Computation of hybrid static potentials, flux tubes and the spectrum of heavy hybrid mesons

"QWG 2019 – The 13th International Workshop on Quarkonium" – Torino, Italy

Marc Wagner

Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik

mwagner@th.physik.uni-frankfurt.de

http://th.physik.uni-frankfurt.de/∼mwagner/

in collaboration with Lasse Müller, Owe Philipsen, Christian Reisinger, Carolin Riehl

May 16, 2019

Main goals / literature

(1) Compute hybrid static potentials, i.e. potentials of a static quark antiquark pair ($\bar{Q}Q$), where the flux tubes are excited with quantum numbers different from the ground state.
$\rightarrow$ SU(3) lattice gauge theory

(2) Use these potentials to approximately compute the spectra of $\bar{b}b$ and $\bar{c}c$ hybrid mesons.
$\rightarrow$ Born-Oppenheimer approximation, SU(3) lattice gauge theory, quantum mechanics

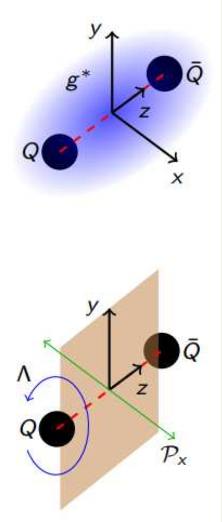
(3) Explore hybrid static potential flux tubes by computing the chromoelectric and chromomagnetic energy density.
$\rightarrow$ SU(2) and SU(3) lattice gauge theory

- The talk summarizes
  [L. Müller, M.W., Acta Phys. Polon. Supp. **11**, 551 (2018) [arXiv:1803.11124]]
  [L. Müller, O. Philipsen, C. Reisinger, M. Wagner, arXiv:1811.00452]
  [S. Capitani, O. Philipsen, C. Reisinger, C. Riehl. M.W., Phys. Rev. D **99**, 034502 (2019) [arXiv:1811.11046 [hep-lat]]]

- For work from other groups using a similar approach cf. e.g.
  [K. J. Juge, J. Kuti, C. J. Morningstar, Nucl. Phys. Proc. Suppl. **63**, 326 (1998) [hep-lat/9709131]
  [C. Michael, Nucl. Phys. A **655**, 12 (1999) [hep-ph/9810415]
  [G. S. Bali *et al.* [SESAM and TχL Collaborations], Phys. Rev. D **62**, 054503 (2000) [hep-lat/0003012]
  [K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. **90**, 161601 (2003) [hep-lat/0207004]
  [C. Michael, Int. Rev. Nucl. Phys. **9**, 103 (2004) [hep-lat/0302001]
  [G. S. Bali, A. Pineda, Phys. Rev. D **69**, 094001 (2004) [hep-ph/0310130]
  [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815 [hep-lat]]]

Hybrid static potentials: quantum numbers

- (Hybrid) static potential states can be characterized by the following quantum numbers:
  - Absolute total angular momentum with respect to the $\bar{Q}Q$ separation axis (z axis): $\Lambda = 0, 1, 2, \ldots \equiv \Sigma, \Pi, \Delta, \ldots$
  - Parity combined with charge conjugation: $\eta = +, - = g, u$.
  - Relection along an axis perpendicular to the $\bar{Q}Q$ separation axis (x axis): $\epsilon = +, -$.
- For $\Lambda \geq 1$ potentials are degenerate with respect to ϵ, i.e. $V_{\Lambda_\eta^+}(r) = V_{\Lambda_\eta^-}(r)$
  $\rightarrow$ use quantum numbers Λ_η^ϵ for $\Lambda = \Sigma$
  $\rightarrow$ use quantum numbers Λ_η for $\Lambda = \Pi, \Delta, \ldots$
- The ordinary static potential has quantum numbers $\Lambda_\eta^\epsilon = \Sigma_g^+$.
- We study hybrid static potentials with quantum numbers $\Lambda_\eta^\epsilon = \Sigma_g^-, \Sigma_u^+, \Sigma_u^-, \Pi_g, \Pi_u, \Delta_g, \Delta_u$.



