Quark(onium) Transport Coefficients in Heavy Ion Collisions from EFT
Quarkonium Working Group 2019

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Motivation

extraction of nuclear modification factor $R_{AA}$ in a strongly coupled medium

- determined by non-equilibrium evolution of medium and system
- in medium evolution of system given in form of quantum master equation
- master equation depends on two parameters, the transport coefficients $\kappa$ and $\gamma$
Method

**open quantum systems**
allows for rigorous treatment of a quantum system of interest (quarkonium) coupled to a bath or reservoir (quark gluon plasma)

**effective field theories**
use potential non-relativistic QCD (pNRQCD) to describe the interaction of a non-relativistic bound state (heavy quarkonium) with the gluon
Method

open quantum systems and EFTs

- place minimal assumptions on medium, i.e., can be strongly or weakly coupled
- respect heavy quark number conservation
- account explicitly for the non-Abelian nature of QCD, i.e., singlet and octet states
- account for disassociation and recombination
- currently use Bjorken evolution but can also be coupled to hydrodynamic evolution
Results

transport coefficients

- quarkonium in medium evolution governed by two parameters \( \kappa \) and \( \gamma \)
- \( \kappa \) is the heavy quarkonium momentum diffusion coefficient
  - connection between in medium heavy quark and heavy quarkonium dynamics
- \( \gamma \) the dispersive counterpart of \( \kappa \)
  - new parameter

calculation of \( \kappa \) and \( \gamma \)

- can be expressed in terms of correlators which can be measured on the lattice
- can be related to quantities which can be measured in unquenched lattice simulations
Results

Figure: Comparisons of theoretical calculation of $R_{AA}$ vs. experimental results from the CMS collaboration at $\sqrt{s_{NN}} = 2.76$ TeV (left) and $\sqrt{s_{NN}} = 5.02$ TeV (right) Escobedo (2018), CMS (2016, 2018).
Physical Setup I

Heavy Quarkonium

- bound state of heavy quark and heavy antiquark characterized by heavy quark mass $M$, Bohr radius $a_0$, and binding energy $E$
- for bottomonium:
  - $M = 4.78$ GeV
  - $1/a_0 = M C_F \alpha_s (1/a_0)/2 = 1.49$ GeV
  - $E = M(\Upsilon(1S)) - 2M_b = -0.1$ GeV
- hierarchical ordering of scales: $M \gg 1/a_0 \gg E$

Medium

- medium (quark gluon plasma) formed in heavy ion collisions
- characteristic scale: $(\pi) T \sim (\pi) O(100)$ MeV
Physical Setup II

Combined System

- no assumption on coupling, i.e., medium can be either strongly ($T \sim m_D \sim gT$) or weakly ($T \gg m_D \sim gT$) coupled
- characterized by time scales:
  - intrinsic time scale of quarkonium:
    $$\tau_S \sim \frac{1}{E}$$
  - correlation time of the plasma:
    $$\tau_E \sim \frac{1}{\pi T}$$
  - relaxation time:
    $$\tau_R \sim \frac{1}{\Sigma s} \sim \frac{1}{a_0^2(\pi T)^3}$$
Open Quantum Systems

- System coupled to bath with interaction; total Hamiltonian given by:

\[ H = H_S \otimes 1_B + 1_S \otimes H_B + H_I \]

- Time evolution of density matrix given by Liouville–von Neumann equation:

\[ \frac{d}{dt} \rho(t) = -i [H_I(t), \rho(t)] \]

- Integrate to get equation for \( \rho(t) \), insert back into time evolution equation, and trace over reservoir

\[ \frac{d}{dt} \rho_S(t) = -T \int_{t_0}^{t} ds \ \text{Tr}_B \left\{ [H_I(t), [H_I(s), \rho(s)]] \right\} \]
Approximations

► for $\tau_R \gg \tau_E$, several simplifying approximations may be used:
  
