Boltzmann Transport and PNRQCD for Quarkonium Production in Heavy Ion Collisions

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arXiv:1709.03529, XY B.Müller

arXiv:1807.06199, XY W.Ke Y.Xu S.Bass B.Müller

arXiv:1811.07027, XY T.Mehen

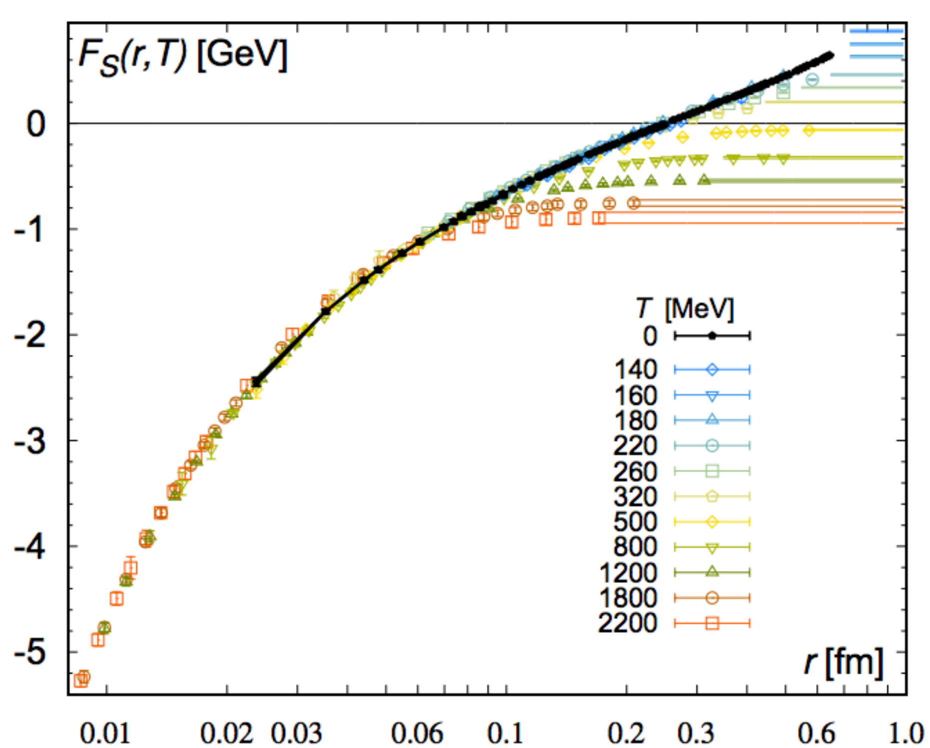
arXiv:1811.09644, XY B.Müller

Introduction: Quarkonium in Heavy Ion

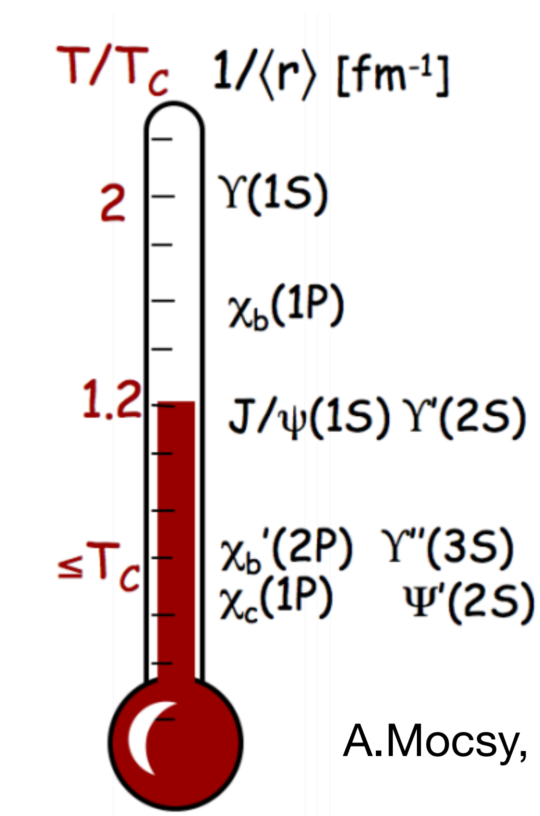
- Heavy ion collisions: quark-gluon plasma, deconfined phase, strongly-coupled fluid with small shear viscosity

- Static plasma screening: real part of attractive potential suppressed —> melting of quarkonium at high temperature —> probe QGP

$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 : \text{Confining part flattened}$$



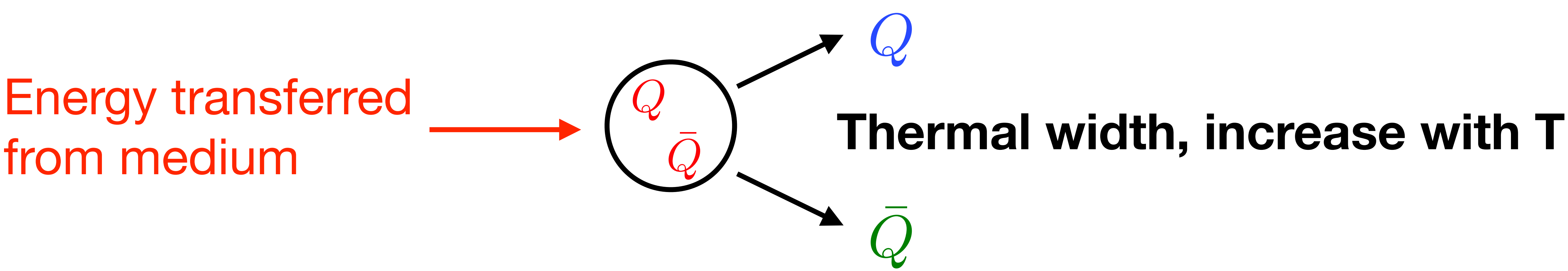
arXiv:1804.10600, A.Bazavov, N.Brambilla P.Petreczky, A.Vairo, J.H.Weber



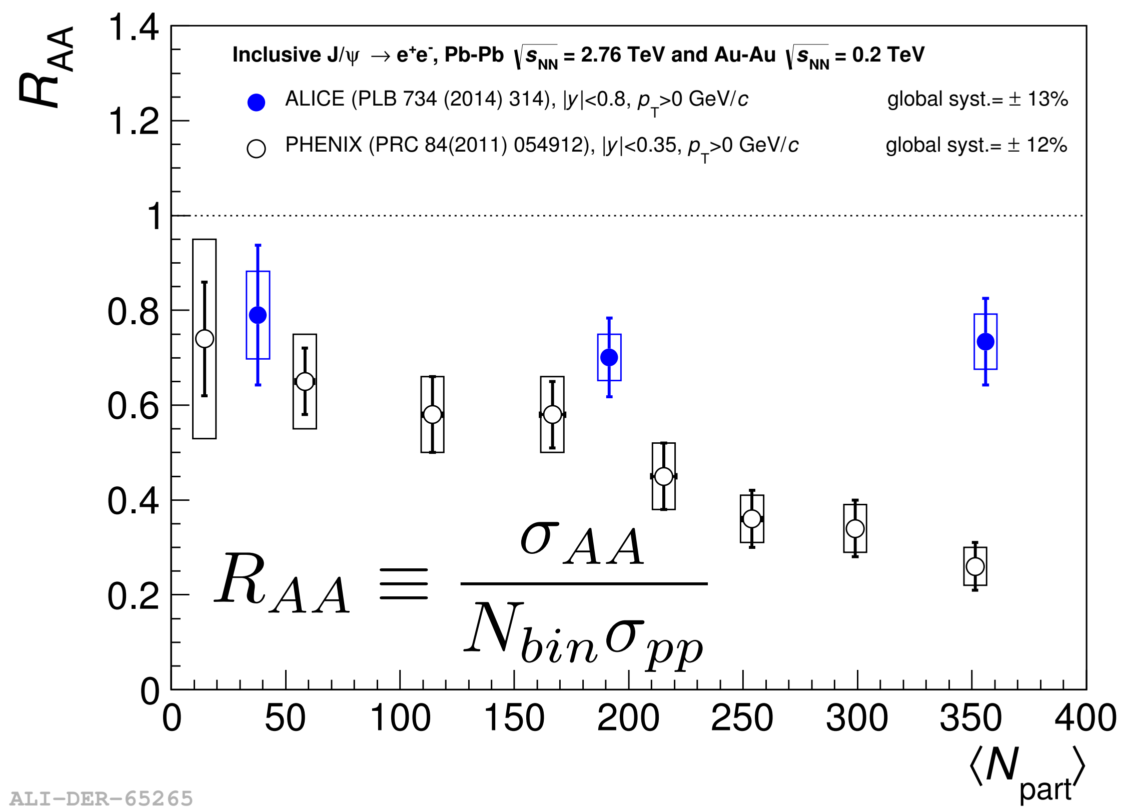
A.Mocsy, arXiv:0811.0337

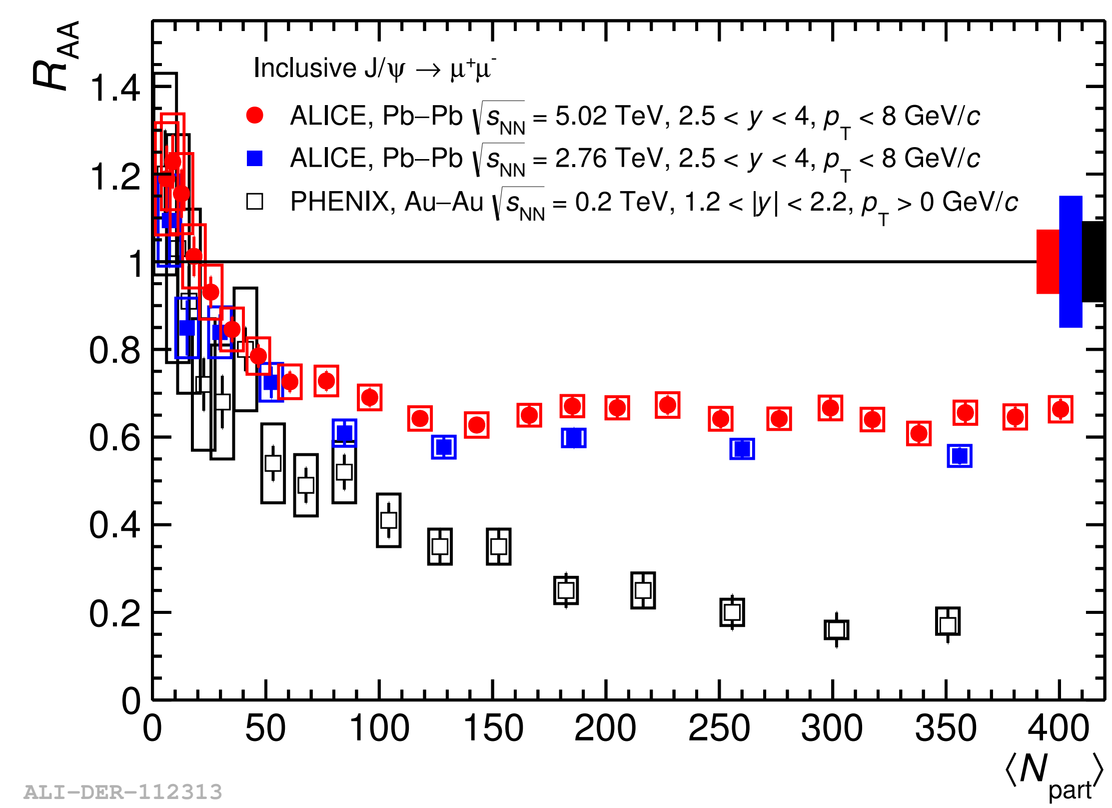
Introduction: Quarkonium inside QGP

- Dynamical plasma screening: dissociation induced by scattering (imaginary part of potential)



- Recombination: J/psi less suppressed at LHC than RHIC, can happen inside QGP below melting temperature





Introduction: Quarkonium Transport inside QGP

- Transport equations: $$\frac{dN}{d\tau} = -\Gamma(T)N + \alpha(\tau)\Gamma(T)N^{\mathrm{eq}}(T)$$

  Dissociation rate calculated from QCD

  Recombination modeled: detailed balance, phenomenological factor

  Treatment of dissociation and recombination not in the same theoretical framework

  Cannot explain how the system approaches detailed balance and thermalization

  Recombination should depend on real-time distributions of open heavy flavors

  Relation to the underlying quantum evolution?

Contents

- Open quantum system: Lindblad equation
- Derivation of Boltzmann transport equation from Lindblad equation
    - Separation of scales
    - Effective field theory of QCD: potential NRQCD
- Phenomenological studies
- Conclusion

Open Quantum System

$$H = H_S + H_E + H_I$$

$$\rho(t=0) = \rho_S \otimes \rho_E$$

System & environment

(Heavy quark pairs & QGP)

Unitary evolution

Time reversible

$$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$$

System & environment

Trace out (integrate out) environment

$$\rho_S(t=0)$$

System(heavy quark pairs)

Non-unitary

Time irreversible

$$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$$

Evolution equation not von Neumann, but Lindblad

Monotonicity of relative entropy under partial trace

$$S(\rho||\sigma) \equiv \mathrm{Tr}(\rho \ln \rho) - \mathrm{Tr}(\rho \ln \sigma)$$

$$S(\rho^{(1)}||\sigma^{(1)}) \leq S(\rho||\sigma)$$

$$\rho^{(1)} \equiv \mathrm{Tr}^{(2)}\rho \qquad \rho = \rho^{(1)} \otimes \rho^{(2)}$$

From Open Quantum System to Transport Equation

Lindblad equation:

$$H_I = \sum_\alpha O_\alpha^{(S)} \otimes O_\alpha^{(E)}$$

Trace perserving

$$\rho_S(t) = \rho_S(0) - i\Big[H_S + \sum_{ab} \sigma_{ab}(t) L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d} \gamma_{ab,cd}(t)\Big(L_{ab}\rho_S(0)L_{cd}^\dagger - \frac{1}{2}\{L_{cd}^\dagger L_{ab}, \rho_S(0)\}\Big)$$

$$\gamma_{ab,cd}(t) \equiv \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1,t_2)\langle a|O_\beta^{(S)}(t_2)|b\rangle\langle c|O_\alpha^{(S)}(t_1)|d\rangle^*$$

$$\sigma_{ab}(t) \equiv \frac{-i}{2}\sum_{\alpha,\beta}\int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1,t_2)\mathrm{sign}(t_1 - t_2)\langle a|O_\alpha^{(S)}(t_1)O_\beta^{(S)}(t_2)|b\rangle$$

$$C_{\alpha\beta}(t_1,t_2) \equiv \mathrm{Tr}_E(O_\alpha^{(E)}(t_1)O_\beta^{(E)}(t_2)\rho_E)$$