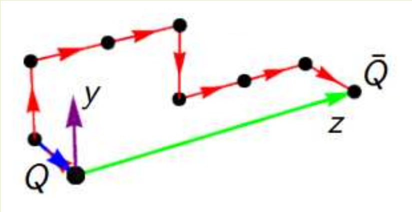
Hybrid static potentials: trial states (1)

- To determine the hybrid static potential with quantum numbers Λ_η^ϵ, compute the temporal correlation functions of suitable trial states,

$$W_{S,S';\Lambda_\eta^\epsilon}(r,t) = \langle \Psi_{\text{hybrid}}(t)|_{S;\Lambda_\eta^\epsilon} |\Psi_{\text{hybrid}}(0)\rangle_{S';\Lambda_\eta^\epsilon} \sim_{t\to\infty} \exp\Big(-V_{\Lambda_\eta^\epsilon}(r)t\Big).$$

- Trial states are

$$|\Psi_{\text{hybrid}}\rangle_{S;\Lambda_\eta^\epsilon} = \bar{Q}(-r/2) a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2) Q(+r/2)|\Omega\rangle$$



with gluonic parallel transporters (on the lattice products of gauge links)

$$a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2) =$$
$$= \frac{1}{4}\sum_{k=0}^{3} \exp\Big(\frac{i\pi\Lambda k}{2}\Big) R\Big(\frac{\pi k}{2}\Big) \bigg(U(-r/2,r_1)\Big(S(r_1,r_2) + \epsilon S_{\mathcal{P}_x}(r_1,r_2)\Big)U(r_2,+r/2) +$$
$$U(-r/2,-r_2)\Big(\eta S_{\mathcal{P}\circ\mathcal{C}}(-r_2,-r_1) + \eta\epsilon S_{(\mathcal{P}\circ\mathcal{C})\mathcal{P}_x}(-r_2,-r_1)\Big)U(-r_1,+r/2)\bigg)$$

generating quantum numbers Λ_η^ϵ.

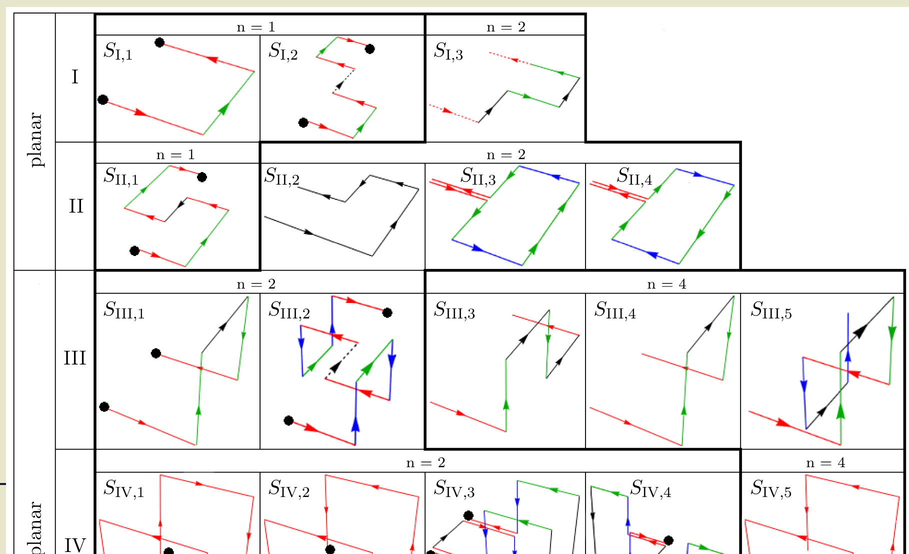
Hybrid static potentials: trial states (2)

- For $a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2)$, which define the trial states

$$|\Psi_{\text{hybrid}}\rangle_{S;\Lambda_\eta^\epsilon} \quad = \quad \bar{Q}(-r/2)a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2)Q(+r/2)|\Omega\rangle,$$

  we have explored many different shapes and variations of their extents.