  ► **Born Approximation**: system has little influence on reservoir over long time scales:

  $$\rho(t) \approx \rho_S(t) \otimes \rho_B$$

  ► **Markov Approximation**: no memory integral, $\rho(s) \rightarrow \rho(t)$

  ► furthermore, behavior of the reservoir correlation functions at large time allows us to take the upper limit to $\infty$

  ► combing above, we have:

  $$\frac{d}{dt} \rho_S(t) = -T \int_0^\infty ds \ Tr_B \left\{ [H_I(t), [H_I(t-s), \rho_S(t) \otimes \rho_B]] \right\}$$
potential Non-Relativistic QCD (pNRQCD)

pNRQCD

- an effective field theory for the strong interaction obtained from full QCD via non-relativistic QCD (NRQCD)
- NRQCD obtained from full QCD by integrating out the hard scale, i.e., the heavy quark mass $M$
- pNRQCD obtained from NRQCD by further integrating out the soft scale $M_v$
- degrees of freedom are singlet and octet states and gluons
- interaction Hamilton describes singlet-octet, octet-singlet, and octet-octet transitions

$$H_I = H_{so} + H_{os} + H_{oo}$$
potential Non-Relativistic QCD (pNRQCD)

pNRQCD

interaction Hamilton describes singlet-octet, octet-singlet, and octet-octet transitions

\[ H_I = H_{so} + H_{os} + H_{oo} \]

\[ H_{so}(t) = -g \sqrt{\frac{1}{2N_c}} \int_{R,r} S^\dagger(R, r) O^a(R, r) r_i E^{i,a}(t, R) \]

\[ H_{os}(t) = -g \sqrt{\frac{1}{2N_c}} \int_{R,r} O^{a\dagger}(R, r) S(R, r) r_i E^{i,a}(t, R) \]

\[ H_{oo}(t) = -\frac{g}{2} d^{abc} \int_{R,r} O^{a\dagger}(R, r) O^c(R, r) r_i E^{i,b}(t, R) \]

where \( R = \frac{r_1 + r_2}{2} \), \( r = r_1 - r_2 \) for quark, anti-quark positions \( r_1, r_2 \)
Evolution Equations

Moving to the Schrödinger picture and decomposing into singlet and octet pieces:

\[
\begin{align*}
\frac{d\rho_s(t)}{dt} &= -i [h_s, \rho_s(t)] - \Sigma_s \rho_s(t) - \rho_s(t) \Sigma_s^\dagger + \Xi_{so}(\rho_o(t)) \\
\frac{d\rho_o(t)}{dt} &= -i [h_o, \rho_o(t)] - \Sigma_o \rho_o(t) - \rho_o(t) \Sigma_o^\dagger + \Xi_{os}(\rho_s(t)) \\
&\quad + \Xi_{oo}(\rho_o(t))
\end{align*}
\]

Where the $\Sigma$ and $\Xi$ contain the reservoir correlators and can be represented diagrammatically.

For $T \gg E$, energy dependent exponentials may be set to 1.
Diagrammatic Evolution Equation of $\rho_s(t)$

$$\frac{d\rho_s(t)}{dt} = -i [h_s, \rho_s(t)] - \Sigma_s \rho_s(t) - \rho_s(t) \Sigma_s^\dagger + \Xi_{so}(\rho_o(t))$$

$$\Sigma_s \rho_s(t) =$$

$$= \frac{g^2}{6N_c} \int_0^\infty dt \, r_j r_j \langle E^{i,a}(t, 0) E^{i,a}(0, 0) \rangle$$

$$\Xi_{so}(\rho_o(t)) =$$

$$= \frac{g^2}{6N_c} \int_0^\infty dt \, [r_j^i \rho_o(t) r^j \langle E^{i,a}(0, 0) E^{i,a}(t, 0) \rangle + \text{h.c.}]$$
Diagramatic Evolution of $\rho_o(t)$

$$\frac{d\rho_o(t)}{dt} = -i [h_o, \rho_o(t)] - \Sigma_o \rho_o(t) - \rho_o(t) \Sigma_o^\dagger + \Xi_{os}(\rho_s(t)) + \Xi_{oo}(\rho_o(t))$$

