Splittings of low-lying charmonium masses at the physical point

Daniel Mohler

Work with C. DeTar, A. Kronfeld, S.h. Lee, J. Simone
Fermilab Lattice and MILC collaborations

Torino,
May 15th, 2019
Well understood from models and well determined in experiment
Spin-dependent mass splittings extremely sensitive to the charm-quark mass and heavy-quark discretization → good benchmark

<table>
<thead>
<tr>
<th>meson</th>
<th>mass</th>
<th>width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c$</td>
<td>2983.9(5)</td>
<td>32.0(8) MeV</td>
</tr>
<tr>
<td>$J/\Psi$</td>
<td>3096.900(6)</td>
<td>92.9(2.8) keV</td>
</tr>
<tr>
<td>$\chi_{c0}$</td>
<td>3414.71(30)</td>
<td>10.8(6) MeV</td>
</tr>
<tr>
<td>$\chi_{c1}$</td>
<td>3510.67(5)</td>
<td>0.84(4) MeV</td>
</tr>
<tr>
<td>$\chi_{c2}$</td>
<td>3556.17(7)</td>
<td>1.97(9) MeV</td>
</tr>
<tr>
<td>$h_c$</td>
<td>3525.38(11)</td>
<td>0.7(4) MeV</td>
</tr>
<tr>
<td>$\eta_c(2S)$</td>
<td>3637.6(1.2)</td>
<td>$11.3^{(+3.2)}_{(-2.9)}$ MeV</td>
</tr>
<tr>
<td>$\Psi(2S)$</td>
<td>3686.097(25)</td>
<td>294(8) keV</td>
</tr>
</tbody>
</table>
We calculate the following mass splittings:

\[ \Delta M_{\text{HF}} = M_{J/\psi} - M_{\eta_c} \]

\[ \Delta M_{1P-1S} = M_{1P} - M_{1S} \]

\[ M_{1P} = \frac{1}{9} (M_{\chi c_0} + 3M_{\chi c_1} + 5M_{\chi c_2}) \]

\[ M_{1S} = \frac{1}{4} (M_{\eta_c} + 3M_{J/\psi}) \]

\[ \Delta M_{\text{spin-orbit}} = \frac{1}{9} (5M_{\chi c_2} - 3M_{\chi c_1} - 2M_{\chi c_0}) \]

\[ \Delta M_{\text{tensor}} = \frac{1}{9} (3M_{\chi c_1} - M_{\chi c_2} - 2M_{\chi c_0}) \]

\[ \Delta M_{1PHF} = M_{1P} - M_{h_c} \]

Corresponds to separate terms in the potential derived from the heavy-quark limit:

\[ V_{\text{tot}} = V(r) + V_S(r)S_Q \cdot S_Q + V_T(r)S_{12} + V_{LS}(r)L \cdot S_Q \]

\[ S_{12} = 3(S_Q \cdot \hat{r})(S_Q \cdot \hat{r}) - S_Q \cdot S_Q \]
Lattice gauge ensembles and important prerequisites