- For the final computation of each hybrid static potential $V_{\Lambda_\eta^\epsilon}(r)$ we have used an optimized set of 3 to 4 creation operators and have solved generalized eigenvalue problems for the corresponding correlation matrices.



Hybrid static potentials: results

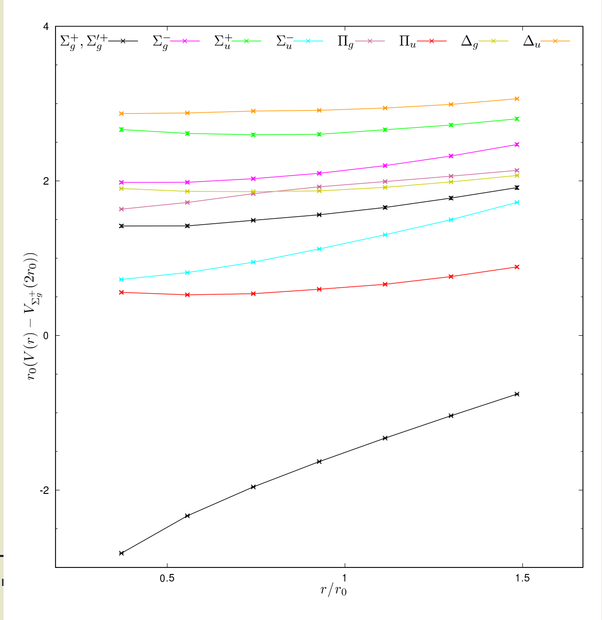
- [S. Capitani, O. Philipsen, C. Reisinger, C. Riehl. M.W., Phys. Rev. D **99**, 034502 (2019) [arXiv:1811.11046 [hep-lat]]]

- E.g. useful for effective field theory studies and predictions of heavy hybrid meson masses.
  [M. Berwein, N. Brambilla, J. Tarrus Castella, A. Vairo, Phys. Rev. D **92**, 114019 (2015) [arXiv:1510.04299]]
  [R. Oncala, J. Soto, Phys. Rev. D **96**, 014004 (2017) [arXiv:1702.03900]]
  [N. Brambilla, G. Krein, J. Tarrus Castella, A. Vairo, Phys. Rev. D **97**, 016016 (2018)[arXiv:1707.09647]]
  [N. Brambilla, W. K. Lai, J. Segovia, J. Tarrus Castella, A. Vairo, Phys. Rev. D **99**, 014017 (2019) [arXiv:1805.07713]]

- Discrepancies to existing results for $V_{\Pi_g}(r)$ and $V_{\Delta_u}(r)$ at small $\bar{Q}Q$ separation $r \leq 0.25\,\text{fm}$.
  [K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. **90**, 161601 (2003) [hep-lat/0207004]]

- Observed degeneracies of $V_{\Sigma'^+_g}(r)$, $V_{\Pi_g}(r)$ and $V_{\Sigma^+_u}(r)$, $V_{\Delta_u}(r)$ at small r expected from pNRQCD.

- Results at small r need to be treated with caution, because of possible glueball decays $\rightarrow$ work in progress.



$\bar{b}b$ and $\bar{c}c$ hybrid meson masses: BO

- Compute $\bar{b}b$ and $\bar{c}c$ hybrid meson masses in two steps.

