Radiative transitions in charmonium from lattice QCD

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Outline

- Introduction and Motivation
- Brief description of lattice methods for radiative transitions (non resonant).
- Recent results from the *Hadron Spectrum Collaboration*
    - Radiative transitions - form factors and decay rates - in charmonium
- Summary

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This is not a review but a summary of recent *preliminary* work from HadSpec Collaboration – APOLOGIES if your favourite topic/transition doesn't appear!

Radiative transitions in charmonium

Long-term goal

A new window on hadron spectroscopy.
Direct determination of elastic and inelastic form factors for hybrids and resonances:

- model-independent determination of observed production/decay rates.
- access to poorly constrained quantities (radii, EM moments) - illuminating structure of heavy hadrons and exotics.
- guide future experimental searches.

Current status

- Early days for lattice study of resonant EM processes
  - first calculation of radiative transition in hadronic resonance, $\pi\gamma^* \to \rho \to \pi\pi$ [HadSpec, 1507.06622]
  - included determination of $\pi \to \rho$ form factor.
- In principle the same method applies to heavy sector
  - to date only transistions b/w stable (ground) states calculated e.g. $J/\psi \longrightarrow \eta_c$.

Introduction

- Interested in

$$\langle h'_{J'}(\lambda', \vec{p}')|j^{\mu}|h_J(\lambda, \vec{p})\rangle$$

  the matrix element between hadron h with spin J, helicity projection λ along $\vec{p}$ and hadron h' with EM vector current ($j^{\mu} = \frac{2}{3}\bar{c}\gamma^{\mu}c$)
- To leading order in α_{em} the matrix elements encode the coupling of hadrons to the photon and are related to $\gamma h \longrightarrow h'$ by contracting with photon polarisation vector

$$\mathcal{M}\Big(\gamma(\lambda_{\gamma}, \vec{q})h_J(\lambda, \vec{p}) \rightarrow h'_{J'}(\lambda', \vec{p}')\Big) = \epsilon_{\mu}(\lambda_{\gamma}, \vec{q})\langle h'_{J'}(\lambda', \vec{p}')|j^{\mu}|h_J(\lambda, \vec{p})\rangle$$

- Express the matrix elements as a sum over form factors with kinematic Lorentz factors

$$\langle h'_{J'}(\lambda', \vec{p}')|j^{\mu}|h_J(\lambda, \vec{p})\rangle = \sum_i K_i^{\mu}\left[h'_{J'}; h_J\right] F_i(Q^2)$$

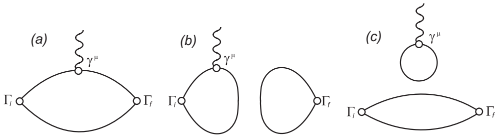
  with Q^2 the photon virtuality.
- Determine the $F_i(Q^2)$ from lattice to determine decay rates.
- $h_J = h'_{J'}$: radiative/charge form factors. $h_J \neq h'_{J'}$: transition form factors.
- Conservation of the current, parity invariance used to constrain the form-factor decomposition.

Calculating radiative transitions

Consider: 3-point function with vector current insertion

$$C^{\mu}_{ij}(\Delta t, t) = \langle 0|\mathcal{O}_i(\Delta t) j^{\mu}(t) \mathcal{O}^{\dagger}_j(0)|0\rangle$$

$$\longrightarrow \sum_{m,n} \frac{1}{2E_m} \frac{1}{2E_n} e^{-E_m(\Delta t - t)} e^{-E_n(t)} \langle 0|\mathcal{O}_i(0)|m\rangle \langle m|j^{\mu}(0)|n\rangle \langle n|\mathcal{O}^{\dagger}_j(0)|0\rangle$$



- Only connected correlators are considered here. Use distillation [PRD80 (2009)] $\longrightarrow$ variationally optimised (projected) operators.
- Use *generalised perambulators* unsmeared at one end for the vector current.
- Wide range of Q^2 needed to access physical point by interpolation - which may be directly inaccessible with discretised momenta. Many momenta combinations required!
- Above thresholds - generalised Lüscher approach in e.g. $D\bar{D}$ scattering yields resonance parameters and transition form factors.