- We use the MILC 2 + 1 flavor ASQTAD ensembles

<table>
<thead>
<tr>
<th>$\approx a$ [fm]</th>
<th>$m'_t/m'_s$</th>
<th>size</th>
<th>$N_{\text{src}}$</th>
<th>$\kappa_c$</th>
<th>$\kappa'_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>0.2</td>
<td>$16^3 \times 48$</td>
<td>2524</td>
<td>0.12237(26)(20)</td>
<td>0.1221</td>
</tr>
<tr>
<td>0.14</td>
<td>0.1</td>
<td>$20^3 \times 48$</td>
<td>2416</td>
<td>0.12231(26)(20)</td>
<td>0.1221</td>
</tr>
<tr>
<td>0.114</td>
<td>0.2</td>
<td>$20^3 \times 64$</td>
<td>4800</td>
<td>0.12423(15)(16)</td>
<td>0.12423</td>
</tr>
<tr>
<td>0.114</td>
<td>0.1</td>
<td>$24^3 \times 64$</td>
<td>3328</td>
<td>0.12423(15)(16)</td>
<td>0.1220, 0.1245, 0.1280</td>
</tr>
<tr>
<td>0.082</td>
<td>0.2</td>
<td>$28^3 \times 96$</td>
<td>1904</td>
<td>0.12722(9)(14)</td>
<td>0.12722</td>
</tr>
<tr>
<td>0.082</td>
<td>0.1</td>
<td>$40^3 \times 96$</td>
<td>4060</td>
<td>0.12714(9)(14)</td>
<td>0.12714</td>
</tr>
<tr>
<td>0.058</td>
<td>0.2</td>
<td>$48^3 \times 144$</td>
<td>2604</td>
<td>0.12960(4)(11)</td>
<td>0.1298</td>
</tr>
<tr>
<td>0.058</td>
<td>0.1</td>
<td>$64^3 \times 144$</td>
<td>1984</td>
<td>0.12955(4)(11)</td>
<td>0.1296</td>
</tr>
<tr>
<td>0.043</td>
<td>0.2</td>
<td>$64^3 \times 192$</td>
<td>3204</td>
<td>0.130921(16)(70)</td>
<td>0.1310</td>
</tr>
</tbody>
</table>

- Uses MILC’s version of the Sommer scale $r_1 = 0.31174(216)$ fm

  Bazavov et al. PRD 85, 114506 (2012)

- Charm-quark hopping parameter $\kappa_c$ (charm-quark mass) from

  Bailey et al. PRD 89, 114504 (2014)
Discretization effects are sizable
  - Fermilab prescription to minimize lattice artefacts in mass splittings
    El-Khadra, Kronfeld, Mackenzie, PRD 55, 3933 (1997)
    Oktay, Kronfeld, PRD 78, 014504 (2008)

Calculation at 5 lattice spacings

(Heavy-)quark mass tuning is crucial
  - Correct runs for mistuned charm-quark mass
  - Correct mistuning of the light-quark masses in the chiral extrapolation

Some mass-splittings are very sensitive to the lattice scale
  - Take into account both direct and indirect scale setting uncertainty

Overall we achieve a much better quantification of systematic uncertainties than in a previous campaign

Burch et al., PRD 81, 034508 (2010)
Chiral and continuum extrapolations

- Fit Ansatz for the extrapolation to the physical point

\[ \Delta M = \Delta M_0 + b(2x_l + x_s) + c_0 f_1(a, \alpha_s) + c_1 f_2(a, \alpha_s) + \cdots, \]

\[ x_l = \frac{m'_l - m_l}{m_s}, \quad x_s = \frac{m'_s - m_s}{m_s} \]

- We determine the mass splitting \( \Delta M_0 \)

- Fit the leading behavior in the light and strange sea-quark mass

<table>
<thead>
<tr>
<th>( \approx a [\text{fm}] )</th>
<th>( am'_l )</th>
<th>( am'_s )</th>
<th>( am_l )</th>
<th>( am_s )</th>
<th>( \alpha_s(2/a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example:</td>
<td>0.14</td>
<td>0.0097</td>
<td>0.0484</td>
<td>0.0015079</td>
<td>0.04185</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>0.0048</td>
<td>0.0484</td>
<td>0.0015180</td>
<td>0.04213</td>
</tr>
</tbody>
</table>

- Add terms for discretization effects
  - Leading and subleading heavy-quark discretization effects (see next slide)
  - Use \( \alpha_s a^2 \) term for sea quarks (improved action; ASQTAD)
Chiral and continuum extrapolations

- Fit Ansatz for the extrapolation to the physical point
  \[
  \Delta M = \Delta M_0 + b(2x_l + x_s) + c_0 f_1(a, \alpha_s) + c_1 f_2(a, \alpha_s) + \cdots ,
  \]
  \[
  x_l = \frac{m'_l - m_l}{m_s}, \quad x_s = \frac{m'_s - m_s}{m_s}
  \]

- We determine the mass splitting \( \Delta M_0 \)

