Heavy Quark Diffusion from the Lattice

Viljami Leino

Technische Universität München, t30f

In collaboration with: Nora Brambilla, Saumen Datta,
Peter Petreczky, and Antonio Vairo

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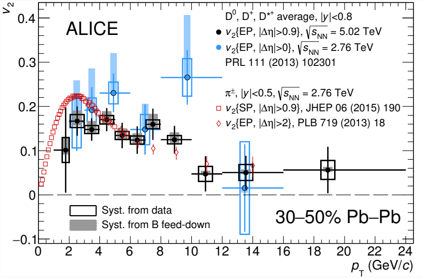
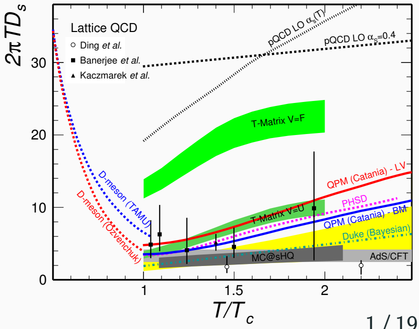
- Charm and bottom quarks much heavier than RHIC/LHC temperatures
- Can be used to probe early time physics
- Experimental results for nuclear suppression factor R_{AA} and elliptic flow v_2 differ from simple perturbative estimates
- Both R_{AA} and v_2 can be calculated from diffusion constant D of heavy quark in medium
- D can be tuned to match experimental results





Figures from: ALICE PRL120 (2017), X. Dong CIPANP (2018)

Heavy Quark in medium

- Heavy quark energy doesn't change much in collision with a thermal quark

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
- Successive collisions with medium are uncorrelated
  $\rightarrow$ Brownian motion
- The physics can be simulated with Langevin dynamics

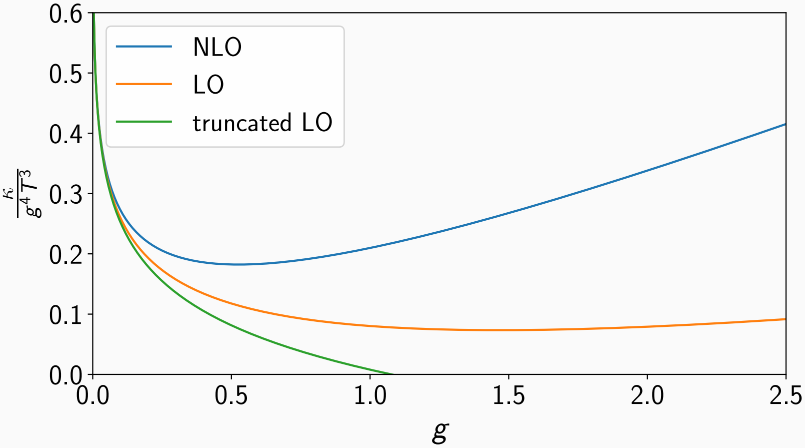
$$\frac{dp_i}{dt} = -\eta_D p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t') \rangle = \kappa\delta(t-t')$$

- κ: strength of stochastic interaction: property of medium
- The drag coefficient $\eta_D = \kappa/(2MT)$
- Relaxation time $\tau_R = 1/\eta_D$
- In position space $\langle x^2(t) \rangle = 6Dt$ with $D = 2T^2/\kappa$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Perturbation theory

- Perturbation theory has poor convergence



- Non-perturbative methods needed

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Directly from HQ current correlator

- Operator of interest: HQ current correlator $\bar{Q}\gamma^i Q$
- problematic observable:
  - Quite insensitive to D (Teaney PRD74 (2006), Petreczky EPJC62 (2008))
  - Narrow transport peak around zero in spectral function
- better approach needed