Some lattice details

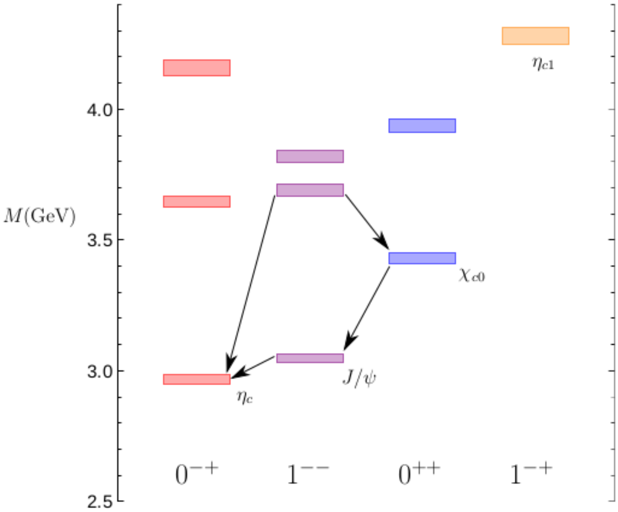
- Dynamical $N_f = 2+1$ anisotropic ensembles, stout-smeared spatial links.
- Tree-level tadpole-improved Clover (Wilson) quarks.
- Relativistic charm. Distillation.
- Operators of definite helicity subduced to the relevant (little group) irrep.

Volume	$\xi = \frac{a_s}{a_t}$	M_π	N_{cfgs}	N_{e-vecs}	N_{tsrcs}
$20^3 \times 128$	3.5	391 MeV	603 (300)	128	1

Related HadSpec work

- Towards radiative transitions in charmonium. O'Hara et al. PoS Lattice2016:120, 2016.
- Exotic and excited-state radiative transitions in charmonium from lattice QCD. Dudek, Edwards & Thomas, PRD 79, 094504, (2009).
- Excited meson radiative transitions from lattice QCD using variationally optimised operators. Shultz, Dudek & Edwards, PRD 91, 114501, (2015).

The charmonium spectrum



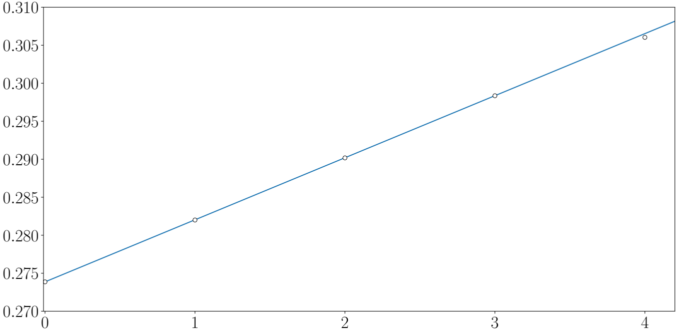
- $20^3 \times 128$, $m_\pi = 391$ MeV.
- Allowed transitions shown (between states of different C).

Lattice dispersion relation

- Finite momenta data needed for form factors and transitions.
- Verify the dispersion relation wrt previous work and theoretical expectations.

$$(a_t E)^2 = (a_t M)^2 + \left(\frac{2\pi}{\xi L/a_s}\right)^2 n^2 \text{ vs } |n_{\vec{p}}|^2$$

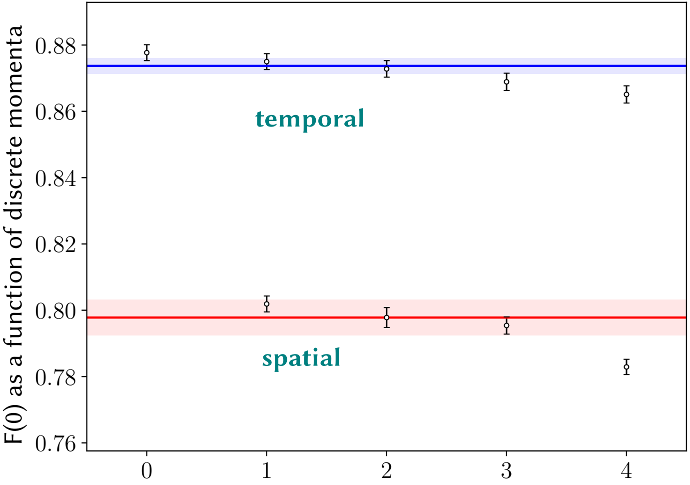
- Extract the anisotropy, ξ, from the slope of the dispersion relation.



