Threshold Effects and the Line Shape of the X(3872) in EFT

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Technische Universität Darmstadt

with
H.-W. Hammer and M. Jansen (TUD)

May 16, 2019

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$$\Gamma_X < 1.2\,\text{MeV}$$

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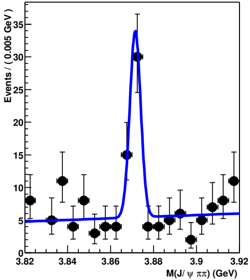

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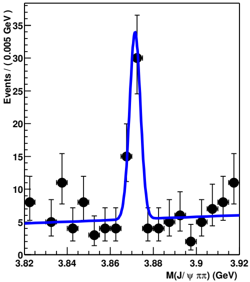

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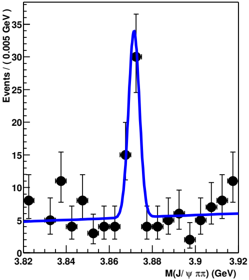

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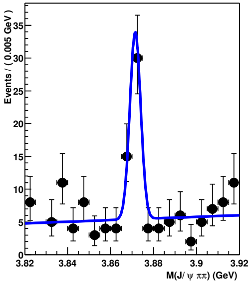

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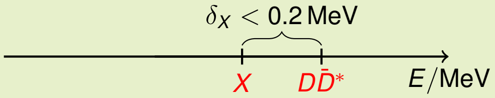

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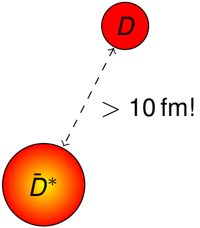

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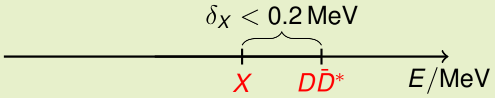

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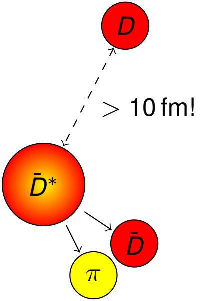

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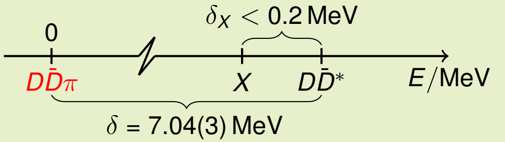

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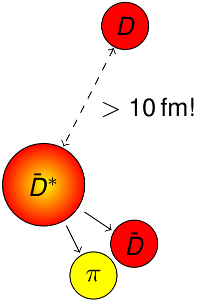

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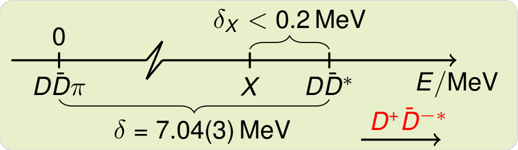

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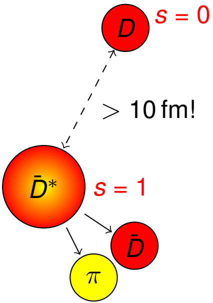

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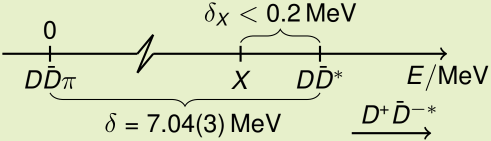

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Universality and the Width

- Universality $a_{D\bar{D}^*} \gg r_{\mathrm{QCD}} \Rightarrow X(3872)$ bound/virtual $\Leftrightarrow$ Need Mass/**Width**!