$|a\rangle$ Eigenstates of H_S

$$L_{ab} = |a\rangle\langle b|$$

Boltzmann transport equation

$$\frac{\partial}{\partial t} f_{nls}(\boldsymbol{x},\boldsymbol{k},t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}_{nls}^{(+)}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}_{nls}^{(-)}(\boldsymbol{x},\boldsymbol{k},t)$$

From Open Quantum System to Transport Equation

Lindblad equation: **Correction to Hamiltonian**

$$\rho_S(t) = \rho_S(0) - i\Big[H_S + \sum_{ab}\sigma_{ab}(t)L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d}\gamma_{ab,cd}(t)\Big(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S(0)\}\Big)$$

Markovian approximation

Wigner transform

Recombination

Dissociation

Boltzmann transport equation

$$\frac{\partial}{\partial t}f_{nls}(\boldsymbol{x},\boldsymbol{k},t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}f_{nls}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}^{(+)}_{nls}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}^{(-)}_{nls}(\boldsymbol{x},\boldsymbol{k},t)$$

arXiv:1811.07027, XY T.Mehen

Two Key Assumptions

1. **System interacts weakly with environment ?**

2. **Markovian assumption (no memory effect) ?**

Effective field theory and separation of scales

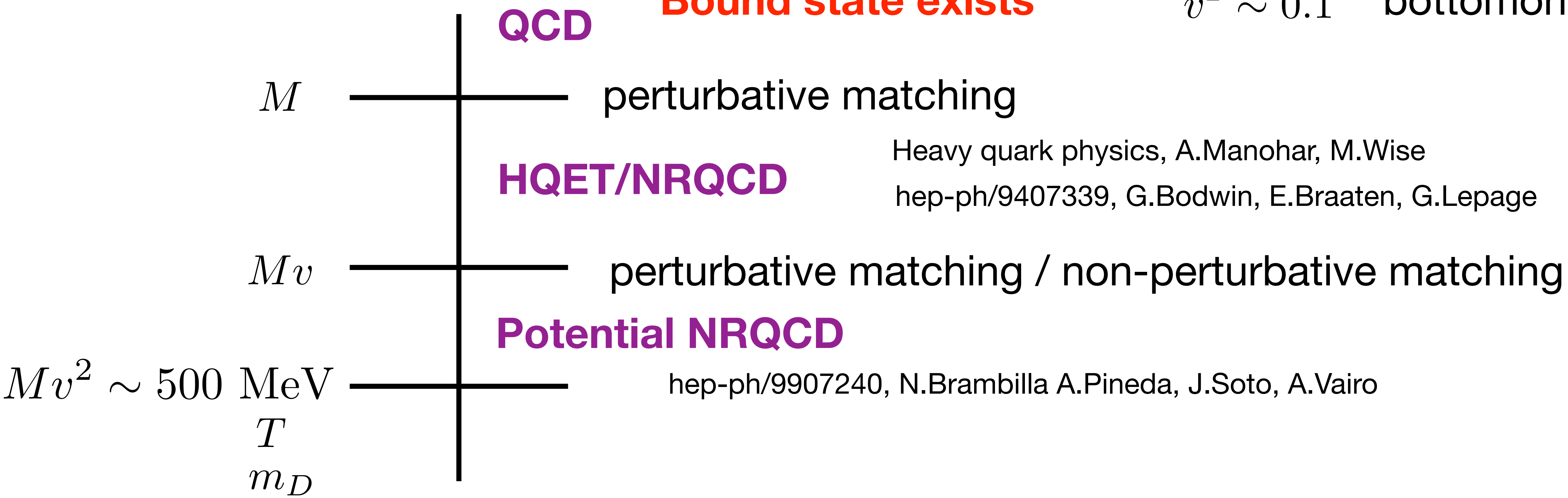
Potential NRQCD

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$

$v^2 \sim 0.3$ charmonium

$v^2 \sim 0.1$ bottomonium

Bound state exists



Heavy quark physics, A.Manohar, M.Wise

hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

hep-ph/9907240, N.Brambilla A.Pineda, J.Soto, A.Vairo

NR & multipole expansions up to linear order of r $\mathrm{S}(\boldsymbol{R},\boldsymbol{r},t)$ $\mathrm{O}(\boldsymbol{R},\boldsymbol{r},t)$

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r\,\mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \text{h.c.}) + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E},\mathrm{O}\} + \cdots\Big)$$

$$H_s = \frac{(i\boldsymbol{\nabla}_{\text{cm}})^2}{4M} + \frac{(i\boldsymbol{\nabla}_{\text{rel}})^2}{M} + V_s^{(0)} + \frac{V_s^{(1)}}{M} + \frac{V_s^{(2)}}{M^2} + \cdots \longrightarrow H_{s,o} = \frac{(i\boldsymbol{\nabla}_{\text{rel}})^2}{M} + V_{s,o}^{(0)}$$

- **In quarkonium c.m. frame, c.m. energy suppressed by at least one power of v when** $v_{\text{med}} \lesssim \sqrt{1-v}$

- **Virial theorem**
- **No hyperfine splitting**

Potential NRQCD

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\Big({\rm S}^\dagger (i\partial_0 - H_s){\rm S} + {\rm O}^\dagger (iD_0 - H_o){\rm O} + \boxed{V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.})} + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots \Big)$$

Dipole interaction $r \sim \dfrac{1}{Mv}$

Weak coupling between quarkonium and QGP: quarkonium small in size

$rMv^2 \sim rT \sim rm_D \sim v$ suppressed

Potential NRQCD

Separation of scales $M \gg Mv \gg Mv^2 \sim T \gtrsim m_D$

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \, \mathrm{Tr}\Big(\mathrm{S}^\dagger (i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger (iD_0 - H_o)\mathrm{O} + V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.}) + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots \Big)$$

Dipole interaction $r \sim \frac{1}{Mv}$

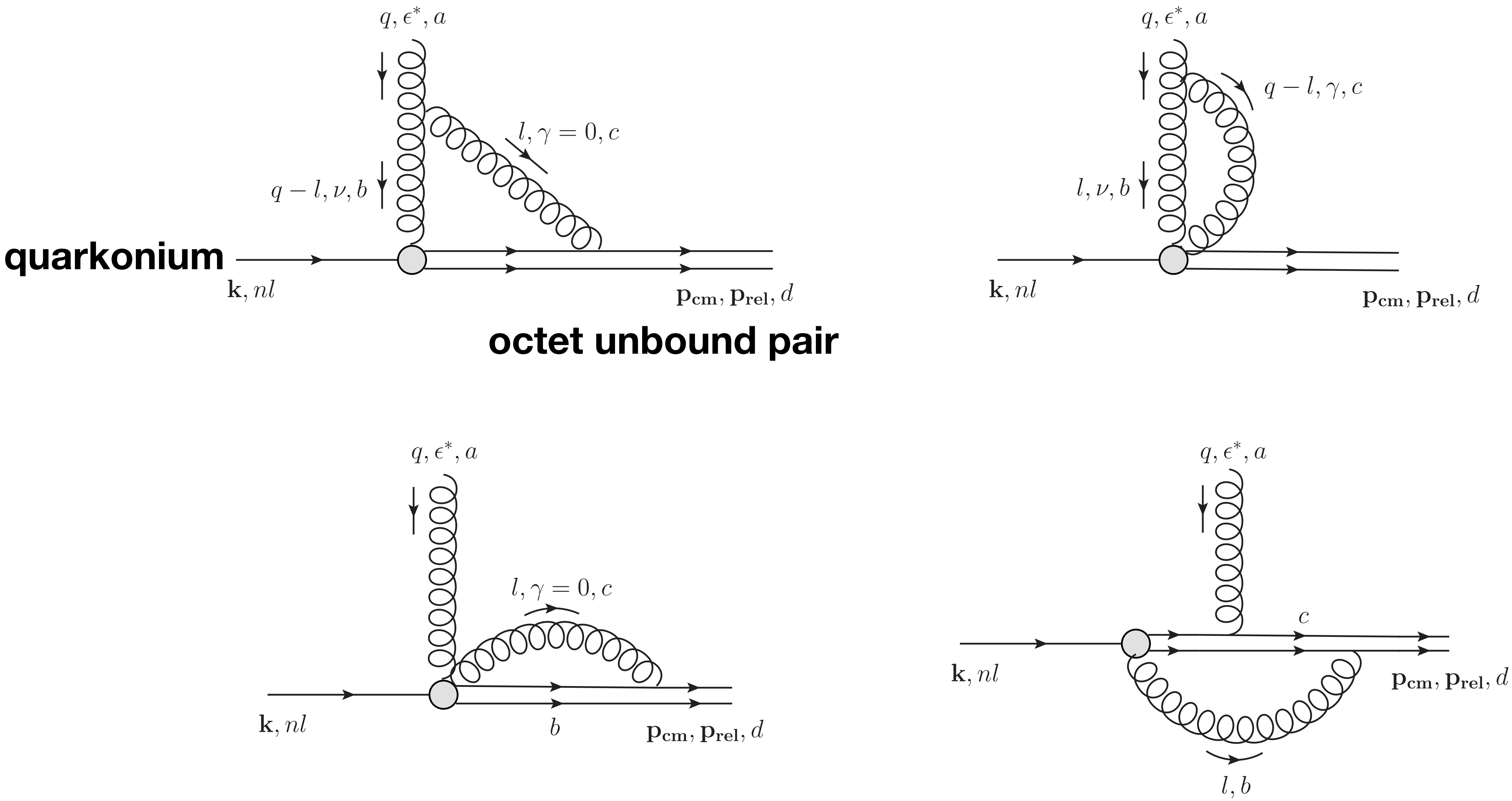
Weak coupling between quarkonium and QGP: quarkonium small in size

$rMv^2 \sim rT \sim rm_D \sim v$ suppressed

Perturbative matching gives $V_A(\mu = Mv) = 1$

Running? large log? no at one loop

Running of Dipole Interaction



$$\frac{0}{\epsilon} + \cdots \qquad \frac{d}{d\mu} V_A(\mu) = 0$$

Mapping Operators

In general theory

$$H_I = \sum_\alpha O_\alpha^{(S)} \otimes O_\alpha^{(E)} \qquad L_{ab} \equiv |a\rangle\langle b|$$

$$\rho_S(t) = \rho_S(0) - i\Big[H_S + \sum_{ab} \sigma_{ab}(t) L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d} \gamma_{ab,cd}(t)\Big(L_{ab}\rho_S(0)L_{cd}^\dagger - \frac{1}{2}\{L_{cd}^\dagger L_{ab}, \rho_S(0)\}\Big)$$

In pNRQCD

$$O_\alpha^{(S)} \to \langle S(\boldsymbol{R},t)|r_i|O^a(\boldsymbol{R},t)\rangle + \langle O^a(\boldsymbol{R},t)|r_i|S(\boldsymbol{R},t)\rangle$$

$$O_\alpha^{(E)} \to \sqrt{\frac{T_F}{N_C}} g E_i^a(\boldsymbol{R},t) \qquad \sum_\alpha \to \int d^3R \sum_i \sum_a$$

Complete set of states $|a\rangle$

$$|\boldsymbol{k}, nl, 1\rangle = a_{nl}^\dagger(\boldsymbol{k})|0\rangle \qquad |\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, 1\rangle = b_{\boldsymbol{p}_{\rm rel}}^\dagger(\boldsymbol{p}_{\rm cm})|0\rangle \qquad |\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a\rangle = c_{\boldsymbol{p}_{\rm rel}}^{a\dagger}(\boldsymbol{p}_{\rm cm})|0\rangle$$

Bound singlet | **Unbound singlet** | **Unbound octet**

Wigner transform —> formulation in phase space, focus on bound state

$$f_{nl}(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(t)|\boldsymbol{k}-\frac{\boldsymbol{k}'}{2}, nl, 1\rangle$$

Need to calculate $\boxed{\langle \boldsymbol{k}_1, n_1 l_1, 1|\rho_S(t)|\boldsymbol{k}_2, n_2 l_2, 1\rangle}$

Dissociation $-\gamma_{ab,cd}\frac{1}{2}\{L^\dagger_{cd}L_{ab},\rho_S(0)\}$

For 1st term: $|d\rangle = |\boldsymbol{k}_1, n_1 l_1, 1\rangle$ $\quad |a\rangle = |c\rangle = |\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1\rangle$ $\quad |b\rangle = |\boldsymbol{k}_3, n_3 l_3, 1\rangle$

Linear order in r : transition between bound singlet & unbound octet

$$\gamma_{ab,cd} = \int d^3R_1 \int d^3R_2 \sum_{i_1,i_2,b_1,b_2} \int_0^t dt_1 \int_0^t dt_2\, C_{\boldsymbol{R}_1 i_1 b_1, \boldsymbol{R}_2 i_2 b_2}(t_1,t_2)$$
$$\langle \boldsymbol{k}_1, n_1 l_1, 1|\langle S(\boldsymbol{R}_1,t_1)|r_{i_1}|O^{b_1}(\boldsymbol{R}_1,t_1)\rangle|\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1\rangle$$
$$\langle \boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1|\langle O^{b_2}(\boldsymbol{R}_2,t_2)|r_{i_2}|S(\boldsymbol{R}_2,t_2)\rangle|\boldsymbol{k}_3, n_3 l_3, 1\rangle$$

Dissociation $-\gamma_{ab,cd}\frac{1}{2}\{L^\dagger_{cd}L_{ab},\rho_S(0)\}$

For 1st term: $|d\rangle = |\boldsymbol{k}_1, n_1 l_1, 1\rangle$ $|a\rangle = |c\rangle = |\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1\rangle$ $|b\rangle = |\boldsymbol{k}_3, n_3 l_3, 1\rangle$

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$$\langle \boldsymbol{k}_1, n_1 l_1, 1|\langle S(\boldsymbol{R}_1,t_1)|r_{i_1}|O^{b_1}(\boldsymbol{R}_1,t_1)\rangle|\boldsymbol{p}_{\rm cm},\boldsymbol{p}_{\rm rel},a_1\rangle$$

$$\langle \boldsymbol{p}_{\rm cm},\boldsymbol{p}_{\rm rel},a_1|\langle O^{b_2}(\boldsymbol{R}_2,t_2)|r_{i_2}|S(\boldsymbol{R}_2,t_2)\rangle|\boldsymbol{k}_3, n_3 l_3, 1\rangle$$

$$\langle \Psi_{\boldsymbol{p}_{\rm rel}}|r_{i_2}|\psi_{n_3 l_3}\rangle \delta^{a_1 b_2} e^{-i(E_{\boldsymbol{k}_3}t_2 - \boldsymbol{k}_3\cdot \boldsymbol{R}_2)} e^{i(E_{\boldsymbol{p}}t_2 - \boldsymbol{p}_{\rm cm}\cdot \boldsymbol{R}_2)}$$

$$\frac{T_F}{N_C} g^2 \langle E^{b_1}_{i_1}(\boldsymbol{R}_1,t_1) E^{b_2}_{i_2}(\boldsymbol{R}_2,t_2)\rangle_T$$

$$= \frac{T_F}{N_C} g^2 \delta^{b_1 b_2} \int \frac{d^4q}{(2\pi)^4} e^{iq_0(t_1-t_2) - i\boldsymbol{q}\cdot(\boldsymbol{R}_1-\boldsymbol{R}_2)} (q_0^2 \delta_{i_1 i_2} - q_{i_1} q_{i_2}) n_B(q_0)(2\pi)\mathrm{sign}(q_0)\delta(q_0^2 - \boldsymbol{q}^2)$$

