Technische Universität München

P-wave heavy quarkonium spectrum

with N^3LL accuracy

Clara Peset

QWG 2019, 15th May 2019

Based on work in collaboration with A. Pineda, M. Stahlhofen and J. Segovia

arxiv:1511.08210, arxiv:1809.09124

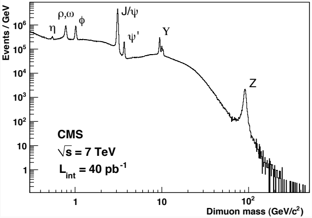
Outline

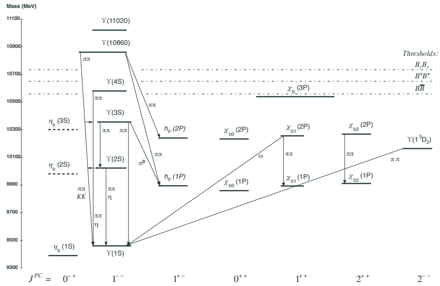
1. Motivation and framework
2. Heavy quarkonium at N^3LL
3. Summary and further applications

Motivation and framework

Motivation: Low energy QCD

QCD predicts a tower of resonances of $q\bar{q}$ bound-states





PDG '17

BUT hadronization in a non-perturbative effect

$\Rightarrow$ exploit **EFTs** to overcome our limitation in describing low energy QCD

pNRQCD at N^3LL

$$\text{QCD} \xrightarrow{m_r} \text{NRQCD} \xrightarrow{m_r\alpha_s} \text{pNRQCD}$$

$$\left.\begin{array}{l}\left(i\partial_0 - \frac{\mathbf{p}^2}{2m_r} - V^{(0)}(r)\right)\phi(\mathbf{r}) = 0 \\ + \text{ corrections to the potential} \\ + \text{ interaction with other low-energy degrees of freedom}\end{array}\right\} \text{pNRQCD.}$$

Pineda, Soto

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Pineda, Soto '98

The singlet potential

$$V_s(\mathbf{p}, \mathbf{r}) = V^{(0)}(r) + \frac{V^{(1,0)}(r)}{m_1} + \frac{V^{(0,1)}(r)}{m_2} + \frac{V^{(2,0)}(\mathbf{p}, \mathbf{r})}{m_1^2} + \frac{V^{(0,2)}(\mathbf{p}, \mathbf{r})}{m_2^2} + \frac{V^{(1,1)}(\mathbf{p}, \mathbf{r})}{m_1 m_2}$$

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N^3LL NNLL NLL

Wilson coefficients of the EFT: **matching** and **renormalization**

scheme dependent

pNRQCD at N^3LL

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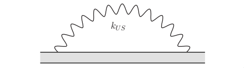
Pineda, Soto '98

Contribution of US gluons:

perturbative for $mv^2 \gg \Lambda_{\mathrm{QCD}}$



- known to 2-loops

Pineda, '02, '11

Applicability of pNRQCD

Strict weak coupling: $E, \Lambda_{\rm QCD} \ll |\mathbf{p}| \ll m_r$

Physical systems:
$$\begin{cases} \text{Bottomonium (1S):} & |\mathbf{p}| \sim m_b v \sim 1.3 \text{ GeV} \\ B_c \text{ (1S):} & |\mathbf{p}| \sim 2 m_r v \sim 0.85 \text{ GeV} \\ \text{Charmonium (1S):} & |\mathbf{p}| \sim m_c v \sim 0.68 \text{ GeV} \end{cases}$$

velocity scales with n quantum number $|\mathbf{p}|_{(nl)} \sim \frac{|\mathbf{p}|_{(1S)}}{n}$

Refinements to apply our EFT to these systems:

- Achieve high orders in PT

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  - resummation of large logarithms ⇒ this talk

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  - resummation of large logarithms ⇒ this talk
  - alternative computation schemes

(see 1302.3528,1806.05197,1809.09124)

Heavy quarkonium at N^3LL

Resummation of Large Logs

Reference scales:

Hard: $\nu_H \sim m_r$, **Soft:** $\nu_S \sim m_r\alpha_s$, **Ultrasoft:** $\nu_{US} \sim m_r\alpha_s^2$

