String breaking from Lattice QCD with N_f=2+1 dynamical fermions

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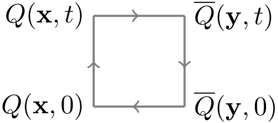
[arXiv:1902.04006]

Static potential and string breaking in QCD

- The static potential $V(r)$ is defined as the energy of the ground state of the system containing a static quark Q and static anti-quark $\overline{Q}$ separated by distance $r = | \mathbf{y} - \mathbf{x} |$

- As a consequence of confinement, the energy between the quark-antiquark pair is contained inside a color flux tube, the so called string
- The time correlation function yields the Wilson loop:

$$\langle W(t, r) \rangle \propto e^{-V(r,t)}$$



If no pair creation from vacuum is allowed, a functional form $V(r)$ is well described by the Cornell parametrization

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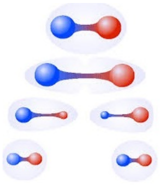
In the full theory with dynamical quarks, the string will break due to creation of a pair of light quarks $q\bar{q}$, which recombine with the static quarks into two static-light mesons B=$q\overline{Q}$ and $\overline{B}$=$\bar{q}Q$



This so called string breaking (SB) is expected as soon as $[V(r) - 2E_B] > 0$

String breaking on the lattice

Since SB is a low energy phenomenon, it is as such not accessible by perturbative QCD, but it can be examined using Lattice QCD.

Expectation: V(r) saturates towards $2E_{m_B}$

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Reasons:

- weak signal-to-noise ratio for distances > 1fm
- Wilson loop not a good ground state observable after SB

SB as a mixing phenomenon

- the string state and the two meson state are both needed to describe the potential
- $|Q\overline{Q}\rangle\,, |B\overline{B}\rangle$ are not QCD eigenstates
- the ground state $|1\rangle$ and first excited state $|2\rangle$ are superpositions of $|Q\overline{Q}\rangle\,, |B\overline{B}\rangle$

$\rightarrow$ avoided level crossing with energy gap ΔE

Use variational technique to extract ground state as well as excited states from a mixing matrix $C(\mathbf{r}, t)$

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Using this method, evidence for SB was first found for the SU(2) Higgs model

Knechtli and Sommer, 1998 [arXiv:hep-lat/9807022],

Philipsen and Wittig, 1998 [arXiv:hep-lat/9807020]

and later for $N_{\mathrm{f}} = 2$ QCD Bali et al., 2005 [arXiv:hep-lat/0505012]

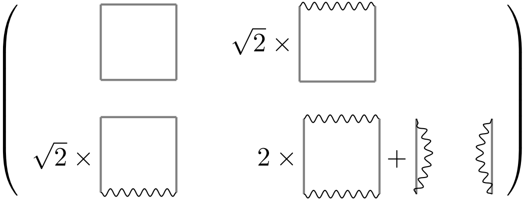
Mixing matrix for $N_\mathrm{f} = 2$

$$\mathcal{O}_W(\mathbf{x},\mathbf{y},t) = \overline{Q}(\mathbf{y},t)\Gamma\mathcal{W}(\mathbf{y},\mathbf{x},t)Q(\mathbf{x},t)$$

$$\mathcal{O}_{B\bar{B}}(\mathbf{x},\mathbf{y},t) = \frac{1}{\sqrt{2}}\sum_{i=u,d}\bar{Q}(\mathbf{y},t)\hat{\Gamma}q^i(\mathbf{y},t)\ \bar{q}^i(\mathbf{x},t)\hat{\Gamma}^\dagger Q(\mathbf{x},t)$$

$$C(\mathbf{r},t) = \begin{pmatrix} \langle\mathcal{O}_W(t)\overline{\mathcal{O}}_W(0)\rangle & \langle\mathcal{O}_{B\overline{B}}(t)\overline{\mathcal{O}}_W(0)\rangle \\ \langle\mathcal{O}_W(t)\overline{\mathcal{O}}_{B\overline{B}}(0)\rangle & \langle\mathcal{O}_{B\overline{B}}(t)\overline{\mathcal{O}}_{B\overline{B}}(0)\rangle \end{pmatrix}$$

