Hybrids and Tetraquarks in EFTs

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May 16, 2019

QWG 2019 - The 13th International Workshop on Heavy Quarkonium

Cavallerizza Reale, Turin
EFT for quarkonium hybrids

Outline

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Outline

- EFT for quarkonium hybrids
  - Spectrum without spin splitting
EFT for quarkonium hybrids

- Spectrum without spin splitting
- Spin-dependent potential and spin splitting

EFT for tetraquarks

Summary and outlook
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- EFT for tetraquarks
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  - Spectrum without spin splitting
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EFT for tetraquarks

Summary and outlook
Exotic quarkonia (XYZ particles) are quarkonium-like bound states that cannot be interpreted as a traditional quarkonium state.

Interpretations for exotic quarkonia:

- Hybrid
- Adjoint tetraquark
- Compact tetraquark
- Diquark-diquark
- Heavy-meson molecule
- Hadroquarkonium

A complete understanding of exotic quarkonia in terms of these pictures has not yet been obtained.

EFT together with lattice QCD: a tool to understand the various pictures in a model-independent way.
Quarkonium hybrids in the EFT picture

- Quarkonium hybrid: $Q\bar{Q}$ in color octet with gluon excitation (light d.o.f. with scale $\Lambda_{QCD}$)
- Separation of scales: $m \gg mv \gg \Lambda_{QCD} \gg mv^2 \Rightarrow$ suitable for EFT description ($m \equiv m_Q$)
- Integrate out d.o.f.: QCD $\rightarrow$ NRQCD $\rightarrow$ pNRQCD $\rightarrow$ BOEFT (Born-Oppenheimer EFT)

$r \sim 1/mv$

$\sim 1/\Lambda_{QCD}$

Scale

$QCD$

$\overline{\Lambda}_{QCD}$

$BOEFT$

$\overline{\Lambda}_{QCD}$

$\overline{\Lambda}_{QCD}$

$mv^2$

$mv$

$m$
**Static limit**

- Dynamics of $Q\bar{Q}$ happens at time scale $\sim 1/mv^2 \gg 1/\Lambda_{QCD} \Rightarrow$ Born Oppenheimer approximation. E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D 90, (2014)
- Interquark potential given by energy eigenvalues with $Q$ and $\bar{Q}$ at fixed positions.
- In the static limit of $Q\bar{Q}$, the system has symmetry group $D_{\infty h}$.
  Irreducible representation of $D_{\infty h}$: $\Lambda^\sigma_{\eta}$.

**Irreducible representations of $D_{\infty h}$**

- $K$: angular momentum of light d.o.f.
  \[ \lambda = \hat{r} \cdot K = 0, \pm 1, \pm 2, \pm 3, \ldots \]
  \[ \Lambda = |\lambda| = 0, 1, 2, 3, \ldots \quad (\Sigma, \Pi, \Delta, \Phi, \ldots) \]
- Eigenvalue of $CP$: $\eta = +1 (g), -1 (u)$
- $\sigma$: eigenvalue of reflection about a plane containing $\hat{r}$ (only for $\Sigma$ states)
In the static limit, when \( r \to 0 \) the symmetry group reduces to \( O(3) \times C \), the quarkonium hybrid turns into a gluelump.

State of light d.o.f. at \( r = 0 \) labeled by \( \kappa = K^{PC} \).

State of light d.o.f. labeled by \( n = (\kappa, \Lambda^\sigma_\eta) \), with \( \kappa \) denoting the \( r \to 0 \) limit.

Static energy \( E_n^{(0)}(r) \): energy eigenvalue of \( H^{(0)} \) (NRQCD Hamiltonian in the limit \( m \to \infty \)), with \( Q \) and \( \bar{Q} \) at fixed positions \( x_1, x_2 \).

