Covariant Verification of a Two-loop test of NRQCD Factorization

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Outline

- Motivation
- Method of calculation
- Results of calculation
- Conclusion

Motivation:

Verification of universality of NRQCD LDMEs to two loops

Gauge-completed NRQCD LDMEs

- Nayak, Qiu, Sterman (NQS) (2005, 2006):
  Color-octet NRQCD long-distance matrix elements (NRQCD LDMEs) should be generalized to include Wilson (eikonal) lines for gauge-invariant definition of them:

NQS, PLB, 2005
NQS, PRD, 2005
NQS, PRD, 2007

$$\langle \mathcal{O}_n^H \rangle = \langle 0 | \chi^\dagger(0) \kappa_{n,c} \psi(0) \mathcal{P}_{H(P)} \psi^\dagger(0) \kappa'_{n,c} \chi(0) | 0 \rangle$$

$\Big\downarrow$ gauge completion $\Phi_\ell^{(A)}(0)$

$$\langle \mathcal{O}_n^H \rangle = \langle 0 | \chi^\dagger(0) \kappa_{n,c} \psi(0) \Phi_\ell^{(A)\dagger}(0)_{cb} \mathcal{P}_{H(P)} \Phi_\ell^{(A)}(0)_{ba} \psi^\dagger(0) \kappa'_{n,a} \chi(0) | 0 \rangle$$

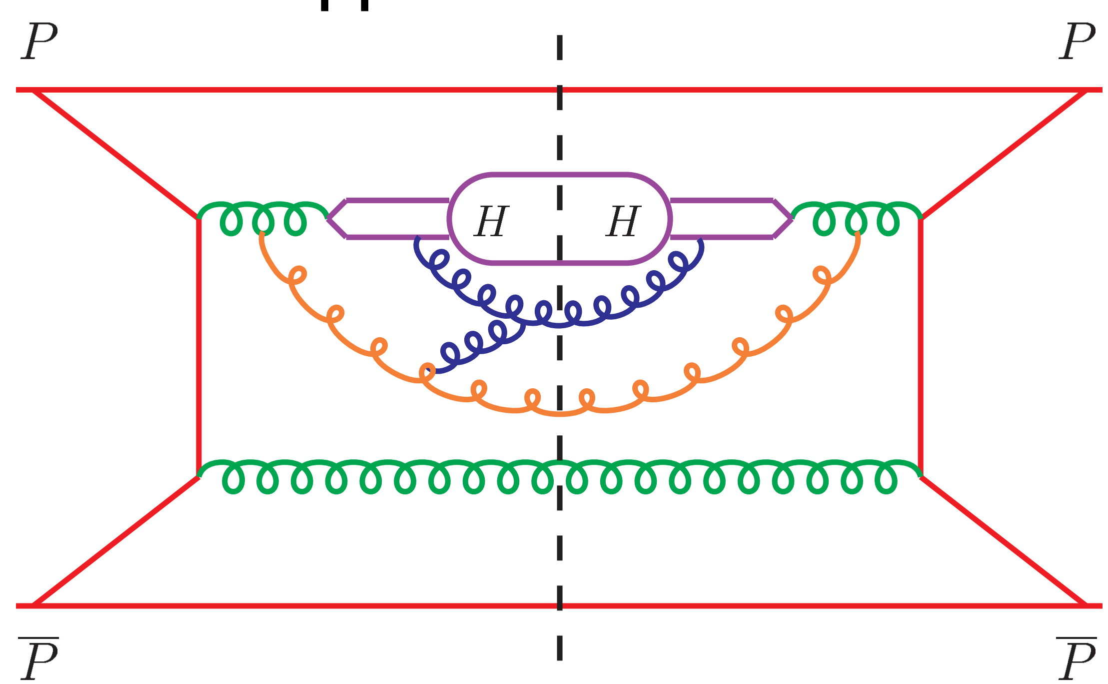
- The Wilson lines $\Phi_\ell^{(A)}(0)_{ba}$ are path-ordered exponential, constructed from the gauge field in adjoint matrix representations $A_\mu^{(A)} = \sum_a T_a^{(A)} A_{\mu,a}$

$$\Phi_\ell^{(A)}(0)_{ba} = \mathrm{P}\exp\left[-ig_s \int_0^\infty d\lambda \ell \cdot A^{(A)}(\ell\lambda) \right]_{ba}$$