$=$



Mixing matrix for $N_\mathrm{f} = 2+1$

including the strange quark

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$$\mathcal{O}_{B_s\bar{B}_s}(\mathbf{x},\mathbf{y},t) = \bar{Q}(\mathbf{y},t)\hat{\Gamma}q^s(\mathbf{y},t)\ \bar{q}^s(\mathbf{x},t)\hat{\Gamma}^\dagger Q(\mathbf{x},t)$$

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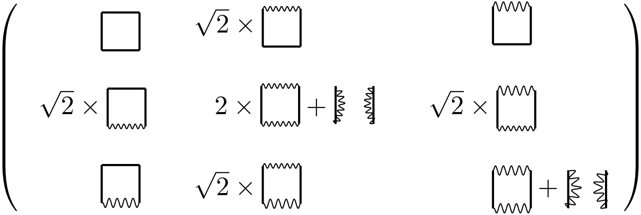

Goal

- examine SB on finer lattice with smaller sea quark mass
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- need for 'all-to-all propagators'

- estimate SB distance using only Wilson loops to determine window to enhance resolution
- ideal playground to explore SB: set of ensembles with $N_\mathrm{f} = 2 + 1$ generated by the CLS effort Bruno et al.[arXiv:1411.3982]
- use of stochastic LapH method to facilitate 'all-to-all propagators' Morningstar et al. [arXiv:1104.3870]

Stochastic LapH method

- based on distillation Peardon et al.[arXiv:0905.2160]
  - the important contributions to the quark propagator are encoded in smaller subspace, spanned by N_{ev} low-lying eigenmodes of covariant 3D Laplace operator
  - projection onto the so-called LapH subspace amounts to a form of quark smearing
- introducing a stochastic estimator in the LapH subspace combined with dilution Foley et al. [arXiv:heplat/0505023] helps reduce rise in computational costs as volume increases

	N_{ev}	line type	dilution scheme	N_r light/strange	source time
N200	192	fixed	(TF,SF,LI8)	5 / 2	32,52
		relative	(TI8,SF,LI8)	2 / 1	-

→ number of inversion required per gauge configuration:

$$832(\text{light}) + 384(\text{strange})$$

Propagators can be reused for other spectroscopy projects, see e.g. Andersen, Bulava, Hörz, Morningstar[arXiv:1808.05007]

Numerical results

id	a[fm]	$N_{\rm s}$	$N_{\rm t}$	m_π[MeV]	m_K[MeV]	$m_\pi L$
N200	0.064	48	128	280	460	4.4

Ensemble used for first exploratory study

- utilize a set of off-axis distances between $17a$ and $21.8a$
- construct variational basis for the string state using 15 and 20 levels of HYP-smeared spatial links, extending $C(\mathrm{r}, t)$ to a 4×4 matrix

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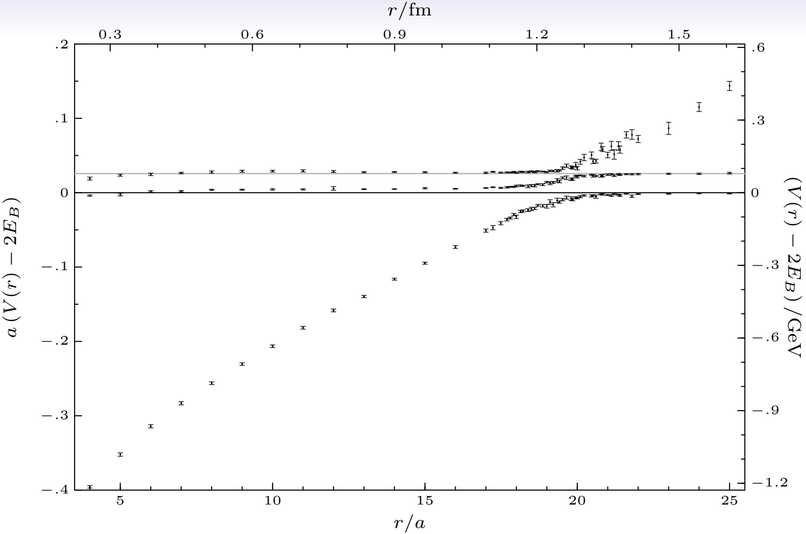
$\rightarrow$ to extract the ground, first and second energy state a generalized eigenvalue problem (GEVP) is solved for each r



