

Charmonium resonances (from lattice QCD)

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Torino, 13-17 May 2019

M. Padmanath, S. Collins, D. Mohler, S. Piemonte, S.P., A. Schafer, S. Weishaeupl : 1811.04116, PRD 2019

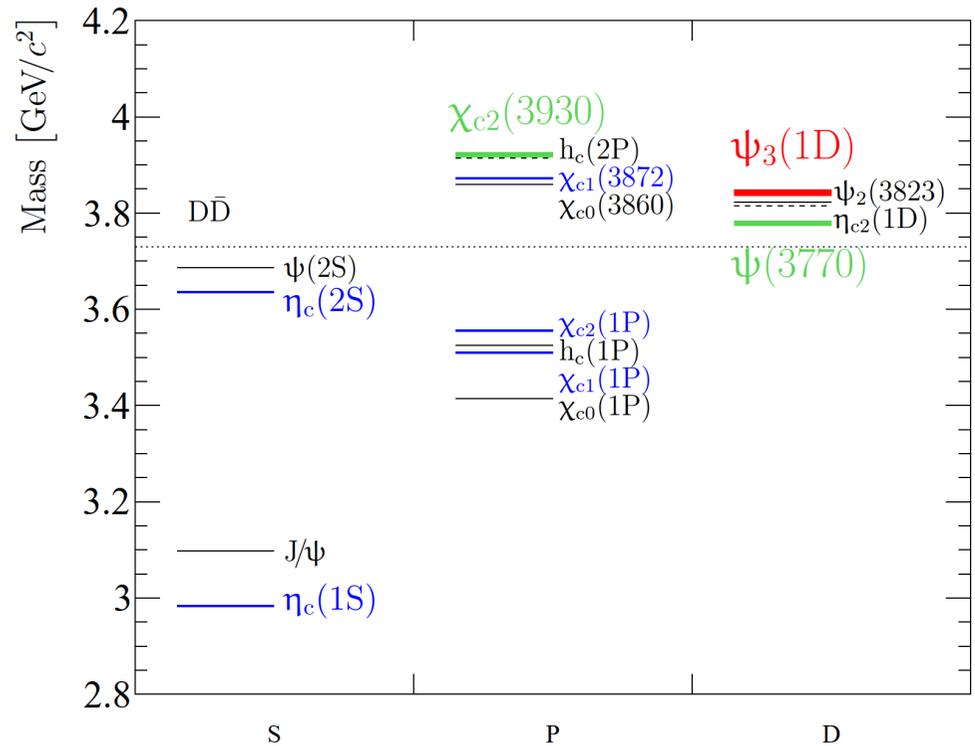
S. Piemonte, S. Collins, M. Padmanath, D. Mohler, S.P. : 1905.03506

Regensburg collaboration RQCD (mostly)

Outline

Lattice QCD study of

- Ground and excited conventional charmonia
 - charmonia with all J^{PC} ($J \leq 3$), momenta $P=0$ and $P \neq 0$
omitting strong decays
determine : m or E , J^{PC}
 - charmonium resonances with $J^{PC} = 1^{--}$ [$\Psi(3770)$] and 3^{--} [$X(3842)$],
taking strong decays to $\underline{D}D$ into account (applying Luscher's method)
determine: m and Γ , J^{PC}
- Very briefly on exotic quarkonium-like channels:
 - pentquark P_c channel
 - tetraquark Z_b channel



taken from
[/lhcb-public.web.cern.ch/lhcb-public](https://lhcb-public.web.cern.ch/lhcb-public)

Conventional charmonia below and above open-charm threshold DD

One of the motivations for lattice QCD study:

Verify lattice QCD treatment of conventional charmonia

Only then apply analogous treatment for more challenging charmonium-like states

First discovery of a charmonium with spin J=3

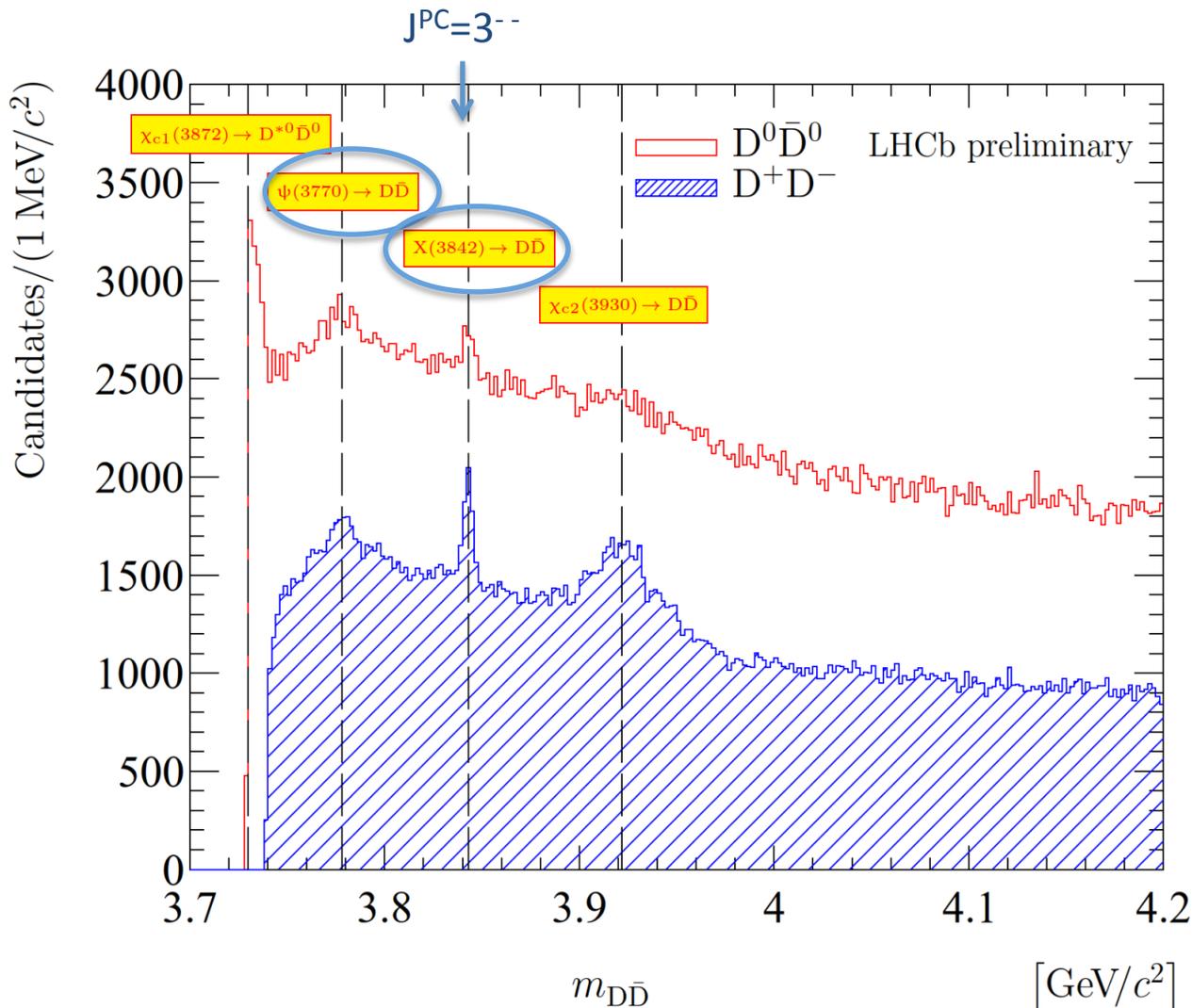
$$m_{X(3842)} = 3842.71 \pm 0.16 \pm 0.12 \text{ MeV}/c^2,$$

$$\Gamma_{X(3842)} = 2.79 \pm 0.51 \pm 0.35 \text{ MeV},$$

LHCb 2019

1903.12240

JHEP 2019



J^{PC} not experimentally measured

LHCb paper:

“The narrow natural width and the mass of the X(3842) state suggest the interpretation as charmonium state with $J^{PC} = 3^{--}$ ”

Quark model quantum numbers:

$$n^{2s+1}l_J = 1^3D_3$$

Lattice ensembles

- CLS ensembles with dynamical u/d and s quarks: $m_\pi \approx 280$ MeV, $m_K \approx 467$ MeV
- two values of m_c to explore dependence of charmonia on the position of \underline{DD} thresholds: $m_D \approx 1762$ MeV, 1927 MeV ($m_D^{\text{exp}} = 1867$ MeV)

$a \approx 0.086$ fm, $N_L = 24$ & 32

$|n\rangle$ denotes eigenstate of H_{QCD}

Info from lattice : E_n, Z_i^n $Z_i^n \equiv \langle 0 | \mathcal{O}_i | n \rangle$

Large number (15-30) of operators with the quantum numbers of desired channel
 quantum number : in continuum: J^{PC}