In terms of Wilson loop:

\[
E_n^{(0)}(r) = \lim_{T \to \infty} \frac{i}{T} \log \langle X_n(T/2) | X_n(-T/2) \rangle
\]

\[
| X_n \rangle = \chi(x_2) \phi(x_2, R) O_n(R) \phi(R, x_1) \psi^\dagger(x_1) | 0 \rangle
\]
LATTICE DETERMINATION OF STATIC ENERGIES

- Ground state: $\Sigma_g^+$
- Gluelump energy $\Lambda_\kappa \equiv E_n(r = 0)$
- Degeneracies at $r \to 0$

Gluelump energies (at $r = 0$):

BOEFT
Successive matchings to obtain BOEFT

- **QCD → NRQCD**: Integrate out modes of scale $m$.

- **NRQCD → weakly-coupled pNRQCD**: Integrate out modes of scale $mv \sim 1/r$ in the short distance regime ($r \ll 1/\Lambda_{QCD}$).

- **weakly-coupled pNRQCD → BOEFT**: Integrate out modes of scale $\Lambda_{QCD}$.

- **BOEFT**: EFT for dynamics of $Q\bar{Q}$ at scale $mv^2 \Rightarrow$ Schrödinger equation

**BOEFT**

- **d.o.f.**: $\int d^3rd^3R \sum_{\kappa\lambda} P^i_{\kappa\lambda} O^a(\mathbf{r}, \mathbf{R}) G^i_{\kappa a}(\mathbf{R})|0\rangle \Psi_{\kappa\lambda}(t, \mathbf{r}, \mathbf{R})$

- $G^i_{\kappa a}(\mathbf{R})$: gluelump operator: $H^{(0)} G^i_{\kappa a}(\mathbf{R})|0\rangle = \Lambda_\kappa G^i_{\kappa a}(\mathbf{R})|0\rangle$ with $r = 0$

- $P^i_{\kappa\lambda}$ projects $G^i_{\kappa a}(\mathbf{R})$ to a representation of $D_{\infty h}$.

- $\Psi_{\kappa\lambda}(t, \mathbf{r}, \mathbf{R})$: wave function of $Q\bar{Q}$ in the quarkonium hybrid
Lagrangian of BOEFT

\[
L_{BOEFT} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda \lambda'} \Psi^\dagger_{\kappa \lambda}(t, r, R) \left\{ i \partial_t - V_{\kappa \lambda \lambda'}(r) + P^i_{\kappa \lambda} \frac{\nabla^2}{m} P^i_{\kappa \lambda'} \right\} \Psi_{\kappa \lambda'}(t, r, R) + \ldots
\]

- \( V_{\kappa \lambda \lambda'}(r) \) is organized as an expansion in \( 1/m \):

\[
V_{\kappa \lambda \lambda'}(r) = V^{(0)}_{\kappa \lambda}(r) \delta_{\lambda \lambda'} + \frac{V^{(1)}_{\kappa \lambda \lambda'}(r)}{m} + \frac{V^{(2)}_{\kappa \lambda \lambda'}(r)}{m^2} + \ldots
\]

- \( V^{(0)}_{\kappa \lambda} \) is related to the static energy by matching:

\[
E^{(0)}_{\eta}(r) = V^{(0)}_o(r) + \Lambda_\kappa + b_{\kappa \lambda} r^2 + \ldots = V^{(0)}_{\kappa \lambda}(r)
\]

\( V^{(0)}_o \): perturbative octet potential in weak-coupled pNRQCD
Hybrid spectrum for lowest lying glue lump

\( \kappa = 1^{+-} \quad (\Lambda_{\eta}^\sigma = \Sigma_u^-, \Pi_u) \)


- Only consider \( V^{(0)}_{1\lambda} \), neglect \( V^{(1)}_{1\lambda\lambda'}, V^{(2)}_{1\lambda\lambda'} \) \ldots
- Use perturbative value \( V^{(0)}_o(r) \) for \( r \lesssim 0.5 \text{ fm} \) and a numerical fit to lattice value \( E^{(0)}_{1\lambda}(r) \) for \( r \gtrsim 0.5 \text{ fm} \)
- The term \( P^{i\dagger}_{\kappa\lambda} \frac{\nabla^2}{m} P^i_{\kappa\lambda} \) mixes \( \Sigma_u^- \) and \( \Pi_u \) states, leading to \( \Lambda \)-doubling

