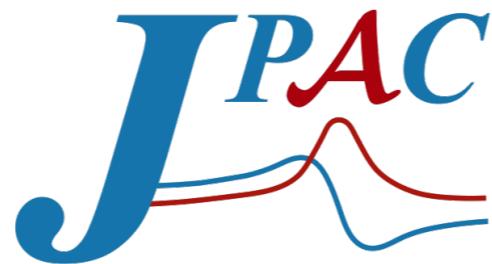


# How to reconstruct resonances (and determine where they come from)

**Adam Szczepaniak, Indiana University/Jefferson Lab**



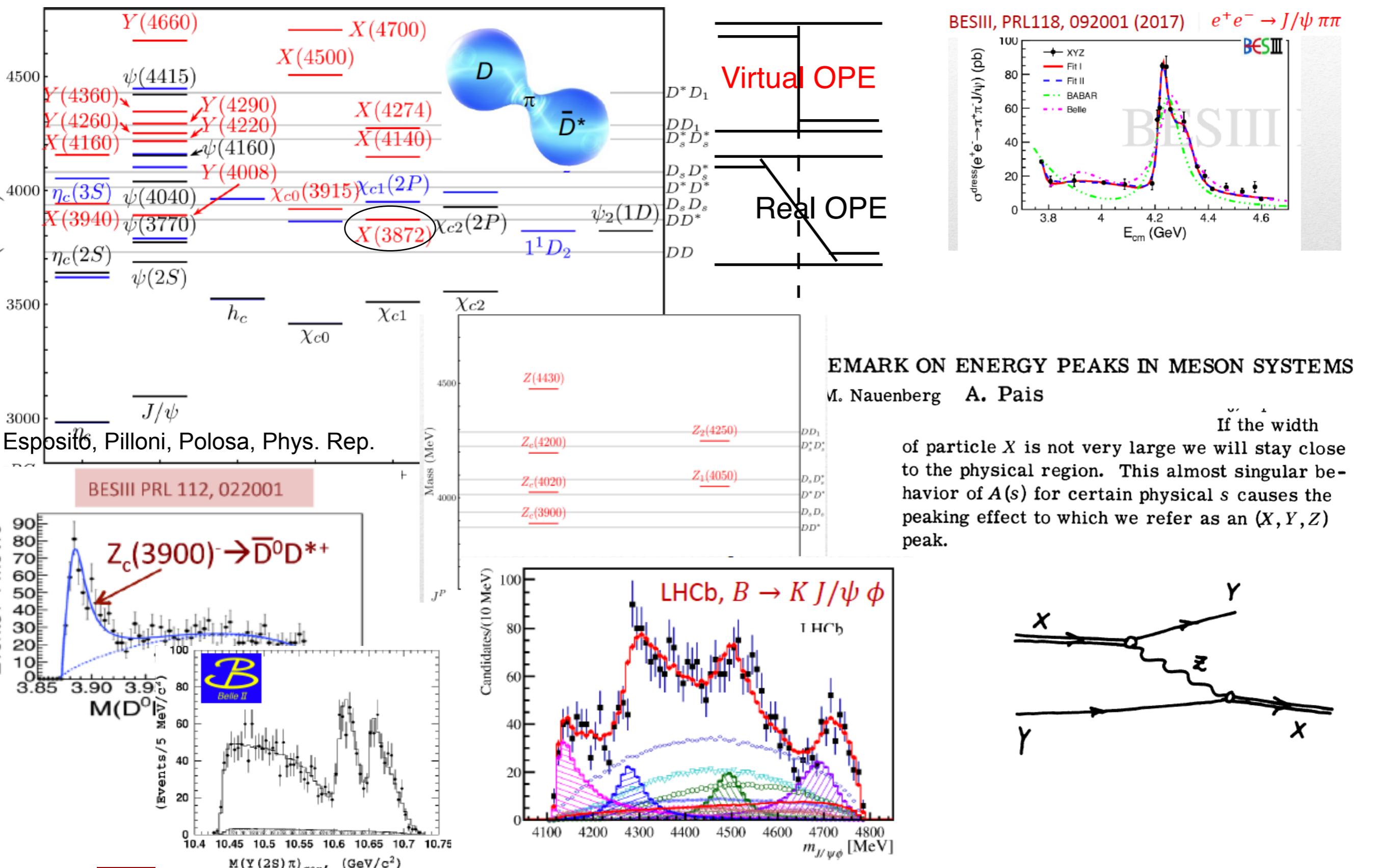
**Join Physics Analysis Center**



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# Signatures of unusual heavy quark resonances

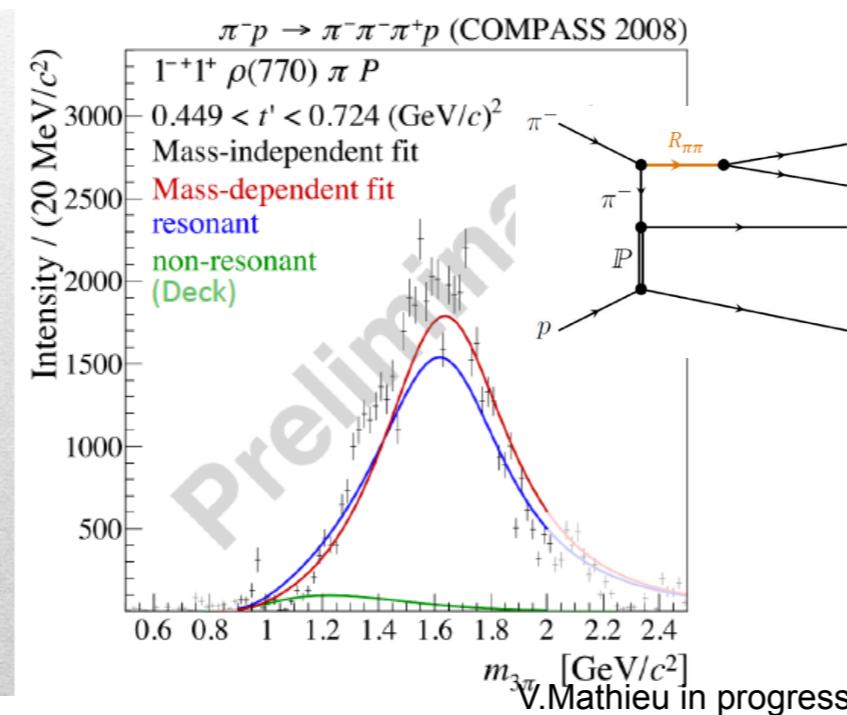
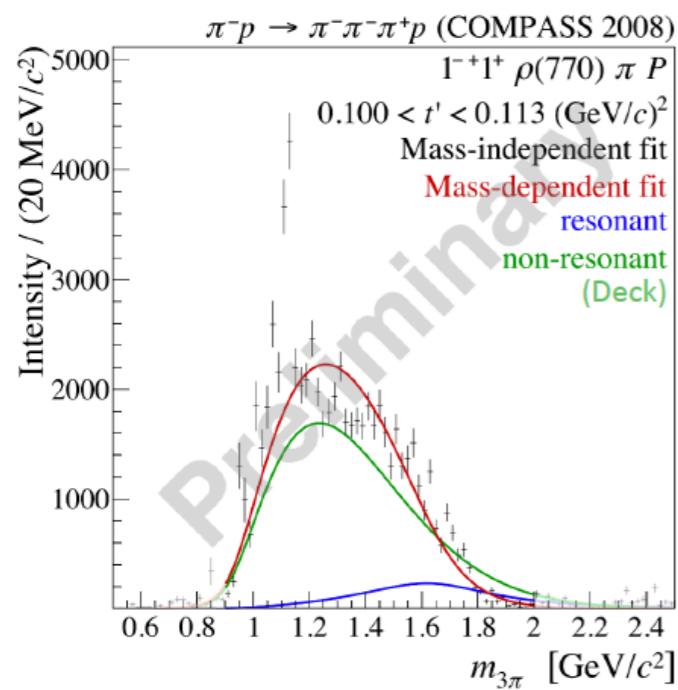


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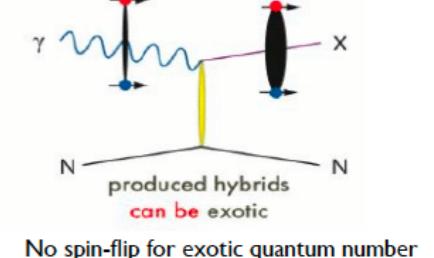
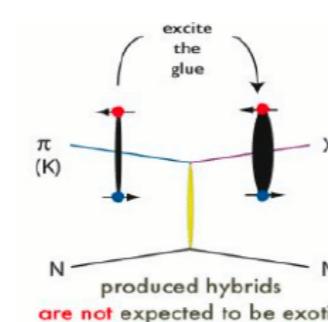


# Signatures of new, unusual light resonances

- Exotic  $J^{PC}=1^{-+}$  (hybrid) mesons expected (VES, GAMS, E852, COMPASS, and theory)
- In low- $t$  pion diffraction (COMPASS) exotic wave production compatible with one pion exchange (but not at high- $t$ )

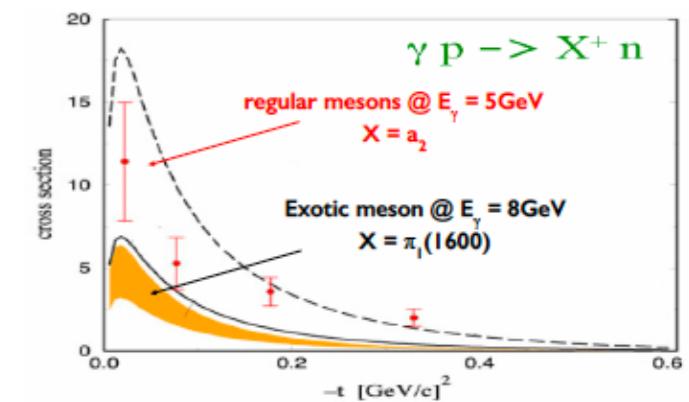
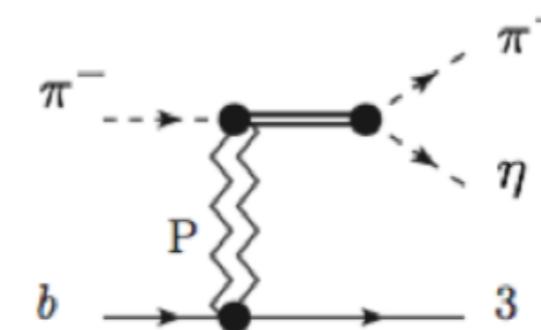
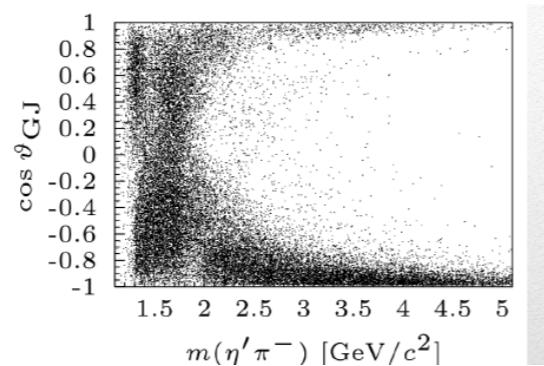


- In photoproduction (GlueX) exotic mesons produced via pion exchange (both good and bad)



A.Afanasev and P.Page et al. PR A57 1998 6771  
A.Szczepaniak and M.Swat PLB 516 2001 72

- Large exotic wave seen in  $\eta^{(')}\pi$  production : Golden Channel



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# Amplitude analysis : connecting data to QCD

Bottom → Up

Top → Bottom

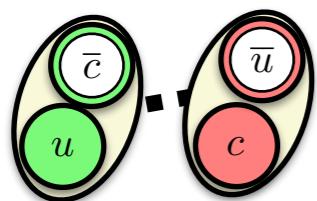
Amplitudes

Rules  
(bubbles,  
regularization,  
renormalization,  
etc. )

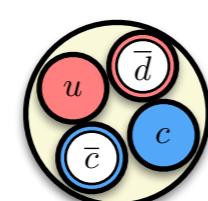


Data

Microscopic model



Mesonic-Molecules



Tetraquarks

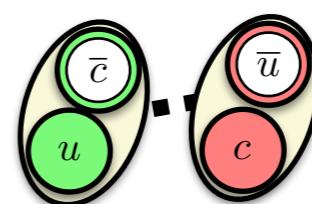
Amplitudes

Data

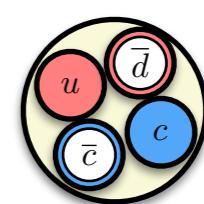
Physical  
interpretation of  
poles, cuts,



Microscopic model



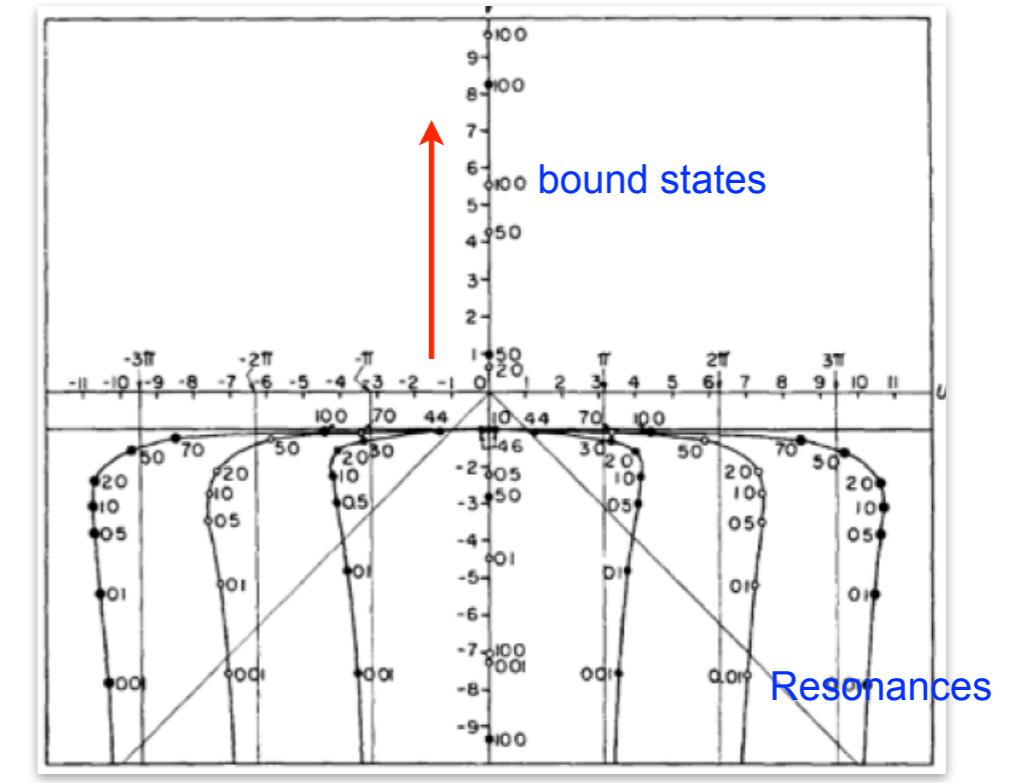
Mesonic-Molecules



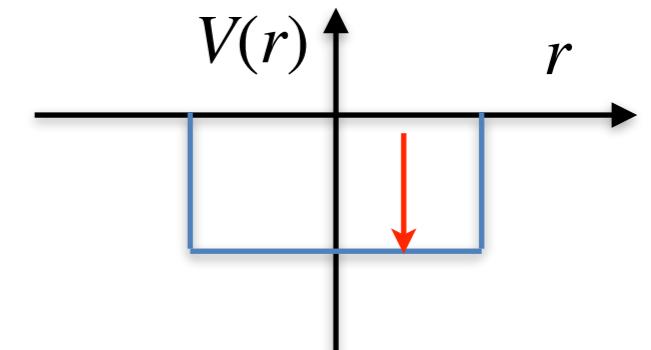
Tetraquarks

# In practice

- Reconstruct amplitudes from its singularities (poles, cuts) Recall that each singularity has its own physical interpretation
- Use data to determine best hypothesis
- Test how singularities depend on parameters (channel couplings, thresholds, etc.) to infer their microscopic origins.



Virtual states

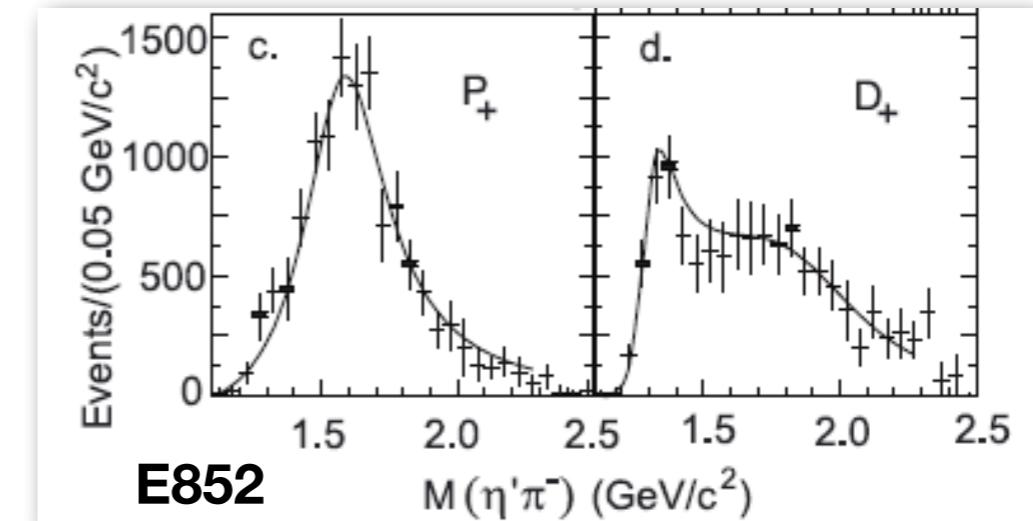


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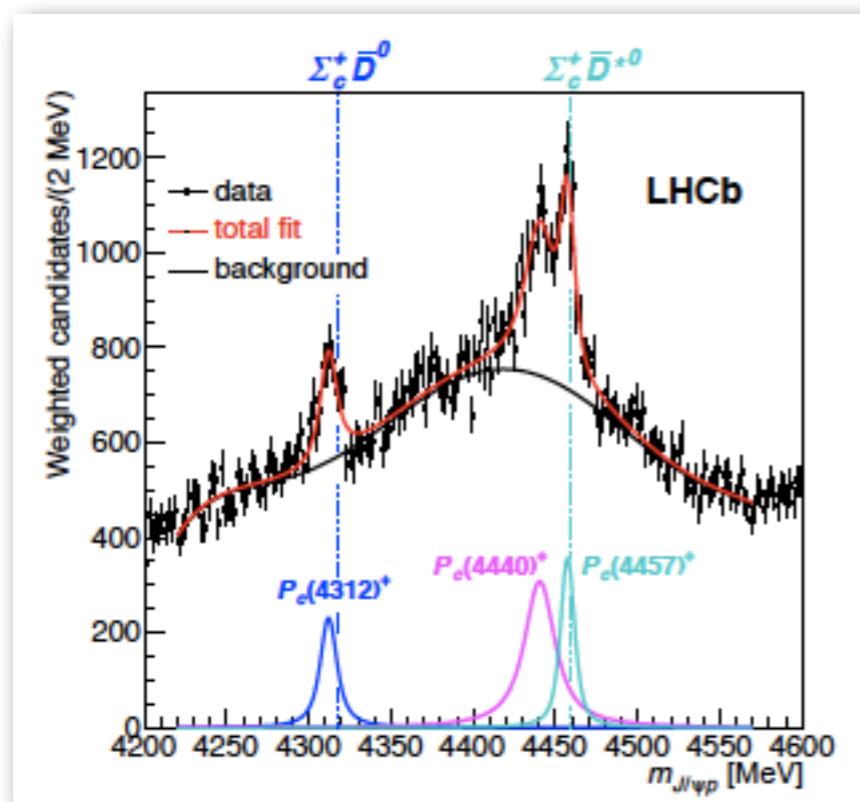


# In this talk

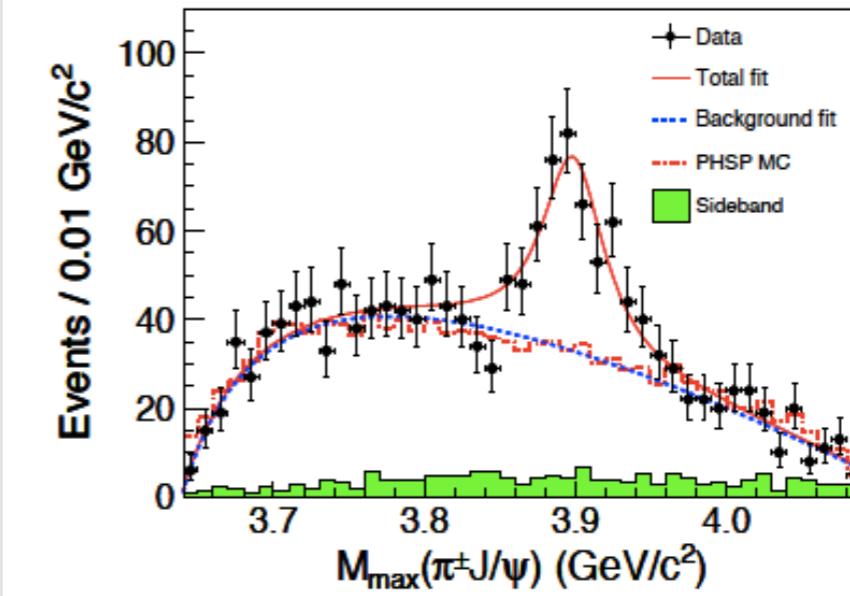
- $J^{PC}=1^{-+}$   $|I|=1$ , light exotic hybrid ?



- $Z_c(3900)$  in  $J/\psi \pi \pi$ ,  $\bar{D}D^*$  ?



