Model independent investigation of the $R_{J/\psi}$ and R_{η_c,χ_{cJ},h_c}

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Outline

1. Motivation
   - The anomaly $R_{J/\psi}$ at LHCb
   - The breakdown of perturbative calculation at minimal momentum recoil point
2. The form factors of B_c into a charmonium within NRQCD
3. Model independent investigation of the $R_{J/\psi}$ and R_{η_c,χ_{cJ},h_c}
4. Summary

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Anomaly in $b \to c\tau\nu$ transitions

- The measurements of R_D and R_{D*} by BABAR, Belle, and LHCb are $R_D = 0.407 \pm 0.039(stat) \pm 0.024(syst)$ and $R_{D*} = 0.306 \pm 0.013(stat) \pm 0.007(syst)$, which are 2.1σ and 3.0σ of deviations from Standard Model, respectively.

$$R_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)} + \tau + \bar{\nu}_\tau)}{\Gamma(B \to D^{(*)} + \ell + \bar{\nu}_\ell)}, \quad \ell = e, \mu$$

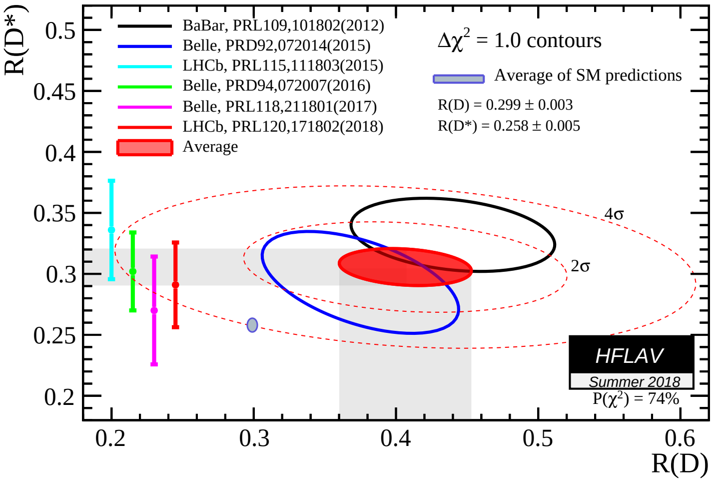
Remind: also seen in Monday talks by Daniel Aloni

- $R_{J/\psi} = 0.71 \pm 0.17(stat) \pm 0.18(syst)$ at LHCb indicated 3σ of deviations from SM, PRL120,121801(2018)

$$R_{J/\psi} = \frac{\Gamma(B \to J/\psi + \tau + \bar{\nu}_\tau)}{\Gamma(B \to J/\psi + \mu + \bar{\nu}_\mu)}$$

