Differential Top-Antitop Production at and beyond Threshold

[arXiv:1712.02220]

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and the WHIZARD team:
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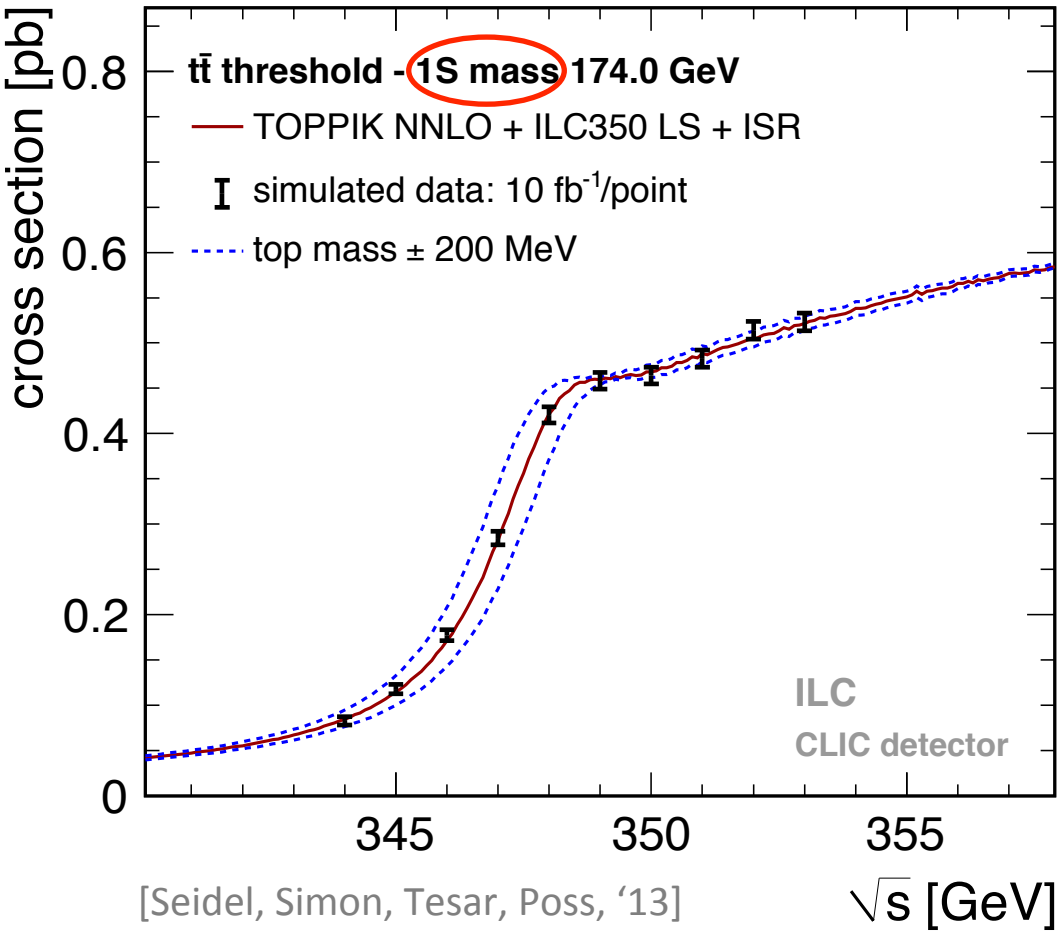
Outline

- Top-antitop threshold scan
- Theory: (v)NRQCD
- Total cross section
- Top-antitop threshold in WHIZARD 
- Summary

Top-antitop threshold scan



[Seidel, Simon, Tesar, Poss, '13]

Experiment (simulation):

$\Delta m_t < 100\,\text{MeV}$ renormalon free!

$\Delta \Gamma_t \sim 30\,\text{MeV}$

$\Delta \alpha_s \sim 0.001$

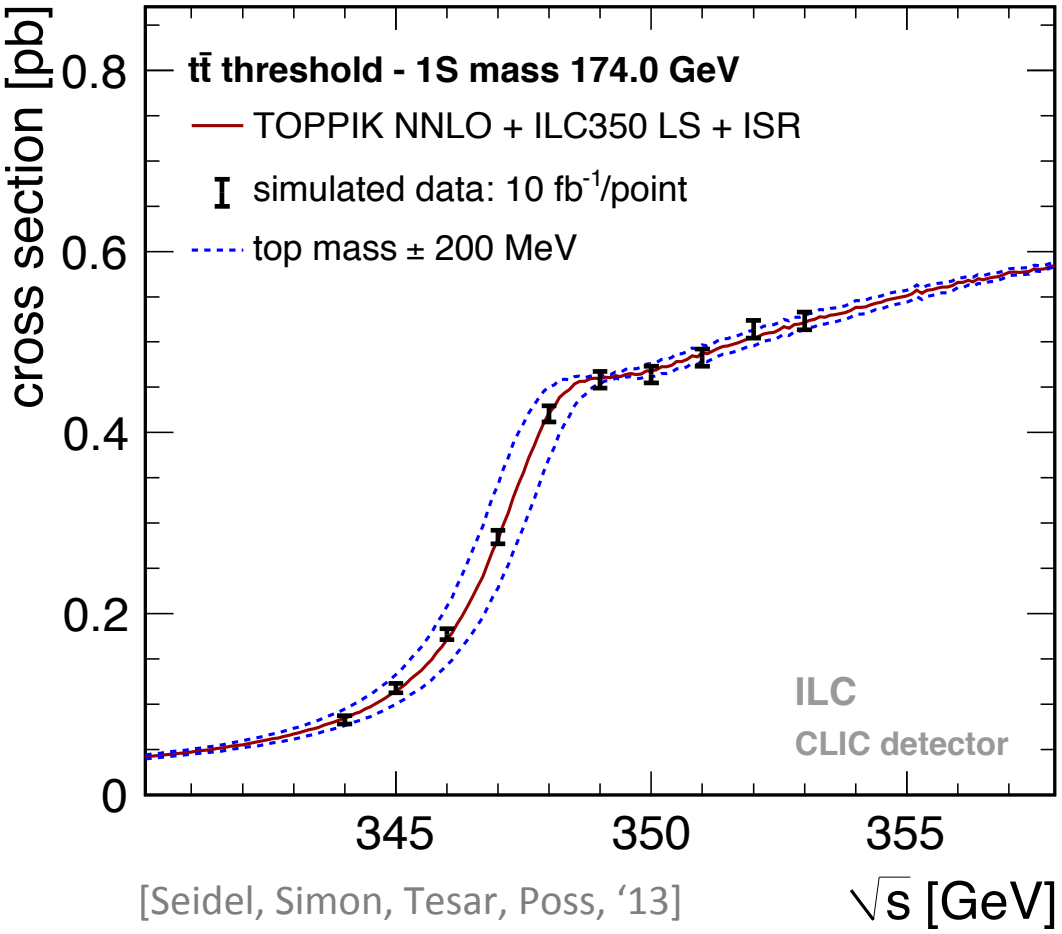
$\Delta y_t / y_t \sim 10\%$

[Martinez, Miquel, '02]
[Seidel, Simon, Tesar, Poss '13]
[Horiguchi et. al. '13]

Theory goal:

$$\Delta\sigma_{\text{tot}}/\sigma_{\text{tot}} \lesssim 3\%$$

Top-antitop threshold scan



[Seidel, Simon, Tesar, Poss, '13]

$$\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{QCD}$$

- Nonpert. effects suppressed
  [Fadin, Khoze, '87]
- No sharp resonance peaks

$$v \sim 0.1 \ll 1$$

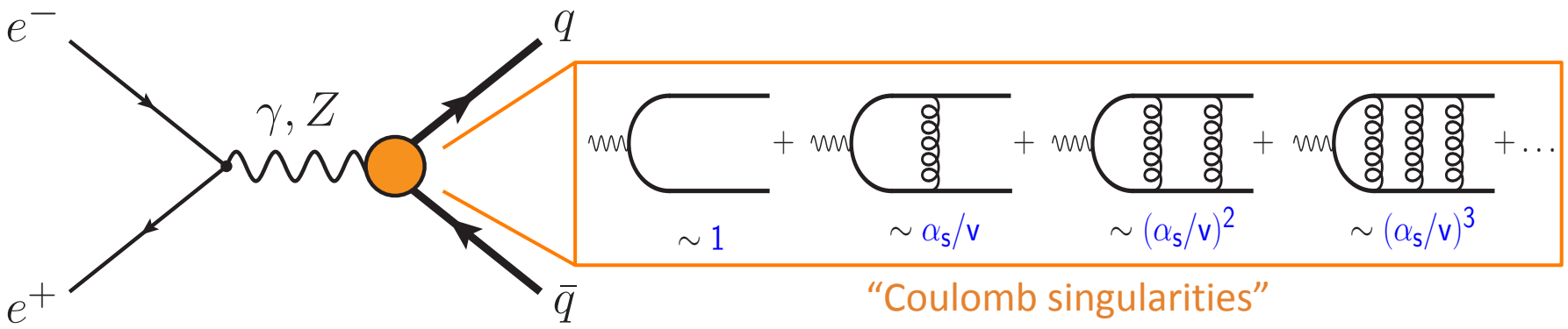
- Nonrelativistic regime
- Multiple scales

Theory: (v)NRQCD

QCD near $t\bar{t}$ threshold: $v \sim \alpha_s \ll 1$ "nonrelativistic bound state"

scales: $m \gg \vec{p} \sim mv \gg E_{kin} \sim mv^2$ $(\sim \Gamma_t \gg \Lambda_{QCD})$

hard soft ultrasoft



$$\sigma \sim \sum_k \left(\frac{\alpha_s}{v}\right)^k \times \left\{1\,(\mathrm{LO});\ \alpha_s, v\,(\mathrm{NLO});\ \alpha_s^2,\ \alpha_s v,\ v^2\,(\mathrm{NNLO});\ \ldots\right\}$$

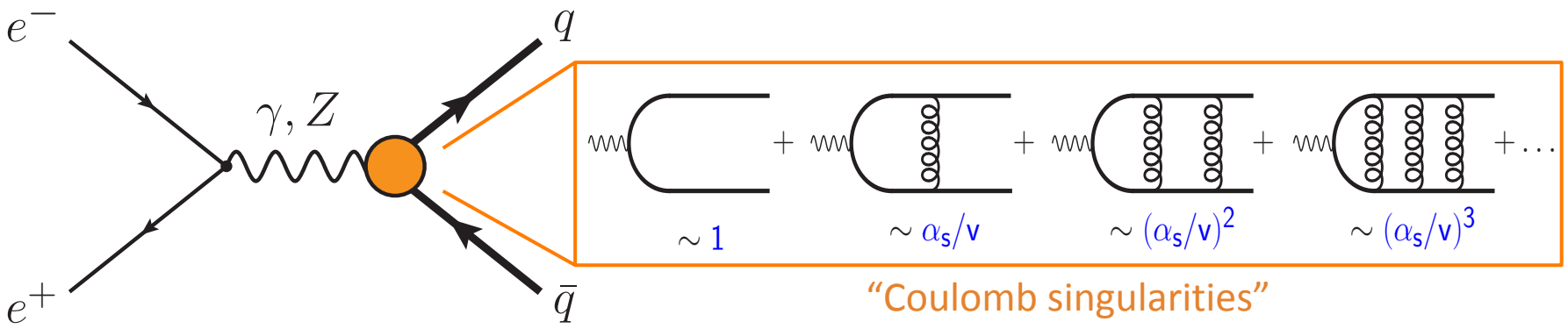
resummation using Schrödinger equation in NRQCD