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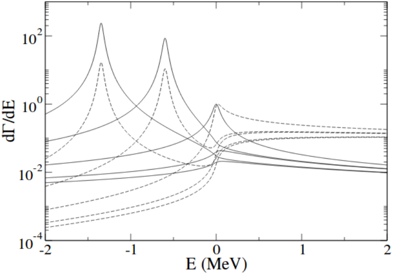

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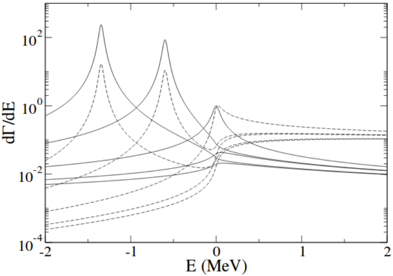

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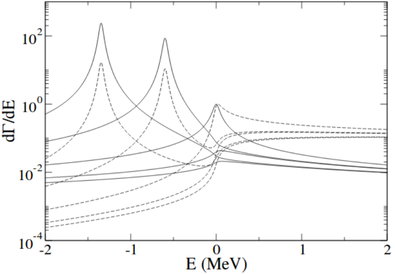

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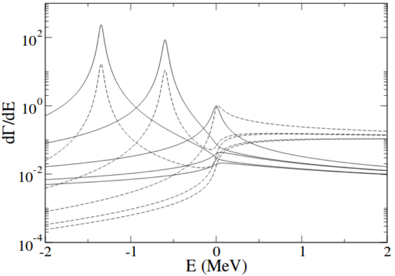

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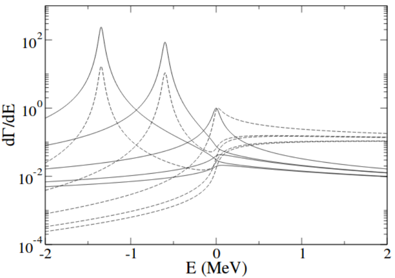

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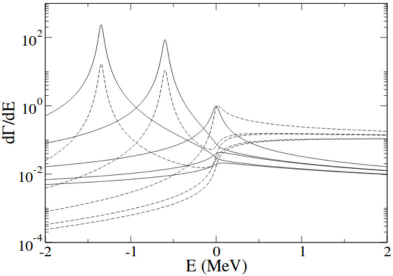

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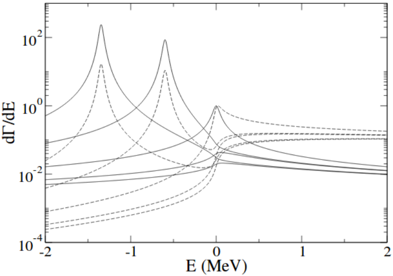

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δ_X, Γ_X from Line Shape!

$\Rightarrow$ **Threshold EFT to extract**

Effective Field Theory

Lagrangian

$$\mathcal{L}_{\text{EFT}} =$$

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- 1-Body: π^0 + D^0 $(+\bar{D}^0)$

Effective Field Theory

Lagrangian

$\mathcal{L}_{\text{EFT}} =$

- 1-Body:
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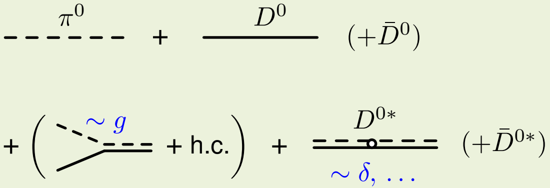

auxiliary field (*p*-wave)

Effective Field Theory

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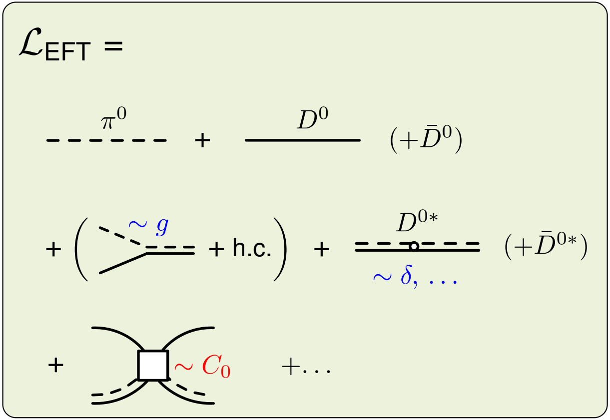

auxiliary field (*p*-wave)

Renormalization (*s*-wave)

2-Body System

The D^{0*} Resonance

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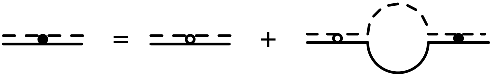
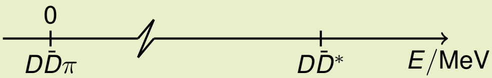
- Full propagator: 