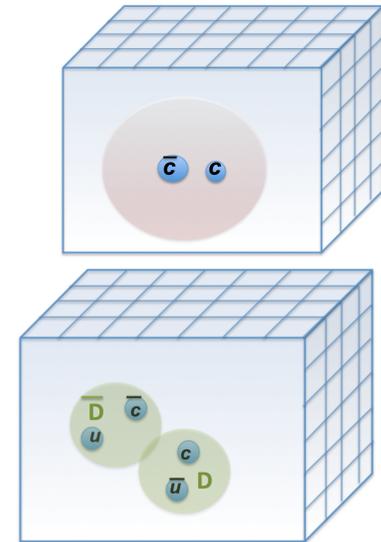
on the lattice: lattice irrep Λ

$$\mathcal{O}^{\bar{c}c} = \bar{c} \Gamma c$$

$$\mathcal{O}^{\bar{D}D} = (\bar{c} \Gamma_1 q) (\bar{q} \Gamma_2 c) = \bar{D} D$$

charm annihilation omitted

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$



M. Padmanath, S. Collins, D. Mohler, S. Piemonte, S.P.,
A. Schafer, S. Weishaeupl : 1811.04116, PRD 2019

charmonia with all J^{PC} ($J \leq 3$) omitting strong decays

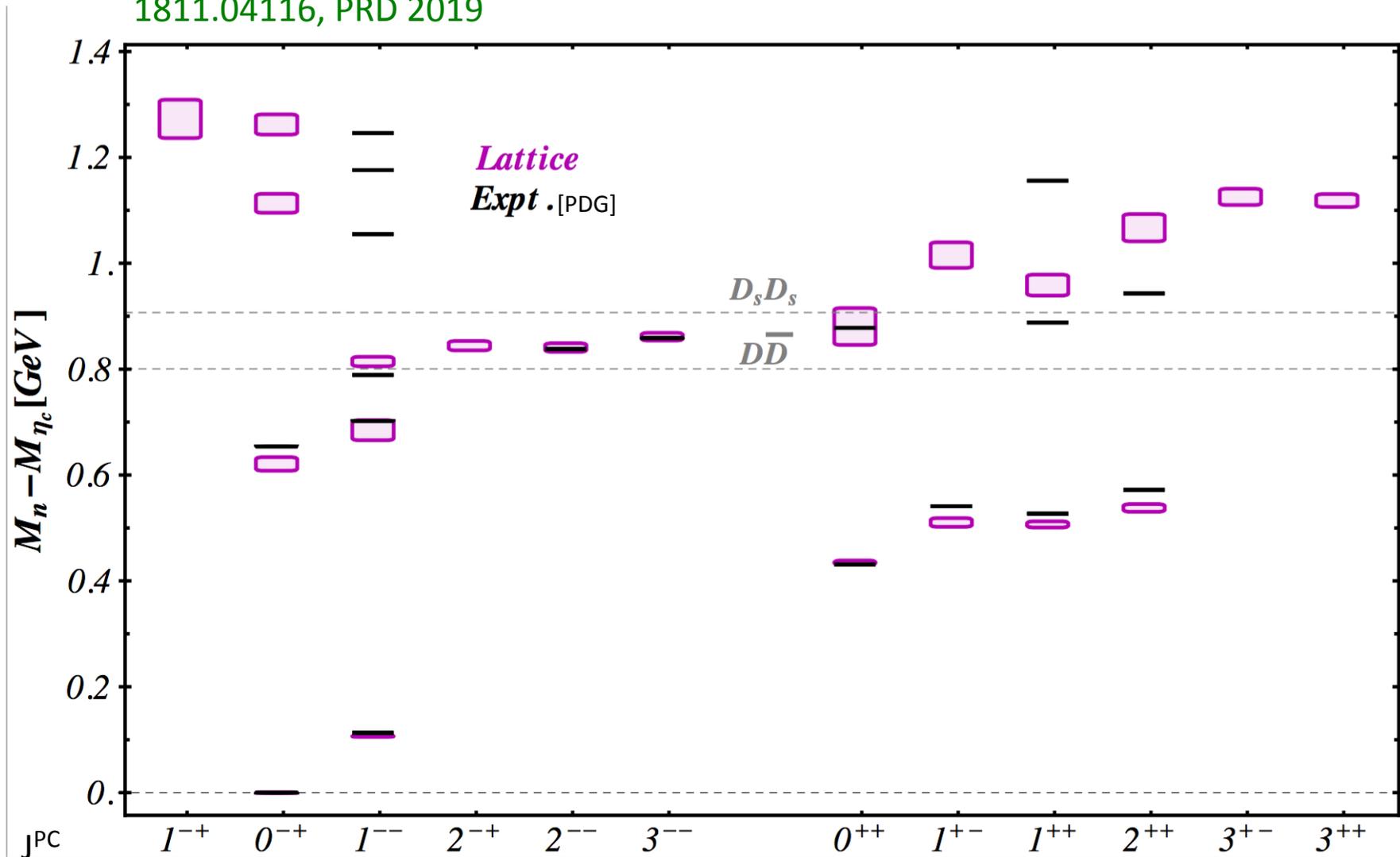
employing only operators $\mathcal{O}^{\bar{c}c} = \bar{c}\Gamma c$
(10-30 for quantum number)

m and J^{PC} of charmonia with $P=0$



(extensive spectra using similar approach HSC: 1610.01073)

1811.04116, PRD 2019



↓
exotic J^{PC}
hybrid candidate

↙ agrees with
Belle 2013
BESIII 2015

↘ agrees with
LHCb X(3842)
1903.12240

↓ agrees with
Belle X(3860)
1704.01872

↓ X(3872) too high without
DD* threshold effect

Charmonia with $P \neq 0$: determining E and J^{PC}

refers to quantum numbers in charmonium's rest frame

- Experiment: charmonium $\rightarrow H_1 H_2$
 - Lorentz transformation of decay products to charmonium's rest frame
 - observed partial wave and q.n. of H_1 and H_2 render J^{PC} of charmonium in its rest frame

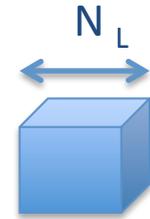
- Lattice: Above strategy can not be followed on the lattice
 - Strategy : following [HSC, Thomas et al, 1107.1930] used for light isovectors
each irrep Λ contains several J^P ; for example: A_1 irrep for $P=1 \times 2\pi/N_L$ contains $J^P= 0+, 1-, 2+, 3-$

$$\begin{array}{ccccc}
 \mathcal{O}^{J^{PC}} = \bar{c} \Gamma c & \xrightarrow{\text{boost to P}} & \mathcal{O}^{J^{PC}, \lambda} & \xrightarrow{\text{subduce to lattice irrep } \Lambda} & \mathcal{O}_{\Lambda}^{[J^{PC}, \lambda]} \\
 \text{good q.n. at rest in continuum} & & \text{good helicity } \lambda & & \text{operator on the lat.} \\
 & & \text{at mom. P in continuum} & &
 \end{array}$$

- determine overlaps $Z_i^n \equiv \langle 0 | \mathcal{O}_i^{[J^{PC}, \lambda]} | n \rangle$ from lattice

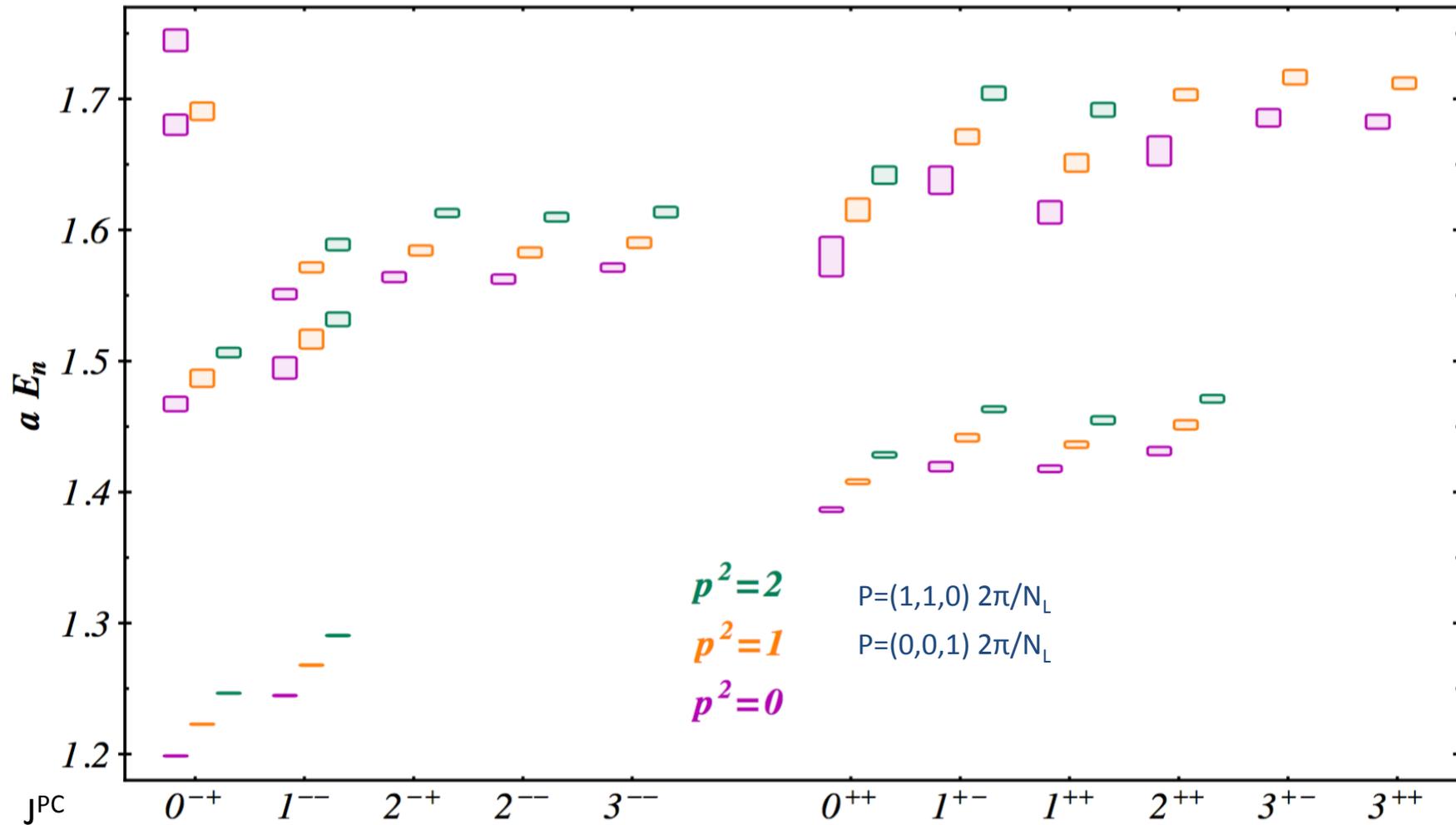
- eigen-state $|J^P, \lambda\rangle$ couples better to $O^{[J, P, \lambda]}$ than to $O^{[J', P', \lambda']}$