  (1) **Compute potentials of two static quarks ($\bar{b}b$ or $\bar{c}c$) in the presence of excited gluons generating quantum numbers Λ_η^ϵ** (i.e. hybrid static potentials) **using lattice gauge theory.**

  (2) **Solve the Schrödinger equation for the relative coordinate of $\bar{b}b$ or $\bar{c}c$ using the potentials from (1) and the mass of either the b or the c quark,**

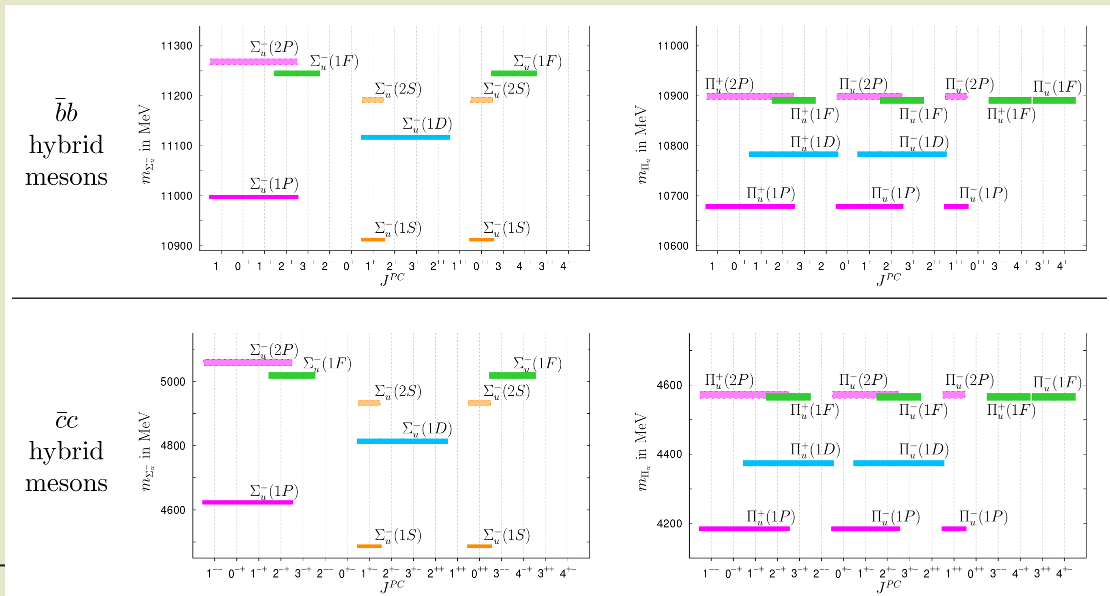
$$\bigg(-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{L(L+1) - 2\Lambda^2 + J_{\Lambda_\eta^\epsilon}(J_{\Lambda_\eta^\epsilon}+1)}{2\mu r^2} + V_{\Lambda_\eta^\epsilon}(r)\bigg)u_{\Lambda_\eta^\epsilon;L,n}(r) \quad = \quad E_{\Lambda_\eta^\epsilon;L,n}u_{\Lambda_\eta^\epsilon;L,n}(r).$$

  Energy eigenvalues $E_{\Lambda_\eta^\epsilon;L,n}$ correspond to masses of $\bar{b}b$ and $\bar{c}c$ hybrid mesons.
  [E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438 [hep-ph]]]

((1) + (2) $\rightarrow$ Born-Oppenheimer approximation).

$\bar{b}b$ and $\bar{c}c$ hybrid meson masses: results

- [S. Capitani, O. Philipsen, C. Reisinger, C. Riehl. M.W., Phys. Rev. D **99**, 034502 (2019) [arXiv:1811.11046 [hep-lat]]]
- Potentials computed $m_{b,c} \rightarrow \infty$ ($\rightarrow$ systematic errors $\approx 30 \ldots 60$ MeV).
- Work in progress: computation of $1/m_{b,c}$ and spin corrections.
  See also [N. Brambilla, W. K. Lai, J. Segovia, J. Tarrus Castella, A. Vairo, Phys. Rev. D **99**, 014017 (2019) [arXiv:1805.07713]] ... and possibly next talk by W. K. Lai.