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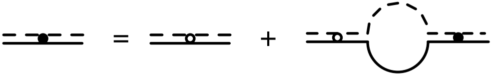
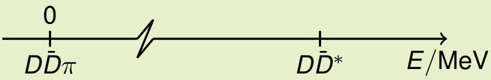
- Full propagator: 

$$\sim \left[-a_1{}^{-1} + \frac{1}{2} r_1 k^2 + \mathcal{O}(k^4) - ik^3\right]^{-1}$$



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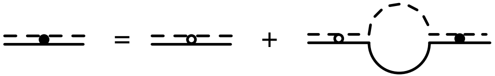
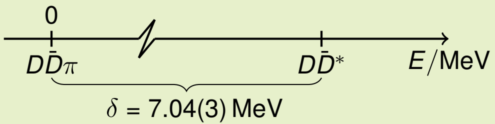


$$\sim \left[\underbrace{-a_1^{-1}}_{(-300\,\mathrm{MeV})^{-3}} + \underbrace{\tfrac{1}{2} r_1}_{-17\,\mathrm{GeV}} k^2 + \mathcal{O}(k^4) - i k^3 \right]^{-1}$$

$g, \delta \Rightarrow$



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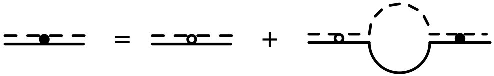
- Full propagator: 

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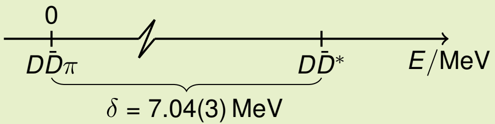
$g, \delta \Rightarrow \quad (-300\,\mathrm{MeV})^{-3} \quad -17\,\mathrm{GeV}$

- D^{0*} shallow ($\delta \ll m_\pi$) $\Rightarrow$ **Scale separation** $K_{\mathrm{lo}} \ll K_{\mathrm{hi}}$
  [Bertulani *et al.*, NPA **712** (2002)], [Bedaque *et al.*, PLB **569** (2003)]



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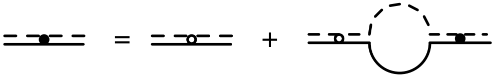
- Full propagator: 

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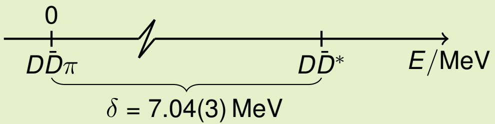

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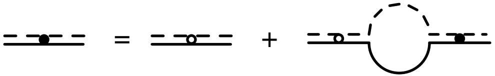

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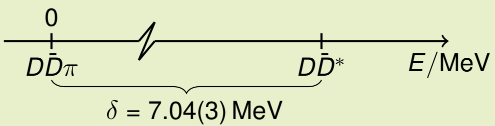
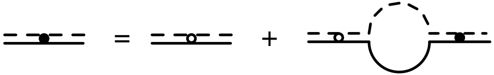


- Full propagator: 

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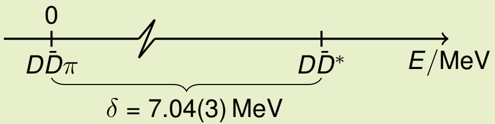
- **Radiative decay width** via $a_1 \in \mathbb{C}$



3-Body System

LO Width (pole position)

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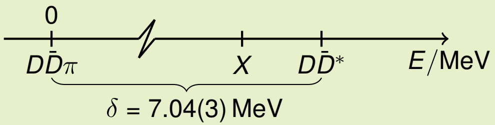

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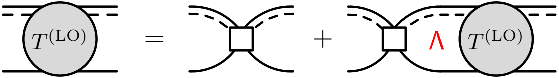
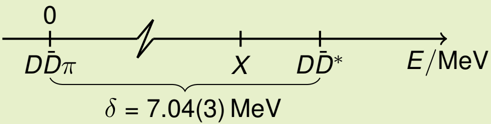
- LO: Iterate **contact force** $C_0(\Lambda)$ and LO propagator!