BESIII



- $P_c(4312)$  in  $\Lambda_b \rightarrow J/\psi p K$



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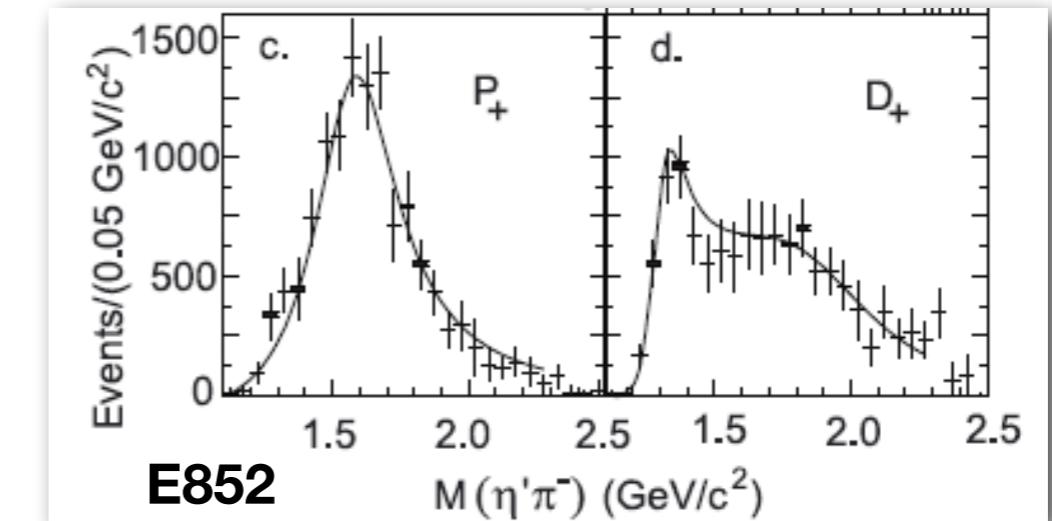
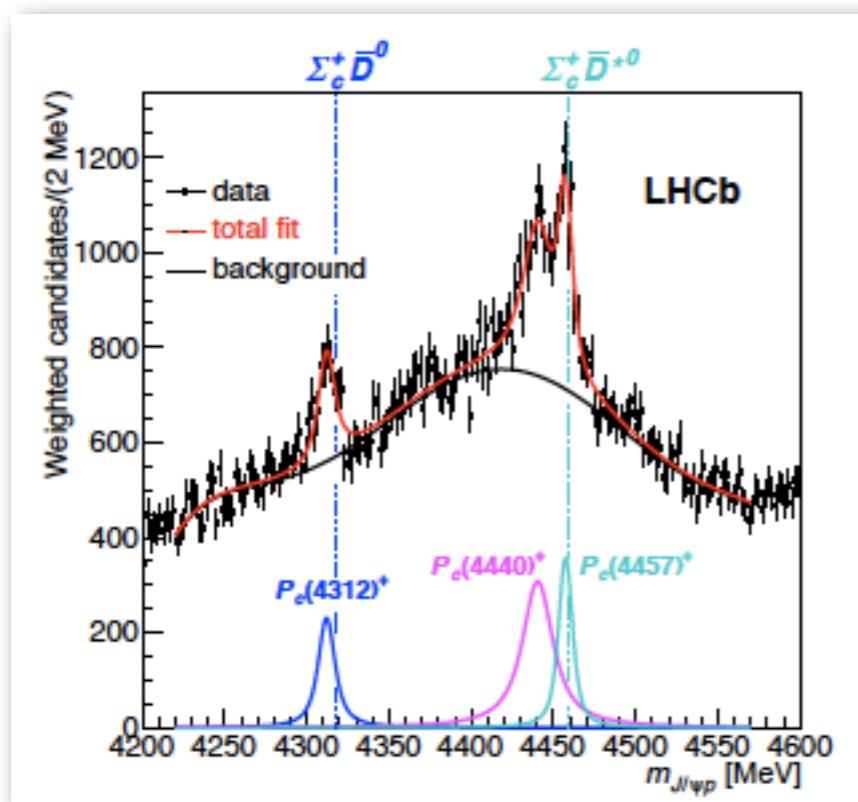
# In this talk

- $J^{PC}=1^{-+}$   $I=1$ , light exotic hybrid ?

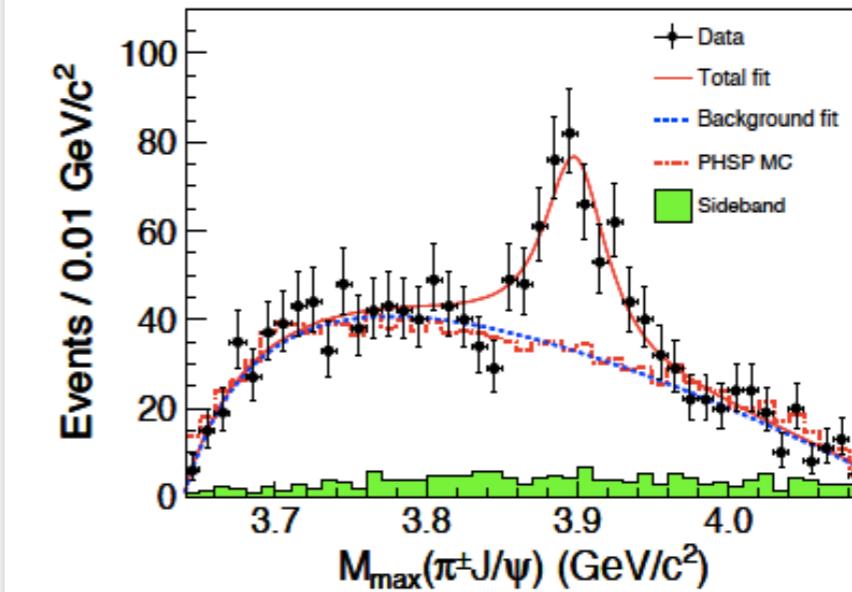
Yes : “Normal resonance”

- $Z_c(3900)$  in  $J/\psi \pi \pi, \bar{D}D^*$  ?

Inconclusive



BESIII



- $P_c(4312)$  in  $\Lambda_b \rightarrow J/\psi p K$

No : Unbound

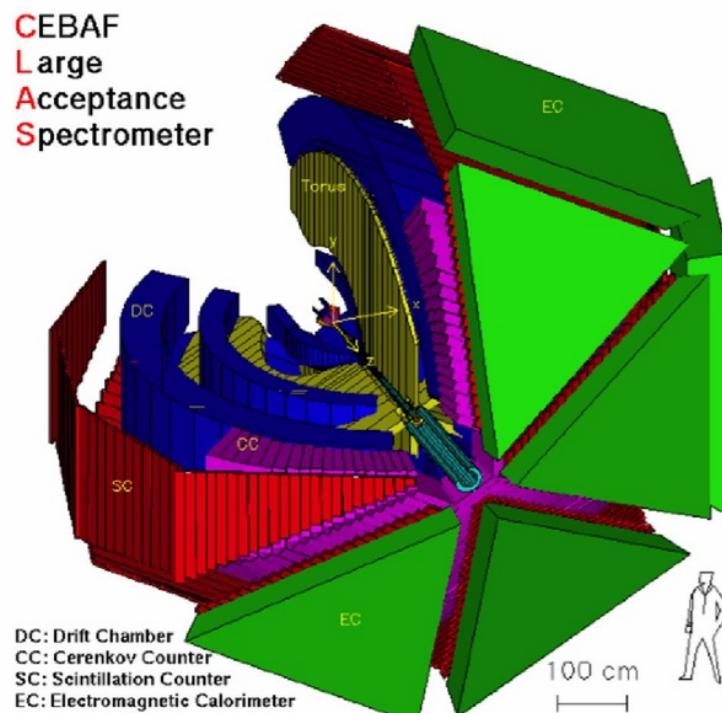
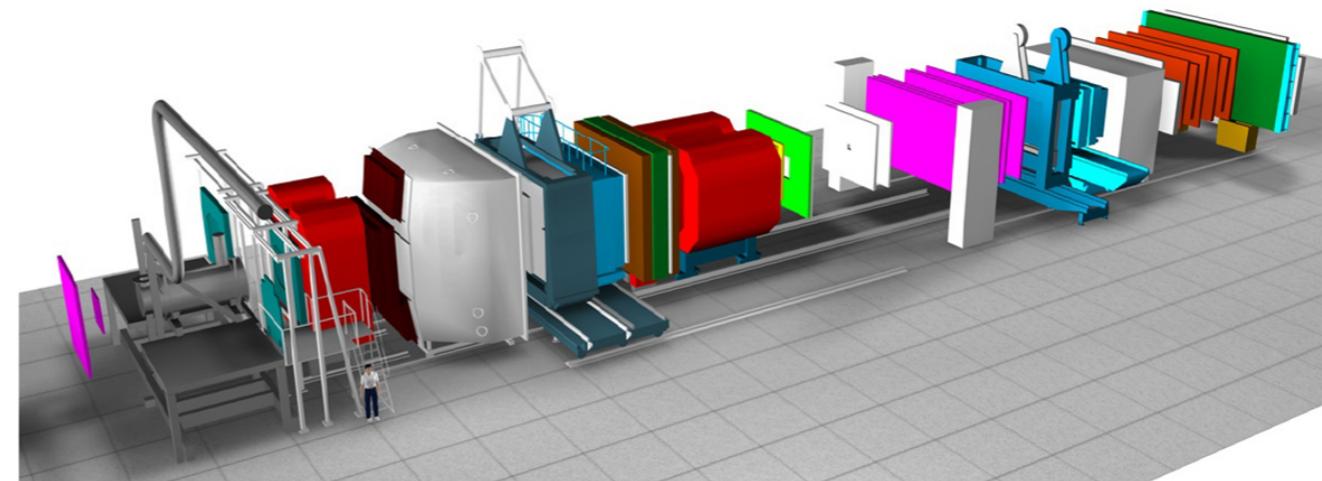
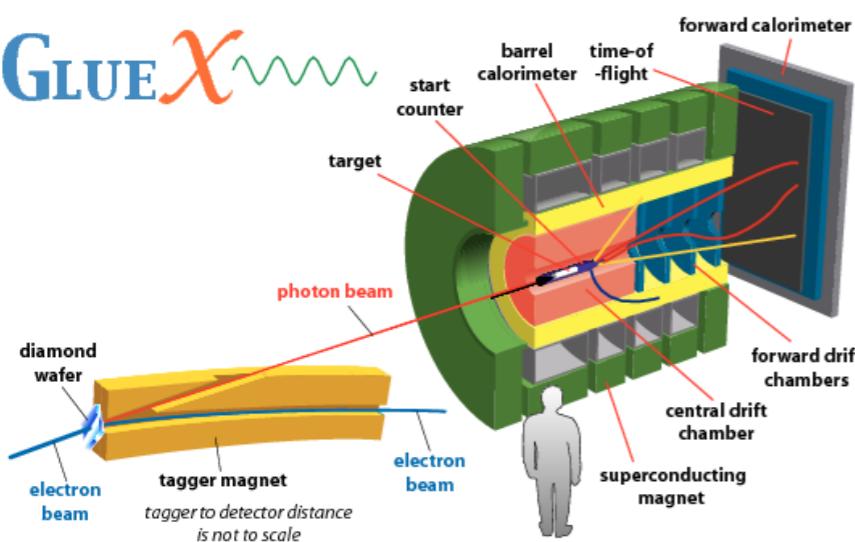


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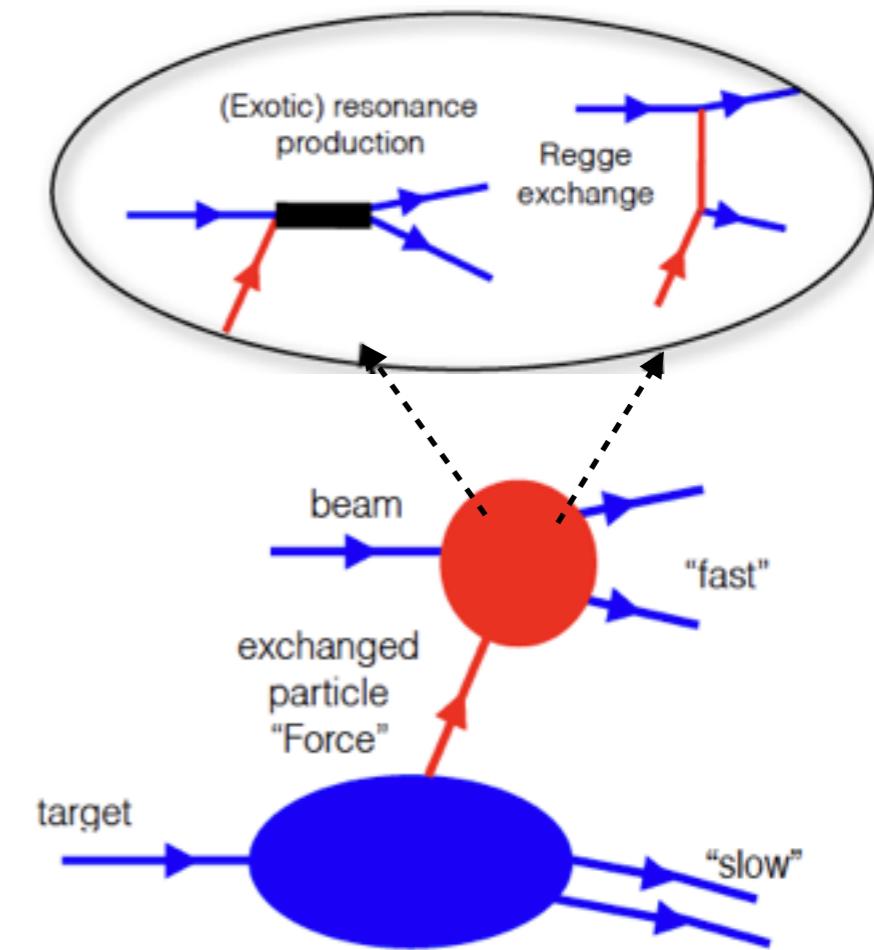


# Spectroscopy from peripheral production

GLUE $\chi$



- Need to establish factorization between beam and target fragmentation (Regge factorization)
- Single Regge pole exchange dominate over cut other singularities (cuts, daughters)

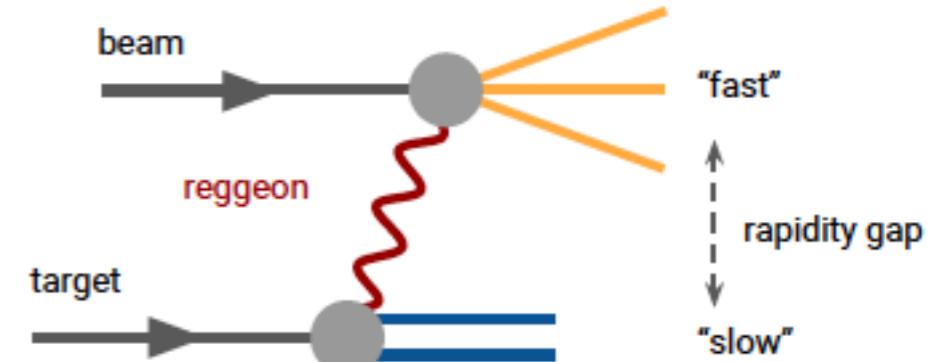


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# Global Regge analysis

- Test Regge pole hypothesis and estimate corrections (daughters, cuts)
- Factorizable Regge pole exchange



$$\mathcal{R}(s, t) \equiv \left( \frac{1 - z_s}{2} \frac{\nu}{-t} \right)^{\frac{1}{2}|\mu - \mu'|} \left( \frac{1 + z_s}{2} \right)^{\frac{1}{2}|\mu + \mu'|}$$

