Hybrids and Tetraquarks in EFTs

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Cavallerizza Reale, Turin

Outline

- EFT for quarkonium hybrids

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  - Spectrum without spin splitting

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  - Spin-dependent potential and spin splitting

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- EFT for tetraquarks

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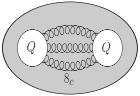
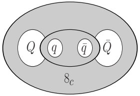
  - Spectrum without spin splitting
  - Spin-dependent potential and spin splitting
- EFT for tetraquarks
- Summary and outlook

Exotic quarkonia

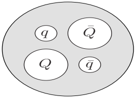
- Exotic quarkonia (XYZ particles) are quarkonium-like bound states that cannot be interpreted as a traditional quarkonium state.
- Interpretations for exotic quarkonia:



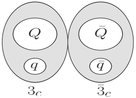
hybrid



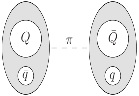
adjoint tetraquark



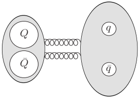
compact tetraquark



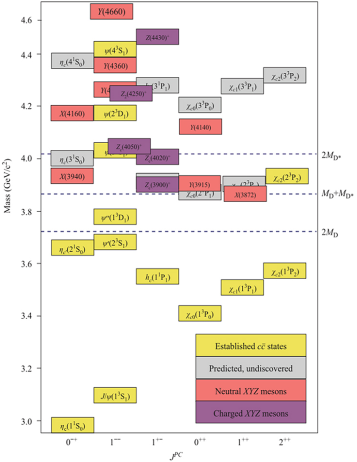
diquark-diquark



heavy-meson molecule



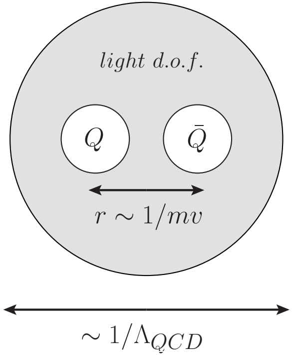
hadroquarkonium

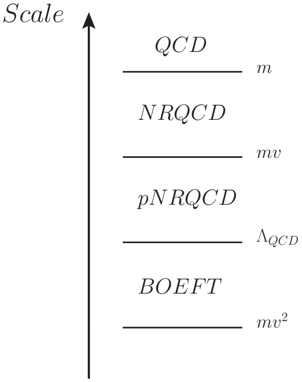


- A complete understanding of exotic quarkonia in terms of these pictures has not yet been obtained.
- EFT together with lattice QCD: a tool to understand the various pictures in a model-independent way.

Quarkonium hybrids in the EFT picture

- Quarkonium hybrid: $Q\bar{Q}$ in color octet with gluon excitation (light d.o.f. with scale Λ_{QCD})
- Separation of scales: $m \gg mv \gg \Lambda_{QCD} \gg mv^2 \implies$ suitable for EFT description ($m \equiv m_Q$)
- Integrate out d.o.f.: QCD $\rightarrow$ NRQCD $\rightarrow$ pNRQCD $\rightarrow$ BOEFT (Born-Oppenheimer EFT)



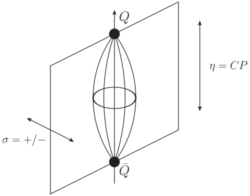


Static limit

- Dynamics of $Q\bar{Q}$ happens at time scale $\sim 1/mv^2 \gg 1/\Lambda_{QCD}$ $\Rightarrow$ Born Oppenheimer approximation. E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D 90, (2014)
- Interquark potential given by energy eigenvalues with Q and $\bar{Q}$ at fixed positions.
- In the static limit of $Q\bar{Q}$, the system has symmetry group $D_{\infty h}$. Irreducible representation of $D_{\infty h}$: Λ_η^σ.

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots$ $(\Sigma,\ \Pi,\ \Delta,\ \Phi,\ \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g),\ -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)



Static energies

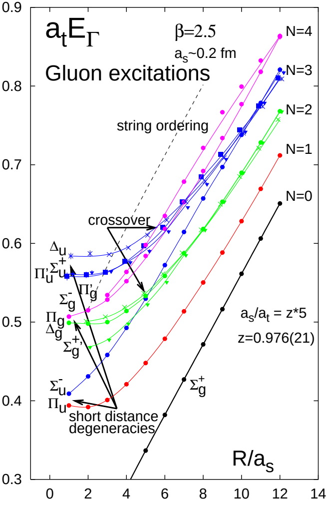
- In the static limit, when $r \to 0$ the symmetry group reduces to $O(3) \times C$, the quarkonium hybrid turns into a gluelump.
  State of light d.o.f. at $r = 0$ labeled by $\kappa = K^{PC}$.
- State of light d.o.f. labeled by $n = (\kappa, \Lambda_\eta^\sigma)$, with κ denoting the $r \to 0$ limit.
- Static energy $E_n^{(0)}(r)$: energy eigenvalue of $H^{(0)}$ (NRQCD Hamiltonian in the limit $m \to \infty$), with Q and $\bar{Q}$ at fixed positions $\boldsymbol{x}_1$, $\boldsymbol{x}_2$.
- In terms of Wilson loop:

$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_n(T/2)|X_n(-T/2)\rangle$$

$$|X_n\rangle = \chi(\boldsymbol{x}_2)\phi(\boldsymbol{x}_2, \boldsymbol{R})O_n(\boldsymbol{R})\phi(\boldsymbol{R}, \boldsymbol{x}_1)\psi^\dagger(\boldsymbol{x}_1)|0\rangle$$

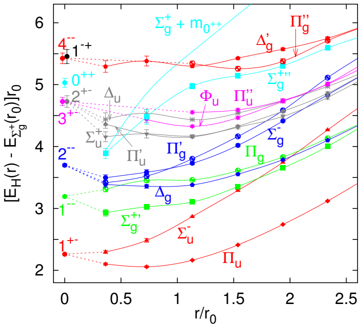
Λ_η^σ	K^{PC}	O_n
Σ_u^-	1^{+-}	$\hat{\boldsymbol{r}} \cdot \boldsymbol{B}$, $\hat{\boldsymbol{r}} \cdot (\boldsymbol{D} \times \boldsymbol{E})$
Π_u	1^{+-}	$\hat{\boldsymbol{r}} \times \boldsymbol{B}$, $\hat{\boldsymbol{r}} \times (\boldsymbol{D} \times \boldsymbol{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\boldsymbol{r}} \cdot \boldsymbol{E}$, $\hat{\boldsymbol{r}} \cdot (\boldsymbol{D} \times \boldsymbol{B})$
Π_g	1^{--}	$\hat{\boldsymbol{r}} \times \boldsymbol{E}$, $\hat{\boldsymbol{r}} \times (\boldsymbol{D} \times \boldsymbol{B})$
Σ_g^-	2^{--}	$(\hat{\boldsymbol{r}} \cdot \boldsymbol{D})(\hat{\boldsymbol{r}} \cdot \boldsymbol{B})$
Π_g'	2^{--}	$\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} \cdot \boldsymbol{D})\boldsymbol{B} + \boldsymbol{D}(\hat{\boldsymbol{r}} \cdot \boldsymbol{B}))$
Δ_g	2^{--}	$(\hat{\boldsymbol{r}} \times \boldsymbol{D})^i(\hat{\boldsymbol{r}} \times \boldsymbol{B})^j + (\hat{\boldsymbol{r}} \times \boldsymbol{D})^j(\hat{\boldsymbol{r}} \times \boldsymbol{B})^i$
Σ_u^+	2^{+-}	$(\hat{\boldsymbol{r}} \cdot \boldsymbol{D})(\hat{\boldsymbol{r}} \cdot \boldsymbol{E})$
Π_u'	2^{+-}	$\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} \cdot \boldsymbol{D})\boldsymbol{E} + \boldsymbol{D}(\hat{\boldsymbol{r}} \cdot \boldsymbol{E}))$
Δ_u	2^{+-}	$(\hat{\boldsymbol{r}} \times \boldsymbol{D})^i(\hat{\boldsymbol{r}} \times \boldsymbol{E})^j + (\hat{\boldsymbol{r}} \times \boldsymbol{D})^j(\hat{\boldsymbol{r}} \times \boldsymbol{E})^i$

Lattice determination of static energies



K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

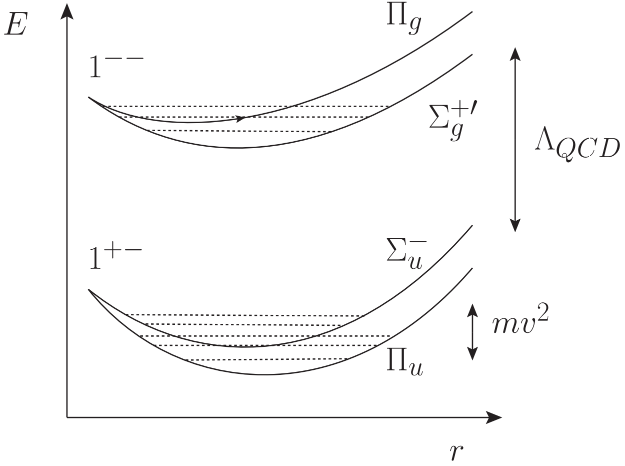
- Ground state: Σ_g^+
- Gluelump energy $\Lambda_\kappa \equiv E_n(r=0)$
- Degeneracies at $r \to 0$