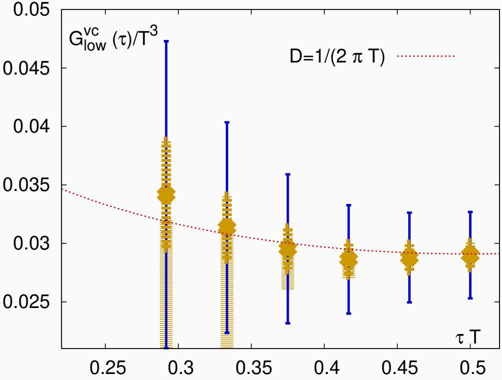


fig: Petreczky EPJC62 (2008)

Euclidean Correlator derivation

- In the $M \to \infty$ limit we can define:

$$\kappa = \frac{1}{3T\chi}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty} M_{\rm kin}^2 \int_{-\infty}^{\infty} {\rm d}t\, e^{i\omega(t-t')}\int {\rm d}^3x \left\langle \frac{1}{2}\left\{\frac{\hat{\mathcal{J}}^i(t,x)}{{\rm d}t}, \frac{\hat{\mathcal{J}}^i(0,0)}{{\rm d}t}\right\}\right\rangle\right]$$

- Where $\hat{\mathcal{J}}^i(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$ is the heavy quark current
- The heavy quark force in static limit:

$$M\frac{{\rm d}\hat{\mathcal{J}}^i}{{\rm d}t} = \left\{\phi^\dagger E^i\phi - \theta^\dagger E^i\theta\right\}$$

- Where ϕ, θ are HQ and $\mathrm{H\bar{Q}}$ operators, E^i color-electric field
- Now the euclidean correlator is defined as:

$$G_{\rm E}(\tau) = -\frac{1}{3T\chi}\sum_{i=1}^{3}\lim_{M\to\infty}\int {\rm d}^3x \left\langle \left[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\right](\tau,x)\left[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\right](0,0)\right\rangle$$

Euclidean correlator

- After simplifying the propagators of ϕ and θ in $M \to \infty$:

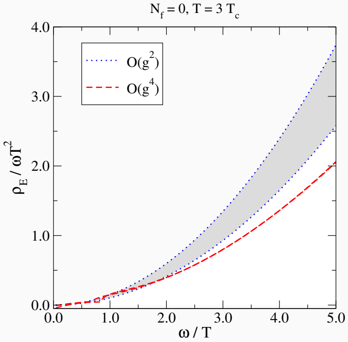
$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}\,[U(\beta,\tau) g E_i(\tau,0) U(\tau,0) g E_i(0,0)] \rangle}{\langle \mathrm{Re\,Tr}\,[U(\beta,0)] \rangle}$$

- Related to the Diffusion coefficient by:

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho(\omega) \frac{\cosh\left(\frac{\beta}{2} - \tau\right)\omega}{\sinh\frac{\beta\omega}{2}}$$

$$\kappa = \lim_{\omega\to 0} \frac{2T}{\omega}\rho(\omega)$$

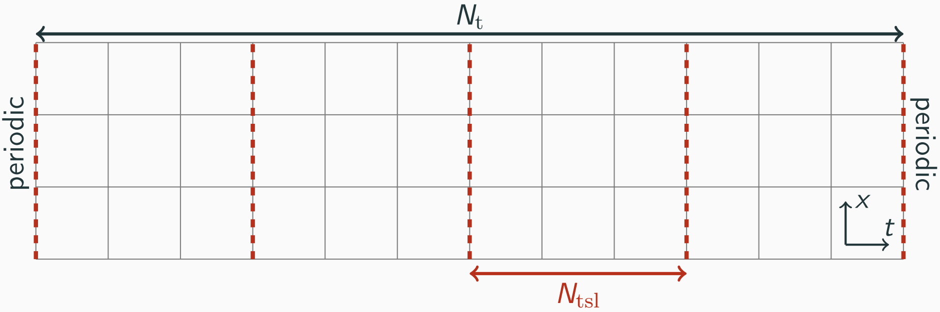
- In general inversion problem is ill-defined