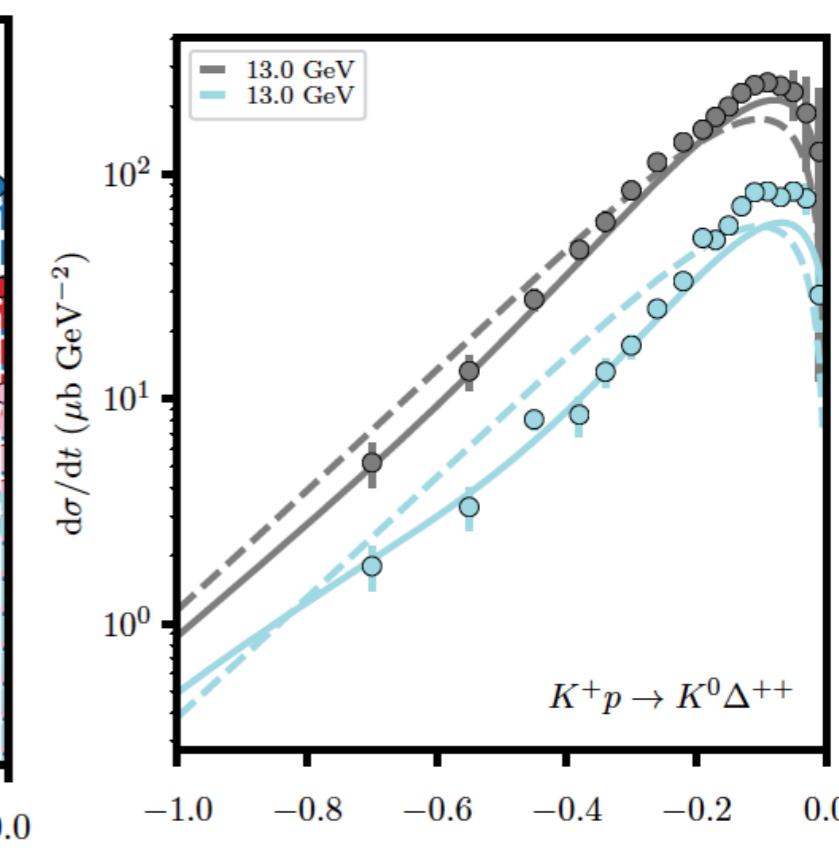
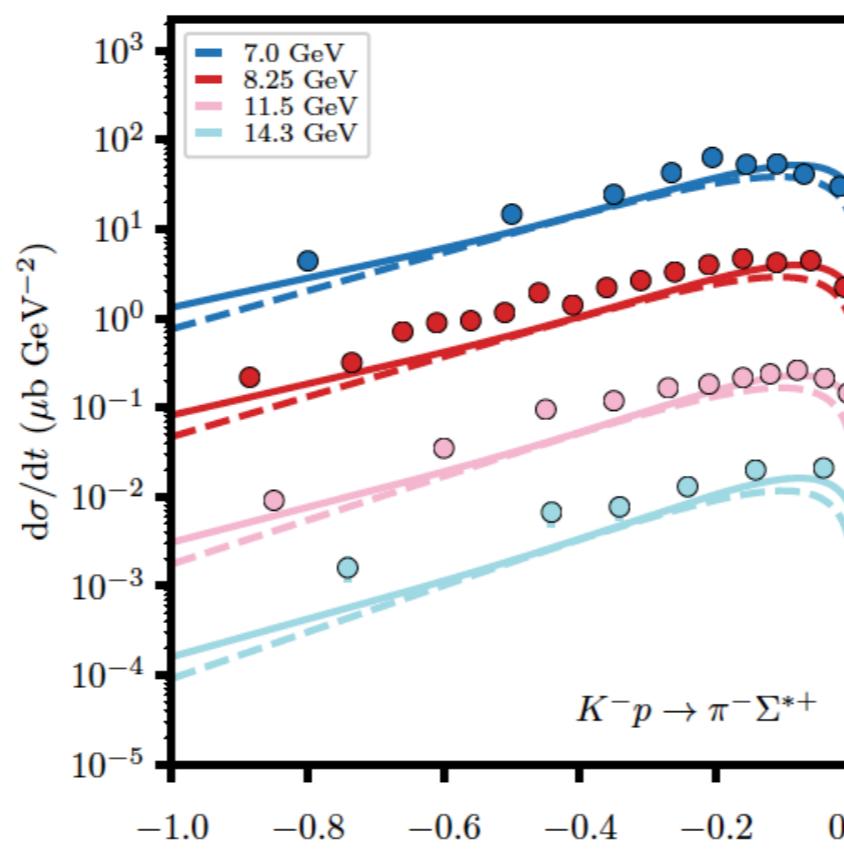
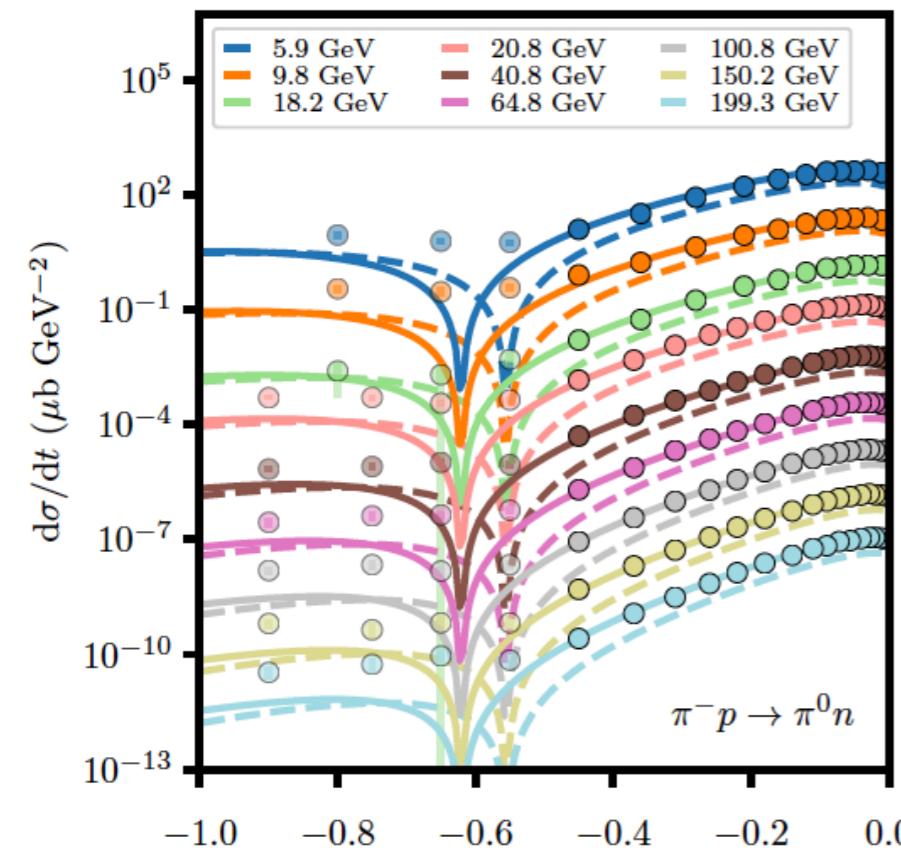
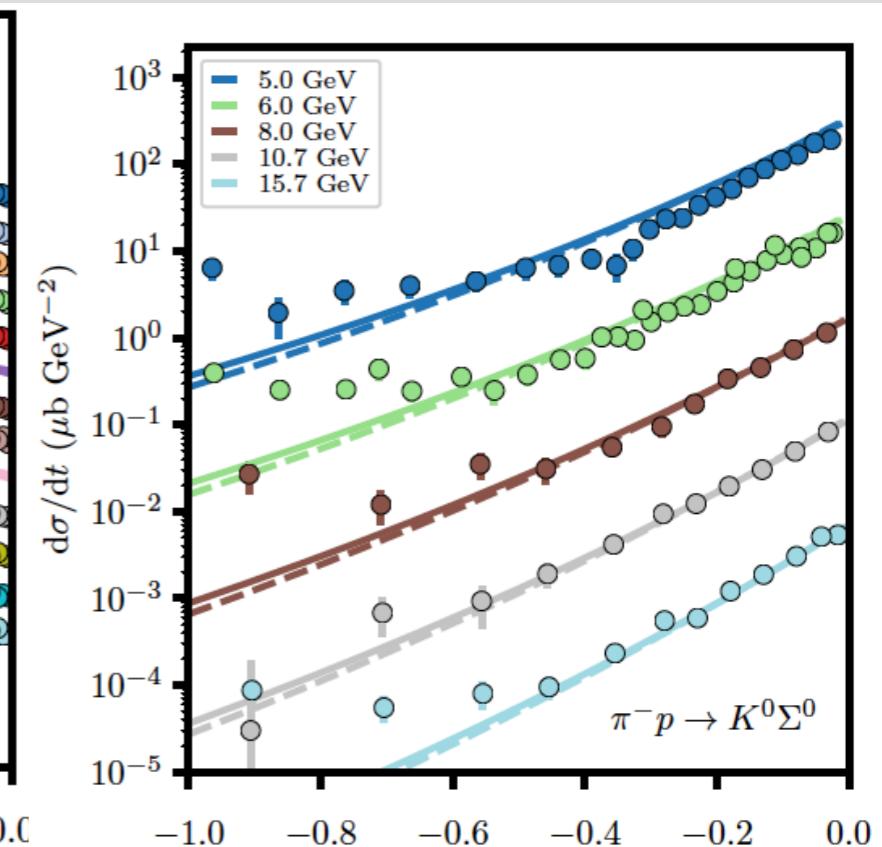
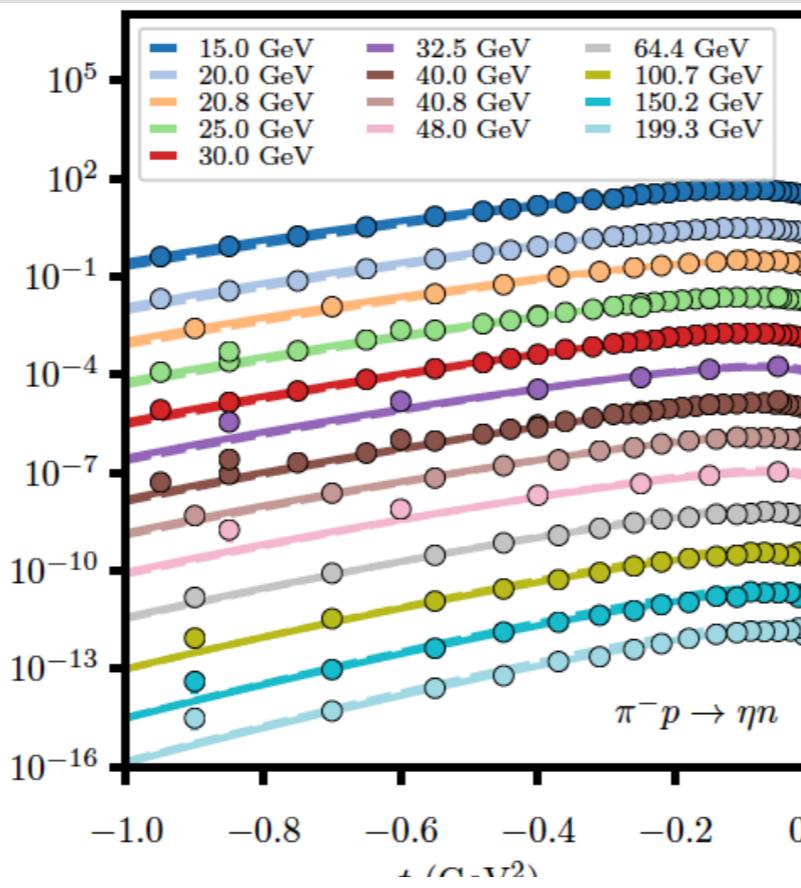
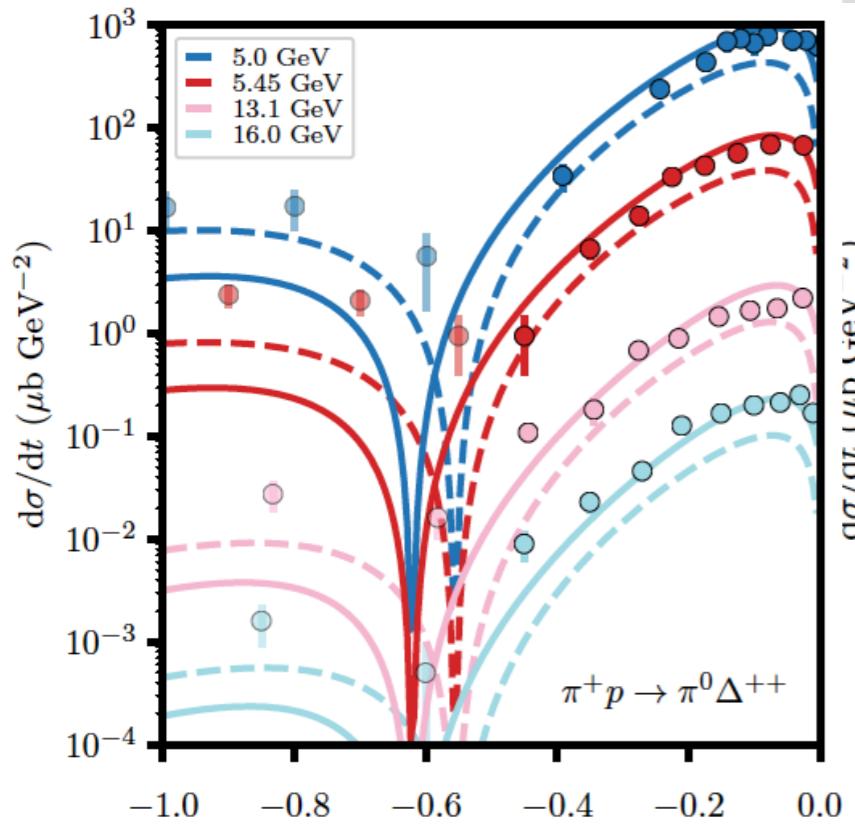
$$A_{\mu_4 \mu_3 \mu_2 \mu_1} = \mathcal{R}(s, t) \sqrt{-t}^{|\mu_1 - \mu_3|} \sqrt{-t}^{|\mu_2 - \mu_4|} \hat{\beta}_{\mu_1 \mu_3}^{e13}(t) \hat{\beta}_{\mu_2 \mu_4}^{e24}(t) \mathcal{F}_e(s, t)$$

$$\mathcal{F}_e(s, t) = -\frac{\zeta_e \pi \alpha_e^1}{\Gamma(\alpha_e(t) - l_e + 1)} \frac{1 + \zeta_e e^{-i\pi\alpha_e(t)}}{2 \sin \pi\alpha_e(t)} \left( \frac{s}{s_0} \right)^{\alpha_e(t)}$$

- $N_{\text{Data}}=1271$ ,  $N_{\text{par}}=9$   
(6 SU(3) couplings, 1 mixing angle, 2 exp. slopes )

$$\mathcal{F}_e(s, t) \xrightarrow{t \rightarrow m_e^2} \frac{(s/s_0)^{J_e}}{m_e^2 - t}.$$

# Global Regge pole analysis



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# The $\pi_1$ (exotic) meson candidate

$\pi^- p \rightarrow \eta \pi^- p$

$$M = 1370 \pm 16^{+50}_{-30} \text{ MeV / } c^2$$

$$\Gamma = 385 \pm 40^{+65}_{-105} \text{ MeV / } c^2$$

$\pi_1(1400)$  E852, also GAMS,  
VES, Crystal Barrel

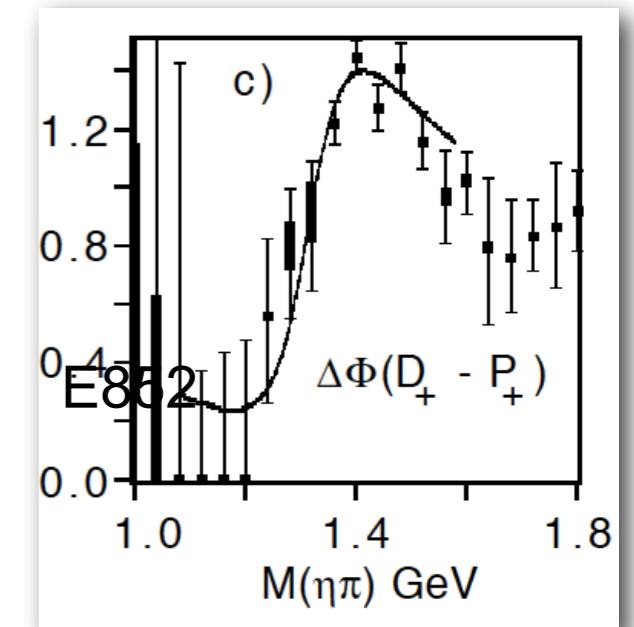
$\pi^- p \rightarrow \eta' \pi^- p$

$$M = 1597 \pm 10^{+45}_{-10} \text{ MeV / } c^2$$

$$\Gamma = 340 \pm 40^{+50}_{-50} \text{ MeV / } c^2$$

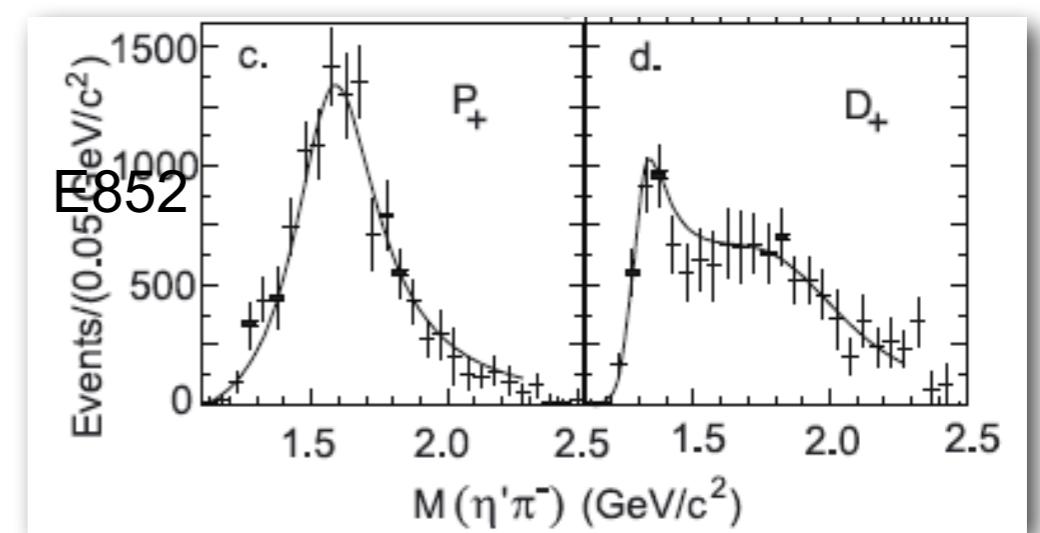
$\pi_1(1600)$

E852, also COMPASS, CLEO



Is it 1400 or 1600 ?

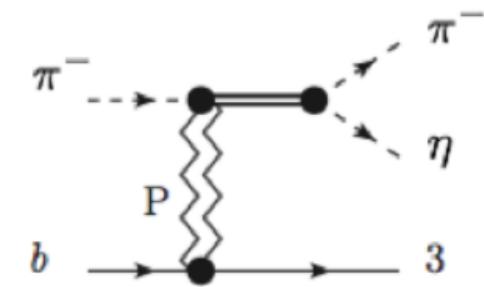
COMPASS consistent with both



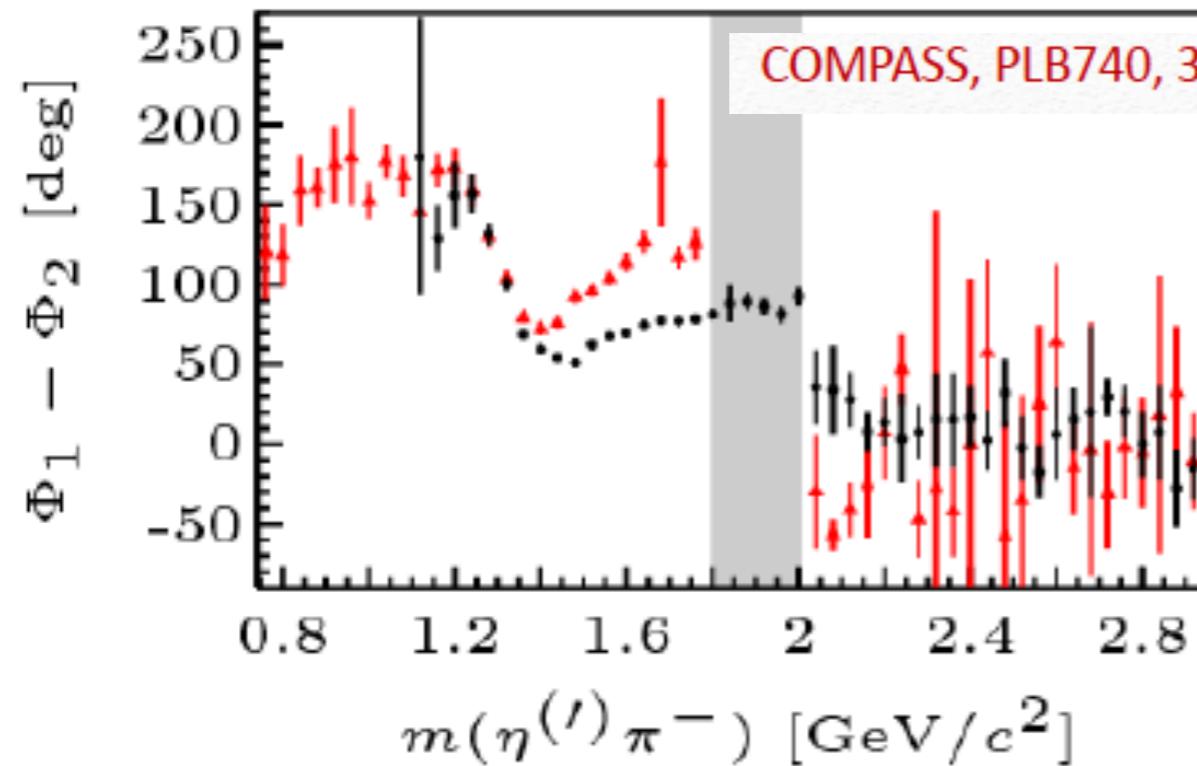
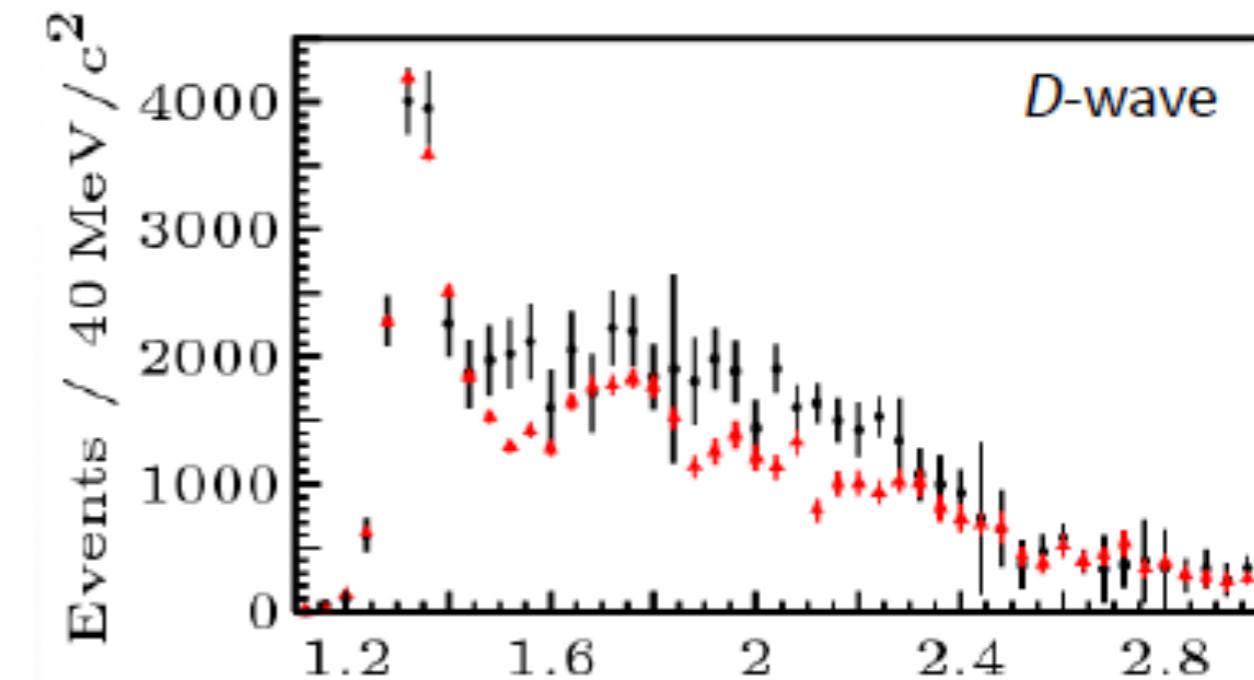
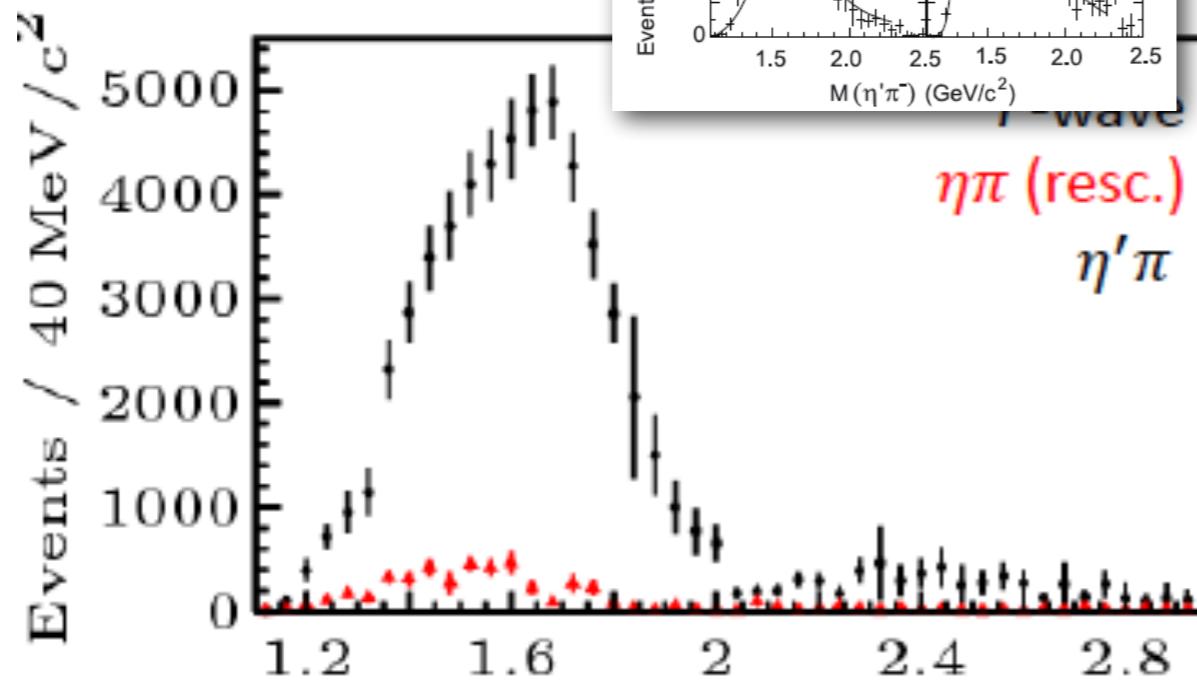
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# COMPASS data



## Data



A sharp drop appears at  $2 \text{ GeV}$  in  $P$ -wave intensity and phase

No convincing physical motivation for it

It affects the position of the  $a'_2(1700)$

We decided to fit up to  $2 \text{ GeV}$  only



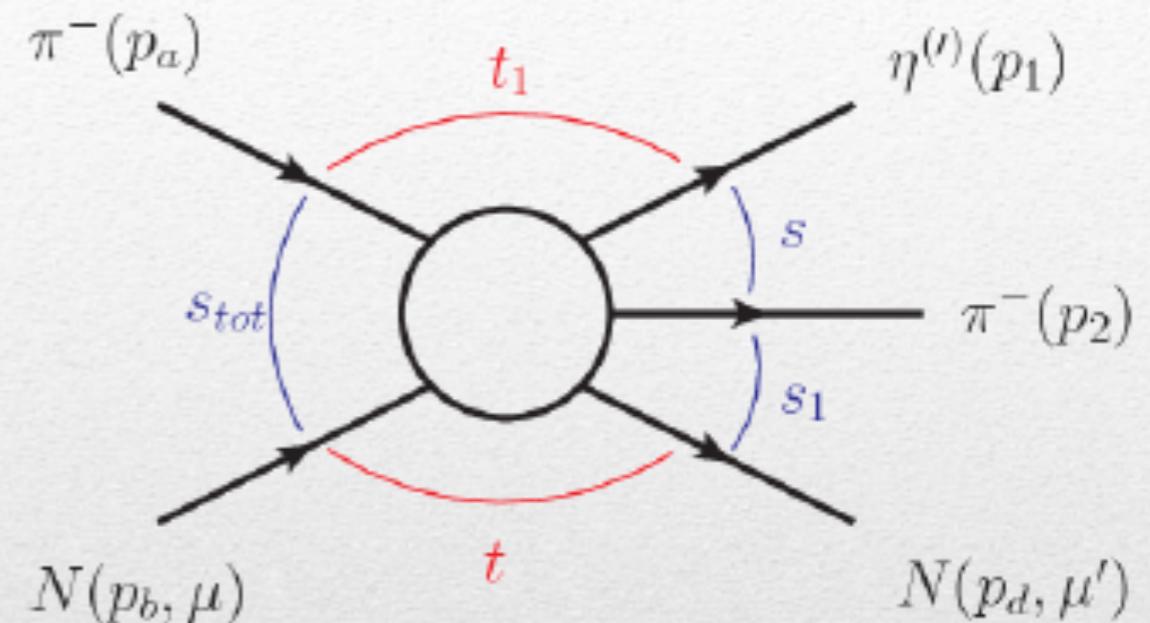
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# 2-meson peripheral production : $\eta\pi$

