Splittings of low-lying charmonium masses at the physical point

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Work with C. DeTar, A. Kronfeld, S.h. Lee, J. Simone
Fermilab Lattice and MILC collaborations

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Low-lying charmonium: A precision benchmark for LQCD

Fermilab Lattice and MILC Collaborations, PR**D** 99, 034509 (2019)

- Well understood from models and well determined in experiment
- Spin-dependent mass splittings extremely sensitive to the charm-quark mass and heavy-quark discretization $\rightarrow$ good benchmark

meson	mass	width
η_c	2983.9(5)	32.0(8) MeV
J/Ψ	3096.900(6)	92.9(2.8) keV
χ_{c0}	3414.71(30)	10.8(6) MeV
χ_{c1}	3510.67(5)	0.84(4) MeV
χ_{c2}	3556.17(7)	1.97(9) MeV
h_c	3525.38(11)	0.7(4) MeV
$\eta_c(2S)$	3637.6(1.2)	$11.3^{(+3.2)}_{(-2.9)}$ MeV
$\Psi(2S)$	3686.097(25)	294(8) keV

Mass splittings and the heavy-quark potential

- We calculate the following mass splittings:

$$\Delta M_{\mathrm{HF}} = M_{J/\psi} - M_{\eta_c}$$

$$\begin{aligned}
\Delta M_{\text{1P-1S}} &= M_{\overline{\mathrm{1P}}} - M_{\overline{\mathrm{1S}}} \\
M_{\overline{\mathrm{1P}}} &= \tfrac{1}{9}(M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}}) \\
M_{\overline{\mathrm{1S}}} &= \tfrac{1}{4}(M_{\eta_c} + 3M_{J/\psi})
\end{aligned}$$

$$\begin{aligned}
\Delta M_{\text{spin-orbit}} &= \tfrac{1}{9}(5M_{\chi_{c2}} - 3M_{\chi_{c1}} - 2M_{\chi_{c0}}) \\
\Delta M_{\text{tensor}} &= \tfrac{1}{9}(3M_{\chi_{c1}} - M_{\chi_{c2}} - 2M_{\chi_{c0}}) \\
\Delta M_{\mathrm{1PHF}} &= M_{\overline{\mathrm{1P}}} - M_{h_c}
\end{aligned}$$

- Corresponds to separate terms in the potential derived from the heavy-quark limit:

$$\begin{aligned}
V_{\mathrm{tot}} &= V(r) + V_S(r)\boldsymbol{S}_Q \cdot \boldsymbol{S}_Q + V_T(r)S_{12} + V_{LS}(r)\boldsymbol{L} \cdot \boldsymbol{S}_Q \\
S_{12} &= 3(\boldsymbol{S}_Q \cdot \hat{\boldsymbol{r}})(\boldsymbol{S}_Q \cdot \hat{\boldsymbol{r}}) - \boldsymbol{S}_Q \cdot \boldsymbol{S}_Q
\end{aligned}$$

Lattice gauge ensembles and important prerequisites

- We use the MILC 2 + 1 flavor ASQTAD ensembles

$\approx a$ [fm]	m'_l/m'_s	size	N_{src}	κ_c	κ'_c
0.14	0.2	$16^3 \times 48$	2524	0.12237(26)(20)	0.1221
0.14	0.1	$20^3 \times 48$	2416	0.12231(26)(20)	0.1221
0.114	0.2	$20^3 \times 64$	4800	0.12423(15)(16)	0.12423
0.114	0.1	$24^3 \times 64$	3328	0.12423(15)(16)	0.1220, 0.1245, 0.1280
0.082	0.2	$28^3 \times 96$	1904	0.12722(9)(14)	0.12722
0.082	0.1	$40^3 \times 96$	4060	0.12714(9)(14)	0.12714
0.058	0.2	$48^3 \times 144$	2604	0.12960(4)(11)	0.1298
0.058	0.1	$64^3 \times 144$	1984	0.12955(4)(11)	0.1296
0.043	0.2	$64^3 \times 192$	3204	0.130921(16)(70)	0.1310

