Model independent investigation of the $R_{J/\psi}$ and $R_{\eta_c,\chi_{cJ},h_c}$

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arXiv:1808.10830 in collaboration with Wei Wang

QWG2019, May 13-17, 2019, Turino, Italy
1 Motivation
   - The anomaly $R_{J/\psi}$ at LHCb
   - The breakdown of perturbative calculation at minimal momentum recoil point

2 The form factors of $B_c$ into a charmonium within NRQCD

3 Model independent investigation of the $R_{J/\psi}$ and $R_{\eta_c, \chi_{cJ}, h_c}$

4 Summary
Outline

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4 Summary
Anomaly in $b \rightarrow c\tau\nu$ transitions

- The measurements of $R_D$ and $R_{D^*}$ by BABAR, Belle, and LHCb are $R_D = 0.407 \pm 0.039\,(\text{stat}) \pm 0.024\,(\text{syst})$ and $R_{D^*} = 0.306 \pm 0.013\,(\text{stat}) \pm 0.007\,(\text{syst})$, which are $2.1\sigma$ and $3.0\sigma$ of deviations from Standard Model, respectively.

$$R_{D^*} = \frac{\Gamma(B \rightarrow D^{(*)} + \tau + \bar{\nu}_\tau)}{\Gamma(B \rightarrow D^{(*)} + \ell + \bar{\nu}_\ell)}, \quad \ell = e, \mu$$

**Remind:** also seen in Monday talks by Daniel Aloni

- $R_{J/\psi} = 0.71 \pm 0.17\,(\text{stat}) \pm 0.18\,(\text{syst})$ at LHCb indicated $3\sigma$ of deviations from SM, PRL120,121801(2018)

$$R_{J/\psi} = \frac{\Gamma(B \rightarrow J/\psi + \tau + \bar{\nu}_\tau)}{\Gamma(B \rightarrow J/\psi + \mu + \bar{\nu}_\mu)}$$
Motivation

The anomaly $R_{J/\psi}$ at LHCb

$R_D$ and $R_{D^*}$ by HFLAV

Combined fitting in 2018

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Publication</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>PRL 109, 101802 (2012)</td>
</tr>
<tr>
<td>Belle</td>
<td>PRD 92, 072014 (2015)</td>
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<tr>
<td>LHCb</td>
<td>PRL 115, 111803 (2015)</td>
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<tr>
<td>Belle</td>
<td>PRD 94, 072007 (2016)</td>
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<td>Belle</td>
<td>PRL 118, 211801 (2017)</td>
</tr>
<tr>
<td>LHCb</td>
<td>PRL 120, 171802 (2018)</td>
</tr>
</tbody>
</table>

Average

Average of SM predictions

$\Delta \chi^2 = 1.0$ contours

$R_D = 0.299 \pm 0.005$

$R_{D^*} = 0.258 \pm 0.005$

HFLAV

Summer 2018

$P(\chi^2) = 74\%$
Very recent measurements of $R_D$ and $R_{D^*}$ by Belle

$0.2\sigma$ and $1.1\sigma$ of deviations respectively, 1904.08794
Recently measurement of $R_{J/\psi}$ by LHCb
Cohen et.al,1807.02730
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4 Summary
Feynman diagrams for semileptonic $B/B_s/B_c$ decays

- The largest uncertainty is from the form factors (see review paper Bifani et.al., 1809.06229)
- The breakdown of perturbative calculation at minimal momentum recoil point
The breakdown of perturbative calculation at minimal momentum recoil point

- The initial and final hadrons overlap effects at minimal momentum recoil point are nonperturbative

![Diagram](image_url)
The form factors of $B_c$ into a charmonium within NRQCD

The definition of the form factors of $B_c$ into a S-wave charmonium

$$
\langle \eta_c(p) | J_\nu^\mu | B_c(P) \rangle = f_0^{\eta_c}(q^2) \frac{m_{B_c}^2 - m_{\eta_c}^2}{q^2} q^\mu + f_+^{\eta_c}(q^2)(P^\mu + p^\mu - \frac{m_{B_c}^2 - m_{\eta_c}^2}{q^2} q^\mu),
$$

$$
\langle J/\psi(p, \epsilon^*) | J_\nu^\mu | B_c(P) \rangle = - \frac{2V_{J/\psi}(q^2)}{m_{B_c} + m_{J/\psi}} \epsilon_{\mu \nu \rho \sigma} \epsilon^*_{\nu} p_{\rho} P_{\sigma},
$$