Theory: (v)NRQCD

QCD near $t\bar{t}$ threshold: $v \sim \alpha_s \ll 1$ "nonrelativistic bound state"

scales: $m \gg \vec{p} \sim mv \gg E_{kin} \sim mv^2$ $(\sim \Gamma_t \gg \Lambda_{QCD})$

hard soft ultrasoft



$$\sigma \sim \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times \left\{1\,(\mathrm{LL});\ \alpha_s, v\,(\mathrm{NLL});\ \alpha_s^2, \alpha_s v, v^2\,(\mathrm{NNLL}); \ldots\right\}$$

resummation of large logs using RG in **vNRQCD**

Theory: (v)NRQCD

vNRQCD

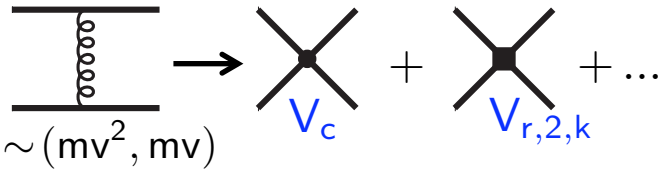
[Luke, Manohar, Rothstein, '00]

nonrel. quark: $k^\mu \sim (mv^2, mv)$

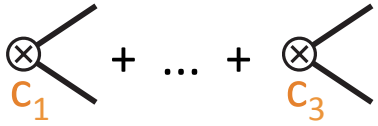
soft gluon: $k^\mu \sim mv$

ultrasoft gluon: $k^\mu \sim mv^2$

potentials:



effective currents:



S-wave $\sim v^0$ P-wave $\sim v$

power counting in $v \sim \alpha_s$

Theory: (v)NRQCD

vNRQCD

[Luke, Manohar, Rothstein, '00]

nonrel. quark: $k^\mu \sim (mv^2, mv)$

soft gluon: $k^\mu \sim mv$

ultrasoft gluon: $k^\mu \sim mv^2$

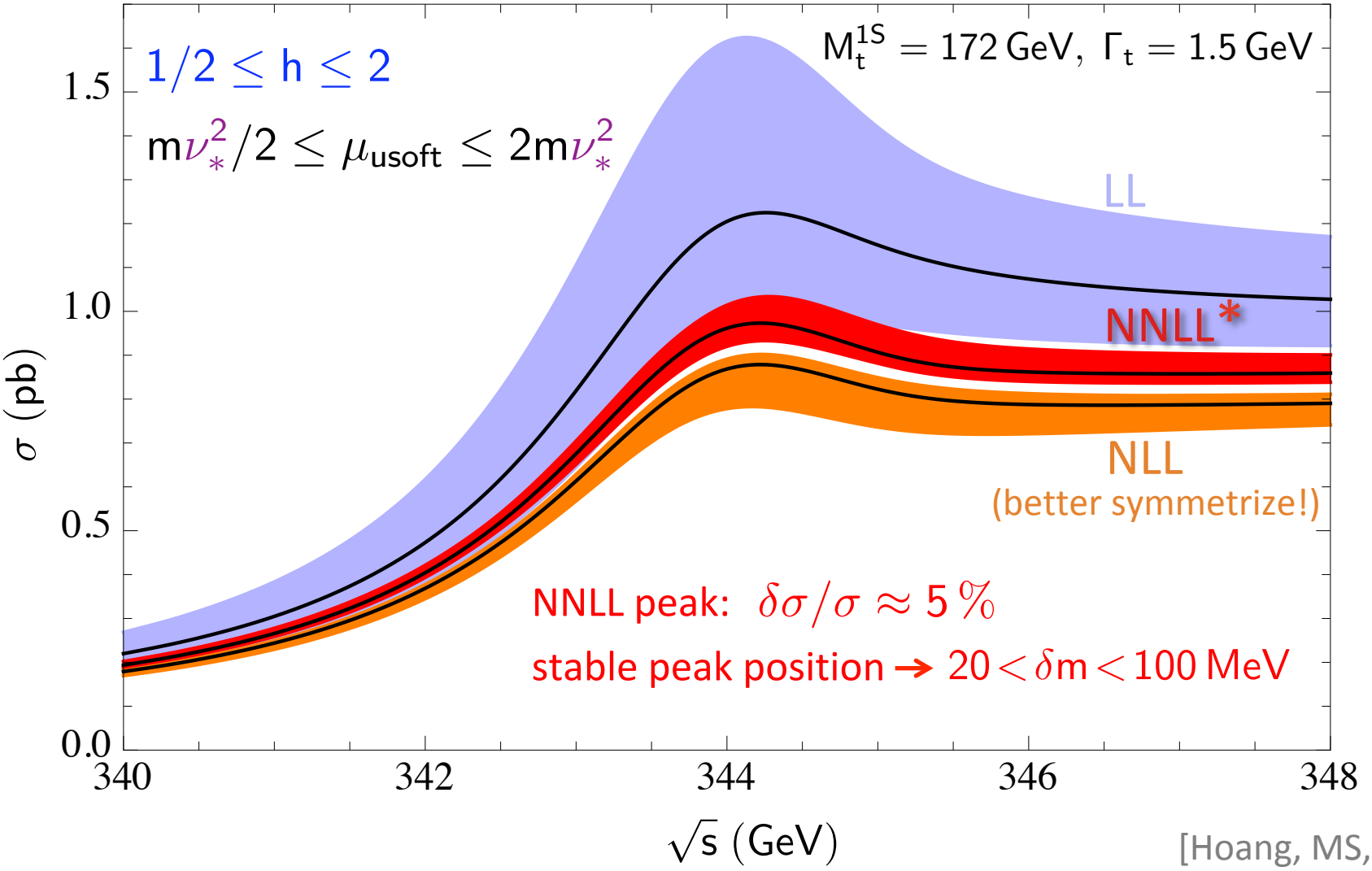
potentials: $k^\mu \sim (mv^2, mv)$ → V_c + $V_{r,2,k}$ + ...

effective currents: c_1 + ... + c_3 + ...

power counting in $v \sim \alpha_s$

$\sigma_{tot} \sim \mathrm{Im}$ [c_1 + V_c + ... + $V_{r,2,k}$ + ...]

Total cross section

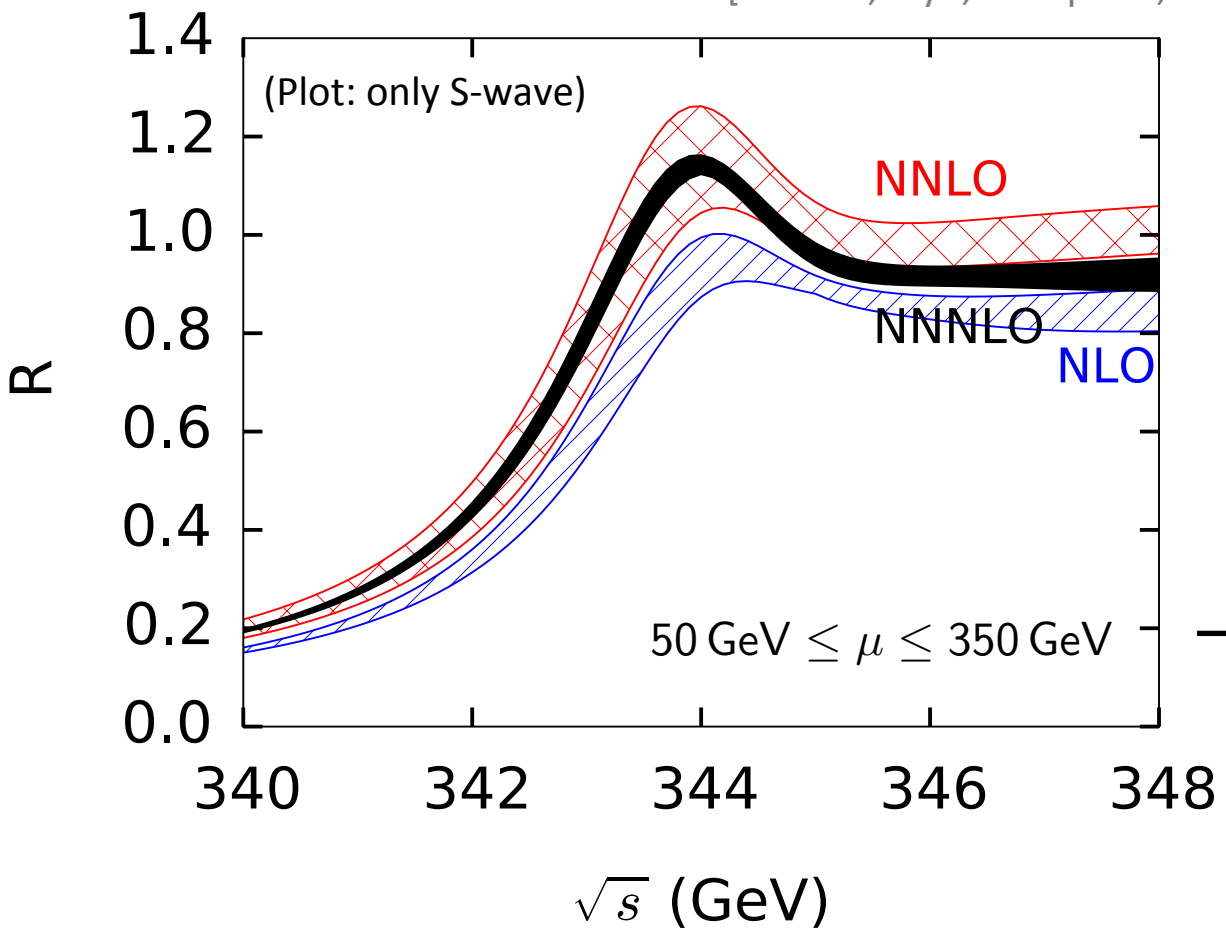


[Hoang, MS, '13]

Total cross section

"Fixed order" state of the art:

[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser, '15]



N³LO peak: $\delta\sigma/\sigma \approx 3\,\%$

peak position →

$\delta \mathrm{m} < 50\,\mathrm{MeV}$

⟶ talk by Thomas Rauh

To do: Combine N³LO + NNLL ⟶ W.I.P.

