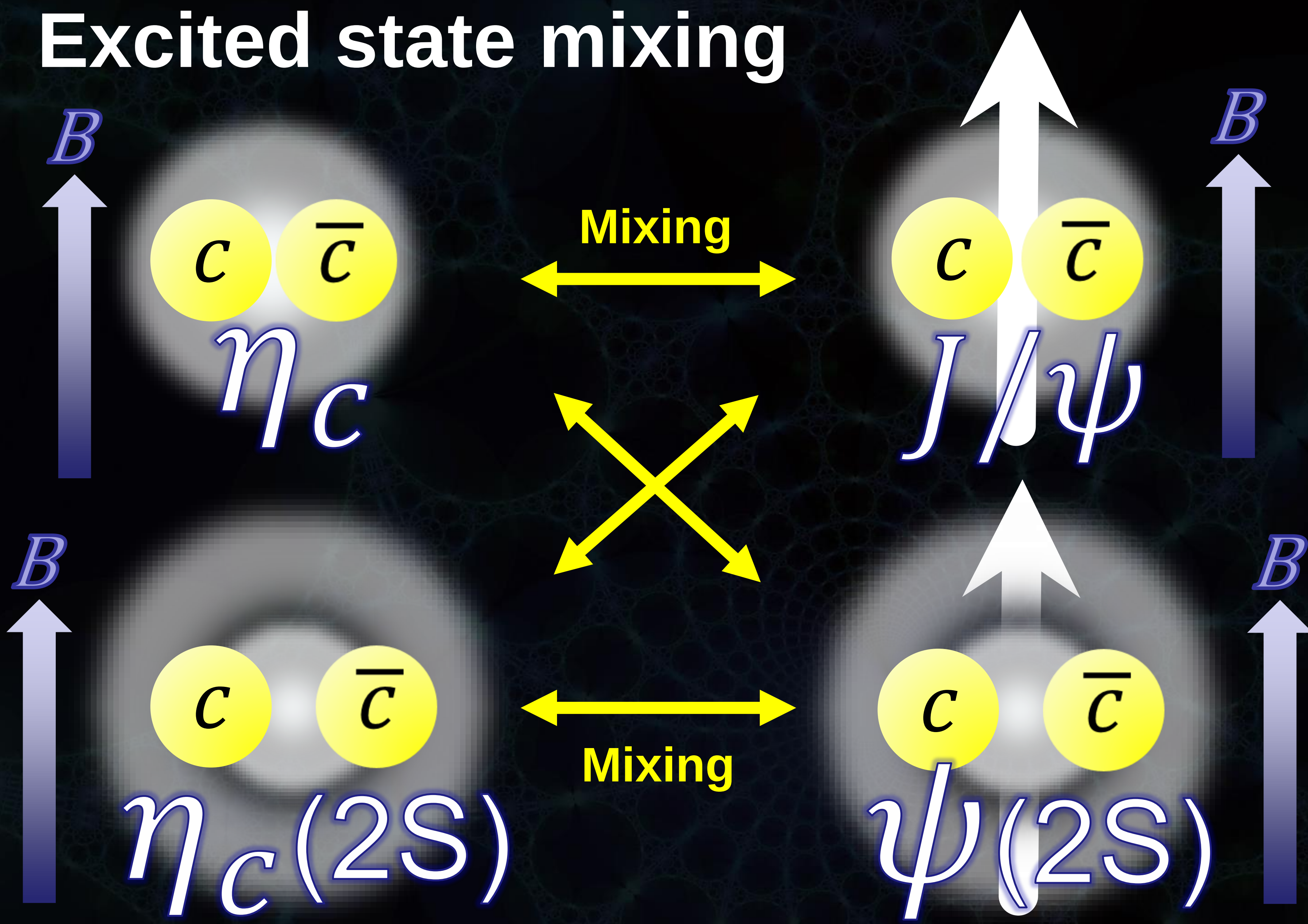
Excited states

$\sqrt{\langle r^2 \rangle} \sim 0.32\mathrm{fm}$ $\sqrt{\langle r^2 \rangle} \sim 0.77\mathrm{fm}$