$$
\langle J/\psi(p, \epsilon^*) | J_A^\mu | B_c(P) \rangle = -i[2m_{J/\psi} A_0^{J/\psi}(q^2) \frac{\epsilon^* \cdot q}{q^2} q^\mu + (m_{B_c} + m_{J/\psi}) A_1^{J/\psi}(q^2)(\epsilon^* \mu - \frac{\epsilon^* \cdot q}{q^2} q^\mu) - A_2^{J/\psi}(q^2) \frac{\epsilon^* \cdot q}{m_{B_c} + m_{J/\psi}} (P^\mu + p^\mu - \frac{m_{B_c}^2 - m_{J/\psi}^2}{q^2} q^\mu)],
$$
The definition of the form factors of $B_c$ into a P-wave charmonium

$$
\langle \chi_{c0}(p) | J_A^\mu | B_c(P) \rangle = t_0^{\chi_{c0}}(q^2) \frac{m_{B_c}^2 - m_{\chi_{c0}}^2}{q^2} q^\mu + i t_0^{\chi_{c0}}(q^2) (P^\mu + p^\mu - \frac{m_{B_c}^2 - m_{\chi_{c0}}^2}{q^2} q^\mu),
$$

$$
\langle \chi_{c1}(p, \varepsilon^*) | J^\mu_V | B_c(P) \rangle = -i [2 m_{\chi_{c1}} A_0^{\chi_{c1}}(q^2) \frac{\varepsilon^* \cdot q}{q^2} q^\mu + (m_{B_c} + m_{\chi_{c1}}) A_1^{\chi_{c1}}(q^2) (\varepsilon^* \mu - \frac{\varepsilon^* \cdot q}{q^2} q^\mu)
$$

$$
- A_2^{\chi_{c1}}(q^2) \frac{\varepsilon^* \cdot q}{m_{B_c} + m_{\chi_{c1}}} (P^\mu + p^\mu - \frac{m_{B_c}^2 - m_{\chi_{c1}}^2}{q^2} q^\mu)] ,
$$

$$
\langle \chi_{c1}(p, \varepsilon^*) | J_A^\mu | B_c(P) \rangle = \frac{2 V^{\chi_{c1}}(q^2)}{m_{B_c} + m_{\chi_{c1}}} \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{\nu}^* p_{\rho} P_{\sigma} ,
$$

$$
\langle \chi_{c2}(p, \varepsilon^*) | J_A^\mu | B_c(P) \rangle = [2 m_{\chi_{c2}} A_0^{\chi_{c2}}(q^2) \frac{\varepsilon^* \alpha \beta q_{\beta}}{q^2} q^\mu + (m_{B_c} + m_{\chi_{c2}}) A_1^{\chi_{c2}}(q^2) (\varepsilon^* \mu \alpha - \frac{\varepsilon^* \alpha \beta q_{\beta}}{q^2} q^\mu)
$$

$$
- A_2^{\chi_{c2}}(q^2) \frac{\varepsilon^* \alpha \beta q_{\beta}}{m_{B_c} + m_{\chi_{c2}}} (P^\mu + p^\mu - \frac{m_{B_c}^2 - m_{\chi_{c2}}^2}{q^2} q^\mu)] - iP_{\alpha},
$$

$$
\langle \chi_{c2}(p, \varepsilon^*) | J^\mu_V | B_c(P) \rangle = \frac{2 V^{\chi_{c2}}(q^2)}{m_{B_c} (m_{B_c} + m_{\chi_{c2}})} \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{\nu}^* p_{\rho} P_{\sigma} P_{\alpha} .
$$
The form factors of $B_c$ into a charmonium at maximum recoil region

- NLO QCD+Relativistic corrections for S-wave charmonium
  Analytic expression
  Bell et. al., NPB164, 189 (2007); Qiao et. al., JHEP08, 087 (2012); Qiao et. al., PRD87, 014009 (2013);
  R.L. Zhu et. al., PRD95, 094012 (2017)
  K factor: 0.2-0.5

- NLO Relativistic corrections for P-wave charmonium
  see R.L. Zhu, NPB931, 359 (2018)
  K factor: 0.15-0.3
The form factors of $B_c$ into a charmonium within NRQCD

Lytle et.al, 1605.05645; HPQCD, 1611.01987; currently only four form factors data
Inputs
in a model independent way

- Lattice QCD simulations for the form factors of $B_c$ to a charmonium. (limited currently)
- NRQCD calculations (breakdown at minimum momentum recoil region)
- HQET (heavy quark effective theory) predictions (valid only for minimum momentum recoil region)
- QFT’s analyticity
- Lattice QCD data + NRQCD calculations + HQET predictions + QFT’s analyticity (combined)
HQET

in minimum momentum recoil region

- The splitting between $J^P = 0^-$ and $J^P = 1^-$ mesons is small because the color magnetic interaction is suppressed by $1/m_Q$.
- Heavy quark flavor and spin symmetry when $m_Q \to \infty$.