ℓ is the velocity of the source, and its direction is arbitrary

Eikonal line

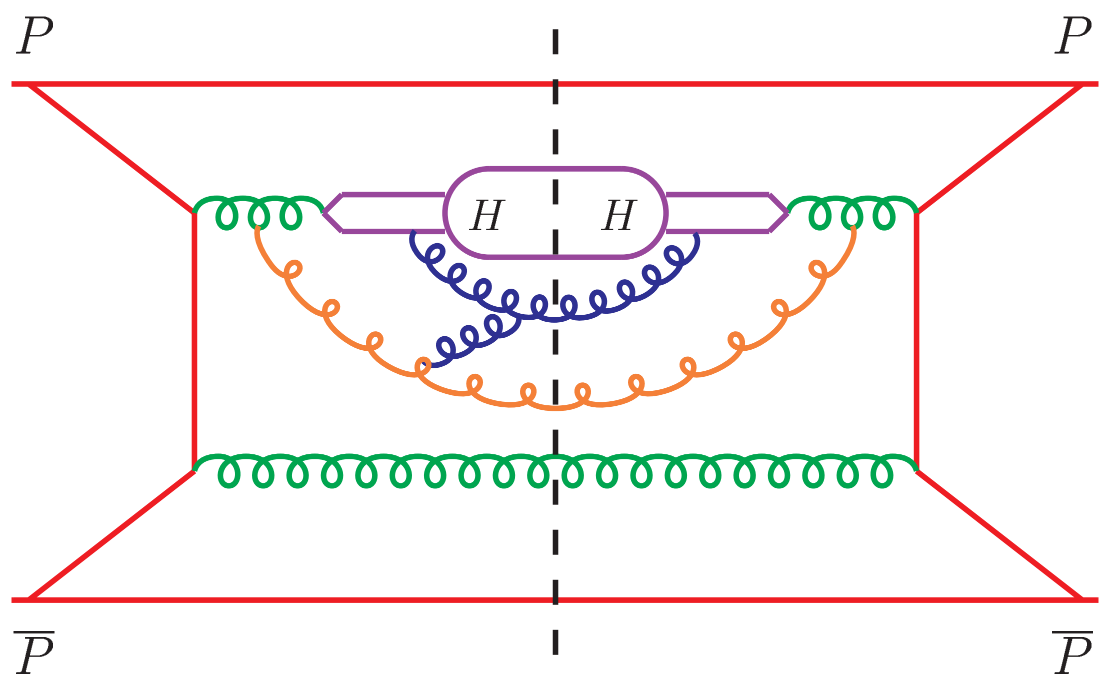
- The operator $\Phi_\ell^{(A)}(0)$ represents eikonal lines in Feynman diagrams
- This eikonal line makes NRQCD LDMEs gauge invariant
- A semi-hard gluon (orange) with energy of order m_Q in $Q\bar{Q}$ rest frame can connect to the $Q\bar{Q}$ pair through soft gluon exchanges (blue). The eikonal line is the soft approximation for it



Bodwin, QWG 2013

- The soft gluons are nonperturbative so they must be absorbed into the NRQCD LDMEs, and the only way to do is to introduce the Wilson line part of the LDMEs

Universality of NRQCD LDMEs



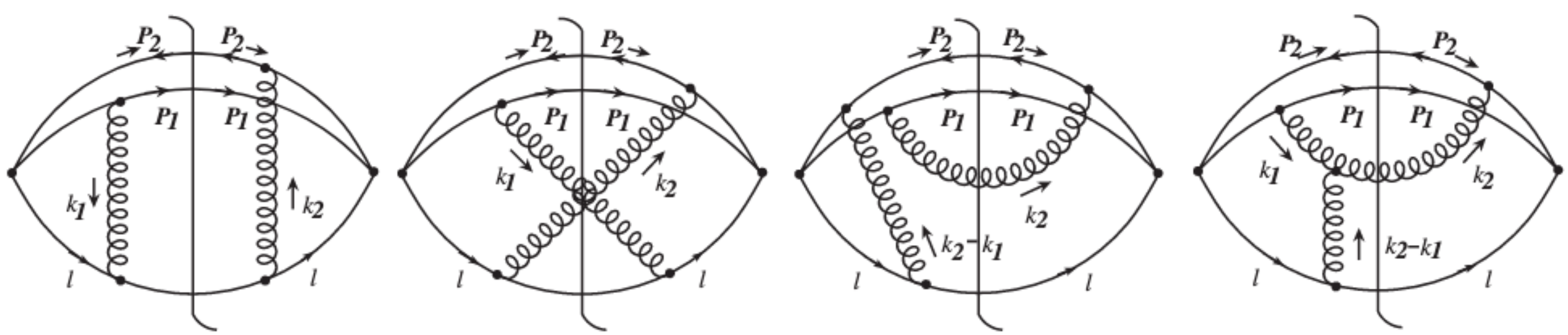
- The direction of Wilson line is the direction of the semi-hard gluon
- If the corresponding NRQCD LDME were to depend on the direction of the Wilson line, then there would be a different LDME for each direction of the hard gluon, that is, no universality of the LDME.
- So, as a necessary condition for NRQCD factorization to hold, we require the IR poles of gauge-completed NRQCD LDMEs to be independent of the direction of Wilson line ℓ

Covariant check of NRQCD factorization

- Using the light-cone variable integrations, Nayak, Qiu, and Sterman computed the IR poles of the fragmentation function for $g \to Q\bar{Q}^{[1]} + X$ to two loops.

NQS, PLB, 2005
NQS, PRD, 2005
NQS, PRD, 2007

- NQS found that there are non-canceled IR poles from the non-topologically factorized diagrams at NNLO, and they are independent of ℓ