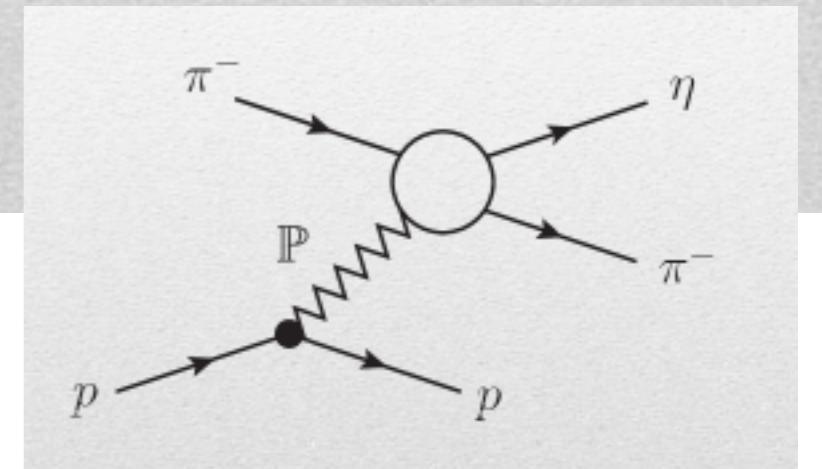
$$\pi^- p \rightarrow \eta^{(\prime)} \pi^- p$$

- Process is at fixed  $s_{tot}$ , and integrated  $t$ . Interested in resonances in  $s$
- Recoil proton kinematically decouples from final state  $\eta\pi$
- Expand amplitude into partial waves



$$A_{\mu'\mu}(s_{tot}, s, t, s_1, t_1) = \sum_{LM\epsilon} a_{LM,\mu'\mu}^\epsilon(s_{tot}, t, s) Y_{LM}^\epsilon(\theta, \phi)$$

$$a_{LM,\mu'\mu}^\epsilon(s_{tot}, t, s) \rightarrow a_{L,M=\pm 1}^1(t, s)$$



# Coupled channel: the model

A. Rodas, AP et al. (JPAC), to appear

Two channels,  $i, k = \eta\pi, \eta'\pi$

Two waves,  $J = P, D$

37 fit parameters

$$D_{ki}^J(s) = [K^J(s)^{-1}]_{ki} - \frac{s}{\pi} \int_{s_k}^{\infty} ds' \frac{\rho N_{ki}^J(s')}{s'(s' - s - i\epsilon)}$$

$$K_{ki}^J(s) = \sum_R \frac{g_k^{(R)} g_i^{(R)}}{m_R^2 - s} + c_{ki}^J + d_{ki}^J s$$

1 K-matrix pole for the P-wave  
2 K-matrix poles for the D-wave

$$\rho N_{ki}^J(s') = g \delta_{ki} \frac{\lambda^{J+1/2} (s', m_{\eta'}^2, m_\pi^2)}{(s' + s_R)^{2J+1+\alpha}} \quad n_k^J(s) = \sum_{n=0}^3 a_n^{J,k} T_n \left( \frac{s}{s + s_0} \right)$$

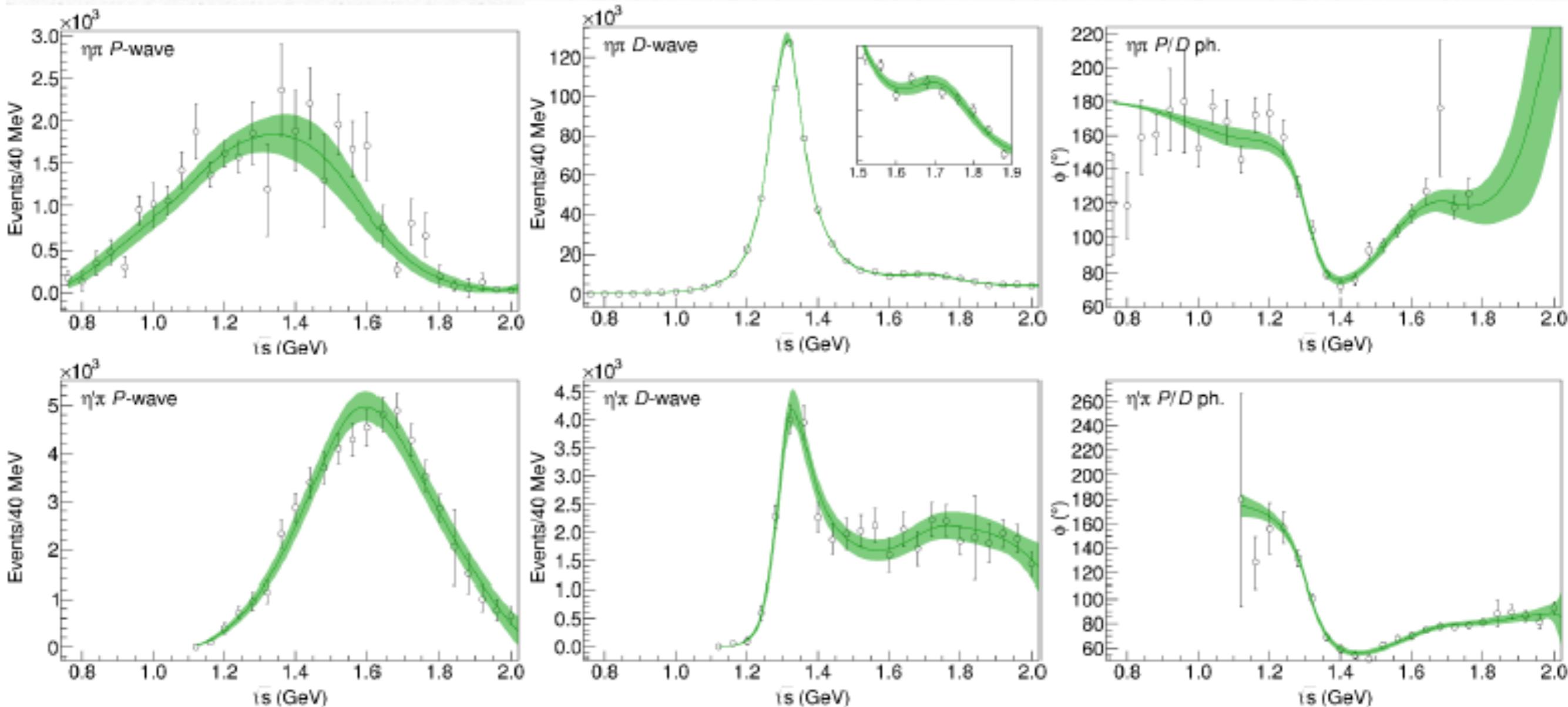
Left-hand scale (Blatt-Weisskopf radius)  $s_R = s_0 = 1 \text{ GeV}^2$   
 $\alpha = 2$  as in the single channel, 3rd order polynomial for  $n_k^J(s)$



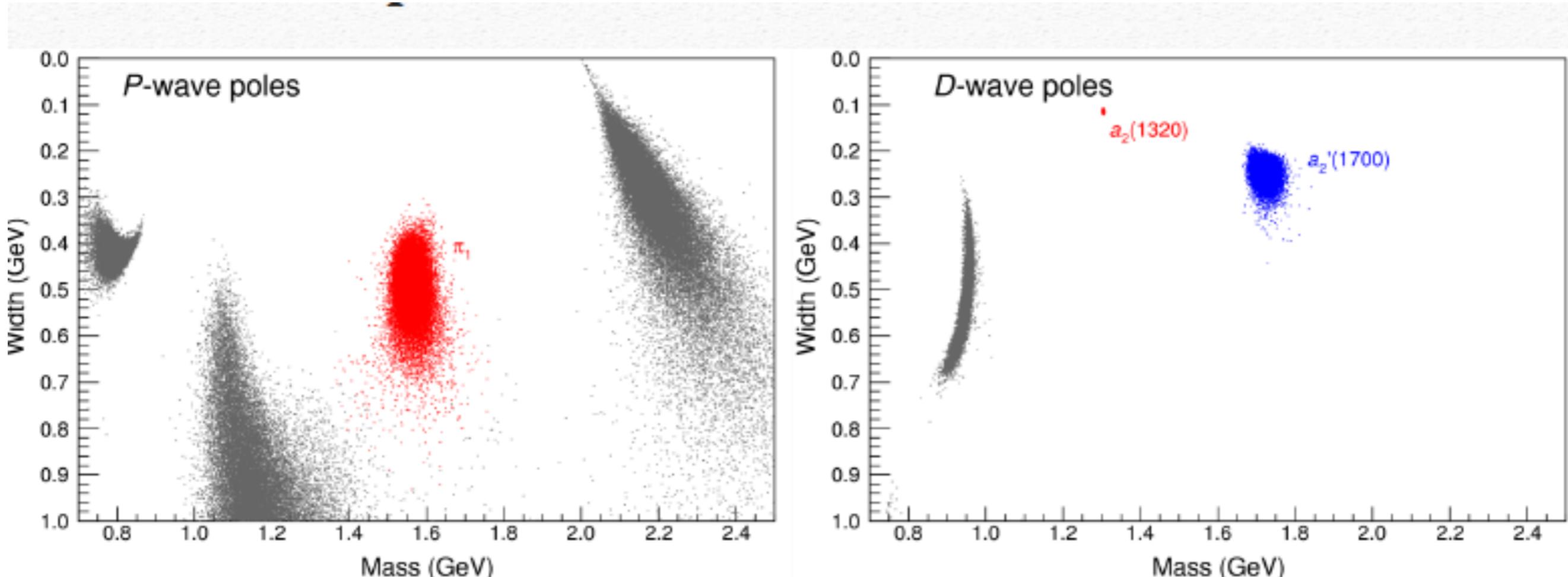
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# Fit



$\chi^2/\text{dof} = 162/122 \sim 1.3$ , statistical error estimated via 50k bootstraps  
 Bands show the  $2\sigma$  error

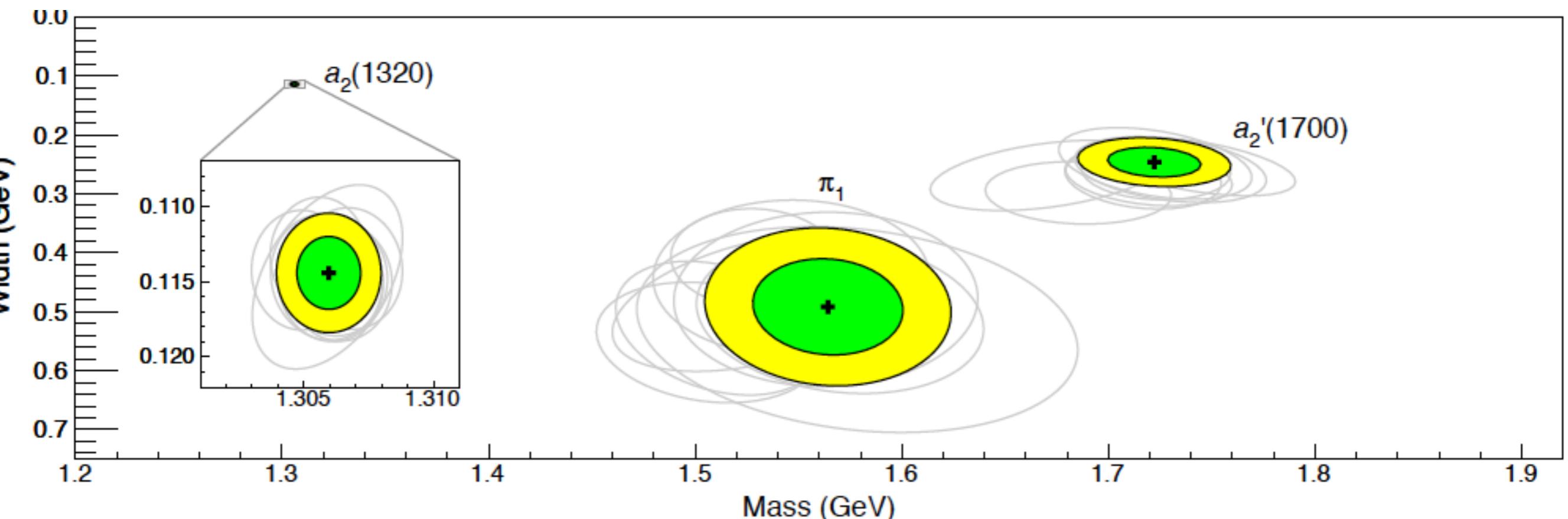


We can identify the poles in the region  $m \in [1.2, 2]$  GeV,  $\Gamma \in [0, 1]$  GeV

Two stable isolated poles are identifiable in the  $D$ -wave  
Only one is stable in the  $P$ -wave

# Bootstrap + model variations

17



The variance of the bootstrapped poles gives the statistical error

Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	$1306.0 \pm 0.8$	$114.4 \pm 1.6$
$a_2'(1700)$	$1722 \pm 15$	$247 \pm 17$
$\pi_1$	$1564 \pm 24$	$492 \pm 54$



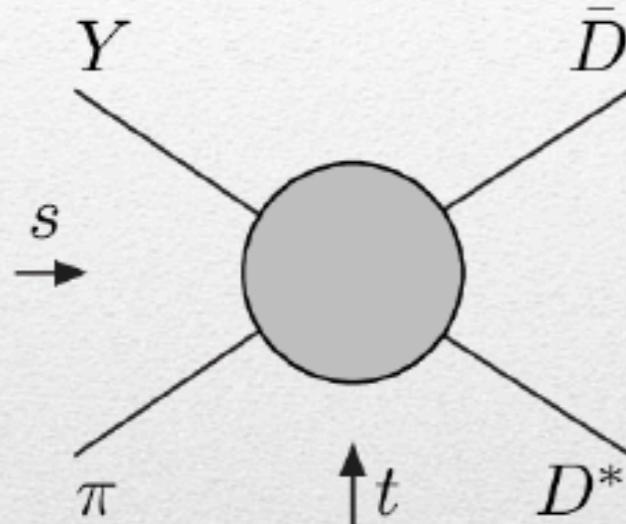
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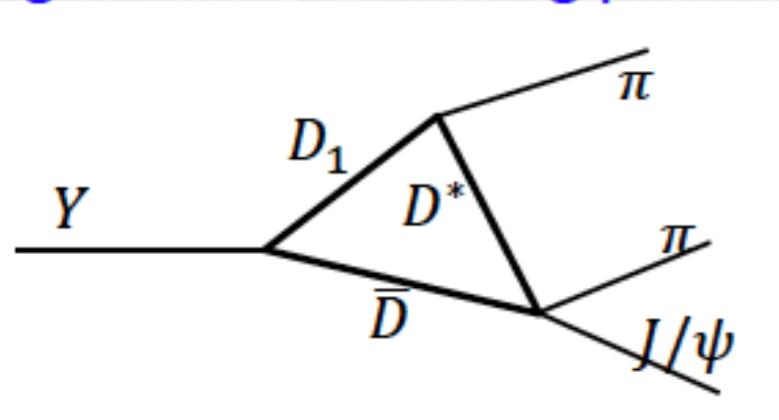
# Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to different singularities → different natures

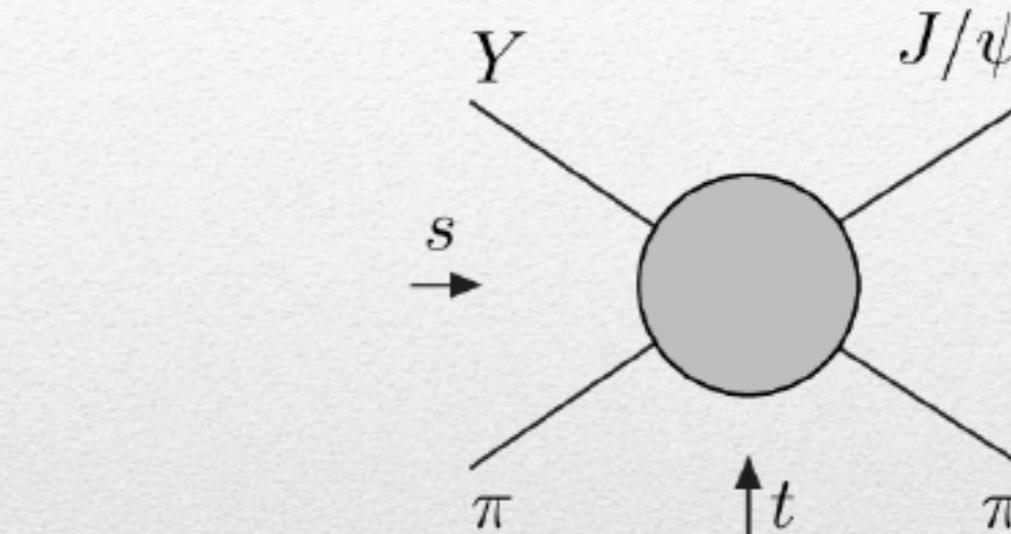
AP *et al.* (JPAC), arXiv:1612.06490



Triangle rescattering,  
logarithmic branching point



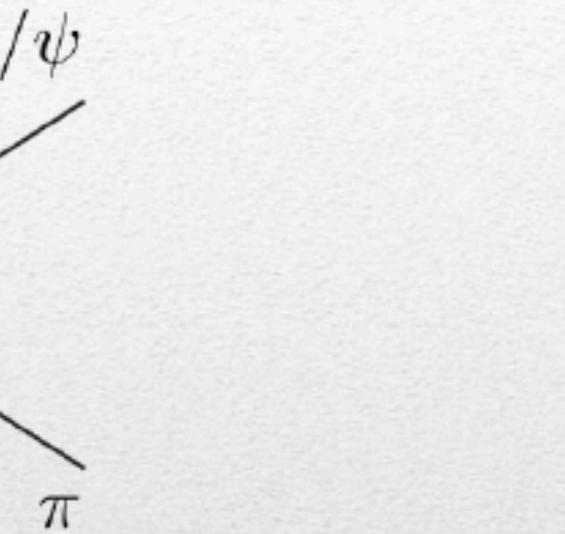
Szczepaniak, PLB747, 410-416  
Szczepaniak, PLB757, 61-64  
Guo *et al.* PRD92, 071502



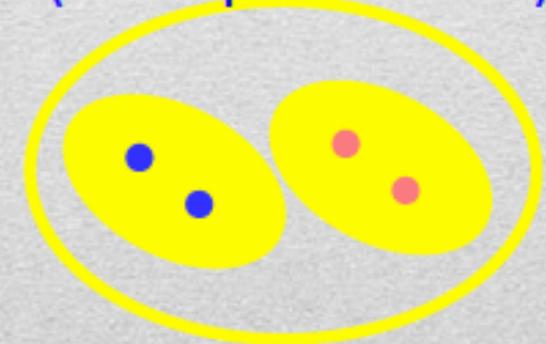
(anti)bound state,  
II/IV sheet pole  
«molecule»



Tornqvist, Z.Phys. C61, 525  
Swanson, Phys.Rept. 429  
Hanhart *et al.* PRL111, 132003