- No $\omega \to 0$ transport-peak like with direct HQ-current measurement

Following Caron-Huot *et.al.*JHEP04 (2009), fig from: Burnier *et.al.*JHEP08 (2010)

Multilevel algorithm

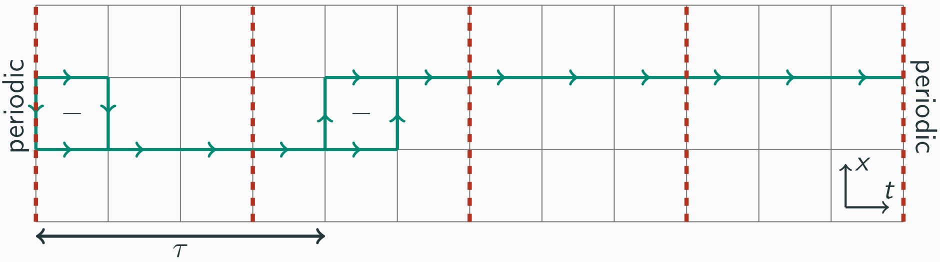


- Algorithm for quenched simulations
- At large N_t observable like Polyakov loop can have poor signal
- Idea: Divide the lattice to temporal slices of size N_tsl
- update each sub-lattice independently keeping boundaries fixed
- Average over different boundary configurations

$\rightarrow$ Allows reaching better statistics with less configurations

Lüscher and Weisz JHEP09 (2001)

Euclidean $\langle EE \rangle$ correlator



$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,\tau)gE_i(\tau,0)U(\tau,0)gE_i(0,0)]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,0)]\rangle}$$

- Renormalization: $Z_{\mathrm{E}} = 1 + g_0^2 \times 0.137718569\ldots + \mathcal{O}(g_0^4)$

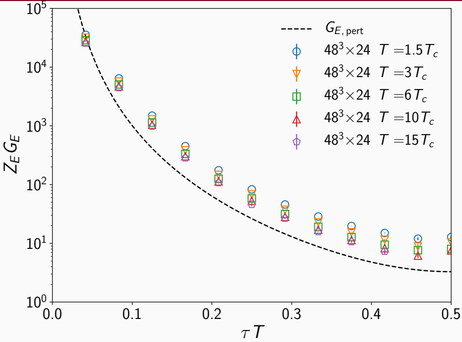
(Christensen and Laine PLB02 (2016))

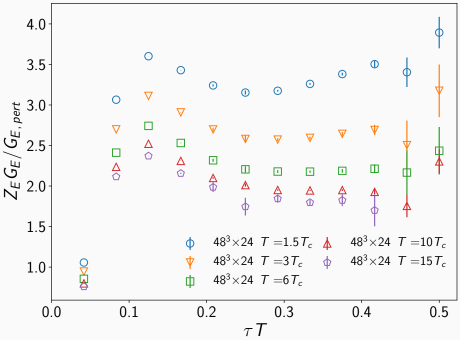
Lattice parameters

- Most simulations with spatial lattice size 48^3
- Vary the temporal size from 12 to 24 to perform continuum limit
- For multilevel 4 sub-lattices with 2000 updates
- Temperatures between $(1.1\,T_c)$ $1.5\,T_c - 10\,T_c$ $(15\,T_c)$
- Set β for each N_t , T with (Francis *et.al.*PRD91 (2015))
- Previous similar studies (Meyer NJP13 (2011), Ding *et.al.*JPG38 (2011), Banarjee *et.al.*PRD85 (2012), Francis *et.al.*PRD92 (2015))

T/T_c	$N_t \times N_s^3$	β	N_{conf}
1.1	12×48^3	6.407	225
	16×48^3	6.621	310
	20×48^3	6.795	230
1.5	12×48^3	6.639	1126
	16×48^3	6.872	1567
	20×48^3	7.044	466
	24×48^3	7.192	2316
3	12×48^3	7.193	450
	16×48^3	7.432	310
	24×48^3	7.774	576
6	12×48^3	7.774	225
	16×48^3	8.019	313
	24×48^3	8.367	525
10	12×48^3	8.211	1126
	16×48^3	8.458	1525
	20×48^3	8.651	963
	24×48^3	8.808	1790
15	24×48^3	9.159	52