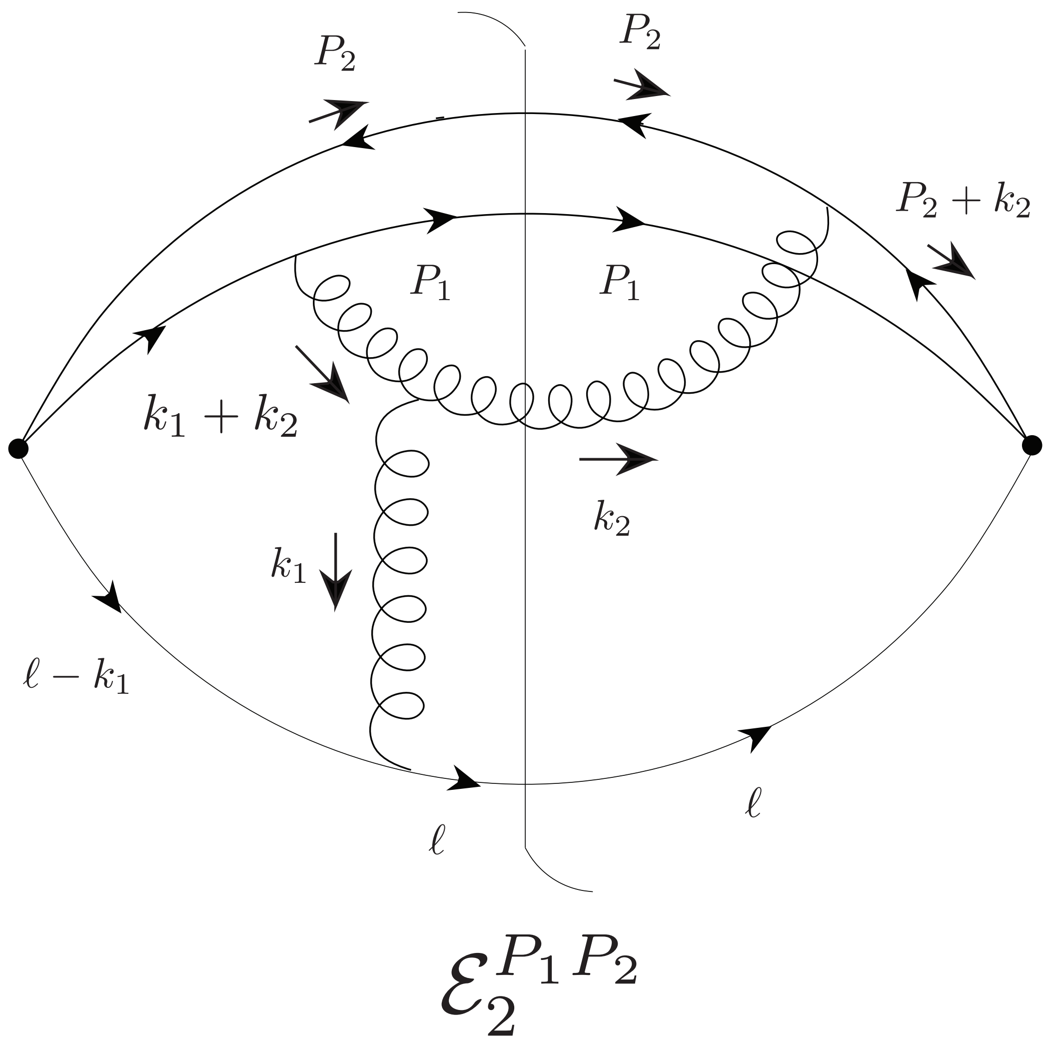
Examples of the non-topologically factorized two loop diagrams

- Our motivation is to provide independent check of the complicated light-cone calculation using the Lorentz covariant integrations.

Method of calculations:

Lorentz covariant integrations

Kinematics



One of NNLO diagrams of NRQCD LDMEs
for octet to singlet transition

- P_1 and P_2 are the momenta of quark and antiquark (top), respectively, and the eikonal gluon (bottom) carries ℓ momentum
- $\sum_i k_i$ is the total momentum carried by the soft gluons making octet $Q\bar{Q}$ pair into color singlet
- our k_1 is $k_1 - k_2$ in NQS
- The available Lorentz invariant quantities are

$$\underbrace{P_2^2 = P_1^2, \quad \ell^2 = 0, \quad a = \frac{P_1 \cdot P_2}{P_1^2},}_{\ell\text{-independent}} \quad \underbrace{c = \frac{P_1 \cdot \ell}{P_1^2}, \quad d = \frac{P_2 \cdot \ell}{P_1^2}}_{\ell\text{-dependent}}$$

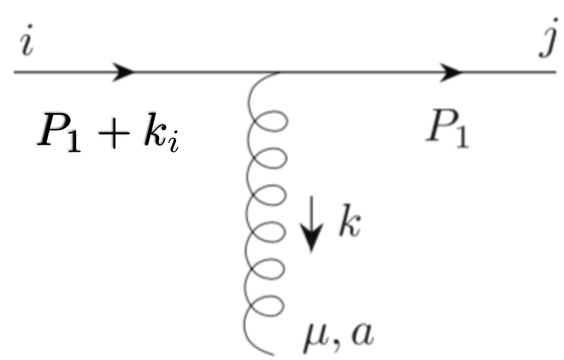
$\boxed{\text{We will show the } \ell\text{-independence of the remaining IR poles}}$

Soft approximation

- Since we are considering IR poles, we take the soft approximation for k_i, because the soft approximation reproduces all IR poles.

- In this approximation, perturbative Feynman rules for quark-gluon interaction are modified as

 * replace Q and $\bar{Q}$ propagators and vertices with eikonal propagators and vertices



$$\frac{i}{P_1 \cdot \boldsymbol{k}_i + i\varepsilon}, \quad \pm i g_s (T^a)_{ji} P_1^\mu,$$

+ for quark and - for antiquark

Covariant phase-space integrations

- The soft approximation removes the kinematic bound on the phase space, so the phase-space integrations of final-state real gluons are given by
$$\int \frac{d^d k_i}{(2\pi)^d} 2\pi\delta(k_i^2)\theta(k_i^0) f(k_i) \equiv \int_{k_i} \mathrm{PS}\, f(k_i)$$