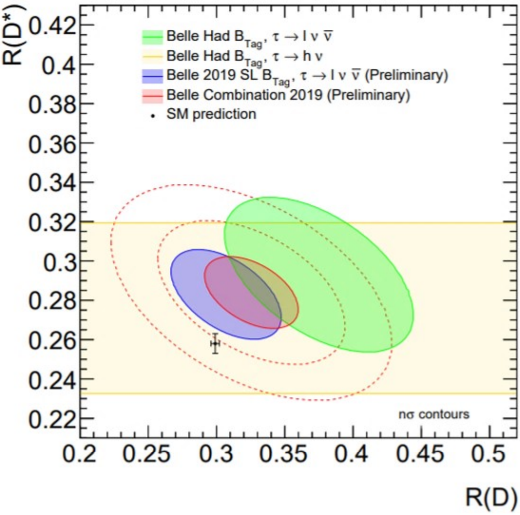
R_D and R_{D*} by HFLAV

Combined fitting in 2018



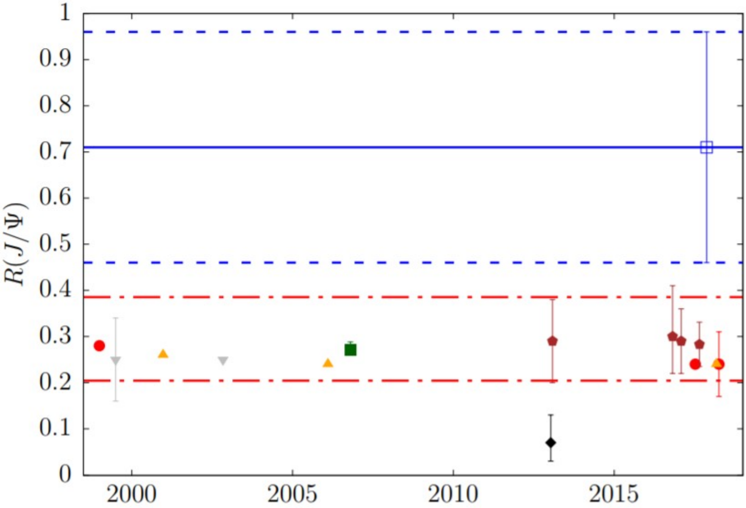
Very recent measurements of R_D and R_{D*} by Belle

0.2σ and 1.1σ of deviations respectively, 1904.08794



Recent measurement of $R_{J/\psi}$ by LHCb

Cohen et.al,1807.02730



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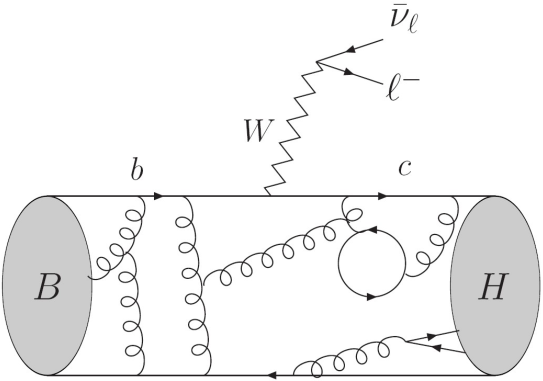
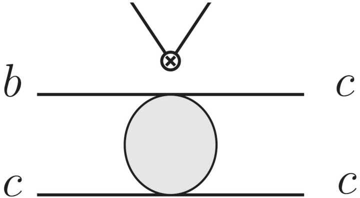
Feynman diagrams for semileptonic B/Bs/Bc decays

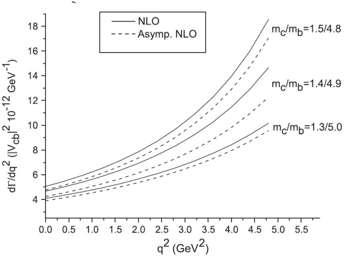
- The largest uncertainty is from the form factors (see review paper Bifani et.al.,1809.06229)
- The breakdown of perturbative calculation at minimal momentum recoil point



The breakdown of perturbative calculation at minimal momentum recoil point

- The initial and final hadrons overlap effects at minimal momentum recoil point are nonperturbative





The definition of the form factors of B_c into a S-wave charmonium

$$
\begin{aligned}
\langle \eta_c(p)|J_V^{\mu}|B_c(P)\rangle &= f_0^{\eta_c}(q^2)\frac{m_{B_c}^2-m_{\eta_c}^2}{q^2}q^{\mu} + f_+^{\eta_c}(q^2)(P^{\mu}+p^{\mu}-\frac{m_{B_c}^2-m_{\eta_c}^2}{q^2}q^{\mu}) ,\\
\langle J/\psi(p,\varepsilon^*)|J_V^{\mu}|B_c(P)\rangle &= -\frac{2V^{J/\psi}(q^2)}{m_{B_c}+m_{J/\psi}}\epsilon^{\mu\nu\rho\sigma}\varepsilon_{\nu}^{*}p_{\rho}P_{\sigma} ,\\
\langle J/\psi(p,\varepsilon^*)|J_A^{\mu}|B_c(P)\rangle &= -i[2m_{J/\psi}A_0^{J/\psi}(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^{\mu} + (m_{B_c}+m_{J/\psi})A_1^{J/\psi}(q^2)(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^{\mu})\\
&\quad -A_2^{J/\psi}(q^2)\frac{\varepsilon^*\cdot q}{m_{B_c}+m_{J/\psi}}(P^{\mu}+p^{\mu}-\frac{m_{B_c}^2-m_{J/\psi}^2}{q^2}q^{\mu})] ,
\end{aligned}
$$