Multiscale computations give rise to large logarithms

$$\text{e.g.} \quad \langle \alpha_s \ln(m_r r) \rangle \sim \alpha_s \ln \frac{\nu_H}{\nu_S} \sim \alpha_s \ln \frac{1}{\alpha_s} \sim \mathcal{O}(1)$$

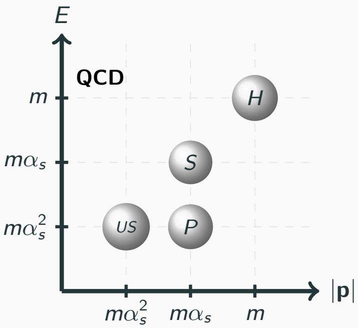
$\Rightarrow$ breakdown of power counting

Solution: solve the **R**enormalization **G**roup **E**quation to resum large logs

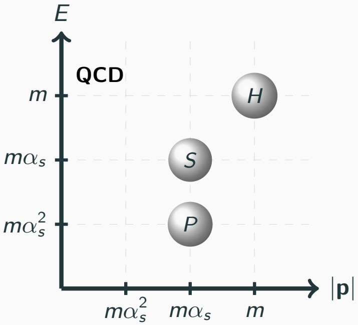
$$\frac{dC(\mu)}{d \ln \mu} = \Gamma C(\mu) \;\&\; C(\nu_h) = C_0 \Rightarrow \boxed{C(\nu_S, \nu_H) \sim \frac{\alpha_s(\nu_H)}{\alpha_s(\nu_S)} \sim \sum_n \alpha_s^n \ln^n \frac{\nu_H}{\nu_S}}$$

Log resummation and the method of regions

- **QCD:**



Gluons

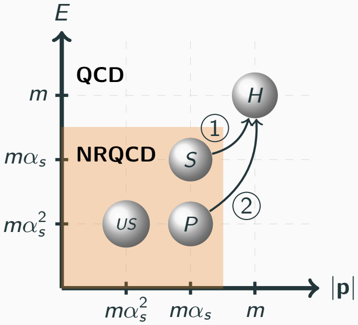


Heavy quarks

Log resummation and the method of regions

- **NRQCD:** $E, |\mathbf{p}|, \Lambda_{\mathrm{QCD}} \ll \nu_S \ll \nu_H \sim m$

① **Soft running** & ② **Potential running**



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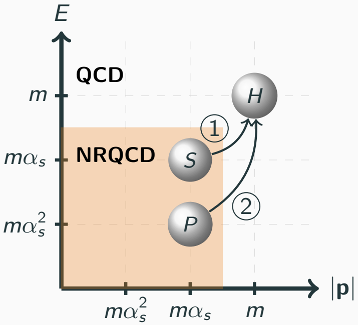
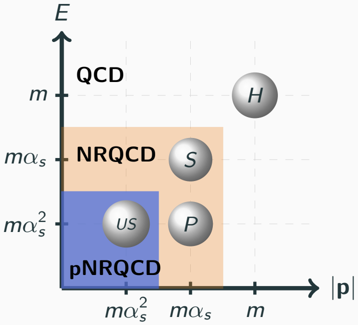


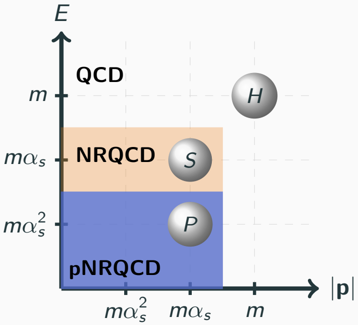
Heavy quarks

Log resummation and the method of regions

- **pNRQCD:** $E, \Lambda_{QCD}, \frac{\mathbf{p}^2}{m} \ll \nu_{US} \ll |\mathbf{p}| \ll \nu_P, \nu_S \ll \nu_H \sim m$



Massless d.o.f.