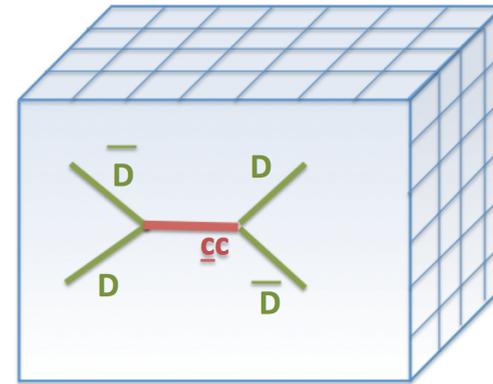
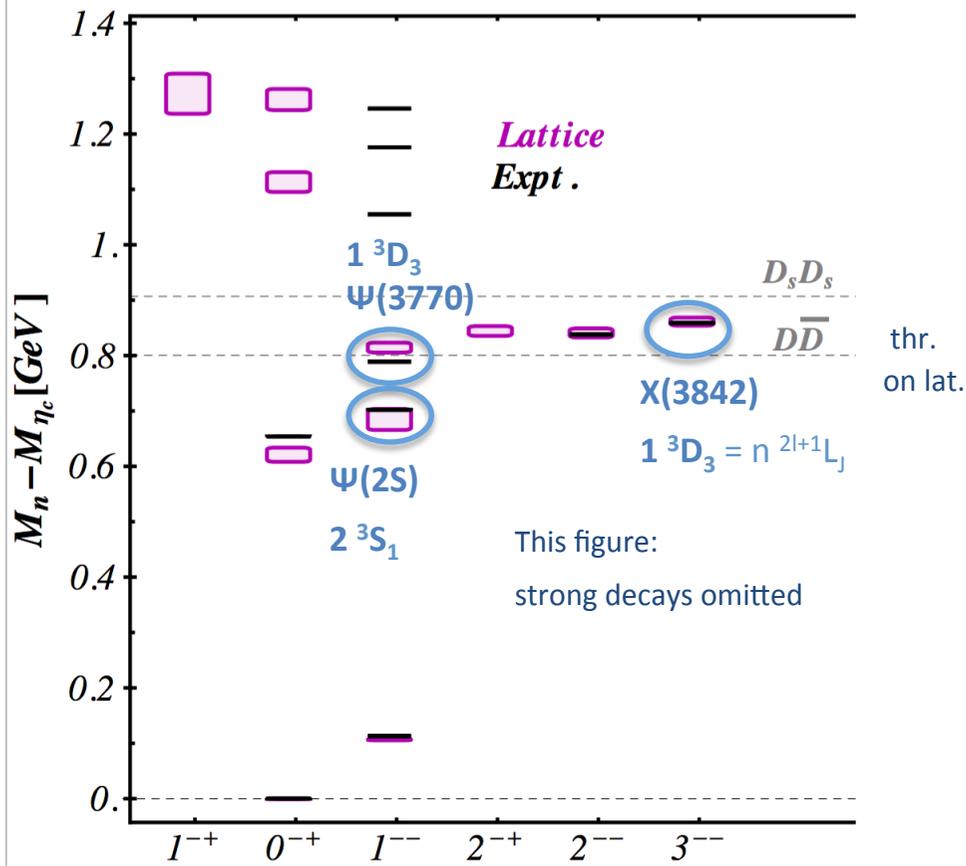
E and J^{PC} of charmonia at $P=0$ and $P \neq 0$



$$E \approx \sqrt{m^2 + P^2}$$

J^{PC} denote quantum numbers in particle's rest frame





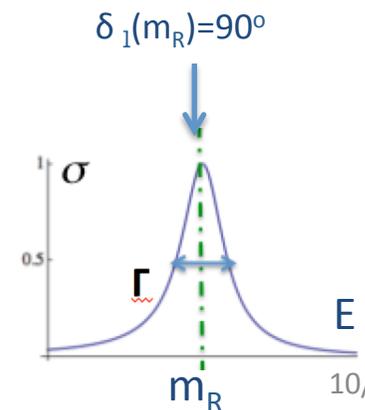
Strategy

- simulate $\underline{D}D$ scattering on the lattice
- determine scattering amplitude

$$S_l(E) = \exp[2i\delta_l(E)], \quad l = 1, 3$$

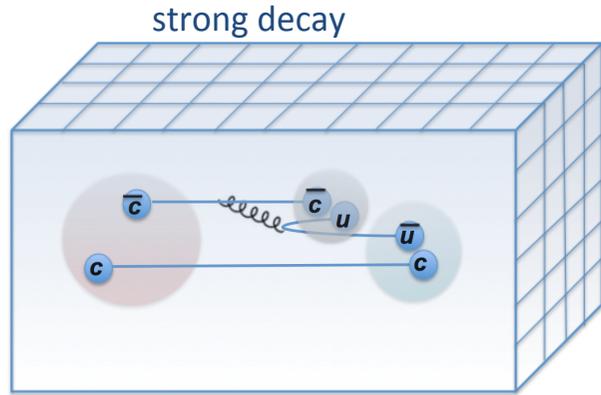
$$\sigma(E) \propto |S(E) - 1|^2 \propto |t(E)|^2$$

- m_R and Γ_R from Breit-Wigner type fits

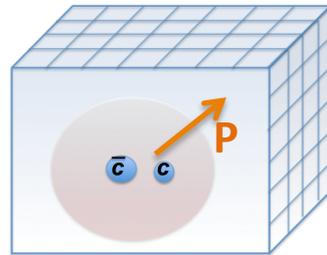


Charmonia with $J^{PC}=1^{-}$ and 3^{-} taking into account their strong transitions to $\underline{D}D$

- only 1 previous lattice study extracted width of charmonium resonances (0^{++} and 1^{-})
Lang, Leskovec, Mohler, S.P. , 1503.05363, JHEP 2015
- Present work:
S. Piemonte, S. Collins, M. Padmanath, D. Mohler, S.P. : 1905.03506



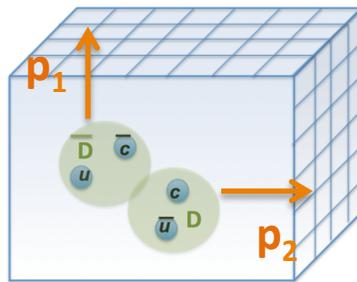
Operators



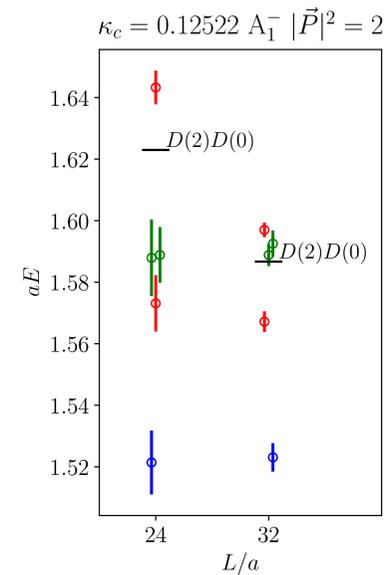
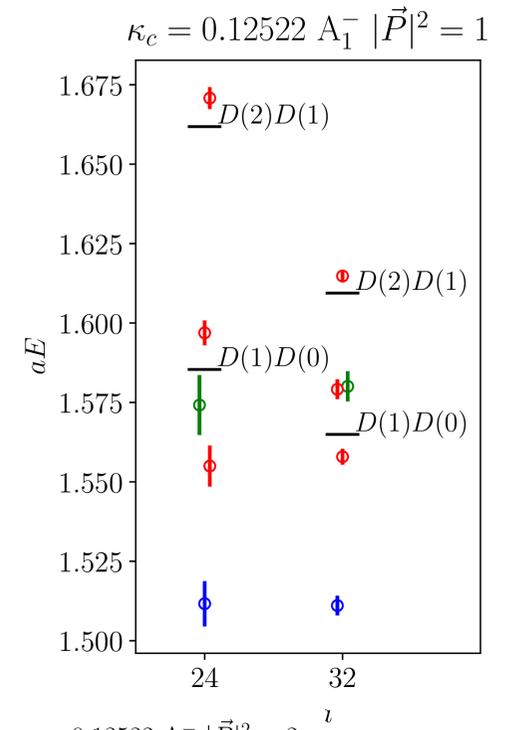
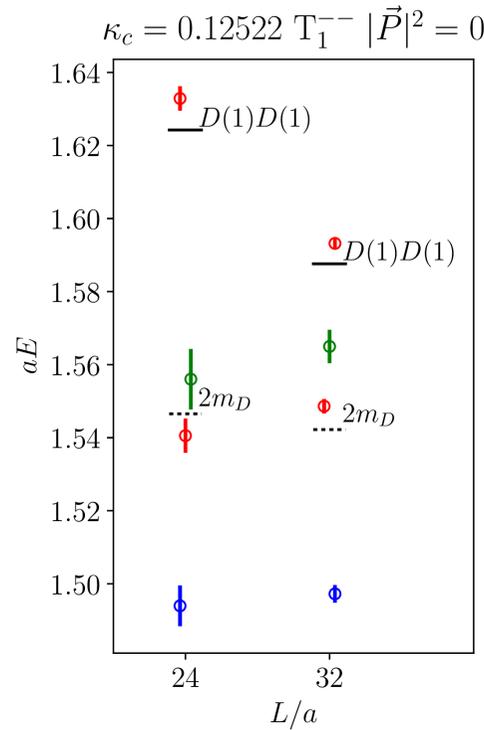
$$\mathcal{O}^{\bar{c}c} = (\bar{c}\Gamma c)_{\vec{P}}$$

$$\begin{aligned} \mathcal{O}^{\bar{D}D} &= (\bar{c}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 c)_{\vec{p}_2} \\ &= \bar{D}(\vec{p}_1) D(\vec{p}_2) \end{aligned}$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2 \quad \begin{array}{l} P: 0 \\ (0,0,1) 2\pi/N_L \\ (1,1,0) 2\pi/N_L \end{array}$$