The definition of the form factors of B_c into a P-wave charmonium

$$\langle \chi_{c0}(p)|J_A^\mu|B_c(P)\rangle = f_0^{\chi_{c0}}(q^2)\frac{m_{B_c}^2 - m_{\chi_{c0}}^2}{q^2}q^\mu + f_+^{\chi_{c0}}(q^2)(P^\mu + p^\mu - \frac{m_{B_c}^2 - m_{\chi_{c0}}^2}{q^2}q^\mu)\,,$$

$$\begin{aligned}\langle \chi_{c1}(p,\varepsilon^*)|J_V^\mu|B_c(P)\rangle &= -i[2m_{\chi_{c1}}A_0^{\chi_{c1}}(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_{B_c}+m_{\chi_{c1}})A_1^{\chi_{c1}}(q^2)(\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu) \\ &\quad -A_2^{\chi_{c1}}(q^2)\frac{\varepsilon^*\cdot q}{m_{B_c}+m_{\chi_{c1}}}(P^\mu + p^\mu - \frac{m_{B_c}^2 - m_{\chi_{c1}}^2}{q^2}q^\mu)]\,,\end{aligned}$$

$$\langle \chi_{c1}(p,\varepsilon^*)|J_A^\mu|B_c(P)\rangle = \frac{2V^{\chi_{c1}}(q^2)}{m_{B_c}+m_{\chi_{c1}}}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p_\rho P_\sigma\,,$$

$$\begin{aligned}\langle \chi_{c2}(p,\varepsilon^*)|J_A^\mu|B_c(P)\rangle &= [2m_{\chi_{c2}}A_0^{\chi_{c2}}(q^2)\frac{\varepsilon^{*\alpha\beta}q_\beta}{q^2}q^\mu + (m_{B_c}+m_{\chi_{c2}})A_1^{\chi_{c2}}(q^2)(\varepsilon^{*\mu\alpha} - \frac{\varepsilon^{*\alpha\beta}q_\beta}{q^2}q^\mu) \\ &\quad -A_2^{\chi_{c2}}(q^2)\frac{\varepsilon^{*\alpha\beta}q_\beta}{m_{B_c}+m_{\chi_{c2}}}(P^\mu + p^\mu - \frac{m_{B_c}^2 - m_{\chi_{c2}}^2}{q^2}q^\mu)]\frac{-iP_\alpha}{m_{B_c}}\,,\end{aligned}$$

$$\langle \chi_{c2}(p,\varepsilon^*)|J_V^\mu|B_c(P)\rangle = \frac{2V^{\chi_{c2}}(q^2)}{m_{B_c}(m_{B_c}+m_{\chi_{c2}})}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_{\nu\alpha}p_\rho P_\sigma P_\alpha\,.$$

The form factors of B_c into a charmonium at maximum recoil region

- NLO QCD+Relativisitc corrections for S-wave charmonium
  Analytic expression
  Bell et. al.,NPB164,189(2007);Qiao et. al.,JHEP08,087(2012);Qiao et. al.,PRD87,014009(2013); R.L. Zhu et. al.,PRD95, 094012(2017)

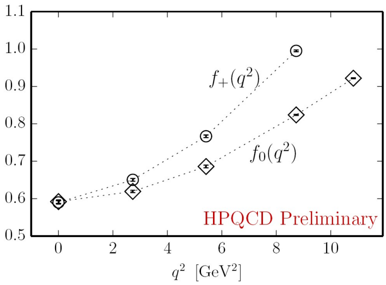
  K factor: 0.2-0.5

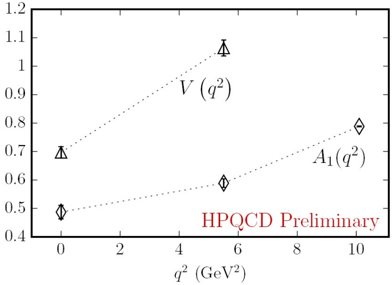
- NLO Relativistic corrections for P-wave charmonium
  see R.L. Zhu, NPB931,359(2018)

  K factor: 0.15-0.3

The form factors of B_c into a charmonium within Lattice QCD

Lytle et.al, 1605.05645; HPQCD, 1611.01987; currently only four form factors data





Inputs

in a model independent way

- Lattice QCD simulations for the form factors of B_c to a charmonium. (limited currently)
- NRQCD calculations (breakdown at minimum momentum recoil region)
- HQET(heavy quark effective theory) predictions (valid only for minimum mentum recoil region)
- QFT's analyticity
- Lattice QCD data + NRQCD calculations + HQET predictions + QFT's analyticity (combined)