Used < thermal correlator, can also use > because of $1 + n_B(q_0) + n_B(-q_0) = 0$

Sign fixed by energy conservation

Dissociation $-\gamma_{ab,cd}\frac{1}{2}\{L^\dagger_{cd}L_{ab},\rho_S(0)\}$

Markovian approximation: $t \longrightarrow \infty$ when doing time integral

Valid when environment correlation time << system relaxation time

T^{-1} dissociation frequency

correlation scale T rate $(grT)^2T \sim T\frac{\alpha_s T^2}{(Mv)^2} \lesssim \alpha_s v^2 T$

Putting everything together, make Wigner transform:

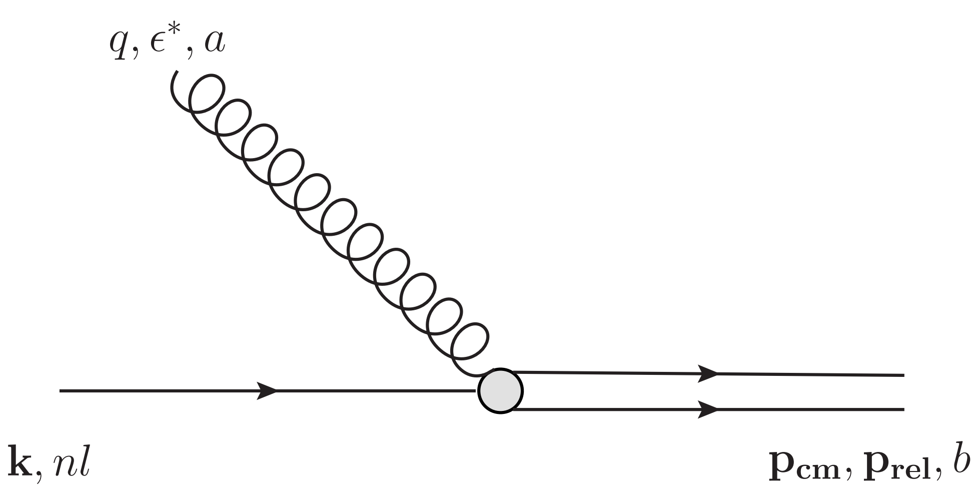
Spatial & time integrals give delta functions (E&p conservation)

E&p conservation

$$t\int\frac{d^3p_{\rm cm}}{(2\pi)^3}\frac{d^3p_{\rm rel}}{(2\pi)^3}\frac{d^3q}{(2\pi)^3 2q}n_B(q)(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\rm cm}+\boldsymbol{q})\delta(E_k-E_p+q)$$

Phase space measure

$$\frac{2}{3}C_F q^2 g^2|\langle\psi_{nl}|\boldsymbol{r}|\Psi_{\boldsymbol{p}_{\rm rel}}\rangle|^2 f_{nl}(\boldsymbol{x},\boldsymbol{k},t=0)$$



Amplitude squared

For Coulomb potential and neglect octet repulsive potential, get Peskin-Bhanot result

Recombination $\gamma_{ab,cd} L_{ab} \rho_S(0) L^{\dagger}_{cd}$

Phase space measure **E&p conservation**

$$t \int \frac{d^3 p_{\rm cm}}{(2\pi)^3} \frac{d^3 p_{\rm rel}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} (1 + n_B(q)) \sum_{a,i} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\rm cm} + \boldsymbol{q}) \delta(-|E_{nl}| + q - \frac{\boldsymbol{p}_{\rm rel}^2}{M})$$

$$\frac{2T_F}{3N_C} q^2 g^2 \langle \psi_{nl} | r_i | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle \int d^3 r\, \psi_{nl}(\boldsymbol{r}) r_i \Psi^*_{\boldsymbol{p}_{\rm rel}}(\boldsymbol{r}) f_{Q\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}_{\rm cm}, \boldsymbol{r}, \boldsymbol{p}_{\rm rel}, a, t=0)$$

$p_{\rm rel} \sim a_B^{-1} \sim Mv$

Amplitude squared, mixed with distribution function hard to implement numerically

When can we take distribution function out?

Uniformly distributed when r < Bohr radius a_B $\sqrt{Dt} \gg a_B$

$$D \sim \frac{1}{\alpha_s^2 T} \qquad t \sim \frac{a_B}{v_{\rm rel}} \sim \frac{1}{p_{\rm rel} v} \qquad p_{\rm rel} \ll \frac{Mv}{\alpha_s^2 v^2}$$

Molecular chaos assumption $f_{Q\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}_{\rm cm}, \boldsymbol{r}, \boldsymbol{p}_{\rm rel}, a, t) = \frac{1}{9} f_Q(\boldsymbol{x}_1, \boldsymbol{p}_1, t) f_{\bar{Q}}(\boldsymbol{x}_2, \boldsymbol{p}_2, t)$

$$t \frac{1}{9} \int \frac{d^3 p_{\rm cm}}{(2\pi)^3} \frac{d^3 p_{\rm rel}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} (1 + n_B(q)) f_Q(\boldsymbol{x}_1, \boldsymbol{p}_1, t) f_{\bar{Q}}(\boldsymbol{x}_2, \boldsymbol{p}_2, t)$$

Spin! $g_s = \frac{3}{4}, \frac{1}{4}$

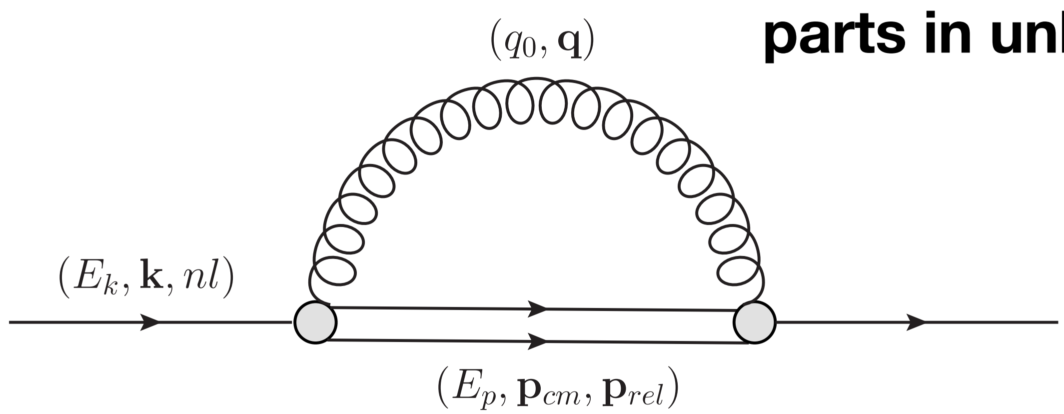
$$(2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\rm cm} + \boldsymbol{q}) \delta(-|E_{nl}| + q - \frac{\boldsymbol{p}_{\rm rel}^2}{M}) \frac{2}{3} C_F q^2 g^2 |\langle \psi_{nl} | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle|^2$$

Numerical implementation when |x1-x2| large

Correction of Potential $-i\sum_{ab}\sigma_{ab}(t)[L_{ab},\rho_S(0)]$

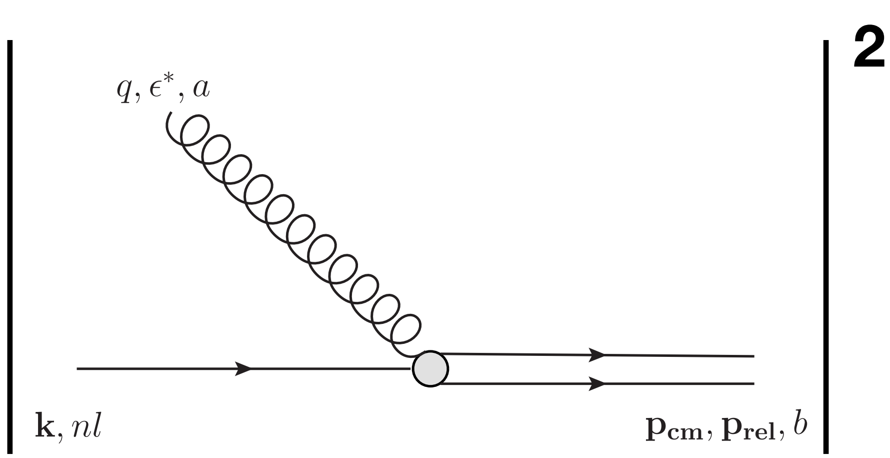
$$\sum_{a,b}\sigma_{ab}L_{ab}\to t\sum_{n,l}\int\frac{d^3k}{(2\pi)^3}\mathrm{Re}\Big\{-ig^2C_F\sum_{i_1,i_2}\int\frac{d^4q}{(2\pi)^4}\int\frac{d^4p_{\rm cm}}{(2\pi)^4}\int\frac{d^3p_{\rm rel}}{(2\pi)^3}$$
$$(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\rm cm}-\boldsymbol{q})\delta(E_k-p^0_{\rm cm}-q^0)$$
$$(q_0^2\delta_{i_1i_2}-q_{i_1}q_{i_2})\Big(\frac{i}{q_0^2-\boldsymbol{q}^2+i\epsilon}+n_B(|q_0|)(2\pi)\delta(q_0^2-\boldsymbol{q}^2)\Big)$$
$$\langle\psi_{nl}|r_{i_1}\frac{i|\Psi_{\boldsymbol{p}_{\rm rel}}\rangle\langle\Psi_{\boldsymbol{p}_{\rm rel}}|}{p^0_{\rm cm}-E_p+i\epsilon}r_{i_2}|\psi_{nl}\rangle\Big\}L_{|\boldsymbol{k},nl,1\rangle\langle\boldsymbol{k},nl,1|}$$

parts in unbound singlet / octet neglected

= Re 

$\theta(t)=(1+\mathrm{sign}(t))/2$

= Im —> dissociation rate



Boltzmann Transport Equation

Phase space free streaming

$$\rho_S(t) = \rho_S(0) - it(H_{eff}\rho_S(0) - \rho_S(0)H_{eff}) + \cdots$$

↓ Wigner transform

$$f_{nl}(\boldsymbol{x},\boldsymbol{k},t) = f_{nl}(\boldsymbol{x},\boldsymbol{k},0) - it\int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} (E_{\boldsymbol{k}+\frac{\boldsymbol{k}'}{2}} - E_{\boldsymbol{k}-\frac{\boldsymbol{k}'}{2}}) \langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(0)|\boldsymbol{k}-\frac{\boldsymbol{k}'}{2}, nl, 1\rangle + t\mathcal{C}^{(+)}_{nl} - t\mathcal{C}^{(-)}_{nl}$$

$$E_{\boldsymbol{k}\pm\frac{\boldsymbol{k}'}{2}} = -|E_{nl}| + \frac{(\boldsymbol{k}\pm\frac{\boldsymbol{k}'}{2})^2}{4M}$$

Add spin dependence —> transport equation:

$$\frac{\partial}{\partial t} f_{nls}(\boldsymbol{x},\boldsymbol{k},t) + \frac{\boldsymbol{k}}{2M}\cdot\nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}^{(+)}_{nls}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}^{(-)}_{nls}(\boldsymbol{x},\boldsymbol{k},t)$$

Open quantum system
Effective field theory: separation of scales

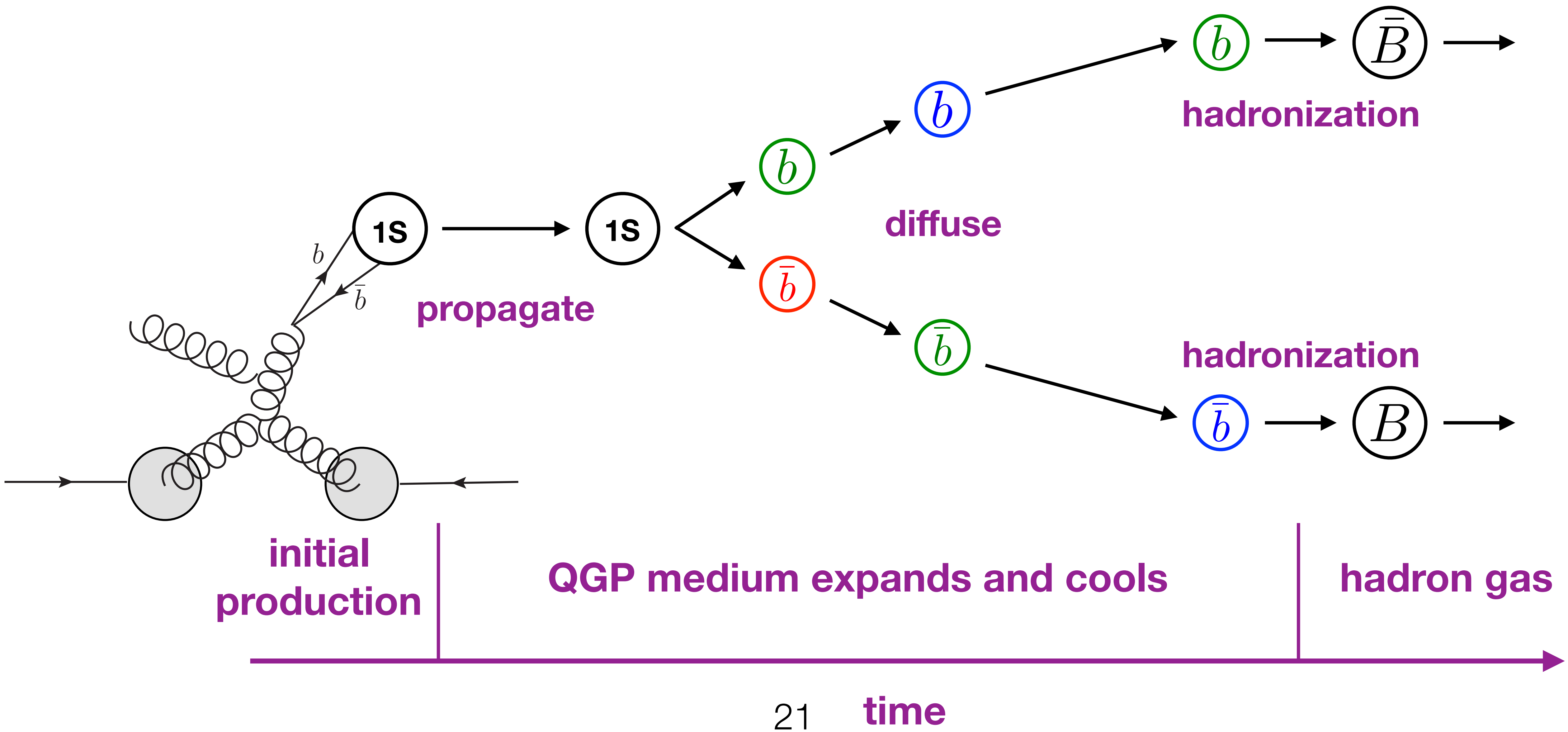
weak coupling between quarkonium & QGP
Markovian approximation