### Lowest-lying quarkonium hybrid multiplets

<table>
<thead>
<tr>
<th>Multiplet</th>
<th>( l )</th>
<th>( J^{PC}(s = 0) )</th>
<th>( J^{PC}(s = 1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_1 )</td>
<td>1</td>
<td>1--</td>
<td>(0, 1, 2)--++</td>
</tr>
<tr>
<td>( H_2 )</td>
<td>1</td>
<td>1++</td>
<td>(0, 1, 2)+--</td>
</tr>
<tr>
<td>( H_3 )</td>
<td>0</td>
<td>0++</td>
<td>1+-</td>
</tr>
<tr>
<td>( H_4 )</td>
<td>2</td>
<td>2++</td>
<td>(1, 2, 3)+--</td>
</tr>
</tbody>
</table>
Lowest-lying charmonium hybrid spectrum. Bands are results from EFT, points with error bars are lattice data from L. Liu et al. (Hadron Spectrum), JHEP, 07, 126 (2012).


For a study of mixing of quarkonium hybrids with and decay to traditional quarkonia, see R. Oncala, J. Soto, Phys. Rev. D96 (2017)
Spin-dependent potential to $\mathcal{O}(1/m^2)$


The spin-dependent potentials to $\mathcal{O}(1/m^2)$ are obtained by performing the matching to $\mathcal{O}(1/m^2)$:

$1/m$:

\[
\begin{array}{c}
\begin{array}{c}
\bullet \\
O_n \quad O_n^\dagger
\end{array}
\end{array}
\quad + \quad
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m}
\end{array}
\end{array}
\ \ = \ 
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m}
\end{array}
\end{array}
\quad + \quad
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m}
\end{array}
\end{array}
\quad + \quad \ldots \quad = \\
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m}
\end{array}
\end{array}
\end{array}
\]

$1/m^2$:

\[
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m^2}
\end{array}
\quad + \quad
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m^2}
\end{array}
\end{array}
\quad + \quad
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m^2}
\end{array}
\end{array}
\quad + \quad \ldots \\
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m^2}
\end{array}
\end{array}
\end{array}
\]

\[
= \quad
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m^2}
\end{array}
\end{array}
\quad + \quad
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m^2}
\end{array}
\end{array}
\quad + \quad \ldots
\end{array}
\]

\[
= \quad
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m^2}
\end{array}
\end{array}
\quad = \\
\begin{array}{c}
\begin{array}{c}
\bullet \\
\frac{1}{m^2}
\end{array}
\end{array}
\]

- Lattice result from Wilson loops not available
- Do our best: match weakly-coupled pNRQCD to BOEFT in the small $r$ regime (multipole expansion)
- We work to the accuracy LO in the multipole expansion for the $1/m^2$-potentials, and NLO in the multipole expansion for the $1/m$-potentials.
For $\kappa = 1^{+-}$, result of matching for the spin-dependent potential to $O(1/m^2)$:

\[
V_{1\lambda\lambda'}^{(1)} SD (r) = V_{1SK} (r) \left( P_{1\lambda}^i K_{1j}^i P_{1\lambda'}^j \right) \cdot S \\
+ V_{1SKb} (r) \left[ (r \cdot P_{1\lambda}^i) \left( r^i K_{1j}^i P_{1\lambda'}^j \right) \cdot S - (r^i K_{1j}^i P_{1\lambda'}^j) \cdot S (r \cdot P_{1\lambda}) \right]
\]