HQET

in minimum momentum recoil region

- The splitting between $J^P = 0^-$ and $J^P = 1^-$ mesons is small because the color magnetic interaction is suppressed by $1/m_Q$.
- Heavy quark flavor and spin symmetry when $m_Q \to \infty$.

$$|D^*, +1\rangle = |D, +1/2, +1/2\rangle$$

$$|D^*, 0\rangle = \frac{1}{\sqrt{2}}\left(|D, +1/2, -1/2\rangle + |D, -1/2, +1/2\rangle\right)$$

$$|D^*, -1\rangle = |D, -1/2, -1/2\rangle$$

$$|D\rangle = \frac{1}{\sqrt{2}}\left(|D, +1/2, -1/2\rangle - |D, -1/2, +1/2\rangle\right)$$

$$\langle D, s_h', s_m', v' |\, \bar{c}\,\Gamma\, b \left|\overline{B}, s_h, s_m, v\right\rangle$$

$$\approx \langle c, s_h', v' |\, \bar{c}\,\Gamma\, b\, | b, s_h, v\rangle \langle \mathrm{muck}, s_m', v' | \mathrm{muck}, s_m, v\rangle$$

$$\approx \langle c, s_h', v' |\, \bar{c}\,\Gamma\, b\, | b, s_h, v\rangle\, \xi_{s_m' s_m}(v', v)$$

Acknowledgement to Howard Georgi

HQET calculation

in minimum momentum recoil region (v is the bottom quark velocity; v' is the produced charm quark velocity; $\omega = v \cdot v' = 1$; $\xi_H(\omega)$ is Isgur-Wise functions)

$$
\begin{aligned}
\langle \eta_c(v')|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle &= \xi_H(\omega)[v^\mu + v'^\mu]\,, \\
\langle J/\psi(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle &= -\xi_H(\omega)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu v'_\rho v_\sigma\,, \\
\langle J/\psi(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle &= -i\xi_H(\omega)[(1+\omega)\varepsilon^{*\mu} - \varepsilon^*\cdot v v'^\mu]\,,
\end{aligned}
$$

$$
\begin{aligned}
\langle h_c(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle &= i\xi_E(\omega)[(\omega-1)\varepsilon^{*\mu} - \varepsilon^*\cdot v v'^\mu]\,, \\
\langle h_c(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle &= \xi_E(\omega)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu v'_\rho v_\sigma\,, \\
\langle \chi_{c0}(v')|\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle &= -\xi_E(\omega)[v^\mu - v'^\mu]\,,
\end{aligned}
$$

$$
\begin{aligned}
\langle \chi_{c1}(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle &= \frac{i\xi_F(\omega)}{\sqrt{6}}[(\omega^2-1)\varepsilon^{*\mu} - \varepsilon^*\cdot v(3v^\mu - (\omega-2)v'^\mu)]\,, \\
\langle \chi_{c1}(v',\varepsilon^*)\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle &= \frac{(\omega+1)\xi_F(\omega)}{\sqrt{6}}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu v'_\rho v_\sigma\,, \\
\langle \chi_{c2}(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle &= -i\xi_F(\omega)v_\alpha[(1+\omega)\varepsilon^{*\alpha\mu} - \varepsilon^{*\alpha\beta}v_\beta v'^\mu]\,, \\
\langle \chi_{c2}(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle &= \xi_F(\omega)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_{\alpha\nu}v^\alpha v'_\rho v_\sigma\,.
\end{aligned}
$$

HQET results

at minimum momentum recoil point ($\omega = v \cdot v' = 1$)

$$f_0^{\eta_c}(\omega) = \frac{(\omega+1)\xi_H(\omega)\sqrt{m_{B_c}}\sqrt{m_{\eta_c}}}{m_{B_c}+m_{\eta_c}},$$ 0.94 （w=1)

$$f_+^{\eta_c}(\omega) = \frac{\xi_H(\omega)\left(m_{B_c}+m_{\eta_c}\right)}{2\sqrt{m_{B_c}}\sqrt{m_{\eta_c}}},$$ 1.07

$$V^{J/\psi}(\omega) = \frac{\xi_H(\omega)\left(m_{B_c}+m_{J/\psi}\right)}{2\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}},$$ 1.06

