Charm-bottom and heavy-light tetraquarks from lattice QCD

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QWG 2019 - The 13th International Workshop on Heavy Quarkonium Turin, 16.05.2019



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Heavy flavor tetraquarks - a challenge to theory

*Mitchell, Ohlsen





Many models and interpretations exist. Very difficult to address on the lattice due to $c\bar{c}$ or $b\bar{b}$ with $q\bar{q}.$

A case for doubly heavy tetraquarks:

Away from the X, Y, Z states, the heavy hadron spectrum suggests a binding mechanism for ground state tetraquarks, $qq'\bar{Q}\bar{Q}'$ ($J^P = 1^+$).

Assumptions:

- Q-spin decouples, $[\bar{Q}\bar{Q}]_3 \leftrightarrow Q$ (good approx. for Q = b)
- q's prefer to be in $\{qq\}_{\bar{3}}$

Observations in Q and q:

- $[ar{Q}ar{Q}]_3^{m_Q
 ightarrow\infty}$ becomes compact
- HQS relates qq'Q and $qq'\bar{Q}\bar{Q}'$
- B(qqQ) with $(qC\gamma_5 q)$ lightest
- $m_B(\{ud\}) < m_B(\{us\})$

a HQS ą 4QQ ดิ $m_0 \rightarrow \infty$, a (QQ)₂ compact ā Q' Q $[\bar{Q}\bar{Q}]$

Question: Combining $(\overline{qC\gamma_5 q'})(\overline{Q}C\gamma_i \overline{Q'})$ diquarks, do they form stable $ud\overline{b}\overline{b}$, $\ell s\overline{b}\overline{b}$, $ud\overline{c}\overline{b}$ tetraquarks?

 $\{qq\}$

Answer in the simple HQS-GDQ picture

 \rightarrow Single-*b* baryon as analogous system to tetraquark.



HQS: $[\bar{Q}\bar{Q}]$ behaves like single Q:

▶ Good approx. in $(\Xi_{bb}^* - \Xi_{bb})/(B^* - B)$ and $(\Omega_{bb}^* - \Omega_{bb})/(B_s^* - B_s)$

Spectrum as guide for diquark effect:

► {
$$ud$$
}: $\Lambda_b - B_{sp} \sim -145 \text{MeV} \leftrightarrow [ud]$: $\Sigma_b - B_{sp} \sim 49 \text{MeV}$
► { ℓs }: $\Xi_b - B_{sp} - \sim -106 \text{MeV} \leftrightarrow [\ell s]$: $\Xi'_b - B_{sp} \sim 36 \text{MeV}$
 $B_{sp} = \frac{1}{4} [3B_{s=0} + B_{s=1}] \sim \text{spin averaged "threshold"}$

Old idea: Stable multiquarks pointed out previously *Ader et al. ('82); *Manohar, Wise ('93); ...

Renewed interest from phenomenology *Karliner, Rosner ('17); *Eichten, Quigg ('17); *Czarnecki, Leng, Voloshin ('18); *Mehen ('17); *Maiani ('19); ...



Lattice work *Guerrieri et al. ('15); *Bicudo, Wagner et al ('11-'19); Bali, Herzegger ('11); ...

 \Rightarrow These studies typically identify $ud\bar{b}\bar{b}$ as favorable channel.

HQS-GDQ picture, consequences for $qq'\bar{Q}'\bar{Q}$ tetraquarks:

- $J^P = 1^+$ ground state tetraquark below meson-meson threshold
- Deeper binding with heavier quarks in the $\bar{Q}'\bar{Q}$ diquark
- Deeper binding for lighter quarks in the qq' diquark

Goal: Determine $\Delta E = E_{\text{tetra}} - E_{\text{meson-meson}}$ for $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$ and $ud\bar{c}\bar{b}$ \Rightarrow Verify, quantify predictions of binding mechanism in mind

Direct lattice calculation of doubly heavy $J^P = 1^+$ tetraquarks

Step I: Set up a basis of operators

Diquark-Diquark:

$$D = \left((q_a)^T (C\gamma_5) q'_b \right) \times \left[\bar{Q}_a (C\gamma_i) (\bar{Q'}_b)^T - a \leftrightarrow b \right]$$