$\eta_c(1S)$ $\eta_c(2S)$

Excited state mixing

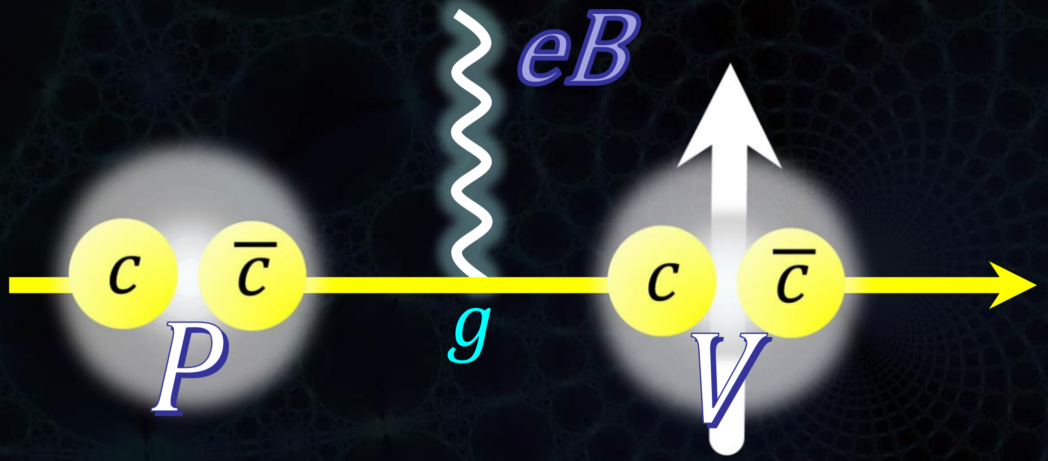


S. Cho, K. Hattori, S. H. Lee, K. Morita, and S. Ozaki, PRL113, 172301 (2014)

Spin mixing from hadron EFT

- Pseudoscalar (spin 0) and Vector (spin 1) are mixed by P-V-γ vertex

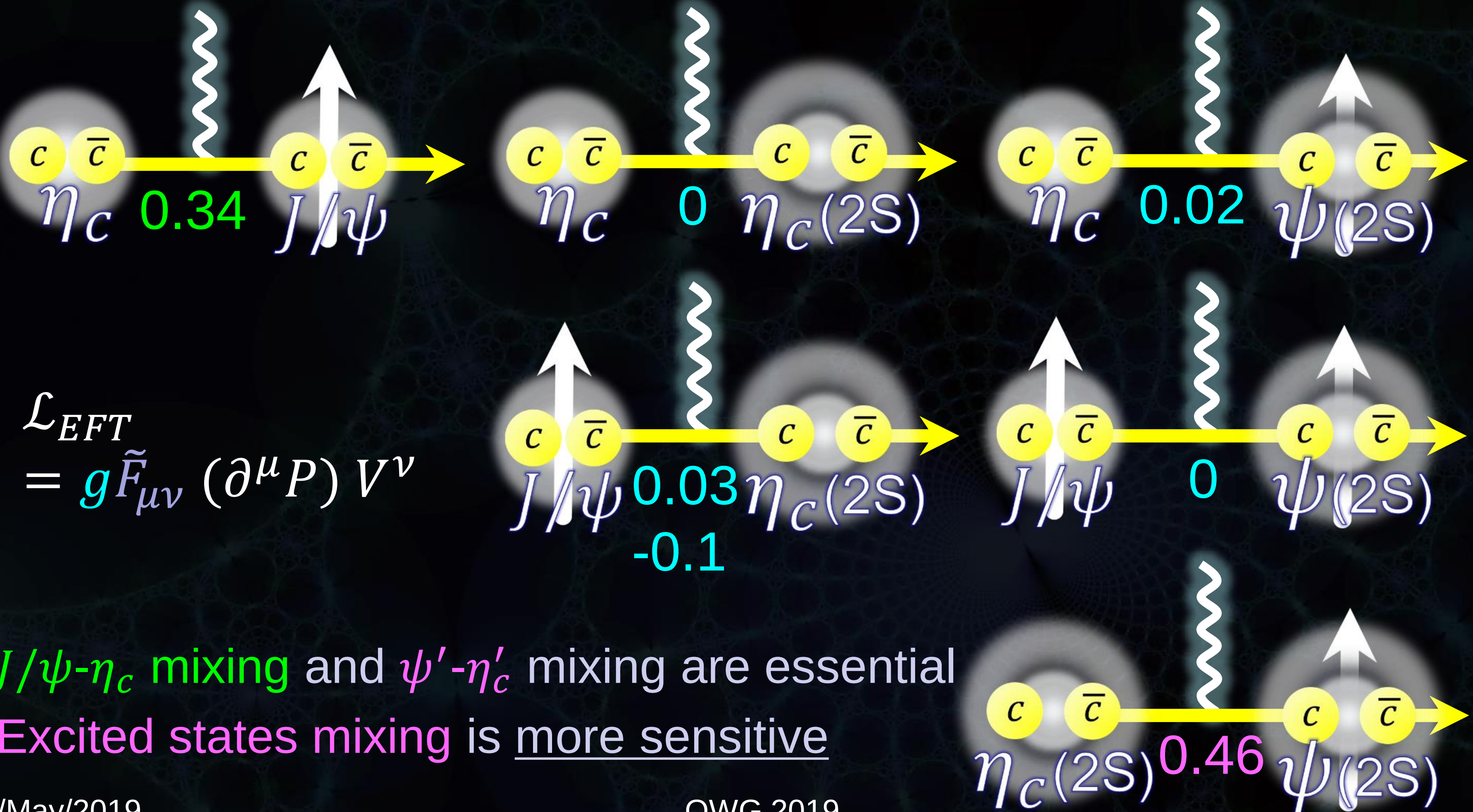
$$\mathcal{L}_{EFT} = g\tilde{F}_{\mu\nu}\,(\partial^{\mu}P)\,V^{\nu}$$



