EFT approach to the pair production of unstable tops near threshold

Thomas Rauh

AEC, University of Bern

M. Beneke, A. Maier, TR, P. Ruiz-Femenía, arXiv:1711.10429
JHEP 1802 (2018) 125

M. Beneke, A. Maier, J. Piclum, TR, arXiv:1506.06865
Nucl.Phys. B 899 (2015) 180

M. Beneke, J. Piclum, TR, arXiv:1312.4792
Nucl.Phys. B 880 (2014) 414

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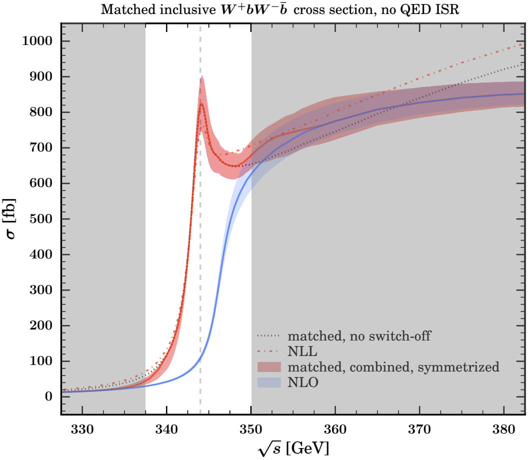
FOR FUNDAMENTAL PHYSICS

Top threshold scan

Motivation

Consider the top threshold region of the $e^+e^- \to W^+W^-\bar{b}bX$ cross section:



[Bach et al. 2017]

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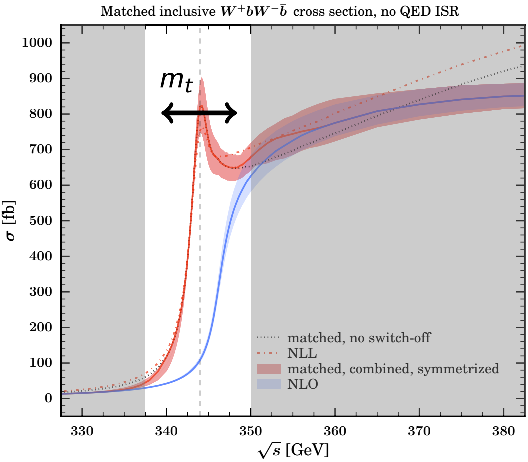
- Allows extremely precise determination of the top quark mass

  $$\delta\overline{m}_t(\overline{m}_t) \approx 50\,\mathrm{MeV}$$

- Sensitive to Γ_t, α_s, y_t



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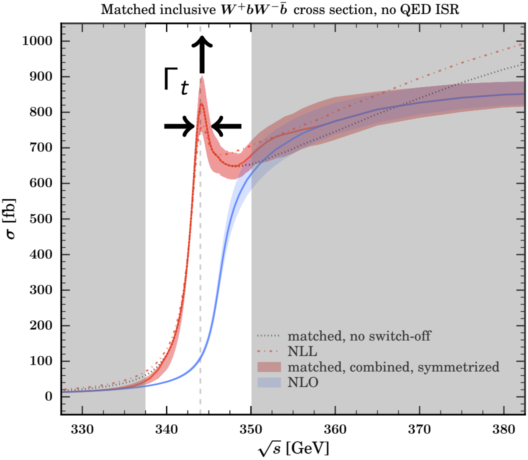

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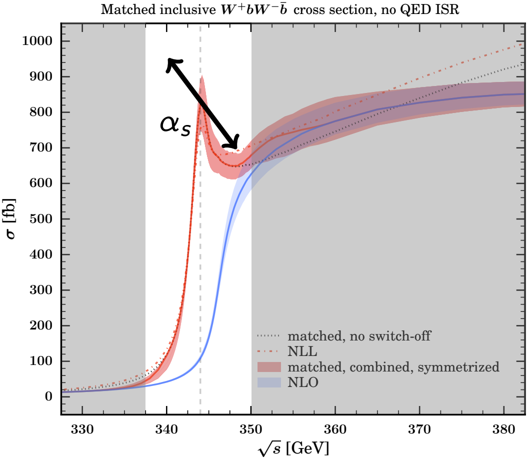

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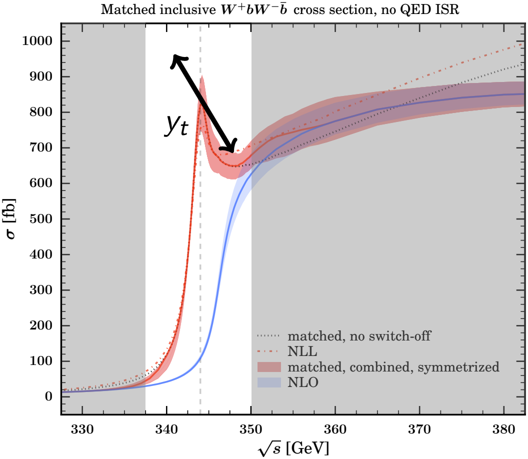

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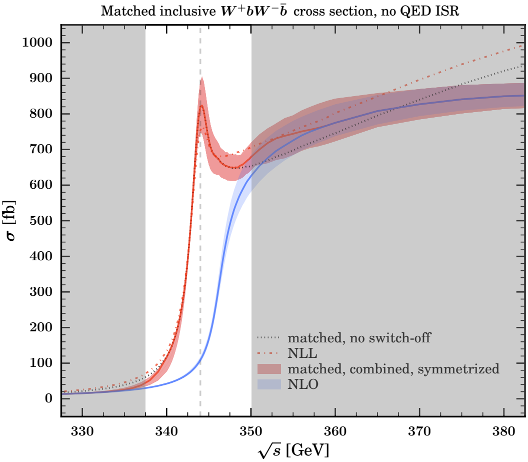
Requires very precise theory predictions:

- Known at NNNLO in QCD, scale uncertainty of 3%
  [Beneke at al. 2015]
- NLO non-QCD up to 15%
- NNLO non-QCD and N^{3}LO Higgs effects in this talk



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Top quarks near threshold

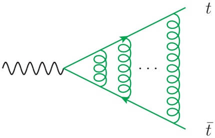
Relevant scales and Coulomb effects

Near threshold tops are non-relativistic with velocity $v \sim \alpha_s$

- Multiple scales are relevant:

hard	m_t	top mass
soft	$m_t v$	momentum
ultrasoft	$m_t v^2$	energy

- Coulomb singularities $(\alpha_s/v)^n$ from n exchanges of potential gluons



$k^0 \sim m_t v^2, \mathbf{k} \sim m_t v$

- Conventional perturbation theory in α_s fails
- Coulomb singularities must be resummed to all orders
- Done with potential non-relativistic QCD (PNRQCD)

[Pineda, Soto 1998; Beneke, Signer, Smirnov 1999; Brambilla, Pineda, Soto, Vairo 2000; Beneke, Kiyo, Schuller 2013]

QCD cross section

Born approximation

Normalized cross section:

$$R(s) = \frac{\sigma(e^+e^- \to t\bar{t}X)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi e_t^2 f(s)\,\mathrm{Im}\left[\Pi^{(v)}(s)\right]$$

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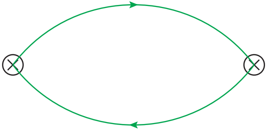

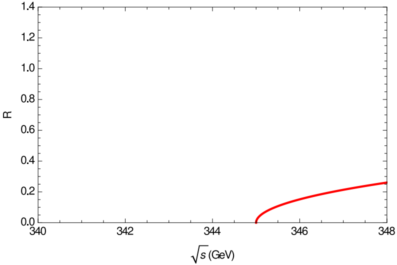

QCD cross section

Resummed cross section at LO

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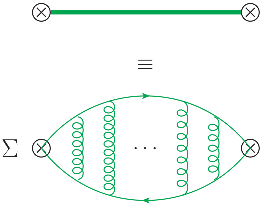
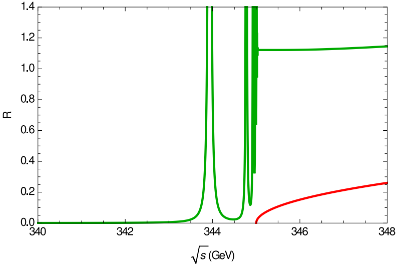


$\Gamma_t = 0$



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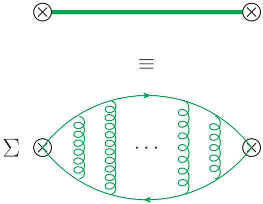
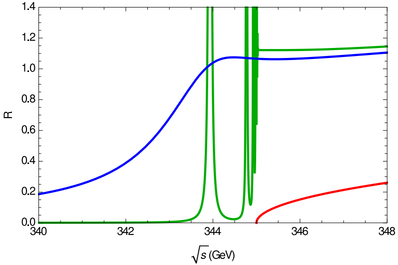


$\Gamma_t \neq 0$



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Resummed cross section at NNNLO

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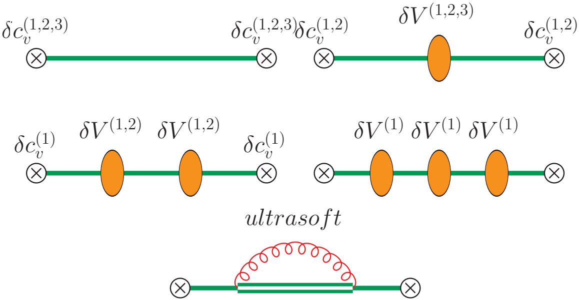

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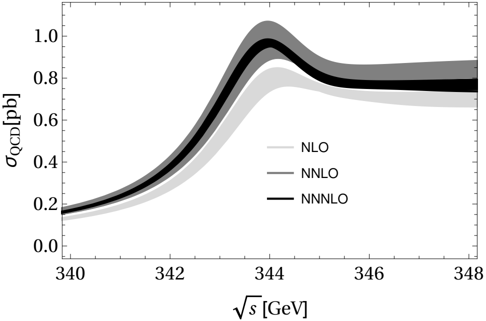

- NNNLO S-wave [Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]
- NLO P-wave [Beneke, Piclum, TR 2013]
- QQbar_Threshold code [Beneke, Kiyo, Maier, Piclum 2016; Beneke, Maier, TR, Ruiz-Femenía 2017]

- Stabilization of perturbative expansion at NNNLO
- 3% uncertainty due to scale variation from 50 to 350 GeV
- Similar conclusions at NNLL (5% uncertainty) [Hoang, Stahlhofen 2013]

Non-QCD effects

Top-quark decay width

$$\mathcal{L}_{\text{bilinear}} = \psi^\dagger \left[i\partial^0 + \frac{\vec{\partial}^2}{2m_t} + \frac{i\Gamma_t}{2} + \frac{(\vec{\partial}^2 + i m_t \Gamma_t)^2}{8m_t^3} + \ldots \right] \psi + \text{anti-quark}$$

- $\psi^\dagger \frac{i\Gamma_t}{2} \psi$: same order as kinetic term, shifts $E \to E + i\Gamma_t$ $(E = \sqrt{s} - 2m_t)$
  Causes divergences at NNLO: $\sigma \supset \sigma_0 \mathrm{Im}\left[\frac{E}{\epsilon}\right] \to \sigma_0 \mathrm{Im}\left[\frac{E+i\Gamma_t}{\epsilon}\right]$