3-Body System

LO Width (pole position)

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- D^{0*} @ resonance $\Rightarrow$ **Resume constant width** $\Gamma[D^{0*}]$ @ LO!

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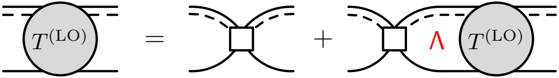
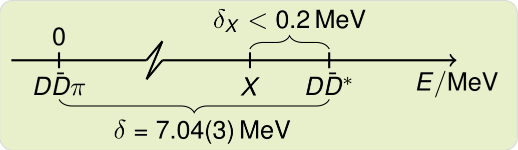


- **Renormalization**: Demand X(3872) pole w/ real part $\delta - \delta_X$



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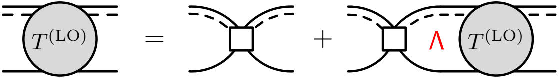
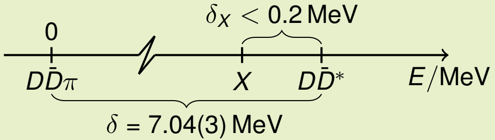


- **Renormalization**: Demand X(3872) pole w/ real part $\delta - \delta_X$

$$\Gamma_X^{(\mathrm{LO})} = \Gamma[D^{0*}] = 53.6\,\mathrm{keV}$$

const. in δ_X



3-Body System

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- Width corrections $\sim Z_X \sim \sqrt{\delta_X/\delta} < 0.17$ (tiny!)

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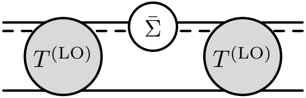
1. D^{0*} **self-energy**: $Z_X \times$ 

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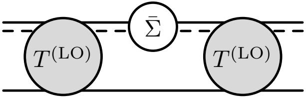
1. D^{0*} **self-energy**: $Z_X \times$  $\sim \Gamma[D^{0*}] \times \sqrt{\frac{\delta_X}{\delta}}$

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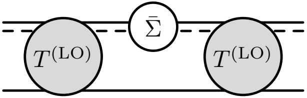
1. D^{0*} **self-energy**: $Z_X \times$ 

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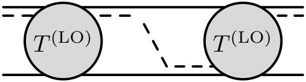
2. **Pion exchanges**: $Z_X \times$ 

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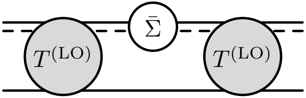
1. D^{0*} **self-energy**: $Z_X \times$ 

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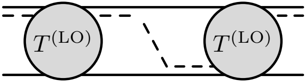
2. **Pion exchanges**: $Z_X \times$ 

$\sim \underbrace{\left(\frac{m_\pi}{2m_D}\right)^2 \delta}_{\sim \Gamma[D^{0*}]} \times \sqrt{\frac{\delta_X}{\delta}}$

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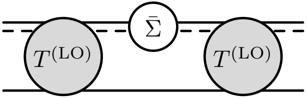
1. D^{0*} **self-energy**: $Z_X \times$ 

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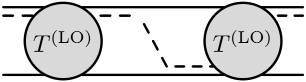
2. **Pion exchanges**: $Z_X \times$ 

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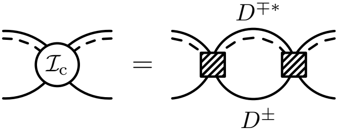
3. **Charged mesons**: 

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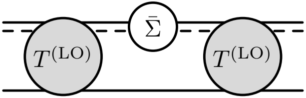
1. D^{0*} **self-energy**: $Z_X \times$ 

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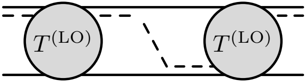
2. **Pion exchanges**: $Z_X \times$ 