Coupled with Transports of Open Heavy Flavor

heavy quark $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_Q(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_Q - \mathcal{C}_Q^+ + \mathcal{C}_Q^-$$

anti-heavy quark $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_{\bar{Q}} - \mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^-$$

each quarkonium state
nl = 1S, 2S,1P etc. $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_{nls}^+ - \mathcal{C}_{nls}^-$$



Coupled with Transport of Open Heavy Flavor

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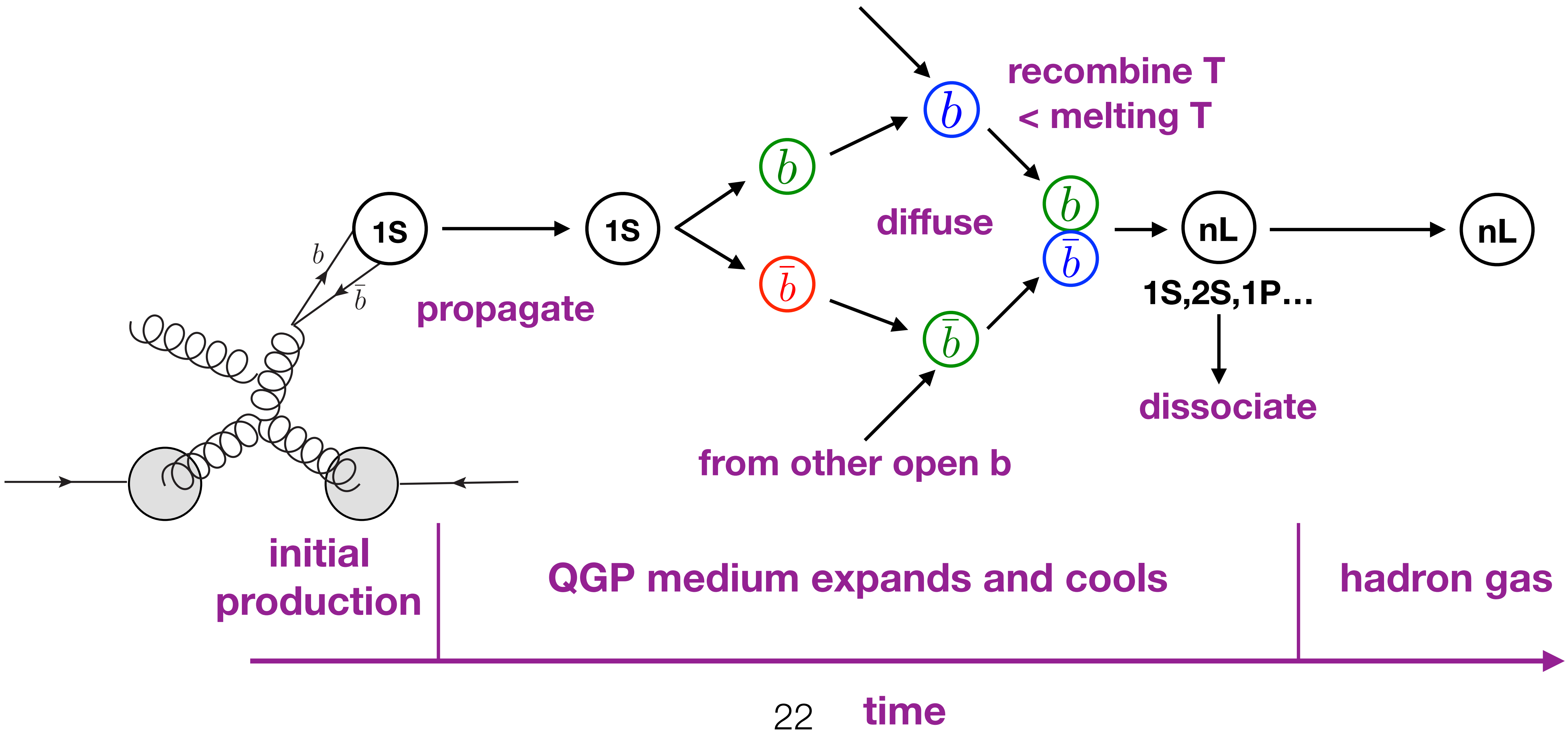
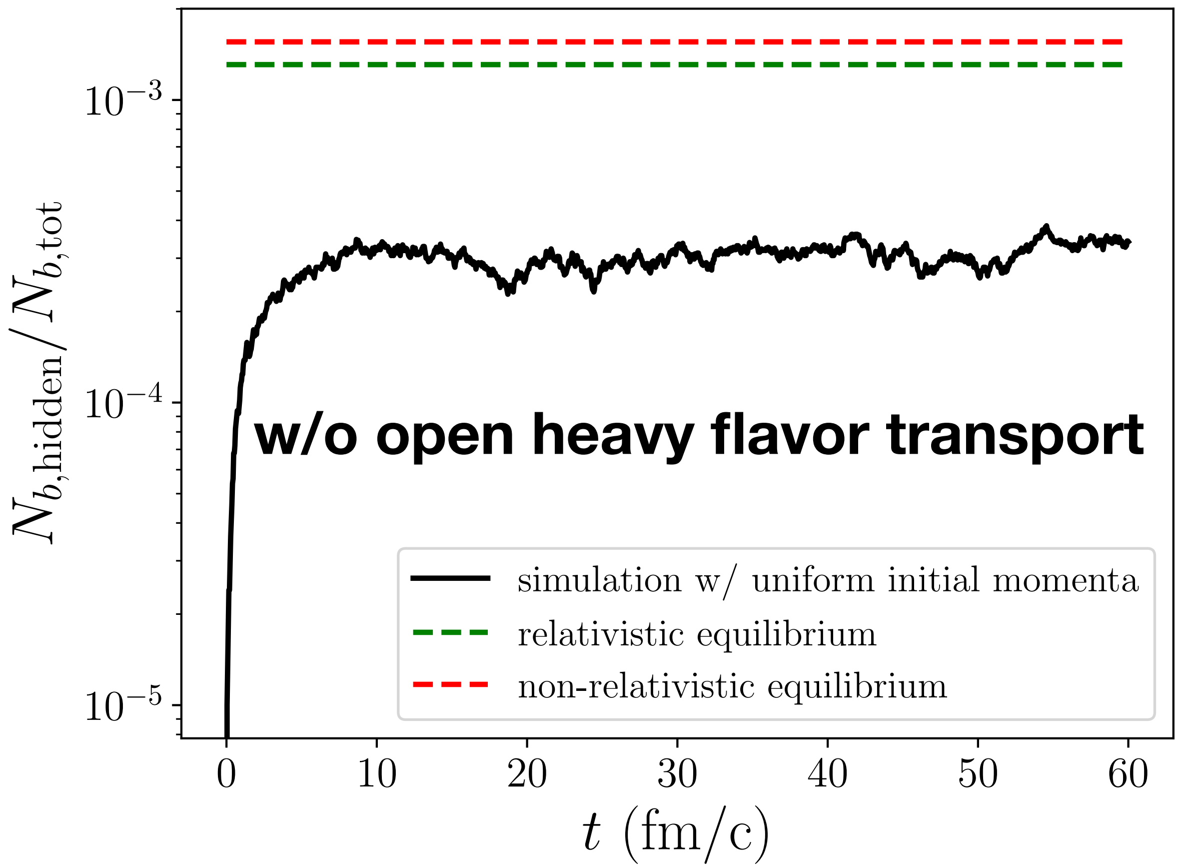
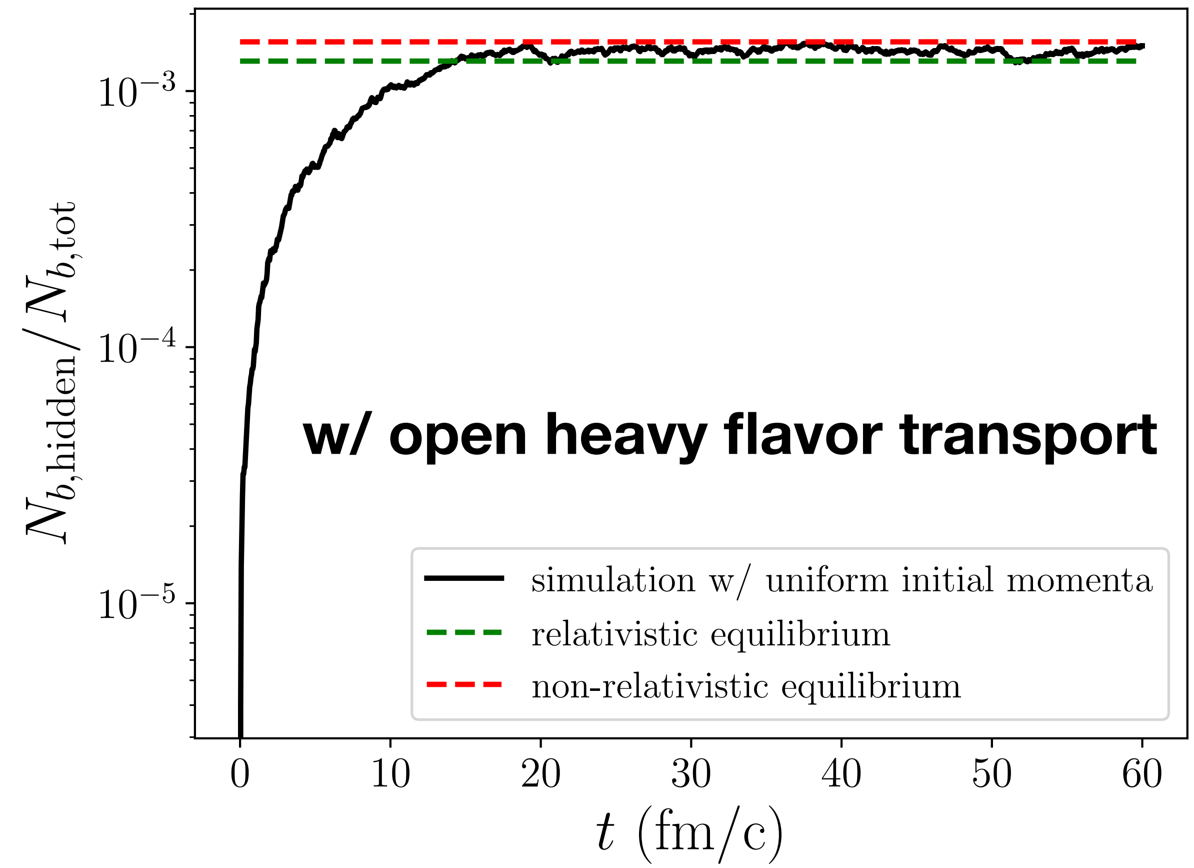
nl = 1S, 2S,1P etc. $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_{nls}^+ - \mathcal{C}_{nls}^-$$



Detailed Balance and Thermalization

Setup:
- QGP box w/ const T=300 MeV, 1S state & b quarks, total b flavor = 50 (fixed)
- Initial momenta sampled from uniform distributions 0-5 GeV
- Turn on/off open heavy quark transport





Dissociation-recombination interplay drives to detailed balance

Heavy quark energy gain/loss necessary to drive kinetic equilibrium of quarkonium

Collision Event Simulation

- Initial production:

  PYTHIA 8.2: NRQCD factorization — Sjostrand, et al, Comput. Phys.Commun.191 (2015) 159; Bodwin, Braaten, Lepage Phys. Rev. D 51, 1125 (1995)

  Nuclear PDF: EPS09 (cold nuclear matter effect) — Eskola, Paukkunen, Salgado, JHEP 0904 (2009) 065

  Trento, sample position, hydro. initial condition

  Moreland, Bernhard, Bass, Phys. Rev. C 92, no. 1, 011901 (2015)

- Medium background: 2+1D viscous hydrodynamics (calibrated)

  Song, Heinz, Phys.Rev.C77,064901(2008)
  Shen, Qiu, Song, Bernhard, Bass, Heinz, Comput. Phys. Commun.199,61 (2016)
  Bernhard, Moreland, Bass, Liu, Heinz, Phys. Rev. C 94,no.2,024907(2016)

- Study bottomonium (larger separation of scales); include 1S 2S; ~26% 2S feed-down to 1S in hadronic phase (from PDG); initial production ratio 1S : 2S ~ between 3:1 to 4:1 (PYTHIA)