⇒Coupling constant g is determined by $J/\psi \to \eta_c\gamma$ decay

Excited states mixing from hadron EFT

- Coupling constant is $g \propto g_{\gamma PV}/(m_P + m_V)$



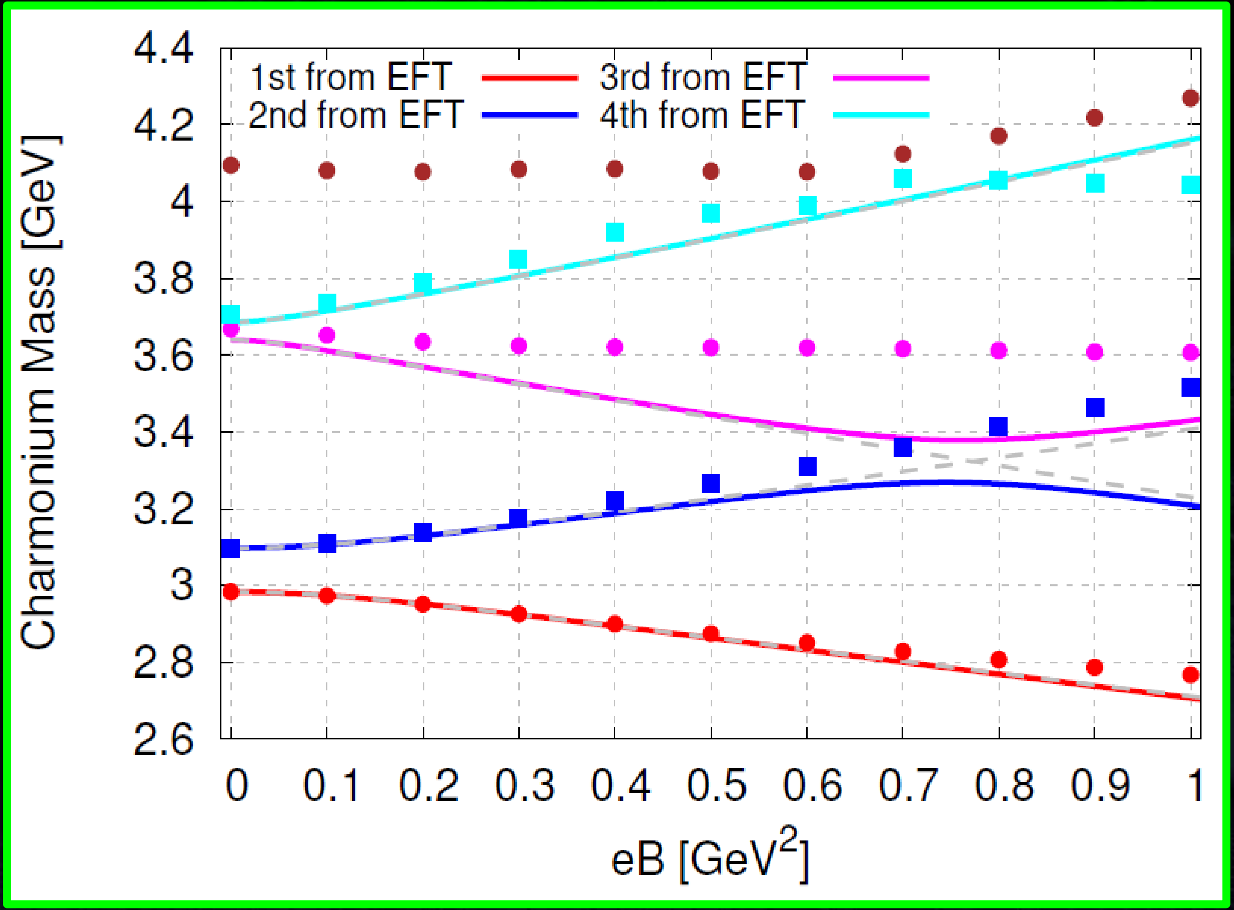
$\mathcal{L}_{EFT}$
$= g\tilde{F}_{\mu\nu}\,(\partial^\mu P)\,V^\nu$

J/ψ-η_c mixing and ψ'-η_c' mixing are essential

Excited states mixing is more sensitive

T. Yoshida and KS, PRD94, 074043 (2016)

Results from hadron EFT and potential model



— Hadron EFT

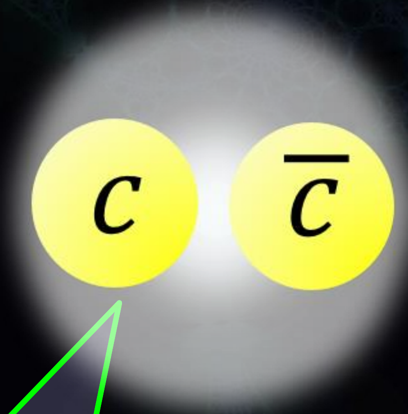
▪ Potential model

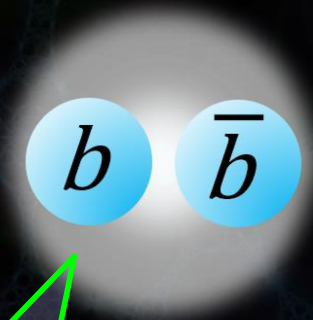
In weak B-field, two methods obtained consistent results

In strong B-field, EFT shows underestimation ⇒ Quark d.o.f ?

