# Doubly charmed baryons from the lattice.

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# Outline

Interest

- ⋆ Understanding the internal structure ($Q\ell \to$ HQET, $Q\bar{Q} \to$ (p)NRQCD, $QQ\ell \to$ ??).
- ⋆ SU(3) flavour symmetry.
- ⋆ Precision prediction of the spectrum $\to$ demonstration of lattice techniques.

Initial study.

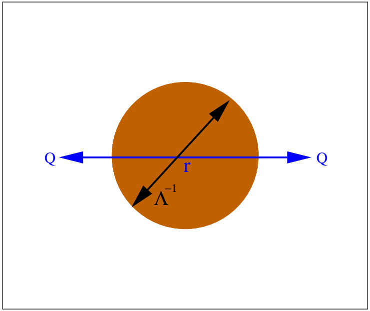
$cc\ell$, $c\ell\ell$: spectrum.

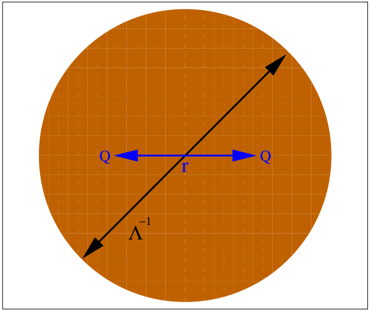
New work (in progress): spin + flavour splittings.

Outlook

# Introduction

$cc\ell$ baryons, heavy-light- or charmonium-like?




Charmonium-like: $r \gg \bar{\Lambda}^{-1}$

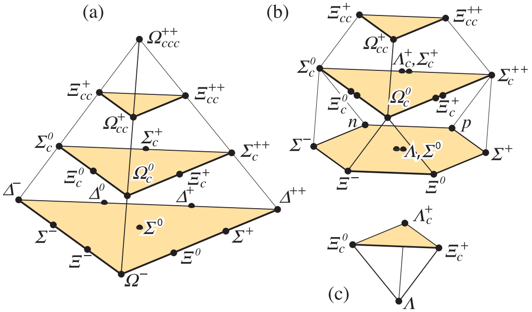
heavy-light-like $r \ll \bar{\Lambda}^{-1}$

$c(c\ell) \sim c\bar{c}$

$(cc)\ell \sim \bar{h}\ell$

[pNRQCD,hep-ph/0506065]: $M^{J=\frac{3}{2}}_{QQq} - M^{J=\frac{1}{2}}_{QQq} = \frac{3}{4}\left(M^{J=1}_{\overline{Q}q} - M^{J=0}_{\overline{Q}q}\right)$

# $c\ell\ell$, $c\ell s$, $css$, $ccs$, $cc\ell$ baryons




SU(4) representations:

$$4 \otimes 4 \otimes 4 =$$

$$20_S \oplus 20_M \oplus 20_M \oplus \overline{4}_A$$

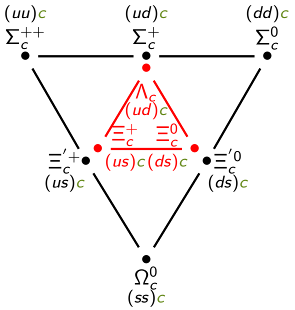
$$J^P = \frac{3}{2}^{\mp} \quad \frac{1}{2}^{\mp} \quad \frac{1}{2}^{\pm}$$

No charges and $m_u = m_d$: $(\Sigma_c^{++}, \Sigma_c^{+}, \Sigma_c^{0}, \Xi_c'^{+}, \Xi_c'^{0}, \Omega_c^0) \to (\Sigma_c, \Xi_c', \Omega_c)$, $(\Xi_{cc}^{++}, \Xi_{cc}^{+}, \Omega_{cc}^{+}) \to (\Xi_{cc}, \Omega_{cc})$ and $(\Lambda_c^+, \Xi_c^+, \Xi_c^0) \to (\Lambda_c, \Xi_c)$.

Stable to strong decay: $J^P = \frac{1}{2}^+$, $\Lambda_c$, $\Xi_c'$, $\Xi_c$, $\Omega_c$, $\Xi_{cc}$, $J^P = \frac{3}{2}^+$, $\Omega_c^*$.

Small widths: $J^P = \frac{1}{2}^+$, $\Sigma_c$ ($\Gamma = 1.9$ MeV), $J^P = \frac{3}{2}^+$, $\Sigma_c^*$ ($\Gamma = 15$ MeV), $\Xi_c^*$ ($\Gamma = 2.1$ MeV).