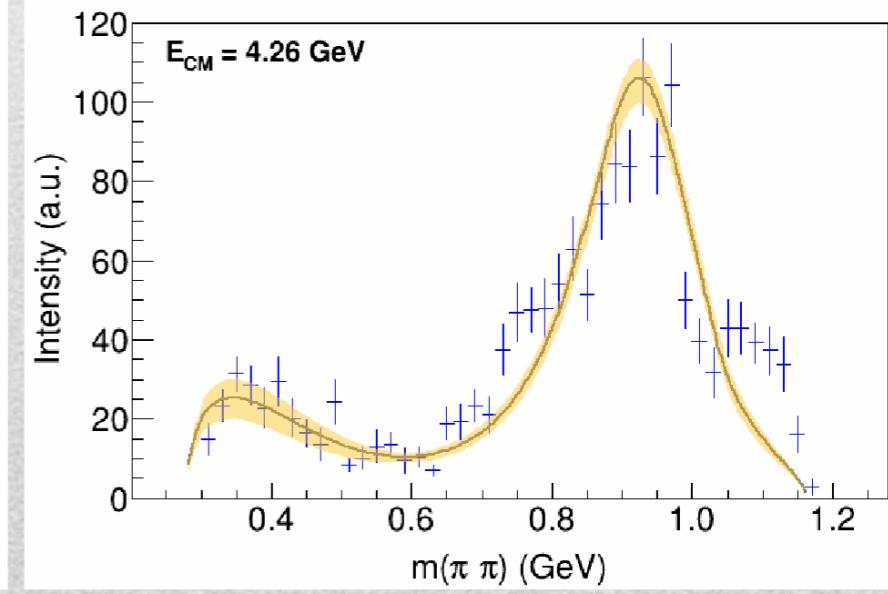
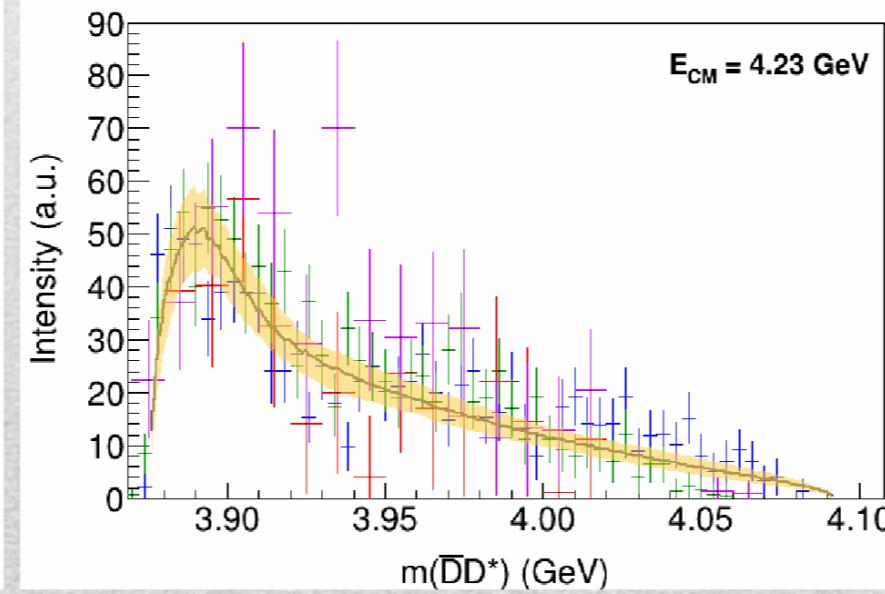
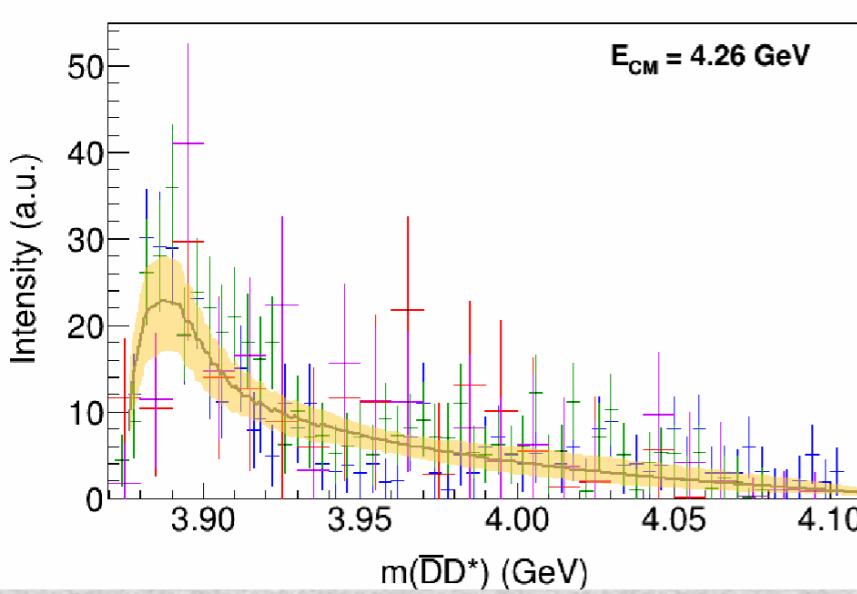
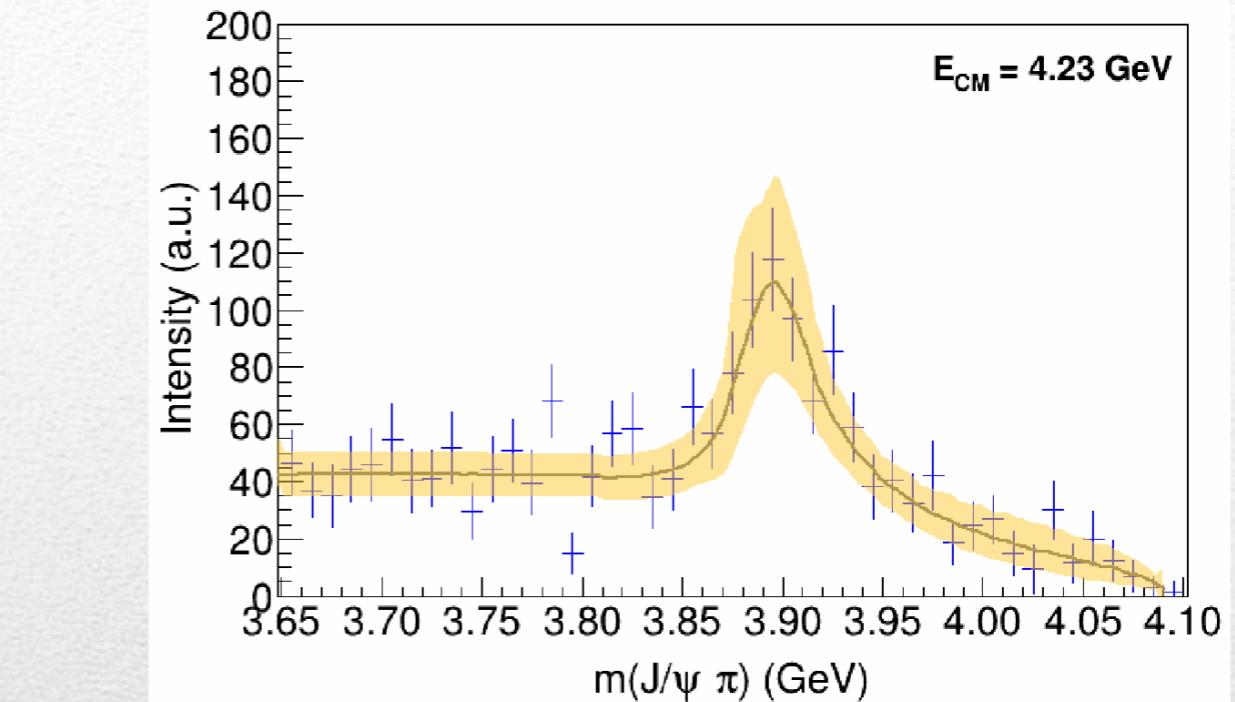
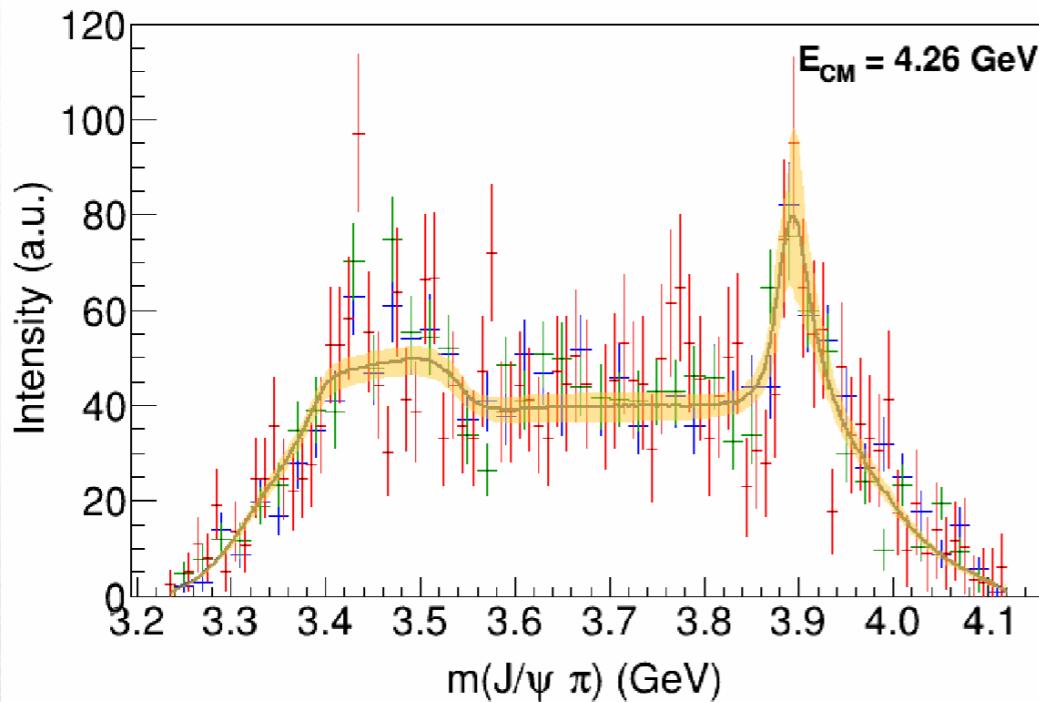


Resonance,  
III sheet pole  
«compact state»

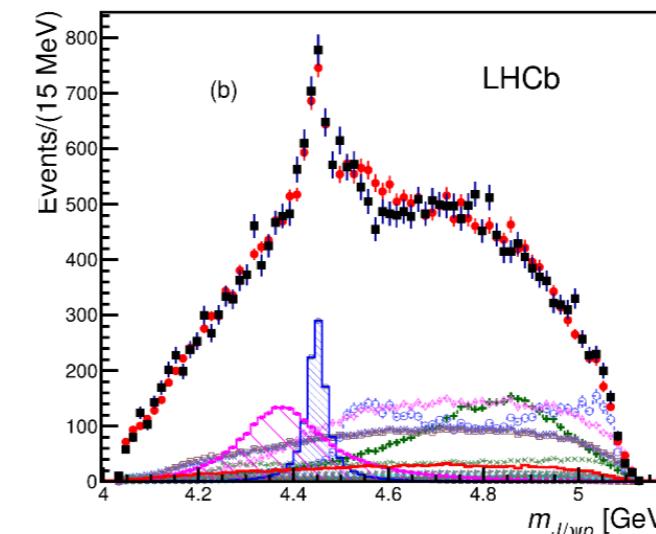
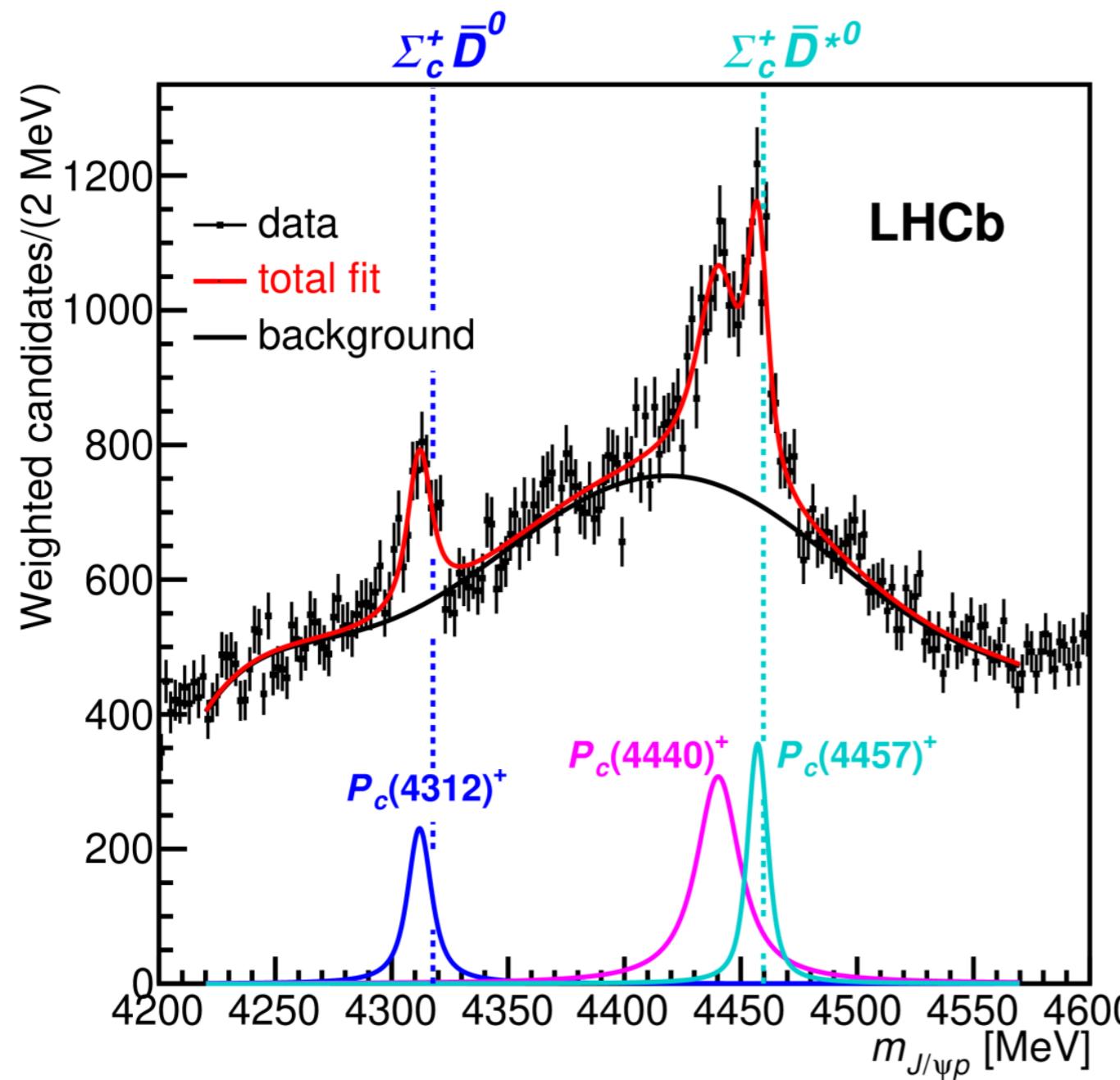


Maiani *et al.*, PRD71, 014028  
Faccini *et al.*, PRD87, 111102  
Esposito *et al.*, Phys.Rept. 668

# Fit: III



# New pentaquarks ?



The lowest  $P_c(4312)$  appears as an isolated peak at the  $\Sigma_c^+ D^0$  threshold

A detailed study of the line-shape can provide insight on its nature.

Is the resolution good enough to distinguish between, molecules, unbound virtual states, or compact pentaquarks?



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# New pentaquarks ?

$$\frac{dN}{d\sqrt{s}} = \rho(s)[|A(s)|^2 + B(s)]$$

$A(s)$  = assumed  
in a  
single p.w

$B(s)$  = higher p.w's

$$A(s) = P(s)T(s)$$

$$T^{-1}(s) = M(s) - ik(s)$$

Case A

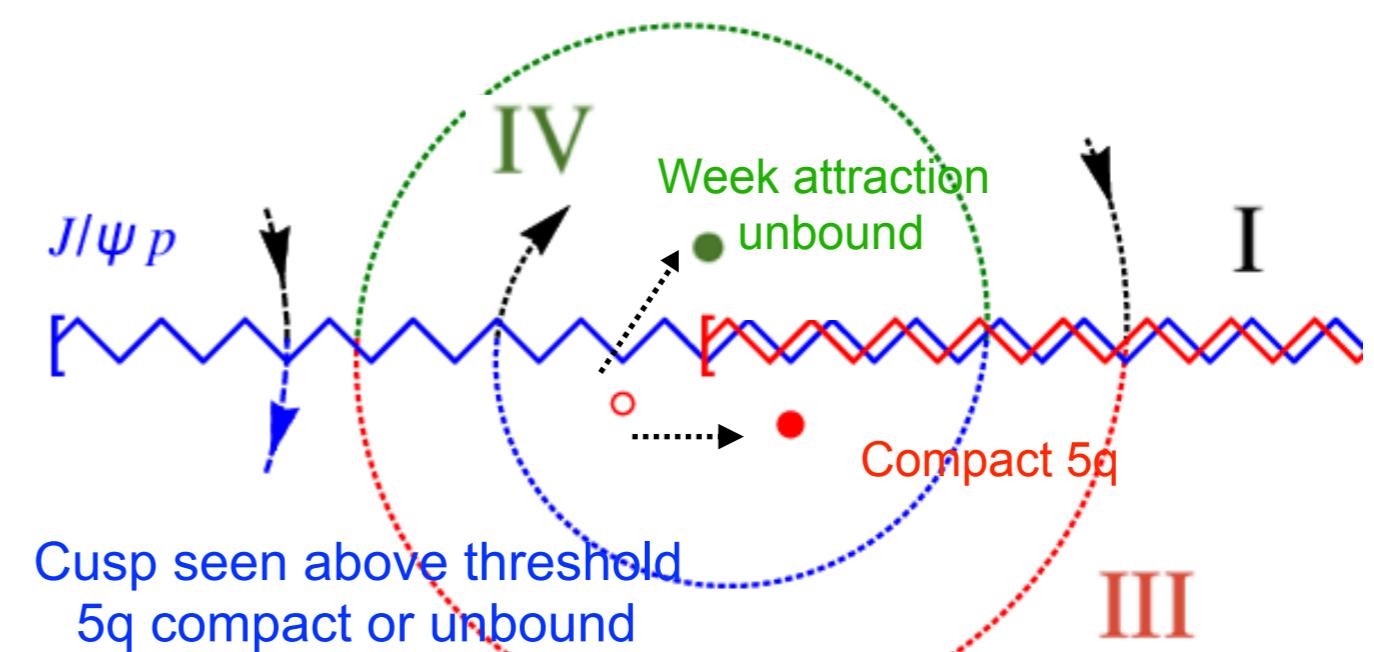
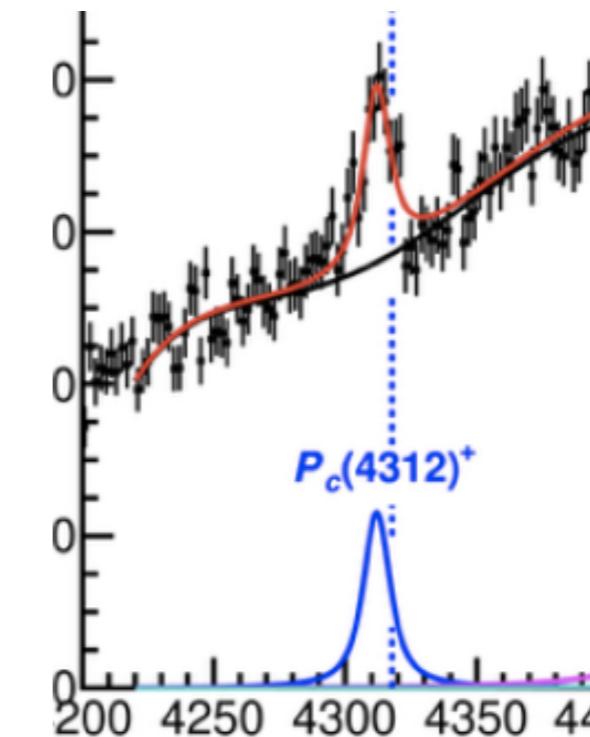
$M(s) = M$  virtual or bound states  
(sheet IV or II but no sheet III)

$M(s) = 2 \times 2$  scattering length matrix

Case B additional compact  
states (sheet III)

$$M(s) = M + Cs$$

$M = 2 \times 2$  scattering length  
+ effective range



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# New pentaquarks ?

$$\frac{dN}{d\sqrt{s}} = \rho(s)[A(s)|^2 + B(s)]$$

$A(s)$  = assumed  
in a  
single p.w

$B(s)$  = higher p.w's

Case A

$$M(s) = M \quad \text{virtual or bound states  
(sheet IV or II)}$$

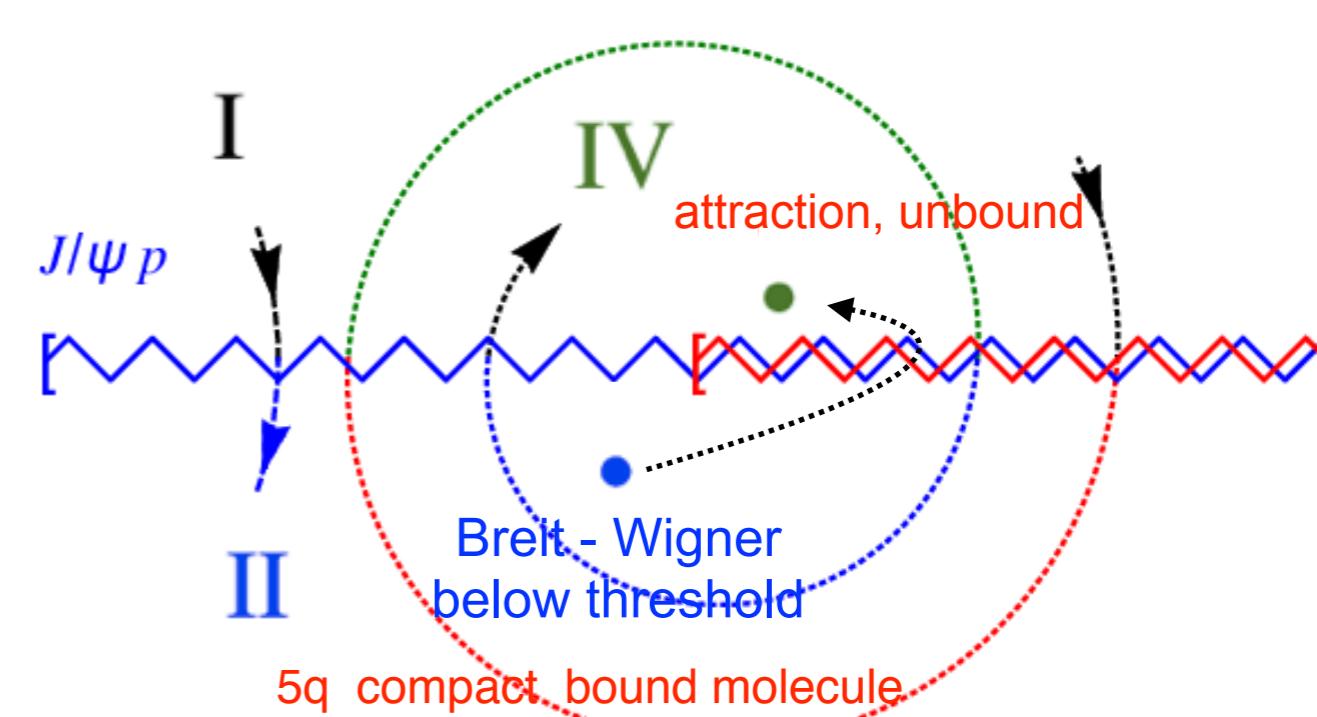
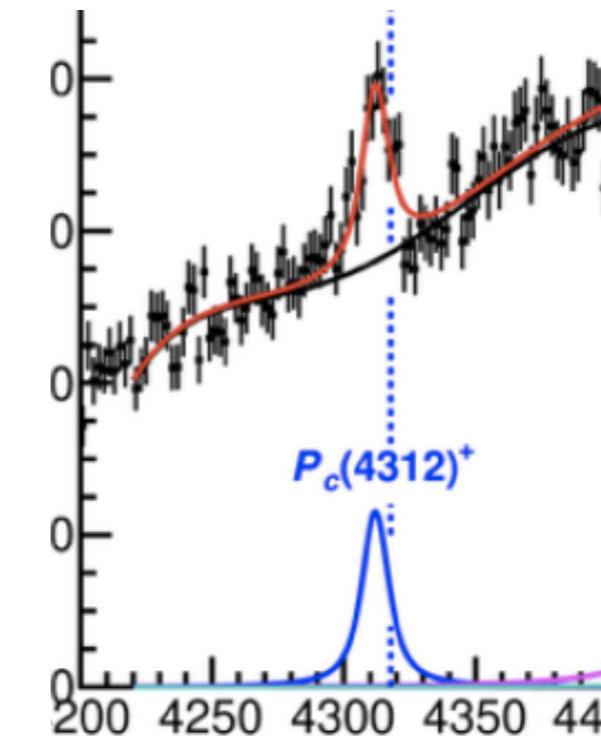
$M(s) = 2 \times 2$  scattering length matrix