\[
|D^*, +1\rangle = |D, +1/2, +1/2\rangle \\
|D^*, 0\rangle = \frac{1}{\sqrt{2}} (|D, +1/2, -1/2\rangle + |D, -1/2, +1/2\rangle) \\
|D^*, -1\rangle = |D, -1/2, -1/2\rangle \\
|D\rangle = \frac{1}{\sqrt{2}} (|D, +1/2, -1/2\rangle - |D, -1/2, +1/2\rangle)
\]

\[
\langle D, s'_h, s'_m, v' | \bar{c} \Gamma b | \bar{B}, s_h, s_m, v \rangle \\
\approx \langle c, s'_h, v' | \bar{c} \Gamma b | b, s_h, v \rangle \langle \text{muck}, s'_m, v' \ | \text{muck}, s_m, v \rangle \\
\approx \langle c, s'_h, v' | \bar{c} \Gamma b | b, s_h, v \rangle \xi_{s'_m s_m} (v', v)
\]

Acknowledgement to Howard Georgi
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HQET calculation

in minimum momentum recoil region ($\nu$ is the bottom quark velocity; $\nu'$ is the produced charm quark velocity; $\omega = \nu \cdot \nu' = 1$; $\xi_H(\omega)$ is Isgur-Wise functions)

\[
\langle \eta_c(\nu')|\bar{c}_{\nu'}\gamma^\mu b_\nu|B_c(\nu)\rangle = \xi_H(\omega)[\nu^\mu + \nu'^\mu], \\
\langle J/\psi(\nu', \epsilon^*)|\bar{c}_{\nu'}\gamma^\mu b_\nu|B_c(\nu)\rangle = -\xi_H(\omega)\epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^* \nu' \rho \nu_\sigma, \\
\langle J/\psi(\nu', \epsilon^*)|\bar{c}_{\nu'}\gamma^\mu \gamma^5 b_\nu|B_c(\nu)\rangle = -i\xi_H(\omega)[(1 + \omega)\epsilon^{*\mu} - \epsilon^* \cdot \nu \nu'^\mu],
\]

\[
\langle h_c(\nu', \epsilon^*)|\bar{c}_{\nu'}\gamma^\mu b_\nu|B_c(\nu)\rangle = i\xi_E(\omega)[(\omega - 1)\epsilon^{*\mu} - \epsilon^* \cdot \nu \nu'^\mu], \\
\langle h_c(\nu', \epsilon^*)|\bar{c}_{\nu'}\gamma^\mu \gamma^5 b_\nu|B_c(\nu)\rangle = \xi_E(\omega)\epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^* \nu' \rho \nu_\sigma, \\
\langle \chi_{c0}(\nu')|\bar{c}_{\nu'}\gamma^\mu \gamma^5 b_\nu|B_c(\nu)\rangle = -\xi_E(\omega)[\nu^\mu - \nu'^\mu],
\]

\[
\langle \chi_{c1}(\nu', \epsilon^*)|\bar{c}_{\nu'}\gamma^\mu b_\nu|B_c(\nu)\rangle = \frac{i\xi_F(\omega)}{\sqrt{6}}[(\omega^2 - 1)\epsilon^{*\mu} - \epsilon^* \cdot \nu(3\nu^\mu - (\omega - 2)\nu'^\mu)], \\
\langle \chi_{c1}(\nu', \epsilon^*)|\bar{c}_{\nu'}\gamma^\mu \gamma^5 b_\nu|B_c(\nu)\rangle = \frac{(\omega + 1)\xi_F(\omega)}{\sqrt{6}}\epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^* \nu' \rho \nu_\sigma, \\
\langle \chi_{c2}(\nu', \epsilon^*)|\bar{c}_{\nu'}\gamma^\mu \gamma^5 b_\nu|B_c(\nu)\rangle = -i\xi_F(\omega)\nu_\alpha[(1 + \omega)\epsilon^{*\alpha\mu} - \epsilon^* \alpha \beta \nu_\beta \nu'^\mu], \\
\langle \chi_{c2}(\nu', \epsilon^*)|\bar{c}_{\nu'}\gamma^\mu b_\nu|B_c(\nu)\rangle = \xi_F(\omega)\epsilon^{\mu\nu\rho\sigma} \epsilon^*_{\alpha \nu} \nu^\alpha \nu' \rho \nu_\sigma.
\]
HQET results
at minimum momentum recoil point \((\omega = \mathbf{v} \cdot \mathbf{v}' = 1)\)