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- $\psi^\dagger \frac{i\Gamma_t \vec{\partial}^2}{4m_t^2} \psi$: time dilatation, reduces toponium width $\Gamma_n = 2\Gamma_t - \frac{\Gamma_t \alpha_s^2 C_F^2}{4n^2} + \ldots$
  Non-Hermitian Hamiltonian $H \Rightarrow$ eigenstates do not form a basis

$$H\,|n\rangle = \mathcal{E}_n\,|n\rangle\,, \qquad H^\dagger\,|\tilde{m}\rangle = \tilde{\mathcal{E}}_m\,|\tilde{m}\rangle\,, \qquad \tilde{\mathcal{E}}_n = \mathcal{E}_n^* = (E_n - i\Gamma_n/2)^*$$

$\uparrow$ exponentially decaying states

$\uparrow$ exponentially growing states

$$\langle n\,|\,\tilde{m}\rangle = \delta_{nm}$$

Non-relativistic Green function: $G(E) = \left\langle \vec{0} \middle| \hat{G}(E) \middle| \vec{0} \right\rangle = \sum_n \frac{\psi_n(\vec{0})\psi_{\tilde{n}}^*(\vec{0})}{\mathcal{E}_n - E}$

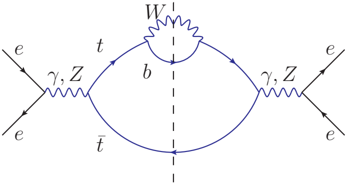
Non-QCD effects

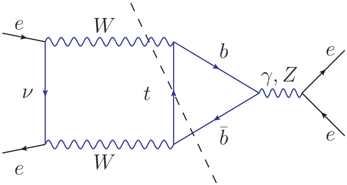
Non-resonant contributions

The physical final state is $W^+W^-\bar{b}bX$

- $\Gamma_t \sim m_t\alpha \sim m_t\alpha_s^2$ is not suppressed with respect to the ultrasoft scale
- Narrow width approximation is unphysical!
- Top decay modifies cross section in non-perturbative way (smearing of toponium resonances)

Top instability implies existence of contributions to the cross section from hard subgraphs that connect to the initial and final state





Non-QCD effects

Effective theory setup

Contributions can be organized systematically within Unstable Particle Effective Theory [Beneke, Chapovsky, Signer, Zanderighi 2003-4]

$$\sigma(s) \sim \mathrm{Im}\left\{ \sum_{k,l} C^{(k)} C^{(l)} \int d^4x \, \langle e^- e^+ | \mathrm{T}[i\mathcal{O}^{(k)\dagger}(0) \, i\mathcal{O}^{(l)}(x)] | e^- e^+ \rangle_{\mathrm{EFT}} \right.$$

$$\left. + \sum_{k} C_{4e}^{(k)} \, \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle_{\mathrm{EFT}} \right\}$$

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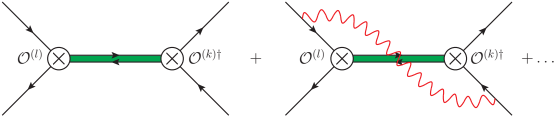

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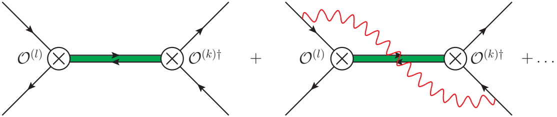

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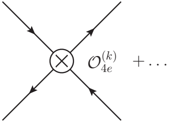
Non-resonant contribution from $W^+W^-b\bar{b}$ production in hard process



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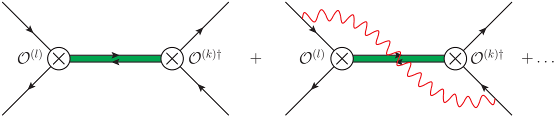

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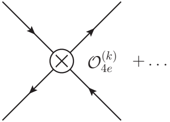
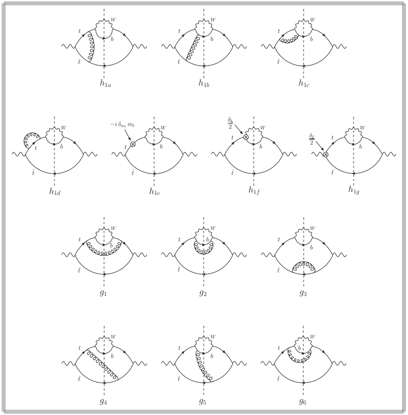


Both parts contain spurious divergences! Only the sum is finite. Calculations must be done in the same regularization scheme.

NNLO non-resonant contribution

Contains endpoint divergences when the hard tops go on-shell [Jantzen, Ruiz-Femenía '13]



'Squared contribution': Gluon corrections to h1, endpoint divergent but UV & IR finite

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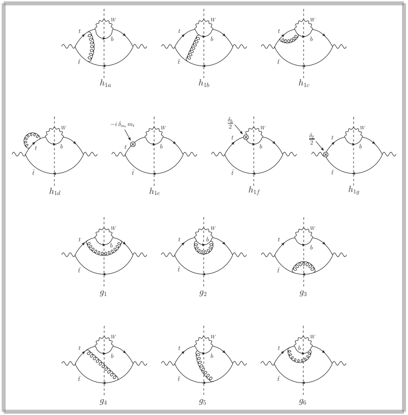

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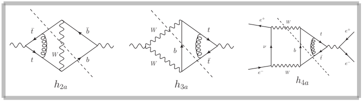