- Uses MILC's version of the Sommer scale $r_1 = 0.31174(216)$ fm

`Bazavov et al. PRD 85, 114506 (2012)`

- Charm-quark hopping parameter κ_c (charm-quark mass) from

`Bailey et al. PRD 89, 114504 (2014)`

Systematic uncertainties and features of the calculation

- Discretization effects are sizable
  - Fermilab prescription to minimize lattice artefacts in mass splittings

    El-Khadra, Kronfeld, Mackenzie, PRD 55, 3933 (1997)
    Oktay, Kronfeld, PRD 78, 014504 (2008)
  - Calculation at 5 lattice spacings
- (Heavy-)quark mass tuning is crucial
  - Correct runs for mistuned charm-quark mass
  - Correct mistuning of the light-quark masses in the chiral extrapolation
- Some mass-splittings are very sensitive to the lattice scale
  - Take into account both direct and indirect scale setting uncertainty
- Overall we achieve a much better quantification of systematic uncertainties than in a previous campaign

  Burch *et al.*, PRD 81, 034508 (2010)

Chiral and continuum extrapolations

- Fit Ansatz for the extrapolation to the physical point

$$\Delta M = \Delta M_0 + b(2x_l + x_s) + c_0 f_1(a, \alpha_s) + c_1 f_2(a, \alpha_s) + \cdots ,$$

$$x_l = \frac{m_l' - m_l}{m_s}, \qquad x_s = \frac{m_s' - m_s}{m_s}$$

- We determine the mass splitting ΔM_0
- Fit the leading behavior in the light and strange sea-quark mass

Example:

$\approx a$ [fm]	am_l'	am_s'	am_l	am_s	$\alpha_s(2/a)$
0.14	0.0097	0.0484	0.0015079	0.04185	0.35885
0.14	0.0048	0.0484	0.0015180	0.04213	0.36042

- Add terms for discretization effects
  - Leading and subleading heavy-quark discretization effects (see next slide)
  - Use $\alpha_s a^2$ term for sea quarks (improved action; ASQTAD)

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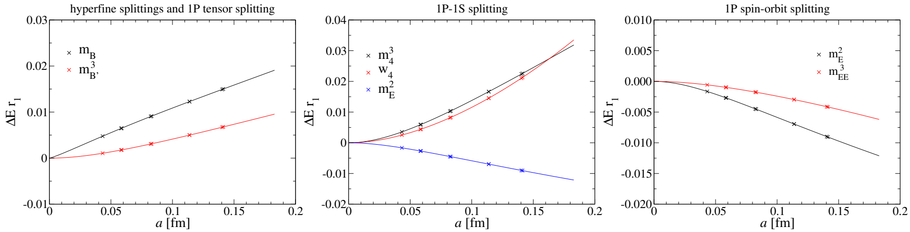
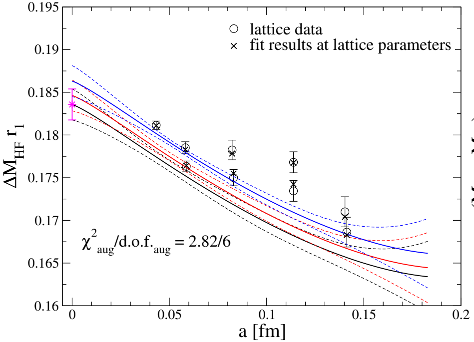
Illustration of heavy-quark discretization effects