# Gell-Mann Okubo formulae: charm spectator




Sextet

$$m_{\Sigma_c^{(*)}} = m_0 - \tfrac{2}{3}A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Xi_c^{\prime(*)}} = m_0 + \tfrac{1}{3}A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Omega_c^{(*)}} = m_0 + \tfrac{4}{3}A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

Anti-triplet

$$m_{\Lambda_c} = m_0 - \tfrac{2}{3}B\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

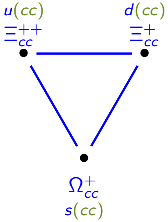
$$m_{\Xi_c} = m_0 + \tfrac{1}{3}B\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

Sextet: $J^P = \frac{3}{2}, \frac{1}{2}$, Anti-triplet: $J^P = \frac{1}{2}$

$$\begin{aligned}\delta m_\ell = m_s - m_{u/d} &\propto M_K^2 - M_\pi^2 + O((\delta m_\ell)^2) \\ &\propto 1 - M_\pi^2/X_\pi^2 + O((\delta m_\ell)^2)\end{aligned}$$

$$X_\pi = \tfrac{1}{3}(2M_K^2 + M_\pi^2)$$

# Gell-Mann Okubo formulae: charm spectator




**Triplet**

$$m_{\Xi_{cc}^{(*)}} = m_0 - \tfrac{1}{3} C \delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Omega_{cc}^{(*)}} = m_0 + \tfrac{2}{3} C \delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

Flavour singlet combinations

$(c\ell\ell)$ $\quad \tfrac{1}{6}(3m_{\Sigma_c} + 2m_{\Xi'_c} + m_{\Omega_c})$ $\qquad (cc\ell)$ $\quad \tfrac{1}{3}(m_{\Omega_{cc}} + 2m_{\Xi_{cc}})$

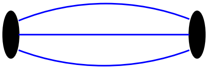
$\quad \tfrac{1}{3}(2m_{\Xi_c} + m_{\Lambda_c})$

At $\mathrm{O}(\delta m_\ell^2)$: new coefficient for each member of the multiplet.

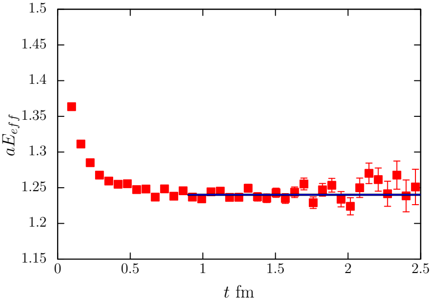
# Extracting masses on the lattice

Time dependence of two-point functions:

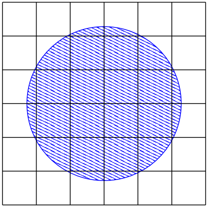
$$C_{2pt}(t) = T^{\pm}_{\gamma\gamma'} \langle \mathcal{O}_\gamma(t) \mathcal{O}^\dagger_{\gamma'}(0) \rangle = |\langle 0 | \mathcal{O}_\gamma | B_\gamma \rangle|^2 e^{-m_B t} + \ldots$$



$$aE_{eff} = \ln \left( \frac{C_{2pt}(t)}{C_{2pt}(t+1)} \right)$$



# Extracting masses on the lattice

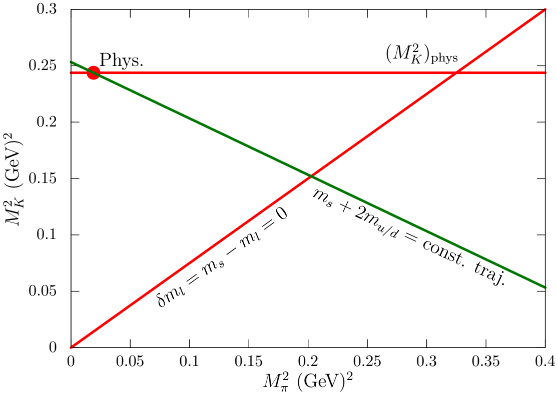


Systematics:

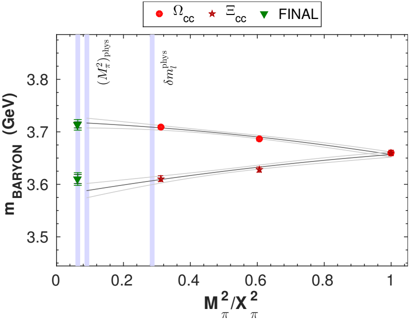
- Excited state pollution.
- Volume: exponentially suppressed $\sim e^{-Lm_\pi}$, $Lm_\pi \gtrsim 4$.
- Discretisation effects: $\mathcal{O}(a^2)$, $am_c \lesssim 0.5$. Some splittings less affected.
- Physical point extrapolation: $m_\pi \rightarrow m_\pi^{phys}$, $m_K \rightarrow m_K^{phys}$.