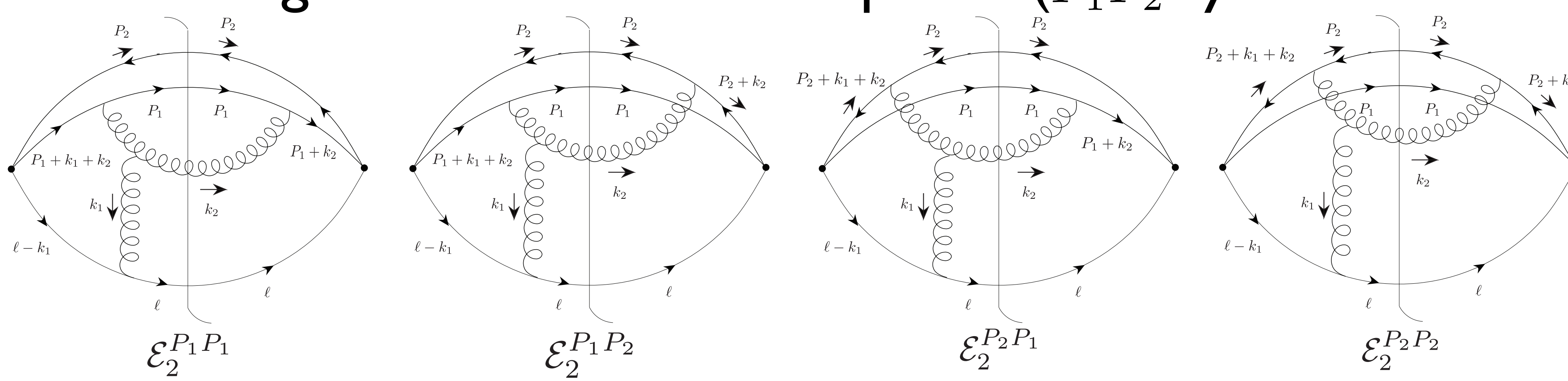
- Applying Feynman parameters, we can show that the phase-space integrations are written in the following forms, where p and M^2 are functions of external momenta and Feynman parameters

- In fact, the phase-space integrations for our problems can be done in terms of the following two integral tables:

$$\int_{k_i} \mathrm{PS} \frac{1}{(2p\cdot k_i + M^2 \pm i\varepsilon)^s} = \frac{1}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma(s-d+2)\Gamma(\frac{d}{2}-1)}{\Gamma(s)} \frac{1}{(p^2 \pm i\varepsilon)^{\frac{d}{2}-1}} \frac{1}{(M^2 \pm i\varepsilon)^{s-d+2}},$$
$$\int_{k_i} \mathrm{PS} \frac{k_i^\mu}{(2p\cdot k_i + M^2 \pm i\varepsilon)^s} = \frac{1}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma(s-d+1)\Gamma(\frac{d}{2})}{\Gamma(s)} \frac{p^\mu}{(p^2 \pm i\varepsilon)^{\frac{d}{2}}} \frac{1}{(M^2 \pm i\varepsilon)^{s-d+1}}.$$

Method of calculations

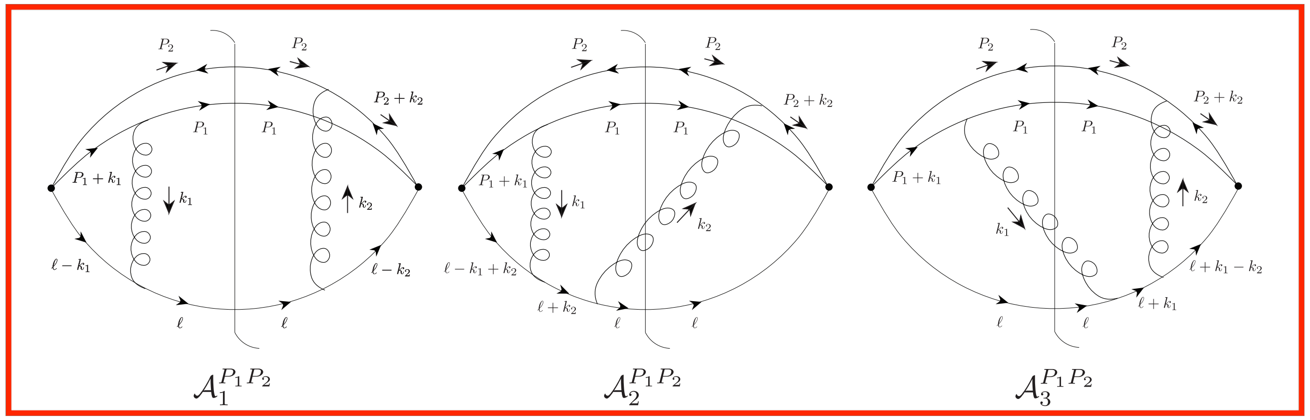
- Compute cut diagrams with soft approximation, with Feynman gauge
- Identify the IR poles by introducing covariant UV cutoffs
- Use dimensional regularizations to regulate IR and UV divergences
- Perform loop and phase-space integrals in covariant way and the remaining parameter integrations using standard techniques for multi-loop integrals such as MB integrals and sector decomposition
- Sum over all of gluon connections to quarks ($P_1 P_2$ symmetrization)