\[
V_{1\lambda\lambda'}^{(2)} SD (r) = V_{1SLa} (r) \left( P_{1\lambda}^i L_{Q \bar{Q}} P_{1\lambda'}^i \right) \cdot S + V_{1SLb} (r) P_{1\lambda}^i \left( L_{Q \bar{Q}} S^i + S^i L_{Q \bar{Q}} \right) P_{1\lambda'}^j \\
+ V_{1S2} (r) S^2 \delta_{\lambda\lambda'} + V_{1S12a} (r) S_{12} \delta_{\lambda\lambda'} + V_{1S12b} (r) P_{1\lambda}^i P_{1\lambda'}^j \left( S^i S^j_{12} + S^i S^j_{21} \right)
\]

where \( (K_{1}^{ij})^k = i \epsilon^{ijk} \)

- Operators absent in traditional quarkonia appear.
- Leading spin-dependent operators appear at order $1/m$, as opposed to $1/m^2$ in traditional quarkonia.
- $V_i (r) = V_{oi} (r) + V_{i}^{np} (r)$, a sum of a perturbative part and a nonperturbative part.
- The nonperturbative part has a multipole expansion:
  \[
  V_{i}^{np} (r) = V_{i}^{np} (0) + V_{i}^{np} (1) r^2 + \ldots
  \]
- In general, $V_{i}^{np} (j)$ has a factorized form: $V_{i}^{np} (j) = c_{ij} U_{ij}$.
  - $c_{ij}$: product of perturbative matching coefficients in weakly-coupled pNRQCD
  - $U_{ij}$: nonperturbative purely gluonic correlator (can be calculated on the lattice)
- Example: $V_{SK}^{np} (0) = \frac{c_F}{12} U_B$

\[
U_B = \lim_{T \to \infty} \frac{i e^{-i \Lambda T}}{T} \int_{-T/2}^{T/2} dt \langle 0 | G^{ia \dagger} (T/2) \phi^{ab} (T/2, t) g B^{jc} (t) \phi^{de} (T/2, t) G^{ke} (-T/2) | 0 \rangle h^{bcd} \epsilon^{ijk}
\]

For analogous expressions of other $V_{i}^{np} (j)$, see N. Brambilla, W. K. Lai, J. Segovia, J. Tarrús Castellà and A. Vairo, TUM-EFT 96/17 (in preparation).
For $\kappa = 1^{+-}$, result of matching for the spin-dependent potential to $O(1/m^2)$:

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\]

\[
V_{1\lambda\lambda'}^{(2)}(r)_{SD} = V_{1SLa}(r) \left( P_{1\lambda}^i L^{iQ}_{Q\bar{Q}} P_{1\lambda'}^j \right) \cdot S + V_{1SLb}(r) P_{1\lambda}^i \left( L^{iQ}_{Q\bar{Q}} S_{1}^{j} + S_{1}^{i} L^{j}_{Q\bar{Q}} \right) P_{1\lambda'}^j
\]

\[
+ V_{1S2}(r) S^2_{\lambda\lambda'} + V_{1S12a}(r) S_{12\delta\lambda\lambda'} + V_{1S12b}(r) P_{1\lambda}^i P_{1\lambda'}^j \left( S_{1}^{i} S_{2}^{j} + S_{2}^{i} S_{1}^{j} \right)
\]

where \((K_{ij})^k = i\epsilon^{ikj}\)

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For $\kappa = 1^{+-}$, result of matching for the spin-dependent potential to $O(1/m^2)$:

$$V^{(1)}_{1\lambda\lambda' SD}(r) = V_{SK}(r) \left( P_{1\lambda}^i K_{1}^{ij} P_{1\lambda'}^j \right) \cdot S + V_{SKb}(r) \left[ (r \cdot P_{1\lambda}^i) \left( r^i K_{1}^{ij} P_{1\lambda'}^j \right) \cdot S - (r^i K_{1}^{ij} P_{1\lambda'}^j) \cdot S (r \cdot P_{1\lambda'}) \right]$$