Hybrid flux tubes: computation

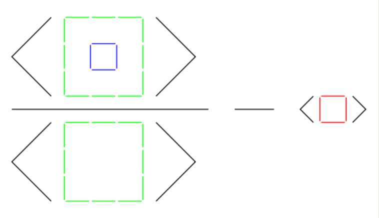
- We are interested in

$$\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x}) \quad = \quad \langle 0_{\Lambda_\eta^\epsilon}(r)|F^2_{\mu\nu}(\mathbf{x})|0_{\Lambda_\eta^\epsilon}(r)\rangle - \langle\Omega|F^2_{\mu\nu}|\Omega\rangle .$$

  - $F^2_{\mu\nu}(\mathbf{x})$, $F^2_{\mu\nu}$: squared chromoelectric/chromomagnetic field strength.
  - $|0_{\Lambda_\eta^\epsilon}(r)\rangle$: hybrid static potential (ground) state (r denotes the $\bar{Q}Q$ separation).
  - $|\Omega\rangle$: vacuum state.

- The sum over the six independent $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$ is proportional to the chromoelectric/chromomagnetic energy density of hybrid static potential flux tubes.

- With lattice gauge theory $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$ can be computed via



$$\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x}) \quad = \quad \pm\frac{\langle\tilde{W}(r,t_2,t_0)P_{\mu\nu}(\mathbf{x},t_1)\rangle_U}{\langle\tilde{W}(r,t_2,t_0)\rangle_U} \mp \langle P_{\mu\nu}\rangle_U .$$

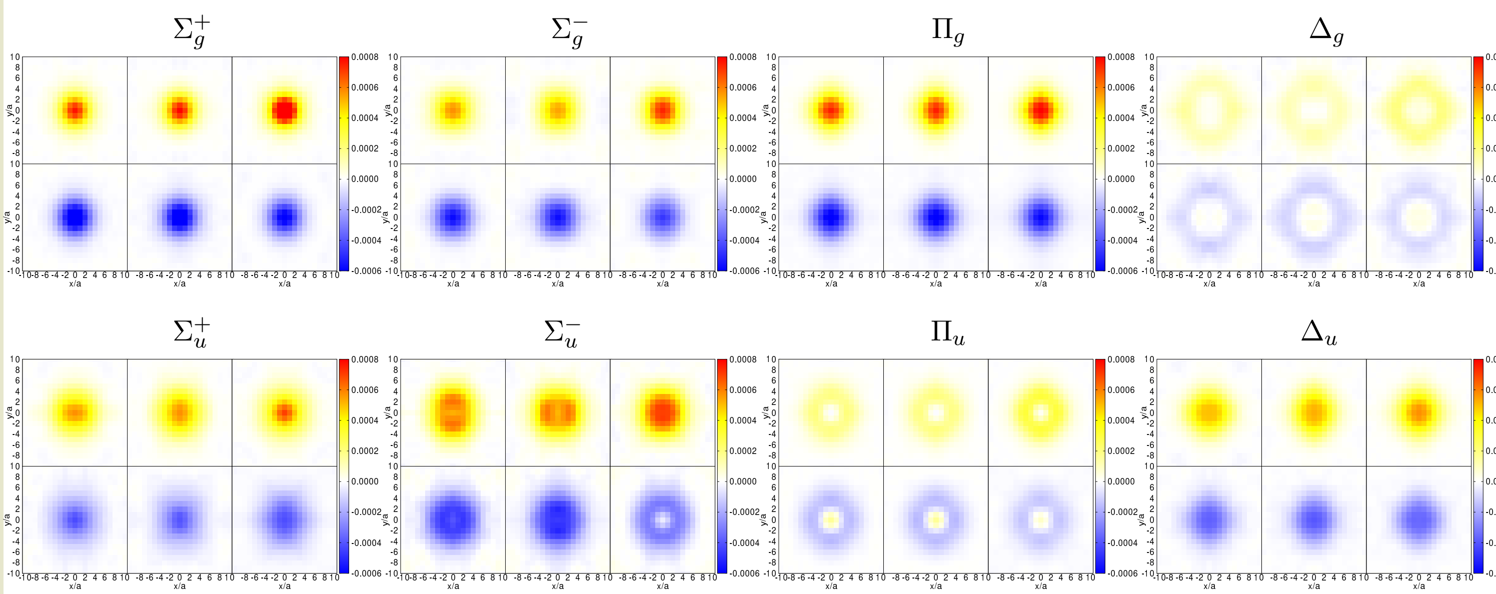
  - $P_{\mu\nu}(\mathbf{x})$, $P_{\mu\nu}$: plaquette, i.e. lattice gauge theory expression for $F^2_{\mu\nu}(\mathbf{x})$.
  - $\tilde{W}(r,t_2,t_0)$: "Wilson loop" (spatial extent r, temporal extent $t_2 - t_1$), with spatial parallel transporters as in the hybrid static potential trial states.

