Quark(onium) Transport Coefficients in Heavy Ion Collisions from EFT

Quarkonium Working Group 2019

Peter Vander Griend

with N. Brambilla, M.A. Escobedo, A. Vairo

Technical University of Munich

15 May 2019

based on arXiv:1903.08063

Motivation

extraction of nuclear modification factor R_{AA} in a strongly coupled medium

- determined by non-equilibrium evolution of medium and system
- in medium evolution of system given in form of quantum master equation
- master equation depends on two parameters, the transport coefficients κ and γ

Method

open quantum systems

allows for rigorous treatment of a quantum system of interest (quarkonium) coupled to a bath or reservoir (quark gluon plasma)

effective field theories

use potential non-relativistic QCD (pNRQCD) to describe the interaction of a non-relativistic bound state (heavy quarkonium) with the gluon

Method

open quantum systems and EFTs

- place minimal assumptions on medium, i.e., can be strongly or weakly coupled
- respect heavy quark number conservation
- account explicitly for the non-Abelian nature of QCD, i.e., singlet and octet states
- account for disassociation and recombination
- currently use Bjorken evolution but can also be coupled to hydrodynamic evolution

Results

transport coefficients

- quarkonium in medium evolution governed by two parameters κ and γ
- κ is the heavy quark~~onium~~ momentum diffusion coefficient
  - connection between in medium heavy quark and heavy quarkonium dynamics
- γ the dispersive counterpart of κ
  - new parameter

calculation of κ and γ

- can be expressed in terms of correlators which can be measured on the lattice
- can be related to quantities which can be measured in unquenched lattice simulations

Results

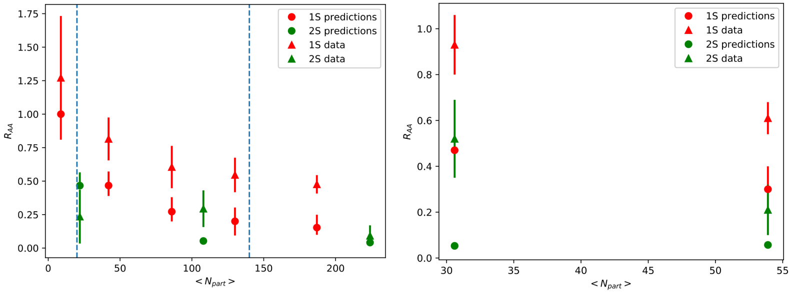


Figure: Comparisons of theoretical calculation of R_{AA} vs. experimental results from the CMS collaboration at $\sqrt{s_{NN}} = 2.76$ TeV (left) and $\sqrt{s_{NN}} = 5.02$ TeV (right) Escobedo (2018), CMS (2016, 2018).

Physical Setup I

Heavy Quarkonium

- bound state of heavy quark and heavy antiquark characterized by heavy quark mass M, Bohr radius a_0, and binding energy E
- for bottomonium:
  - $M = 4.78$ GeV
  - $1/a_0 = MC_F\alpha_s(1/a_0)/2$=1.49 GeV
  - $E = M(\Upsilon(1S)) - 2M_b = -0.1$ GeV
- hierarchical ordering of scales: $M \gg 1/a_0 \gg E$

Medium

- medium (quark gluon plasma) formed in heavy ion collisions
- characteristic scale: $(\pi)T \sim (\pi)\mathcal{O}(100)$ MeV

Physical Setup II

Combined System

- no assumption on coupling, i.e., medium can be either strongly ($T \sim m_D \sim gT$) or weakly ($T \gg m_D \sim gT$) coupled
- characterized by time scales:
  - intrinsic time scale of quarkonium:

$$\tau_S \sim \frac{1}{E}$$

  - correlation time of the plasma:

$$\tau_E \sim \frac{1}{\pi T}$$

  - relaxation time:

$$\tau_R \sim \frac{1}{\Sigma s} \sim \frac{1}{a_0^2 (\pi T)^3}$$

Open Quantum Systems

- system coupled to bath with interaction; total Hamiltonian given by:

$$H = H_S \otimes \mathbb{1}_B + \mathbb{1}_S \otimes H_B + H_I$$

- time evolution of density matrix given by Liouville–von Neumann equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -i\,[H_I(t), \rho(t)]$$

- integrate to get equation for $\rho(t)$, insert back into time evolution equation, and trace over reservoir

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S(t) = -\mathrm{T}\int_{t_0}^{t} \mathrm{d}s\ \mathrm{Tr}_B\Big\{\big[H_I(t), [H_I(s), \rho(s)]\,\big]\Big\}$$