Gluelump energies (at $r = 0$):

M. Foster and C. Michael, Phys. Rev. D59 (1999)

BOEFT



BOEFT

Successive matchings to obtain BOEFT

- QCD $\to$ NRQCD: Integrate out modes of scale m.
  W. Caswell, G. Lepage, Phys. Lett. 167B (1986); G. Bodwin, E. Braaten, G. Lepage, Phys. Rev. D51 (1995)
- NRQCD $\to$ weakly-coupled pNRQCD: Integrate out modes of scale $mv \sim 1/r$ in the short distance regime ($r \ll 1/\Lambda_{QCD}$).
  A. Pineda, J. Soto, Nucl. Phys. Proc. Suppl. 64 (1998); N. Brambilla, A. Pineda, J. Soto, A. Vairo, Nucl. Phys. B566 (2000)
- weakly-coupled pNRQCD $\to$ BOEFT: Integrate out modes of scale Λ_{QCD}.
  M. Berwein, N. Brambilla, J. Tarrús Castellà, A. Vairo, Phys. Rev. D92 (2015); N. Brambilla, G. Krein, J. Tarrús Castellà, A. Vairo, Phys. Rev. D97 (2018); R. Oncala, J. Soto, Phys. Rev. D96 (2017)
- BOEFT: EFT for dynamics of $Q\bar{Q}$ at scale mv^2 $\Rightarrow$ Schrödinger equation

BOEFT

- d.o.f.: $\int d^3r d^3R \sum_{\kappa\lambda} P^i_{\kappa\lambda} O^{a\dagger}(\boldsymbol{r}, \boldsymbol{R}) G^{ia}_{\kappa}(\boldsymbol{R})|0\rangle \Psi_{\kappa\lambda}(t, \boldsymbol{r}, \boldsymbol{R})$
- $G^{ia}_{\kappa}(\boldsymbol{R})$: gluelump operator: $H^{(0)} G^{ia}_{\kappa}(\boldsymbol{R})|0\rangle = \Lambda_\kappa G^{ia}_{\kappa}(\boldsymbol{R})|0\rangle$ with $r = 0$
- $P^i_{\kappa\lambda}$ projects $G^{ia}_{\kappa}(\boldsymbol{R})$ to a representation of $D_{\infty h}$.
- $\Psi_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R})$: wave function of $Q\bar{Q}$ in the quarkonium hybrid

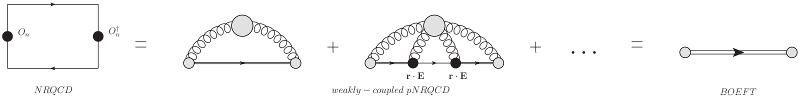
Lagrangian of BOEFT

$$L_{BOEFT} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \left\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{m} P^i_{\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R}) + \dots$$

- $V_{\kappa\lambda\lambda'}(r)$ is organized as an expansion in $1/m$:

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m^2} + \dots$$

- $V^{(0)}_{\kappa\lambda}$ is related to the static energy by matching:



$$E^{(0)}_n(r) = V^{(0)}_o(r) + \Lambda_\kappa + b_{\kappa\lambda} r^2 + \dots = V^{(0)}_{\kappa\lambda}(r)$$

$V^{(0)}_o$: perturbative octet potential in weak-coupled pNRQCD

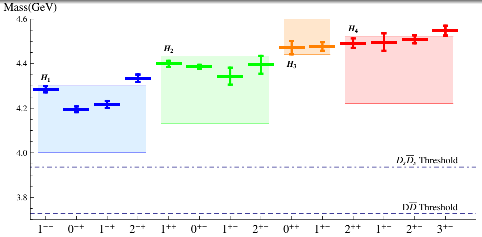
Hybrid spectrum for lowest lying gluelump $\kappa = 1^{+-}$ $(\Lambda_\eta^\sigma = \Sigma_u^-, \Pi_u)$