Bottomonia

Charmonium vs Bottomonium



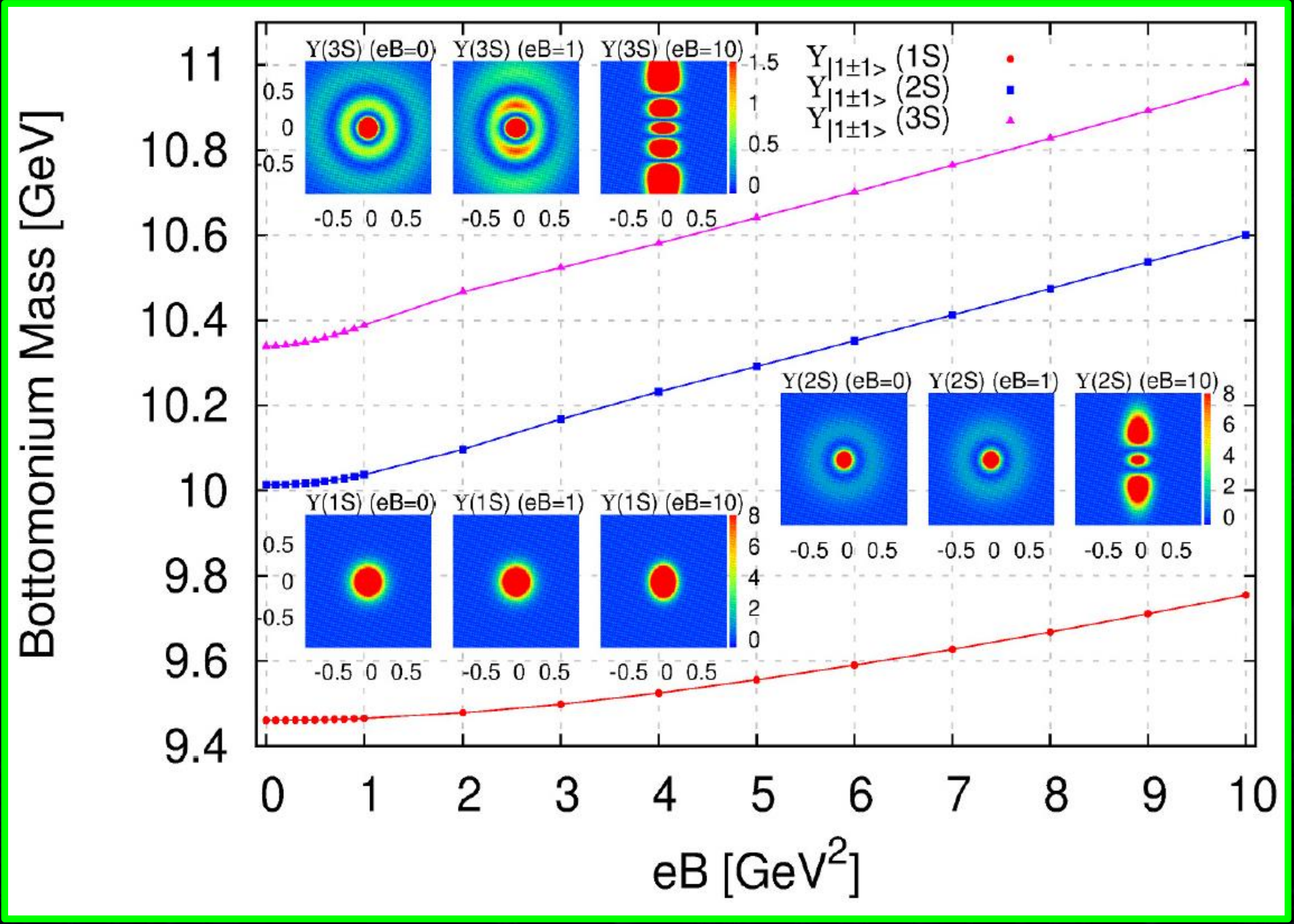


$$|Q_c| = (2/3)e$$
$$\mu_c \propto \frac{gQ}{2m_c}$$