\[
\begin{align*}
    f_0^{\eta_c}(\omega) &= \frac{(\omega + 1)\xi_H(\omega)\sqrt{m_{B_c}}\sqrt{m_{\eta_c}}}{m_{B_c} + m_{\eta_c}}, \\
    f_+^{\eta_c}(\omega) &= \frac{\xi_H(\omega)(m_{B_c} + m_{\eta_c})}{2\sqrt{m_{B_c}}\sqrt{m_{\eta_c}}}, \\
    V^{J/\psi}(\omega) &= \frac{\xi_H(\omega)(m_{B_c} + m_{J/\psi})}{2\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}}, \\
    A_0^{J/\psi}(\omega) &= \frac{\xi_H(\omega)(m_{B_c} + m_{J/\psi})}{2\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}}, \\
    A_1^{J/\psi}(\omega) &= \frac{(\omega + 1)\xi_H(\omega)\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}}{m_{B_c} + m_{J/\psi}}, \\
    A_2^{J/\psi}(\omega) &= \frac{\xi_H(\omega)(m_{B_c} + m_{J/\psi})}{2\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}}. \\
\end{align*}
\]

0.94 \ (w=1) \quad 1.07 \quad 1.06 \quad 1.06 \quad 0.94 \quad 1.08
Analyticity and dispersion constraints

- Lots of works on the analytic properties of the Fourier transform of the 2-point correlator.
- B/Bs/Bc decays form factors are widely studied by the analyticity and dispersion relations. (Z-series)
  See literature: Boyd et.al, PRL74, 4603(1995); Cohen et.al, 1807.02730; Murphy et.al, 1808.05932; Berns et.al, 1808.07360
Analyticity and dispersion constraints
for all the physical region

Introduce two currents:

\[ j^\mu_V = \bar{c}\gamma^\mu b, \quad j^\mu_A = \bar{c}\gamma^\mu\gamma^5 b. \]

Two-point correlation function of two currents:

\[
\Pi^{\mu\nu}(q^2) = i \int d^4x \, e^{i\mathbf{q}\cdot\mathbf{x}} \langle 0 | T j^\mu(x) j^{\dagger\nu}(0) | 0 \rangle \\
= \left( \frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) \Pi_T(q^2) + \frac{q^\mu q^\nu}{q^2} \Pi_L(q^2),
\]

Dispersion relation:

\[
\Pi_I(q^2) = \frac{1}{\pi} \int_0^\infty dt \frac{\text{Im} \Pi_I(t)}{t - q^2}.
\]
Analyticity and dispersion constraints for all the physical region

Inserting the hadron bases:

\[
\text{Im} \Pi_{P}^{BP} = \frac{1}{2} \int \frac{d^3 p_B}{(2\pi)^3 2E_B} \frac{d^3 p_P}{(2\pi)^3 2E_P} (2\pi)^4 \delta^4 (q - p_B - p_P) \frac{\lambda}{3q^2} |A_{I,V}^P|^2
\]

\[
= \frac{1}{48\pi} \frac{\lambda^{3/2}}{q^4} |A_{I,V}^P|^2 ,
\]

Operator Product Expansion (OPE):

\[
i \int dx \ e^{i q \cdot x} \langle 0 | T \ j^\mu (x) j^{\dagger \nu} (0) | 0 \rangle
\]

\[
= \left( \frac{q^\mu q^\nu}{q^2} - g^\mu^\nu \right) \sum_{n=1}^{\infty} C_{T,n}(q) \langle 0 | : \mathcal{O}_n(0) : | 0 \rangle
\]

\[
+ \frac{q^\mu q^\nu}{q^2} \sum_{n=1}^{\infty} C_{L,n}(q) \langle 0 | : \mathcal{O}_n(0) : | 0 \rangle ,
\]
Analyticity and dispersion constraints for all the physical region

The inequality:

$$\text{Im} \Pi_i^{BV}(t) \leq \text{Im} \Pi_i(t).$$

Lead to the z-series:

$$F_i(t) = \frac{1}{B(t) \phi_i(t)} \sum_k \alpha^i_k z^k(t).$$

- $F_i(t)$ represents one form factors. $t_{\pm} \equiv m_{Bc}^2 \pm m_H^2$.

$$z(t) \equiv z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}},$$

- $B(t)$ represents the Blaschke factor from the low-lying poles.
Results

Form factors of $B_c$ to $J/\psi$
Results
Form factors of $B_c$ to $J/\psi$
Results