$$V^{(2)}_{1\lambda\lambda' SD}(r) = V_{SLa}(r) \left( P_{1\lambda}^i L_{Q\bar{Q}} P_{1\lambda'}^j \right) \cdot S + V_{SLb}(r) P_{1\lambda}^i \left( L_{Q\bar{Q}} S_j^i + S_i^j L_{Q\bar{Q}} \right) P_{1\lambda'}^j$$

$$+ V_{S2}(r) S^2_{12} \delta_{\lambda\lambda'} + V_{S12a}(r) S_1^2 \delta_{\lambda\lambda'} + V_{S12b}(r) P_{1\lambda}^i P_{1\lambda'}^j \left( S_i^1 S_j^2 + S_i^2 S_j^1 \right)$$

where $(K_{1}^{ij})^k = i\epsilon^{ijk}$

- Operators absent in traditional quarkonia appear.
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- $V_i(r) = V_{oi}(r) + V_{i}^{np}(r)$, a sum of a perturbative part and a nonperturbative part.
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Quarkonium hybrid spectrum with spin splitting

Method:

- First, solve the Schrödinger equation with the static potential $V_{1\lambda}^{(0)}(r)$.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{SK} = V_{SK}^{np, (0)} + V_{SK}^{np, (1)} r^2$</td>
<td></td>
</tr>
<tr>
<td>$V_{SKb} = V_{SK}^{np, (0)}$</td>
<td></td>
</tr>
<tr>
<td>$V_{SLa} = V_{o, SL} + V_{SLa}^{np, (0)}$</td>
<td></td>
</tr>
<tr>
<td>$V_{SLb} = V_{SLb}^{np, (0)}$</td>
<td></td>
</tr>
<tr>
<td>$V_{S2} = V_{o, S2} + V_{S2}^{np, (0)}$</td>
<td></td>
</tr>
<tr>
<td>$V_{S12, a} = V_{o, S12}$</td>
<td></td>
</tr>
<tr>
<td>$V_{S12, b} = V_{S12, b}^{np, (0)}$</td>
<td></td>
</tr>
</tbody>
</table>
**Method:**

- First, solve the Schrödinger equation with the static potential $V_{1\lambda}^{(0)}(r)$.
- Apply perturbative theory to the spin-dependent terms, 2nd order for the $V_{SK}^{np(0)}$-term, 1st order for the rest.

\[
\begin{align*}
V_{SK} &= V_{SK}^{np(0)} + V_{SK}^{np(1)} r^2 \\
V_{SKb} &= V_{SKb}^{np(0)} \\
V_{SLa} &= V_{SLa}^{np(0)} \\
V_{SLb} &= V_{SLb}^{np(0)} \\
V_{S2} &= V_{S2}^{np(0)} \\
V_{S12a} &= V_{S12a}^{np(0)} \\
V_{S12b} &= V_{S12b}^{np(0)}
\end{align*}
\]
Quarkonium hybrid spectrum with spin splitting

Method:

- First, solve the Schrödinger equation with the static potential $V^{(0)}_{1\lambda}(r)$.
- Apply perturbative theory to the spin-dependent terms, 2nd order for the $V^{np(0)}_{SK}$-term, 1st order for the rest.
- Six parameters $V^{np(0)}_{SK}$, $V^{np(1)}_{SK}$, $V^{np(0)}_{SLa}$, $V^{np(0)}_{SLb}$, $V^{np(0)}_{S2}$, $V^{np(0)}_{S12b}$ are fitted to lattice data of charmonium hybrid spectrum.

\[
\begin{align*}
V_{SK} &= V^{np(0)}_{SK} + V^{np(1)}_{SK} r^2 \\
V_{SKb} &= V^{np(0)}_{SKb} \\
V_{SLa} &= V_o SL + V^{np(0)}_{SLa} \\
V_{SLb} &= V^{np(0)}_{SLb} \\
V_{S2} &= V_o S2 + V^{np(0)}_{S2} \\
V_{S12a} &= V_o S_{12} \\
V_{S12b} &= V^{np(0)}_{S12b}
\end{align*}
\]
Quarkonium hybrid spectrum with spin splitting