Eigen-energies E_n



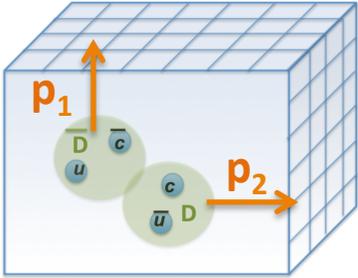
Plots for
 $m_D \approx 1762 \text{ MeV}$

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^+(0) | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

Relation between eigen-energies E and $S_1(\mathbb{E}) = \exp[2i \delta_1(E)]$

$P=0$ or $P \neq 0$, 3D space

$E \rightarrow E_{cm}$



L

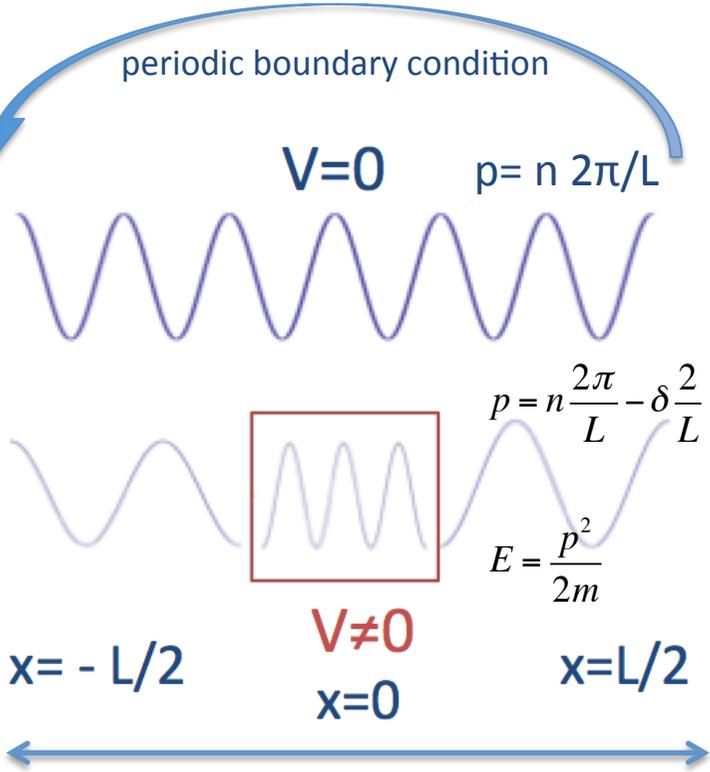
$l=1,3$ appear together due to finite cubic lattice (lattice artifact)

$$f^{\Lambda, \vec{P}}(\delta_1(E_{cm}), \delta_3(E_{cm}), E_{cm}, L) = 0$$

1D space, quantum mechanics

periodic boundary condition

$V=0$ $p = n \frac{2\pi}{L}$



$E = \frac{p^2}{2m}$

$x = -L/2$ $V \neq 0$ $x = L/2$
 $x=0$

L

$$f(\delta(E), E, L) = 0$$

$E \rightarrow \delta(E)$

known functions

Fit of phase shifts for $l=1,3$ from E_n

$$E_{cm} = \sqrt{s} = 2\sqrt{m_D^2 + p^2}$$

p = relative momenta of D-mesons in CMF

$$\frac{p^{2l+1} \cot(\delta_l)}{\sqrt{s}} = \frac{m^2 - s}{G^2} \quad \text{Breit-Wigner}$$

$$\delta_l(m_R) = 90^\circ$$

Fit forms:

$$l=1 \quad \frac{p^3 \cot(\delta_1)}{\sqrt{s}} = \left(\frac{G_1^2}{m_1^2 - s} + \frac{G_2^2}{m_2^2 - s} \right)^{-1}$$

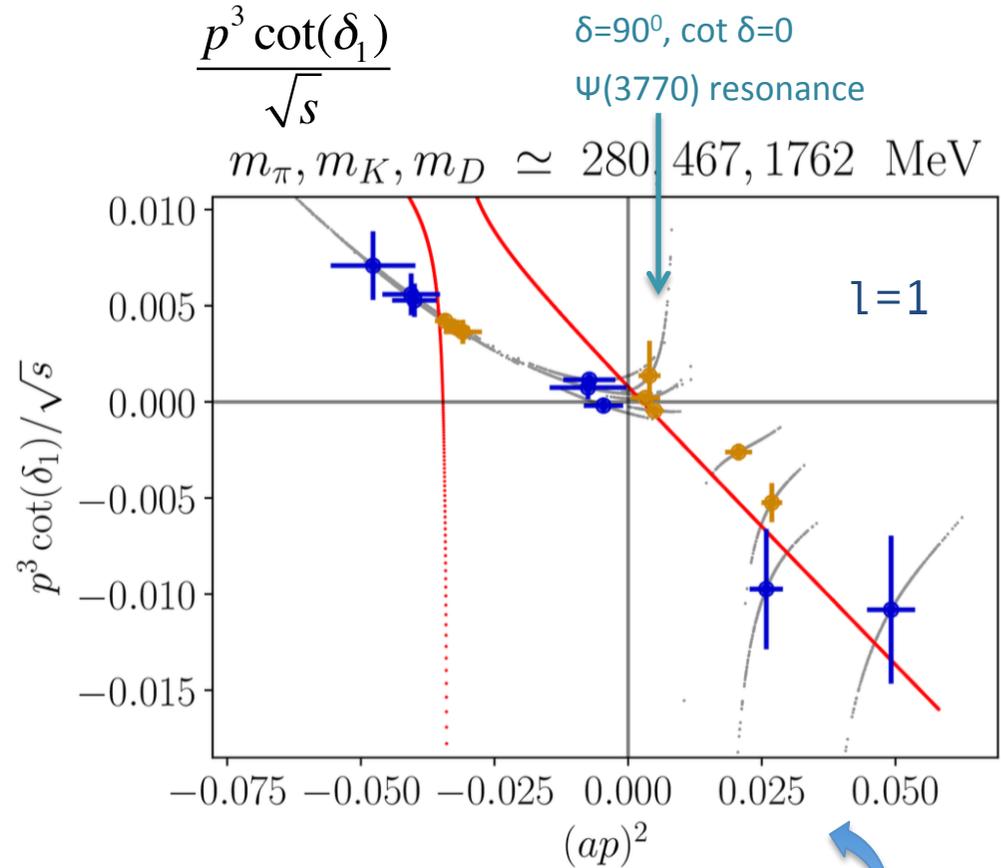
$\Psi(25)$ $\Psi(3770)$

$$l=3 \quad \frac{p^7 \cot(\delta_3)}{\sqrt{s}} = \frac{m_3^2 - s}{g_3^2}$$

$X(3842)$

Result:

$$\frac{p^{2l+1} \cot(\delta)}{\sqrt{s}} = \begin{cases} \left(\frac{[0.63(33)]^2}{[1.4966(30)]^2 - s} + \frac{[3.69(37)]^2}{[1.5457(32)]^2 - s} \right)^{-1} & l = 1 \\ \frac{[1.568(11)]^2 - s}{[0.07(3)]^2} & l = 3 \end{cases}$$



Scattering amplitude $t(E)$ in complex energy plane

$$S_l(E) = \exp[2i\delta_l(E)] = 1 + 2i\rho t_l(E)$$

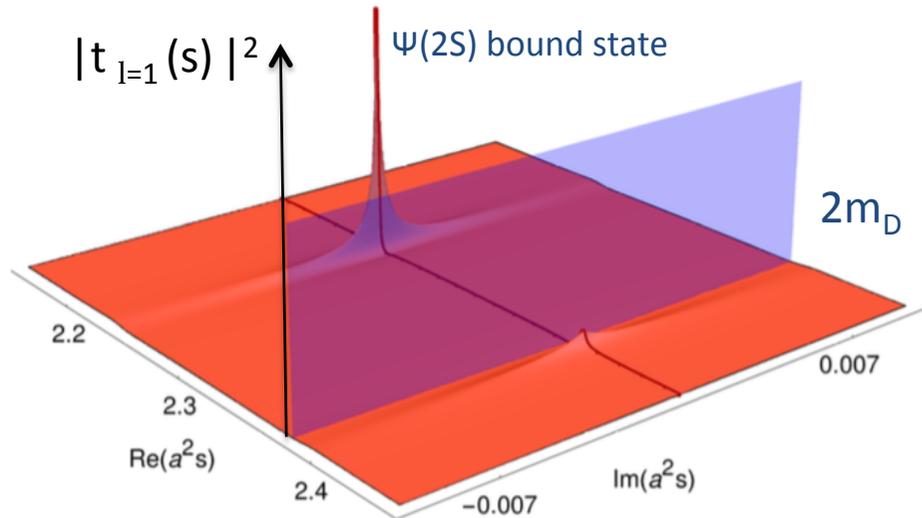
$$t_l(s) = \frac{1}{\rho \cot(\delta_l) - i\rho}$$