Dimeson:
$$M = (\bar{b}_a \gamma_5 u_a) (\bar{b}_b \gamma_i d_b) - (\bar{b}_a \gamma_5 d_a) (\bar{b}_b \gamma_i u_b)$$

Step II: Solve the GEVP and get the energies

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \quad F(t)\nu = \lambda(t)F(t_0)\nu,$$
$$G_{\mathcal{O}_1\mathcal{O}_2} = \frac{C_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t)C_{VV}(t)}, \ \lambda(t) = Ae^{-\Delta E(t-t_0)}.$$

*3 \times 3 GEVP for $\bar{Q}'\,\bar{Q};\,2\,\times\,2$ for $\bar{Q}\bar{Q}.$ More γ 's possible but not beneficial in $\bar{b}\bar{b}$.

Further lattice energy levels studies:

- Similar set-up to this one *Junnarkar, Mathur, Padmanath ('18)
- Non-local sink operators *Leskovec, Meinel, Plaumer, Wagner ('19)
- ▶ Distillation $(ud\bar{c}\bar{c}, \ell s\bar{c}\bar{c})$ *HadronSpectrum Coll. ('17)

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Roadmap:

- Determine $\Delta E_{tetra} \Rightarrow$ Binding correlator $\sim e^{-\Delta E t}$
- ▶ Quark mass dependence $qq', \bar{Q}\bar{Q}' \Rightarrow$ Verify, quantify predictions
- Finite volume effects $_* \Rightarrow$ Scattering or stable state?
- Energy level systematics $_* \Rightarrow$ Precision *no binding correlator

Lattice action:

• $N_f = 2 + 1$ Wilson-Clover fermions with Iwasaki gauge action

Valence-quarks:

- Wilson-Clover quarks for u = d, s
- Fermilab/Tsukuba relativistic effective HQ action for c
- NRQCD for b and non-relativitic, unphysical Q' = b'

PACS-CS,'09	$32^3 imes 64$	$a^{-1} = 2.194[{ m GeV}]$	$m_{s,lat}=m_{s,phys}$
m_{π} [MeV]	415	299	163
$m_{\pi}L$	6.1	4.4	2.4
n _{conf}	400	800	187

PhysRevLett.118.142001 (2017)



Physical point: $\Delta E_{ud\bar{b}\bar{b}} = 189(10)(3)$ MeV and $\Delta E_{ls\bar{b}\bar{b}} = 98(7)(3)$ MeV

- Bound ground state tetraquark below meson-meson threshold \checkmark
- Deeper binding with heavier $ar{Q}'ar{Q}$ diquarks, $\sim 1/m_Q$
- Deeper binding for lighter quarks in the qq' diquark \checkmark



Setting the heavy quark mass $m_{b'}$ to unphysical values we map out the heavy quark mass dependence of the binding energy.

- \blacktriangleright Bound ground state tetraquark below meson-meson threshold \checkmark
- Deeper binding with heavier $ar{Q}'ar{Q}$ diquarks, $\sim 1/m_Q$ \checkmark
- \blacktriangleright Deeper binding for lighter quarks in the qq' diquark \checkmark



Previously: Most likely bound tetraquark in charm quark region is $ud\bar{c}\bar{b}$

Calculation indeed reveals evidence for doubly heavy tetraquarks:

•
$$\Delta E_{ud \, b \bar{b}} \simeq 190$$
 MeV and $\Delta E_{ls \, b \bar{b}} \simeq 100$ MeV

• $\Delta E_{ud\bar{c}\bar{b}} \sim 15 - 61$ MeV (current status).

Finite volume corrections

Large energy shifts are possible due to the finite lattice volume.

Scenario I: Scattering state The finite volume energy belongs to a scattering state, the corrections go as

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + \frac{a}{L^3} + \mathcal{O}(\frac{1}{L^4})\right]$$

Scenario II: Stable state



*M. Hansen

The corrections are exponentially suppressed with $\kappa = \sqrt{E_{b,\infty}^2 + p^2}$

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + A e^{-\kappa L}
ight]$$

An in-depth study of volume effects is absolutely important and gives insight into the nature of the states observed.