Case B

additional compact  
states (sheet III)

$$M(s) = M + Cs$$

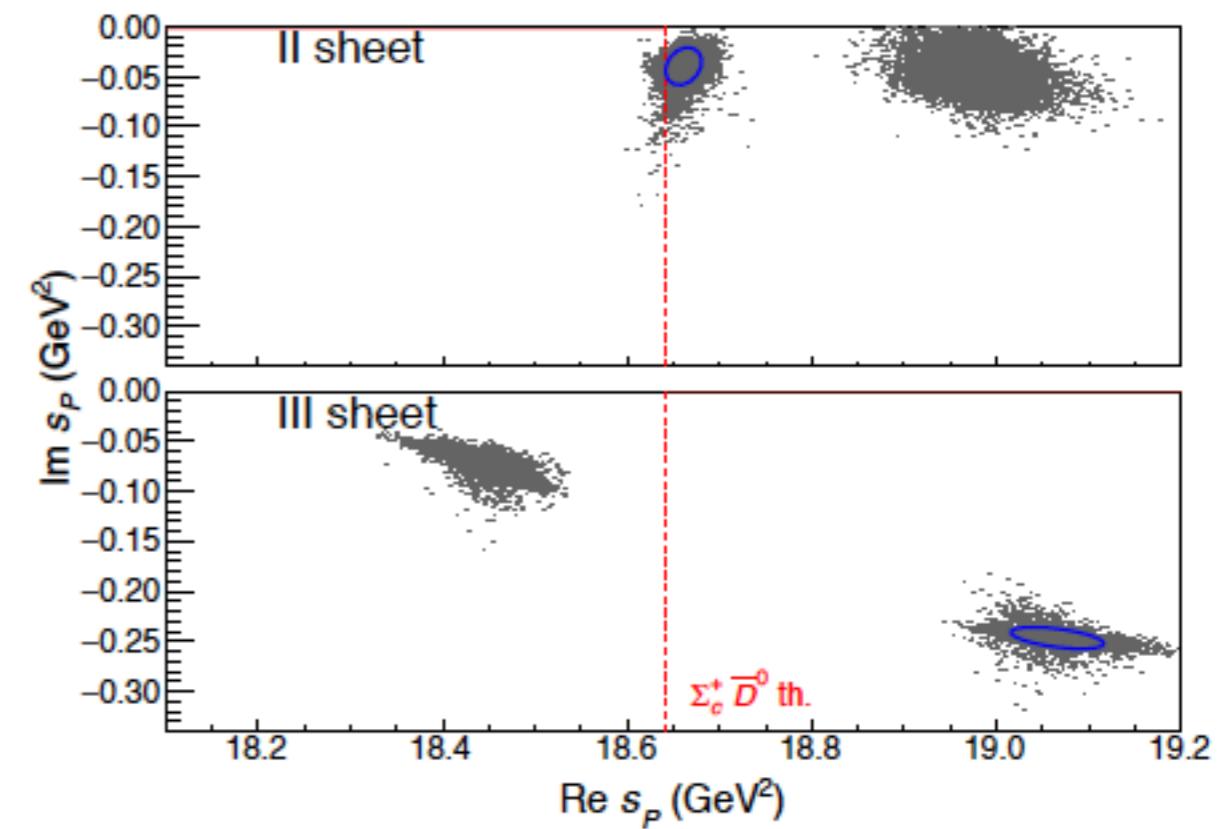
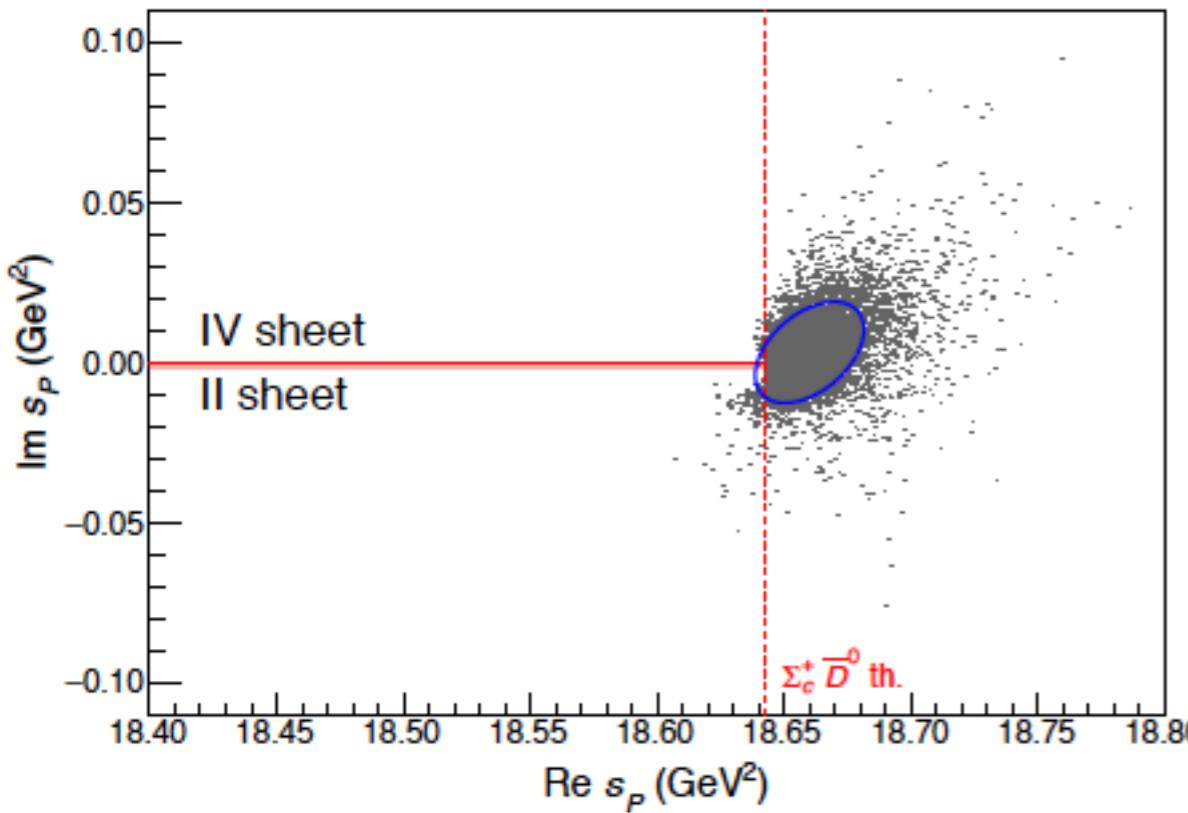
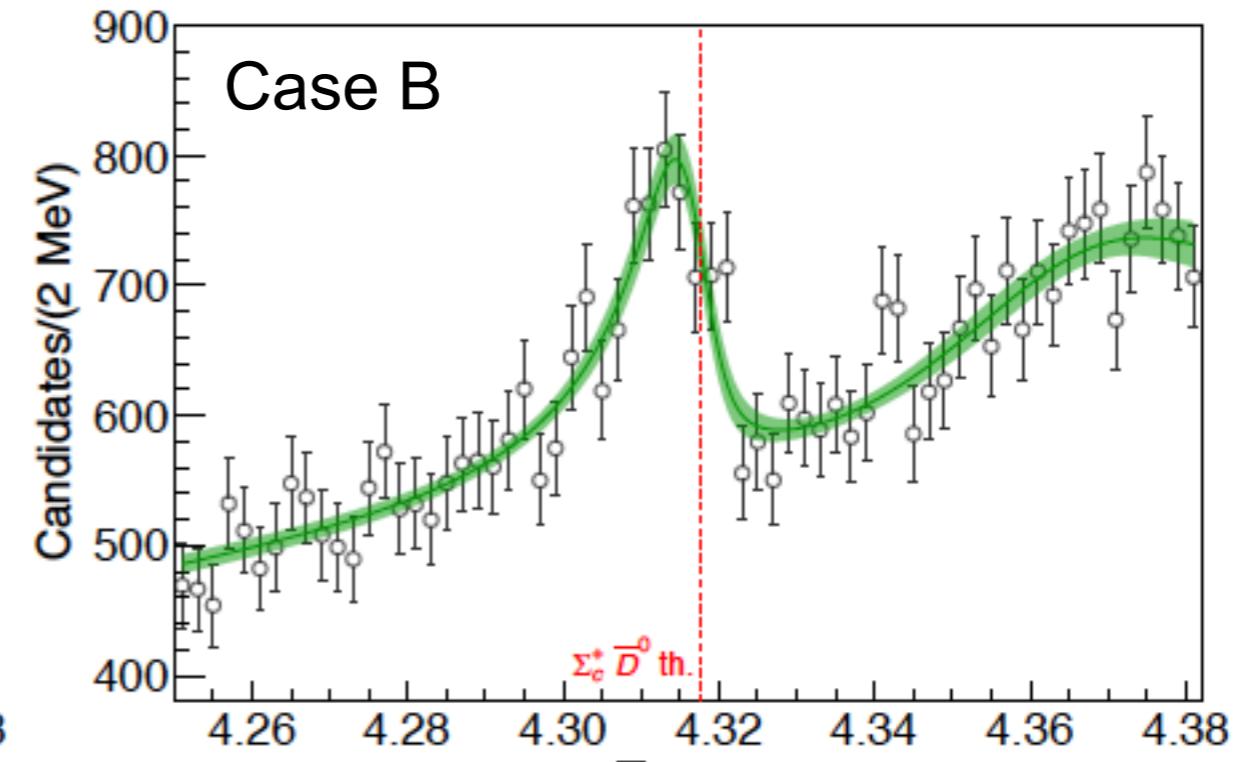
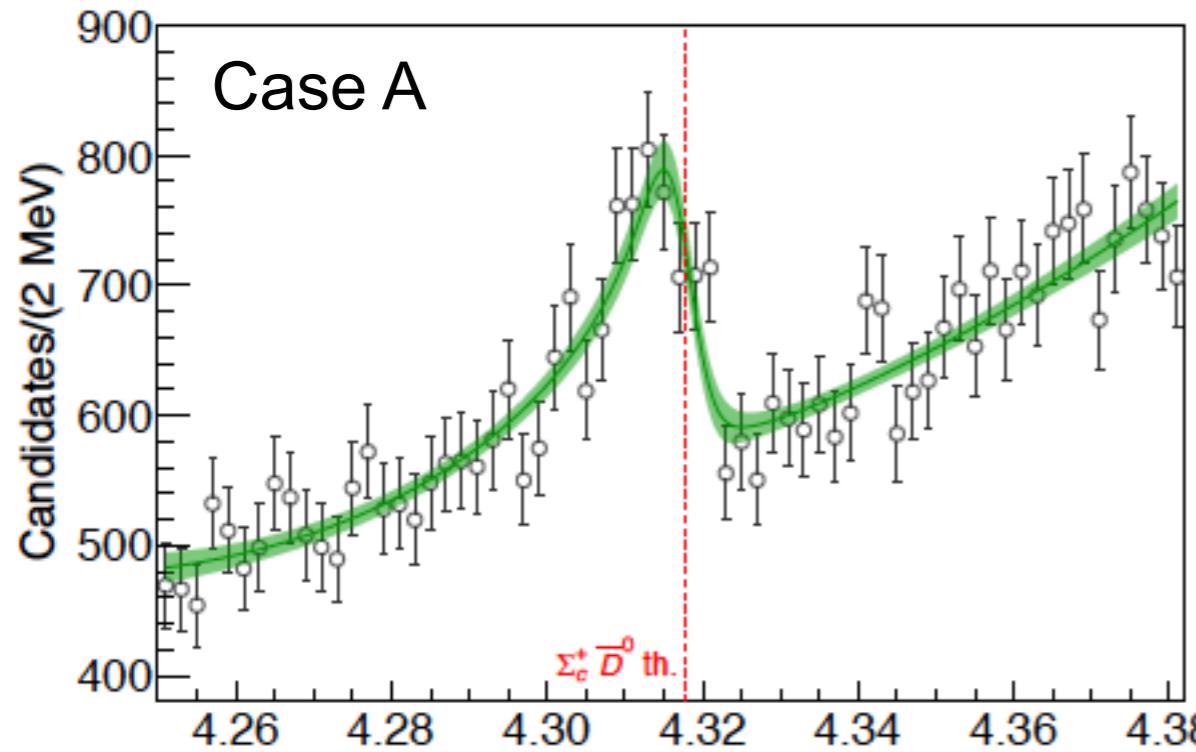
$M = 2 \times 2$  scattering length  
+ effective range



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# New pentaquarks ?

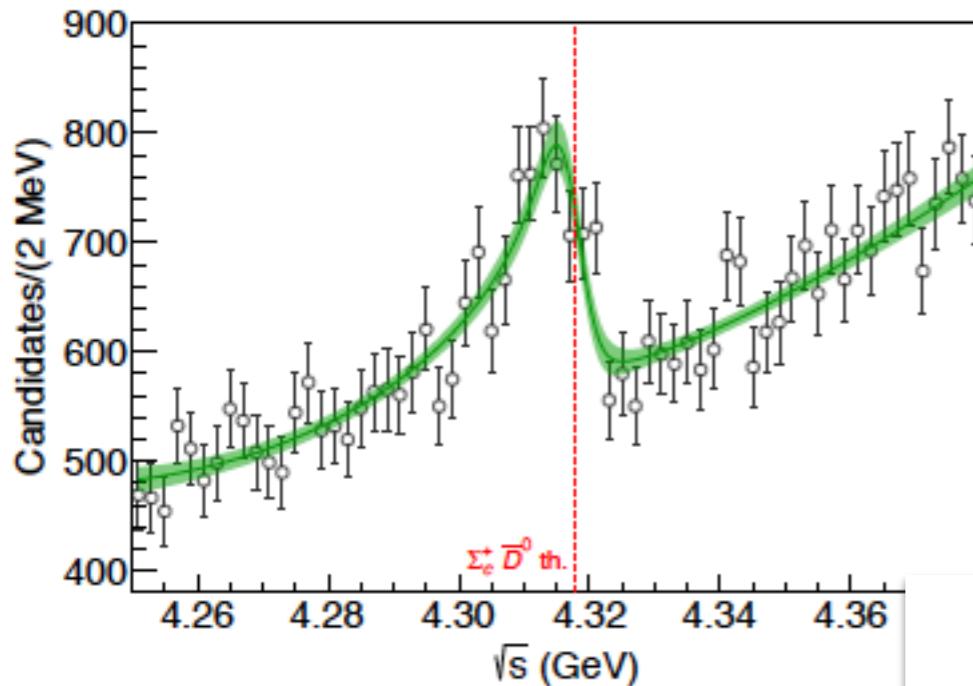


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# New pentaquarks ?

Case A



Decrease coupling  
between  $J/\psi$  p and  
 $\Sigma^+\bar{D}$  channels

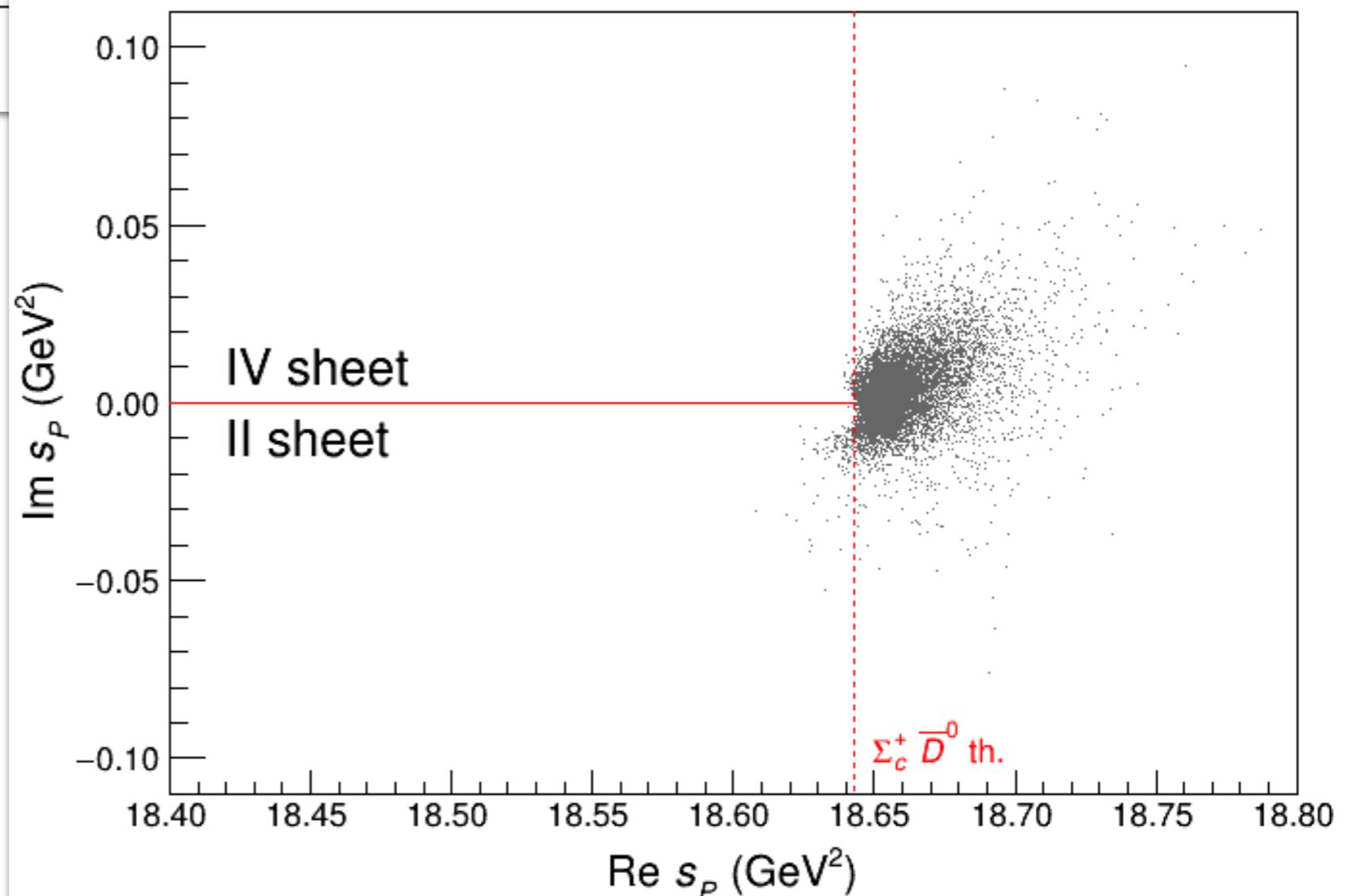
IV sheet pole moves  
onto real axis (virtual  
state)

II sheet pole moves onto  
real axis (bound state)

Virtual state in (>90%)

$$T^{-1} = M - i\rho(s)$$

$M = 2 \times 2$  scattering  
length

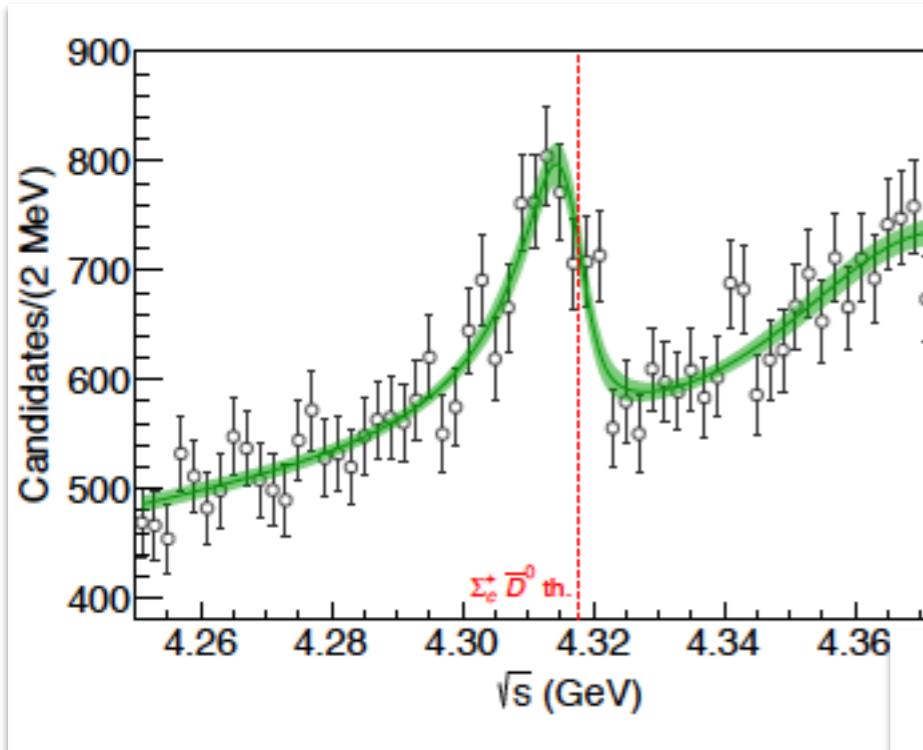


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# New pentaquarks ?