Hybrid flux tubes: results (1)

- $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$, SU(2), mediator plane (x-y plane with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,\mathrm{fm}$.
  [L. Müller, O. Philipsen, C. Reisinger, M. Wagner, in preparation]

- See also [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815 [hep-lat]]] for results for $\Lambda_\eta^\epsilon = \Sigma_g^+, \Sigma_u^+, \Pi_u$.

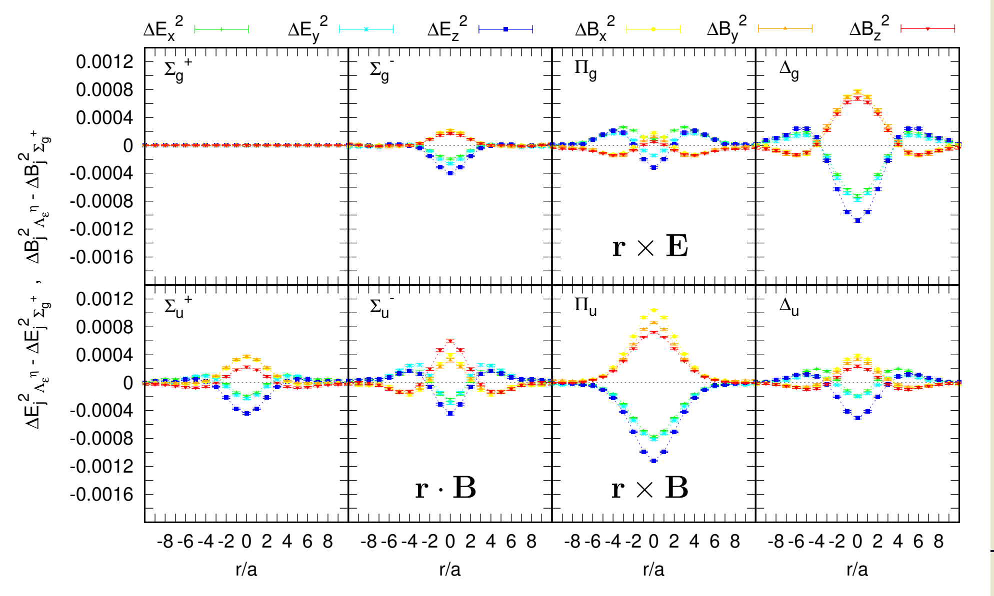
ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2



Hybrid flux tubes: results (2)

- $\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x}) - \Delta F^2_{\mu\nu,\Sigma^+_g}(r;\mathbf{x})$, SU(2), mediator axis ($x$ axis with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,$fm.
  [L. Müller, O. Philipsen, C. Reisinger, M. Wagner, in preparation]

- Chromoelectric and chromomagnetic field stengths reflect typical operators used to study hybrid static potentials, e.g. in pNRQCD.
  [M. Berwein, N. Brambilla, J. Tarrus Castella, A. Vairo, Phys. Rev. D **92**, 114019 (2015) [arXiv:1510.04299]]