Top-antitop threshold in WHIZARD

[arXiv:1712.02220]

Threshold resummation in Monte Carlo event generator:

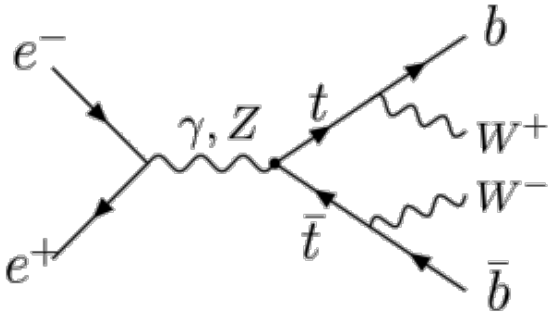
- ✓ Realistic final state: $W^+W^-b\bar{b}$
- ✓ Study arbitrary differential observables/ experimental cuts
- ✓ Include (interferences with) background
- ✓ Smooth transition between threshold and continuum region
- ✓ QED ISR, beam structure, polarization effects

Top-antitop threshold in WHIZARD

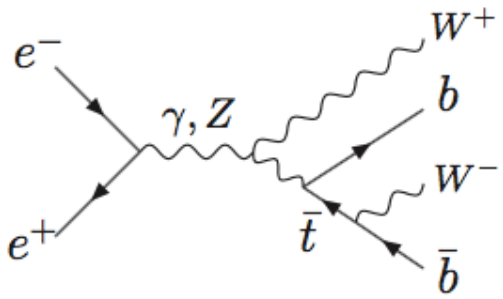
Resolve realistic final state: $W^+W^-b\bar{b}$

$$v \sim \alpha_s \sim \sqrt{\alpha_{em}}$$

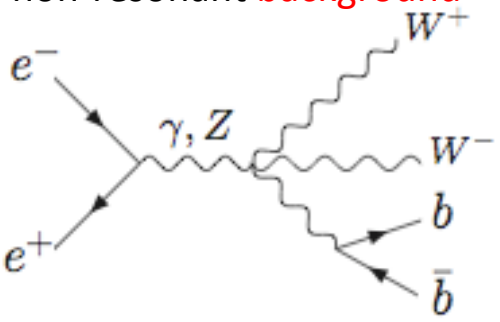
double-resonant "signal"



single-resonant background



non-resonant background



Cross section:

$\mathcal{O}(v)$ → LO

$\mathcal{O}(\alpha_{em})$ → NLO

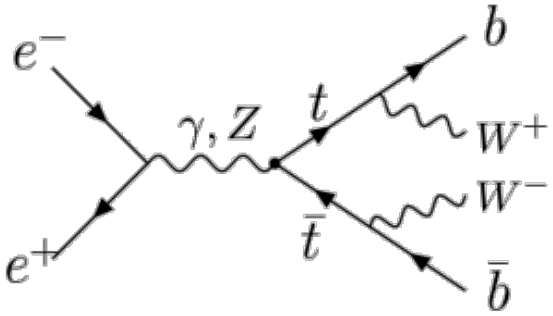
$\mathcal{O}(\alpha_{em}^2)$ → N³LO

Top-antitop threshold in WHIZARD

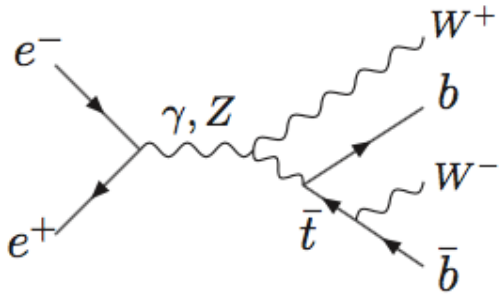
Resolve realistic final state: $W^+W^-b\bar{b}$

$$v \sim \alpha_s \sim \sqrt{\alpha_{em}}$$

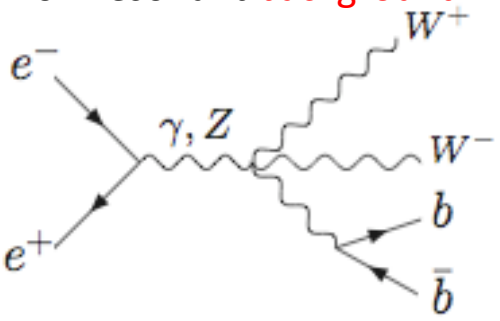
double-resonant "signal"



single-resonant background



non-resonant background



Cross section:

$\mathcal{O}(v)$ → LO

$\mathcal{O}(\alpha_{em})$ → NLO

$\mathcal{O}(\alpha_{em}^2)$ → N³LO

NLO QCD corrections (virtual+real):

$\mathcal{O}(v\alpha_s)$ → NLO

$\mathcal{O}(\alpha_{em}\alpha_s)$ → NNLO

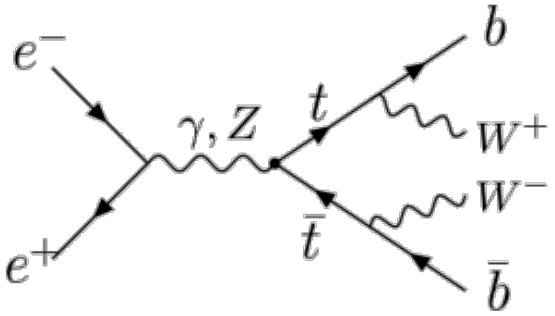
$\mathcal{O}(\alpha_{em}^2\alpha_s)$ → N⁴LO

Top-antitop threshold in WHIZARD

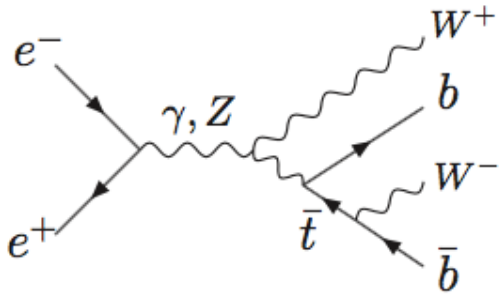
Resolve realistic final state: $W^+W^-b\bar{b}$

$$v \sim \alpha_s \sim \sqrt{\alpha_{em}}$$

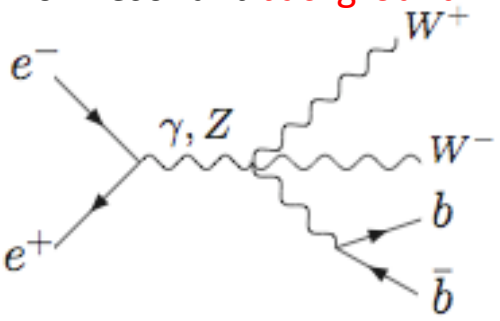
double-resonant "signal"



single-resonant background



non-resonant background



Cross section:

$\mathcal{O}(v) \rightarrow$ LO

$\mathcal{O}(\alpha_{em}) \rightarrow$ NLO

$\mathcal{O}(\alpha_{em}^2) \rightarrow$ N^{3}LO

NLO QCD corrections (virtual+real):

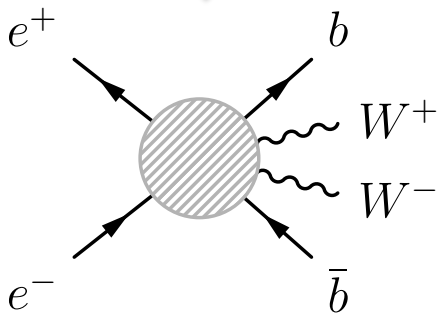
$\mathcal{O}(v\alpha_s) \rightarrow$ NLO

$\mathcal{O}(\alpha_{em}\alpha_s) \rightarrow$ NNLO

$\mathcal{O}(\alpha_{em}^2\alpha_s) \rightarrow$ N^{4}LO

WHIZARD MC generator:

[Choukoufé Nejad, Kilian, Lindert, Pozzorini, Reuter, Weiss, '16; Kilian, Ohl, Reuter, '11]

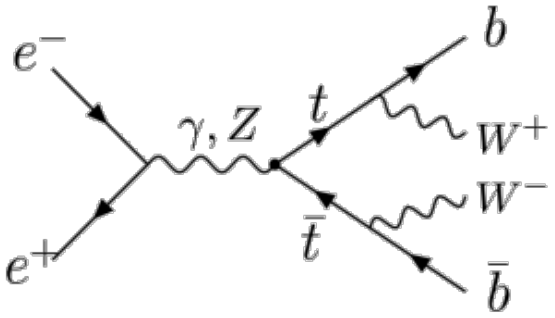


$+\mathcal{O}(\alpha_s)$

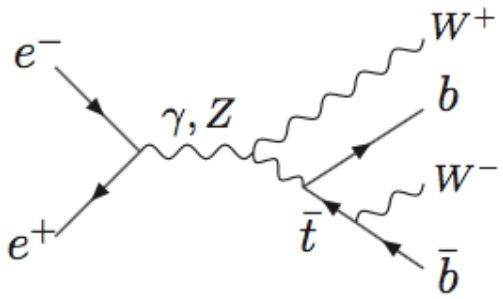
Top-antitop threshold in WHIZARD

Resolve realistic final state: $W^+W^-b\bar{b}$

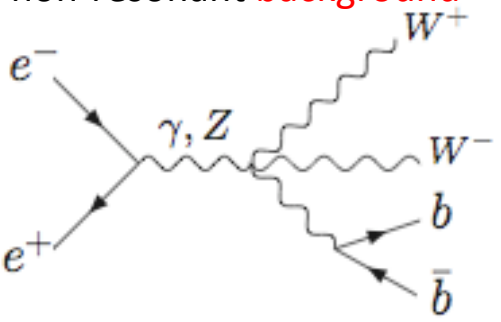
double-resonant "signal"



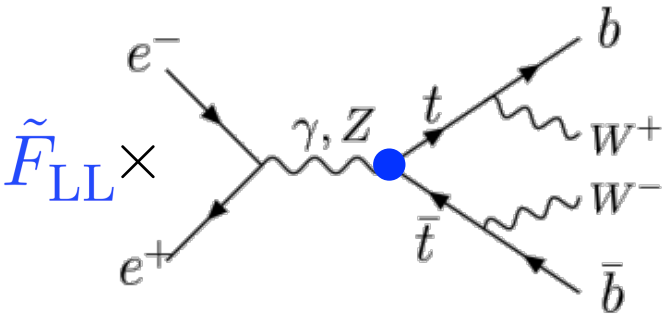
single-resonant background



non-resonant background



Idea: add threshold resummation via form factor:



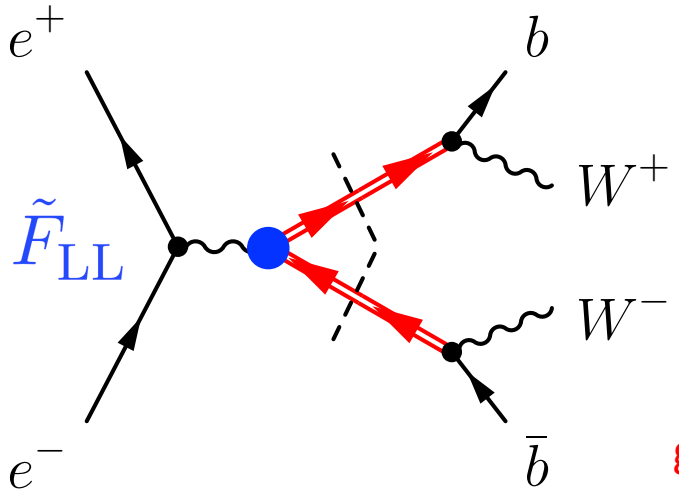
$$\tilde{F}_{\mathrm{LL}} = \left[c_{1,3}(\nu)\, G^{S,P}(0, p_t, E+i\Gamma_t, \nu) - 1\right]$$

(N)LL S/P-wave G-function from TOPPIK code [Jezabek, Teubner]

avoids double counting

Top-antitop threshold in WHIZARD

Ensure gauge invariance: Double Pole Approximation



gauge invariant onshell MEs

$$\mathcal{M}_{\mathrm{fact}} = \tilde{F}_{\mathrm{LL}} \sum_{h_t, h_{\bar{t}}} \frac{i}{p_t^2 - m^2 - im\Gamma_t} \frac{i}{p_{\bar{t}}^2 - m^2 - im\Gamma_t} \mathcal{M}^{h_t, h_{\bar{t}}}_{\mathrm{prod}}(\hat{p}_t, \hat{p}_{\bar{t}}) \mathcal{M}^{h_t}_{\mathrm{dec},t}(\hat{p}_t) \mathcal{M}^{h_{\bar{t}}}_{\mathrm{dec},\bar{t}}(\hat{p}_{\bar{t}})$$

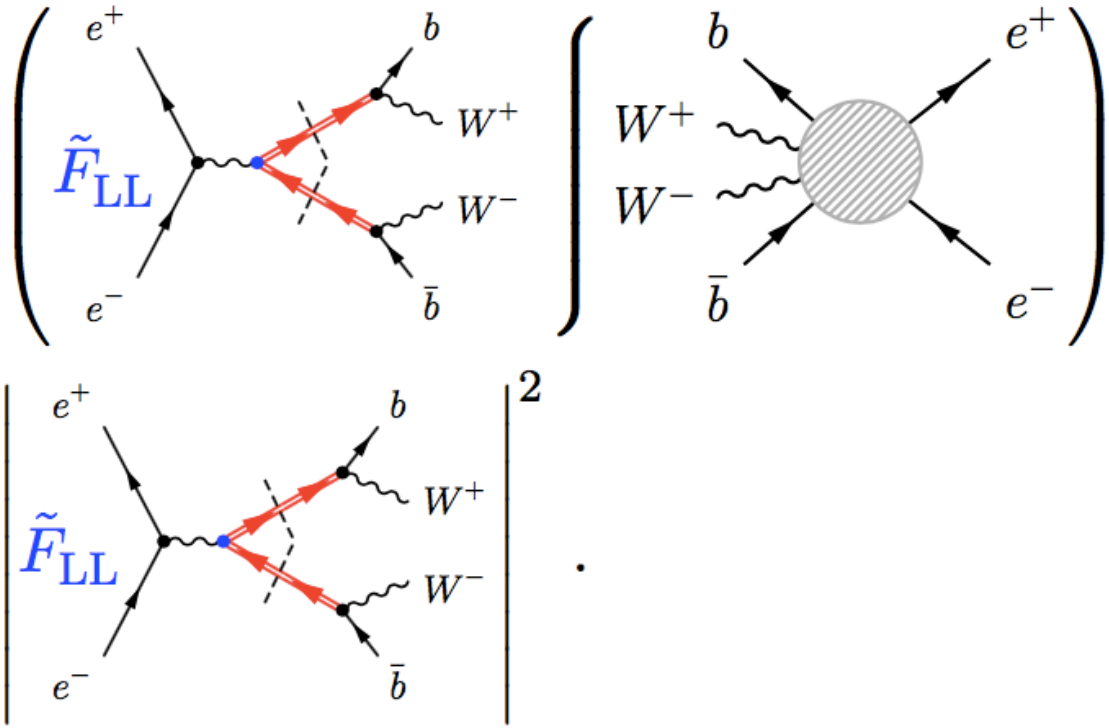
- For $\sqrt{s} \geq 2m$ use onshell projection: $\hat{p}_t^2 = \hat{p}_{\bar{t}}^2 = m^2$
- For $\sqrt{s} < 2m$ use momenta $\hat{p}_t$, $\hat{p}_{\bar{t}}$ as if $\sqrt{s} = 2m$
- Directions of original 3-momenta retained!

Top-antitop threshold in WHIZARD

Combine LO + LL:

$$\left[\tilde{F}_{\mathrm{LL}} \equiv F_{\mathrm{LL}} - 1\right]$$

$$\sigma_{\mathrm{LO+LL}} = \sigma_{\mathrm{LO}} + \left(\tilde{F}_{\mathrm{LL}} \right) \left| \right|^2 + \left| \tilde{F}_{\mathrm{LL}} \right|^2 .$$

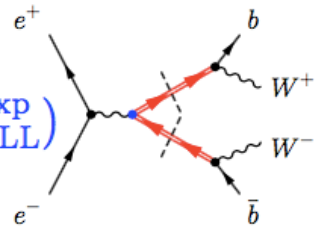


Top-antitop threshold in WHIZARD

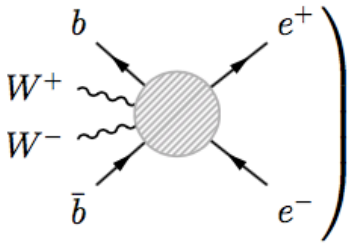
Combine NLO + NLL:

$$\left[\tilde{F}_{\mathrm{NLL}} \equiv F_{\mathrm{NLL}} - 1\right]$$

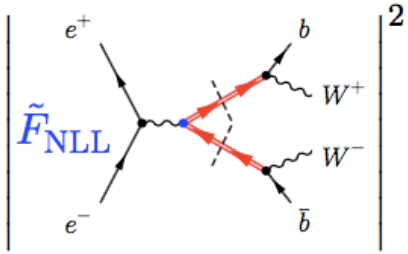
$$\sigma_{\mathrm{NLO+NLL}} = \sigma_{\mathrm{NLO}} + \left((F_{\mathrm{NLL}} - F_{\mathrm{NLL}}^{\mathrm{exp}}) \right.$$



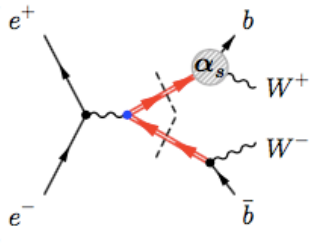
$$\left. \right) \left|\mathcal{M}_{\mathrm{NLO}}\right|^2 +$$



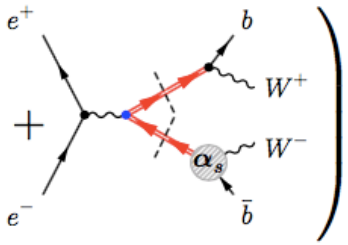
$$+ \left| \tilde{F}_{\mathrm{NLL}} \right.$$



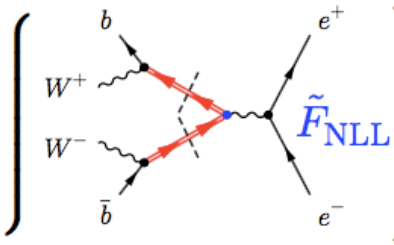
$$\left. \right|^2 + \left(\tilde{F}_{\mathrm{NLL}} \left(\right. \right.$$



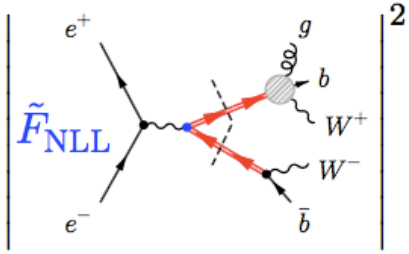
$$+$$



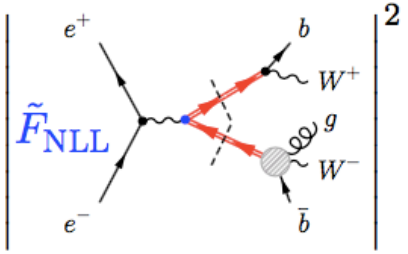
$$\left. \right) \left(\right.$$



$$\left. \right) + \left| \tilde{F}_{\mathrm{NLL}} \right.$$



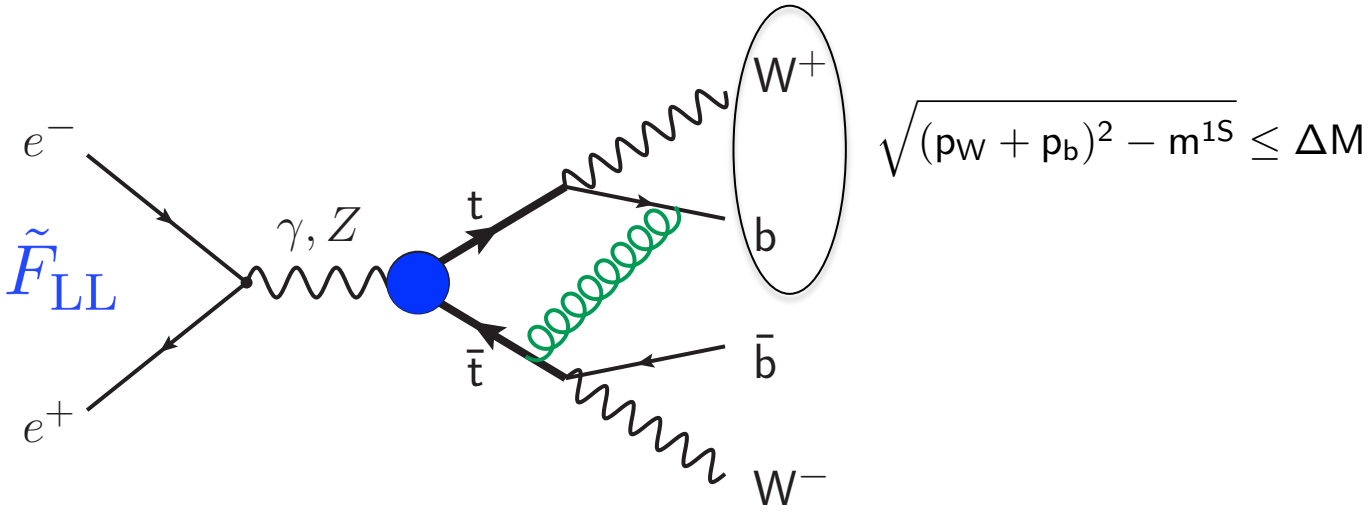
$$\left. \right|^2 + \left| \tilde{F}_{\mathrm{NLL}} \right.$$



$$\left. \right|^2 .$$

Top-antitop threshold in WHIZARD

Caveat: Omitting some NLL ultrasoft final state interactions, e.g.