M. Berwein, N. Brambilla, J. Tarrús Castellà, A. Vairo, Phys. Rev. D92 (2015)

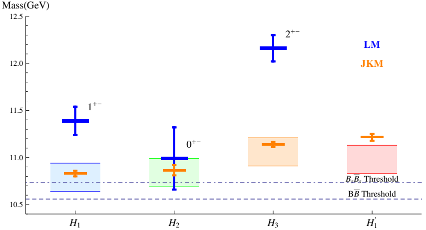
- Only consider $V_{1\lambda}^{(0)}$, neglect $V_{1\lambda\lambda'}^{(1)}$, $V_{1\lambda\lambda'}^{(2)}$ ...
- Use perturbative value $V_o^{(0)}(r)$ for $r \lesssim 0.5$ fm and a numerical fit to lattice value $E_{1\lambda}^{(0)}(r)$ for $r \gtrsim 0.5$ fm
- The term $P_{\kappa\lambda}^{i\dagger} \frac{\nabla_r^2}{m} P_{\kappa\lambda'}^{i}$ mixes Σ_u^- and Π_u states, leading to Λ-doubling

Lowest-lying quarkonium hybrid multiplets

Multiplet	l	$J^{PC}(s=0)$	$J^{PC}(s=1)$
H_1	1	1^{--}	$(0,1,2)^{-+}$
H_2	1	1^{++}	$(0,1,2)^{+-}$
H_3	0	0^{++}	1^{+-}
H_4	2	2^{++}	$(1,2,3)^{+-}$



Lowest-lying charmonium hybrid spectrum. Bands are results from EFT, points with error bars are lattice data from L. Liu et al. (Hadron Spectrum), JHEP, 07, 126 (2012).



Lowest-lying bottomonium hybrid spectrum. Bands are results from EFT, points with error bars are lattice data from K. J. Juge, J. Kuti, and C. J. Morningstar, Phys. Rev. Lett., 82, 4400 (1999) (JKM) and X. Liao and T. Manke, Phys. Rev., D65, 074508 (2002) (LM).

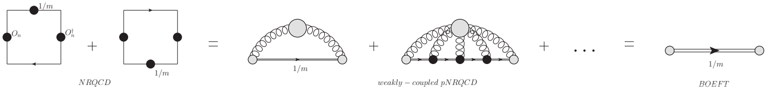
- For a study of mixing of quarkonium hybrids with and decay to traditional quarkonia, see R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Spin-dependent potential to $\mathcal{O}(1/m^2)$

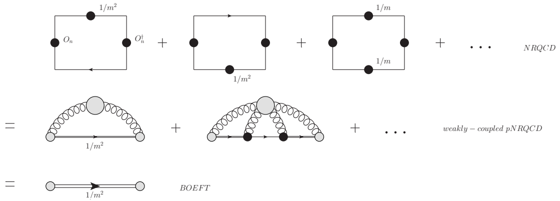
N. Brambilla, W. K. Lai, J. Segovia, J. Tarrús Castellà, A. Vairo, Phys. Rev. D 99 (2019)

The spin-dependent potentials to $\mathcal{O}(1/m^2)$ are obtained by performing the matching to $\mathcal{O}(1/m^2)$:

$1/m$:



$1/m^2$:



- Lattice result from Wilson loops not available
- Do our best: match weakly-coupled pNRQCD to BOEFT in the small r regime (multipole expansion)
- We work to the accuracy LO in the multipole expansion for the $1/m^2$-potentials, and NLO in the multipole expansion for the $1/m$-potentials.

For $\kappa = 1^{+-}$, result of matching for the spin-dependent potential to $\mathcal{O}(1/m^2)$:

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{K}^{ij}_1 P^{j}_{1\lambda'}\right)\cdot\boldsymbol{S}$$
$$+V_{1\,SKb}(r)\left[\left(\boldsymbol{r}\cdot P^{\dagger}_{1\lambda}\right)\left(r^i\boldsymbol{K}^{ij}P^{j}_{1\lambda'}\right)\cdot\boldsymbol{S}-\left(r^i\boldsymbol{K}^{ij}P^{j\dagger}_{1\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot P_{1\lambda'}\right)\right]$$
$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SLa}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{L}_{Q\bar{Q}}P^{i}_{1\lambda'}\right)\cdot\boldsymbol{S}+V_{1\,SLb}(r)P^{i\dagger}_{1\lambda}\left(L^i_{Q\bar{Q}}S^j+S^iL^j_{Q\bar{Q}}\right)P^{j}_{1\lambda'}$$
$$+V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'}+V_{1\,S_{12}a}(r)S_{12}\delta_{\lambda\lambda'}+V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^{j}_{1\lambda'}\left(S^i_1S^j_2+S^i_2S^j_1\right)$$