$$|Q_b| = (1/3)e$$
$$\mu_b \propto \frac{gQ}{2m_b}$$

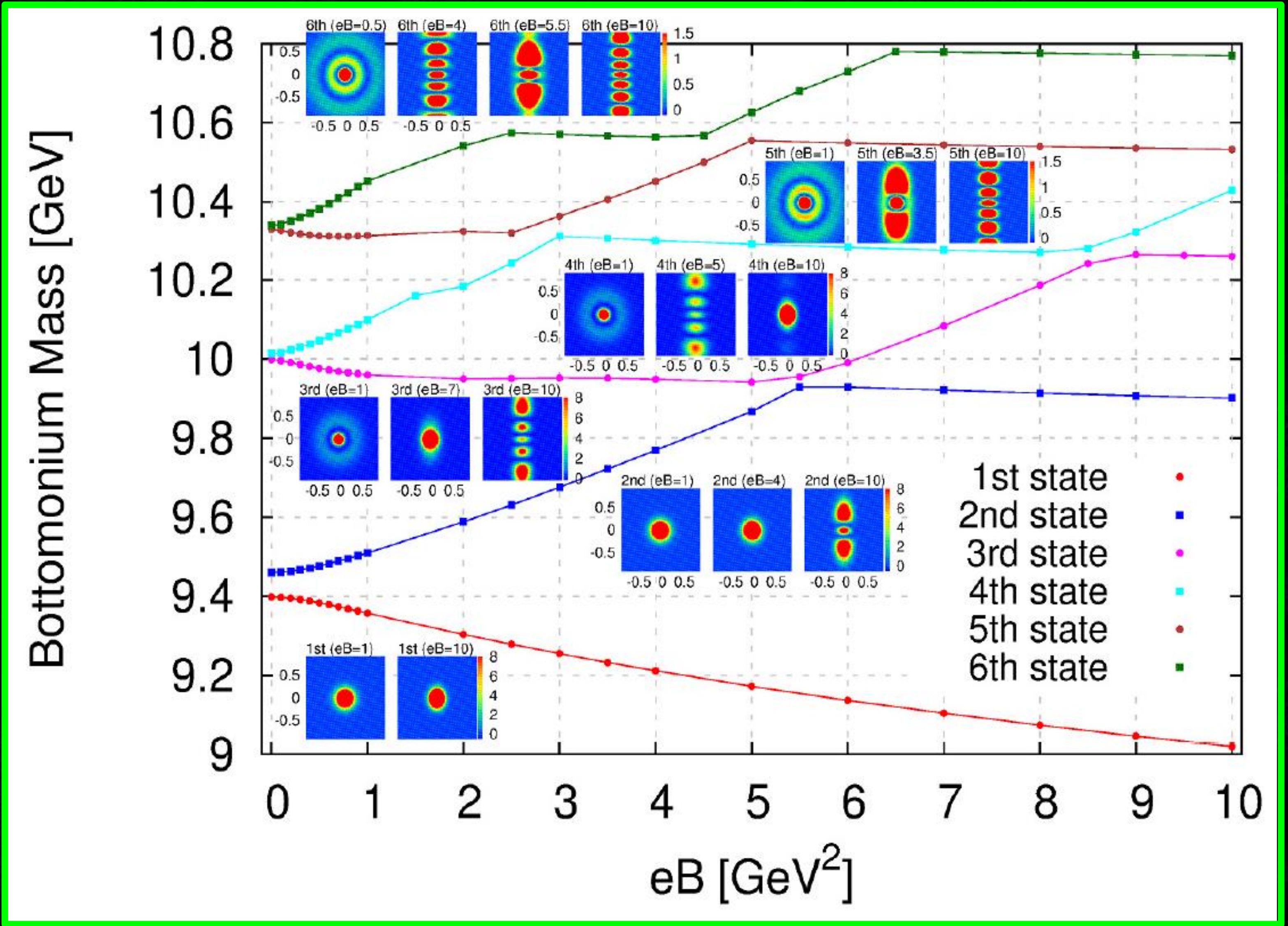
⇒Bottomonium is insensitive to B-field than charmonium