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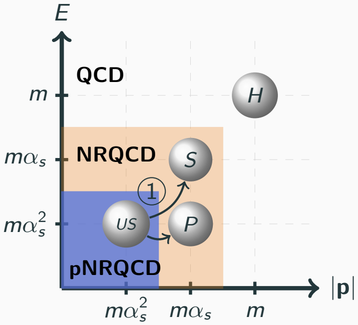
① **Ultrasoft running**

② **Potential running:** needs correlation of scales to keep hierarchy: $\nu_{US} = \frac{\nu_S^2}{\nu_H}$



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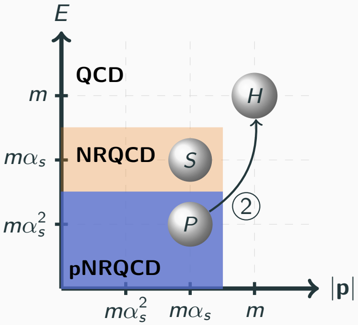

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$$V_s(\nu_H, \nu_S, \nu_P) = V_s(c(\nu_H, \nu_S, \nu_P), \nu_H, \nu_S, \nu_{US})$$

Energy levels

$$E(n, l, s, s^-, j) = E_n^C \left(1 + \frac{\alpha_s}{\pi} P_1 + \left(\frac{\alpha_s}{\pi}\right)^2 P_2^{\mathrm{NNLL}} + \left(\frac{\alpha_s}{\pi}\right)^3 P_3^{\mathrm{N^3LL}}\right),$$

- Hard work over many years:

see Dani Moreno's talk for a bibliographical historic account

N^3LL state-of-the-art

- **On the path to N^3LL:**

① **Ultrasoft running:**
  - 1 and 2-loop self energy with an ultrasoft gluon ✓

② **Soft running:**
  - Momentum-dependent $\frac{1}{m^2}$-potentials ✓
  - delta-like ($l = 0$) potentials ⚠

③ **Potential running:**
  - Only in delta-like potentials ✓

$\Rightarrow$ **P-wave @N^3LL**

N^{3}LL: p-wave spin-averaged splitting

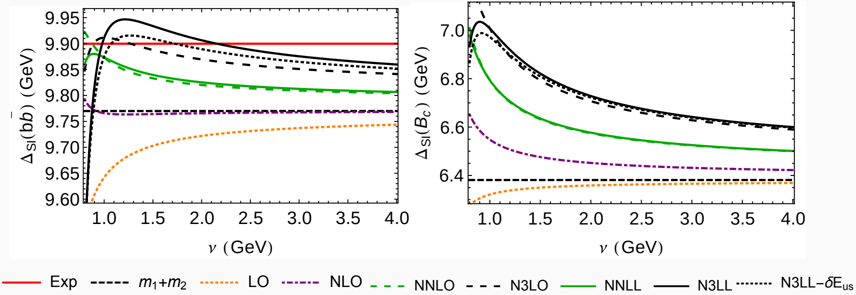
$$\Delta_{\rm SI} = \frac{1}{12}\left(5M_{\chi_{b2}} + 3M_{\chi_{b1}} + M_{\chi_{b0}}\right) + \frac{1}{4}M_{h_b}$$

Only sensitive to the ultrasoft resummation

- RS' scheme at $\nu_f = 1$ GeV, at $\nu_h = 2m_r$ $\nu_S = \nu$ and $\nu_{US} = 1$ GeV

$m_{b,\rm RS'}(\nu_f)$ determined elsewhere: see tomorrow's talk

Bottomonium B_c



N^3LL: p-wave fine splitting

- splitting produced by the total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$

$$
\begin{aligned}
E(1^3P_j) - E(1^3P_{j'}) &= \frac{\alpha^4 C_F^4 m_r^3}{96 m_1 m_2}(j(j+1) - j'(j'+1)) \\
&\times \Bigg\{ -\frac{3}{5} c_F^{(1)} c_F^{(2)} ((j+1)j + (j'+1)j' - 7) \\
&\left(1 + \frac{\alpha}{2\pi}\left(2\left(\frac{209}{36} - \frac{\pi^2}{3}\right)\beta_0 + 2(2\beta_0 - C_A)\ln\frac{\nu}{\alpha C_F m_r} - \frac{17 C_A}{3}\right)\right) \\
&+ \left(c_F^{(1)}\left(\frac{m_2}{m_1} + 1\right) + c_F^{(2)}\left(\frac{m_1}{m_2} + 1\right) - \frac{m_1^2 + m_2^2}{2 m_1 m_2}\right) \\
&\times \left(1 + \frac{\alpha}{2\pi}\left(C_A\left(\log\left(\frac{\alpha C_F m_r}{\nu}\right) - \frac{19}{3}\right) + \beta_0\left(4\log\left(\frac{\nu}{\alpha C_F m_r}\right) - \frac{2\pi^2}{3} + \frac{215}{18}\right)\right)\right) \\
&+ \frac{\alpha}{4\pi}\frac{C_A\left(m_1^2 + m_2^2\right)}{m_1 m_2}\left(\ln\frac{\alpha C_F m_r}{\nu} - 1\right)\Bigg\}
\end{aligned}
$$