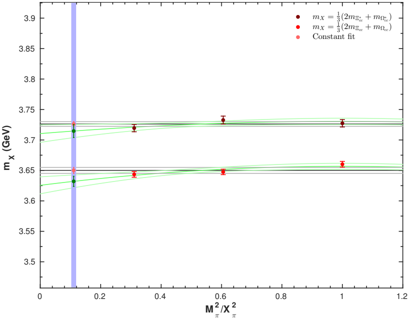
# Initial study: RQCD [Perez-Rubio,1503.08440]

- ⋆ $N_f = 2 + 1$ QCDSF configurations [1102.5300].
- ⋆ Single lattice spacing $a \sim 0.075$ fm, $am_c \sim 0.5$.
- ⋆ Two volumes, $Lm_\pi = 3.1 - 4.3$.
- ⋆ Start from $N_f = 3$ ($m_{u/d} = m_s$) and keep $2m_{u/d} + m_s$ fixed.



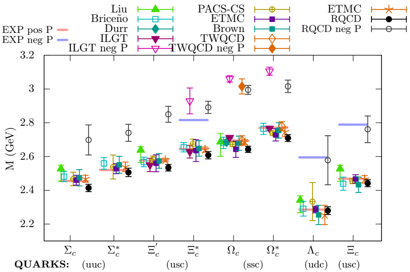
# $SU(3)$ flavour breaking: [RQCD,1503.08440]

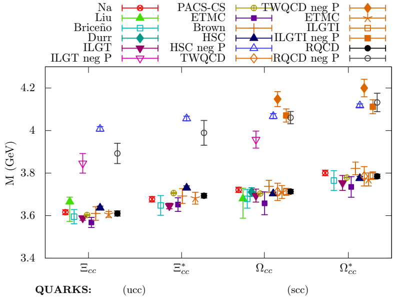



- 3 values of $m_\pi = 459$ MeV, 354 MeV and 255 MeV.
- $\delta m_\ell = m_s - m_{u/d} \propto 1 - M_\pi^2/X_\pi^2 + O((\delta m_l)^2)$
- Small $(\delta m_\ell)^2$ contributions.

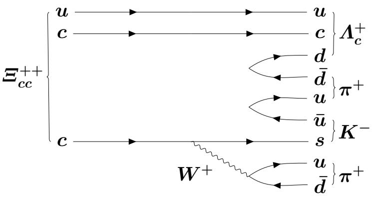
# Spectrum

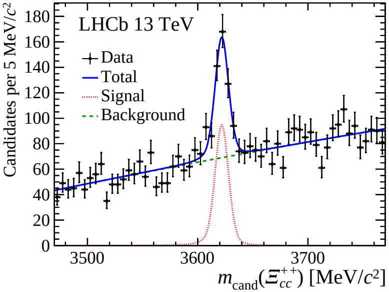



- Variety of $N_f$, quark actions, etc.
- RQCD: Some $c\ell\ell$ states below experiment.
- Negative parity: $J^P = \frac{1}{2}^-, \frac{3}{2}^-$: [ILGTI,1211.6277], [HSC,1502.01845], [RQCD,1503.08440], [TWQCD,1701.02581], [ILGTI,1807.00174]
- Continuum, chiral extrapolation: [Briceno,1207.3536], [ETMC,1406.4310], [Brown,1409.0497], [ILGTI,1807.00174].

# Expt: $\Xi_{cc}^{++}$




[LHCb,1707.01621]

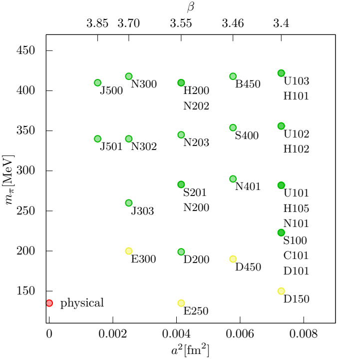
$\Xi_{cc}^{++}$ observed in the $\Lambda_c^+ K^- \pi^+ \pi^-$ mass spectrum.