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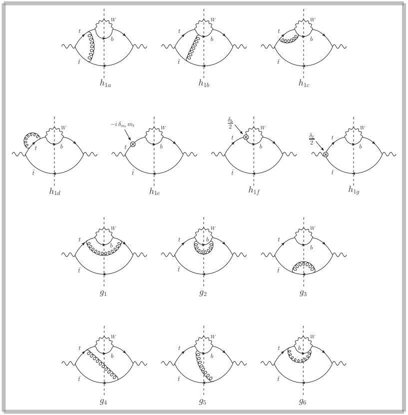

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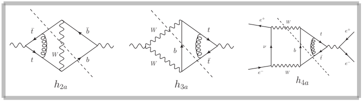

'Interference contribution': endpoint & UV divergent

+ O(100) endpoint finite diagrams (not drawn)

'Automated contribution': endpoint finite but UV divergent, computed with automated tools (MadGraph)

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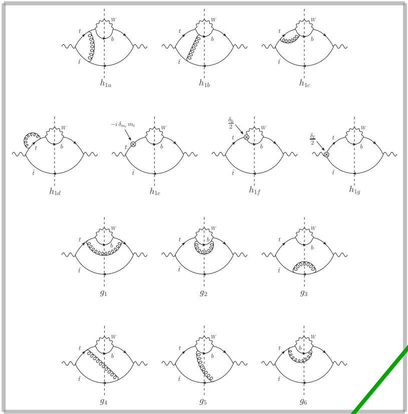

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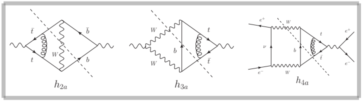

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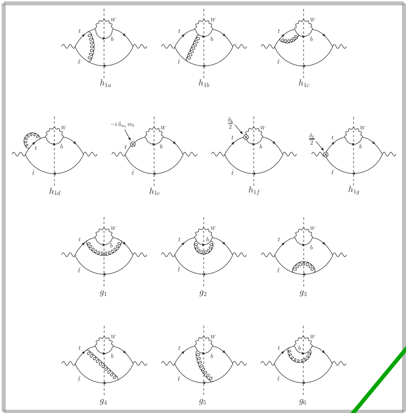

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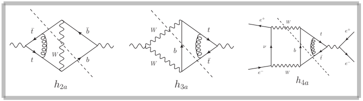

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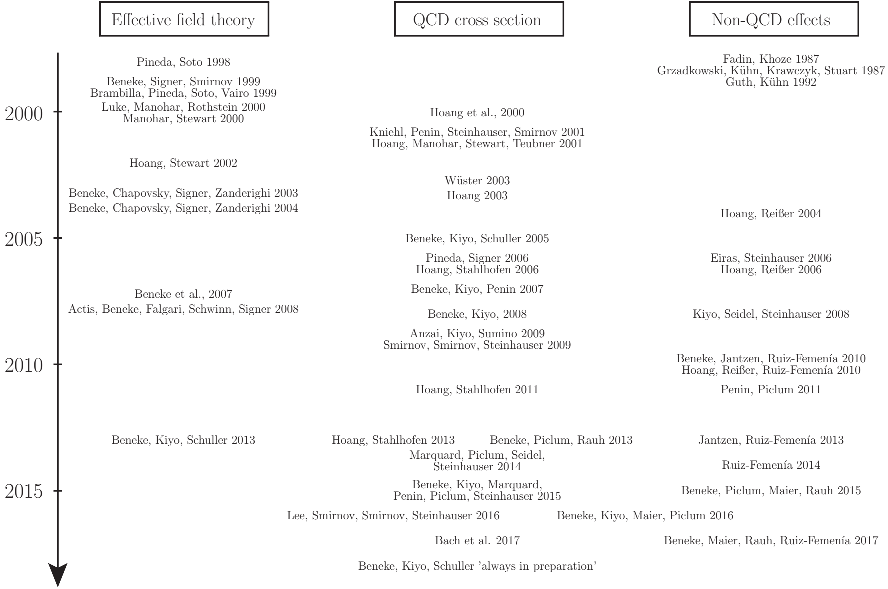
Work beyond NNLO QCD



Effective field theory

Pineda, Soto 1998
Beneke, Signer, Smirnov 1999
Brambilla, Pineda, Soto, Vairo 1999
Luke, Manohar, Rothstein 2000
Manohar, Stewart 2000
Hoang, Stewart 2002
Beneke, Chapovsky, Signer, Zanderighi 2003
Beneke, Chapovsky, Signer, Zanderighi 2004
Beneke et al., 2007
Actis, Beneke, Falgari, Schwinn, Signer 2008
Beneke, Kiyo, Schuller 2013

QCD cross section

Hoang et al., 2000
Kniehl, Penin, Steinhauser, Smirnov 2001
Hoang, Manohar, Stewart, Teubner 2001
Wüster 2003
Hoang 2003
Beneke, Kiyo, Schuller 2005
Pineda, Signer 2006
Hoang, Stahlhofen 2006
Beneke, Kiyo, Penin 2007
Beneke, Kiyo, 2008
Anzai, Kiyo, Sumino 2009
Smirnov, Smirnov, Steinhauser 2009
Hoang, Stahlhofen 2011
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Beneke, Piclum, Rauh 2013
Marquard, Piclum, Seidel,
Steinhauser 2014
Beneke, Kiyo, Marquard,
Penin, Piclum, Steinhauser 2015
Lee, Smirnov, Smirnov, Steinhauser 2016
Beneke, Kiyo, Maier, Piclum 2016
Bach et al. 2017
Beneke, Kiyo, Schuller 'always in preparation'