- Fit the leading behavior in the light and strange sea-quark mass

<table>
<thead>
<tr>
<th>( \approx a ) [fm]</th>
<th>( am'_l )</th>
<th>( am'_s )</th>
<th>( am_l )</th>
<th>( am_s )</th>
<th>( \alpha_s(2/a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.14</td>
<td>0.0097</td>
<td>0.0484</td>
<td>0.0015079</td>
<td>0.04185</td>
<td>0.35885</td>
</tr>
<tr>
<td>0.14</td>
<td>0.0048</td>
<td>0.0484</td>
<td>0.0015180</td>
<td>0.04213</td>
<td>0.36042</td>
</tr>
</tbody>
</table>

- Add terms for discretization effects
  - Leading and subleading heavy-quark discretization effects (see next slide)
  - Use \( \alpha_s a^2 \) term for sea quarks (improved action; ASQTAD)
Chiral and continuum extrapolations

- Fit Ansatz for the extrapolation to the physical point

\[ \Delta M = \Delta M_0 + b(2x_l + x_s) + c_0 f_1(a, \alpha_s) + c_1 f_2(a, \alpha_s) + \cdots, \]

\[ x_l = \frac{m'_l - m_l}{m_s}, \quad x_s = \frac{m'_s - m_s}{m_s} \]

- We determine the mass splitting \( \Delta M_0 \)

- Fit the leading behavior in the light and strange sea-quark mass

<table>
<thead>
<tr>
<th>( \approx a \quad [\text{fm}] )</th>
<th>( am'_l )</th>
<th>( am'_s )</th>
<th>( am_l )</th>
<th>( am_s )</th>
<th>( \alpha_s(2/a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example:</td>
<td>0.14</td>
<td>0.0097</td>
<td>0.0484</td>
<td>0.0015079</td>
<td>0.04185</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>0.0048</td>
<td>0.0484</td>
<td>0.0015180</td>
<td>0.04213</td>
</tr>
</tbody>
</table>

- Add terms for discretization effects
  - Leading and subleading heavy-quark discretization effects (see next slide)
  - Use \( \alpha_s a^2 \) term for sea quarks (improved action; ASQTAD)
Shapes and sizes of expected heavy-quark discretization uncertainties

- NRQCD power counting using \( v^2 = 0.3 \) and \( mv^2 \approx 420 \text{ MeV} \approx 1P-1S \) splitting
- terms denoted by the masses in the short-distance coefficients

Oktay, Kronfeld, PRD 78, 014504 (2008)

- Fits use priors of the size expected from the power counting
Results

- Black curve corresponds to physical pion masses
- Black crosses show fit results evaluated at lattice parameters
Results

- Black curve corresponds to physical pion masses
- Black crosses show fit results evaluated at lattice parameters
Black curve corresponds to physical pion masses
Black crosses show fit results evaluated at lattice parameters
Uncertainty budget

- We quote the statistical uncertainty after extrapolation and additional systematic uncertainties separately.

- Systematic uncertainties on the mass splittings in MeV; (*) denotes small compared to statistical uncertainty.

<table>
<thead>
<tr>
<th>Source</th>
<th>1P-1S</th>
<th>1S HF</th>
<th>1P spin-orbit</th>
<th>1P tensor</th>
<th>1P HF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope in $\kappa_c$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>(*)</td>
</tr>
<tr>
<td>Chiral-continuum fit shape</td>
<td>(*)</td>
<td>1.5</td>
<td>(*)</td>
<td>1.6</td>
<td>(*)</td>
</tr>
<tr>
<td>Lattice scale</td>
<td>3.3</td>
<td>1.6</td>
<td>0.9</td>
<td>0.1</td>
<td>(*)</td>
</tr>
<tr>
<td>Total</td>
<td>3.3</td>
<td>2.2</td>
<td>0.9</td>
<td>1.6</td>
<td>&lt;0.1</td>
</tr>
</tbody>
</table>

- Additional stability checks:
  - Variations of the interpolator basis
  - Variations of the fitting procedure
  - Dependence of the results on the continuum fit priors used
Comparison to previous Collaboration results