Method:

- First, solve the Schrödinger equation with the static potential $V_{1\lambda}^{(0)}(r)$.
- Apply perturbative theory to the spin-dependent terms, 2nd order for the $V_{SK}^{np(0)}$-term, 1st order for the rest.
- Six parameters $V_{SK}^{np(0)}$, $V_{SK}^{np(1)}$, $V_{SLa}^{np(0)}$, $V_{SLb}^{np(0)}$, $V_{S2}^{np(0)}$, $V_{S12b}^{np(0)}$ are fitted to lattice data of charmonium hybrid spectrum.
- Since the parameters are factorized into a perturbative part, which has known flavor dependence, and a nonperturbative purely gluonic part, which is flavor independent, we can use the fitted values of the parameters to predict the spin splitting in bottomonium hybrids.

$$V_{SK} = V_{SK}^{np(0)} + V_{SK}^{np(1)} r^2$$
$$V_{SKb} = V_{SKb}^{np(0)}$$
$$V_{SLa} = V_{o SL} + V_{SLa}^{np(0)}$$
$$V_{SLb} = V_{SLb}^{np(0)}$$
$$V_{S2} = V_{o S2} + V_{S2}^{np(0)}$$
$$V_{S12a} = V_{o S12}$$
$$V_{S12b} = V_{S12b}^{np(0)}$$
<table>
<thead>
<tr>
<th>Multiplet</th>
<th>Mass (GeV)</th>
<th>Spin Average</th>
<th>Perturbative</th>
<th>Total</th>
<th>Lattice</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₁ multiplet</td>
<td>4.393 GeV</td>
<td>4.393 GeV</td>
<td>Perturbative</td>
<td>Total</td>
<td>Lattice</td>
</tr>
<tr>
<td>H₂ multiplet</td>
<td>4.472 GeV</td>
<td>4.472 GeV</td>
<td>Perturbative</td>
<td>Total</td>
<td>Lattice</td>
</tr>
<tr>
<td>H₃ multiplet</td>
<td>4.575 GeV</td>
<td>4.575 GeV</td>
<td>Perturbative</td>
<td>Total</td>
<td>Lattice</td>
</tr>
<tr>
<td>H₄ multiplet</td>
<td>4.638 GeV</td>
<td>4.638 GeV</td>
<td>Perturbative</td>
<td>Total</td>
<td>Lattice</td>
</tr>
</tbody>
</table>

Features of the results

- Perturbative contributions in spin triplets have a pattern opposite to that in the lattice data, and also to that of ordinary quarkonia. This is due to the repulsive nature of the perturbative heavy-quark-antiquark octet potential.

Nonperturbative parameters obtained from fitting to lattice data:

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{SK}^{np(0)} / \Lambda_{QCD}^2$</td>
<td>+1.03</td>
</tr>
<tr>
<td>$V_{SK}^{np(1)} / \Lambda_{QCD}^4$</td>
<td>-0.51</td>
</tr>
<tr>
<td>$V_{SLa}^{np(0)} / \Lambda_{QCD}^3$</td>
<td>-1.32</td>
</tr>
<tr>
<td>$V_{SLb}^{np(0)} / \Lambda_{QCD}^3$</td>
<td>+2.44</td>
</tr>
<tr>
<td>$V_{S2}^{np(0)} / \Lambda_{QCD}^3$</td>
<td>-0.33</td>
</tr>
<tr>
<td>$V_{S12b}^{np(0)} / \Lambda_{QCD}^3$</td>
<td>-0.39</td>
</tr>
</tbody>
</table>

($\Lambda_{QCD} = 0.5 \text{ GeV}$)
Features of the results

- Perturbative contributions in spin triplets have a pattern opposite to that in the lattice data, and also to that of ordinary quarkonia. This is due to the repulsive nature of the perturbative heavy-quark-antiquark octet potential.