Target	This work	Charmonium on $24^3 \times 128$	D meson
3.5	3.476(3)	3.50(2)	3.454(6)

Renormalising the vector current

- Local vector current not conserved; multiplicatively renormalised using the pseudoscalar charge form-factor.
- On an anisotropic lattice, $a_s \neq a_t$ these can be different.
- $Z_V = 1/F_{\eta_c}(0)$ and $Z_V^s = 1.251(4)$; $Z_V^t = 1.144(3)$.



Improving the vector current

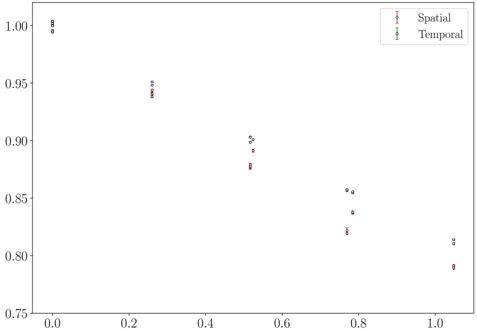
- Discretisation on the lattice introduces errors of the order of the lattice spacing, which can be systematically reduced by *improvement*.
- Process involves adding higher order terms to the lattice action which disappear as $a \longrightarrow 0$
- Field redfinitions lead to a redefinition of the vector current

$$\begin{aligned} j_0 &= Z_V^t \left(\bar{\psi}\gamma_0\psi + \frac{1}{4}\frac{\nu_s}{\xi}(1-\xi)\, a_s \delta_j \left(\bar{\psi}\sigma_{0j}\psi\right)\right) \\ j_k &= Z_V^s \left(\bar{\psi}\gamma_k\psi + \frac{1}{4}(1-\xi)\, a_t \delta_0 \left(\bar{\psi}\sigma_{0k}\psi\right)\right) \end{aligned}$$

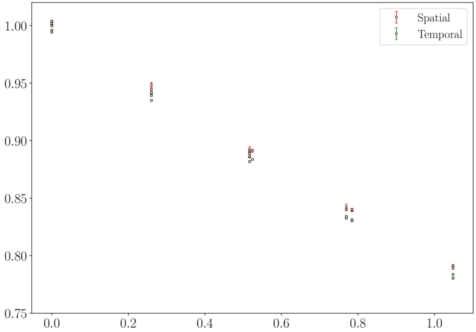
Improved currents for improved form factors

$Q^2(GeV)$ dependence of the η_c spatial and temporal form factors

Unimproved currents



Improved currents



preliminary

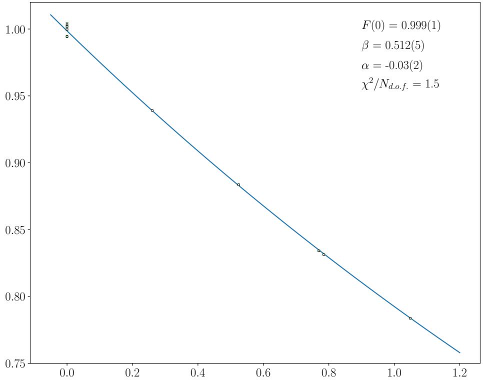
Form factor and transition results

Preliminary

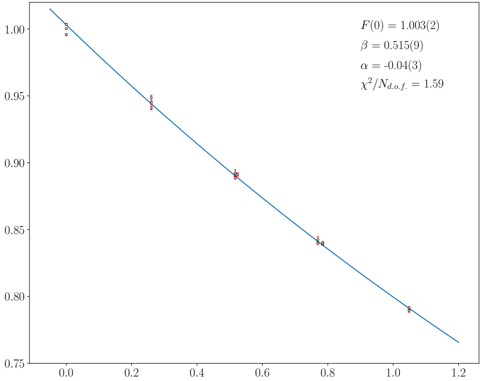
η_c FORM FACTOR

The matrix element is $\langle \eta_c(\vec{p}\,')|j^\mu|\eta_c(\vec{p})\rangle = (\vec{p}+\vec{p}\,')^\mu F_{\eta_c}(Q^2)$.