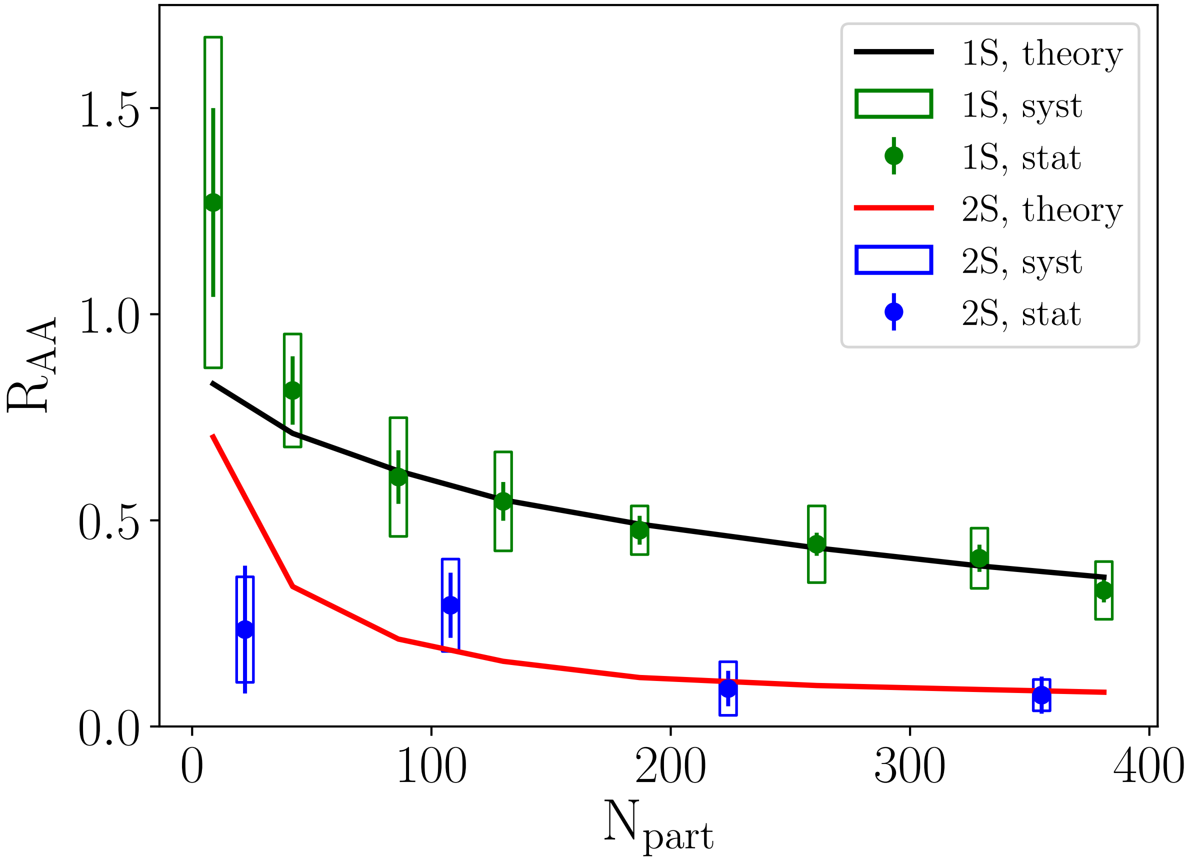
Upsilon in 2760 GeV PbPb Collision

Fix $\alpha_s = 0.3$

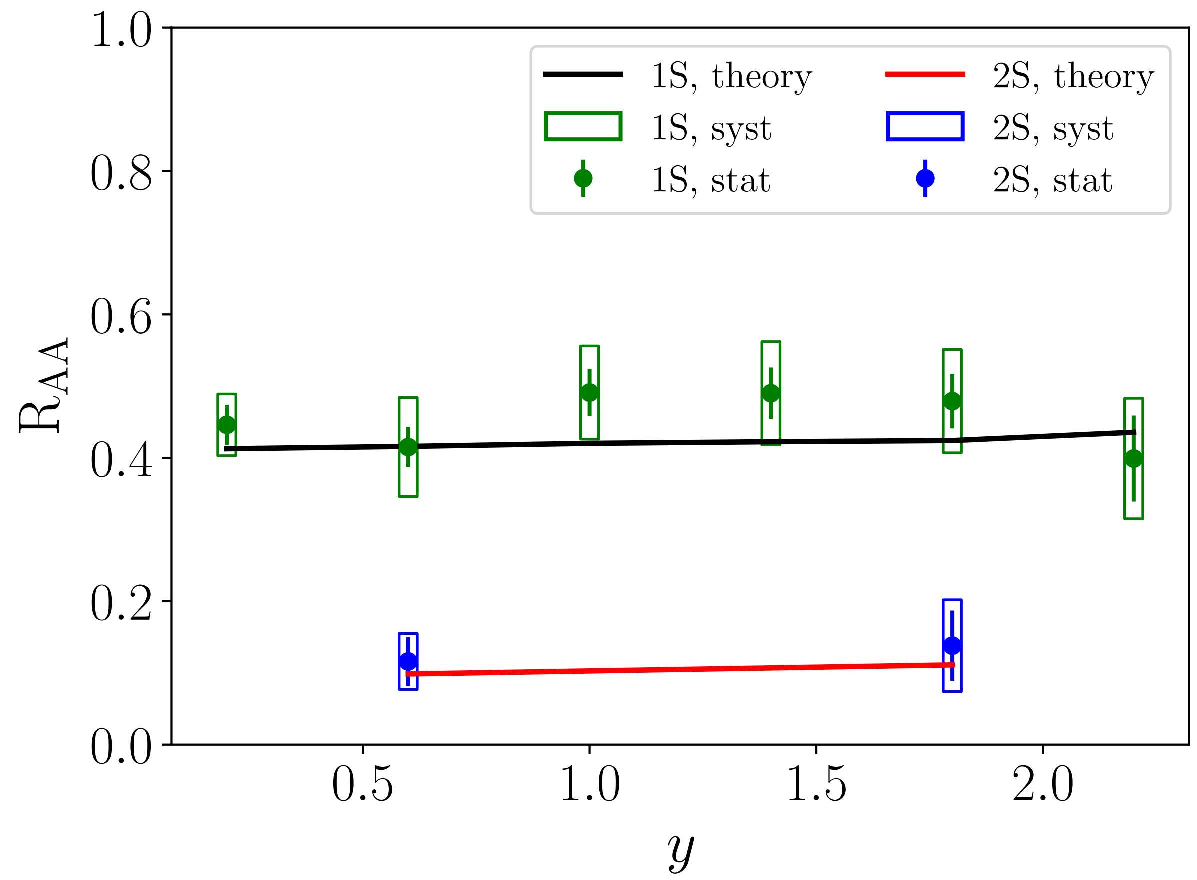
Tune $T_{\text{melt}}(2S) = 210 \text{ MeV}$

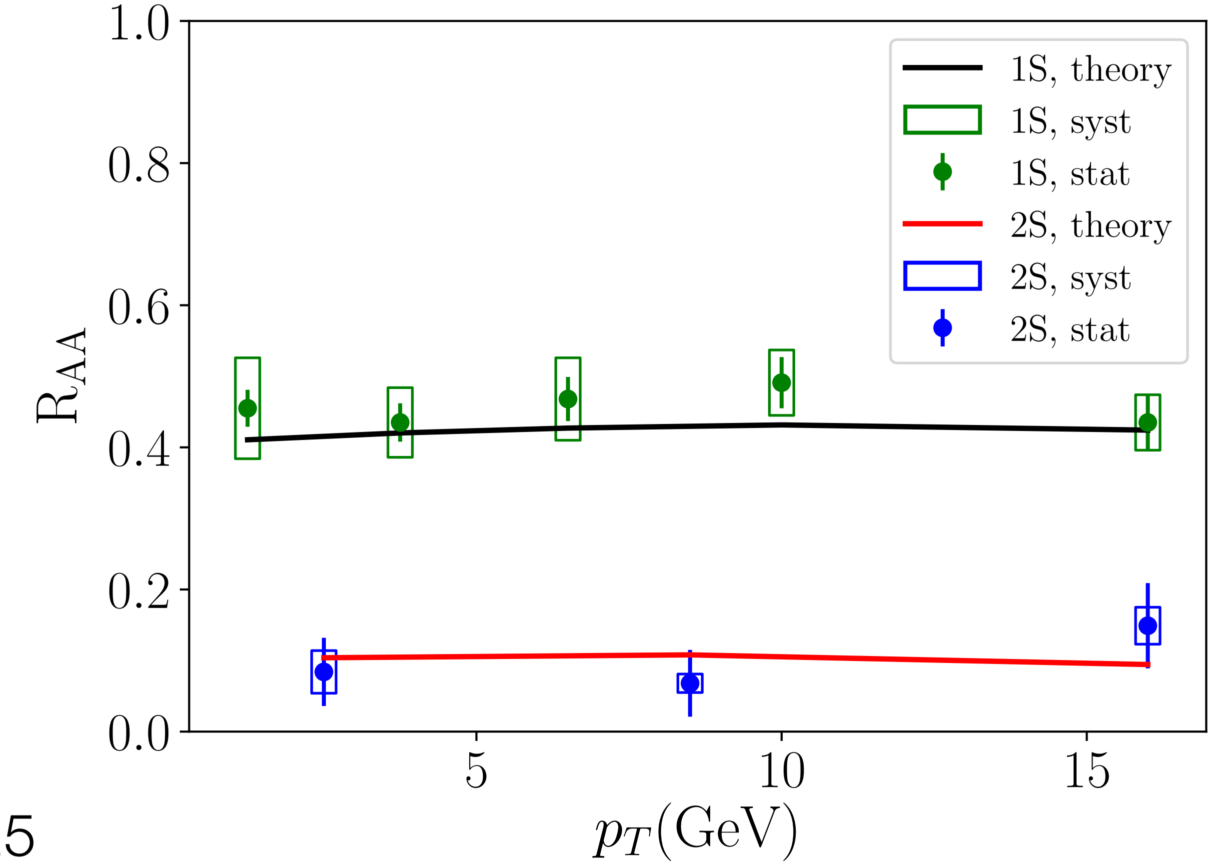
Tune $V_s = -C_F \dfrac{0.42}{r}$

Cold nuclear matter effect ~ 0.87 (PYTHIA + nPDF)