Approximations

- for $\tau_R \gg \tau_E$, several simplifying approximations may be used:
  - **Born Approximation**: system has little influence on reservoir over long time scales:

$$\rho(t) \approx \rho_S(t) \otimes \rho_B$$

  - **Markov Approximation**: no memory integral, $\rho(s) \to \rho(t)$
- furthermore, behavior of the reservoir correlation functions at large time allows us to take the upper limit to ∞
- combing above, we have:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S(t) = -\mathrm{T}\int_0^{\infty} \mathrm{d}s\ \mathrm{Tr}_B\Big\{\big[H_I(t), [H_I(t-s), \rho_S(t) \otimes \rho_B]\big]\Big\}$$

potential Non-Relativistic QCD (pNRQCD)

pNRQCD

- an effective field theory for the strong interaction obtained from full QCD via non-relativistic QCD (NRQCD)
- NRQCD obtained from full QCD by integrating out the hard scale, i.e., the heavy quark mass M
- pNRQCD obtained from NRQCD by further integrating out the soft scale Mv
- degrees of freedom are singlet and octet states and gluons
- interaction Hamilton describes singlet-octet, octet-singlet, and octet-octet transitions

$$H_I = H_{so} + H_{os} + H_{oo}$$

potential Non-Relativistic QCD (pNRQCD)

pNRQCD

interaction Hamilton describes singlet-octet, octet-singlet, and octet-octet transitions

$$H_I = H_{so} + H_{os} + H_{oo}$$

$$H_{so}(t) = -g\sqrt{\frac{1}{2N_c}}\int_{\mathbf{R},\mathbf{r}} S^\dagger(\mathbf{R},\mathbf{r})O^a(\mathbf{R},\mathbf{r})r_i E^{i,a}(t,\mathbf{R})$$

$$H_{os}(t) = -g\sqrt{\frac{1}{2N_c}}\int_{\mathbf{R},\mathbf{r}} O^{a\dagger}(\mathbf{R},\mathbf{r})S(\mathbf{R},\mathbf{r})r_i E^{i,a}(t,\mathbf{R})$$

$$H_{oo}(t) = -\frac{g}{2}d^{abc}\int_{\mathbf{R},\mathbf{r}} O^{a\dagger}(\mathbf{R},\mathbf{r})O^c(\mathbf{R},\mathbf{r})r_i E^{i,b}(t,\mathbf{R})$$

where $\mathbf{R} = \frac{\mathbf{r}_1+\mathbf{r}_2}{2}$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ for quark, anti-quark positions $\mathbf{r}_1$, $\mathbf{r}_2$

Evolution Equations

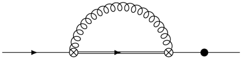
moving to the Schrödinger picture and decomposing into singlet and octet pieces:

$$\begin{aligned}\frac{\mathrm{d}\rho_s(t)}{\mathrm{d}t} &= -i\,[h_s, \rho_s(t)] - \Sigma_s\rho_s(t) - \rho_s(t)\Sigma_s^\dagger + \Xi_{so}(\rho_o(t)) \\ \frac{\mathrm{d}\rho_o(t)}{\mathrm{d}t} &= -i\,[h_o, \rho_o(t)] - \Sigma_o\rho_o(t) - \rho_o(t)\Sigma_o^\dagger + \Xi_{os}(\rho_s(t)) \\ &\quad + \Xi_{oo}(\rho_o(t))\end{aligned}$$

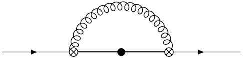
where the Σ and Ξ contain the reservoir correlators and can be represented diagrammatically
for $T \gg E$, energy dependent exponentials may be set to 1

Diagrammatic Evolution Equation of $\rho_s(t)$

$$\frac{\mathrm{d}\rho_s(t)}{\mathrm{d}t} = -i\,[h_s, \rho_s(t)] - \Sigma_s \rho_s(t) - \rho_s(t)\Sigma_s^\dagger + \Xi_{so}(\rho_o(t))$$

$\Sigma_s \rho_s(t) =$ 

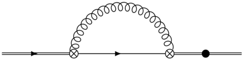
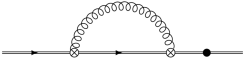
$$= \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}t\, r_j r_j \left\langle E^{i,a}(t, \mathbf{0}) E^{i,a}(0, \mathbf{0}) \right\rangle$$

$\Xi_{so}(\rho_o(t)) =$ 

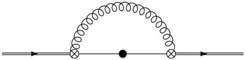
$$= \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}t\, \left[r^j \rho_o(t) r^j \left\langle E^{i,a}(0, \mathbf{0}) E^{i,a}(t, \mathbf{0}) \right\rangle + \text{h.c.} \right]$$