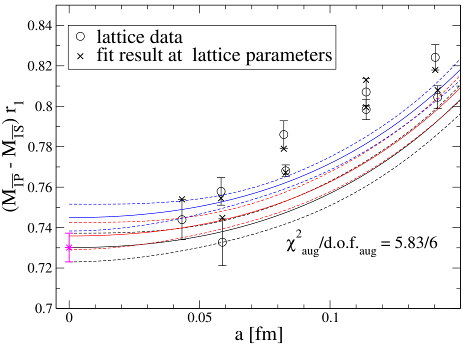


- Shapes and sizes of expected heavy-quark discretization uncertainties
  - NRQCD power counting using $v^2 = 0.3$ and $mv^2 \approx 420$ MeV $\approx$ 1P-1S splitting
  - terms denoted by the masses in the short-distance coefficients

    `Oktay, Kronfeld, PRD 78, 014504 (2008)`
  - Fits use priors of the size expected from the power counting

Results





- Black curve corresponds to physical pion masses
- Black crosses show fit results evaluated at lattice parameters

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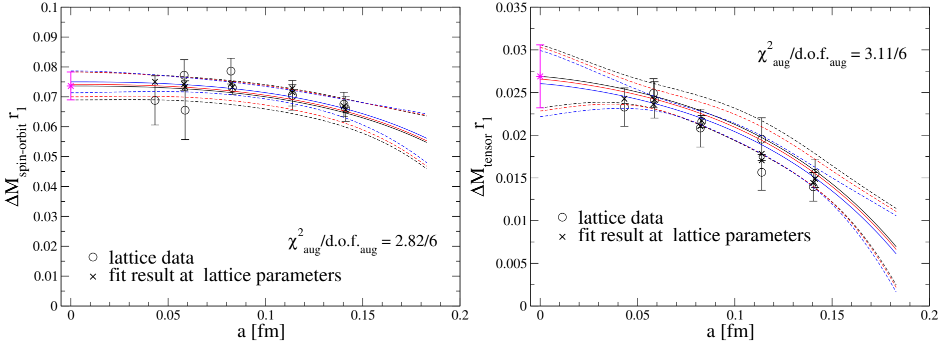

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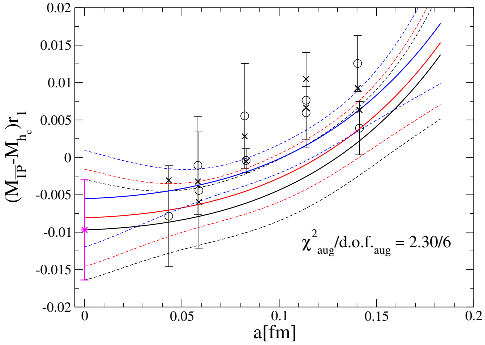

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Uncertainty budget

- We quote the statistical uncertainty after extrapolation and additional systematic uncertainties separately
- Systematic uncertainties on the mass splittings in MeV;
  (*) denotes small compared to statistical uncertainty

Source	1P-1S	1S HF	1P spin-orbit	1P tensor	1P HF
Slope in κ_c	0.2	0.2	0.1	0.2	(*)
Chiral-continuum fit shape	(*)	1.5	(*)	1.6	(*)
Lattice scale	3.3	1.6	0.9	0.1	(*)
Total	3.3	2.2	0.9	1.6	$<$0.1

- Additional stability checks:
  - Variations of the interpolator basis
  - Variations of the fitting procedure
  - Dependence of the results on the continuum fit priors used

Comparison to previous Collaboration results

Comparison to Burch *et al.*, PRD 81, 034508 (2010)

Mass difference	This analysis [MeV]	Burch *et al.* [MeV]
1S hyperfine	$116.2 \pm 1.1 \pm 2.2^{-1.5}_{-4.0}$	$116.0 \pm 7.4^{+2.6}_{-0}$
1P-1S splitting	$462.2 \pm 4.5 \pm 3.3$	$473 \pm 12^{+10}_{-0}$
1P spin-orbit	$46.6 \pm 3.0 \pm 0.9$	$43.3 \pm 6.6^{+1.0}_{-0}$
1P tensor	$17.0 \pm 2.3 \pm 1.6$	$15.0 \pm 2.3^{+0.3}_{-0}$
1P hyperfine	$-6.1 \pm 4.2 \pm 0.1$	–