temporal current



spatial current



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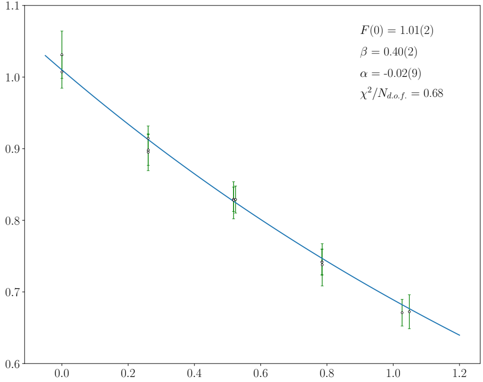
- Fit: $F(Q^2) = F(0)exp\left[-\frac{Q^2}{16\beta^2}\left(1+\alpha Q^2\right)\right]$.
- From slope of the form factor at $Q^2 = 0$ get charge radius: $\langle r^2\rangle = 6/16\beta^2$.
- Yielding $\sqrt{\langle r_s^2\rangle} = 0.234(4)\mathrm{fm}$ & $\sqrt{\langle r_t^2\rangle} = 0.236(2)\mathrm{fm}$

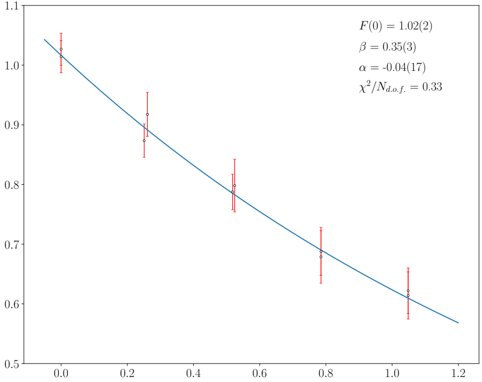
χ_{c0} FORM FACTOR

As for η_c yielding $\sqrt{\langle r_s^2 \rangle} = 0.34(3)\text{fm}$ $\sqrt{\langle r_t^2 \rangle} = 0.30(2)\text{fm}$

from temporal current



from spatial current

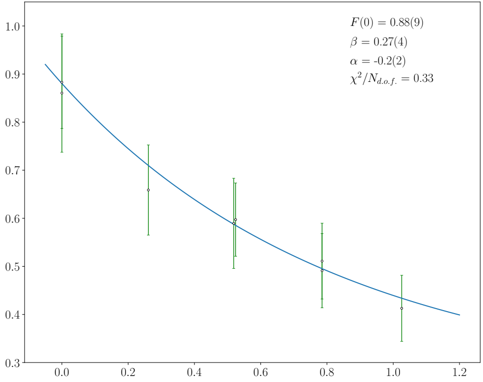


preliminary

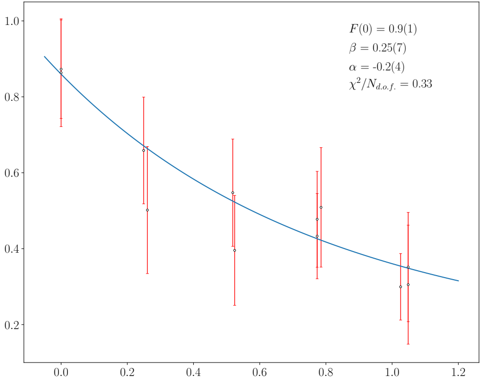
η_c' form factor

As for η_c yielding $\sqrt{\langle r_s^2 \rangle} = 0.49(13)\mathrm{fm}$ & $\sqrt{\langle r_t^2 \rangle} = 0.45(7)\mathrm{fm}$

from temporal current



from spatial current



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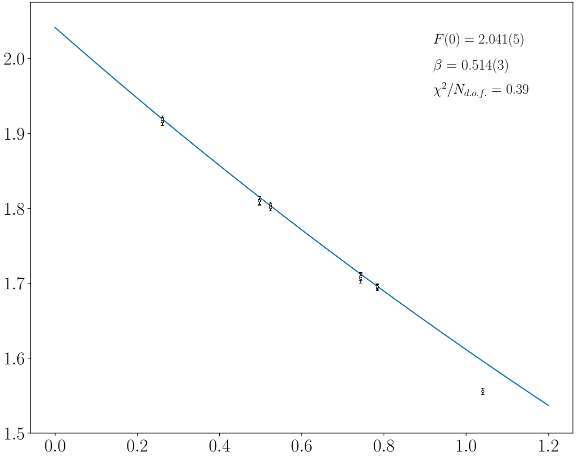
$J/\psi \longrightarrow \eta_c \gamma$

- Described by a single form factor:
  $\langle \eta_c(\vec{p}')|j^\mu|J/\psi(\lambda,\vec{p})\rangle = \epsilon^{\mu\nu\rho\sigma} p'_\nu p_\rho \epsilon_\sigma(\lambda,\vec{p}) \frac{2}{m_{\eta_c}+m_{J/\psi}} F(Q^2)$
- Extract form factor at zero virtuality via $F(Q^2) = F(0)exp\left(-Q^2/16\beta^2\right)$.