where $\left(K^{ij}_1\right)^k = i\epsilon^{ikj}$

- Operators absent in traditional quarkonia appear.
- Leading spin-dependent operators appear at order $1/m$, as opposed to $1/m^2$ in traditional quarkonia.
- $V_i(r) = V_{oi}(r) + V^{np}_i(r)$, a sum of a perturbative part and a nonperturbative part.
- The nonperturbative part has a multipole expansion: $V^{np}_i(r) = V^{np\,(0)}_i + V^{np\,(1)}_i r^2 + \dots$.
- In general, $V^{np\,(j)}_i$ has a factorized form: $V^{np\,(j)}_i = c_{ij}U_{ij}$.
  c_{ij}: product of perturbative matching coefficients in weakly-coupled pNRQCD
  U_{ij}: nonperturbative purely gluonic correlator (can be calculated on the lattice)
- Example: $V^{np\,(0)}_{SK} = \frac{c_F}{12}U_B$

$$U_B = \lim_{T\to\infty}\frac{ie^{i\Lambda T}}{T}\int_{-T/2}^{T/2}dt\,\langle 0|G^{ia\dagger}(T/2)\phi^{ab}(T/2,t)gB^{jc}(t)\phi^{de}(T/2,t)G^{ke}(-T/2)|0\rangle h^{bcd}\epsilon^{ijk}$$

For analogous expressions of other $V^{np\,(j)}_i$, see N. Brambilla, W. K. Lai, J. Segovia, J. Tarrús Castellà and A. Vairo, TUM-EFT 96/17 (in preparation).

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$$+ V_{1\,SKb}(r)\left[\left(\boldsymbol{r}\cdot P^\dagger_{1\lambda}\right)\left(r^i\boldsymbol{K}^{ij}P^j_{1\lambda'}\right)\cdot\boldsymbol{S} - \left(r^i\boldsymbol{K}^{ij}P^{j\dagger}_{1\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot P_{1\lambda'}\right)\right]$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SLa}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{L}_{Q\bar{Q}}P^i_{1\lambda'}\right)\cdot\boldsymbol{S} + V_{1\,SLb}(r)P^{i\dagger}_{1\lambda}\left(L^i_{Q\bar{Q}}S^j + S^iL^j_{Q\bar{Q}}\right)P^j_{1\lambda'}$$
$$+ V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)S_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^j_{1\lambda'}\left(S^i_1S^j_2 + S^i_2S^j_1\right)$$

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Quarkonium hybrid spectrum with spin splitting

Method:

- First, solve the Schrödinger equation with the static potential $V_{1\lambda}^{(0)}(r)$.

$$
\begin{aligned}
V_{SK} &= V_{SK}^{np\,(0)} + V_{SK}^{np\,(1)} r^2 \\
V_{SKb} &= V_{SKb}^{np\,(0)} \\
V_{SLa} &= V_{o\,SL} + V_{SLa}^{np\,(0)} \\
V_{SLb} &= V_{SLb}^{np\,(0)} \\
V_{S^2} &= V_{o\,S^2} + V_{S^2}^{np\,(0)} \\
V_{S_{12}a} &= V_{o\,S_{12}} \\
V_{S_{12}b} &= V_{S_{12}b}^{np\,(0)}
\end{aligned}
$$

Quarkonium hybrid spectrum with spin splitting

Method:

- First, solve the Schrödinger equation with the static potential $V_{1\lambda}^{(0)}(r)$.
- Apply perturbative theory to the spin-dependent terms, 2nd order for the $V_{SK}^{np\,(0)}$-term, 1st order for the rest.

$$
\begin{aligned}
V_{SK} &= V_{SK}^{np\,(0)} + V_{SK}^{np\,(1)} r^2 \\
V_{SKb} &= V_{SKb}^{np\,(0)} \\
V_{SLa} &= V_{o\,SL} + V_{SLa}^{np\,(0)} \\
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V_{S^2} &= V_{o\,S^2} + V_{S^2}^{np\,(0)} \\
V_{S_{12}a} &= V_{o\,S_{12}} \\
V_{S_{12}b} &= V_{S_{12}b}^{np\,(0)}
\end{aligned}
$$

Quarkonium hybrid spectrum with spin splitting

Method:

- First, solve the Schrödinger equation with the static potential $V_{1\lambda}^{(0)}(r)$.
- Apply perturbative theory to the spin-dependent terms, 2nd order for the $V_{SK}^{np\,(0)}$-term, 1st order for the rest.
- Six parameters $V_{SK}^{np\,(0)}$, $V_{SK}^{np\,(1)}$, $V_{SLa}^{np\,(0)}$, $V_{SLb}^{np\,(0)}$, $V_{S^2}^{np\,(0)}$, $V_{S_{12}b}^{np\,(0)}$ are fitted to lattice data of charmonium hybrid spectrum.

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- Since the parameters are factorized into a perturbative part, which has known flavor dependence, and a nonperturbative purely gluonic part, which is flavor independent, we can use the fitted values of the parameters to predict the spin splitting in bottomonium hybrids.