Non-QCD effects

Fadin, Khoze 1987
Grzadkowski, Kühn, Krawczyk, Stuart 1987
Guth, Kühn 1992
Hoang, Reißer 2004
Eiras, Steinhauser 2006
Hoang, Reißer 2006
Kiyo, Seidel, Steinhauser 2008
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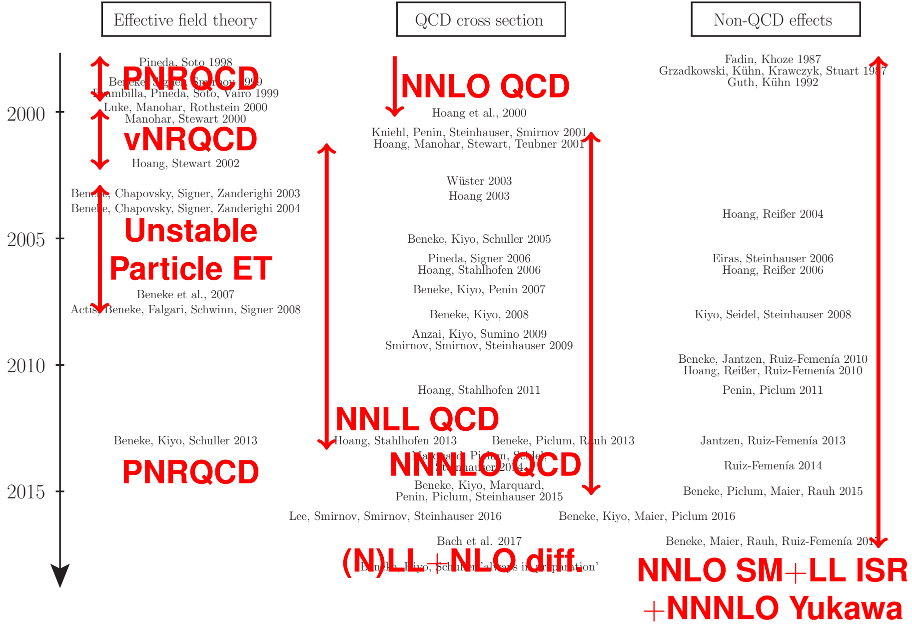
Hoang, Reißer, Ruiz-Femenía 2010
Penin, Piclum 2011
Jantzen, Ruiz-Femenía 2013
Ruiz-Femenía 2014
Beneke, Piclum, Maier, Rauh 2015
Beneke, Maier, Rauh, Ruiz-Femenía 2017

Work beyond NNLO QCD



2000

2005

2010

2015

Effective field theory

Pineda, Soto 1998
Beneke, Signer, Smirnov 1999
Brambilla, Pineda, Soto, Vairo 1999
Luke, Manohar, Rothstein 2000
Manohar, Stewart 2000
Hoang, Stewart 2002
Beneke, Chapovsky, Signer, Zanderighi 2003
Beneke, Chapovsky, Signer, Zanderighi 2004
Beneke et al., 2007
Actis, Beneke, Falgari, Schwinn, Signer 2008
Beneke, Kiyo, Schuller 2013

PNRQCD

vNRQCD

Unstable Particle ET

PNRQCD

QCD cross section

Hoang et al., 2000
Kniehl, Penin, Steinhauser, Smirnov 2001
Hoang, Manohar, Stewart, Teubner 2001
Wüster 2003
Hoang 2003
Beneke, Kiyo, Schuller 2005
Pineda, Signer 2006
Hoang, Stahlhofen 2006
Beneke, Kiyo, Penin 2007
Beneke, Kiyo, 2008
Anzai, Kiyo, Sumino 2009
Smirnov, Smirnov, Steinhauser 2009
Hoang, Stahlhofen 2011
Hoang, Stahlhofen 2013
Beneke, Piclum, Rauh 2013
Marquard, Piclum, Seidel,
Steinhauser 2014
Beneke, Kiyo, Marquard,
Penin, Piclum, Steinhauser 2015
Lee, Smirnov, Smirnov, Steinhauser 2016
Beneke, Kiyo, Maier, Piclum 2016
Bach et al. 2017
Beneke, Kiyo, Schuller 'always in preparation'

NNLO QCD

NNLL QCD

NNNLO QCD

(N)LL+NLO diff.

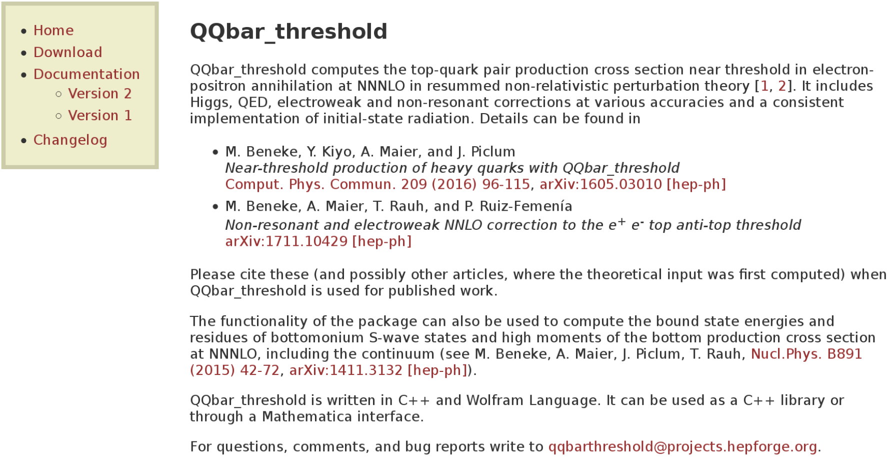
Non-QCD effects

Fadin, Khoze 1987
Grzadkowski, Kühn, Krawczyk, Stuart 1987
Guth, Kühn 1992
Hoang, Reißer 2004
Eiras, Steinhauser 2006
Hoang, Reißer 2006
Kiyo, Seidel, Steinhauser 2008
Beneke, Jantzen, Ruiz-Femenía 2010
Hoang, Reißer, Ruiz-Femenía 2010
Penin, Piclum 2011
Jantzen, Ruiz-Femenía 2013
Ruiz-Femenía 2014
Beneke, Piclum, Maier, Rauh 2015
Beneke, Maier, Rauh, Ruiz-Femenía 2017