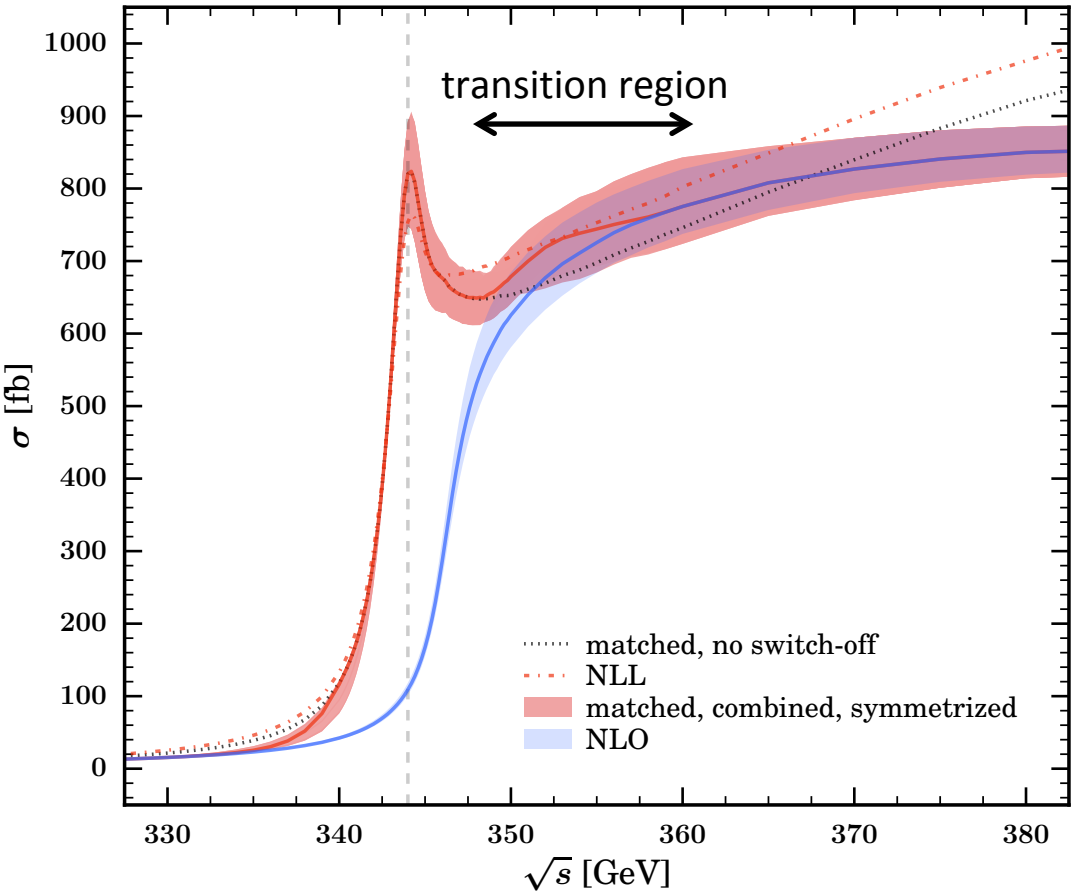
- Simple form factor implementation not enough for full NLL
- Missing NLL contributions $\sim \alpha_s \Gamma_t / \Delta M$

  suppressed for sufficiently inclusive observables!

  [Fadin, Khoze, Martin '94] [Melnikov, Yakovlev '94]
  [Hoang, Reisser, Ruiz-Femenia, '10]
- For arbitrary observables at least NLO + LL precision

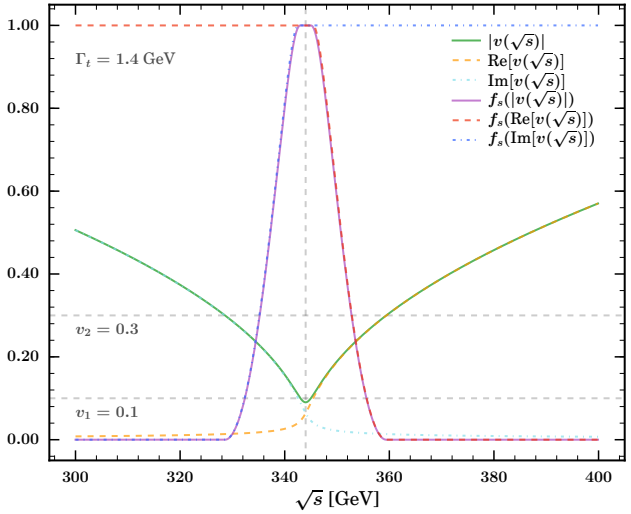
Top-antitop threshold in WHIZARD

Matching NLO+NLL with relativistic NLO continuum:



- Usual scale variations
- Symmetrize
- Variation of switch-off parameters $0.1 \leq v_1 < v_2 \leq 0.4$
- Take envelope

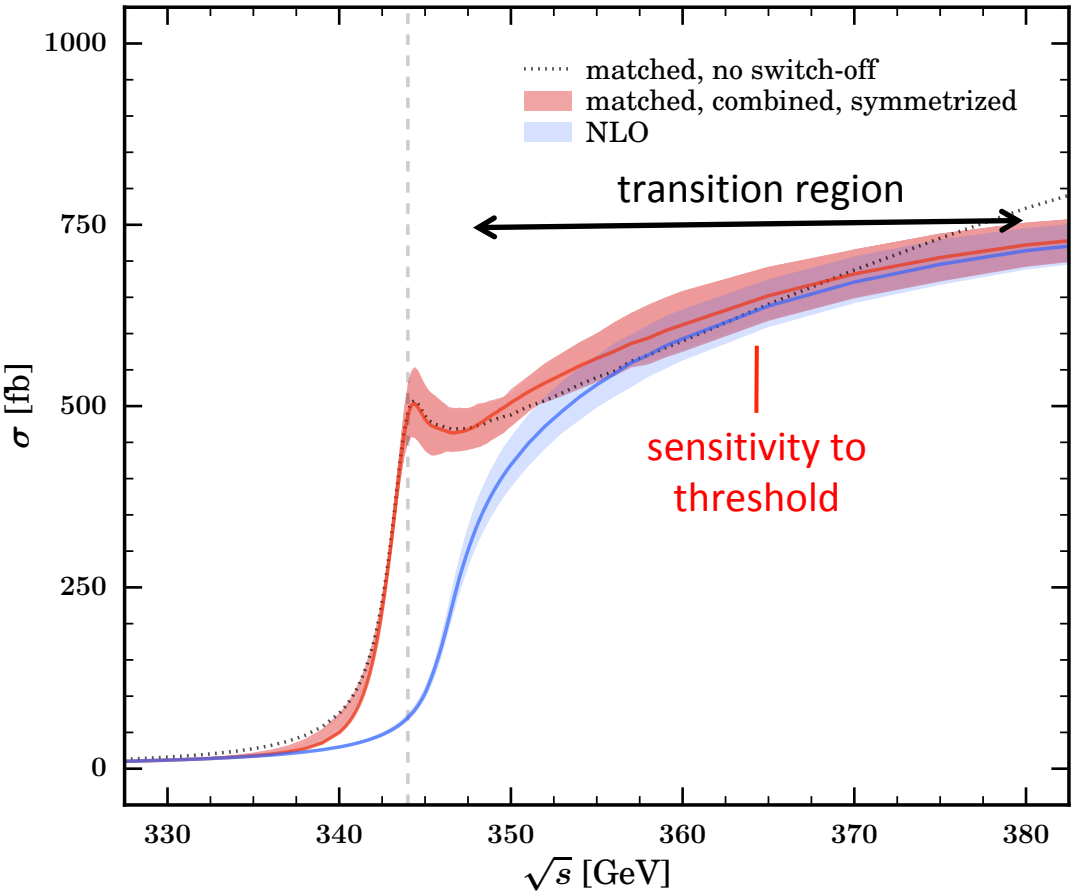
switch-off function f_s:



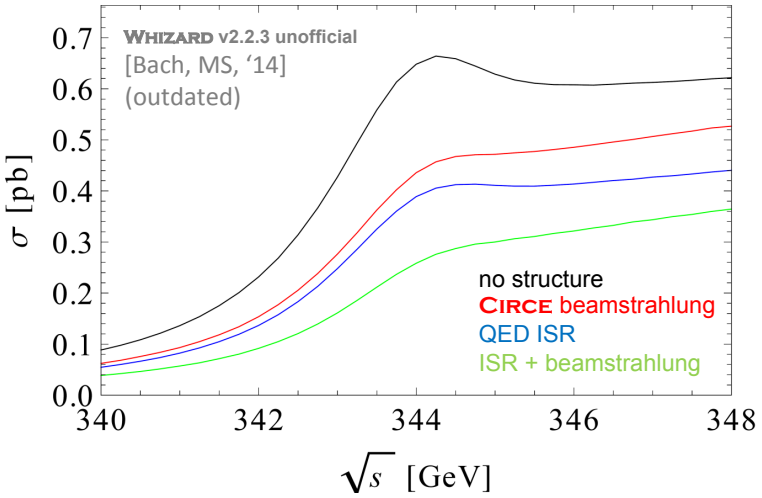
$$\sigma_{\text{matched}} = \sigma_{\text{NLO}}[\alpha_{\text{H}}] + \sigma_{\text{resum}}[f_s\alpha_{\text{H}}, f_s\alpha_{\text{S}}, f_s\alpha_{\text{US}}] - \sigma^{\text{expand}}_{\text{resum}}[f_s\alpha_{\text{H}}]$$

Top-antitop threshold in WHIZARD

Including QED ISR via convolution with structure function:



Further beam effects straightforward:

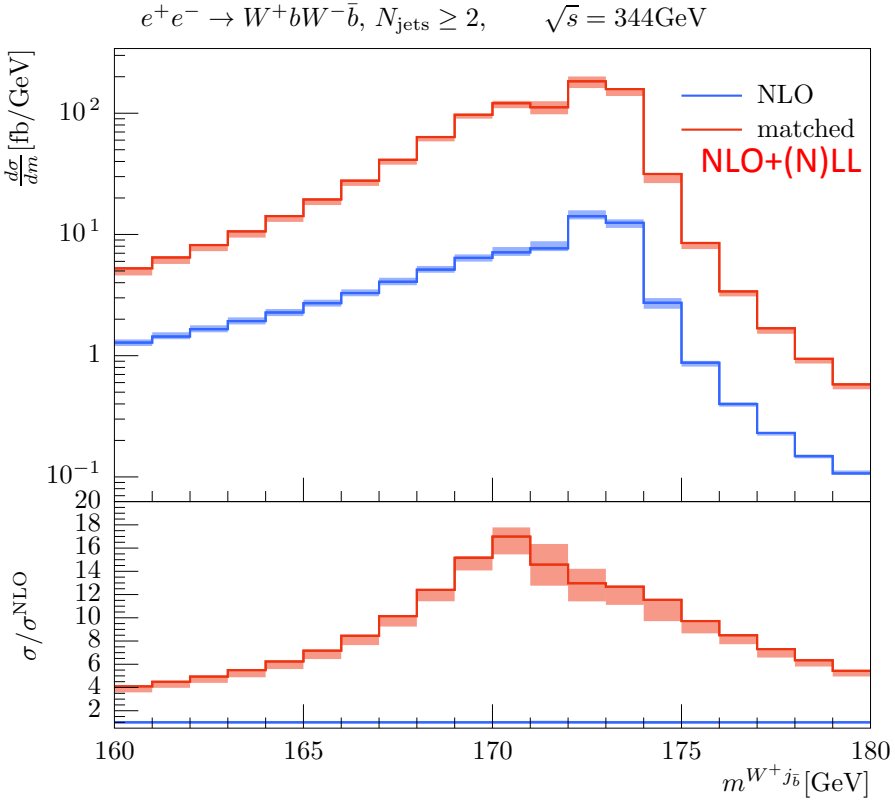


... polarization of colliding leptons can also be taken into account.