$$\rho = \frac{2p}{\sqrt{s}} = \sqrt{1 - 4\frac{m_D^2}{s}}$$

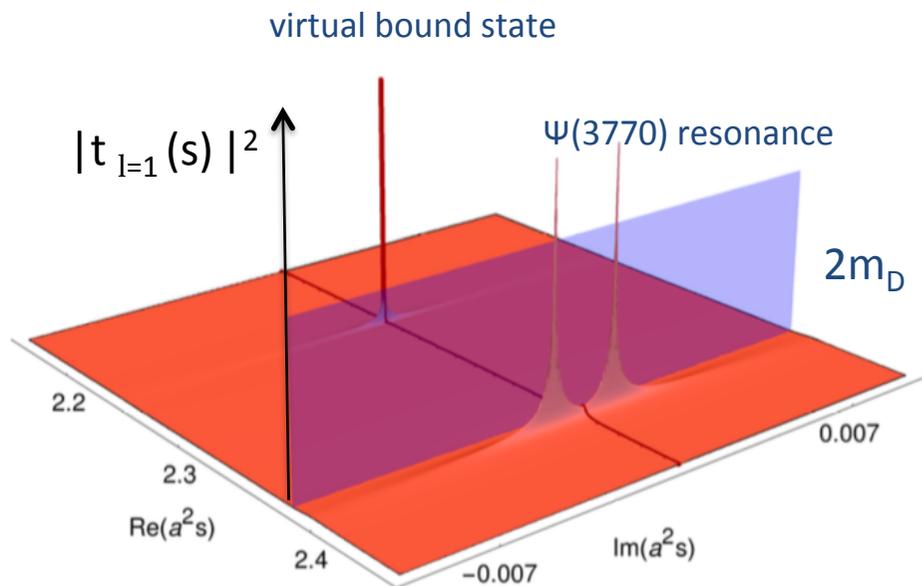
poles in $t(s)$ related to
resonance and bound states

Fig for $l=1$ and $m_D \approx 1762$ MeV:

one resonance, one bound state, one virtual bound state



(a) I Riemann sheet



(b) II Riemann sheet

Results at $m_D=1762$ MeV

Resonances

$\Psi(3770)$ $J^{PC}=1^{--}$, $l=1$

$X(3842)$ $J^{PC}=3^{--}$, $l=3$

Mass:

$$\delta_1(E_{cm}=m_R)=90^\circ$$

$$\cot[\delta_1(m_R)]=0$$

Width or coupling g:

$\Psi(3770) \rightarrow \underline{D}D$, $l=1$

$$\frac{p^3 \cot \delta_1}{\sqrt{s}} \Big|_{s \simeq m^2} = \frac{6\pi}{g^2} (m^2 - s)$$

$$\Gamma = \frac{g^2 p^3}{6\pi s}$$

	gg
lat	$16.0^{+2.1}_{-0.2}$
exp	18.7 ± 0.9

$X(3842)$: too narrow to resolve

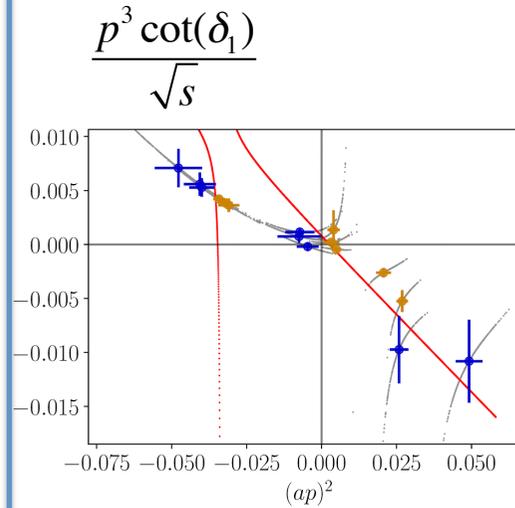
Bound state

$\Psi(2S)$ $J^{PC}=1^{--}$, $l=1$

Mass:

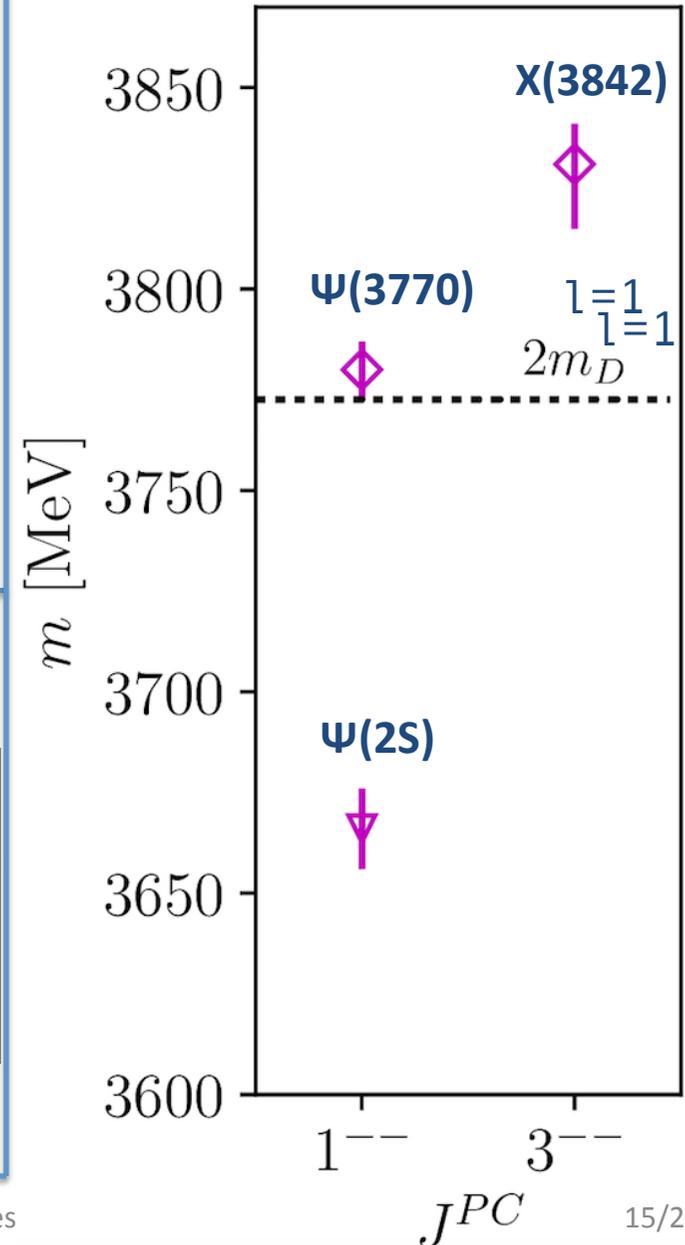
$$|t(E_{cm}=m_B)| = \infty$$

m_B given by the pole on the first sheet



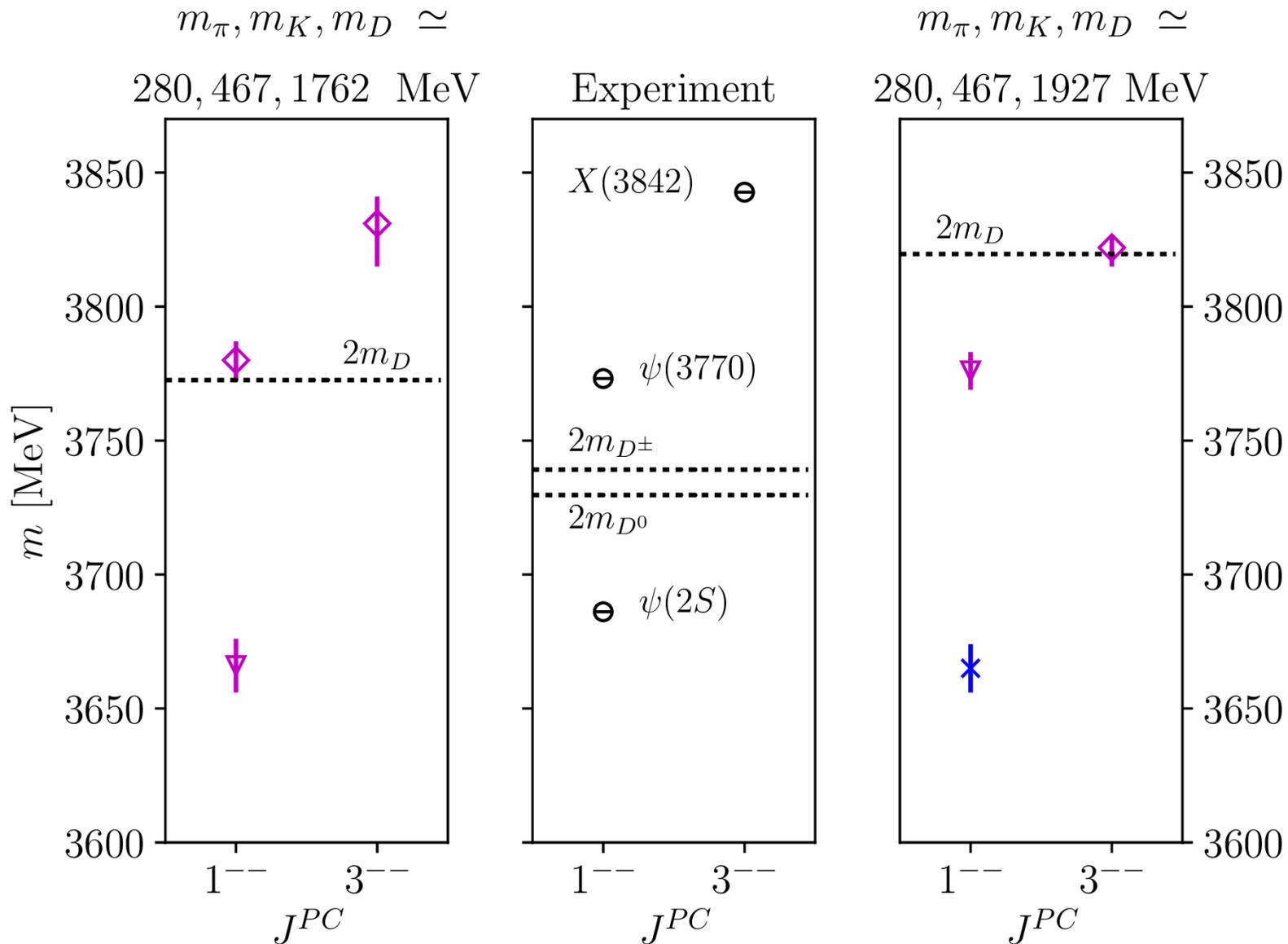
$$m_\pi, m_K, m_D \simeq$$

280, 467, 1762 MeV



$$m = m^{\text{lat}} - M_{\text{av}}^{\text{lat}} + M_{\text{av}}^{\text{exp}} \quad M_{\text{av}} = \frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$$