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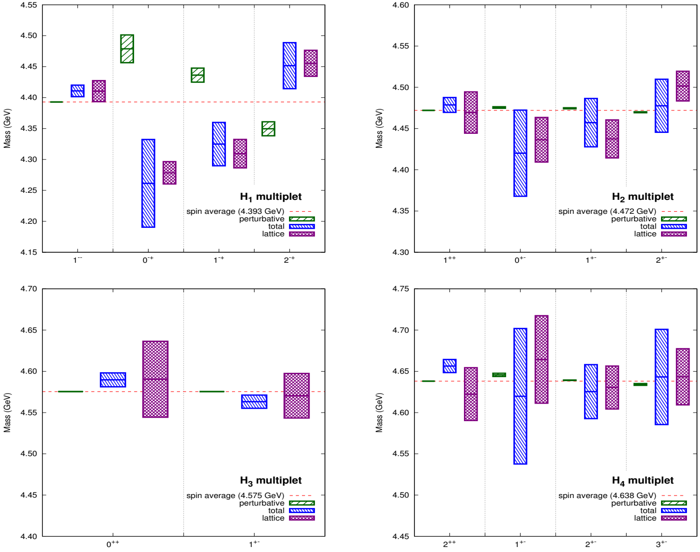

Lowest-lying charmonium hybrid spectrum with spin splitting. Lattice data are from G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP, 12, 089 (2016).

Features of the results

- Perturbative contributions in spin triplets have a pattern opposite to that in the lattice data, and also to that of ordinary quarkonia. This is due to the repulsive nature of the perturbative heavy-quark-antiquark octet potential.

Nonperturbative parameters obtained from fitting to lattice data:

$V_{SK}^{np\,(0)}/\Lambda_{QCD}^2$	$+1.03$
$V_{SK}^{np\,(1)}/\Lambda_{QCD}^4$	-0.51
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$V_{SLb}^{np\,(0)}/\Lambda_{QCD}^3$	$+2.44$
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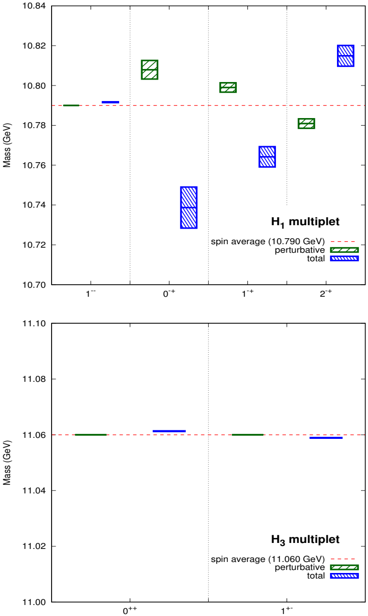

Lowest-lying bottomonium hybrid spectrum predicted by BOEFT with spin splitting.

EFT FOR TETRAQUARKS

- The formalism of EFT for tetraquarks parallels that of hybrids. The only difference is that the sector of light d.o.f. has more structure: $\kappa = \{K^{PC}, f\}$. f: flavor quantum numbers (isospin, strangeness, etc.)

Lagrangian of BOEFT (LO in $1/m$)

$$L_{BOEFT} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \Big\{ \Big[i\partial_t + \frac{\nabla^2_{\boldsymbol{r}}}{m} - V_o(r) - \Lambda_\kappa - b_{\kappa\lambda} r^2 + \dots \Big] \delta_{\lambda\lambda'} + C^{\rm nad}_{\kappa\lambda\lambda'} \Big\} \Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R})$$

- We need static energies from the lattice, with insertions of light-quark bilinear operators in the static Wilson loop:

$$O_n = O_n^a T^a,\ \boldsymbol{q} = (u, d)$$

Λ_η^σ	K^{PC}	$O_n^a(I=0, I=1)$
Σ_g^+	0^{++}	$\bar{\boldsymbol{q}} T^a(1, \boldsymbol{\tau})\boldsymbol{q}$
Σ_u^+	0^{+-}	$\bar{\boldsymbol{q}}\gamma^0 T^a(1, \boldsymbol{\tau})\boldsymbol{q}$
Σ_u^-	0^{-+}	$\bar{\boldsymbol{q}}\gamma_5 T^a(1, \boldsymbol{\tau})\boldsymbol{q}$
Σ_g^+	1^{--}	$\bar{\boldsymbol{q}}(\boldsymbol{r}\cdot\boldsymbol{\gamma}) T^a(1, \boldsymbol{\tau})\boldsymbol{q}$
Π_g	1^{--}	$\bar{\boldsymbol{q}}(\boldsymbol{r}\times\boldsymbol{\gamma}) T^a(1, \boldsymbol{\tau})\boldsymbol{q}$
Σ_g^+	1^{++}	$\bar{\boldsymbol{q}}(\boldsymbol{r}\cdot\boldsymbol{\gamma})\gamma_5 T^a(1, \boldsymbol{\tau})\boldsymbol{q}$
Π_g	1^{++}	$\bar{\boldsymbol{q}}(\boldsymbol{r}\times\boldsymbol{\gamma})\gamma_5 T^a(1, \boldsymbol{\tau})\boldsymbol{q}$