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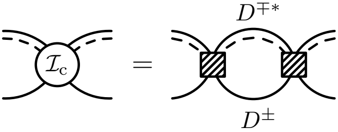
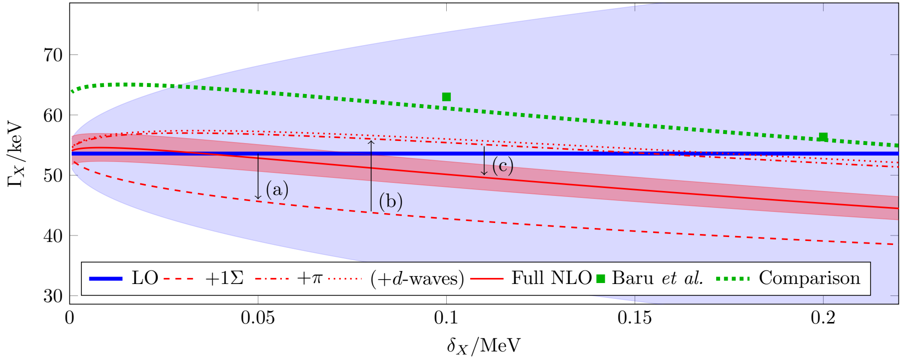
3. **Charged mesons**: 

$\Rightarrow \Gamma[D^{0*}] \times \sqrt{\frac{\delta_X}{\Delta M_{\text{charged}}}}$

Results

Width (pole position)



[MS, Jansen, Hammer (2018), accepted by PRD]

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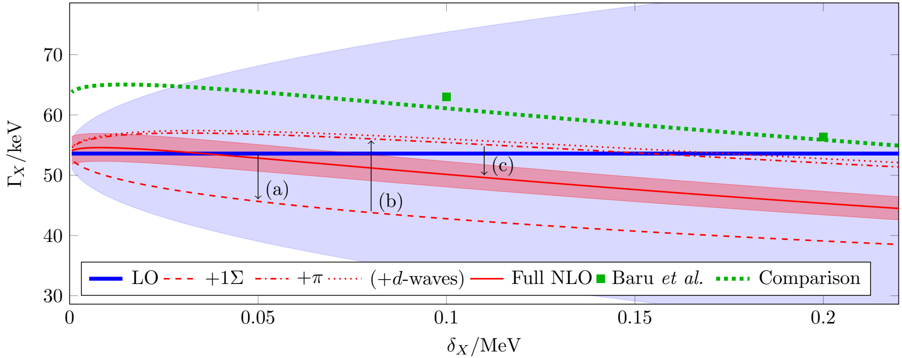

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- **Fast convergence**! LO=NLO @ $\delta_X \approx 40\,\text{keV}$!

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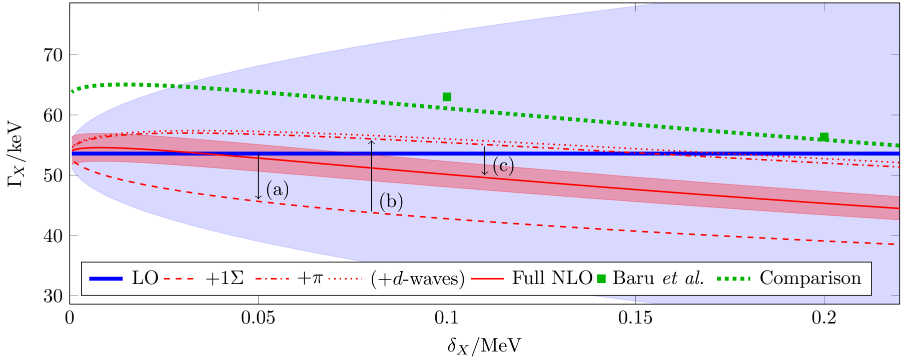
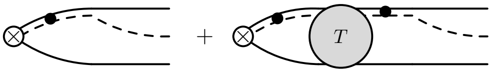


[MS, Jansen, Hammer (2018), accepted by PRD]

- **Fast convergence**! LO=NLO @ $\delta_X \approx 40$ keV!
- Agreement with Baru *et al.* ⇒ No need for *d*-waves, charged pions...
  [Baru *et al.*, PRD **84** (2011)]

Results

Line Shape for $X(3872) \to D\bar{D}\pi$

- Short-distance production: 