Masses of charmonium resonances and bound states



Briefly on exotic quarkonium-like channels

Lattice study of P_c pentaquark channel

$$P_c = uud\bar{c}c \rightarrow (uud) (\bar{c}c)$$

light-baryon charmonium

$$\rightarrow (uuc) (\bar{c}d)$$

charmed-baryon charmed-meson

Question we address: Do P_c resonances appear in one-channel $p J/\psi$ scattering on the lattice (in approximation where this channel is decoupled from other channels)

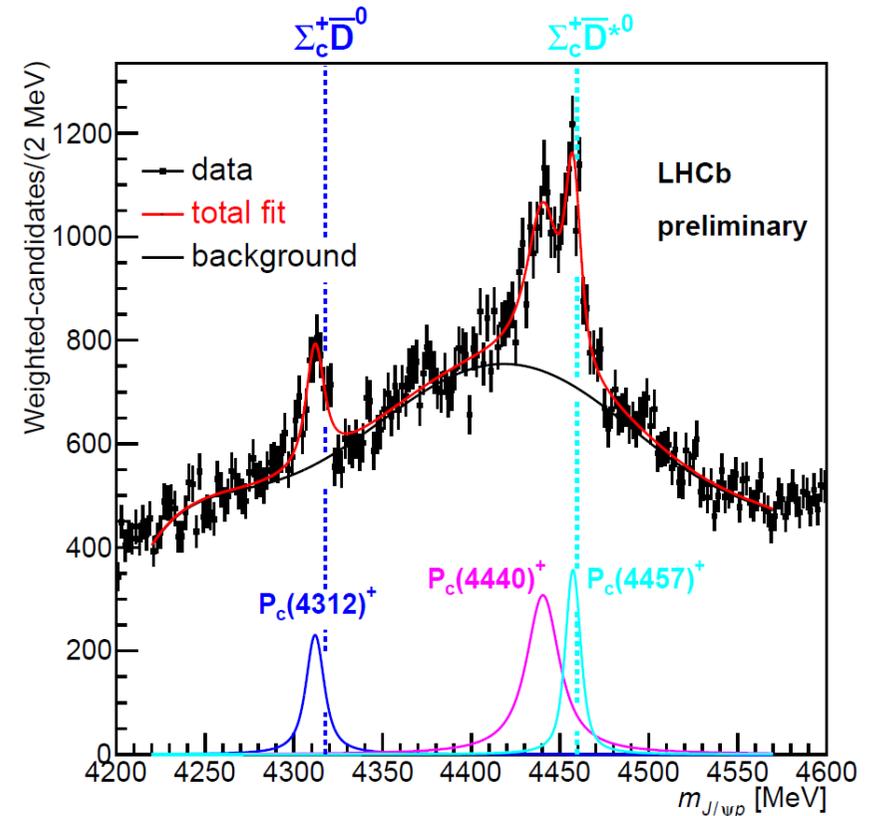
$$p J/\psi \rightarrow P_c \rightarrow p J/\psi$$

We simulate this scattering and cover also the energy region of P_c for the first time.

The answer from our lattice simulation : No.

U. Skerbis, S. Prelovsek, 1811.02285 to appear in PRD

This indicates that the coupling of $p J/\psi$ channel with other two-hadron channels is likely responsible for P_c resonances in experiment.



This is in line with LHCb results, where P_c 's are found near other thresholds. This by itself indicates that other channels are important.

Lattice study of Z_b tetraquark channel

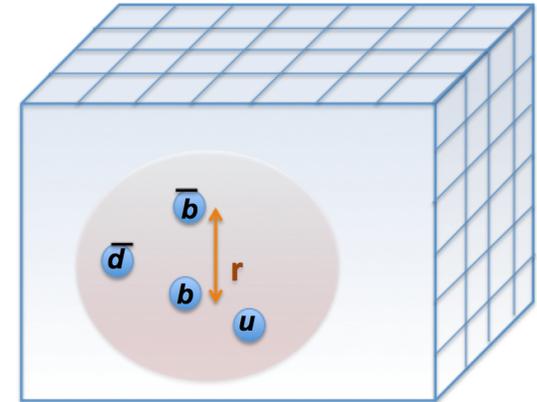
Lattice simulation of Z_b channel with static b and \bar{b}

Inspired by [Bicudo, Peters, Wagner \[1602.07621, 1709.03306\]](#)

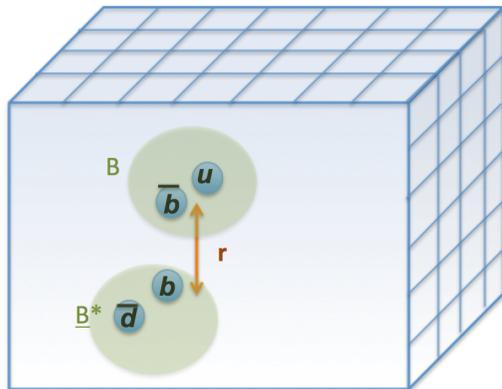
Extracted $E_n(r)$

ongoing study with [J. Petkovic](#) and [H. Bahtiyar](#)

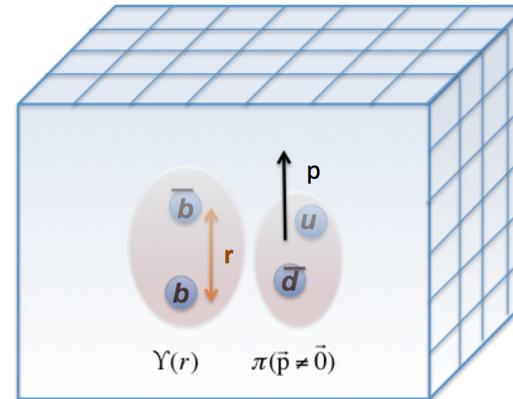
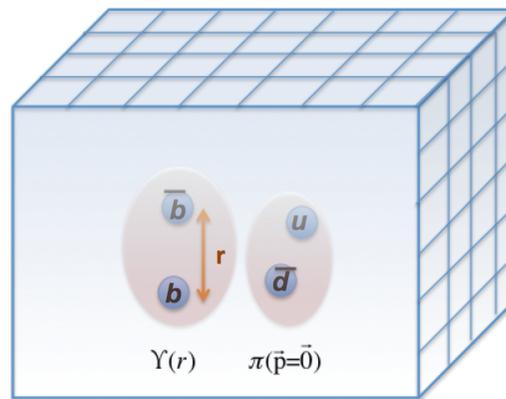
conclusions concerning Z_b from this approach: coming soon



Fock components incorporated:



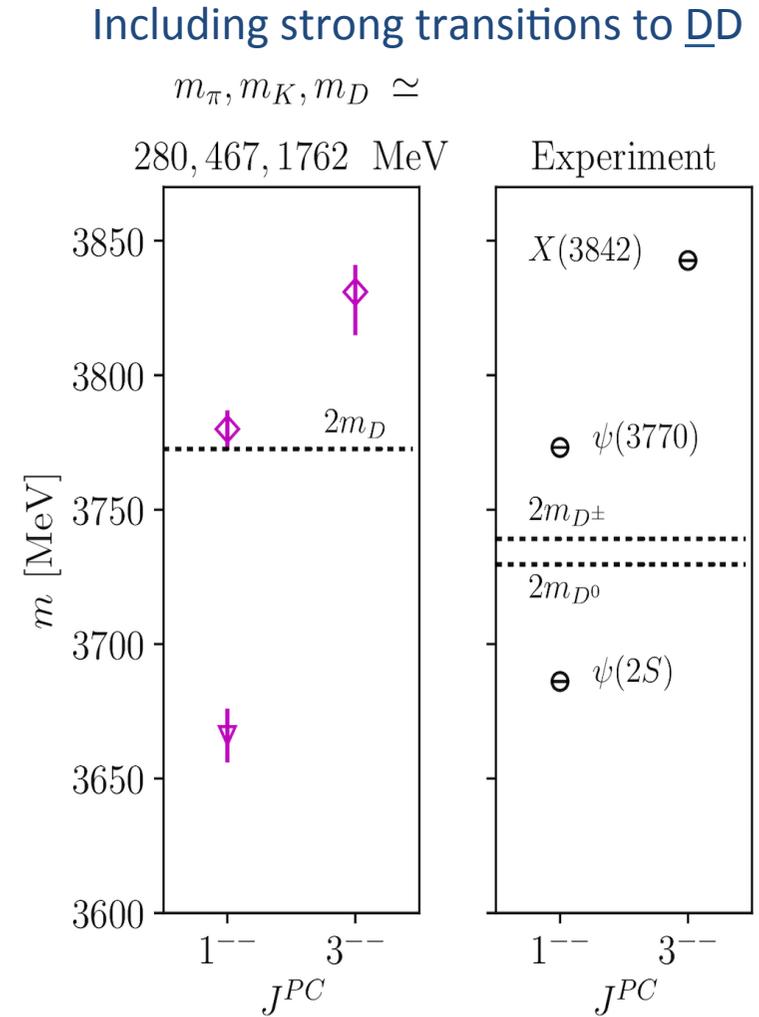
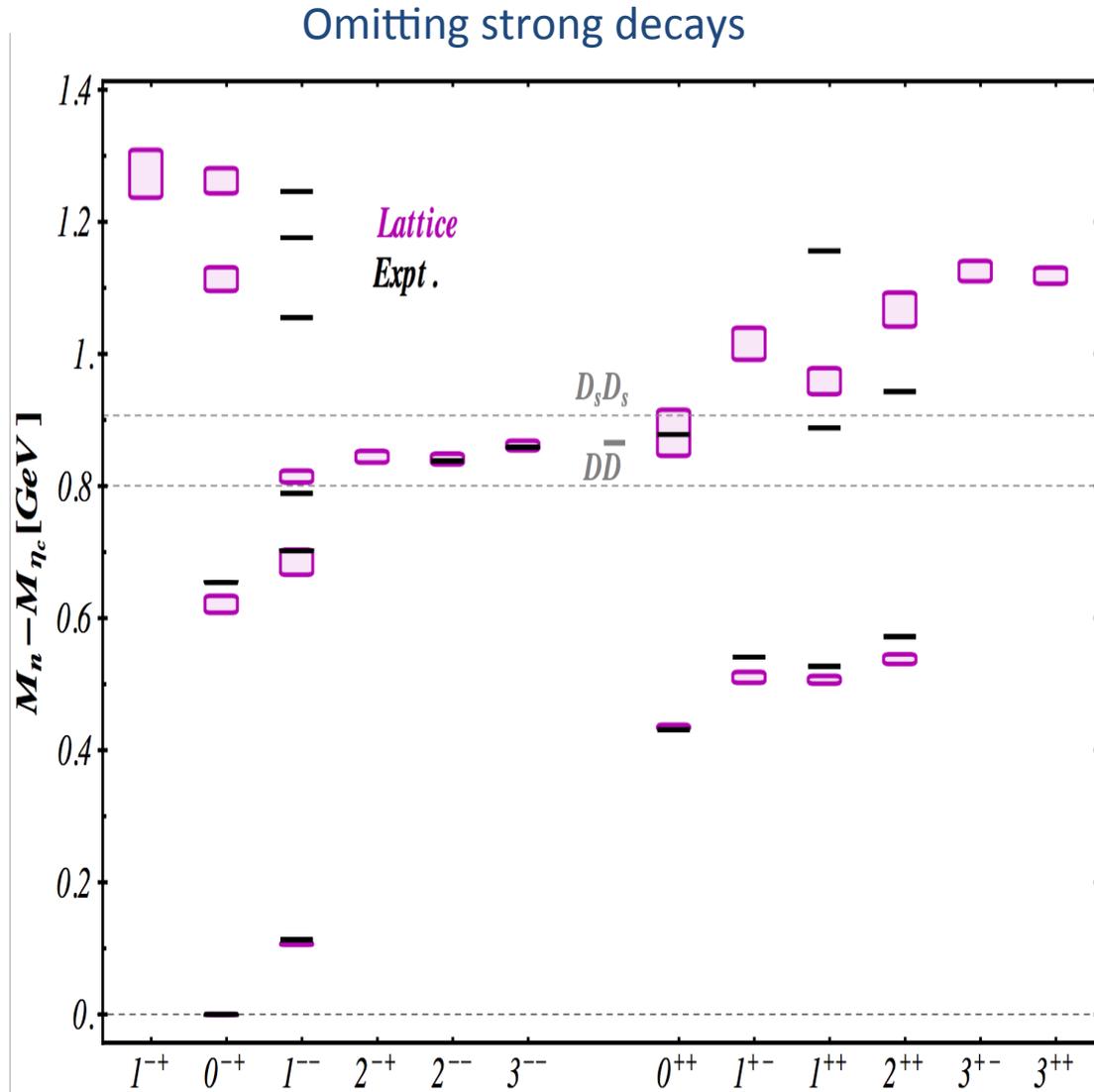
sizable attraction
found in this channel



new

Conclusions

Charmonium spectrum from a lattice study at a single lattice spacing and $m_\pi \approx 280$ MeV (not extrapolated to physical point)



$\Psi(3770) \rightarrow D\bar{D}$ coupling/width
in agreement with exp

Backup

PDG 2018

$n^{2s+1}\ell_J \quad J^{PC}$	$l = 0$ $c\bar{c}$	$l = 0$ $b\bar{b}$
$1^1S_0 \quad 0^{-+}$	$\eta_c(1S)$	$\eta_b(1S)$
$1^3S_1 \quad 1^{--}$	$J/\psi(1S)$	$\Upsilon(1S)$
$1^1P_1 \quad 1^{+-}$	$h_c(1P)$	$h_b(1P)$
$1^3P_0 \quad 0^{++}$	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$
$1^3P_1 \quad 1^{++}$	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$
$1^3P_2 \quad 2^{++}$	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$
$1^3D_1 \quad 1^{--}$	$\psi(3770)$	
$1^3D_3 \quad 3^{--}$	$X(3842)$	
$2^1S_0 \quad 0^{-+}$	$\eta_c(2S)$	$\eta_b(2S)$
$2^3S_1 \quad 1^{--}$	$\psi(2S)$	$\Upsilon(2S)$

	J^{PC}	lat (present work) $\kappa_c = 0.12522$	lat (present work) $\kappa_c = 0.12315$	exp $\bar{D}^0 D^0 / D^+ D^-$	lat [16]
m_D [MeV]		1762(2)	1927(2)	$\bar{m}_D \simeq 1867$ MeV	1763(22)(18)*
m_{D_s} [MeV]		1818(1)	1981(1)	1968.34(7)	
M_{av} [MeV]		2820(3)	3103(3)	3068.6(2)	3119(9)(33)*
m_π [MeV]		280	280	$\bar{m}_\pi \simeq 137$ MeV	266
$\psi(3770)$	1^{--}	resonance	bound st.	resonance [44]	resonance
g		$16.0^{(+2.1)}_{(-0.2)}$	$18.9^{(+0.8)}_{(-0.7)}$	18.7(9)	13.2(1.2)
$m - M_{av}$ [MeV]		711(7)	707(7)	704.25(35)	715(7)
$m - 2m_D$ [MeV]		9(7)	-43(8)	38.52(35)	
m [MeV]		3780(7)	3776(7)	3773.13(35)^a	3784(7)
$\psi(2S)$	1^{--}	bound st.	bound st.	bound st. [44]	bound st.
$m - M_{av}$ [MeV]		597(10)	596(9)	617.347(25)	605(6)
$m - 2m_D$ [MeV]		-105(11)	-154(10)	-48.383(25)	
m [MeV]		3666(10)	3665(9)	3686.097(25)	3674(6)
X(3842)	3^{--}	resonance	resonance	resonance [21]	
$m - M_{av}$ [MeV]		$762^{(+10)}_{(-16)}$	$754^{(+4)}_{(-7)}$	773.9(2)	
$m - 2m_D$ [MeV]		$59^{(+11)}_{(-16)}$	$4^{(+9)}_{(-3)}$	108.2(2)	
m [MeV]		$3831^{(+10)}_{(-16)}$	$3822^{(+4)}_{(-7)}$	3842.7(2)	