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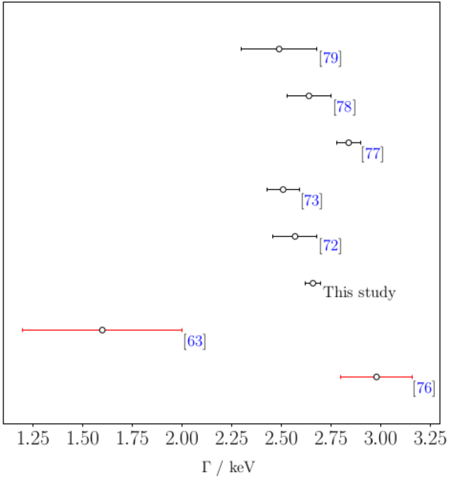
$J/\psi \longrightarrow \eta_c\gamma$: A comparison with selected results

$$\Gamma(J/\psi \longrightarrow \eta_c\gamma) = \frac{64\alpha}{27}\frac{|\mathbf{q}|^3}{(m_{\eta_c}+m_{J/\psi})^2}|\hat{F}(0)|^2 \ ; \ F(Q^2) = 2\times\frac{2}{3}e\times\hat{F}(Q^2).$$



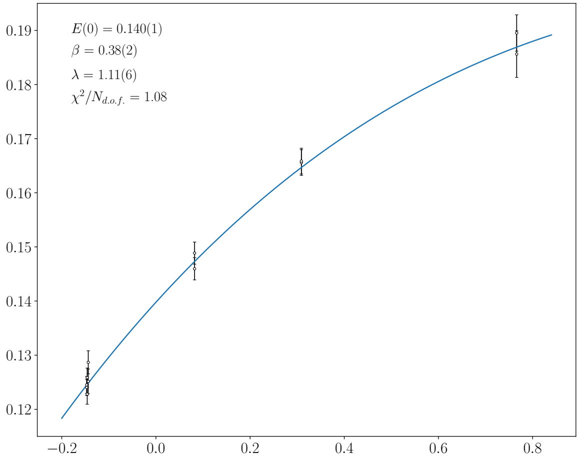
- (72), (73) quenched lattice by HadSpec
- (77) $N_f = 2$ twisted mass lattice, Chen et al. PRD84:034503 (2011)
- (78): $N_f = 2$ twisted mass, Becirevic & Sanfilippo JHEP01 (2013)
- (79) $N_f = 2+1$ improved staggered, physical point, Donald et al PRD86:094501 (2012)

- (63) PDG (2018)
- (76) KEDR Phys.Lett.B738 (2014)

Comparison is indicative only - systematic error budgets in lattice results very different.

$\chi_{c0} \longrightarrow J/\psi\gamma$

- Characterised by 2 form factors
- Electric dipole $E_1(Q^2)$ is physically relevant: $\langle \chi_{c0}(\vec{p}\,')|j^\mu|J/\psi(\lambda,\vec{p})\rangle = E_1(Q^2)\epsilon^\mu(\lambda,\vec{p})$
- $\Gamma(\chi_{c0} \longrightarrow J/\psi\gamma) = \alpha \frac{|q|}{m^2_{\chi_{c0}}} \frac{16}{9} |\hat{E}_1(0)|^2.$



preliminary

Summary

- Charmonium radiative transitions and form factors calculated on lattice.
- Methods working well, as in light sector. Preliminary results for stable ground states in agreement with other work and can be precisely determined.
- Many improvements and issues to address:
  - Better statistics, more time sources, to reliably determine the excited and hybrid states (done in e.g. 1204.5425) - underway. Lighter pions and larger volumes also possible.
  - Better control of systematics including e.g. fitting systematics.
- With these improvements we can be more adventurous!

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Thanks for listening!