$\Sigma_o \rho_o(t) =$

$$= \frac{g^2}{6N_c(N^2_c - 1)} \int_0^\infty dt \left[ 1 + \frac{N^2_c - 4}{2} \right] r^j r^j \langle E^{i,a}(t, 0) E^{i,a}(0, 0) \rangle$$

$\Xi_{os}(\rho_s(t)) =$

$$= \frac{g^2}{6N_c} \int_0^\infty dt \left[ r^j \rho_s(t) r^j \langle E^{i,a}(0, 0) E^{i,a}(t, 0) \rangle + \text{h.c.} \right]$$

$\Xi_{os}(\rho_s(t)) =$

$$= \frac{g^2 (N^2_c - 4)}{12N_c(N^2_c - 1)} \int_0^\infty dt \left[ r^j \rho_s(t) r^j \langle E^{i,a}(0, 0) E^{i,a}(t, 0) \rangle + \text{h.c.} \right]$$
Master Equation

evolution equation

► can be written as

\[
\frac{d\rho}{dt} = -i[H, \rho] + \sum_{n,m} h_{nm} \left( L_i^n \rho L_i^{m\dagger} - \frac{1}{2} \{ L_i^{m\dagger} L_i^n, \rho \} \right),
\]

where \( H, \rho, \) and \( L_i \) are 2×2 matrices

► \( L_i \) depend on two reservoir correlators:

\[
\kappa = \frac{g^2}{6N_c} \int_0^\infty dt \langle \{ E^{a,i}(t,0), E^{a,i}(0,0) \} \rangle,
\]

\[
\gamma = -\frac{ig^2}{6N_c} \int_0^\infty dt \langle [ E^{a,i}(t,0), E^{a,i}(0,0) ] \rangle
\]

► as shown in Casalderrey-Solana, Teaney (2006), \( \kappa \) is the heavy quark momentum diffusion coefficient occurring in a Langevin equation
Langevin Dynamics

- description of Brownian motion, i.e., a particle moving randomly due to uncorrelated interactions with its environment

\[
\frac{d p_i}{dt} = -\eta_D p_i + \dot{\xi}_i(t), \quad \langle \dot{\xi}_i(t)\dot{\xi}_j(t') \rangle = \kappa \delta_{ij} \delta(t - t'), \quad \eta_D = \frac{\kappa}{2MT},
\]

where \( p_i \) is the momentum of the particle (heavy quark), \( \eta_D \) is the drag coefficient, and \( \dot{\xi}_i \) encodes the random, uncorrelated interactions of the particle with the medium

- integration of force-force correlator along the Schwinger-Keldysh contour gives \( \kappa \) in correlator form
Extraction of Transport Coefficients

- from the definitions of $\Sigma_s$, $\kappa$, and $\gamma$

\[ \Sigma_s = \frac{r^2}{2}(\kappa + i\gamma) \]

- from which

\[
\begin{align*}
    r^2\kappa &= \Sigma_s + \Sigma_s^\dagger = -2\text{Im}(-i\Sigma_s) \\
    r^2\gamma &= -i\Sigma_s + i\Sigma_s^\dagger = 2\text{Re}(-i\Sigma_s)
\end{align*}
\]

- relating the real and imaginary parts of the self energy to the width and the mass shift and projecting onto $1S$ states gives

\[
\Gamma(1S) = 3a_0^2\kappa, \quad \delta M(1S) = \frac{3}{2}a_0^2\gamma
\]

thus relating both $\kappa$ and $\gamma$ to quantities measurable on the lattice in unquenched simulations.
Previous Calculations of $\kappa$