CMS Phys.Lett. B
770 (2017) 357-379

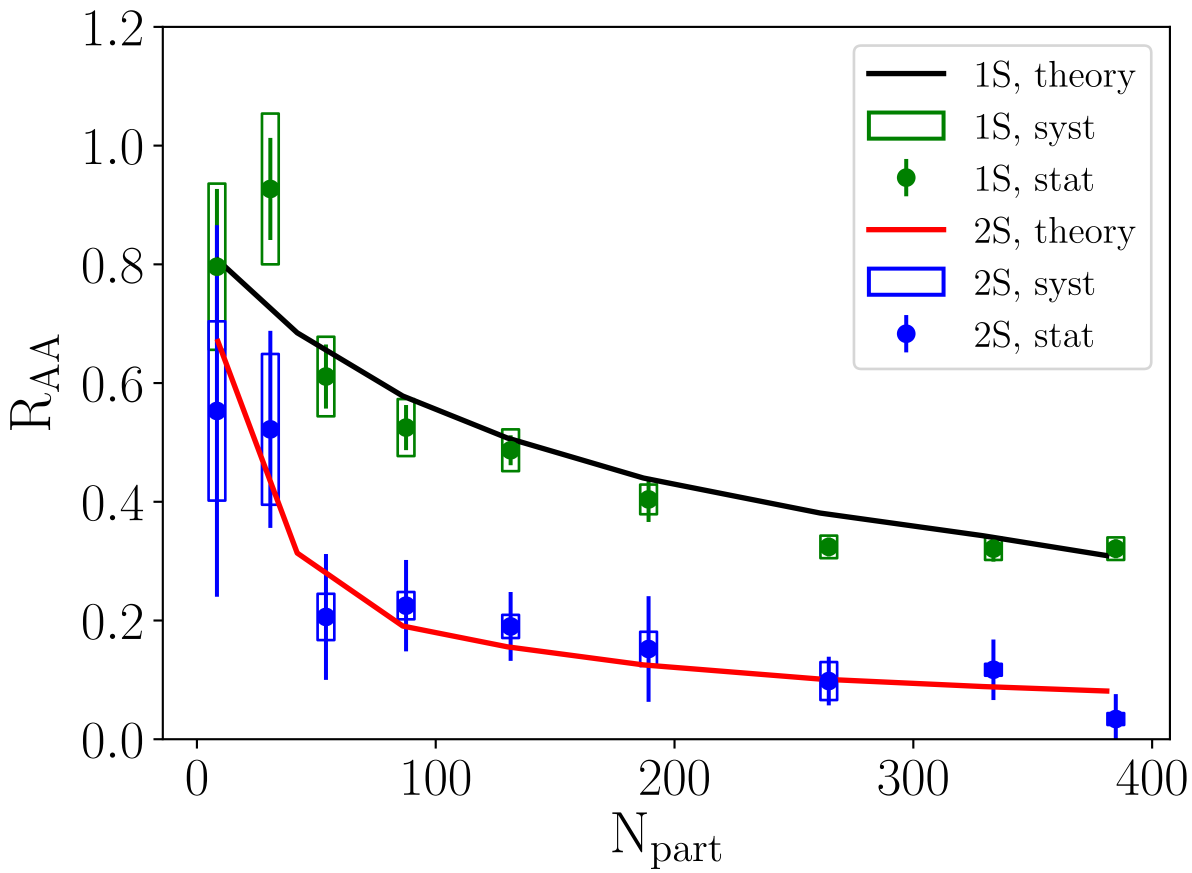




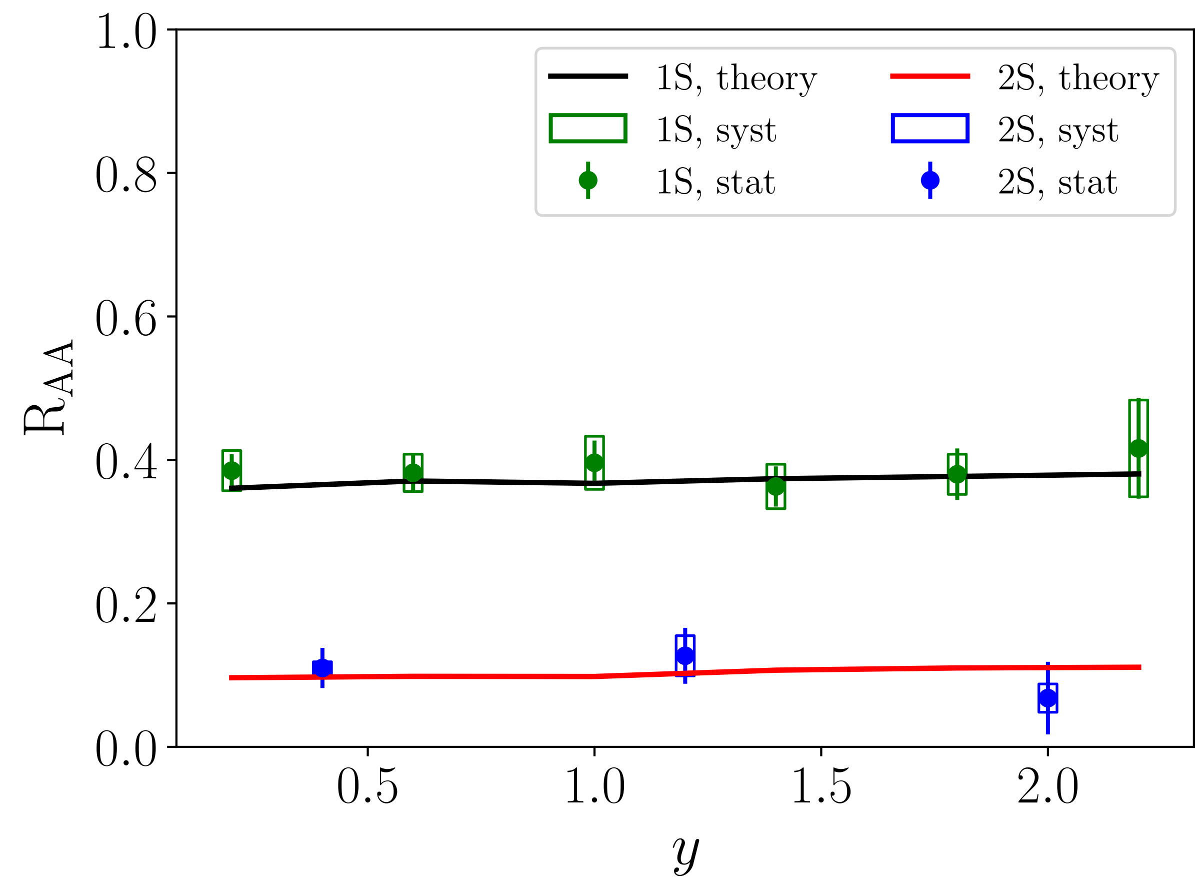
Upsilon in 5020 GeV PbPb Collision

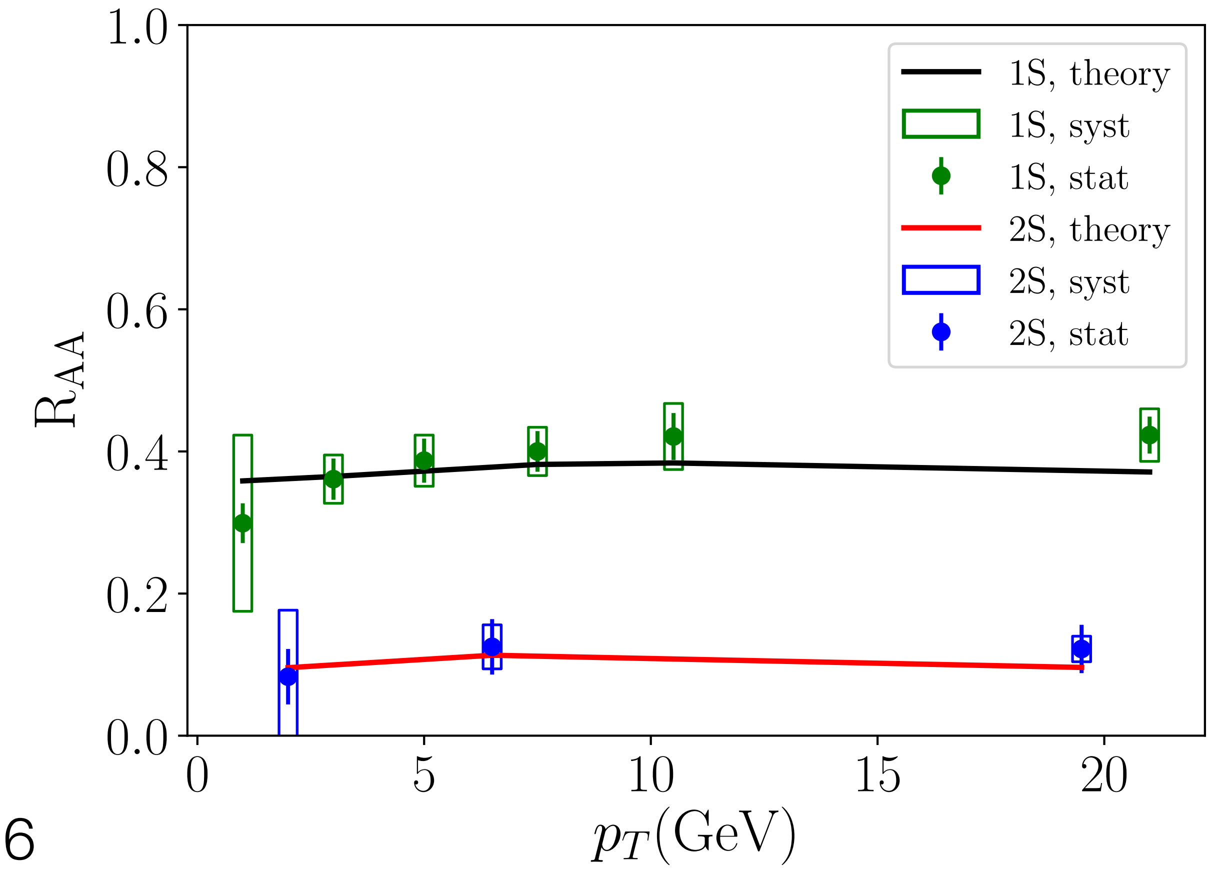
Use same set of parameters

Cold nuclear matter effect ~ 0.85 (PYTHIA + nPDF)



CMS arXiv:1805.09215

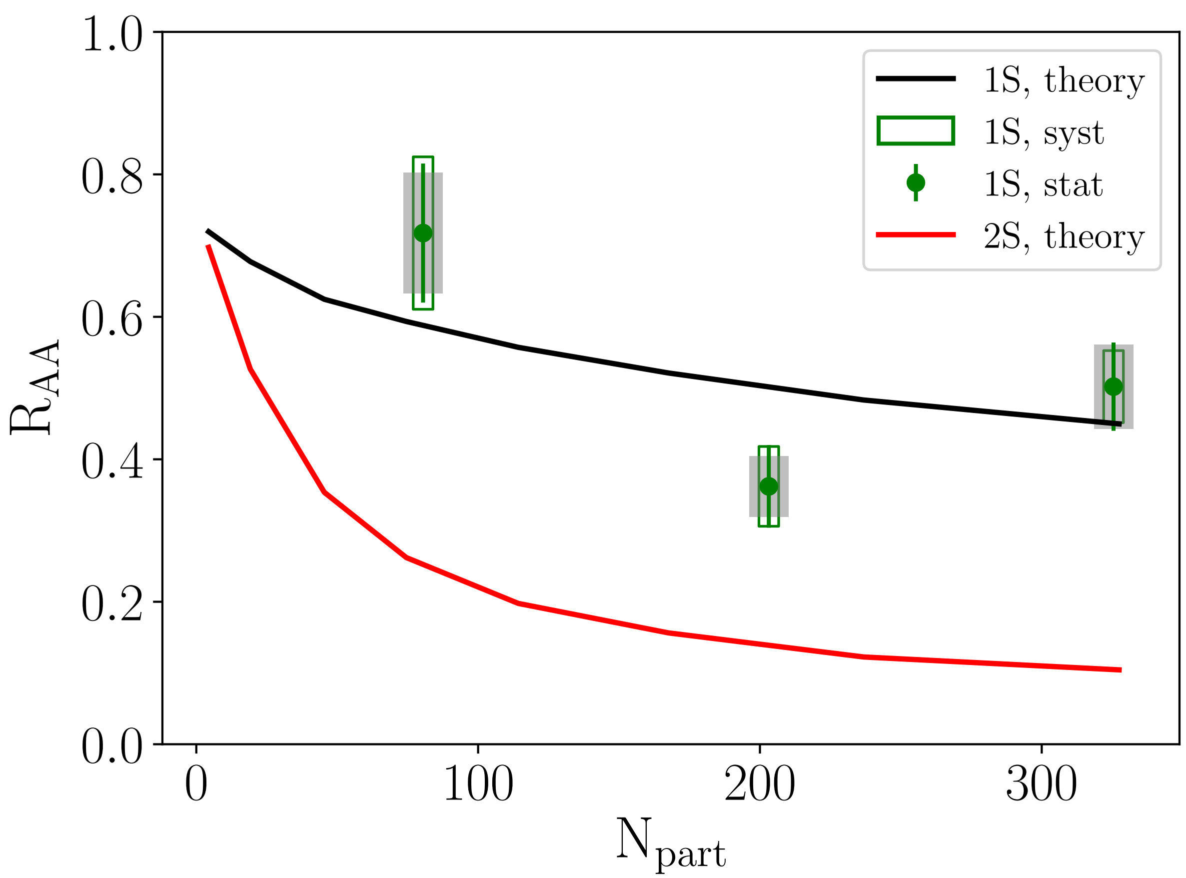


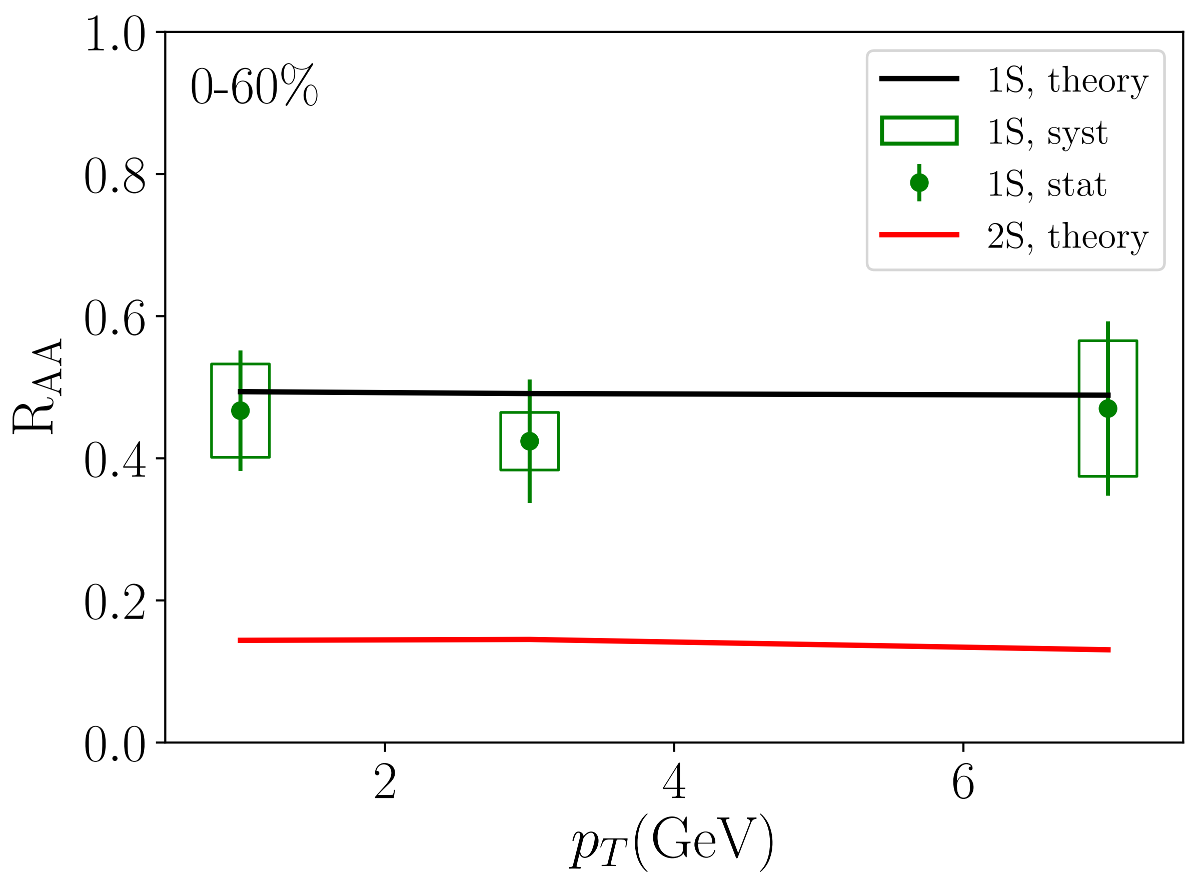


Upsilon in 200 GeV AuAu Collision

Use same set of parameters

Cold nuclear matter effect ~ 0.72 (use p-Au data of STAR)