Lattice correlator



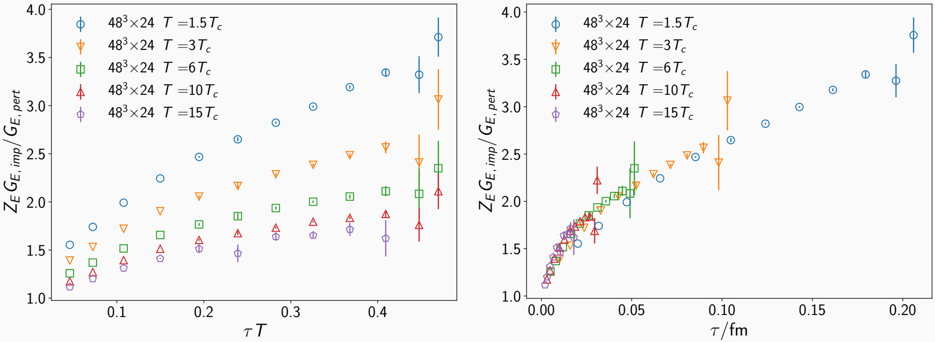


Two possible ways to normalize the data

$$G_{E,\,\mathrm{pert}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\right]$$

$$G_{E,\,\mathrm{norm}} = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{g^2(\mu_\omega) C_F \omega^3}{6\pi} \frac{\frac{\omega}{T}\cosh\left(\tau T - \frac{1}{2}\right)}{\sinh\frac{\omega}{2T}}$$

Tree level improvement

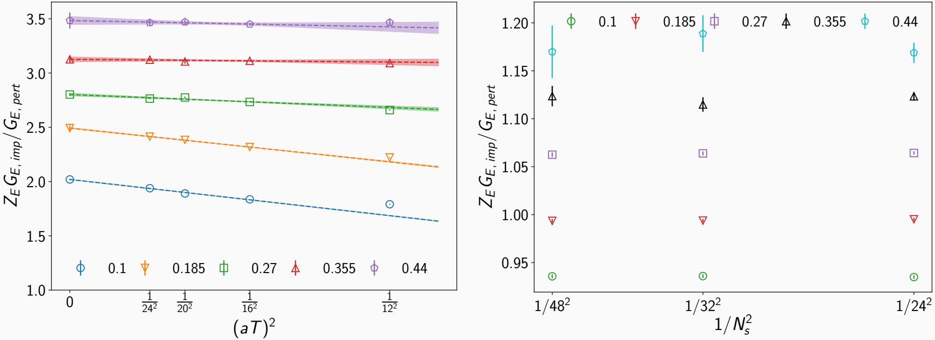


$$G_{E,\,\mathrm{LOlat}} = \frac{1}{3a^4}\int_{-\pi}^{\pi}\frac{\mathrm{d}^3 q}{(2\pi)^3}\,\frac{e^{\bar{q}N_\tau(1-\tau T)}+e^{\bar{q}N_\tau \tau T}}{e^{\bar{q}N_\tau}-1}\,\frac{\tilde{q}^2}{\sinh\bar{q}},$$

$$\bar{q} = 2\mathrm{arsinh}\left(\frac{\tilde{q}}{2}\right), \quad \tilde{q}^2 = \sum_{i=1}^{3} 4\sin^2\left(\frac{q_i}{2}\right)$$