Case B



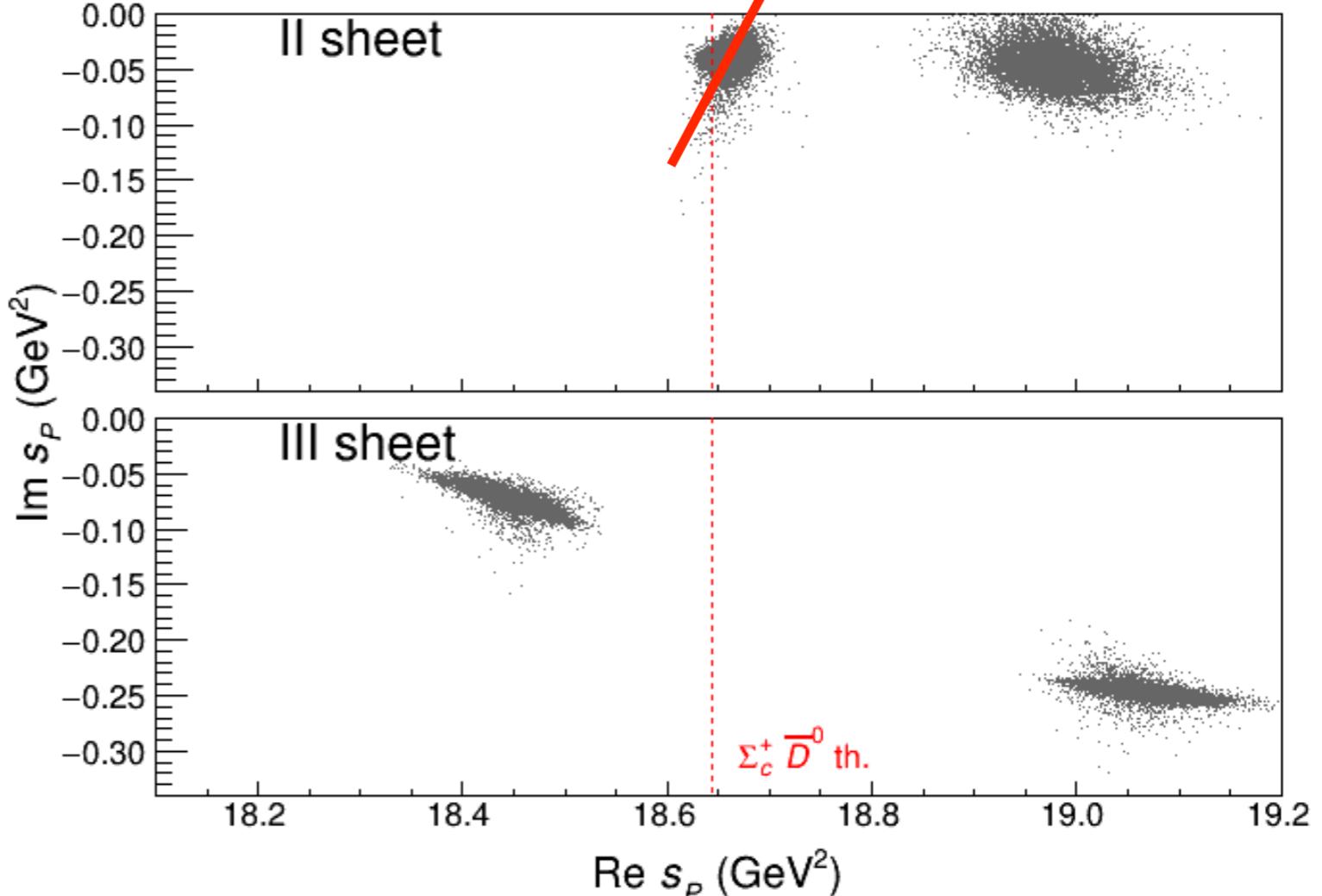
Decrease coupling  
between  $J/\psi$  p and  
 $\Sigma^+ \bar{D}$  channel

Remove imaginary parts

Peak generated by the  
II sheet pole which is  
“eaten” by a zero on IV  
sheet. Same with the  
lower III she pole

$$T^{-1} = M - i\rho(s)$$

M = 2 x 2 scattering  
length + effective  
range matrix

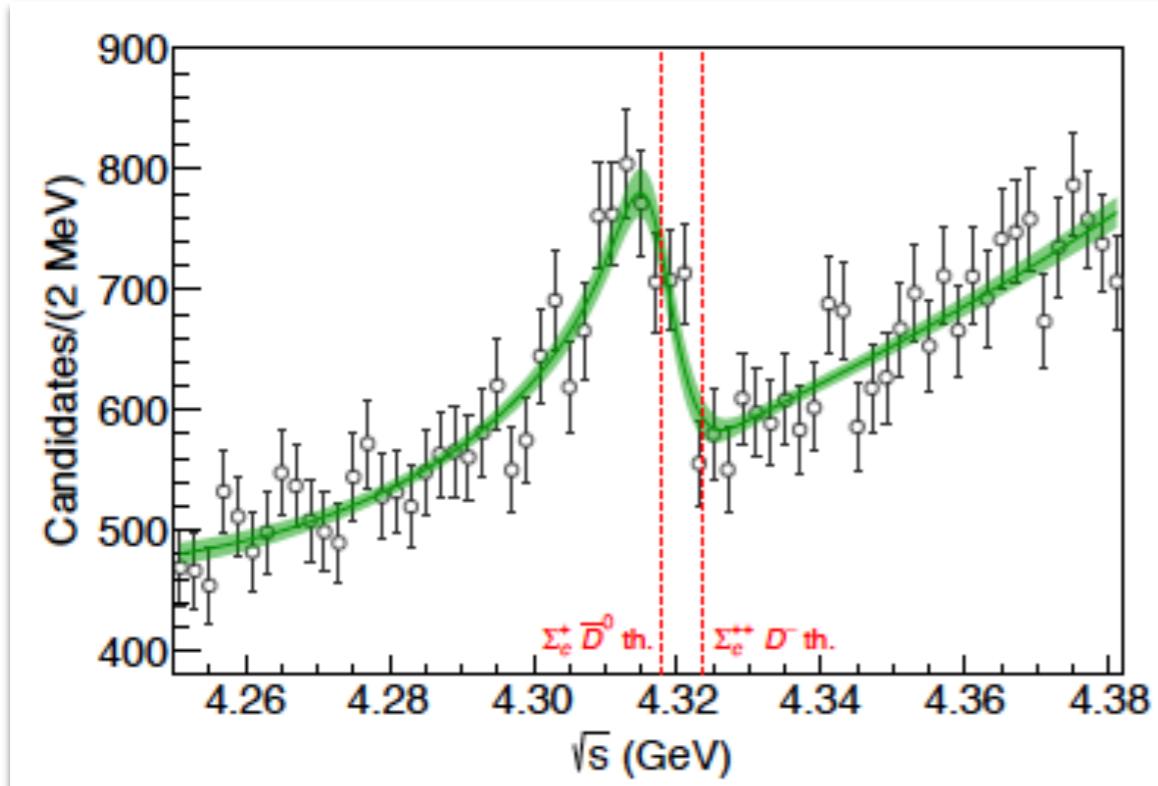


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# New pentaquarks ?

3 channel fit



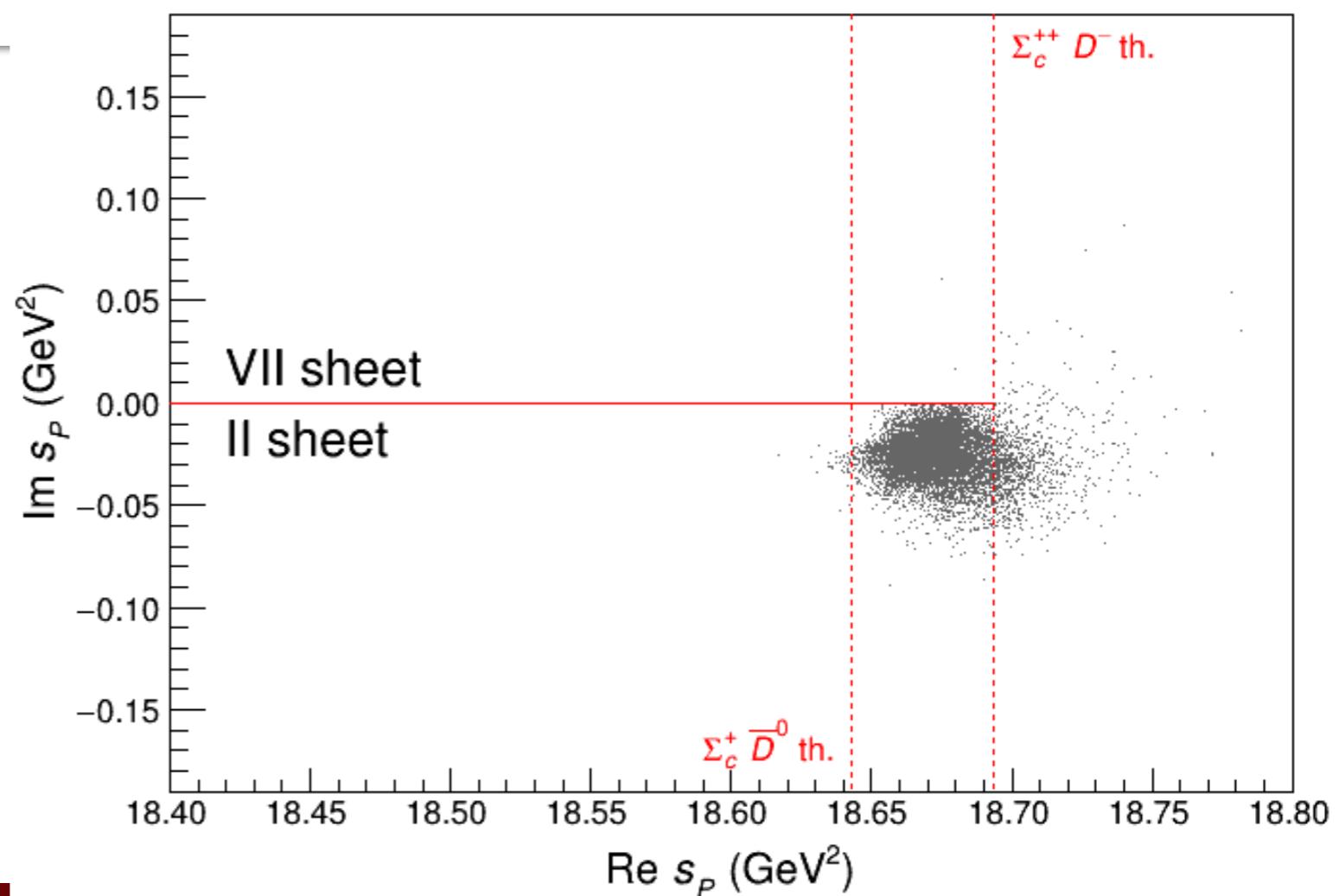
$$T^{-1} = M - i\rho(s)$$

$M = 3 \times 3$  scattering length matrix

Decrease coupling  
between  $J/\psi$  p and  
 $\Sigma^+ \bar{D}$  channels

Remove imaginary parts

Consistent with  
unbound state

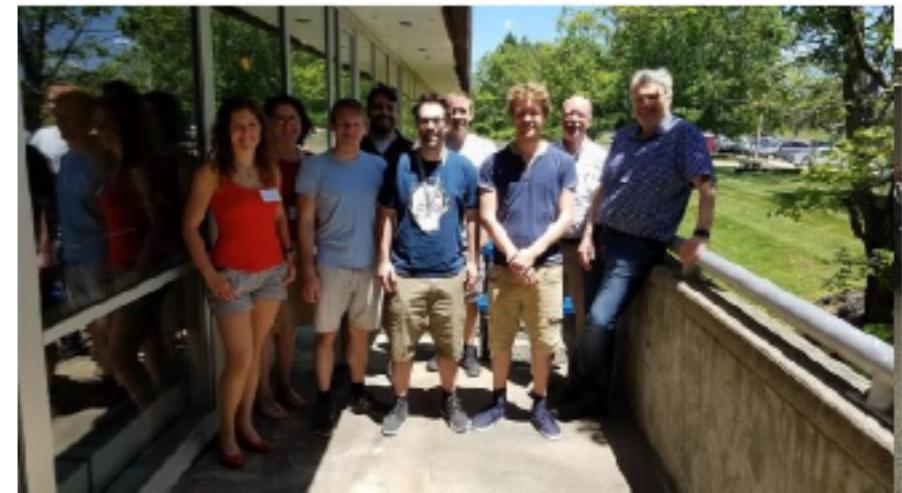


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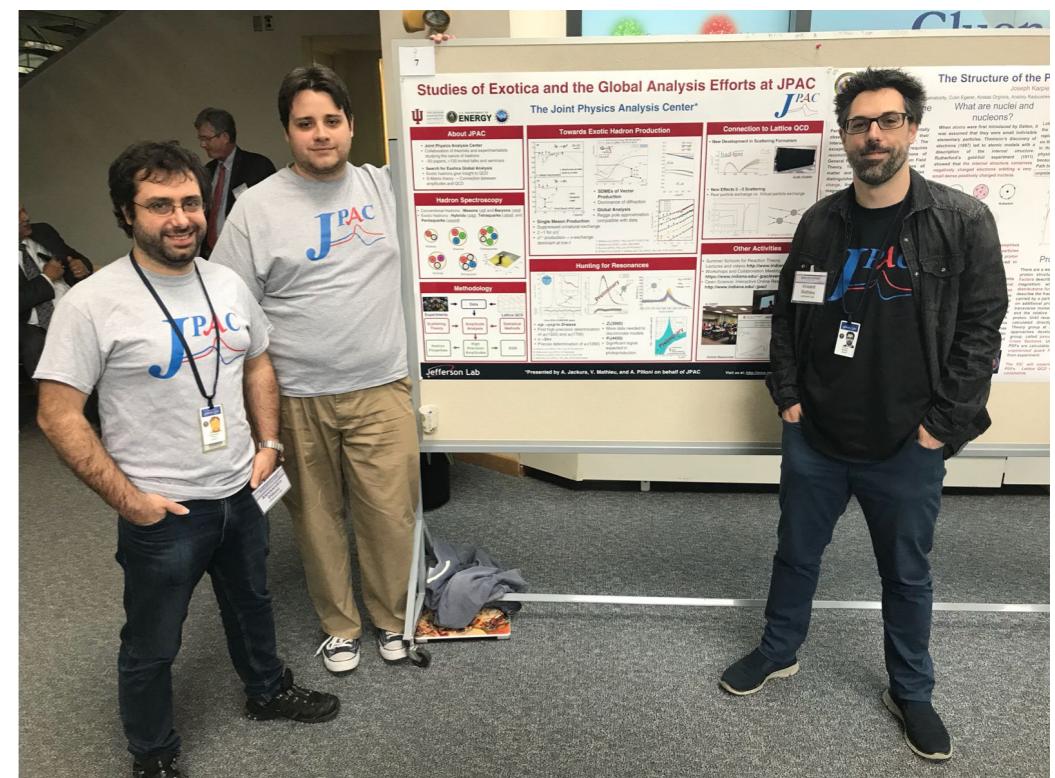
# Joint Physics Analysis Center

- JPAC: theory, phenomenology and analysis tools in support of experimental data from JLab12 and other accelerator laboratories.
- Contribute to education of new generation of practitioners in physics of strong interactions.



<https://jpac.jlab.org>

<http://www.indiana.edu/~jpac/>



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# Spares

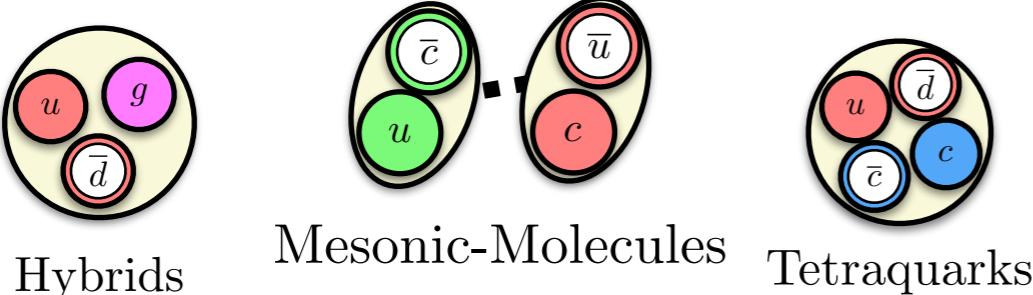
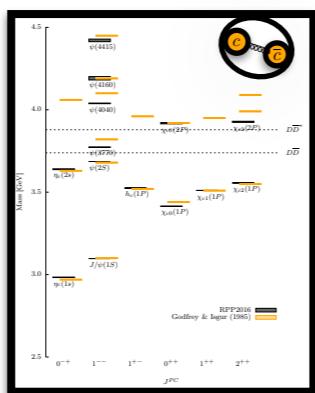
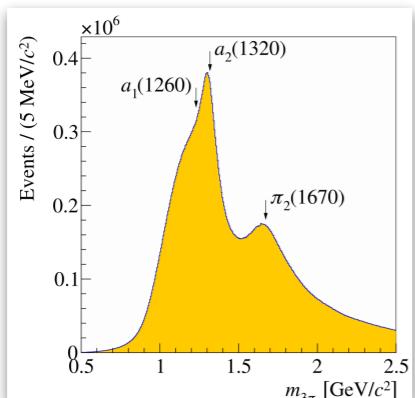


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# Amplitude analysis : connecting data to QCD

Experimental or lattice signatures  
(**real axis data**: cross section  
bumps and dips, energy levels)



What is the interpretation (constituent  
quarks, molecules, ...) ?

$$d\sigma \propto |T(s, t, \dots)|^2$$

Luscher quantization  
condition

Amplitude analysis



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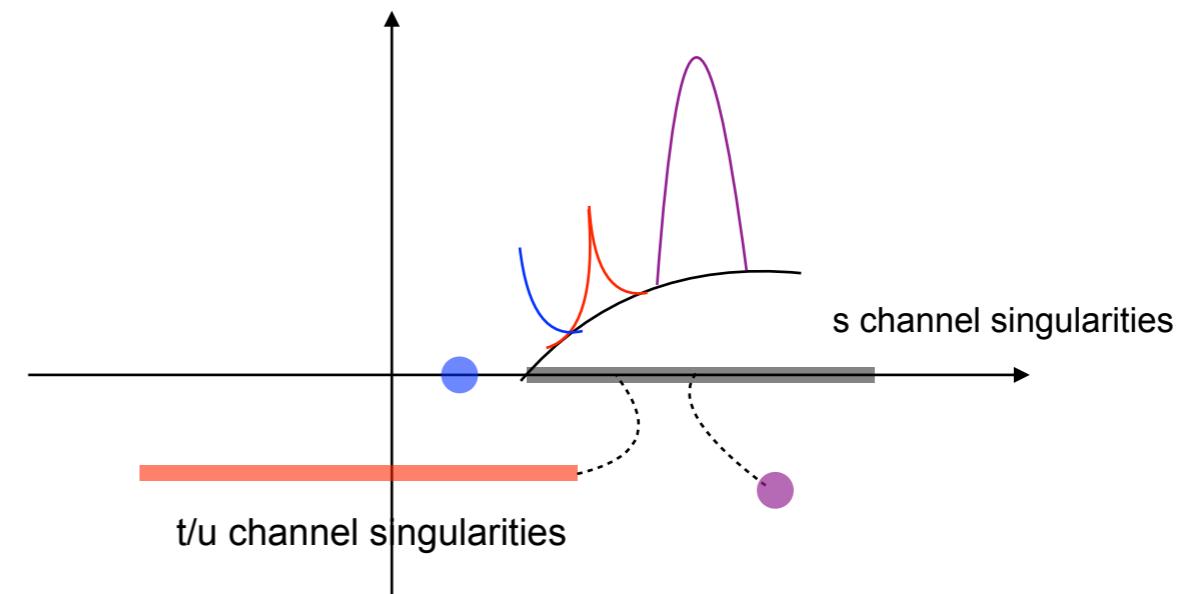
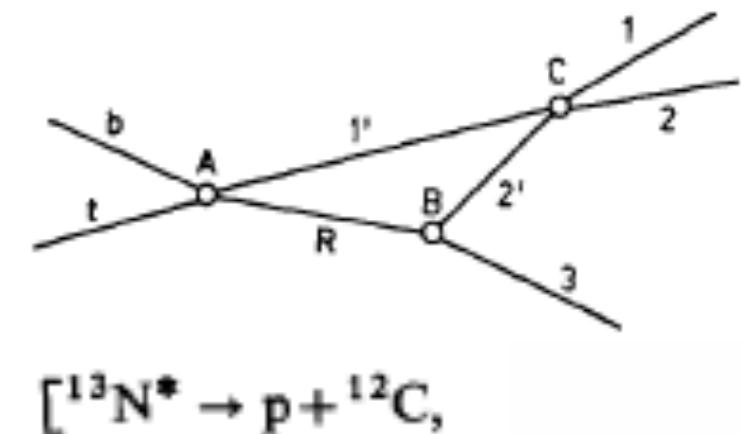
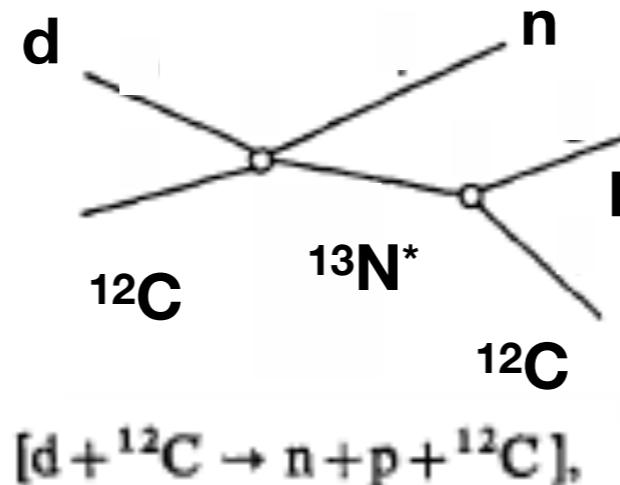
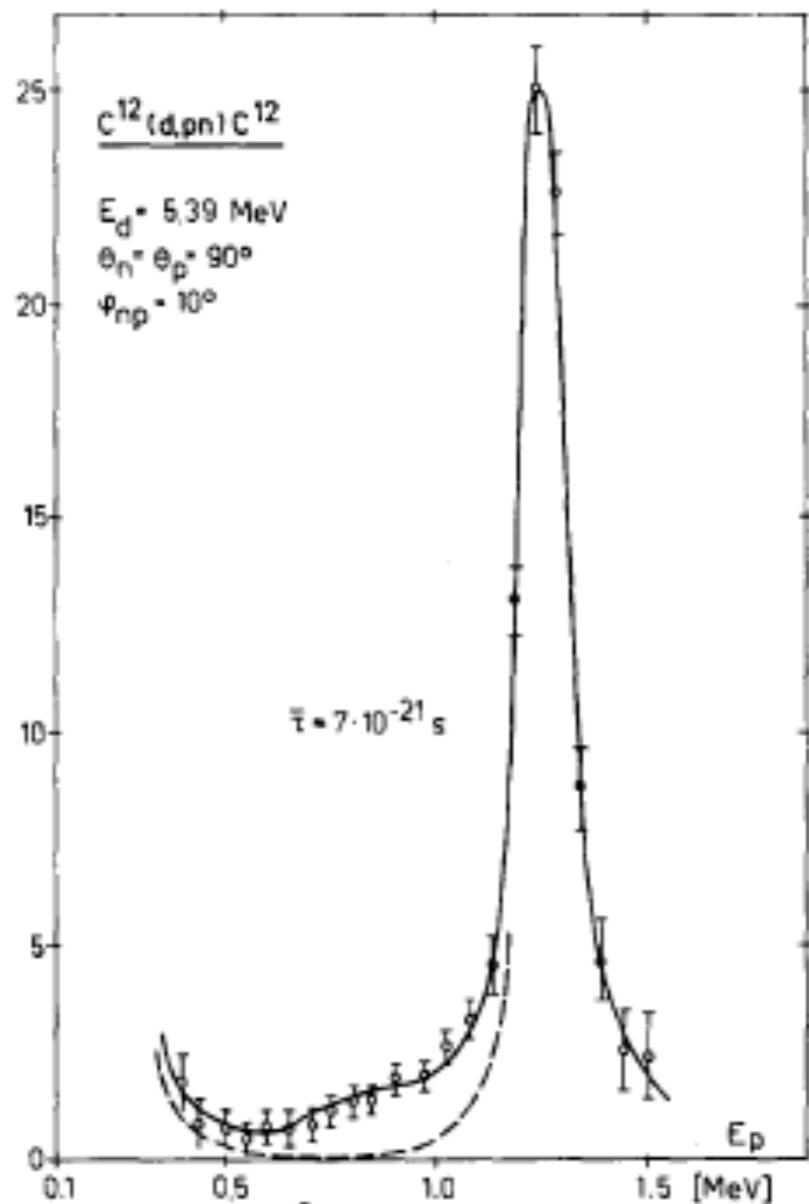