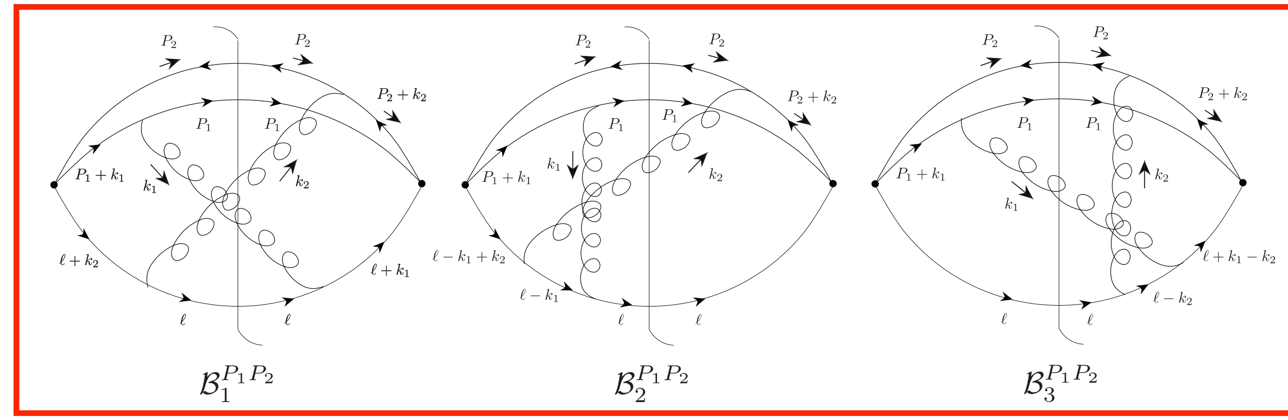


- We need only the 2 times of real parts of the diagrams because we should sum the corresponding complex conjugate cut diagrams
- Therefore, for diagram type $\mathcal{I}$, we compute $\mathcal{I} \equiv 2\mathrm{Re}\left(\sum_{i,j} \mathcal{I}^{P_i P_j}\right)$

Two-loop Abelian diagrams

- We have 4 types of Abelian two-loop diagrams:

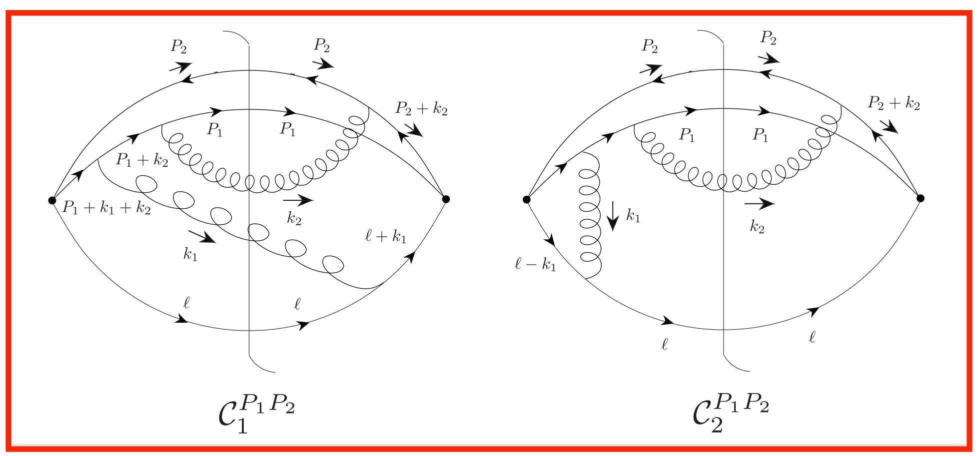


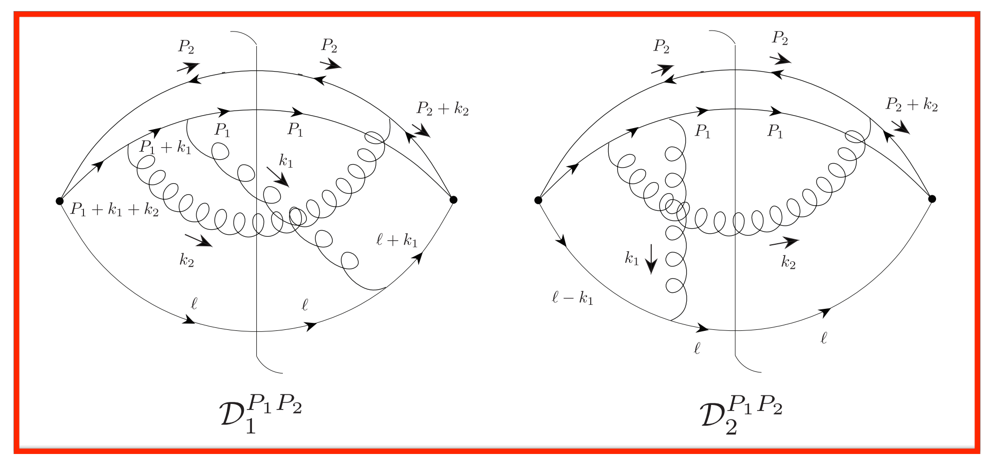


$$\mathcal{A}^{P_1P_2} \equiv \sum_{\text{cut } i} \mathcal{A}_i^{P_1P_2}$$

no single gluon connection between quarks

$$\mathcal{B}^{P_1P_2} \equiv \sum_{\text{cut } i} \mathcal{B}_i^{P_1P_2}$$





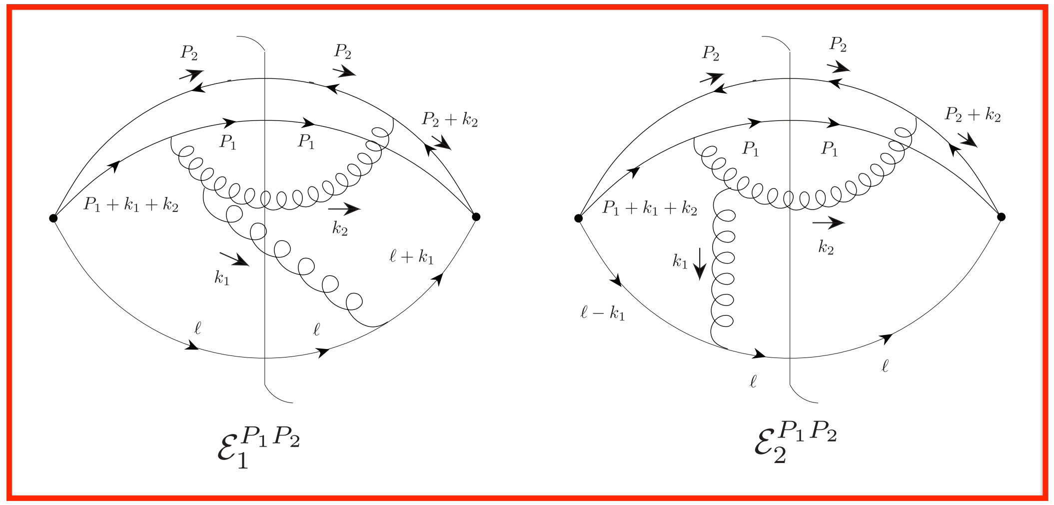
$$\mathcal{C}^{P_1P_2} \equiv \sum_{\text{cut } i} \mathcal{C}_i^{P_1P_2}$$