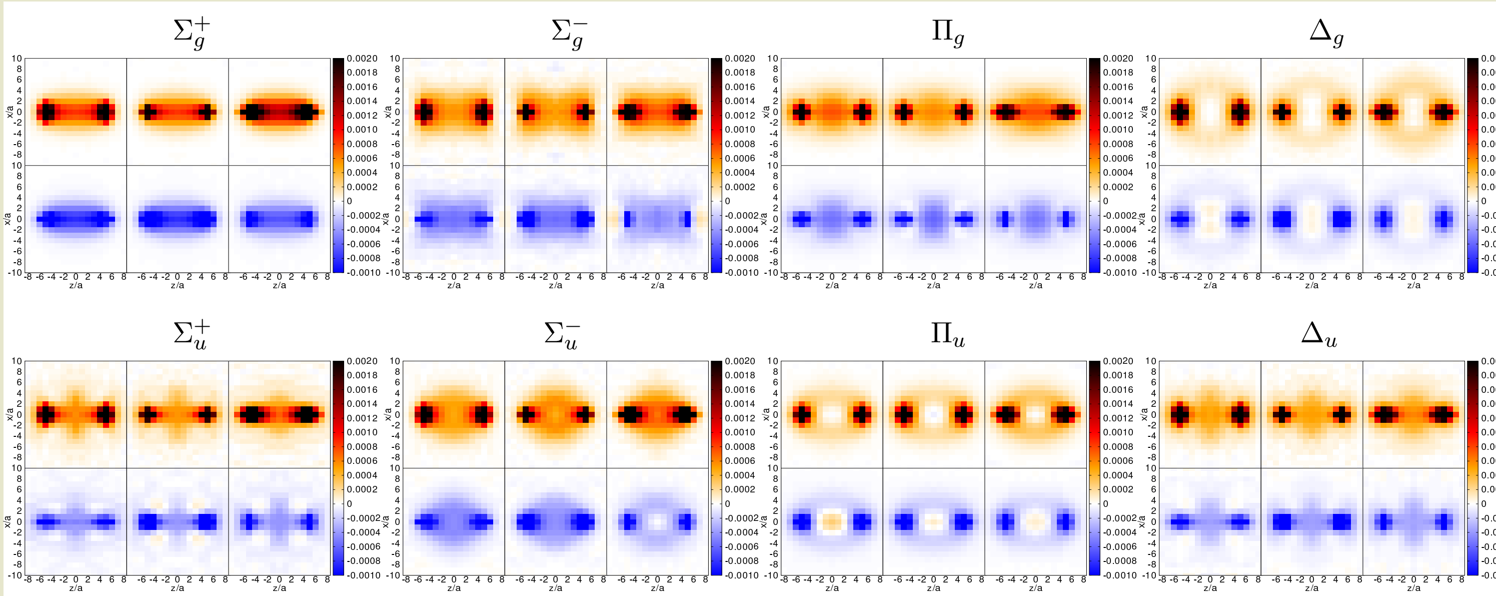


Hybrid flux tubes: results (3)

- $\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x})$, SU(2), separation plane (x-z plane with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,\mathrm{fm}$.
  [L. Müller, O. Philipsen, C. Reisinger, M. Wagner, in preparation]

- See also [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815 [hep-lat]]] for results for $\Lambda^\epsilon_\eta = \Sigma^+_g, \Sigma^+_u, \Pi_u$.

ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2



Summary

(1) Compute hybrid static potentials, i.e. potentials of a static quark antiquark pair ($\bar{Q}Q$), where the flux tubes are excited with quantum numbers different from the ground state.
$\rightarrow$ SU(3) lattice gauge theory

(2) Use these potentials to approximately compute the spectra of $\bar{b}b$ and $\bar{c}c$ hybrid mesons.
$\rightarrow$ Born-Oppenheimer approximation, SU(3) lattice gauge theory, quantum mechanics

(3) Explore hybrid static potential flux tubes by computing the chromoelectric and chromomagnetic energy density.
$\rightarrow$ SU(2) and SU(3) lattice gauge theory