*From now on: No more binding correlator

Signs of stability

κ_l	L	Т	$m_{\pi}[{ m MeV}]$	$m_{\pi}L$	<i>L</i> [fm]	n _{conf}	status
0.13781	32	64	164	2.4	2.88	71	preliminary
	48	64		3.6	4.32	113	preliminary
	64	64		4.8	5.76	32	pending

 \Rightarrow New volumes for a well understood/tuned setup. (add. $m_{\pi} \simeq 180, 200 \text{MeV}$)



Short- and long-distance agreement are signs of the $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$ (not shown here) and $ud\bar{c}\bar{b}$ being stable states_{*}. Further work needed!

*See e.g. 1705.09239.

Solidifying conclusions





Finite volume scaling \rightarrow stable states in QCD?

To Do: Further statistics and study is needed to firmly establish this conclusion

Wall-local correlators \rightarrow approach to ground state from below. Systematic?

To Do: Extend and include correlator that approaches from above

Experimental detection possibilities

 $J^P = 1^+$ doubly heavy tetraquarks are a new type of exotic predicted in QCD. Many possible decay channels exist, examples:

$ud\overline{b}\overline{b} \longrightarrow B^+D^0$	$us\bar{b}\bar{b} \longrightarrow B^+D^0_s$	$udar{c}ar{b}\longrightarrowar{D}^0ar{D}^0$
$\longrightarrow J/\psi B^+ K^0$	$\longrightarrow B_s \bar{D}^+$	$us\bar{c}\bar{b}\longrightarrow\pi^-K^+B^0$
	$\longrightarrow J/\psi B_s K^+$	$dsar{c}ar{b}\longrightarrow D^-B^+\gamma$

Highest experimental detection probability at LHCb. *Gershon, Poluetkov



Project prospects and summary

Pheno. intuition hints at doubly heavy tetraquarks based on HQS and good diquarks.

In direct calculations we find evidence of $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$ and $ud\bar{c}\bar{b} J^P = 1^+$ tetraquarks (single volume L[fm]=2.88)



Varying the quark masses in all $qq'\bar{Q}\bar{Q}'$ channels broad agreement with the intuitive binding mechanism is seen.

A preliminary study of volume scaling shows signs that $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$ and $ud\bar{c}\bar{b}$ are stable states in QCD. A clear statement is premature.

In our setup, currently ground states energies are reached from *below*. An extension to establish a firm upper bound is desirable.

Outlook for experimental detection (1806.09288, 1810.06657) anthony.francis@cern.ch

Exciting prospects. Let's hunt some exotica!



Thank you for your attention.

Appendix

Phenomenological model

 $\bar{b}'\bar{b}'$:

$$\Delta E_{ud\bar{b}'\bar{b}'} = \frac{C_0}{2r} + C_1^{ud} + C_2^{ud} (2r) + (23 \text{ MeV}) r,$$

$$\Delta E_{\ell s\bar{b}'\bar{b}'} = \frac{C_0}{2r} + C_1^{\ell s} + C_2^{\ell s} (2r) + (24 \text{ MeV}) r,$$

 $\bar{b}'\bar{b}$, r < 1:

$$\Delta E_{ud\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud} (1+r) + (34 \text{ MeV} - 11 \text{ MeV} r),$$

$$\Delta E_{\ell s\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s} (1+r) + (34 \text{ MeV} - 12 \text{ MeV} r)$$

 $\bar{b}'\bar{b}$, r>1:

$$\Delta E_{ud\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud} (1+r) + (34 \text{ MeV} r - 11 \text{ MeV}),$$

$$\Delta E_{\ell s \bar{b}' \bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s} (1+r) + (36 \text{ MeV} r - 11 \text{ MeV})$$

Detection possibilities in experiment: $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$

With such deep ΔE , both $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ tetraquarks decay only weakly



Detection possibilities in experiment: $ud\bar{c}\bar{b}$

At this point $ud\bar{c}\bar{b}$ could decay only weakly or also electromagnetically



Energy of $ud\bar{c}\bar{b}$ at $m_{\pi}[MeV] = 164$