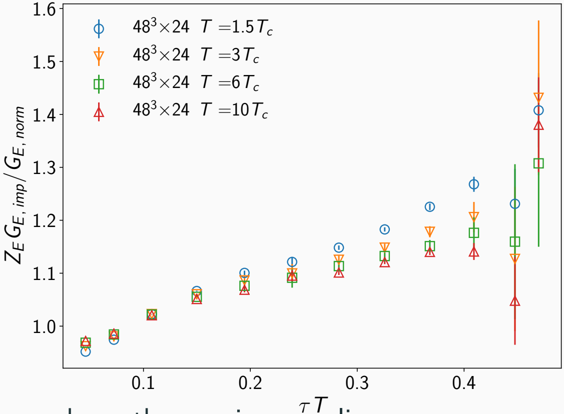
$$G_{E,\,\mathrm{pert}}(\bar{\tau}\bar{T}) = G_{E,\,\mathrm{LOlat}}(\tau T)$$

Continuum limit and finite size effects



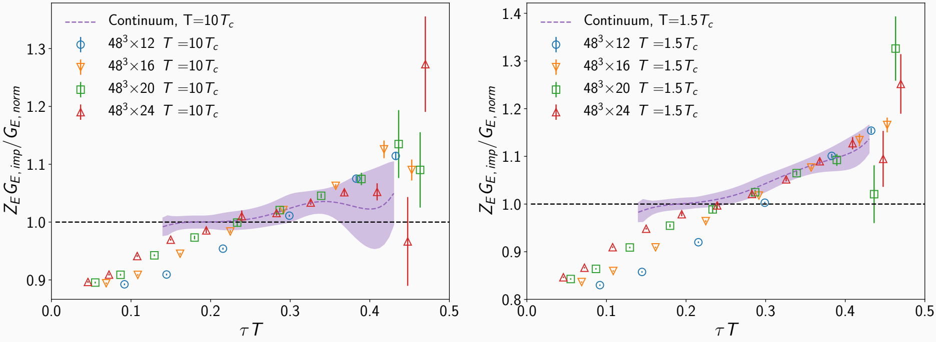
- Use 3 largest lattices for continuum limit
- Check systematics by including the $N_t = 12$ point
- Preliminary: finite size effects seem to be reasonably small

g^2 effects



- $G_{E\,,\mathrm{norm}}$ depends on the running coupling
- What part of the data is running coupling and what thermal effects
- How to set the scale $\mu_\omega \sim c\omega$, c affects absolute value of g^2
- Divide data with running coupling dependent G_E
- assume ρ_{LO} for $\omega > \pi T$, 0 otherwise

g^2 divided continuum limit



$T = 10T_c$ $\qquad$ $T = 1.5T_c$

- Normalize data to 1 at small $\tau T \sim 0.19$
- At smaller τT continuum limit not in control
- At large τT, needs bigger lattices, more statistics
- Limit includes a preliminary estimate of systematic errors

Spectral function

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho(\omega) \frac{\frac{\omega}{T} \cosh\left(\tau T - \frac{1}{2}\right)}{\sinh \frac{\omega}{2T}}$$

$$\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega) \,, \qquad \gamma = -\frac{1}{3N_c} \int_0^\infty \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega}$$

- Assume simple behavior on IR ($\omega \ll T$):

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

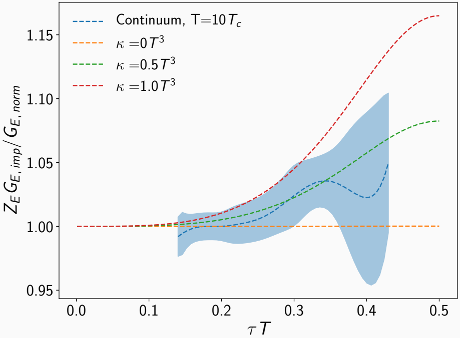
- Perturbative behavior in UV ($\omega \gg T$):

$$\rho^{\mathrm{LO}}_{\mathrm{UV}}(\omega) = \frac{g^2(\mu_\omega) C_F \omega^3}{6\pi} \,, \qquad \rho^{\mathrm{NLO}}_{\mathrm{UV}} \text{ from (Burnier \textit{et.al.}JHEP08 (2010))}$$