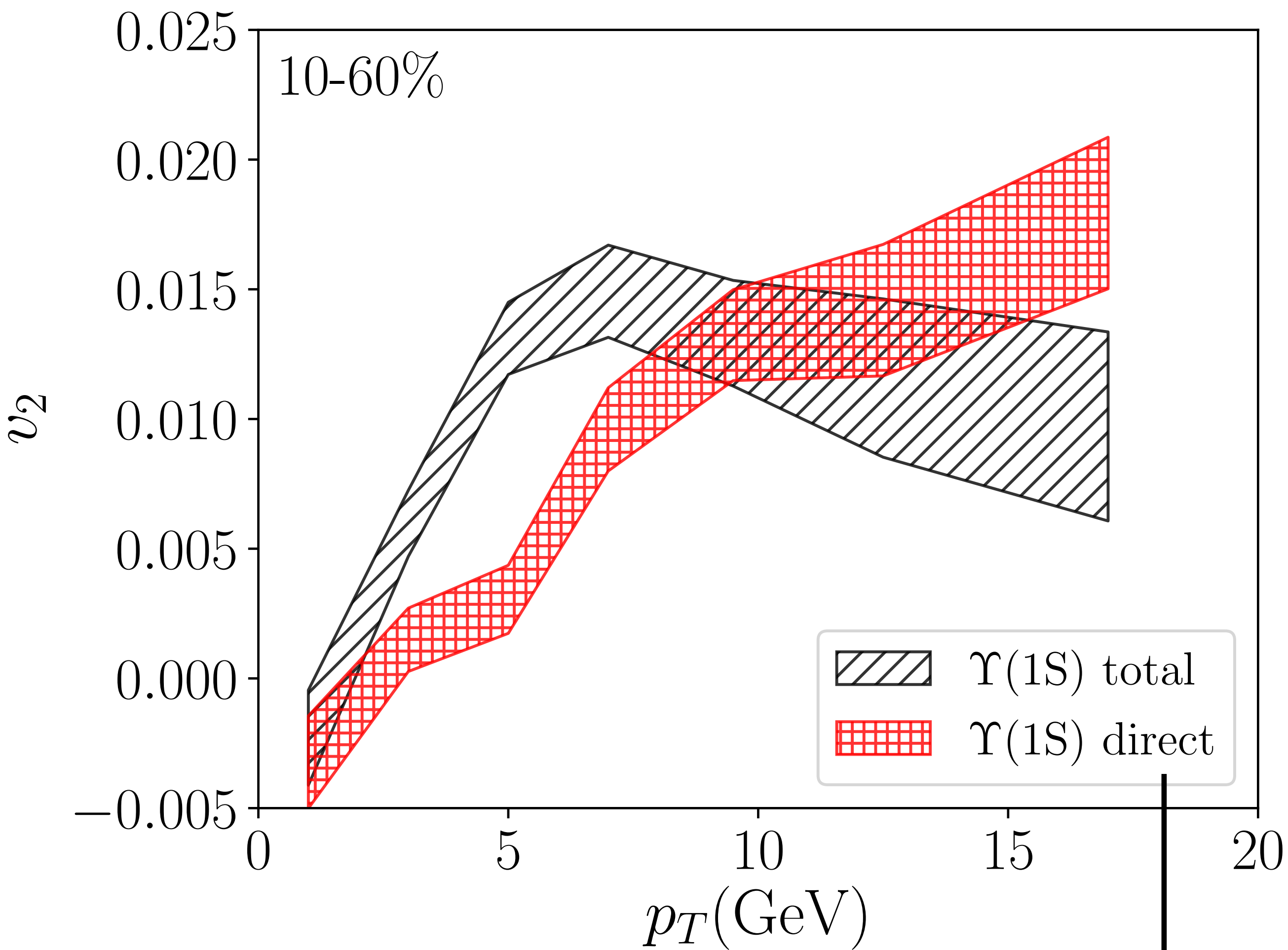


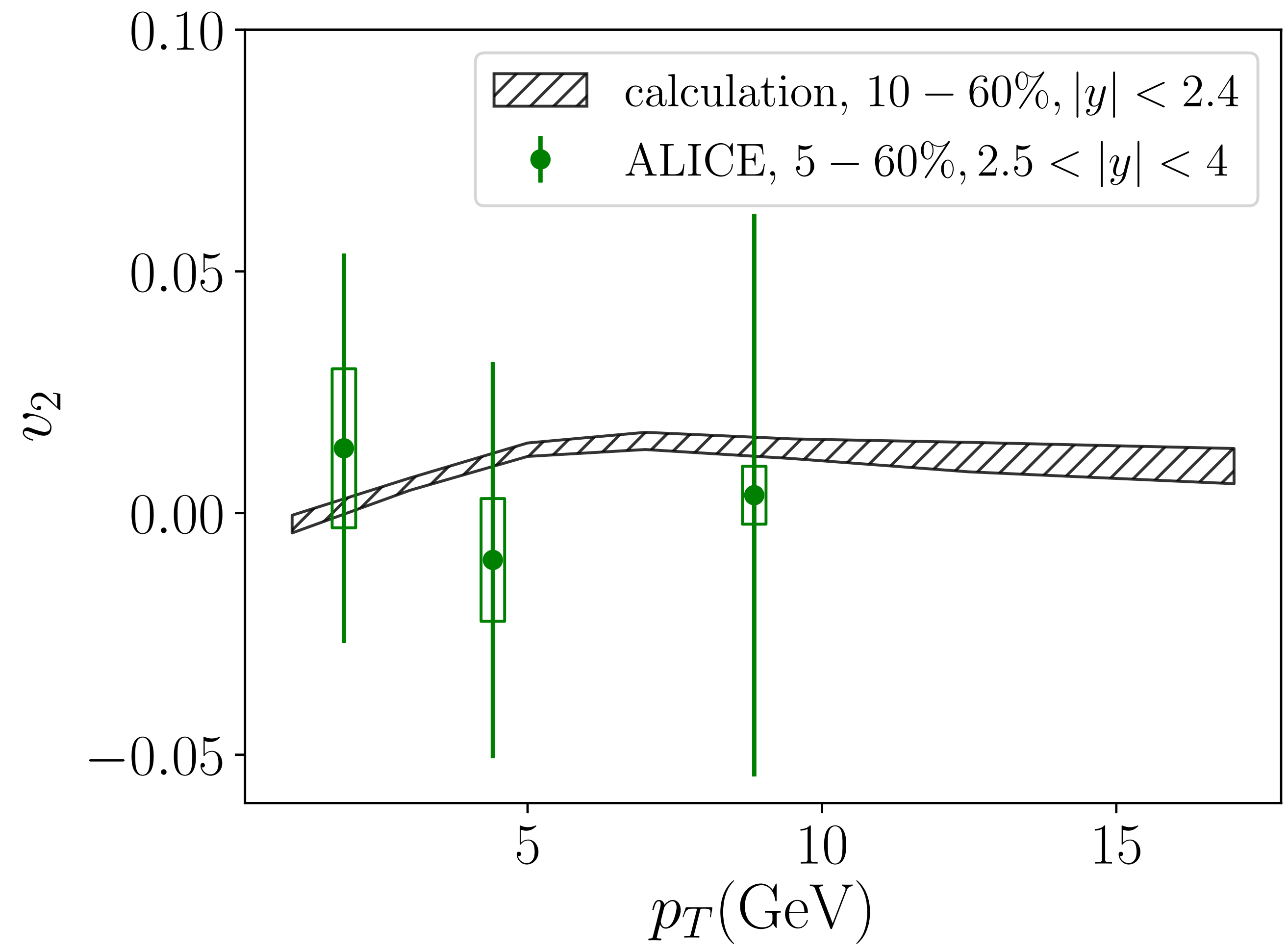
STAR measures 2S+3S

STAR Talks at QM 17&18

Upsilon(1S) Azimuthal Anisotropy in 5020 GeV PbPb

$$E\frac{d^3N}{dp^3} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left(1 + 2v_2\cos(2\phi) + \cdots\right)$$





Produced from initial hard collision

(see talk P.Biswarup, Tu.9:15 am)

Conclusion

- Derivation of transport equations from open quantum system and effective field theory

- **Assumptions used to derive semi-classical transport equations are justified from the separation of scales**

- **Coupled transport equations: detailed balance and thermalization**

- Phenomenological results on bottomonium production in heavy ion collisions

- Future plan: charmonium
  try to connect lattice with transport (lattice calculation of complex potential, see A. Rothkopf Tu.8:45 am)

Backup: Basis Change in Relative Motion

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \, {\rm Tr}\Big({\rm S}^\dagger(i\partial_0 - H_s){\rm S} + {\rm O}^\dagger(iD_0 - H_o){\rm O} + V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.}) + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots\Big)$$

Bra-ket notation in relative motion

$${\rm S}(\boldsymbol{R},\boldsymbol{r},t) = \frac{1}{\sqrt{N_c}} S(\boldsymbol{R},\boldsymbol{r},t) \equiv \frac{1}{\sqrt{N_c}}\langle \boldsymbol{r}|S(\boldsymbol{R},t)\rangle$$

$${\rm O}(\boldsymbol{R},\boldsymbol{r},t) = \frac{1}{\sqrt{T_F}} O^a(\boldsymbol{R},\boldsymbol{r},t)T^a \equiv \frac{1}{\sqrt{T_F}}\langle \boldsymbol{r}|O^a(\boldsymbol{R},t)\rangle T^a$$

Dipole interaction

$$\sqrt{\frac{T_F}{N_C}}\Big(\langle O^a(\boldsymbol{R},t)|\boldsymbol{r}\cdot g\boldsymbol{E}^a(\boldsymbol{R},t)|S(\boldsymbol{R},t)\rangle + {\rm h.c.}\Big)$$

Composite fields

$$|S(\boldsymbol{R},t)\rangle = \int \frac{d^3p_{\rm cm}}{(2\pi)^3} e^{-i(Et-\boldsymbol{p}_{\rm cm}\cdot\boldsymbol{R})}\Big(\sum_{nl} a_{nl}(\boldsymbol{p}_{\rm cm})\otimes|\psi_{nl}\rangle + \int \frac{d^3p_{\rm rel}}{(2\pi)^3} b_{\boldsymbol{p}_{\rm rel}}(\boldsymbol{p}_{\rm cm})\otimes|\psi_{\boldsymbol{p}_{\rm rel}}\rangle\Big)$$

$$|O^a(\boldsymbol{R},t)\rangle = \int \frac{d^3p_{\rm cm}}{(2\pi)^3} e^{-i(Et-\boldsymbol{p}_{\rm cm}\cdot\boldsymbol{R})}\int \frac{d^3p_{\rm rel}}{(2\pi)^3} c^a_{\boldsymbol{p}_{\rm rel}}(\boldsymbol{p}_{\rm cm})\otimes|\psi_{\boldsymbol{p}_{\rm rel}}\rangle \qquad E = -|E_{nl}|,\ \frac{p_{\rm rel}^2}{M}$$

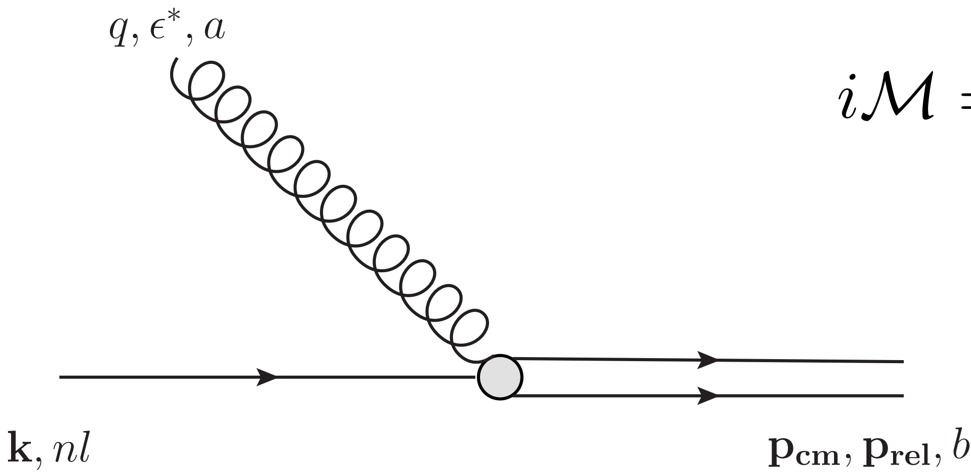
Power counting

Quantization

$$[a_{n_1l_1}(\boldsymbol{p}_{\rm cm1}),\ a^\dagger_{n_2l_2}(\boldsymbol{p}_{\rm cm2})] = (2\pi)^3\delta^3(\boldsymbol{p}_{\rm cm1}-\boldsymbol{p}_{\rm cm2})\delta_{n_1n_2}\delta_{l_1l_2}$$

$$[c^{a_1}_{\boldsymbol{p}_{\rm rel1}}(\boldsymbol{p}_{\rm cm1}),\ c^{a_2\dagger}_{\boldsymbol{p}_{\rm rel2}}(\boldsymbol{p}_{\rm cm2})] = (2\pi)^6\delta^3(\boldsymbol{p}_{\rm cm1}-\boldsymbol{p}_{\rm cm2})\delta^3(\boldsymbol{p}_{\rm rel1}-\boldsymbol{p}_{\rm rel2})\delta^{a_1a_2}$$

Backup: Scattering Amplitudes



$$i\mathcal{M} = g\sqrt{\frac{T_F}{N_c}}(q^0\epsilon^{*i} - q^i\epsilon^{*0})\langle\Psi_{\boldsymbol{p}_{\rm rel}}|r^i|\psi_{nl}\rangle\delta^{ab} \equiv i\epsilon^{*\mu}(\mathcal{M})_\mu$$

Ward identity $\quad q^\mu(\mathcal{M})_\mu = 0$

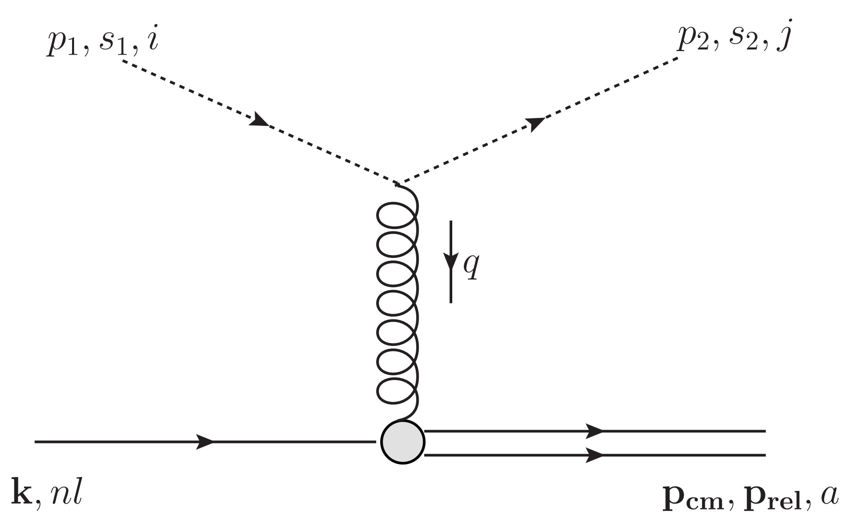
Higher order corrections of

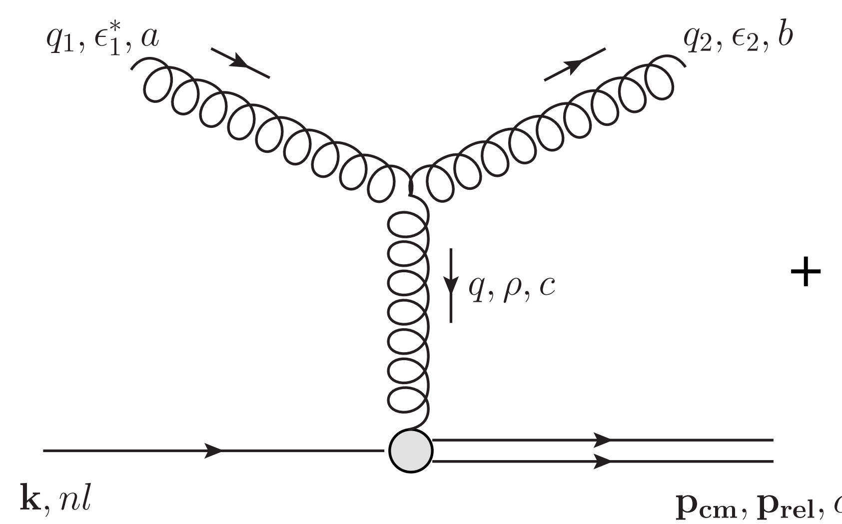
$$\frac{T_F}{N_C}g^2\langle E^{b_1}_{i_1}(\boldsymbol{R}_1, t_1)E^{b_2}_{i_2}(\boldsymbol{R}_2, t_2)\rangle_T$$

⟶ **Inelastic scattering in t-channel**

Ward identity

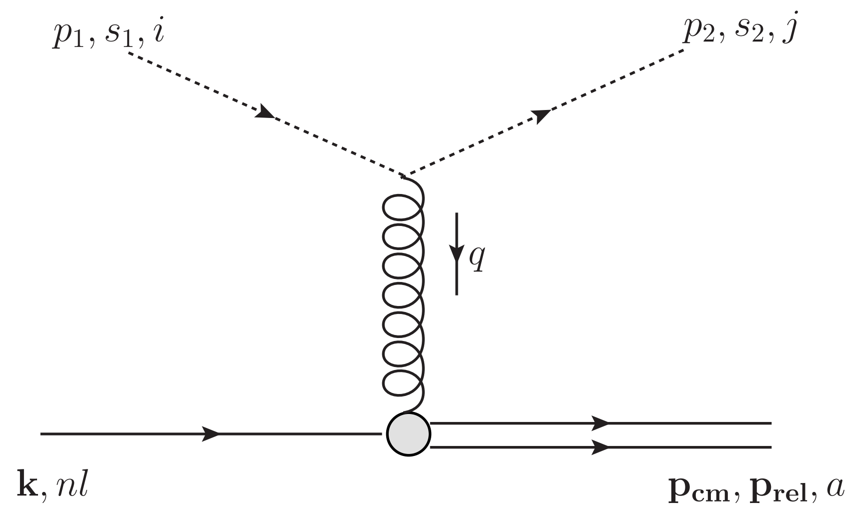
Infrared safe





+ more diagrams

Backup: NLO Amplitudes: Light Quark



Gauge independence: Dirac equation + LO Ward identity

$$i\mathcal{M}_{(b)} = g^2 V_A \sqrt{\frac{T_F}{N_c}} \langle \Psi_{\boldsymbol{p}_{\mathrm{rel}}} | r^k | \psi_{nl} \rangle \Big[\frac{-q^0(\delta^{kl} - \hat{q}^k \hat{q}^l)}{(q^0)^2 - \boldsymbol{q}^2 + i\epsilon} \bar{u}_{s_2}(p_2) \gamma^l T^a u_{s_1}(p_1) + \frac{q^k}{\boldsymbol{q}^2} \bar{u}_{s_2}(p_2) \gamma^0 T^a u_{s_1}(p_1) \Big]$$

$$\sum |\mathcal{M}_{(b)}|^2 \equiv \sum_{a,i,j} \sum_{s_1,s_2} \sum_{u,\bar{u},d,\bar{d}} |\mathcal{M}_{(b)}|^2 = \frac{16}{3} g^4 V_A^2 T_F C_F |\langle \Psi_{\boldsymbol{p}_{\mathrm{rel}}} | \boldsymbol{r} | \psi_{nl} \rangle|^2$$