$R_{J/\psi}$ and $R_{\eta_c,\chi_{cJ},h_c}$

<table>
<thead>
<tr>
<th>$R_H$</th>
<th>LHCb data</th>
<th>z-Series approach</th>
<th>S1</th>
<th>S2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{J/\psi}$</td>
<td>$0.71 \pm 0.17^{(\text{stat})} \pm 0.18^{(\text{syst})}$</td>
<td>$0.25 \pm 0.01$</td>
<td>$0.31 \pm 0.02$</td>
<td>$0.31 \pm 0.01$</td>
</tr>
<tr>
<td>$R_{LJ/\psi}$</td>
<td>$0.23 \pm 0.01$</td>
<td>$0.29 \pm 0.01$</td>
<td>$0.28 \pm 0.01$</td>
<td></td>
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<tr>
<td>$R_{J/\psi}$</td>
<td>$0.28 \pm 0.01$</td>
<td>$0.36 \pm 0.02$</td>
<td>$0.35 \pm 0.01$</td>
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<tr>
<td>$R_{\eta_c}$</td>
<td>$0.31 \pm 0.01$</td>
<td>$0.39 \pm 0.02$</td>
<td>$0.41 \pm 0.02$</td>
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<tr>
<td>$R_{\chi_{c0}}$</td>
<td>$0.09 \pm 0.01$</td>
<td>$0.11 \pm 0.01$</td>
<td>$0.12 \pm 0.01$</td>
<td></td>
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<tr>
<td>$R_{\chi_{c1}}$</td>
<td>$0.09 \pm 0.01$</td>
<td>$0.11 \pm 0.01$</td>
<td>$0.12 \pm 0.01$</td>
<td></td>
</tr>
<tr>
<td>$R_{L\chi_{c1}}$</td>
<td>$0.10 \pm 0.02$</td>
<td>$0.11 \pm 0.02$</td>
<td>$0.13 \pm 0.01$</td>
<td></td>
</tr>
<tr>
<td>$R_{\perp\chi_{c1}}$</td>
<td>$0.09 \pm 0.01$</td>
<td>$0.11 \pm 0.01$</td>
<td>$0.12 \pm 0.01$</td>
<td></td>
</tr>
<tr>
<td>$R_{h_c}$</td>
<td>$0.06^{-0.02}^{+0.03}$</td>
<td>$0.08^{-0.02}^{+0.03}$</td>
<td>$0.08^{-0.02}^{+0.03}$</td>
<td></td>
</tr>
<tr>
<td>$R_{Lh_c}$</td>
<td>$0.06^{-0.02}^{+0.03}$</td>
<td>$0.08^{-0.02}^{+0.03}$</td>
<td>$0.07^{-0.02}^{+0.03}$</td>
<td></td>
</tr>
<tr>
<td>$R_{\perp h_c}$</td>
<td>$0.14^{-0.01}^{+0.02}$</td>
<td>$0.18^{-0.01}^{+0.02}$</td>
<td>$0.17^{-0.01}^{+0.02}$</td>
<td></td>
</tr>
<tr>
<td>$R_{\chi_{c2}}$</td>
<td>$0.04^{-0.01}^{+0.02}$</td>
<td>$0.05^{-0.01}^{+0.02}$</td>
<td>$0.05^{-0.01}^{+0.02}$</td>
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<tr>
<td>$R_{L\chi_{c2}}$</td>
<td>$0.03 \pm 0.01$</td>
<td>$0.04 \pm 0.01$</td>
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<tr>
<td>$R_{\perp\chi_{c2}}$</td>
<td>$0.05^{-0.01}^{+0.02}$</td>
<td>$0.06^{-0.01}^{+0.02}$</td>
<td>$0.07^{-0.01}^{+0.02}$</td>
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</tr>
</tbody>
</table>

$S1$ and $S2$ represent the new physics contributions including

$$O_{V_1} = (\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma_\mu \nu_L), \quad O_{V_2} = (\bar{c}_R\gamma^\mu b_R)(\bar{\tau}_L\gamma_\mu \nu_L).$$  (1)
Summary

- Form factors of $B_c$ to a charmonium at maximum recoil region are in NLO accuracy.
- Form factors of $B_c$ to a charmonium are expanded into Z-series using HQET and Dispersion relations.
- The theoretical uncertainty is reduced, but the SM value of $R_{J/\psi}$ is a significant deviation from the LHCb data.

Outlook
- More bin and polarization dependent measurements.
- More LQCD simulations.
Thank You!