NNLO SM+LL ISR
+NNNLO Yukawa

Public code

Implementation of NNNLO QCD + NNLO SM + LL ISR + NNNLO Yukawa results is available on HEPForge



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QQbar_threshold

QQbar_threshold computes the top-quark pair production cross section near threshold in electron-positron annihilation at NNNLO in resummed non-relativistic perturbation theory [1, 2]. It includes Higgs, QED, electroweak and non-resonant corrections at various accuracies and a consistent implementation of initial-state radiation. Details can be found in

- M. Beneke, Y. Kiyo, A. Maier, and J. Piclum
  Near-threshold production of heavy quarks with QQbar_threshold
  Comput. Phys. Commun. 209 (2016) 96-115, arXiv:1605.03010 [hep-ph]
- M. Beneke, A. Maier, T. Rauh, and P. Ruiz-Femenía
  Non-resonant and electroweak NNLO correction to the e^+ e^- top anti-top threshold
  arXiv:1711.10429 [hep-ph]

Please cite these (and possibly other articles, where the theoretical input was first computed) when QQbar_threshold is used for published work.

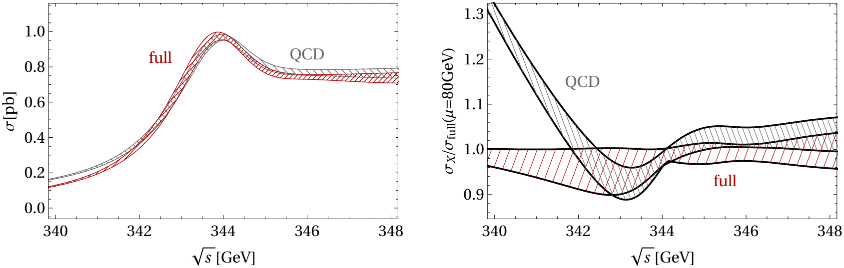
The functionality of the package can also be used to compute the bound state energies and residues of bottomonium S-wave states and high moments of the bottom production cross section at NNNLO, including the continuum (see M. Beneke, A. Maier, J. Piclum, T. Rauh, Nucl.Phys. B891 (2015) 42-72, arXiv:1411.3132 [hep-ph]).

QQbar_threshold is written in C++ and Wolfram Language. It can be used as a C++ library or through a Mathematica interface.

For questions, comments, and bug reports write to qqbarthreshold@projects.hepforge.org.

Non-QCD effects

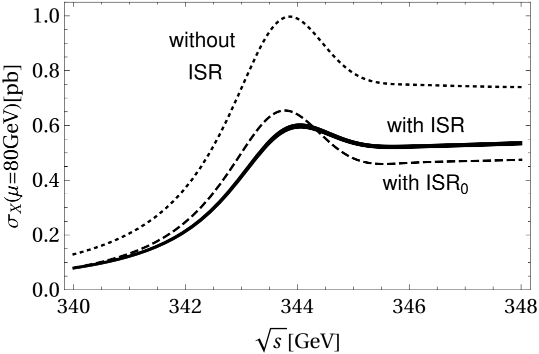
NNLO SM and NNNLO Yukawa contributions



- Uncertainty due to renormalization scale variation between 50 GeV and 350 GeV
- Effects significantly larger than QCD uncertainty
- Shape changes particularly in the important region at and below threshold

Non-QCD effects

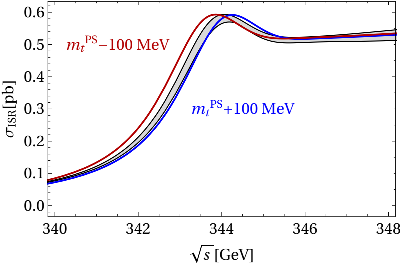
Initial state radiation

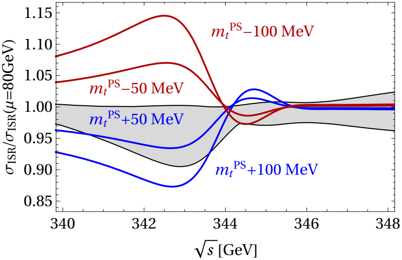


- ISR reduces cross section by 30-45 %
- Band is envelope of different LL accurate implementations
- NLL precision is a must for a lepton collider (not just for ttbar)

Determination of SM parameters

Top-quark mass

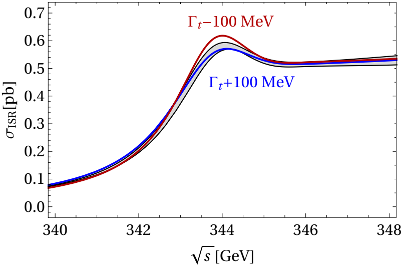


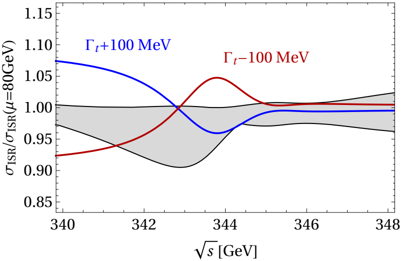


- Estimate theory uncertainty by determining what parameter shift is needed to obtain curves outside the scale variation band
- Naive expectation: $\delta m_t^{\mathrm{PS}} \approx 40\,\mathrm{MeV}$
- Full simulation: [Simon 2016]
  - theory uncertainty: $\delta m_t^{\mathrm{PS}} \approx 40\,\mathrm{MeV}$
  - Statistical uncertainty (CLIC): $\delta m_t^{\mathrm{PS}} = 21\,\mathrm{MeV}$