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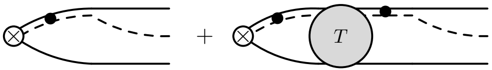
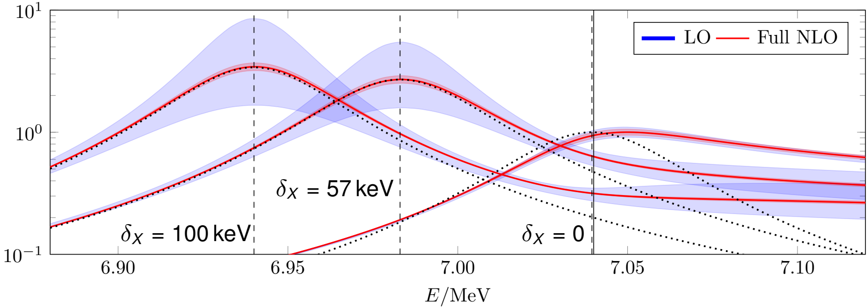
- Short-distance production: 



[MS, Jansen, Hammer, PRD **98** (2019)]

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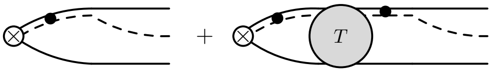
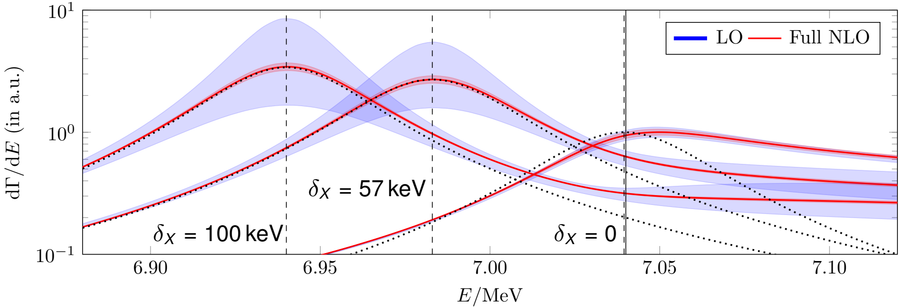
- Short-distance production: 



[MS, Jansen, Hammer, PRD **98** (2019)]

- **Enhancement @ $D^0\bar{D}^{0*}$** threshold $\Longrightarrow$ no Breit-Wigner!

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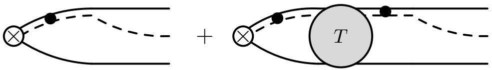
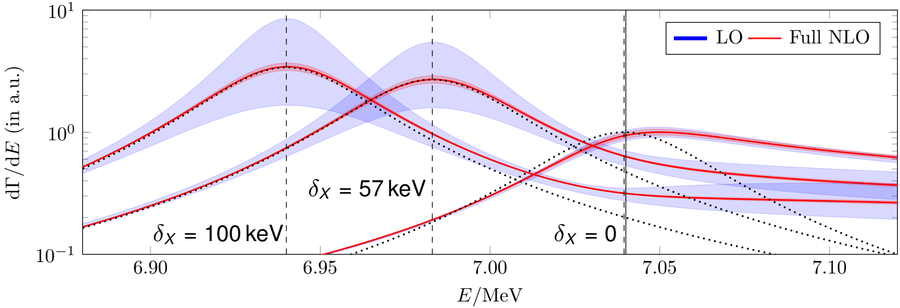


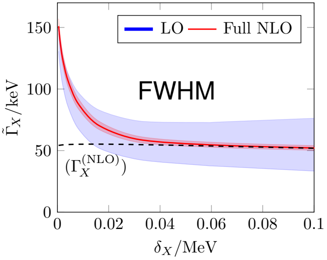


[MS, Jansen, Hammer, PRD **98** (2019)]

- **Enhancement** @ $D^0\bar{D}^{0*}$ threshold $\Longrightarrow$ no Breit-Wigner!
- $\delta_X \to 0$: Maximum **above** threshold!!