N^3LO Brambilla et al '04, N^3LL CP,Pineda,Segovia

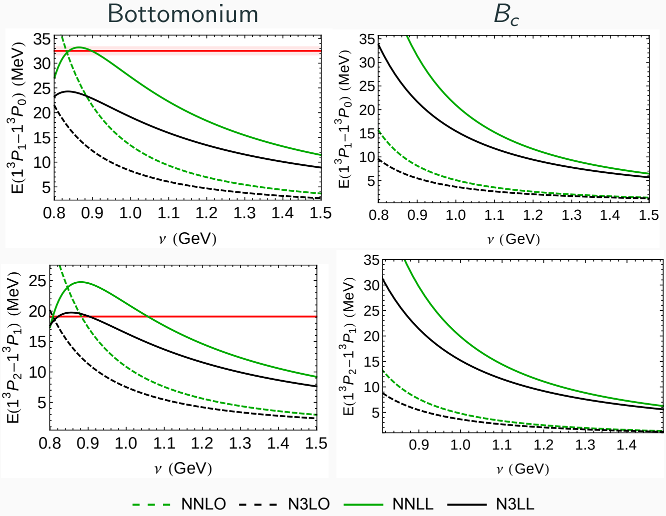
- Resummation encoded in the NRQCD coefficient $c_F^{(i)}(\nu, \nu_h)$

Amoros et al '97, Czarneck '97

N^{3}LL: p-wave fine splitting

Only sensitive to soft running

- RS' masses at $\nu_f = 1$ GeV, at $\nu_h = 2m_r$ $\nu_S = \nu$ and $\nu_{US} = 1$ GeV



N^3LL: p-wave fine splitting

$$\rho = \frac{E(1^3P_2) - E(1^3P_1)}{E(1^3P_1) - E(1^3P_0)}$$

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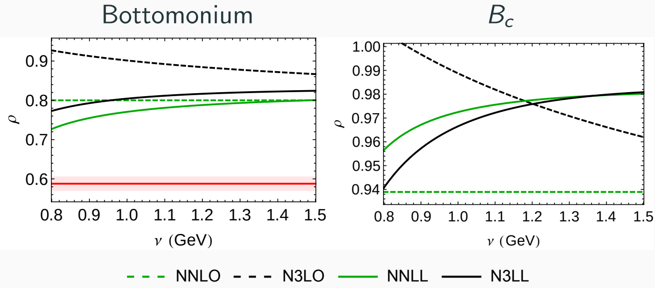


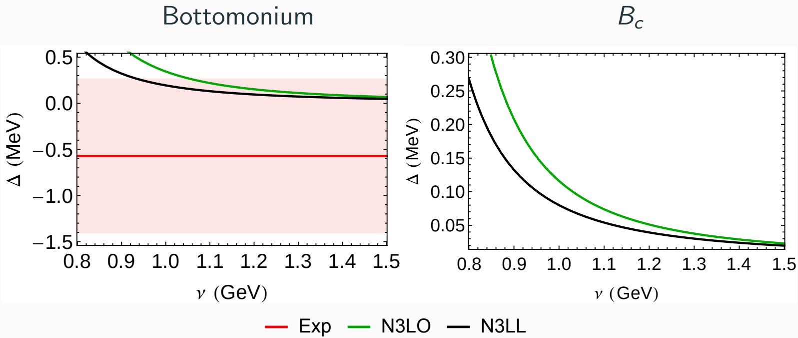
N^{3}LL: p-wave hypefine splitting

Only sensitive to soft running

$$\Delta = M_{h_b} - \frac{1}{9}\left(5M_{\chi_{b2}} + 3M_{\chi_{b1}} + M_{\chi_{b0}}\right) = -\frac{m_r^3 C_F^5 \alpha_s^5}{432\pi m_1 m_2}\left(C_A - 8TFn_f\right) c_F^{(1)} c_F^{(2)}$$