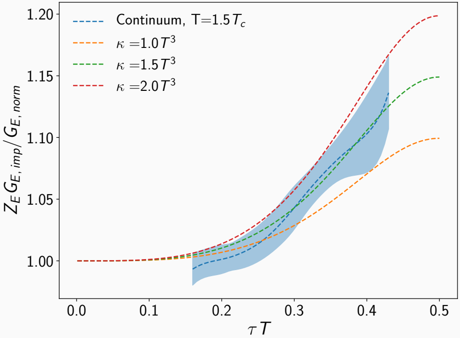
$$\mu_\omega = \max(\omega, \pi T) \qquad \text{or from EQCD}$$

- Use 5-loop running for the coupling

Simple comparison



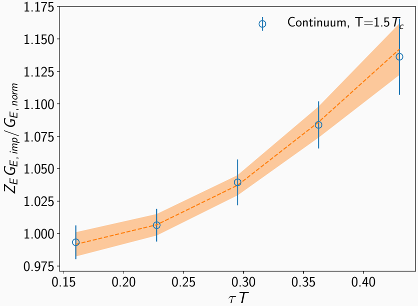
$T = 10\,T_c$

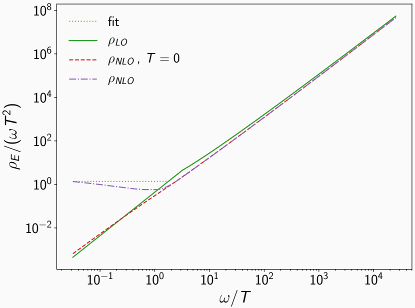


$T = 1.5\,T_c$

- Assume $\rho = \rho_{\mathrm{UV}}^{\mathrm{LO}}$ when $\omega > \pi T$ and $\rho = \rho_{\mathrm{IR}}$ when $\omega < \pi T$

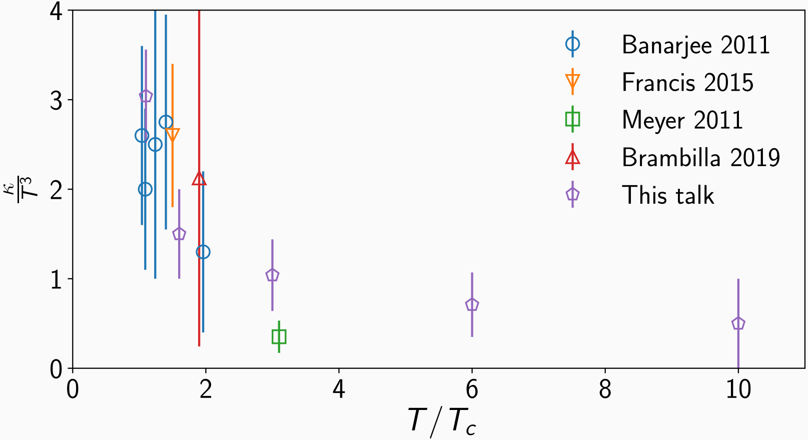
Preliminary fits





- Simple ansatz : $\max(\rho_{\mathrm{IR}}, c\rho_{\mathrm{UV}}^{\mathrm{LO}})$ (Francis *et.al.*PRDD92 (2015), Banerjee *et.al.*PRD85 (2011))
- For $T = 1.5$ we get 1.37 ± 0.49 (Preliminary)
- Other fit forms need to be tried to properly understand the systematics

Comparison to existing results



- The analysis still needs to be refined
  (Preliminary)

Conclusions and Future prospects

- We observe that κ decreases as Temperature increases
- The analysis still need refinement, but the initial results are promising
- Is it possible to extract κ un-quenched using the lattice euclidean field correlator?
    - Gradient flow?
- Is it possible to measure γ from lattice?

Thank You