Determination of SM parameters

Top-quark width

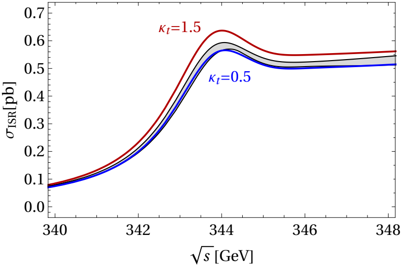


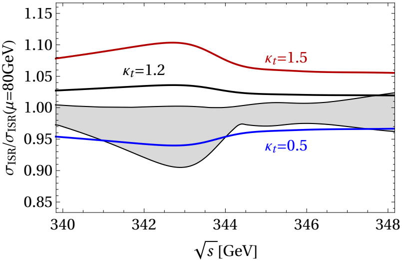


- Estimate theory uncertainty by determining what parameter shift is needed to obtain curves outside the scale variation band
- Naive expectation: $\delta\Gamma_t \approx 60\,\text{MeV}$

Determination of SM parameters

Top-quark Yukawa coupling

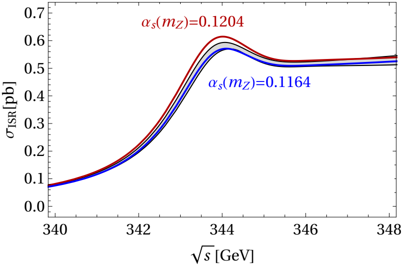


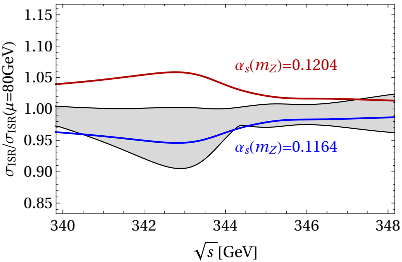


- Estimate theory uncertainty by determining what parameter shift is needed to obtain curves outside the scale variation band
- Naive expectation: $\delta\kappa_t \approx^{+20}_{-25}$ %

Determination of SM parameters

Strong coupling





- Estimate theory uncertainty by determining what parameter shift is needed to obtain curves outside the scale variation band
- Naive expectation: $\delta\alpha_s \approx 0.0015$

Summary

- Determination of several SM parameters possible from scan of the total $e^+e^- \to W^+W^-\bar{b}bX$ cross section near the top threshold
- NNNLO QCD + NNLO SM + LL ISR + NNNLO Yukawa prediction known and available in QQbar_Threshold
- Theoretical uncertainty of 2-5% (energy-dependent)
- Expected theoretical uncertainties for parameters:

$$\delta m_t^{\text{PS}} \approx 40\,\text{MeV} \qquad \delta\Gamma_t \approx 60\,\text{MeV}$$

$$\delta\kappa_t \approx^{+20}_{-25}\,\% \qquad \delta\alpha_s \approx 0.0015$$

Power counting

$$\alpha_{\rm EW} \sim \alpha_t \equiv \frac{\lambda_t^2}{4\pi} \sim \alpha_s^2 \sim v^2,$$

$$\sigma_{\rm QCD\,only} \sim \alpha_{\rm EW}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} 1 & \text{LO} \\ \alpha_s, v & \text{NLO} \\ \alpha_s^2, \alpha_s v, v^2 & \text{NNLO} \\ \alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3 & \text{NNNLO} \end{cases},$$

$$\sigma \sim \alpha_{\rm EW}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} \dfrac{\alpha_{\rm em}}{v} & \text{NLO} \\ \left(\dfrac{\alpha_{\rm em}}{v}\right)^2, \dfrac{\alpha_{\rm em}}{v} \times \{\alpha_s, v\}, \alpha_{\rm EW}, \sqrt{\alpha_{\rm EW}\alpha_t}, \alpha_t & \text{NNLO} \\ \left(\dfrac{\alpha_{\rm em}}{v}\right)^3, \left(\dfrac{\alpha_{\rm em}}{v}\right)^2 \times \{\alpha_s, v\}, \dfrac{\alpha_{\rm em}}{v} \times \{\alpha_s^2, \alpha_s v, v^2, \sqrt{\alpha_{\rm EW}\alpha_t}\}, & \\ \alpha_t \times \{\dfrac{\alpha_{\rm em}}{v}, \alpha_s, v\}, \ldots & \text{NNNLO} \end{cases}$$

$$+ \alpha_{\rm EW}^2 \times \begin{cases} \alpha_{\rm EW} & \text{NLO} \\ \alpha_{\rm EW}\alpha_s & \text{NNLO} \\ \ldots & \text{NNNLO} \end{cases},$$

Organization of the calculation

Split cross section into three separately finite parts (I), (II) and (III):

$$\sigma^{\text{NNLO}} = \underbrace{\left[\sigma_{\text{sq}} + \sigma_{\text{res, rest}}\right]}_{\text{(I)}} + \underbrace{\left[\sigma_{\text{int}}^{\text{(EP div)}} + \sigma_{C^{(k)}_{\text{Abs,bare}}}\right]}_{\text{(II)}} + \underbrace{\left[\sigma_{\text{int}}^{\text{(EP fin)}} + \sigma_{\text{aut}}\right]}_{\text{(III)}}.$$

- (I): computational scheme for ’squared contribution’ fixed by existing QCD results (Dim reg with NDR for γ^5)
- (II): Use freedom of scheme choice to simplify calculation (some parts done in four dimensions)
- (III): Endpoint finite part of ’interference contribution’ must be computed consistent with MadGraph