- measured on the lattice in a quenched simulation Francis, et. al. (2015)
- can be extracted from experimental measurements of the azimuthal anisotropy coefficient $v_2$ at experiments, e.g., ALICE and STAR
- perturbative calculation done to NLO but suffers from convergence issues Caron-Huot, et. al. 2009
- correlator form of $\kappa$ can be related to the spectral function

$$\kappa = \frac{T}{6N_c} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

allowing for an unquenched lattice measurement
Unquenched Calculation of $\kappa$

- lower bound
  - 2+1 flavor measurement of $\Gamma$ of $\Upsilon(1S)$ with $M_b = 4.65$ GeV
    - Kim, et. al. 2018
  - at $T = 407$ MeV, $\Gamma = 23.6$ MeV
  - rescale $\Gamma$ to account for different bottom mass

\[
\Gamma = 23.6 \text{ MeV} \rightarrow \Gamma = \left( \frac{4.65}{4.78} \right)^2 23.6 \text{ MeV} = 22.3 \text{ MeV}
\]

- with $a_0 = 0.67$ GeV$^{-1}$

\[
\frac{\kappa}{T^3} = \frac{\Gamma}{3a_0^2 T^3} = 0.24
\]
Unquenched Calculation of $\kappa$

- upper bound
  - 2 flavor measurement of $\Gamma$ of $\Upsilon(1S)$ Aarts, et. al. 2011 with $M_b = 5$ GeV
  - at $T = 440$ MeV, $\Gamma = 440$ MeV
  - rescale $\Gamma$ to account for different bottom mass

$$\Gamma = 440 \text{ MeV} \rightarrow \Gamma = \left(\frac{5}{4.78}\right)^2 440 \text{ MeV} = 481 \text{ MeV}$$

- with $a_0 = 0.67 \text{ GeV}^{-1}$

$$\frac{\kappa}{T^3} = \frac{\Gamma}{3a_0^2 T^3} = 4.2$$

- combined bounds on unquenched estimate of $\kappa$

$$0.24 \lesssim \frac{\kappa}{T^3} \lesssim 4.2$$
Current Estimates of $\kappa$

Figure: Current estimates of $\kappa$ from (top to bottom) the thermal width, quenched lattice measurement, and experimental measurements.
Calculation of $\gamma$

- one prior perturbative calculation Brambilla, et. al. 2008
  - thermal correction to singlet potential $V_s(r)$
- lattice measurements of the thermal mass shift of the $\Upsilon(1S)$ at $T = 251$ MeV and 407 MeV and the $J/\psi$ at $T = 251$ MeV from Kim, et. al. 2018
  - same mass scaling as for $\kappa$
  - total spread of results to place bounds:

$$-3.8 \leq \frac{\gamma}{T^3} \leq -0.7$$

- correlator form of $\gamma$ can be related to spectral function

$$\gamma = -\frac{1}{3N_c} \int_0^\infty \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega}$$

- allows for lattice measurement
- divergent; infinite vacuum contribution must be subtracted
Current Estimates of $\gamma$

$J/\psi, T = 251$ MeV

$\Upsilon(1S), T = 407$ MeV

$\Upsilon(1S), T = 251$ MeV

$n_f = 3, T = 407$ MeV
(perturbation theory)

$n_f = 3, T = 251$ MeV
(perturbation theory)

Figure: Current estimates of $\gamma$ from the thermal width (black) and perturbation theory (blue).
Conclusions

- to compute nuclear modification factor $R_{AA}$, out of equilibrium evolution of in medium quarkonium needed
- this evolution specified by two parameters, $\kappa$ and $\gamma$
- $\kappa$ is the heavy quark momentum diffusion coefficient describing in medium momentum equilibration of a heavy quark
- $\gamma$ new parameter
- expressions for $\kappa$ and $\gamma$ in terms of gluo-electric correlators allow them to be related to quantities measurable in unquenched lattice simulations
- expressions for $\kappa$ and $\gamma$ in terms of the spectral function allow for them to be measured directly in unquenched lattice simulations
- new unquenched estimates of $\kappa$ in agreement with prior estimates from other sources
Thank you!