Diagramatic Evolution of $\rho_o(t)$

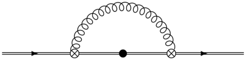
$$\frac{\mathrm{d}\rho_o(t)}{\mathrm{d}t} = -i\left[h_o, \rho_o(t)\right] - \Sigma_o \rho_o(t) - \rho_o(t)\Sigma_o^\dagger + \Xi_{os}(\rho_s(t)) + \Xi_{oo}(\rho_o(t))$$

$\Sigma_o \rho_o(t) =$  $+$ 

$$= \frac{g^2}{6N_c(N_c^2-1)} \int_0^\infty \mathrm{d}t \left[1 + \frac{N_c^2-4}{2}\right] r^j r^j \left\langle E^{i,a}(t,\mathbf{0}) E^{i,a}(0,\mathbf{0})\right\rangle$$

$\Xi_{os}(\rho_s(t)) =$ 

$$= \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}t \, \left[r^j \rho_s(t) r^j \left\langle E^{i,a}(0,\mathbf{0}) E^{i,a}(t,\mathbf{0})\right\rangle + \text{h.c.}\right]$$

$\Xi_{os}(\rho_s(t)) =$ 

$$= \frac{g^2(N_c^2-4)}{12N_c(N_c^2-1)} \int_0^\infty \mathrm{d}t \, \left[r^j \rho_s(t) r^j \left\langle E^{i,a}(0,\mathbf{0}) E^{i,a}(t,\mathbf{0})\right\rangle + \text{h.c.}\right]$$

Master Equation

evolution equation

- can be written as

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \sum_{n,m} h_{nm} \left(L_i^n \rho L_i^{m\dagger} - \frac{1}{2} \left\{ L_i^{m\dagger} L_i^n, \rho \right\} \right),$$

  where H, ρ, and L_i are 2×2 matrices
- L_i depend on two reservoir correlators:

$$\kappa = \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}t \left\langle \left\{ E^{a,i}(t,\mathbf{0}), E^{a,i}(0,\mathbf{0}) \right\} \right\rangle,$$
$$\gamma = -\frac{ig^2}{6N_c} \int_0^\infty \mathrm{d}t \left\langle \left[E^{a,i}(t,\mathbf{0}), E^{a,i}(0,\mathbf{0}) \right] \right\rangle$$

- as shown in Casalderrey-Solana, Teaney (2006), κ is the heavy quark momentum diffusion coefficient occurring in a Langevin equation

Langevin Dynamics

- description of Brownian motion, i.e., a particle moving randomly due to uncorrelated interactions with its environment

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -\eta_D p_i + \xi_i(t),\ \langle \xi_i(t)\xi_j(t') \rangle = \kappa \delta_{ij}\delta(t-t'),\ \eta_D = \frac{\kappa}{2MT},$$

  where p_i is the momentum of the particle (heavy quark), η_D is the drag coefficient, and ξ_i encodes the random, uncorrelated interactions of the particle with the medium
- integration of force-force correlator along the Schwinger-Keldysh contour gives κ in correlator form

Extraction of Transport Coefficients

- from the definitions of Σ_s, κ, and γ

$$\Sigma_s = \frac{r^2}{2}(\kappa + i\gamma)$$

- from which

$$\begin{aligned} r^2\kappa &= \Sigma_s + \Sigma_s^\dagger &&= -2\,\mathrm{Im}(-i\Sigma_s) \\ r^2\gamma &= -i\Sigma_s + i\Sigma_s^\dagger &&= \;\;2\,\mathrm{Re}(-i\Sigma_s) \end{aligned}$$

- relating the real and imaginary parts of the self energy to the width and the mass shift and projecting onto $1S$ states gives

$$\Gamma(1S) = 3a_0^2\kappa, \qquad \delta M(1S) = \frac{3}{2}a_0^2\gamma$$

thus relating both κ and γ to quantities measurable on the lattice in unquenched simulations

Previous Calculations of κ

- measured on the lattice in a quenched simulation Francis, et. al. (2015)
- can be extracted from experimental measurements of the azimuthal anisotropy coefficient v_2 at experiments, e.g., ALICE and STAR
- perturbative calculation done to NLO but suffers from convergence issues Caron-Huot, et. al. 2009
- correlator form of κ can be related to the spectral function

$$\kappa = \frac{T}{6N_c} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

allowing for an unquenched lattice measurement

Unquenched Calculation of κ

- lower bound
  - 2+1 flavor measurement of Γ of $\Upsilon(1S)$ with $M_b = 4.65$ GeV Kim, et. al. 2018
  - at $T = 407$ MeV, $\Gamma = 23.6$ MeV
  - rescale Γ to account for different bottom mass

$$\Gamma = 23.6 \text{ MeV} \to \Gamma = \left(\frac{4.65}{4.78}\right)^2 23.6 \text{ MeV} = 22.3 \text{ MeV}$$