N^{3}LO Brambilla et al '04, N^{3}LL CP,Pineda,Segovia

- Starts at $\mathcal{O}(m_r\alpha^5)$ $\Rightarrow$ we only have one term
- RS' masses at $\nu_f = 1$ GeV, at $\nu_h = 2m_r$ $\nu_S = \nu$ and $\nu_{US} = 1$ GeV



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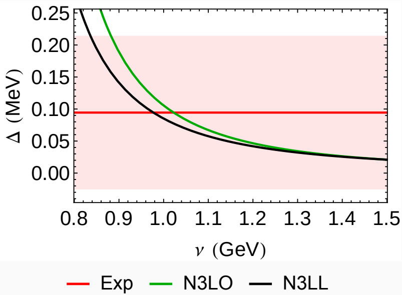
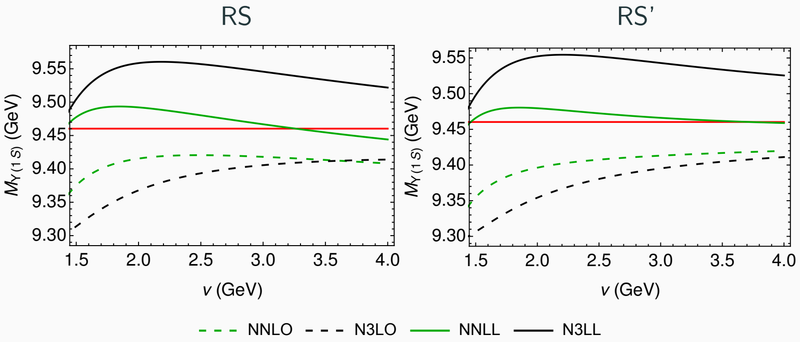
Charmonium



N^3LL: s-wave mass

- RS(') masses at $\nu_f = 2$ GeV, at $\nu_h = 2m_r$ $\nu_S = \nu$ and $\nu_{US} = 1$ GeV



- Complete NNLL.
- Partial N^3LL: full NLL ultrasoft potential but missing delta-like Wilson coefficients

$$\Rightarrow \text{scale variation reduced by} \sim \tfrac{1}{2}!$$

Summary and further applications

Summary and conclusions

- We compute the **N^3LL spectrum** in pNRQCD for **p-waves**
  - The potentials obtained are valid for $\mathbf{mv} \gg \Lambda_{QCD}$
  - The US contribution is valid for $\mathbf{mv^2} \gg \Lambda_{QCD}$
  - The results include terms of $\mathcal{O}(m_r \alpha_s^6 \ln \alpha_s)$
- We apply the results to 1P **bottomonium** and $\mathbf{B_c}$
  - For $b\bar{b}$ our prediction is compatible with experiment although with somewhat large uncertainties
  - Prediction for B_c hyperfine and fine splittings
  - For Δ_{SI} convergence is marginal
- The effect of the ultrasoft scale looks small
- Preliminary (incomplete) results for N^3LL s-wave look promising!

Future perspectives

For the near future:

- Phenomenology N^3LL for s-wave spectrum
- $t\bar{t}$-production near threshold at NNLL+N^3LO

Thank you!

Convergence of the perturbative series

$$\mathcal{L} = \sum_i \frac{1}{m_q^i} C_i \mathcal{O}_i, \qquad C_i(\nu) = \tilde{C}_i + \sum_{n=0}^{\infty} C_{i,n}\alpha_s^{n+1}$$

- Wilson coefficients are **asymptotic** : $\boxed{C_{i,n} \sim n!}$

$\Rightarrow$ **BUT** comply the OPE: $m_q = m_{os} + \tilde{\Lambda}_{QCD}$ is renormalon free

$$m_{os} = m_{\overline{\rm MS}}\left(1 + B_1\alpha_s + B_2\alpha_s^2 + \dots\right), \qquad B_n \sim n!$$

- Redefine the mass such that C_i is **not asymptotic**: **threshold mass**

Renormalon subtracted schemes:

$$m_{\rm RS(')} = m_{OS} - N_m \pi \nu_f \sum_{N=0(1)}^{\infty} \left(\frac{\beta_0}{2}\right)^N \left(\frac{\alpha(n_l,\nu_f)}{\pi}\right)^{N+1} \sum_{n=0}^{\infty} \tilde{\chi}_n \frac{\Gamma(b+N+1-n)}{\Gamma(b+1-n)}$$

Pineda

- Introduction of a new scale $\nu_f \sim m_r\alpha_s$