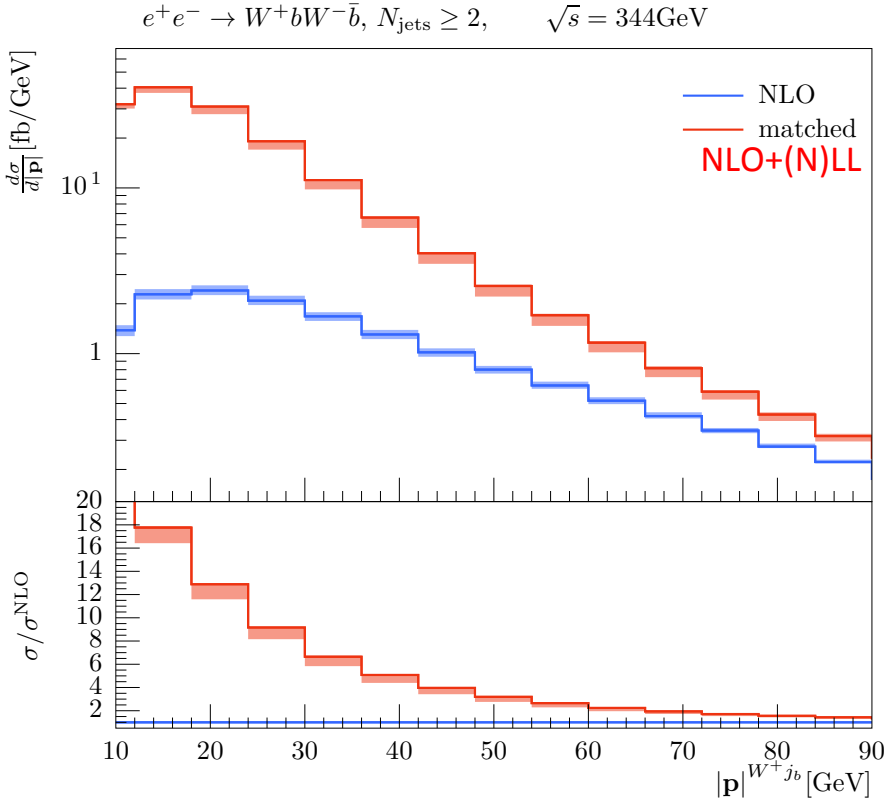
Top-antitop threshold in WHIZARD

Differential observables at peak: $\sqrt{s} = 2m^{1S} = 344\,\text{GeV}$

top inv. mass distribution



top 3-mom distribution

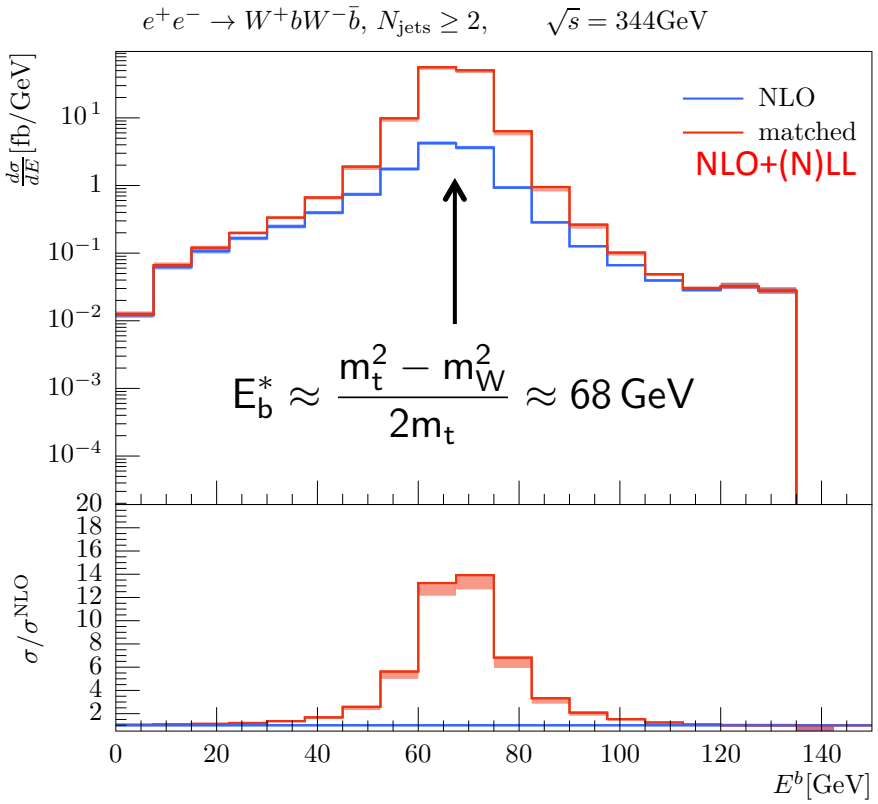


(RIVET event analysis; FASTJET generalized k_T algorithm, R=0.4, p=-1; $E_{jet} > 1$ GeV)