# Triangle singularities

DIRECT DETERMINATION OF A SHORT NUCLEAR LIFETIME ( $\approx 10^{-20}$  s)  
BY THE PROXIMITY SCATTERING METHOD

J. LANG, R. MÜLLER, W. WÖLFLI, R. BÖSCH and P. MARMIER  
*Laboratorium für Kernphysik, Eidg. Techn. Hochschule, Zürich* †

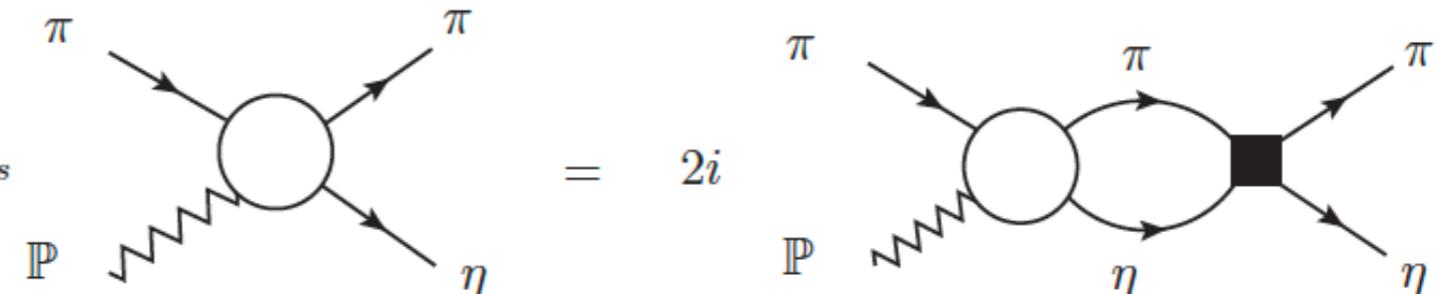
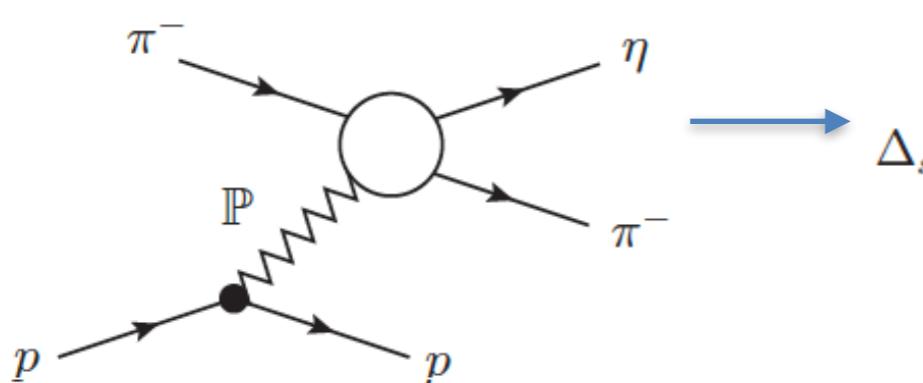
Received 4 February 1966



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$$\Delta_s a_{\ell m_\ell}(s) = 2i \rho_\ell(s) t_\ell^*(s) a_{\ell m_\ell}(s)$$



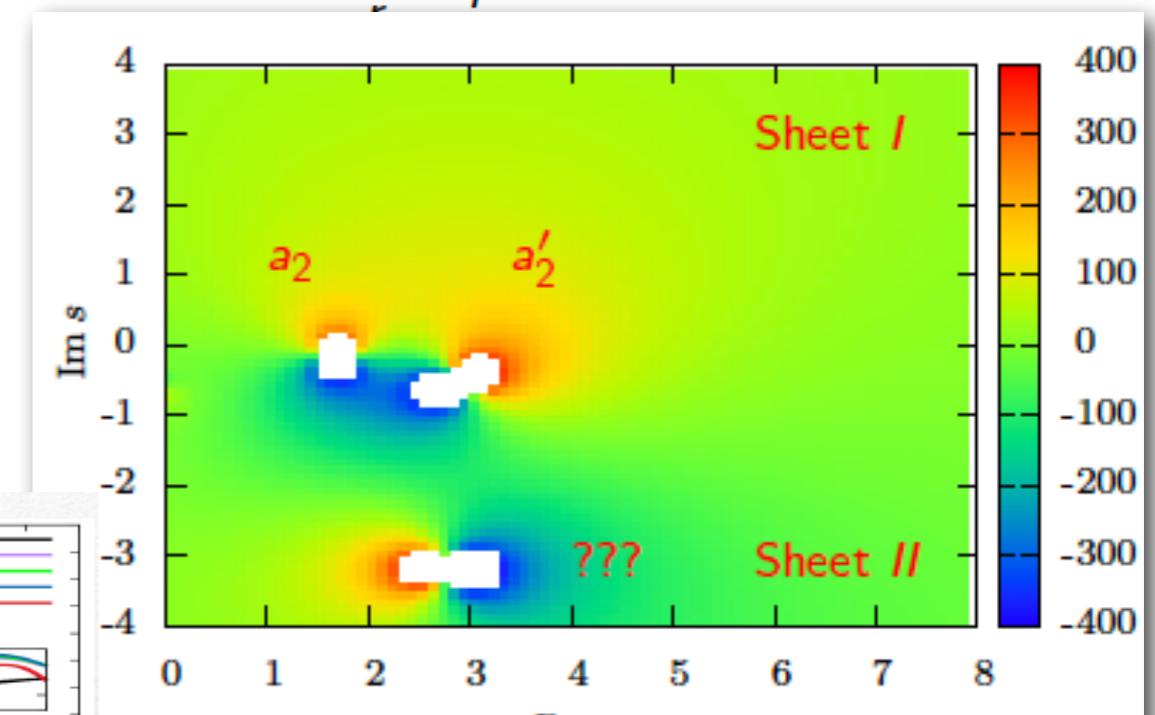
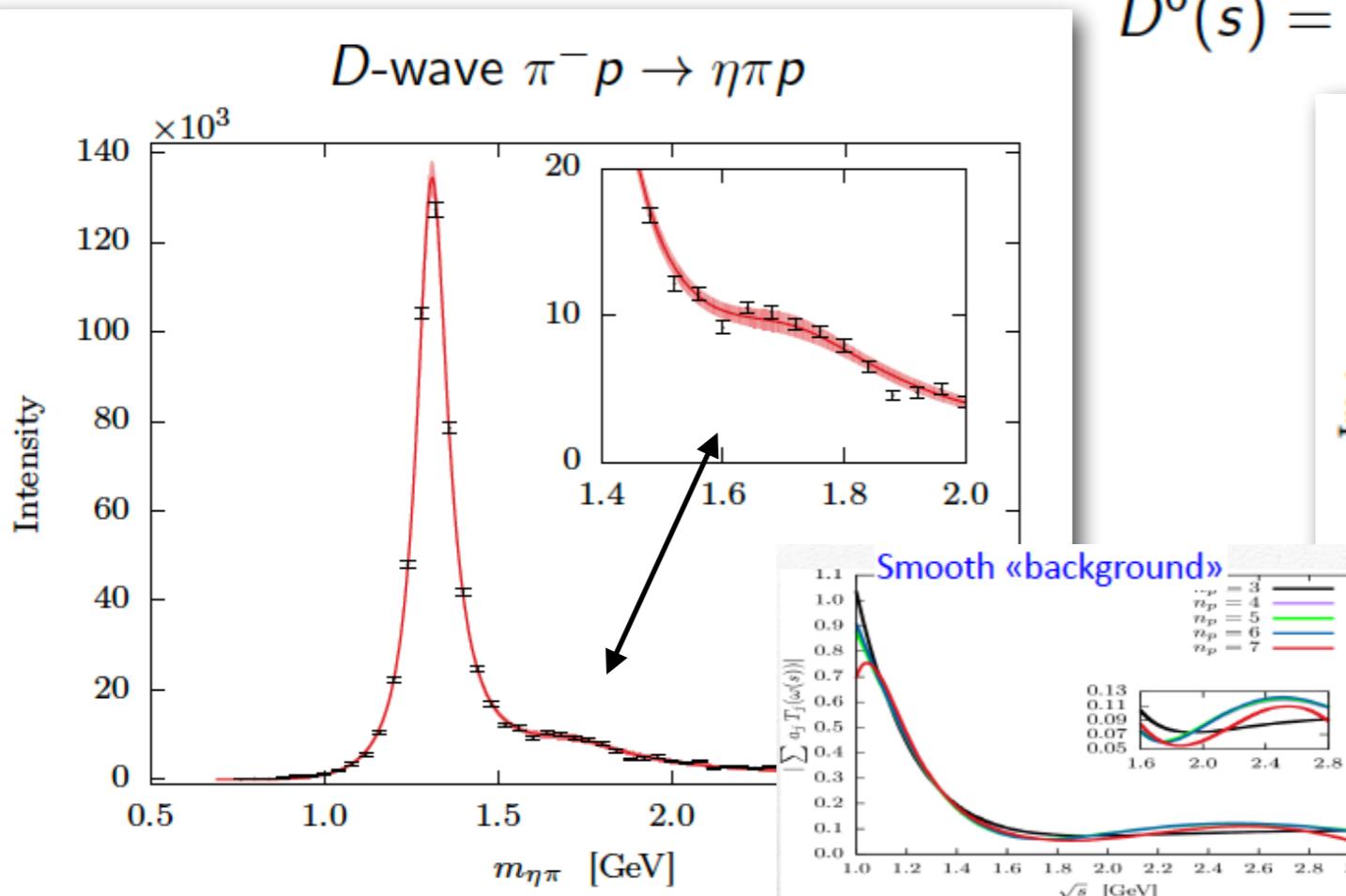
**Production( $s_{\pi\pi}$ ) x Interactions in  $\eta\pi$  ( $s_{\eta\pi}$ )**

**Constrained by unitary**

$$a_{\ell m_\ell} = f_{\ell m_\ell}(s) t_\ell(s)$$

$$f_{\ell m_\ell}(s) = \sum_{n=0} \alpha_n T_n(\omega(s)) \quad t_\ell(s) = N(s)/D(s) \quad D(s) = D^0(s) - \frac{s}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\rho(s') N(s')}{s'(s' - s)}$$

$$D^0(s) = a - bs - \sum_r \frac{c_r}{s_r - s}$$



$$M(1320) = 1.308(2) \text{ GeV}, \Gamma(1320) = 0.113(1) \text{ GeV}$$

$$M(1700) = 1.71(6) \text{ GeV}, \Gamma(1700) = 0.30(6) \text{ GeV}$$



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# Flate Formula

1 Channel

$$f^{-1}(s) = K^{-1}(s) - i\rho(s)$$

Phase space for deal  
~ break up momentum

Analytic near threshold

$$K = \frac{g^2}{m^2 - s} \longrightarrow f(s) = \frac{g^2}{m^2 - s - i\rho(s)} \quad \text{Breit-Wigner}$$

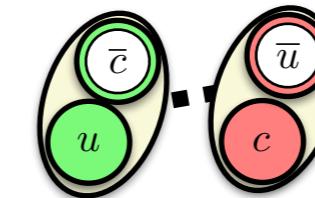
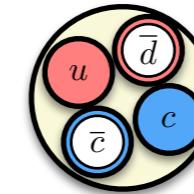
2 Channels

$$f_{11} = \frac{g_1^2}{m^2 - s - ig_1^2\rho_1(s) - ig_2^2\rho_2(s)}$$

$$\mathbf{f}^{-1} = \mathbf{K}^{-1} - i\rho(s)$$
$$\mathbf{K} = \frac{1}{m^2 - s} \begin{pmatrix} g_1^2 & g_1g_2 \\ g_1g_2 & g_2^2 \end{pmatrix}$$

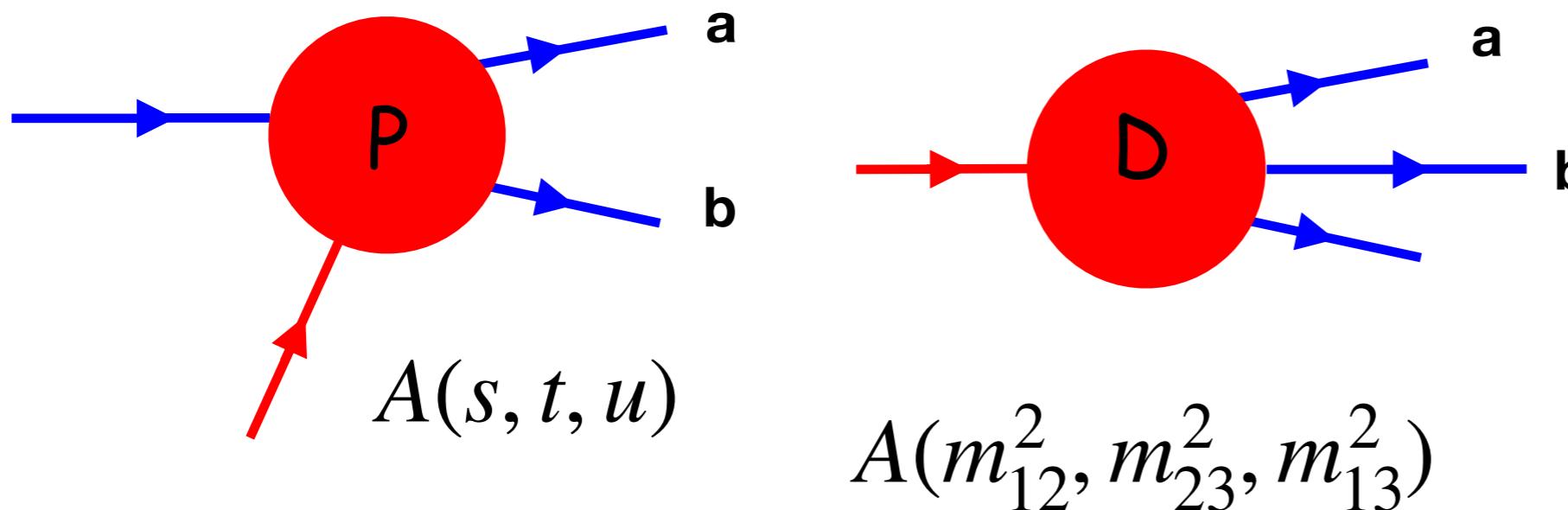
K-matrix pole produces a pole on the complex plane. But its existence is independent from that of decay channels

If  $m^2, g_i \gg 1$ , it becomes an effective range approximation  $\rightarrow$  threshold (bound state of virtual state) poles



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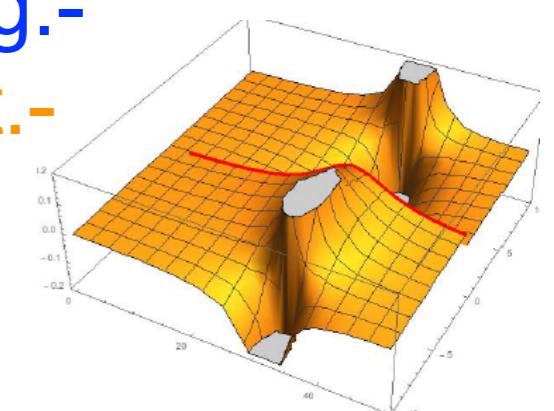


What constrains resonance in e.g. ab?

- A is analytical with unitarity branch points

$$A_l(s) = \int dz P_l(z) A(s, t(z, s), u(z, s))$$

- Partial waves are constrained by unitarity; it exposes singularities hidden under cuts: **poles (resonance)**, **log.-branch points** (“forces” e.g. triangle singularities), **sqrt.-branch points** (multi-particle production), etc.  
→ all singularities have physical origin !



# Systematic Studies

- Change of functional form and parameters in the denominator

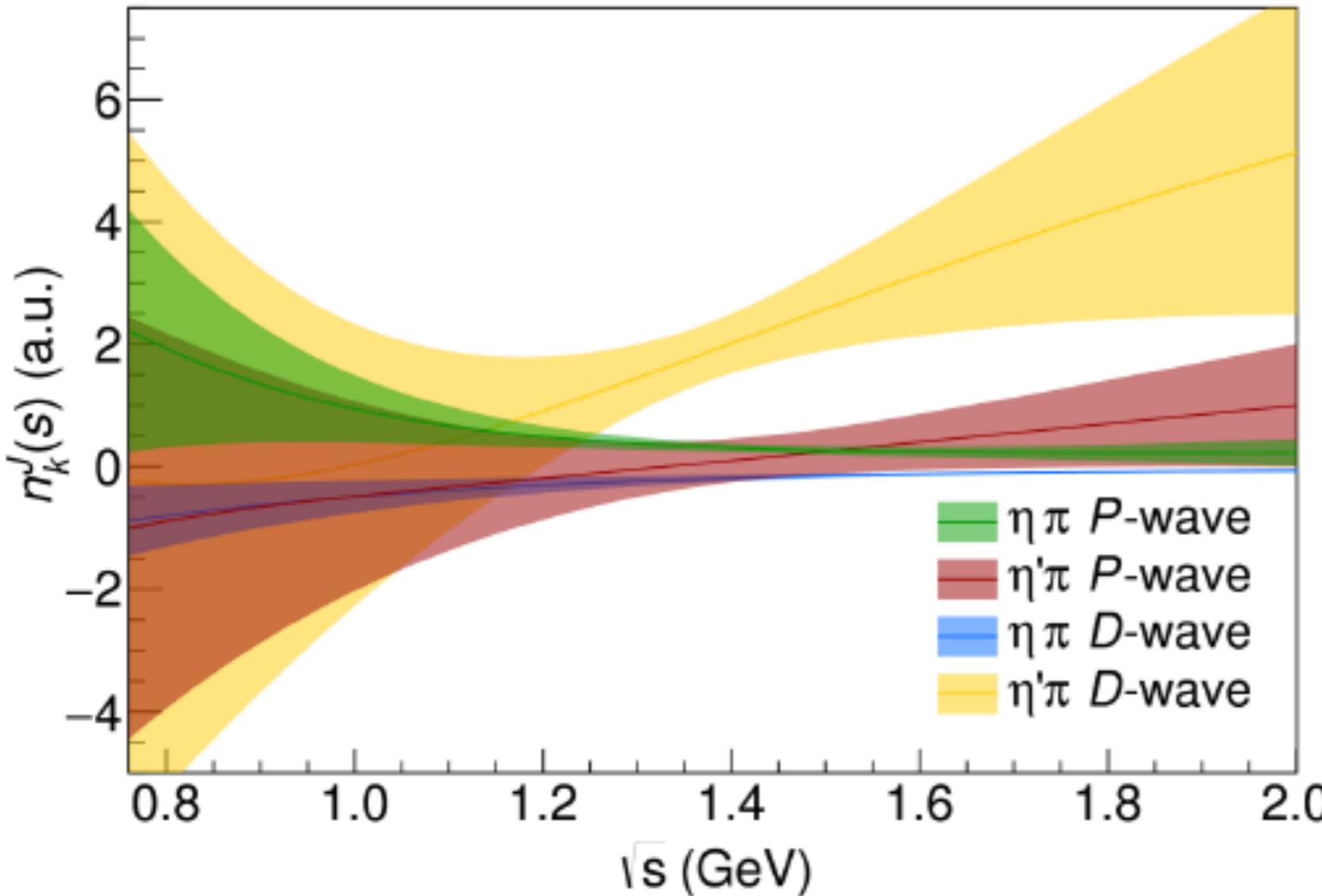
$$\rho N_{ki}^J(s') = g \delta_{ki} \frac{\lambda^{J+1/2} \left( s', m_{\eta^{(\prime)}}^2, m_\pi^2 \right)}{(s' + s_R)^{2J+1+\alpha}}$$

- Default:  $s_R = 1 \text{ GeV}^2$ . We try  $s_R = 0.8, 1.8 \text{ GeV}^2$
- Default:  $\alpha = 2$ . We try  $\alpha = 1$
- We also try a different function:  $\rho N_{ki}^J(s') = g \delta_{ki} \frac{Q_J(z_{s'})}{s'^\alpha \lambda^{1/2}(s', m_{\eta^{(\prime)}}, m_\pi)}$   
with  $\alpha = 2, 1.5, 1$

- Change of parameters in the numerator

- Default:  $t_{\text{eff}} = -0.1 \text{ GeV}^2$ . We try  $t_{\text{eff}} = -0.5 \text{ GeV}^2$
- Default: 3rd order polynomial. We try 4th

## Polynomial in the numerator



The numerator should be smooth and have variation milder than the typical resonance width

This happens indeed