<table>
<thead>
<tr>
<th>Mass difference</th>
<th>This analysis [MeV]</th>
<th>Burch et al. [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S hyperfine</td>
<td>$116.2 \pm 1.1 \pm 2.2^{+1.5}_{-4.0}$</td>
<td>$116.0 \pm 7.4^{+2.6}_{-0}$</td>
</tr>
<tr>
<td>1P-1S splitting</td>
<td>$462.2 \pm 4.5 \pm 3.3$</td>
<td>$473 \pm 12^{+10}_{-0}$</td>
</tr>
<tr>
<td>1P spin-orbit</td>
<td>$46.6 \pm 3.0 \pm 0.9$</td>
<td>$43.3 \pm 6.6^{+1.0}_{-0}$</td>
</tr>
<tr>
<td>1P tensor</td>
<td>$17.0 \pm 2.3 \pm 1.6$</td>
<td>$15.0 \pm 2.3^{+0.3}_{-0}$</td>
</tr>
<tr>
<td>1P hyperfine</td>
<td>$-6.1 \pm 4.2 \pm 0.1$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

- Significantly reduced statistical uncertainties
- Continuum extrapolation and systematic uncertainties more conservative/reliable
- These new results therefore supercede the old results
Comparison to experiment

<table>
<thead>
<tr>
<th>Mass difference</th>
<th>This analysis [MeV]</th>
<th>Experiment [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S hyperfine</td>
<td>$116.2 \pm 1.1 \pm 3.3^{+1.5}_{-4.0}$</td>
<td>$113.0 \pm 0.5$</td>
</tr>
<tr>
<td>1P-1S splitting</td>
<td>$462.2 \pm 4.5 \pm 3.3$</td>
<td>$456.64 \pm 0.14$</td>
</tr>
<tr>
<td>1P spin-orbit</td>
<td>$46.6 \pm 3.0 \pm 0.9$</td>
<td>$46.60 \pm 0.08$</td>
</tr>
<tr>
<td>1P tensor</td>
<td>$17.0 \pm 2.3 \pm 1.6$</td>
<td>$16.27 \pm 0.07$</td>
</tr>
<tr>
<td>1P hyperfine</td>
<td>$-6.1 \pm 4.2 \pm 0.1$</td>
<td>$-0.09 \pm 0.14$</td>
</tr>
</tbody>
</table>

- Last error estimate on the hyperfine splitting from charm-annihilation contributions
- Overall excellent agreement
All results at physical quark masses and in the continuum limit
Lattice numbers exclude annihilation effects
Estimate from data expects a shift of -1.5..-4.5 MeV

Levkova and DeTar, PRD 83 074504, 2011
Conclusions

- Determined mass splittings of low-lying charmonium states from LQCD
- Results at the physical point with improved control of systematic uncertainties
- Charm annihilation contributions are now a considerable uncertainty
  - Rigorous treatments as resonances decaying into light hadrons currently not possible
  - Likely novel methods needed. Interesting developments:
    Hansen, Meyer, Robaina, PRD 96, 094513 (2017)
    Bulava, Hansen, arXiv:1903.11735
- Studying spin-splittings in the $B_c$ spectrum is interesting
  talk by Andrew Lytle
  Mathur, Padmanath, Mondal, PRL 121, 202002 (2018)
- For progress on charmonium resonances: talk by S. Prelovsek
Thank you!
Systematic variations

same as “default” but using sea-quark discretization
effects of order $a^2$ rather than $\alpha_s a^2$
results when omitting the lattice data
at the coarsest lattice spacing
results when omitting the lattice data
at the finest lattice spacing
result using just terms of order $\alpha_s a^2$ and a single
shape for the heavy-quark discretization effects
heavy-quark discretization effects with priors
for $c_i$ half of the default width ($0 \pm 0.5$)
heavy-quark discretization effects with priors
for $c_i$ double the default width ($0 \pm 2$)
$1\sigma$ variation of the $\kappa_c$ slope
used to shift data to physical $\kappa_c$