The challenge to determine J^P

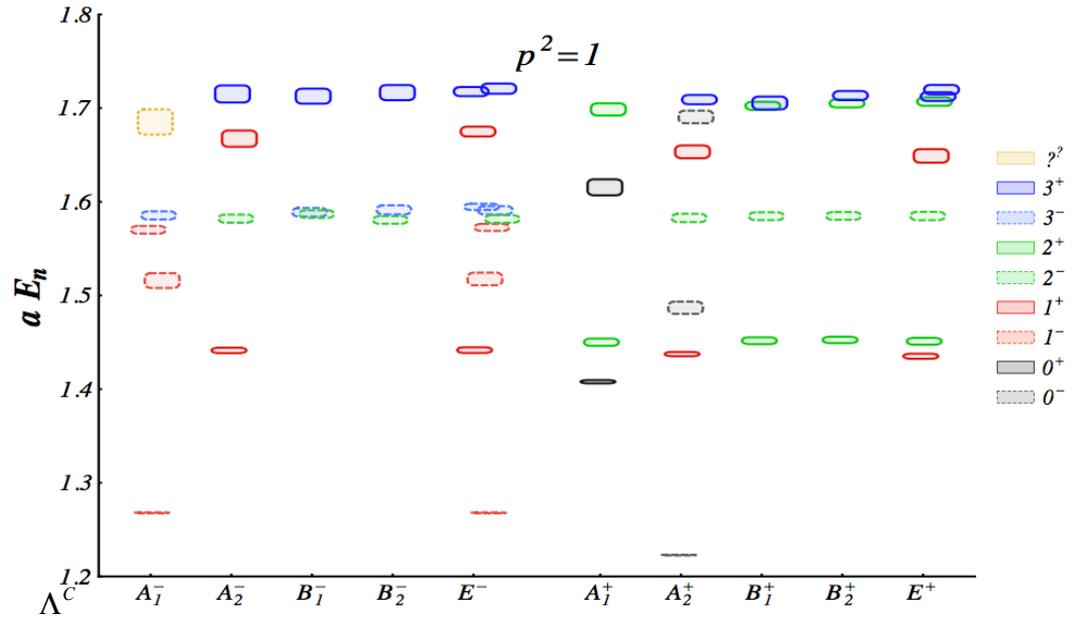
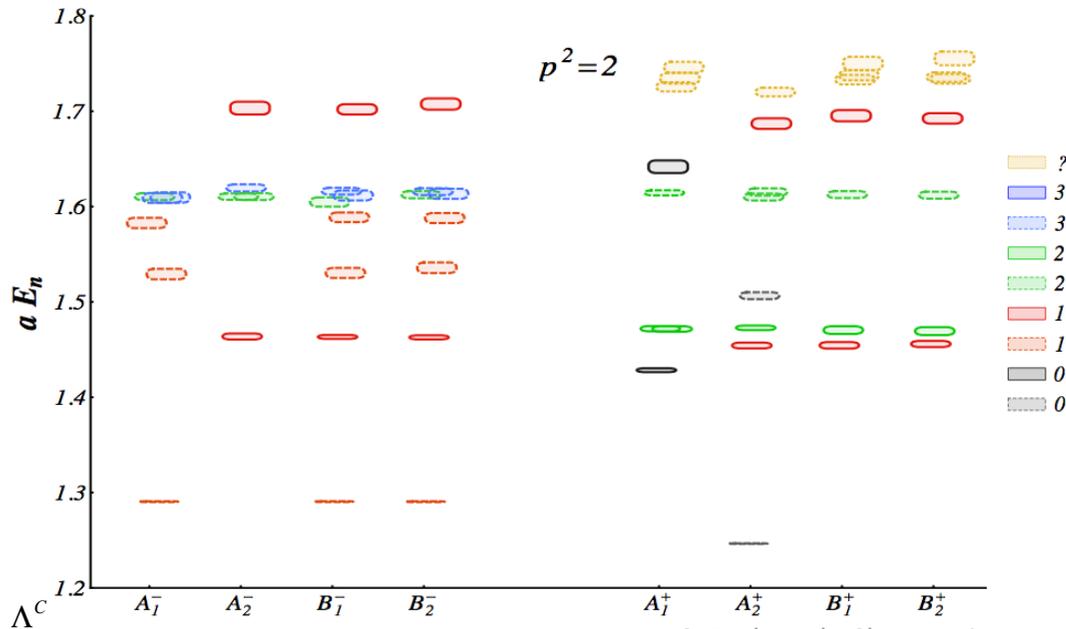


FIG. 11. J^P -identified charmonium spectrum in the moving frame with $\mathbf{p} = (0, 0, 1)$. Irreps Λ^C of group Dic_4 are presented. The colors indicate J^P of states according to the color-coding (21).



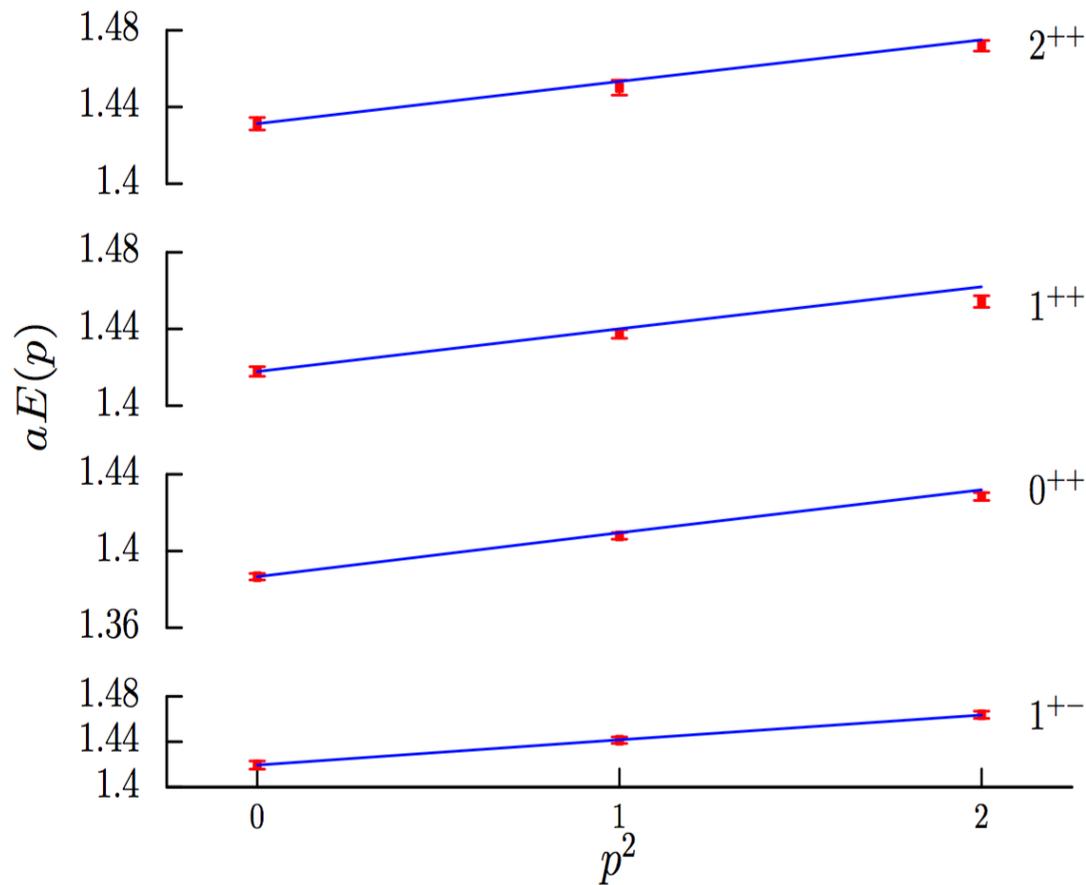
$\mathbf{p} = (0, 0, 1), Dic_4$		
Λ (dim)	$ \lambda ^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	$0^+, 1^-, 2^+, 3^-$
A_2 (1)	0^-	$0^-, 1^+, 2^-, 3^+$
E (2)	1	$1^\pm, 2^\pm, 3^\pm$
	3	3^\pm
B_1 (1)	2	$2^\pm, 3^\pm$
B_2 (1)	2	$2^\pm, 3^\pm$

$\mathbf{p} = (1, 1, 0), Dic_2$		
Λ (dim)	$ \lambda ^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	$0^+, 1^-, 2^+, 3^-$
	2	$2^\pm, 3^\pm$
A_2 (1)	0^-	$0^-, 1^+, 2^-, 3^+$
	2	$2^\pm, 3^\pm$
B_1 (1)	1	$1^\pm, 2^\pm, 3^\pm$
	3	3^\pm
B_2 (1)	1	$1^\pm, 2^\pm, 3^\pm$
	3	3^\pm

1811.04116, PRD 2019

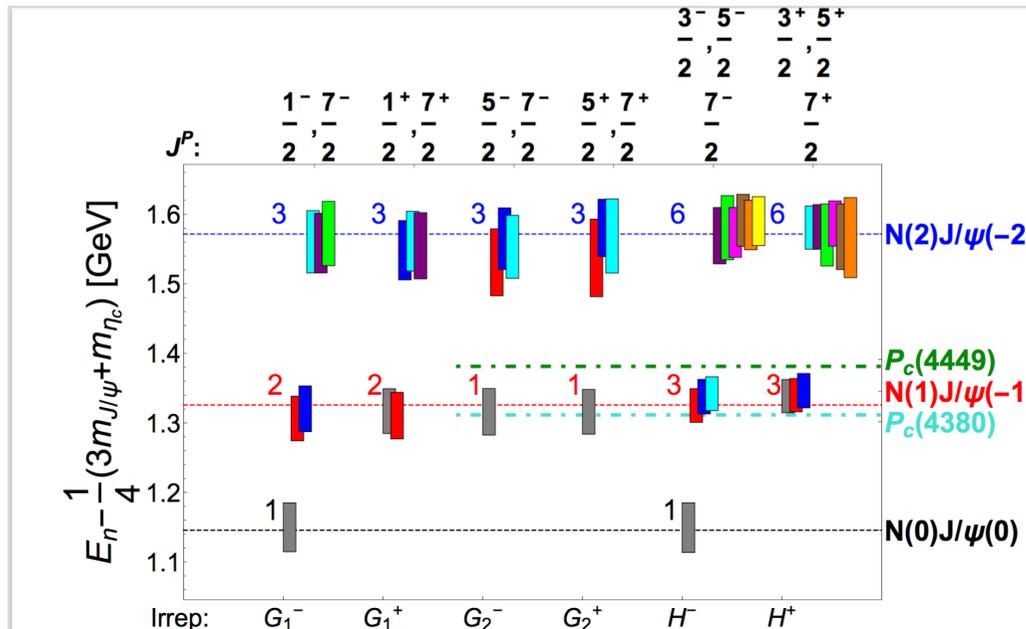
$$\cosh(bE(\mathbf{p})) = \cosh(bM) + \sum_{i=x,y,z} \left[2 \sin\left(\frac{bp_i}{2}\right) \right]^2$$

$$E(\mathbf{p}) = \sqrt{M^2 + \mathbf{p}^2} \text{ for } b \rightarrow 0$$



1811.04116, PRD 2019

proton J/ψ scattering in lattice QCD in P_c channels



U. Skerbis, S. Prelovsek, 1811.02285