3-Body System

Peak parameters



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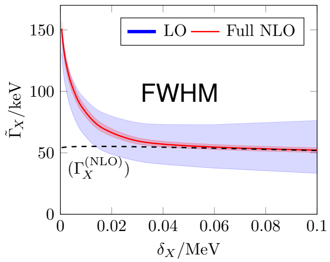

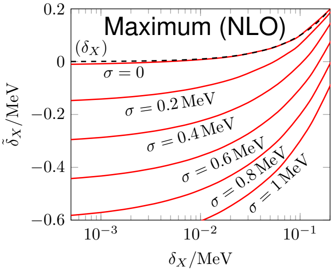

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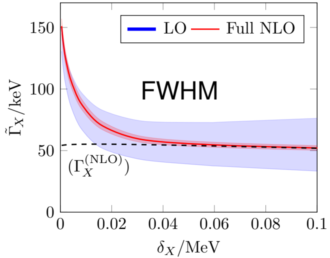

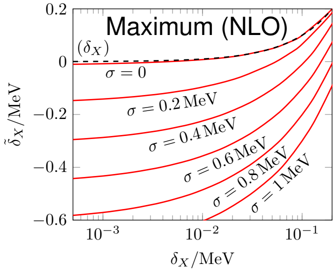

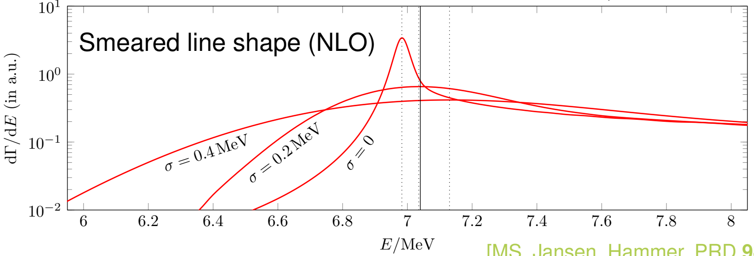

[MS, Jansen, Hammer, PRD **98** (2019)]

Summary and Outlook

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- Line shape $X(3872) \to D^0 \bar{D}^0 \pi^0$ **asymmetric**
- $D^0 \bar{D}^0 \pi^0$ EFT: $X(3872)$ pole $\leftrightarrow$ line shape
- D^{0*} = $D^0 \pi^0$ resonance (**Separation of scales**)
- LO = Zero-range theory ($\Gamma_X = \Gamma[D^{0*}]$) & **NLO corrections tiny**
- Weak binding: **Exp. peak above $D^0 \bar{D}^{0*}$**!!

- **Virtual** $X(3872)$ states
- Predict **momentum distributions** for PANDA (FAIR), Belle2, ...

Effective Field Theory

Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + (\mathcal{L}_{D\pi} + \mathcal{L}_{\bar{D}\pi}) + \mathcal{L}_{D\bar{D}\pi}$$

$$\mathcal{L}_{\text{kin}} = D^\dagger \left[i\,\partial_0 + \frac{\nabla^2}{2m_D} \right] D + \bar{D}^\dagger \left[i\,\partial_0 + \frac{\nabla^2}{2m_D} \right] \bar{D} + \pi^\dagger \left[i\,\partial_0 + \frac{\nabla^2}{2m_\pi} \right] \pi$$

$$\mathcal{L}_{D\pi} = \boldsymbol{D}^\dagger \left[\Delta_0 + \Delta_1\, i\,\partial_{\text{cm}} + \sum_{n\geq 2} \Delta_n \left(i\,\partial_{\text{cm}} \right)^n \right] \boldsymbol{D} + g \left[\boldsymbol{D}^\dagger \cdot \left(\pi \overleftrightarrow{\boldsymbol{\nabla}} D \right) + \text{h.c.} \right]$$

w/ $\overleftrightarrow{\boldsymbol{\nabla}} \equiv \mu \left(m_\pi^{-1} \overleftarrow{\boldsymbol{\nabla}} - m_D^{-1} \overrightarrow{\boldsymbol{\nabla}} \right)$ $\; i\,\partial_{\text{cm}} \equiv i\,\partial_0 + \nabla^2/(2M)$

$$\mathcal{L}_{D\bar{D}\pi} = -C_0\, \frac{1}{2} \left[\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}} \right]^\dagger \cdot \left[\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}} \right] + \ldots$$