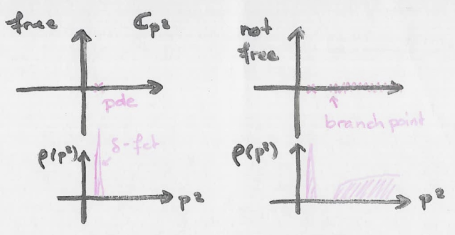
$$A_0^{J/\psi}(\omega) = \frac{\xi_H(\omega)\left(m_{B_c}+m_{J/\psi}\right)}{2\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}},$$ 1.06

$$A_1^{J/\psi}(\omega) = \frac{(\omega+1)\xi_H(\omega)\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}}{m_{B_c}+m_{J/\psi}},$$ 0.94

$$A_2^{J/\psi}(\omega) = \frac{\xi_H(\omega)\left(m_{B_c}+m_{J/\psi}\right)}{2\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}}.$$ 1.08

Analyticity and dispersion constraints

- Lots of works on the analytic properties of the Fourier transform of the 2-point correlator.
- B/Bs/Bc decays form factors are widely studied by the analyticity and dispersion relations. (Z-series)
  See literature: Boyd et.al,PRL74,4603(1995);Cohen et.al,1807.02730;Murphy et.al,1808.05932;Berns et.al,1808.07360



Analyticity and dispersion constraints

for all the physical region

Introduce two currents:

$$j_V^\mu = \bar{c}\gamma^\mu b\,, \qquad\qquad j_A^\mu = \bar{c}\gamma^\mu\gamma^5 b\,.$$

Two-point correlation function of two currents:

$$\begin{aligned}\Pi^{\mu\nu}(q^2) &= i\int d^4x\, e^{i\,q\cdot x}\,\langle 0|\,\mathrm{T}\,j^\mu(x)\,j^{\dagger\,\nu}(0)\,|0\rangle \\ &= (\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu})\Pi_T(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi_L(q^2)\,,\end{aligned}$$

Dispersion relation:

$$\Pi_I(q^2) = \frac{1}{\pi}\int_0^\infty dt\,\frac{\mathrm{Im}\,\Pi_I(t)}{t-q^2}\,.$$

Analyticity and dispersion constraints

for all the physical region

Inserting the hadron bases:

$$\mathrm{Im}\,\Pi^{BP}_{I,V} = \frac{1}{2}\int \frac{d^3 p_B}{(2\pi)^3 2E_B}\frac{d^3 p_P}{(2\pi)^3 2E_P}(2\pi)^4\delta^4(q-p_B-p_P)\frac{\lambda}{3q^2}\left|A^P_{I,V}\right|^2$$
$$= \frac{1}{48\pi}\frac{\lambda^{3/2}}{q^4}\left|A^P_{I,V}\right|^2 ,$$

Operator Product Expansion (OPE):

$$i\int dx\, e^{i\,q\cdot x}\,\langle 0|\,\mathrm{T}\,j^\mu(x)\,j^{\dagger\,\nu}(0)\,|0\rangle$$
$$= \left(\frac{q^\mu q^\nu}{q^2}-g^{\mu\nu}\right)\sum_{n=1}^{\infty} C_{T,n}(q)\,\langle 0|:\mathcal{O}_n(0):|0\rangle$$
$$+\frac{q^\mu q^\nu}{q^2}\sum_{n=1}^{\infty} C_{L,n}(q)\,\langle 0|:\mathcal{O}_n(0):|0\rangle ,$$

Analyticity and dispersion constraints

for all the physical region

The inequality:

$$\mathrm{Im}\Pi_I^{BV}(t) \leq \mathrm{Im}\Pi_I(t)\,.$$

Lead to the z-series:

$$F_i(t) = \frac{1}{B(t)\,\phi_i(t)} \sum_k \alpha_k^i\, z^k(t)\,.$$

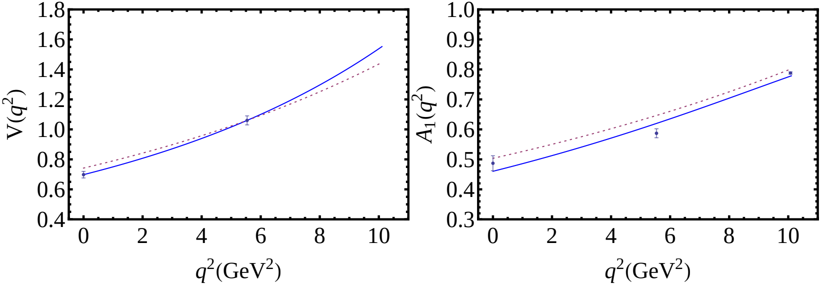
- $F_i(t)$ represents one form factors. $t_\pm \equiv m_{B_c}^2 \pm m_H^2$.