  - with $a_0 = 0.67 \text{ GeV}^{-1}$

$$\frac{\kappa}{T^3} = \frac{\Gamma}{3a_0^2 T^3} = 0.24$$

Unquenched Calculation of κ

- upper bound
  - 2 flavor measurement of Γ of $\Upsilon(1S)$ Aarts, et. al. 2011 with $M_b = 5$ GeV
  - at $T = 440$ MeV, $\Gamma = 440$ MeV
  - rescale Γ to account for different bottom mass

$$\Gamma = 440 \text{ MeV} \to \Gamma = \left(\frac{5}{4.78}\right)^2 440 \text{ MeV} = 481 \text{ MeV}$$

  - with $a_0 = 0.67 \text{ GeV}^{-1}$

$$\frac{\kappa}{T^3} = \frac{\Gamma}{3a_0^2 T^3} = 4.2$$

- combined bounds on unquenched estimate of κ

$$0.24 \lesssim \frac{\kappa}{T^3} \lesssim 4.2$$

Current Estimates of κ

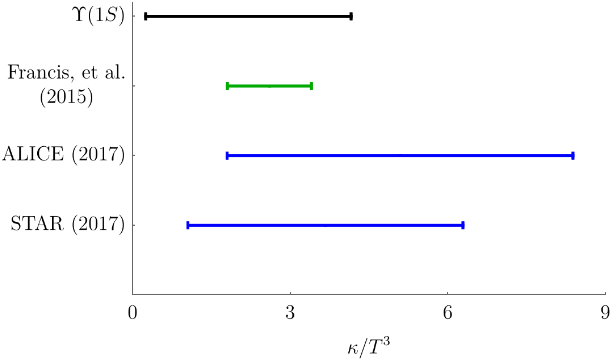


Figure: Current estimates of κ from (top to bottom) the thermal width, quenched lattice measurement, and experimental measurements.

Calculation of γ

- one prior perturbative calculation Brambilla, et. al. 2008
  - thermal correction to singlet potential $V_s(\mathbf{r})$
- lattice measurements of the thermal mass shift of the $\Upsilon(1S)$ at $T = 251\,\text{MeV}$ and $407\,\text{MeV}$ and the J/ψ at $T = 251\,\text{MeV}$ from Kim, et. al. 2018
  - same mass scaling as for κ
  - total spread of results to place bounds:

$$-3.8 \lesssim \frac{\gamma}{T^3} \lesssim -0.7$$

- correlator form of γ can be related to spectral function

$$\gamma = -\frac{1}{3N_c} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{\rho(\omega)}{\omega}$$

  - allows for lattice measurement
  - divergent; infinite vacuum contribution must be subtracted

Current Estimates of γ

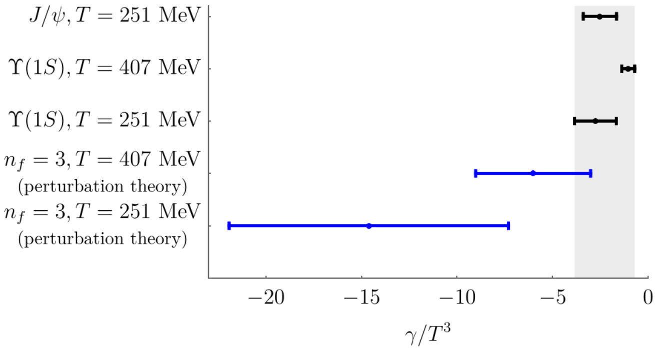


Figure: Current estimates of γ from the thermal width (black) and perturbation theory (blue).

Conclusions

- to compute nuclear modification factor R_{AA}, out of equilibrium evolution of in medium quarkonium needed
- this evolution specified by two parameters, κ and γ
- κ is the heavy quark momentum diffusion coefficient describing in medium momentum equilibration of a heavy quark
- γ new parameter
- expressions for κ and γ in terms of gluo-electric correlators allow them to be related to quantities measurable in unquenched lattice simulations
- expressions for κ and γ in terms of the spectral function allow for them to be measured directly in unquenched lattice simulations
- new unquenched estimates of κ in agreement with prior estimates from other sources

Thank you!