- The discrepancy with the lattice data can be reconciled by the nonperturbative contributions, in particular the contribution from the $V_{SK}^{np(0)}$ term $\sim \Lambda_{QCD}^2/m$, parametrically larger than the perturbative contributions $\sim mv^4$.

Nonperturbative parameters obtained from fitting to lattice data:

<table>
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<th>Term</th>
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<tr>
<td>$V_{SK}^{np(0)}/\Lambda_{QCD}^2$</td>
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($\Lambda_{QCD} = 0.5$ GeV)
Lowest-lying bottomonium hybrid spectrum predicted by BOEFT with spin splitting.
The formalism of EFT for tetraquarks parallels that of hybrids. The only difference is that the sector of light d.o.f. has more structure: $\kappa = \{K^{PC}, f\}$.

**Lagrangian of BOEFT (LO in $1/m$)**

$$L_{BOEFT} = \int d^3R d^3r \sum_\kappa \sum_{\lambda \lambda'} \Psi_{\kappa\lambda}^\dagger(t, r, R) \left\{ i\partial_t + \frac{\nabla^2 r}{m} - V_o(r) - \Lambda_\kappa - b_{\kappa\lambda} r^2 + \ldots \right\} \delta_{\lambda\lambda'}$$

$$+ C_{\kappa\lambda\lambda'}^{nad} \right\} \Psi_{\kappa\lambda'}(t, r, R)$$

We need static energies from the lattice, with insertions of light-quark bilinear operators in the static Wilson loop:

$$O_n = O_n^a T^a, \quad q = (u, d)$$

<table>
<thead>
<tr>
<th>$\Lambda^\sigma$</th>
<th>$K^{PC}$</th>
<th>$O_n^a(I = 0, I = 1)$</th>
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<tr>
<td>$\Sigma^+_q$</td>
<td>0++</td>
<td>$\bar{q} T^a(1, \tau) q$</td>
</tr>
<tr>
<td>$\Sigma^+_u$</td>
<td>0+-</td>
<td>$\bar{q} \gamma^0 T^a(1, \tau) q$</td>
</tr>
<tr>
<td>$\Sigma^-_q$</td>
<td>0-+</td>
<td>$\bar{q} \gamma_5 T^a(1, \tau) q$</td>
</tr>
<tr>
<td>$\Sigma^+_q$</td>
<td>1--</td>
<td>$\bar{q}(r \cdot \gamma) T^a(1, \tau) q$</td>
</tr>
<tr>
<td>$\Pi^+_q$</td>
<td>1--</td>
<td>$\bar{q}(r \times \gamma) T^a(1, \tau) q$</td>
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<tr>
<td>$\Sigma^+_g$</td>
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EFT for quarkonium hybrids  Hybrid spectrum  Spin-dependent potential  EFT for tetraquarks  Summary and outlook

**Summary and outlook**

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Using static energies from the lattice, the leading-order Schrödinger equation gives spectra consistent with the lattice results. We obtained the spin-dependent potential by integrating out the scale $\Lambda_{QCD}$ in the small $r$ regime. Terms in the spin-dependent potential are factorized into a perturbative part and a flavor-independent nonperturbative gluonic correlator. Spin-dependent operators which are absent for traditional quarkonia appear. Remarkably, the leading spin-dependent terms enter at order $1/m$, as opposed to order $1/m^2$ for traditional quarkonia. This would have an important impact on the form of decays and transitions. We fitted the gluonic correlators to the charmonium hybrid spectrum from the lattice. The fitted values are consistent with the power counting. We use them to predict spin splitting in the bottomonium hybrid spectrum. Lattice calculation of the gluonic correlators that appear in the spin-dependent potential will help justify the EFT formalism. See Marc Wagner's talk for related work.