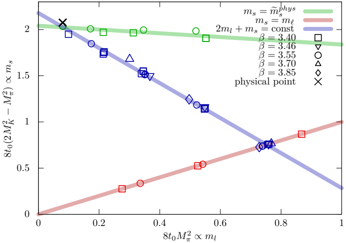
$M = 3621.40 \pm 0.72$ (stat) $\pm 0.27$ (stat) $\pm 0.14$ ($\Lambda_c$) MeV

[SELEX,hep-ex/0406033]: $\Xi_{cc} = 3518.7(1.7)$ MeV.

# Present work

Coordinated Lattice Simulations (CLS): Berlin, CERN, Mainz, UA Madrid, Milano Bicocca, Münster, Odense, Regensburg, Rome I and II, Wuppertal, DESY-Zeuthen.




⋆ Five lattice spacings: $a = 0.09 - 0.04$ fm (open boundary conditions).

⋆ $Lm_\pi \gtrsim 4$ and multiple spatial volumes.

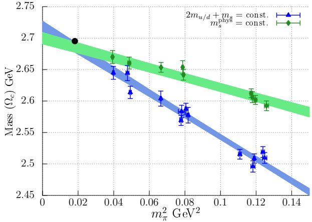
⋆ two trajectories to the physical point + $m_s = m_l$ trajectory. Physical point ensembles.

# Continuum, quark mass extrapolation

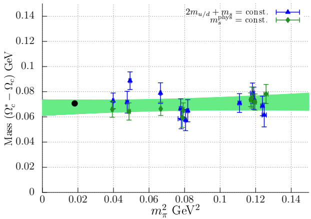
**Very preliminary**: $m_c$ not tuned precisely $\rightarrow$ take mass differences with $\Lambda_c$.
Extrapolate each member of the multiplet separately:

$$Lm_\pi > 3.8, m_\pi \lesssim 350 \text{ MeV}, \qquad M^m = \left(M_0 + b_1^m \overline{M}^2 + b_2 \delta M^2\right)\left(1 + c_1^m a^2 \overline{M}^2\right)$$

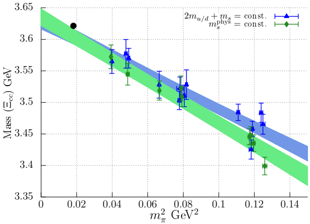
$$\delta M^2 \propto m_s - m_{u/d}, \qquad \overline{M}^2 \propto m_s + 2m_{u/d}$$



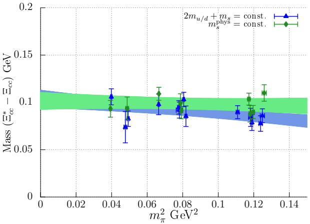

$\Omega_c$




$\Omega_c^* - \Omega_c$



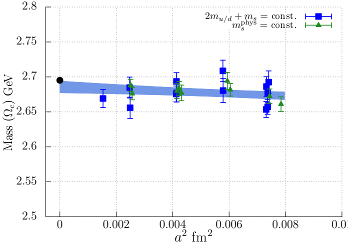

$\Xi_{cc}$



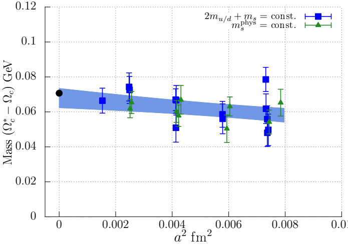

$\Xi_{cc}^* - \Xi_{cc}$

# Continuum, quark mass extrapolation

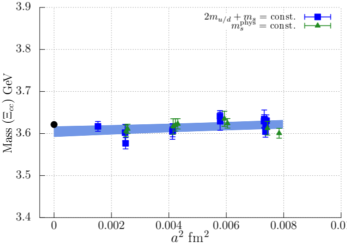
Data points shifted to $m_\pi^{phys}$ and $m_K^{phys}$ using the fit.