- The hierarchy of scales has to be checked by solving for the spectrum using the static energies as the potential.

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  - Lattice calculation of the gluonic correlators that appear in the spin-dependent potential will help justify the EFT formalism. See Marc Wagner's talk for related work.
  - In the future, we will study decays and transitions following what was done in R. Oncala, J. Soto Phys. Rev. D96 (2017). In this case, the relevant mixing potential also involves gluonic correlators which can be computated on the lattice.

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- Like hybrids, we can study tetraquarks in the BOEFT framework. We will need static energies for tetraquarks from the lattice.
- The EFT framework has the potential to describe all exotic quarkonia.

Thank you.

Back up slides

At the scale $\Lambda_{QCD} \ll \mu \ll mv$, we have the weakly-coupled pNRQCD:

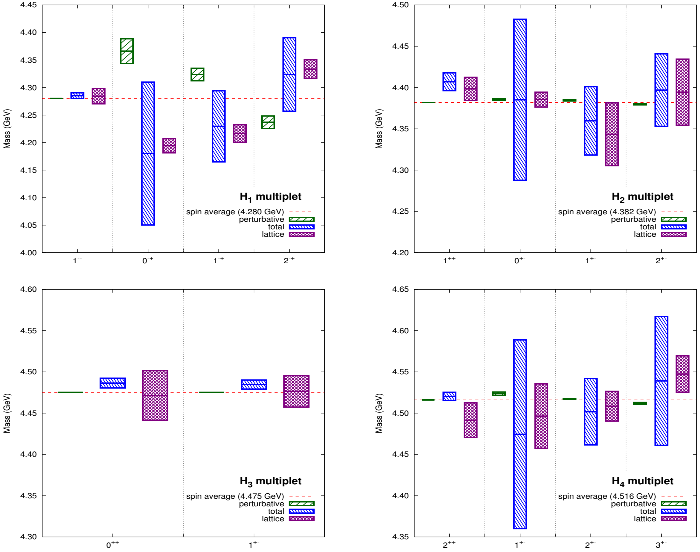
Lagrangian of weakly-coupled pNRQCD

$$L_{\rm pNRQCD} = \int d^3R \Big\{ \int d^3r \Big(\mathrm{Tr}\left[\mathrm{S}^\dagger\,(i\partial_0 - h_s)\,\mathrm{S} + \mathrm{O}^\dagger\,(iD_0 - h_o)\,\mathrm{O}\right]$$
$$+ g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\mathrm{O} + \mathrm{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\mathrm{S} + \frac{1}{2}\mathrm{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\mathrm{O}\}\right] + \frac{g}{4m}\mathrm{Tr}\left[\mathrm{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\mathrm{O}]\right]$$
$$+ \frac{gc_F}{m}\mathrm{Tr}\left[\mathrm{S}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\mathrm{O} + \mathrm{O}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\mathrm{S} + \mathrm{O}^\dagger\boldsymbol{S}_1\cdot\boldsymbol{B}\mathrm{O} - \mathrm{O}^\dagger\boldsymbol{S}_2\mathrm{O}\cdot\boldsymbol{B}\right]$$
$$+ \frac{gc_s}{2m^2}\mathrm{Tr}\left[\mathrm{S}^\dagger(\boldsymbol{S}_1 + \boldsymbol{S}_2)\cdot(\boldsymbol{E}\times\boldsymbol{p})\mathrm{O} + \mathrm{O}^\dagger(\boldsymbol{S}_1 + \boldsymbol{S}_2)\cdot(\boldsymbol{E}\times\boldsymbol{p})\mathrm{S}\right.$$
$$\left.+ \mathrm{O}^\dagger\boldsymbol{S}_1\cdot(\boldsymbol{E}\times\boldsymbol{p})\mathrm{O} + \mathrm{O}^\dagger\boldsymbol{S}_2\mathrm{O}\cdot(\boldsymbol{E}\times\boldsymbol{p})\right]\Big) - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a} + \ldots \Big\}$$