$$z(t) \equiv z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}\,,$$

- $B(t)$ represents the Blaschke factor from the low-lying poles

Results

Form factors of B_c to J/ψ



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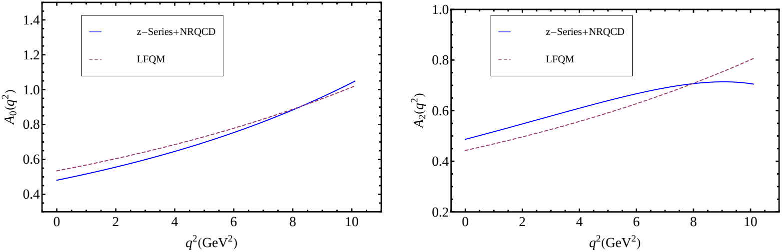

Results

$R_{J/\psi}$ and R_{η_c,χ_{cJ},h_c}

R_H	LHCb data	z-Series approach	S1	S2
$R_{J/\psi}$	$0.71 \pm 0.17(stat) \pm 0.18(syst)$	0.25 ± 0.01	0.31 ± 0.02	0.31 ± 0.01
$R^L_{J/\psi}$		0.23 ± 0.01	0.29 ± 0.01	0.28 ± 0.01
$R^\perp_{J/\psi}$		0.28 ± 0.01	0.36 ± 0.02	0.35 ± 0.01
R_{η_c}		0.31 ± 0.01	0.39 ± 0.02	0.41 ± 0.02
$R_{\chi_{c0}}$		0.09 ± 0.01	0.11 ± 0.01	0.12 ± 0.01
$R_{\chi_{c1}}$		0.09 ± 0.01	0.11 ± 0.01	0.12 ± 0.01
$R^L_{\chi_{c1}}$		0.10 ± 0.02	0.11 ± 0.02	0.13 ± 0.01
$R^\perp_{\chi_{c1}}$		0.09 ± 0.01	0.11 ± 0.01	0.12 ± 0.01
R_{h_c}		$0.06^{+0.03}_{-0.01}$	$0.08^{+0.03}_{-0.01}$	$0.08^{+0.03}_{-0.01}$
$R^L_{h_c}$		$0.06^{+0.02}_{-0.02}$	$0.08^{+0.02}_{-0.02}$	$0.07^{+0.02}_{-0.02}$
$R^\perp_{h_c}$		$0.14^{+0.00}_{-0.01}$	$0.18^{+0.01}_{-0.01}$	$0.17^{+0.01}_{-0.01}$
$R_{\chi_{c2}}$		$0.04^{+0.00}_{-0.01}$	$0.05^{+0.01}_{-0.01}$	$0.05^{+0.01}_{-0.01}$
$R^L_{\chi_{c2}}$		0.03 ± 0.01	0.04 ± 0.01	0.04 ± 0.01
$R^\perp_{\chi_{c2}}$		$0.05^{+0.01}_{-0.00}$	$0.06^{+0.01}_{-0.01}$	$0.07^{+0.01}_{-0.01}$

$S1$ and $S2$ represent the new physics contributions including

$$O_{V_1} = (\bar{c}_L \gamma^\mu b_L)(\bar{\tau}_L \gamma_\mu \nu_L), \quad O_{V_2} = (\bar{c}_R \gamma^\mu b_R)(\bar{\tau}_L \gamma_\mu \nu_L). \tag{1}$$

Summary

- Form factors of B_c to a charmonium at maximum recoil region are in NLO accuracy.
- Form factors of B_c to a charmonium are expanded into Z-series using HQET and Dispersion relations.
- The theoretical uncertainty is reduced, but the SM value of $R_{J/\psi}$ is a significant deviation from the LHCb data.

- Outlook
  - More bin and polarization dependent measurements.
  - More LQCD simulations.

Thank You!