Υ $(J_z = \pm 1)$ in B-field



T. Yoshida and KS, PRD94, 074043 (2016)

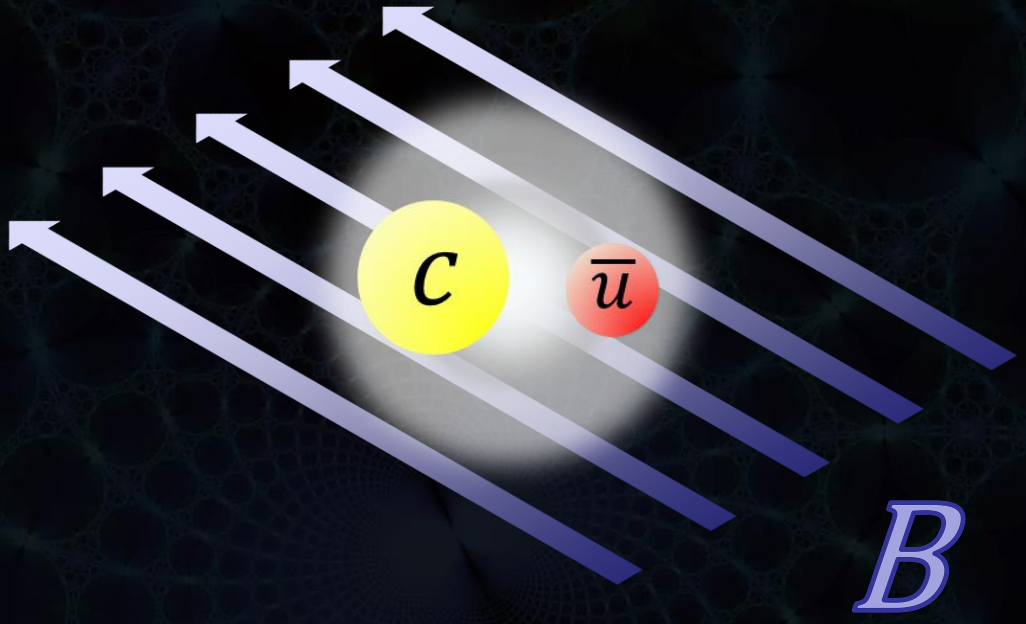
η_b and Υ ($J_z = 0$) in B-field



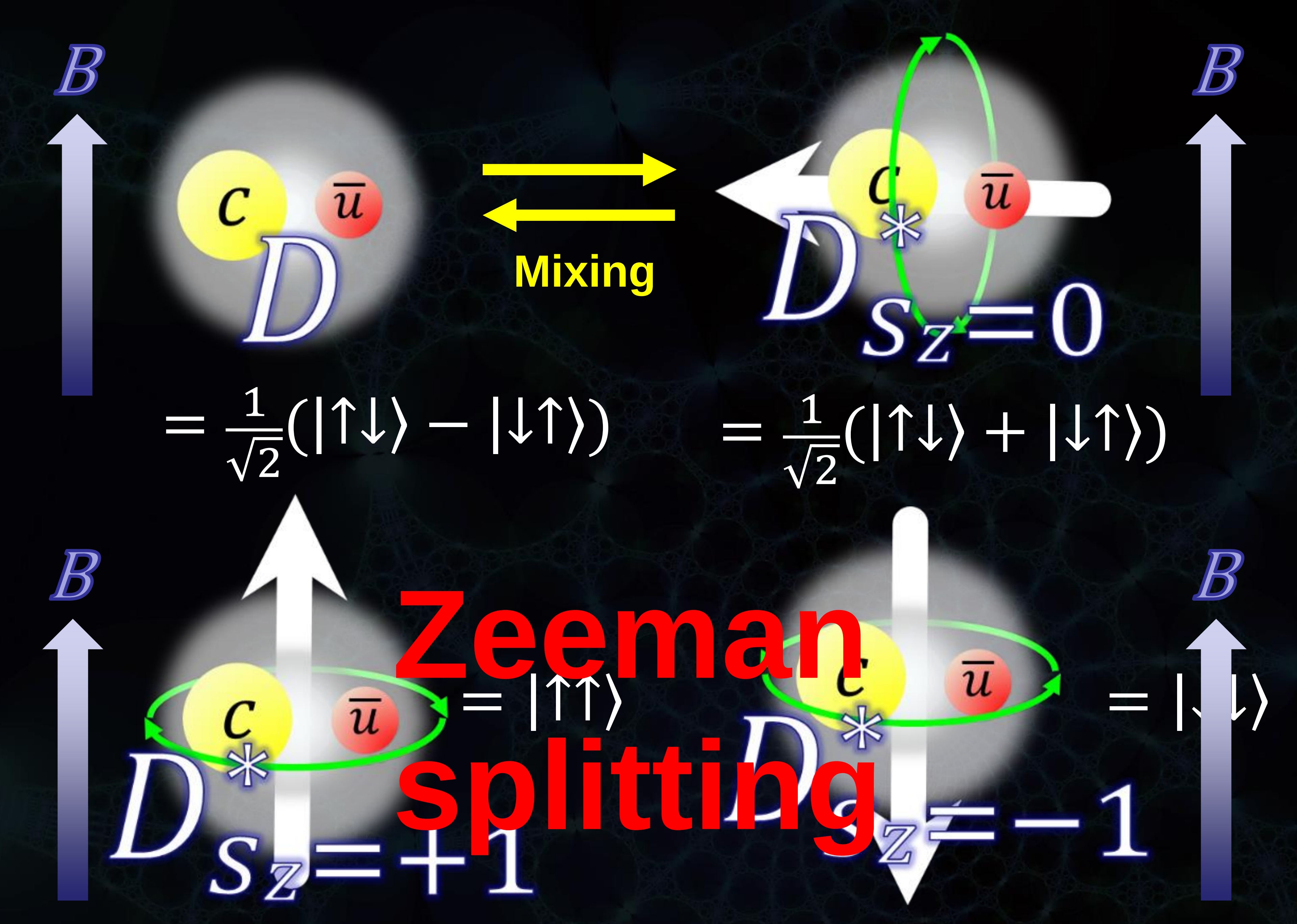
D mesons

D meson properties in B-field

- What's happen D mesons in a magnetic field?