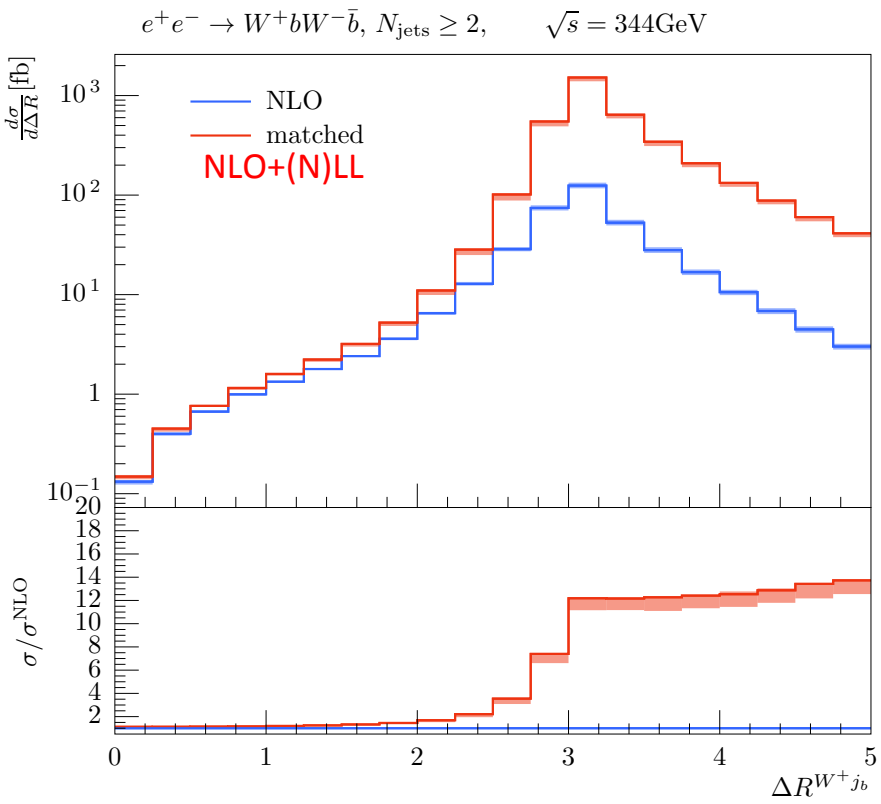
Top-antitop threshold in WHIZARD

Differential observables at peak: $\sqrt{s} = 2m^{1S} = 344\,\text{GeV}$

b-jet energy distribution

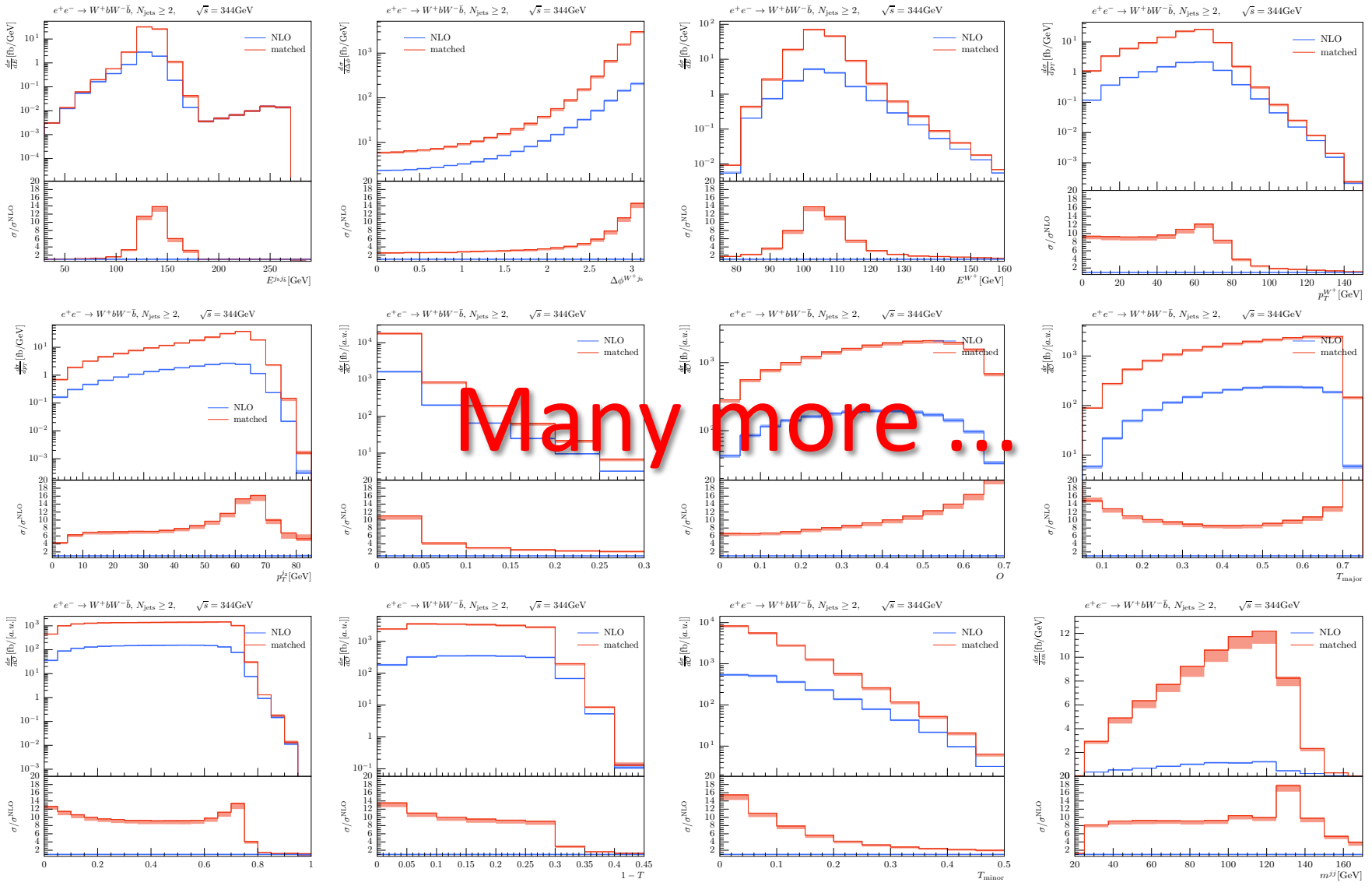


b-W⁺ rapidity separation



(RIVET event analysis; FASTJET generalized k_T algorithm, R=0.4, p=-1; $E_{jet} > 1$ GeV)

Top-antitop threshold in WHIZARD



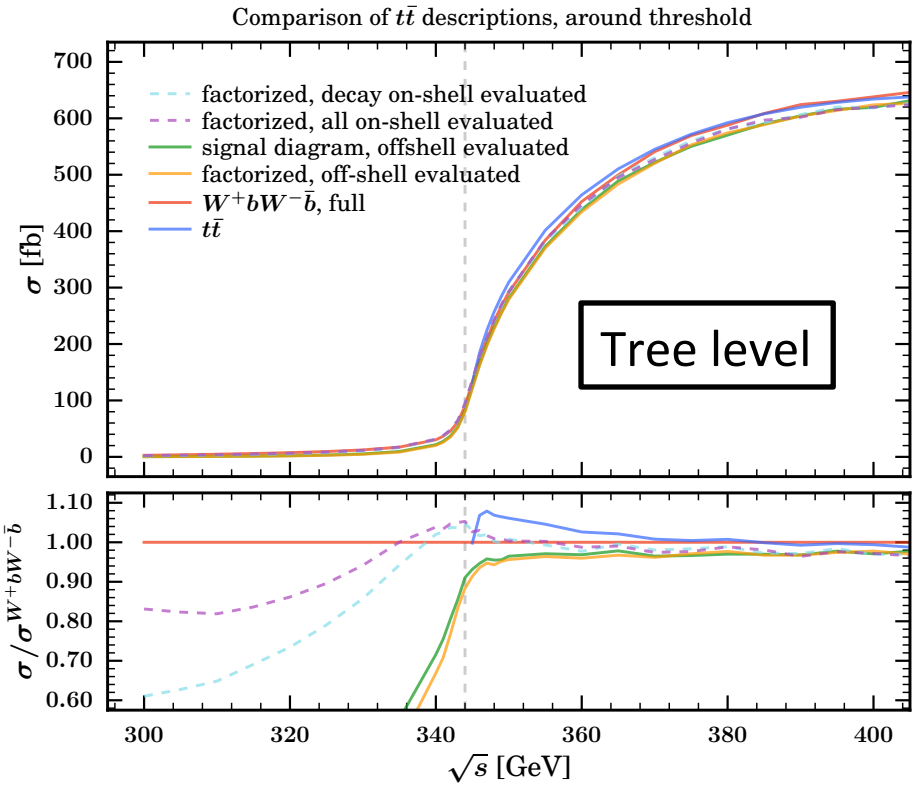
Summary

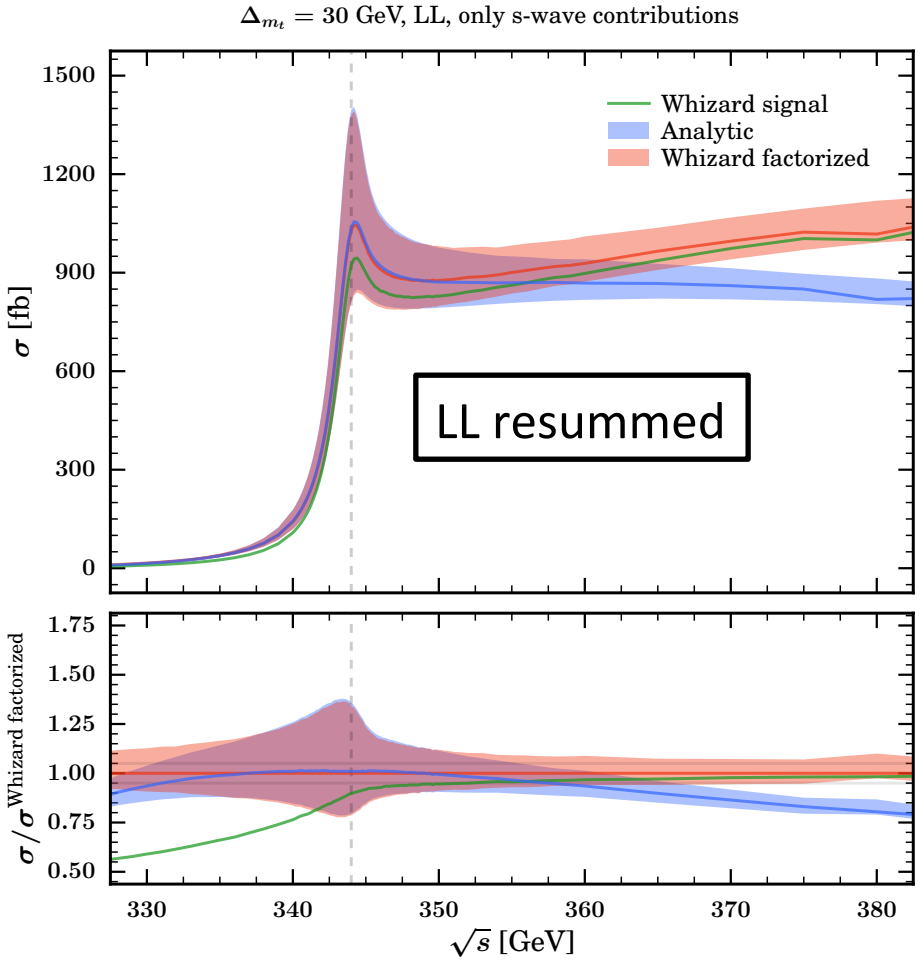
- Total cross section
  - ✓ known to NNLL and N^3LO, to do: N^3LO + NNLL
  - ✓ precise m_t, y_t, α_s, Γ_t from $t\bar{t}$ threshold @ lepton collider
    ⟶ next talk

- NLL + NLO threshold in WHIZARD 
  - ✓ realistic ($W^+W^-b\bar{b}$) final state
  - ✓ fully differential, arbitrary observables!
  - ✓ background/interference effects
  - ✓ Matching to relativistic NLO continuum
  - ✓ ISR, beam structure, polarization effects

Backup

Validation of factorized WHIZARD implementation:





Backup

"Unphysical" scales:

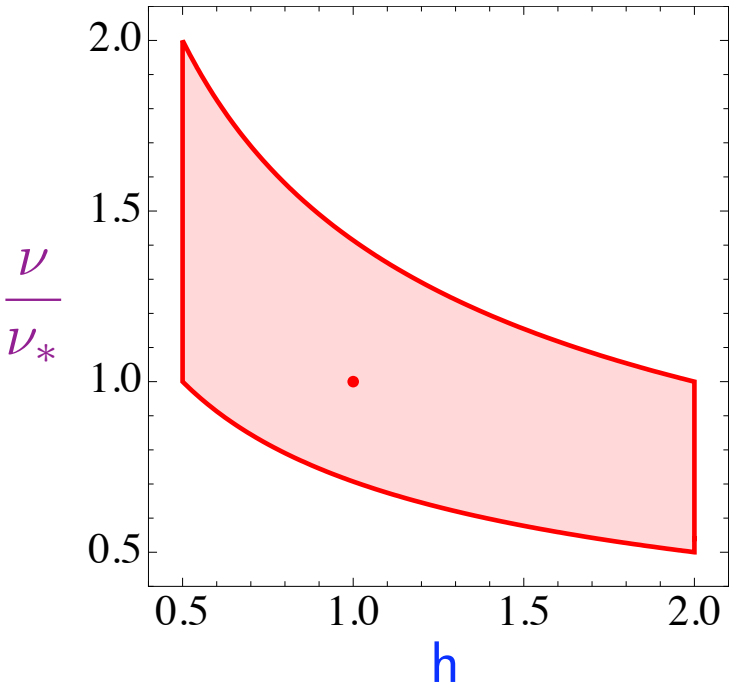
matching scale:

$$\mu_{\text{hard}} = h m$$

renormalization scales:

$$\mu_{\text{soft}} = h m \nu$$

$$\mu_{\text{usoft}} = h m \nu^2$$



default choice:

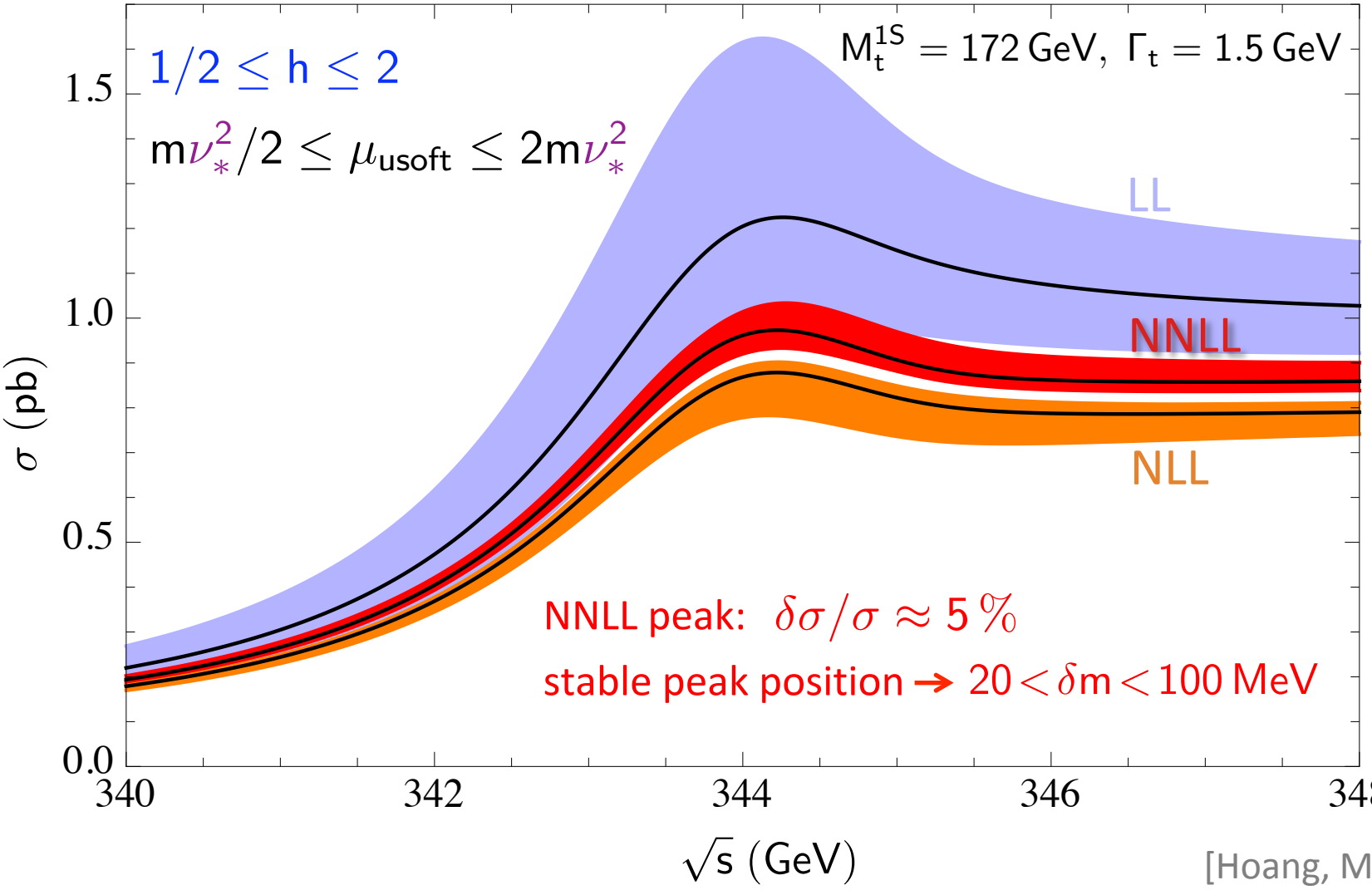
$$h = 1 \qquad \nu_* = 0.05 + |v_{\text{eff}}| \qquad v_{\text{eff}} = \sqrt{\frac{\sqrt{s} - 2m + i\Gamma_t}{m}}$$

scale variation:

$$1/2 \leq h \leq 2$$

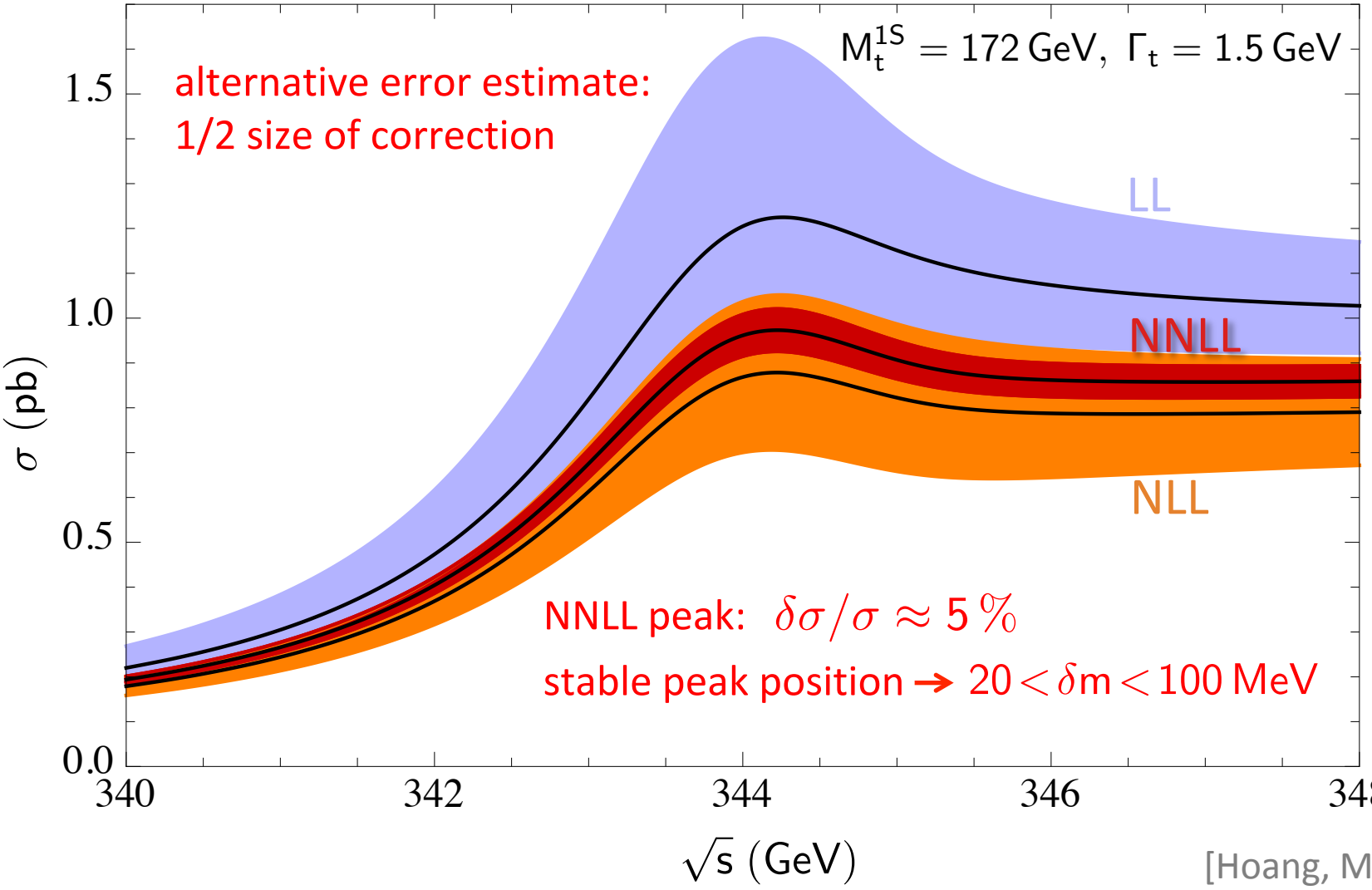
$$m\nu_*^2/2 \leq \mu_{\text{usoft}} \leq 2m\nu_*^2$$

Backup



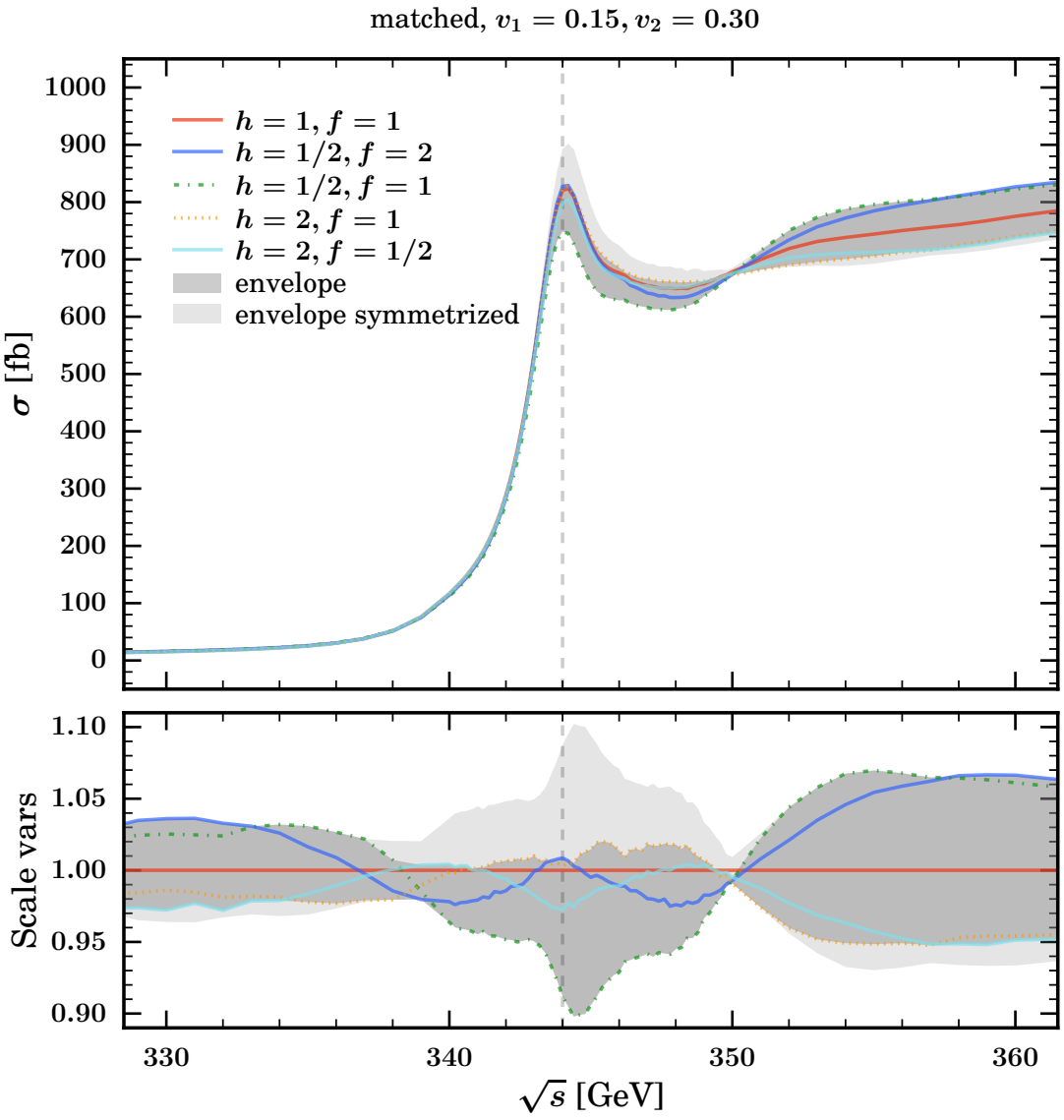
[Hoang, MS, '13]

Backup



[Hoang, MS, '13]

Backup


matched, $v_1 = 0.15, v_2 = 0.30$
$h = 1, f = 1$
$h = 1/2, f = 2$
$h = 1/2, f = 1$
$h = 2, f = 1$
$h = 2, f = 1/2$
envelope
envelope symmetrized
σ [fb]
Scale vars
$\sqrt{s}$ [GeV]