gluon connection between quarks

$$\mathcal{D}^{P_1P_2} \equiv \sum_{\text{cut } i} \mathcal{D}_i^{P_1P_2}$$

IR poles emerge only from the soft gluon connections between two quarks

Two-loop non-Abelian diagrams



$$\mathcal{E}^{P_1P_2} \equiv \sum_{\text{cut } i} \mathcal{E}_i^{P_1P_2}$$

- Rearranging numerator terms due to triple-gluon vertex, we write $\mathcal{E}_1^{P_1P_2}$

$$\mathcal{E}_1^{P_1P_2} = \mathcal{E}_{10} + \mathcal{E}'_{10} + \mathcal{E}_{11} + \mathcal{E}_{12}$$

$$\mathcal{E}_{10} = -g_s^4\mu^{4\epsilon}\int_{k_1}\text{PS}\int_{k_2}\text{PS}\frac{(P_1\cdot\ell)}{[P_1\cdot(k_1+k_2)+i\varepsilon](\ell\cdot k_1-i\varepsilon)[(k_1+k_2)^2+i\varepsilon]},$$

canceled by part of $\mathcal{E}_1^{P_1P_1}$

$$\mathcal{E}'_{10} = -g_s^4\mu^{4\epsilon}\int_{k_1}\text{PS}\int_{k_2}\text{PS}\frac{(P_2\cdot l)}{(P_2\cdot k_2-i\varepsilon)(\ell\cdot k_1-i\varepsilon)[(k_1+k_2)^2+i\varepsilon]},$$

canceled by part of $\mathcal{E}_1^{P_2P_2}$

$$\mathcal{E}_{11} = g_s^4\mu^{4\epsilon}\int_{k_1}\text{PS}\int_{k_2}\text{PS}\frac{(k_1\cdot\ell)(P_1\cdot P_2)+2(k_1\cdot P_1)(P_2\cdot\ell)-2(k_1\cdot P_2)(P_1\cdot\ell)}{(P_2\cdot k_2-i\varepsilon)[P_1\cdot(k_1+k_2)+i\varepsilon](\ell\cdot k_1-i\varepsilon)[(k_1+k_2)^2+i\varepsilon]},$$

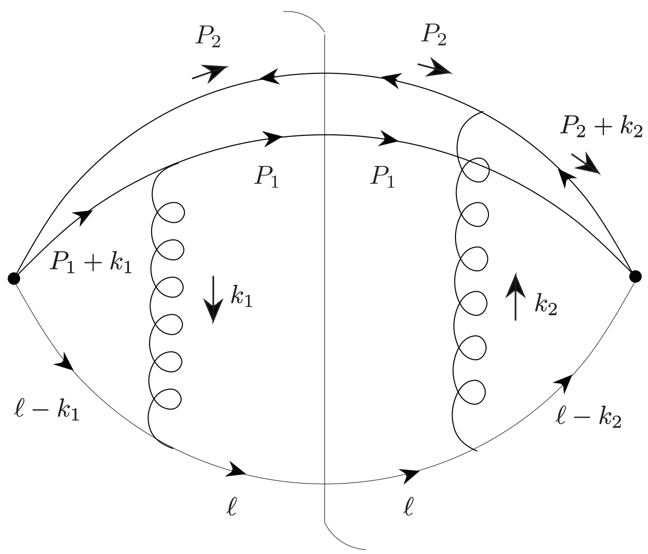
$$\mathcal{E}_{12} = g_s^4\mu^{4\epsilon}\int_{k_1}\text{PS}\int_{k_2}\text{PS}\frac{2(k_2\cdot\ell)(P_1\cdot P_2)}{(P_2\cdot k_2-i\varepsilon)[P_1\cdot(k_1+k_2)+i\varepsilon](\ell\cdot k_1-i\varepsilon)[(k_1+k_2)^2+i\varepsilon]}$$

- In this rearrangement, we can eliminate the collinear to ℓ singularities even before performing integrations, and so we can remove the most singular divergence $1/\epsilon^4$. But, we have to deal with $1/\epsilon^3$ and $1/\epsilon^2$ poles

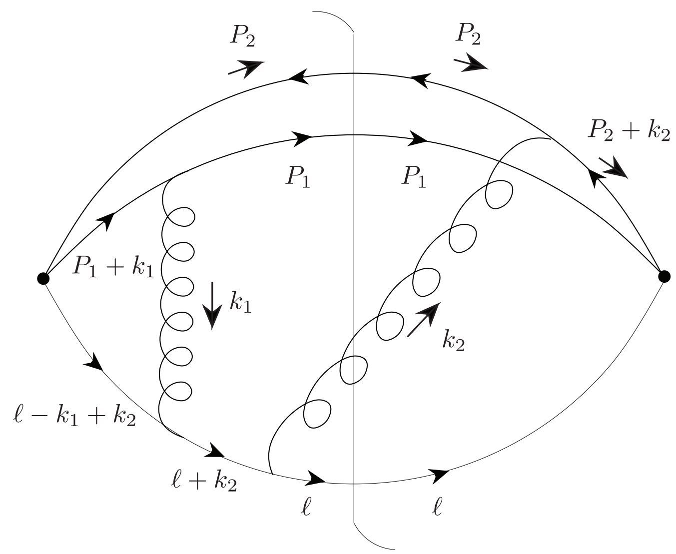
Results of calculations:

uncanceled IR poles

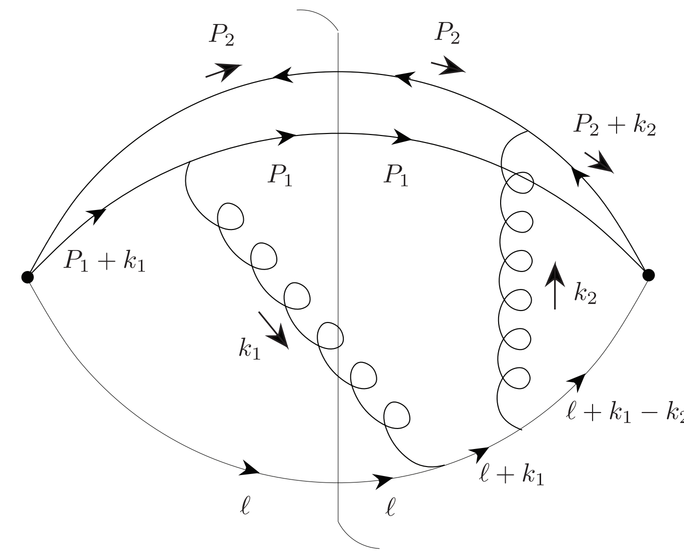
Diagram $\mathcal{A}$



$\mathcal{A}_1^{P_1 P_2}$



$\mathcal{A}_2^{P_1 P_2}$

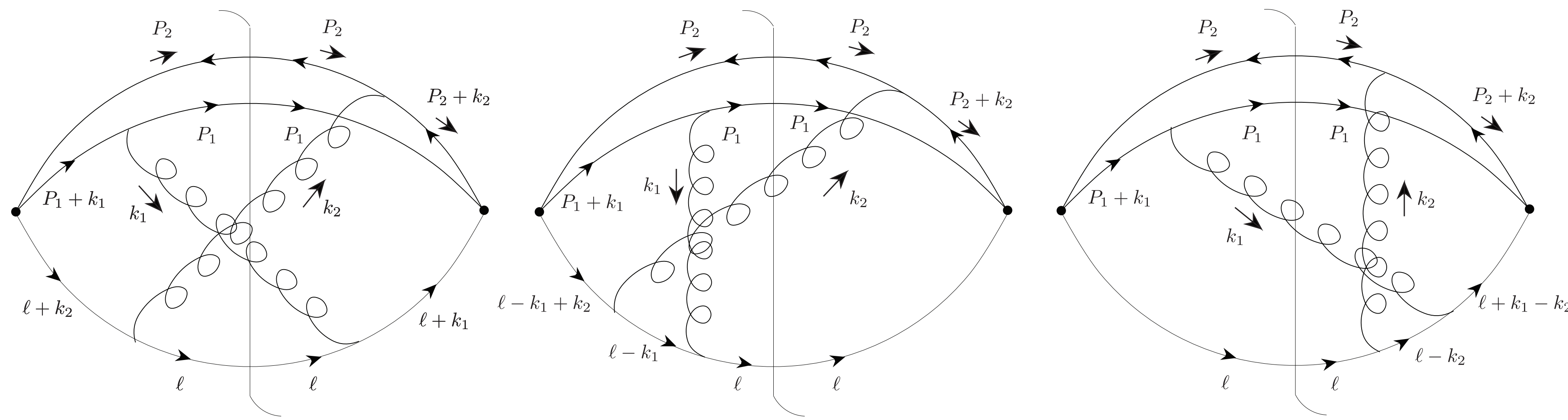


$\mathcal{A}_3^{P_1 P_2}$

- The color factors of these diagrams vanish → no IR poles

$$\mathcal{A} \equiv 2\mathrm{Re}\left(\sum_{i,j} \mathcal{A}^{P_i P_j}\right)_{\mathrm{color}} = 0$$

Diagram $\mathcal{B}$

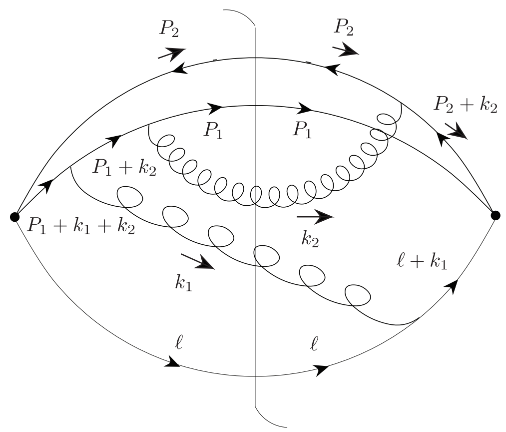


$\mathcal{B}_1^{P_1P_2}$ $\mathcal{B}_2^{P_1P_2}$ $\mathcal{B}_3^{P_1P_2}$

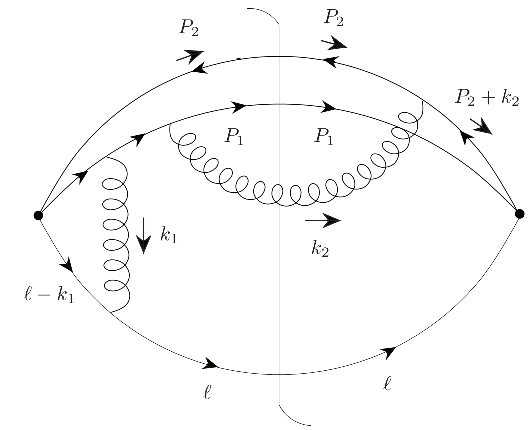
- After P_1P_2 symmetrization and summation over cut diagrams, we find that the remaining poles are pure imaginary, so there are no remaining real IR poles

$$\mathcal{B} \equiv 2\mathrm{Re}\left(\sum_{i,j}\mathcal{B}^{P_iP_j}\right)_{\mathrm{color}} = O(\epsilon^0)$$