Divergence structure

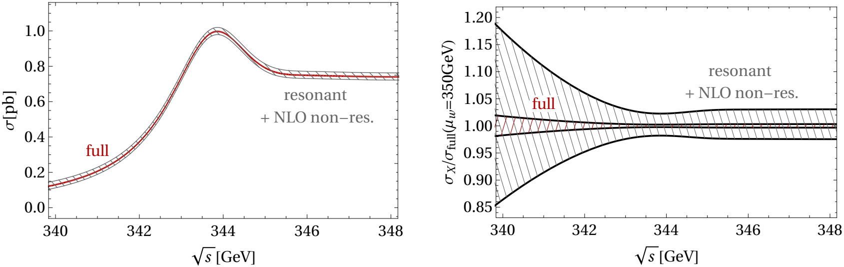
			UV finite	IR finite	EP finite
(I)			✓	✓	✓
	σ_{sq}		✓	✓	–
		$\sigma_{\text{sq}}^{(h_{1a},\dots,h_{1g})}$	✓	–	–
		$\sigma_{\text{sq}}^{(g_1,\dots,g_6)}$	✓	–	$\star$
	$\sigma_{\text{res, rest}}$		✓	✓	–
		σ_{QCD}	✓	✓	–
		$\sigma_{\text{P-wave}}$	✓	✓	–
		σ_H	✓	✓	✓
		$\sigma_{\delta V_{\text{QED}}}$	✓	✓	✓
		σ_Γ	✓	✓	–
		$\sigma_{C_{\text{EW}}^{(k)}}$	✓	✓	✓
		$\sigma_{C^{(k)}_{\text{Abs},Z_t}}$	✓	✓	–
		$\sigma_{\text{IS}}^{\text{conv}}$	✓	✓	✓
(II)			✓	✓	✓
	$\sigma_{\text{int}}^{(\text{EP div})}$		✓	✓	–
	$\sigma_{C^{(k)}_{\text{Abs,bare}}}$		✓	✓	–
(III)			✓	✓	✓
	$\sigma_{\text{int}}^{(\text{EP fin})}$		–	✓	✓
	σ_{aut}		–	✓	✓

Dependence on μ_w scale

Regularizing width-related/endpoint divergences dimensionally splits some of the large logarithms by introducing the scale μ_w

$$\sigma_{\text{full}} \supset \ln v = \underbrace{\ln \frac{\mu_w}{m_t}}_{\subset \sigma_{\text{non-res}}} + \underbrace{\ln \frac{m_t v}{\mu_w}}_{\subset \sigma_{\text{res}}} .$$

The dependence on μ_w cancels exactly at a given order.



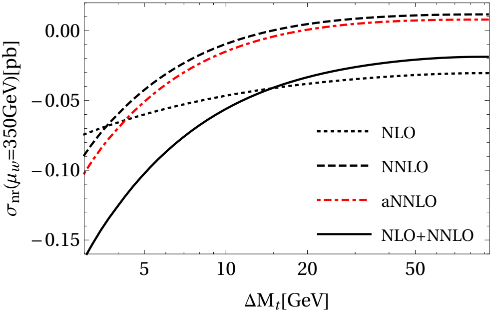
We choose a central scale of $\mu_w = 350\,\text{GeV}$ to minimize the unknown logarithms from the NNNLO non-resonant part.

Invariant mass cut

Consider "loose" invariant mass cuts

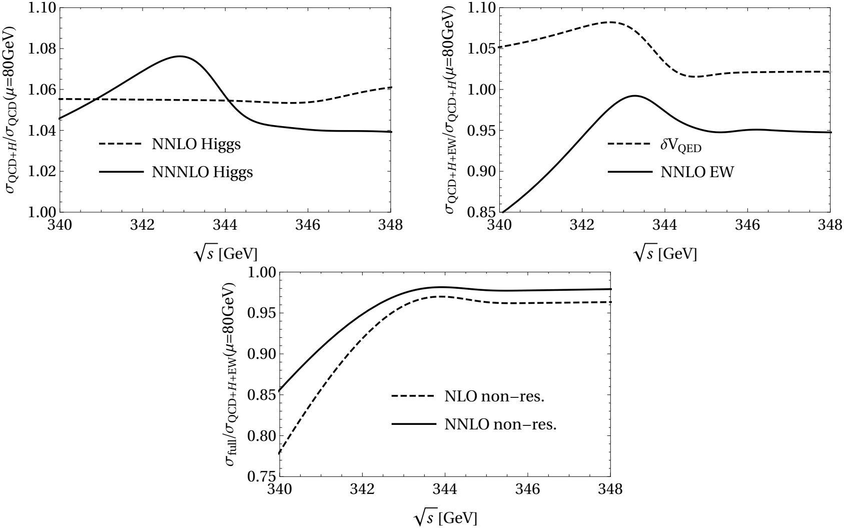
$$(m_t - \Delta M_t)^2 \leq p_{t,\bar{t}}^2 \leq (m_t + \Delta M_t)^2,$$

with $\Delta M_t \gg \Gamma_t$. Since the off-shellness in the resonant part is parametrically of the order Γ_t they only affect the non-resonant part:



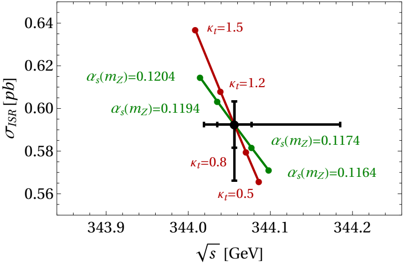
Non-QCD effects

Individual contributions



Determination of SM parameters

Correlation between top Yukawa and strong coupling



Peak height and width

- Estimate theory uncertainty by determining what parameter shift is needed to obtain curves outside the scale variation band
- Naive expectation: $\delta\kappa_t \approx^{+20}_{-25}$ % and $\delta\alpha_s \approx 0.0015$
- Effects from variation of Yukawa coupling and strong coupling very similar
- Need full simulation to see how well they can be disentangled