Effective Field Theory

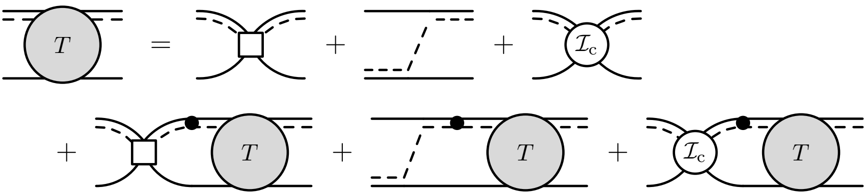
Inputs/Outputs

Table I: Inputs and Outputs of the EFT up to NLO

	Two-Body System		Three-Body System	
	Inputs	**Outputs**	**Inputs**	**Outputs**
LO (κ^0)	δ, δ_{+0}, δ_{++}, Γ_c $\mathcal{B}$	g^2, $\Gamma_{D\pi}$, a_1^{-1}, $r_1/2$ $\Gamma_{D\gamma}$	δ_X	Γ_X, $\mathrm{d}\Gamma/\mathrm{d}E$ (with $\tilde{\delta}_X$, $\tilde{\Gamma}_X$)
NLO (κ^2)	–	–	ν	–

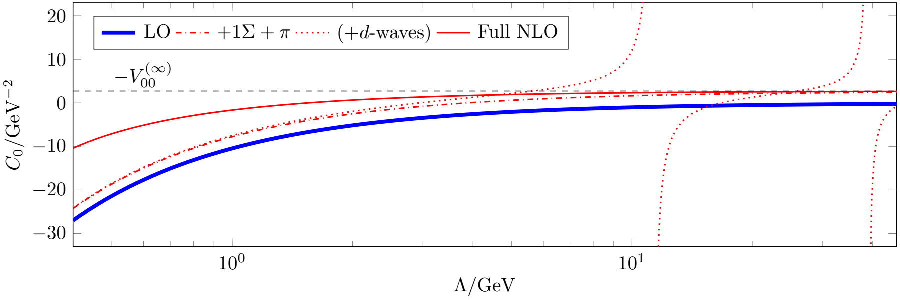
Three-Body System

Amplitude at NLO



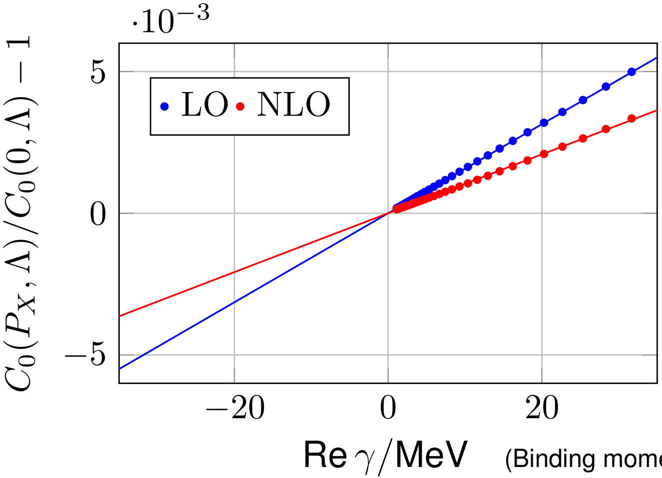
Three-Body System

Renormalization



Virtual States

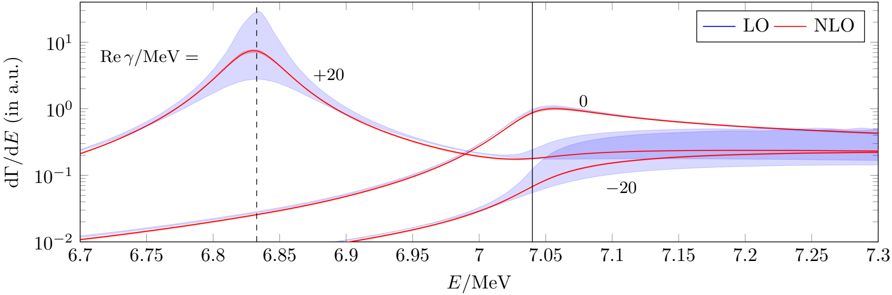
Extrapolation of Coupling



(Binding momentum)

Virtual States

Line Shapes



Virtual States

Line Shapes