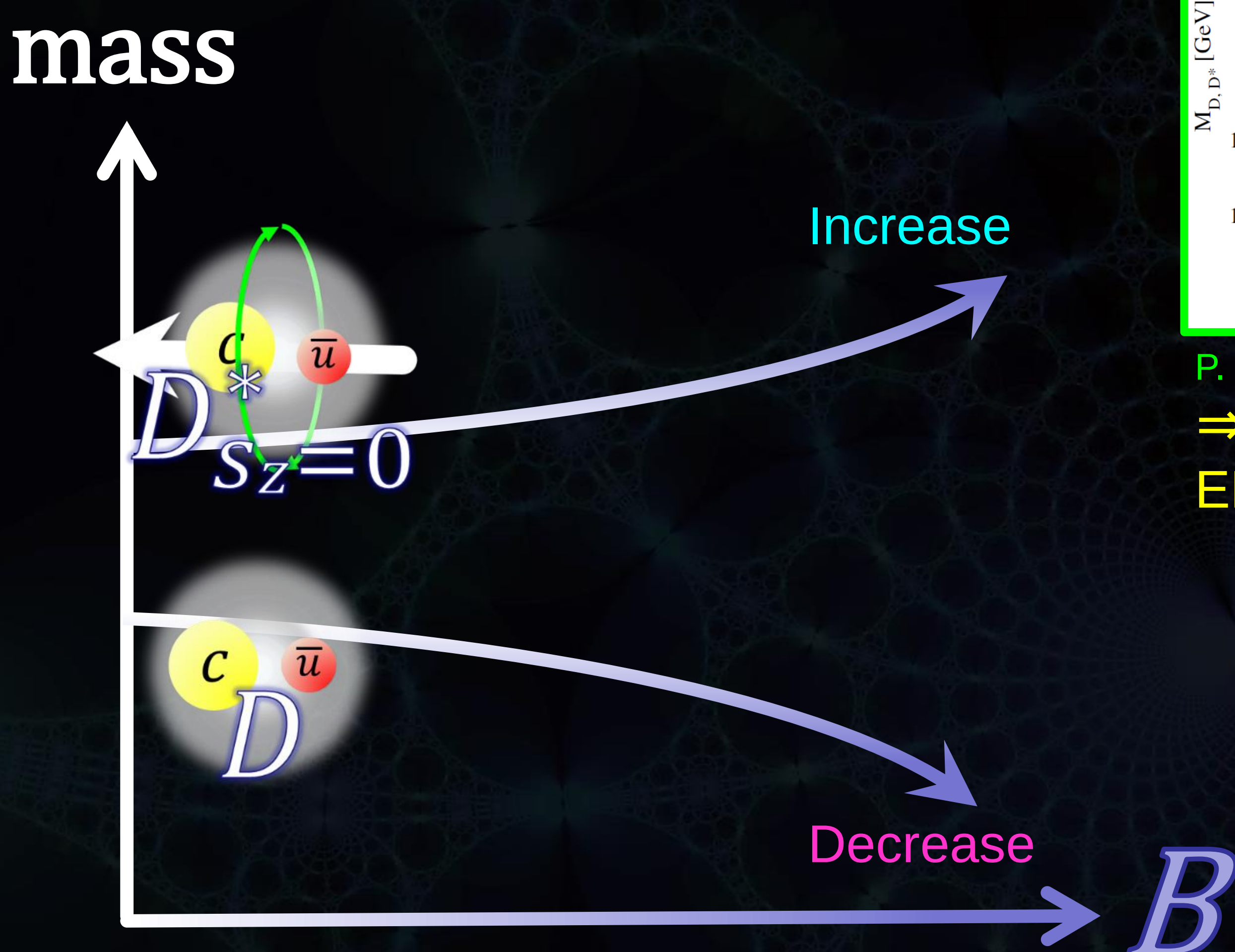


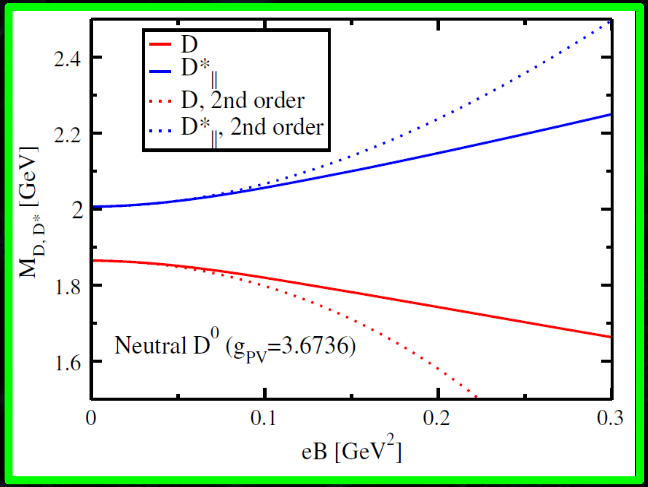
1. Spin mixing
2. Zeeman splitting
3. Quark Landau levels
4. Wave function deformation
5. Magnetic catalysis (B-dependence of $\langle \bar{q} q \rangle$)


B
c
ū
D
Mixing
$D^*_{S_z=0}$
$= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
$= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
$D^*_{S_z=+1}$
$= |\uparrow\uparrow\rangle$
$D^*_{S_z=-1}$
$= |\downarrow\downarrow\rangle$
Zeeman splitting

C.S. Machado et al. PRD88 (2013), J. Alford and M. Strickland, PRD88 (2013), S. Cho et al. PRL113 (2014)