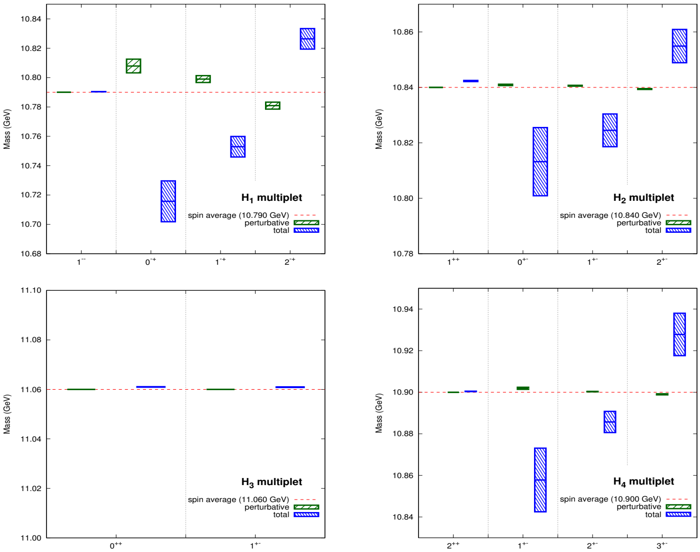
$$h_s = -\frac{\boldsymbol{\nabla}_r^2}{m} + V_s(r)$$
$$h_o = -\frac{\boldsymbol{\nabla}_r^2}{m} + V_o(r)$$

$$V_o(r) = V_o^{(0)}(r) + \frac{V_o^{(1)}(r)}{m} + \frac{V_o^{(2)}(r)}{m^2} + \ldots$$
$$V_o^{(2)}(r) = V_{o\,SD}^{(2)}(r) + V_{o\,SI}^{(2)}(r)$$
$$V_{o\,SD}^{(2)}(r) = V_{o\,SL}(r)\boldsymbol{L}_{Q\bar{Q}}\cdot\boldsymbol{S} + V_{o\,S^2}(r)\boldsymbol{S}^2 + V_{o\,S_{12}}(r)S_{12}$$

$\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$, $S_{12} = 12(\boldsymbol{S}_1\cdot\hat{\boldsymbol{r}})(\boldsymbol{S}_2\cdot\hat{\boldsymbol{r}}) - 4\boldsymbol{S}_1\cdot\boldsymbol{S}_2$



Spectrum of the four lowest-lying charmonium hybrid multiplets. The lattice results from Liu et al. with $m_\pi \approx 400$ MeV are plotted in purple.

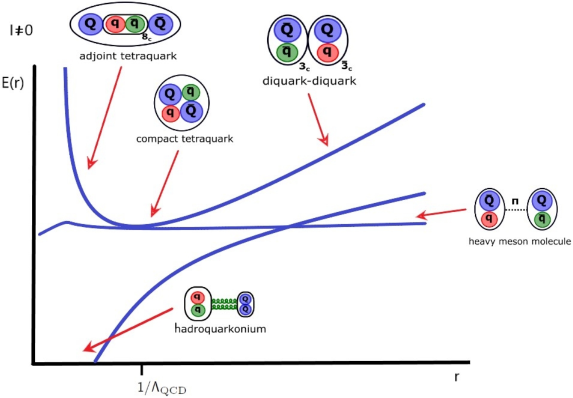


Spectrum of the four lowest-lying bottomonium hybrids. Parameters from fitting to Liu et al.

Nonperturbative matching coefficients determined by fitting charmonium hybrid spectrum obtained from the hybrid EFT to the lattice spectrum from the Hadron Spectrum Collaboration data of Liu et al. and Cheung et al. with pion masses of $m_\pi \approx 400$ MeV and $m_\pi \approx 240$ MeV respectively. The matching coefficients are normalized to their parametric natural size. We take the value $\Lambda_{QCD} = 0.5$ GeV.

	Liu et al.	Cheung et al.
$V_{SK}^{np\,(0)}/\Lambda_{QCD}^2$	$+1.50$	$+1.03$
$V_{SK}^{np\,(1)}/\Lambda_{QCD}^4$	-0.65	-0.51
$V_{SLa}^{np\,(0)}/\Lambda_{QCD}^3$	$+0.81$	-1.32
$V_{SLb}^{np\,(0)}/\Lambda_{QCD}^3$	$+1.18$	$+2.44$
$V_{S^2}^{np\,(0)}/\Lambda_{QCD}^3$	-0.26	-0.33
$V_{S_{12}b}^{np\,(0)}/\Lambda_{QCD}^3$	$+0.69$	-0.39

Static energies for tetraquarks (schematic):



Courtesy J. Tarrús Castellà