Results on 104 configurations (1664 for Wilson loops) of N200

- set of off-axis distances allows for a good resolution
- avoided level crossing between the ground state and the first excited state can be resolved clearly
- expected second avoided crossing due to the formation of two static-strange mesons is evident for the first time from lattice calculations

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- not possible to determine the difference between the first r_c and the second string breaking distance r_{c_s} by eye
- quantification of SB involving three levels is more complex in comparison to $N_{\mathrm{f}} = 2$

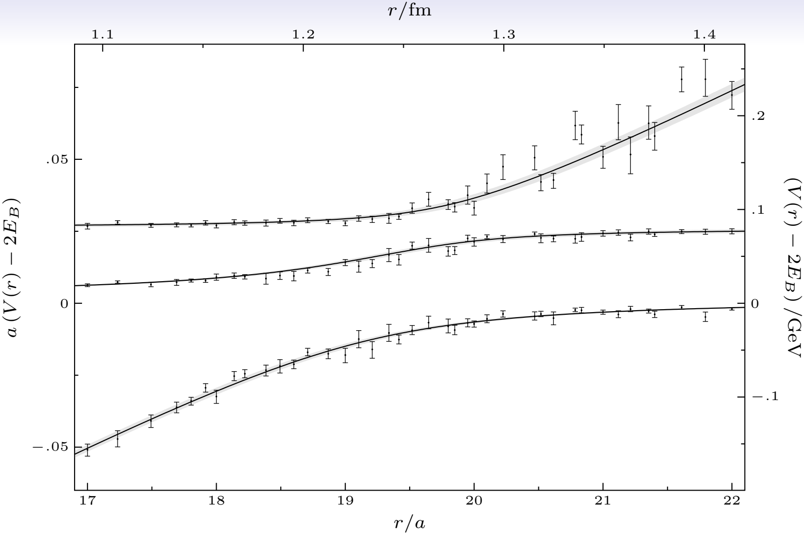
$\rightarrow$ How to quantify the results?

A model for the string breaking spectrum

- describe data using a simple model of a three state system with a Hamiltonian given by:

$$H(r) \quad = \quad \begin{pmatrix} \hat{V}(r) & g_1 & g_2 \\ g_1 & E_1 & 0 \\ g_2 & 0 & E_2 \end{pmatrix}$$

- diagonal elements are a function $\hat{V}(r)$ describing the unbroken string and $\hat{E}_1$, $\hat{E}_2$, the energies of a noninteracting pair of static-light and static-strange mesons
- a suitable choice for string state is the linear part of $\hat{V}_l(r) = \sigma r + V_0$ of the Cornell potential
- eigenvalues of H correspond to the three extracted energy levels $\rightarrow$ after diagonalizing H, fit the model to the data



Uncorrelated six-parameter fit to the string breaking spectrum over fit range $[11, 25]$

- we find for our fit parameters:

$$a\hat{E}_1 = 0.0019(2), \qquad a\hat{E}_2 = 0.0262(6),$$
$$ag_1 = 0.0154(4), \qquad ag_2 = 0.0080(5),$$
$$a^2\sigma = 0.0229(3), \qquad a\hat{V}_0 = -0.434(5)$$

- $g_1 = 47.2(1.4)\,\mathrm{MeV}$ and $g_2 = 24.5(1.6)\,\mathrm{MeV}$ describe the mixing between the gluonic flux tube and the broken string
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$\rightarrow$ use asymptotic states of model to extract two distinct SB distances r_c and r_{c_s} corresponding to the light and strange mixing phenomenon

Quantitative definition of SB distance for $N_\mathrm{f} = 2+1$

r_c is defined as the crossing distance where $\hat{V}(r_c) = \hat{E}_1$ and
r_{c_s} is defined as the crossing distance where $\hat{V}(r_{c_s}) = \hat{E}_2$