Mass shift by spin mixing

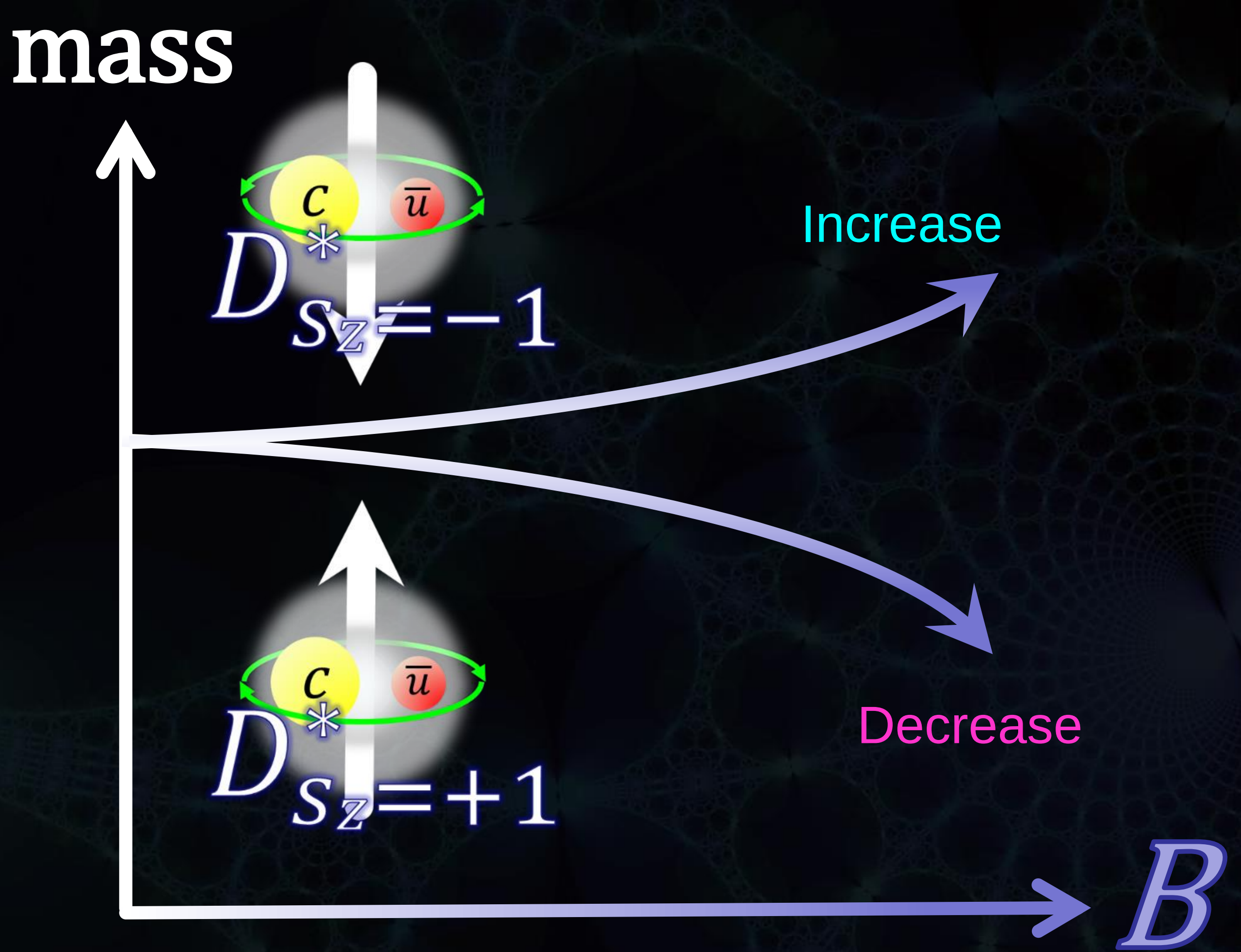


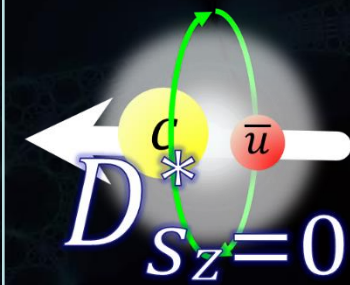
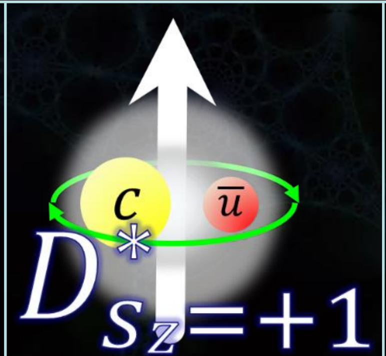
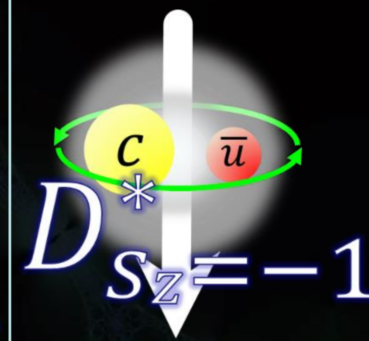
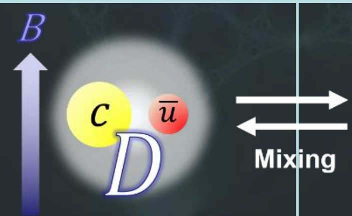
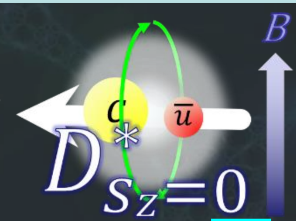


P. Gubler et al. PRD93 (2016)

⇒ Results from Hadron EFT

Mass shift by Zeeman splitting



				
Spin mixing	 Decrease ↘	 Increase ↗	No	No
Zeeman splitting	No	No	Decrease ↘	Increase ↗
Quark LLs	Increase ↗	Increase ↗	Increase ↗	Increase ↗
$\langle \bar{q}q \rangle$	?	?	?	?

D meson mass in B-field w/o MC