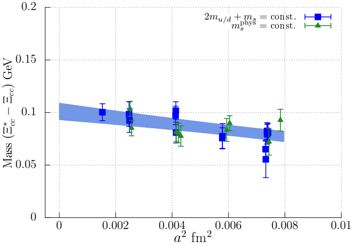

$\Omega_c$




$\Omega_c^* - \Omega_c$

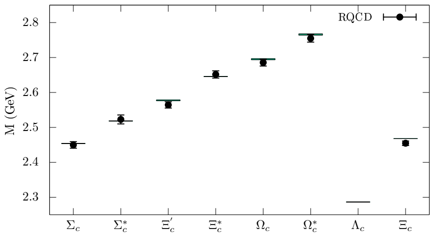


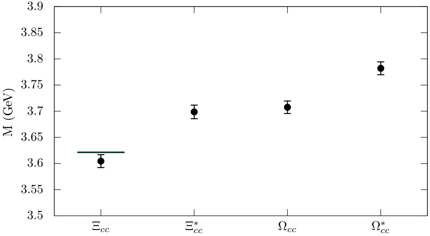

$\Xi_{cc}$




$\Xi_{cc}^* - \Xi_{cc}$

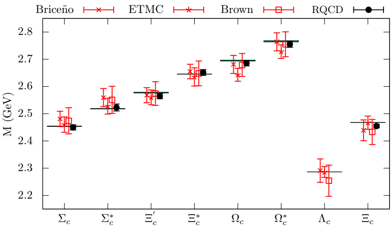
# Spectrum: continuum limit, physical point

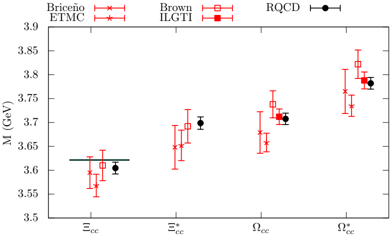



- Lattice operators for $\Xi_c$ and $\Xi'_c$ will have contributions from both states. [Brown,1409.0497] found the mixing effects to be small.

# Spectrum

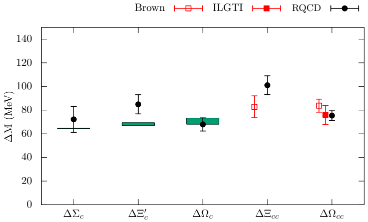



Comparison with other continuum and chirally extrapolated results:

- [Briceno,1207.3536], [ETMC,1406.4310], [Brown,1409.0497], [ILGTI,1807.00174]
- RQCD: (very) preliminary.

# Hyperfine splittings




Heavy-light-like: [pNRQCD,hep-ph/0506065] for $Q-Q$ separation $r \sim 1/m_Q v < 1/\Lambda$.

$$M_{QQq}^{J=\frac{3}{2}} - M_{QQq}^{J=\frac{1}{2}} = \frac{3}{4}\left(M_{\overline{Q}q}^{J=1} - M_{\overline{Q}q}^{J=0}\right)$$

RQCD

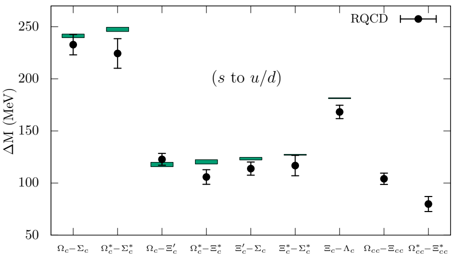
$$\Omega_{cc}^* - \Omega_{cc}/(D_s^{*0} - D_s^0[Expt]) = 0.52(3)$$

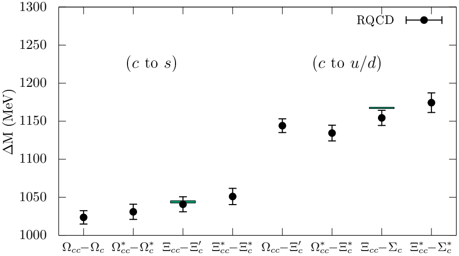
[Brown,1409.0497]

$$\Omega_{cc}^* - \Omega_{cc}/(D_s^{*0} - D_s^0[Expt]) = 0.58(4)$$
$$\Omega_{bb}^* - \Omega_{bb}/(B_s^{*0} - B_s^0[Expt]) = 0.74(12)$$

# Flavour splittings

- $s \to \ell$: splitting is roughly half that for $ss \to \ell\ell$ in $Q\ell\ell$.
- $c \to s$ and $c \to \ell$ splittings very similar across different baryons.



# Outlook

- Reasonable agreement with expt for $\Xi_{cc}$ and $c\ell\ell$ spectrum.
- Agreement with other lattice determinations for other $cc\ell$.
- GMO relations describe the flavour breaking well.
- Work in progress for precision determination: continuum, quark mass limits under control.
- Investigate internal structure by looking at the splittings.
- $m_Q \to m_b$.