- Significantly reduced statistical uncertainties
- Continuum extrapolation and systematic uncertainties more conservative/reliable
- These new results therefore supercede the old results

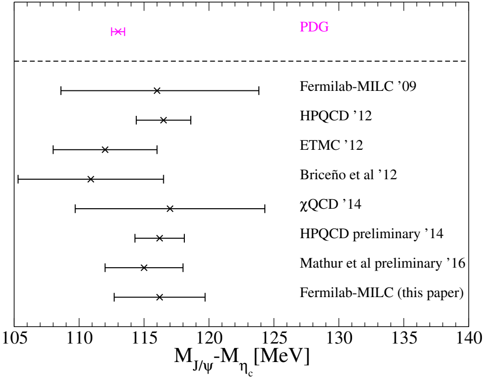
Comparison to experiment

Mass difference	This analysis [MeV]	Experiment [MeV]
1S hyperfine	$116.2 \pm 1.1 \pm 3.3^{-1.5}_{-4.0}$	113.0 ± 0.5
1P-1S splitting	$462.2 \pm 4.5 \pm 3.3$	456.64 ± 0.14
1P spin-orbit	$46.6 \pm 3.0 \pm 0.9$	46.60 ± 0.08
1P tensor	$17.0 \pm 2.3 \pm 1.6$	16.27 ± 0.07
1P hyperfine	$-6.1 \pm 4.2 \pm 0.1$	-0.09 ± 0.14

- Last error estimate on the hyperfine splitting from charm-annihilation contributions
- Overall excellent agreement

A comparison of hyperfine splittings

Fermilab Lattice and MILC Collaborations, PR**D** 99, 034509 (2019)



- All results at physical quark masses and in the continuum limit
- Lattice numbers exclude annihilation effects
- Estimate from data expects a shift of -1.5..-4.5 MeV

Levkova and DeTar, PRD 83 074504, 2011

Conclusions

- Determined mass splittings of low-lying charmonium states from LQCD
- Results at the physical point with improved control of systematic uncertainties
- Charm annihilation contributions are now a considerable uncertainty
  - Rigorous treatments as resonances decaying into light hadrons currently not possible
  - Likely novel methods needed. Interesting developments:

`Hansen, Meyer, Robaina, PRD 96, 094513 (2017)`

`Bulava, Hansen, arXiv:1903.11735`

- Studying spin-splittings in the B_c spectrum is interesting

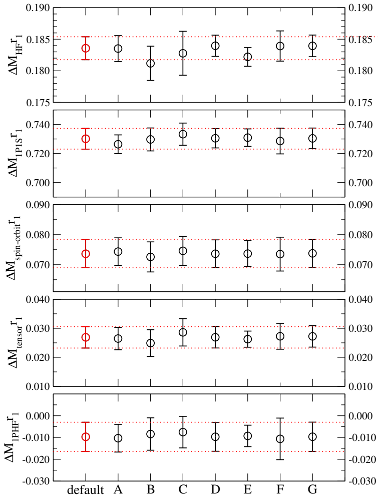
talk by Andrew Lytle

`Mathur, Padmanath, Mondal, PRL 121, 202002 (2018)`

- For progress on charmonium resonances: talk by S. Prelovsek

Thank you!

Systematic variations



A	same as "default" but using sea-quark discretization effects of order a^2 rather than $\alpha_s a^2$
B	results when omitting the lattice data at the coarsest lattice spacing
C	results when omitting the lattice data at the finest lattice spacing
D	result using just terms of order $\alpha_s a^2$ and a single shape for the heavy-quark discretization effects
E	heavy-quark discretization effects with priors for c_i half of the default width (0 ± 0.5)
F	heavy-quark discretization effects with priors for c_i double the default width (0 ± 2)
G	1σ variation of the κ_c slope used to shift data to physical κ_c