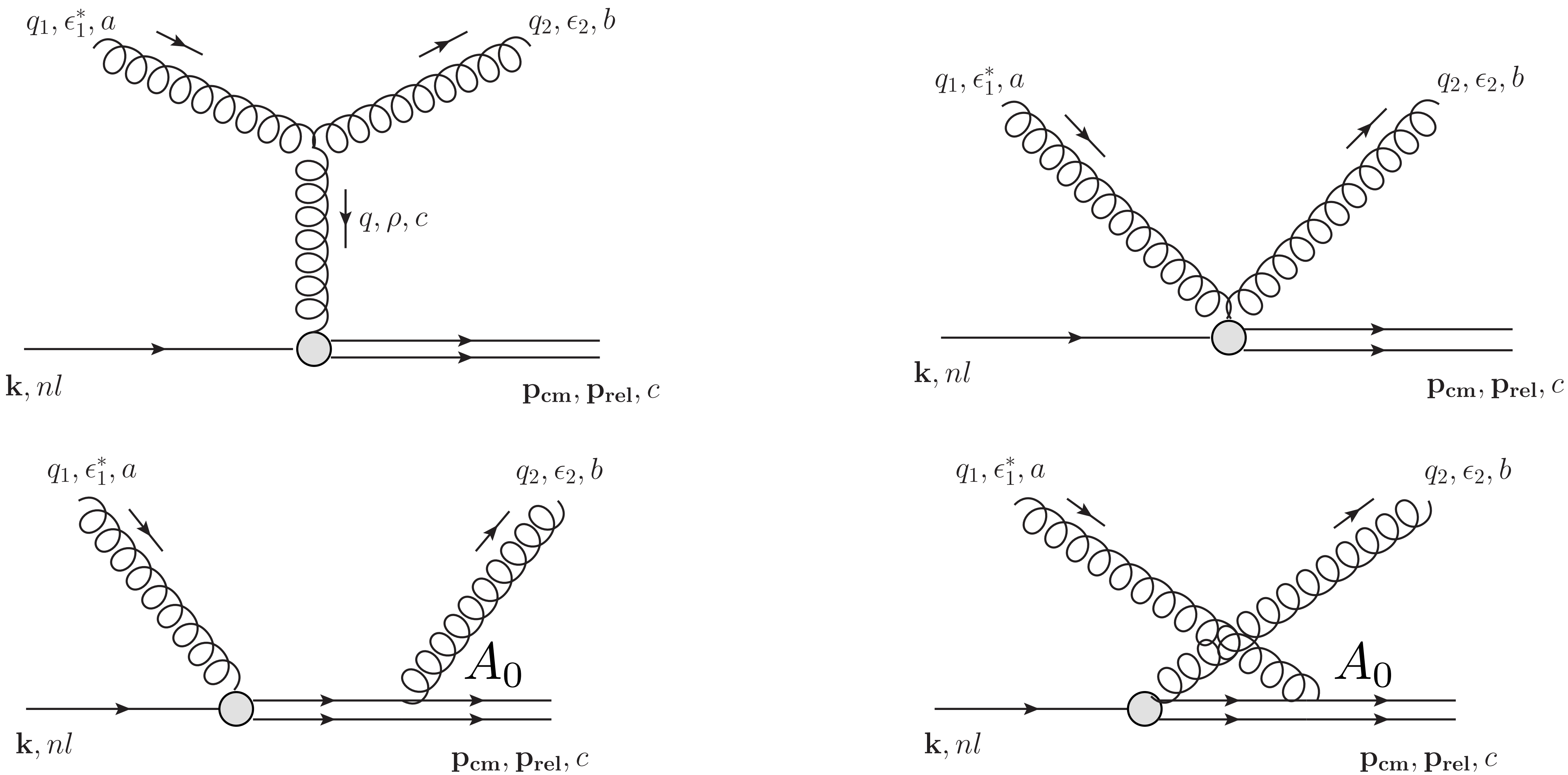
$$\Big[\frac{p_1 p_2 + \boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{\boldsymbol{q}^2} + \frac{2(q^0)^2 (p_1 p_2 - \boldsymbol{p}_1 \cdot \hat{q} \cdot \boldsymbol{p}_2 \cdot \hat{q})}{((q^0)^2 - \boldsymbol{q}^2 + i\epsilon)^2} \Big]$$

Infrared singularity

Infrared safe: finite binding energy

Soft safe: finite binding energy
Collinear divergent

Backup: NLO Amplitudes: Gluon



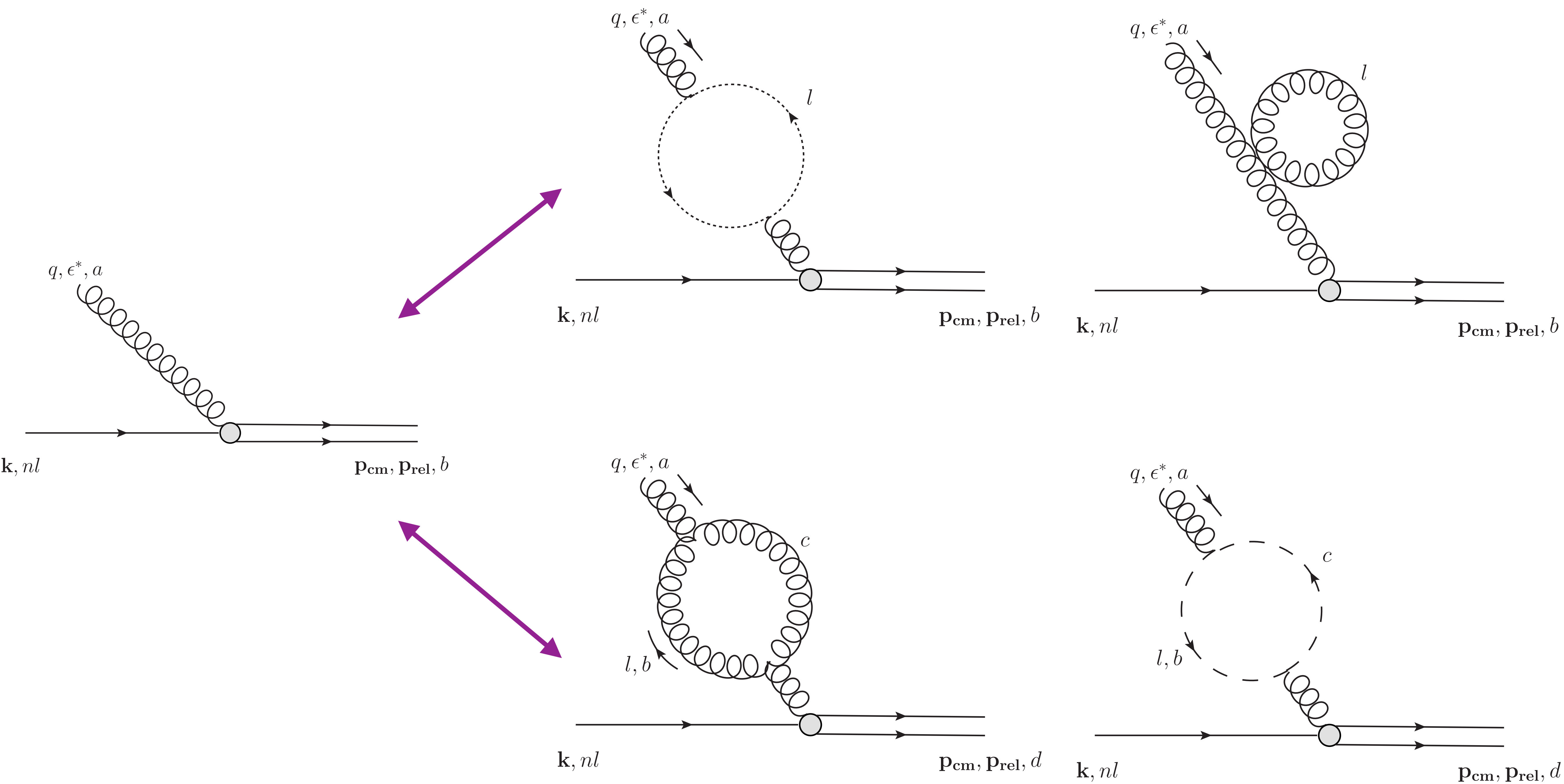
Ward identity $$i(q_1)_\mu \sum \mathcal{M}^{\mu\nu}(\epsilon_2)_\nu = 0 + \mathcal{O}(v^2)$$

Choose Coulomb gauge, only first one contributes

Same collinear divergence as in NLO w/ light quark

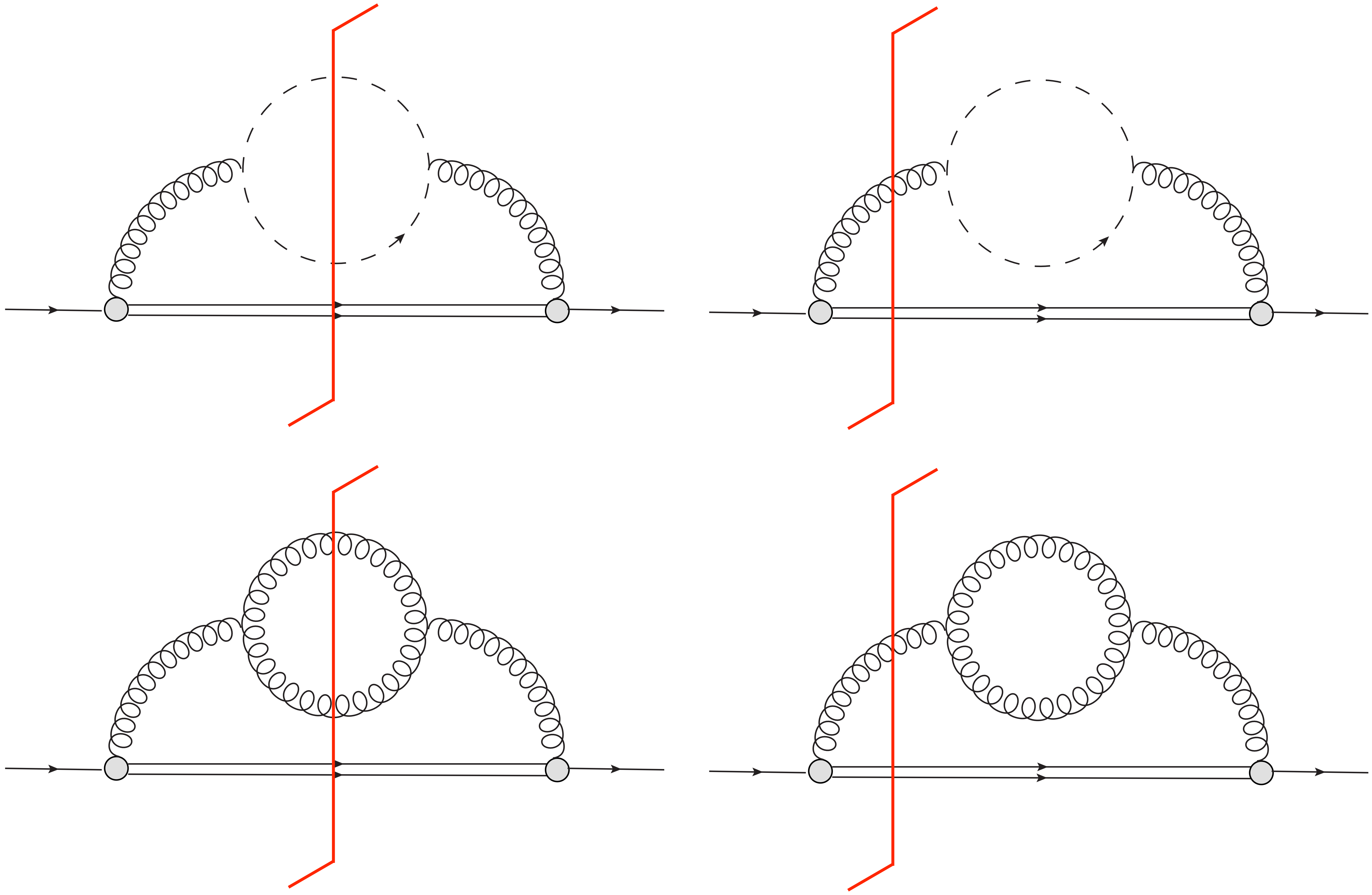
Backup: Collinear Divergence Cancellation: Interference

$$n_B(p_1 - p_2)(n_F(p_1) - n_F(p_2)) = -n_F(p_1)(1 - n_F(p_2))$$



$$n_B(q_1 - q_2)(n_B(q_1) - n_B(q_2)) = -n_B(q_1)(1 + n_B(q_2))$$

Backup: Collinear Divergence Cancellation



Backup: Numerical Implementation

- **Test particle Monte Carlo** $f(\boldsymbol{x},\boldsymbol{p},t)=\sum_i \delta^3(\boldsymbol{x}-\boldsymbol{y}_i(t))\delta^3(\boldsymbol{p}-\boldsymbol{k}_i(t))$

- Each time step: read in hydro-cell velocity, temperature; consider diffusion, dissociation, recombination in particle's rest frame and boost back

- If specific process occurs, sample incoming medium particles and outgoing particles from differential rates, conserving energy momentum

- Recombination term contains $f_Q(\boldsymbol{x}_1,\boldsymbol{p}_1,t)f_{\bar{Q}}(\boldsymbol{x}_2,\boldsymbol{p}_2,t)$

For each HQ, search anti-HQ within a radius, weighted sum

$$f_Q(\boldsymbol{x}_1,\boldsymbol{p}_1,t)f_{\bar{Q}}(\boldsymbol{x}_2,\boldsymbol{p}_2,t)=\sum_{i,j}\frac{e^{-(\boldsymbol{y}_i-\bar{\boldsymbol{y}}_j)^2/2a_B^2}}{(2\pi a_B)^{3/2}}\delta^3(\frac{\boldsymbol{x}_1+\boldsymbol{x}_2}{2}-\frac{\boldsymbol{y}_i+\bar{\boldsymbol{y}}_j}{2})\delta^3(\boldsymbol{p}_1-\boldsymbol{k}_i)\delta^3(\boldsymbol{p}_2-\bar{\boldsymbol{k}}_j)$$