In the future, we will study decays and transitions following what was done in R. Oncala, J. Soto Phys. Rev. D96 (2017). In this case, the relevant mixing potential also involves gluonic correlators which can be computed on the lattice.
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The EFT framework has the potential to describe all exotic quarkonia.
Thank you.
Back up slides
At the scale $\Lambda_{QCD} \ll \mu \ll m v$, we have the weakly-coupled pNRQCD:

### Lagrangian of weakly-coupled pNRQCD

\[
L_{\text{pNRQCD}} = \int d^3 R \left\{ \int d^3 r \left( \text{Tr} \left[ S^\dagger (i \partial_0 - h_s) S + O^\dagger (iD_0 - h_o) O \right] \right. \\
+ g \text{Tr} \left[ S^\dagger r \cdot EO + O^\dagger r \cdot ES + \frac{1}{2} O^\dagger r \cdot \{ E, O \} \right] + \frac{g}{4m} \text{Tr} \left[ O^\dagger L_{QQ} \cdot [B, O] \right] \\
+ \frac{g c_F}{m} \text{Tr} \left[ S^\dagger (S_1 - S_2) \cdot BO + O^\dagger (S_1 - S_2) \cdot BS + O^\dagger S_1 \cdot BO - O^\dagger S_2 O \cdot B \right] \\
+ \frac{g c_s}{2m^2} \text{Tr} \left[ S^\dagger (S_1 + S_2) \cdot (E \times p)O + O^\dagger (S_1 + S_2) \cdot (E \times p)S \right. \\
\left. + O^\dagger S_1 \cdot (E \times p)O + O^\dagger S_2 O \cdot (E \times p) \right) \right\} - \frac{1}{4} G^a_{\mu \nu} G^{\mu \nu} + \ldots
\]

\[
h_s = - \nabla_2^r \frac{1}{m} + V_s(r)
\]

\[
h_o = - \nabla_2^r \frac{1}{m} + V_o(r)
\]

\[
V_o(r) = V_o^{(0)}(r) + \frac{V_o^{(1)}(r)}{m} + \frac{V_o^{(2)}(r)}{m^2} + \ldots
\]

\[
V_o^{(2)}(r) = V_o^{(2)}_{SD}(r) + V_o^{(2)}_{SI}(r)
\]

\[
V_o^{(2)}_{SD}(r) = V_o SL(r) L_{QQ} \cdot S + V_o S^2(r) S^2 + V_o S_{12}(r) S_{12}
\]

\[
S = S_1 + S_2, \quad S_{12} = 12(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - 4S_1 \cdot S_2
\]
Spectrum of the four lowest-lying charmonium hybrid multiplets. The lattice results from Liu et al. with $m_\pi \approx 400$ MeV are plotted in purple.
Spectrum of the four lowest-lying bottomonium hybrids. Parameters from fitting to Liu et al.
Nonperturbative matching coefficients determined by fitting charmonium hybrid spectrum obtained from the hybrid EFT to the lattice spectrum from the Hadron Spectrum Collaboration data of Liu et al. and Cheung et al. with pion masses of $m_\pi \approx 400$ MeV and $m_\pi \approx 240$ MeV respectively. The matching coefficients are normalized to their parametric natural size. We take the value $\Lambda_{QCD} = 0.5$ GeV.

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<td>+1.03</td>
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<tr>
<td>$V_{SPa}^{np}(0)/\Lambda_{QCD}^3$</td>
<td>+0.81</td>
<td>-1.32</td>
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<td>+1.18</td>
<td>+2.44</td>
</tr>
<tr>
<td>$V_{SPc}^{np}(0)/\Lambda_{QCD}^3$</td>
<td>-0.26</td>
<td>-0.33</td>
</tr>
<tr>
<td>$V_{SPd}^{np}(0)/\Lambda_{QCD}^3$</td>
<td>+0.69</td>
<td>-0.39</td>
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Static energies for tetraquarks (schematic):

![Graph showing static energies for tetraquarks](image)

- **Adjoint tetraquark**
- **Diquark-diquark**
- **Compact tetraquark**
- **Hadroquarkonium**
- **Heavy meson molecule**

Courtesy J. Tarrús Castellà