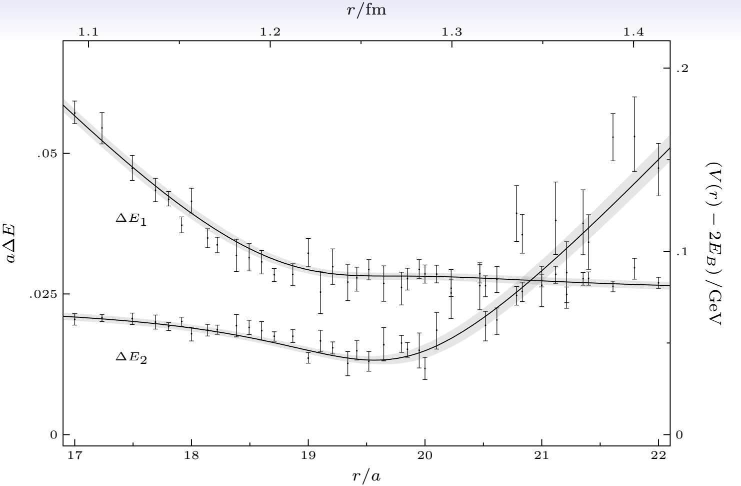
$$r_c = 19.053(82)a = 1.224(15)\mathrm{fm}$$
$$r_{c_s} = 20.114(87)a = 1.293(16)\mathrm{fm}$$

only one previous study of string breaking for $N_\mathrm{f} = 2$ QCD on the lattice exists with $m_\pi \approx 640\mathrm{MeV}$ Bali et al., 2005 [arXiv:hep-lat/0505012]

using the position of the minimal energy gap, they find

$$r_c \approx 1.244(16)\mathrm{fm}$$

due to differing definitions of the string breaking distance, it is not possible to make a statement on quark mass dependence



Energy gap between ground and first-excited state ($\Delta E_1(r)$), as well as first-excited and second-excited state ($\Delta E_2(r)$) from the model fit as well as from our data.

Phenomenology

→ information can be used as input for investigations of quarkonia above threshold and heavy-light and heavy-strange coupled-channel meson scattering

first, simple comparison between our data and the experimentally observed quarkonium spectrum:

use the ground-state energy computed from our model as input potential in Schrödinger equation and extract the bound state energies of quarkonia in the Born-Oppenheimer approximation

m_Q	$l = 0$	$l = 1$	$l = 2$	$l = 3$	$l = 4$
m_b	3	3	2	2	1
m_c	2	1	1	–	–

Number of bound-state solutions $E_{nl} < 0$ for bottomonium and charmonium of the Schrödinger equation.

Summary and outlook

- we observe the effect of the strange sea-quark flavor, which results in a second mixing-phenomenon due to the formation of two static-strange mesons
- we provide a simple model for the mixing scenario that describes the SB data very well and allows for definition of two distinct SB distances for the light and the strange mixing phenomenon

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Steps towards fully quantifying string breaking on the lattice:

- repeat calculation on suitable set of ensembles, varying lattice size, lattice spacing and quark masses

  next step $\rightarrow$ investigate dependence of SB distance and shape of gap on sea quark mass

id	a[fm]	$N_{\rm s}$	$N_{\rm t}$	m_π[MeV]	m_K[MeV]	$m_\pi L$
N202	0.064	48	128	420	420	6.5
N203	0.064	48	128	340	440	5.4
N200	0.064	48	128	280	460	4.4
D200	0.064	64	128	200	480	4.2

Additional details

- excited states in SB region
  - possible excited states we are missing in our operator basis: string excitations, static-light meson excitations, state containing two extra pions
    $\rightarrow$ all higher by a scale of about 500MeV, substantially larger than the gaps observed

- setting $g_3 = 0$ in the Hamiltonian model
  - no constraint on this mixing in the energy spectrum alone, a basis rotation shows any non-zero value of this parameter yields equivalent spectrum to Hamiltonian
  - ensures the diagonal elements of H correspond to the asymptotic energy eigenvalues up to corrections at $O(r^{-1})$ as $r \rightarrow \infty$