Diagram $\mathcal{C}$



$\mathcal{C}_1^{P_1 P_2}$



$\mathcal{C}_2^{P_1 P_2}$

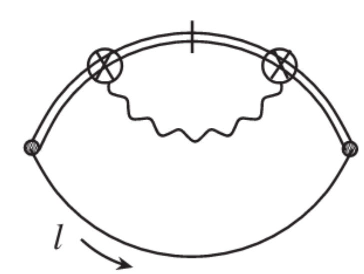
color factors

$$C_{\mathcal{C}_i^{P_1P_2}} = C_{\mathcal{C}_i^{P_1P_1}} = +\frac{iN_c(N_c^2-1)}{8},$$
$$C_{\mathcal{C}_i^{P_2P_1}} = C_{\mathcal{C}_i^{P_2P_2}} = -\frac{iN_c(N_c^2-1)}{8}.$$

- The first diagrams that yield the uncanceled IR poles
- After $P_1 P_2$ symmetrization, we obtain

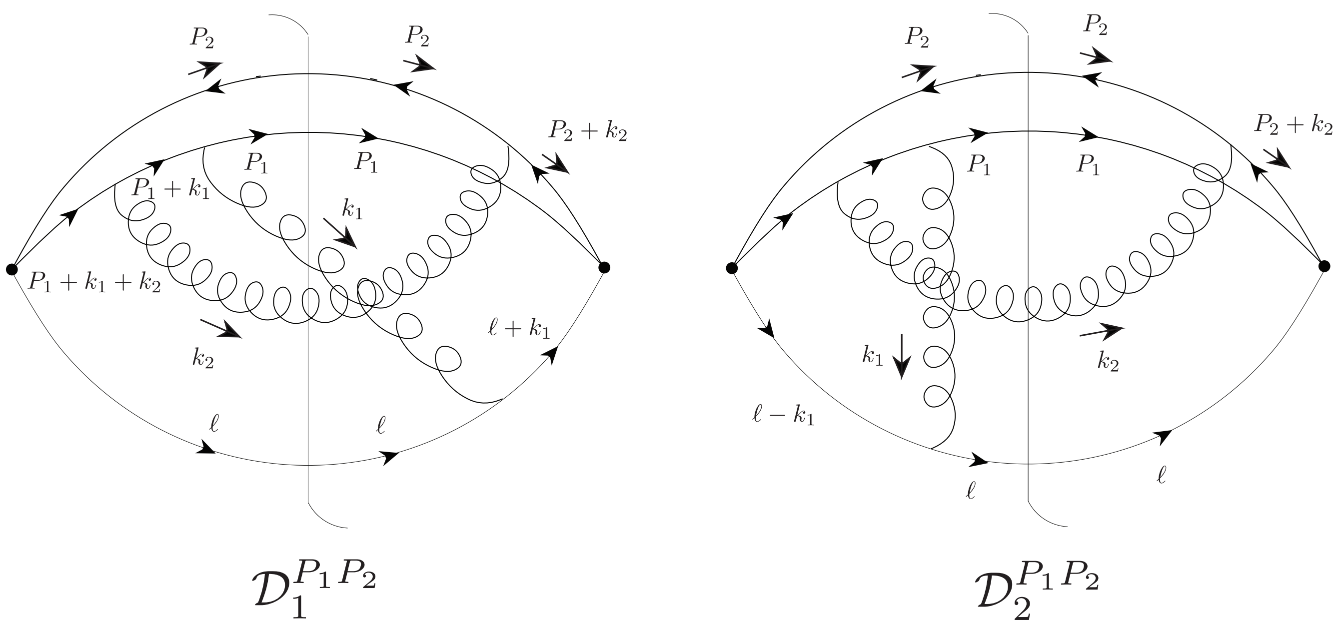
$$\mathcal{C} \equiv 2\mathrm{Re}\left(\sum_{i,j} \mathcal{C}^{P_i P_j}\right)_{\text{color}} = \left[\frac{\tilde{\mu}^2 P_1^2 (2c)^2}{\Lambda^4}\right]^{\epsilon} \left[\frac{1}{\pi^2 \epsilon_{\text{UV}}^2} + \frac{7}{4}\right]_{k_1} \frac{\alpha_s^2}{4\epsilon_{\text{IR}}} \left[\frac{N_c(N_c^2-1)}{4}\right] \left[1 - \frac{a \log\left(a + \sqrt{a^2-1}\right)}{\sqrt{a^2-1}}\right]$$

- We identified the UV and IR poles with additional UV cutoffs
- The UV poles should be absorbed into the short-distance coefficients of LDME, and the IR pole is inherently the 1-loop contribution to LDME (topologically factorized as the LO diagram)



NQS, PRD, 2005

Diagram $\mathcal{D}$



color factors

$$C_{\mathcal{D}_i^{P_1P_2}} = C_{\mathcal{D}_i^{P_1P_1}} = -\frac{iN_c(N_c^2-1)}{8},$$
$$C_{\mathcal{D}_i^{P_2P_1}} = C_{\mathcal{D}_i^{P_2P_2}} = +\frac{iN_c(N_c^2-1)}{8}$$

- Diagrams $\mathcal{D}$ have very similar IR structure to $\mathcal{C}$, but contains no UV poles

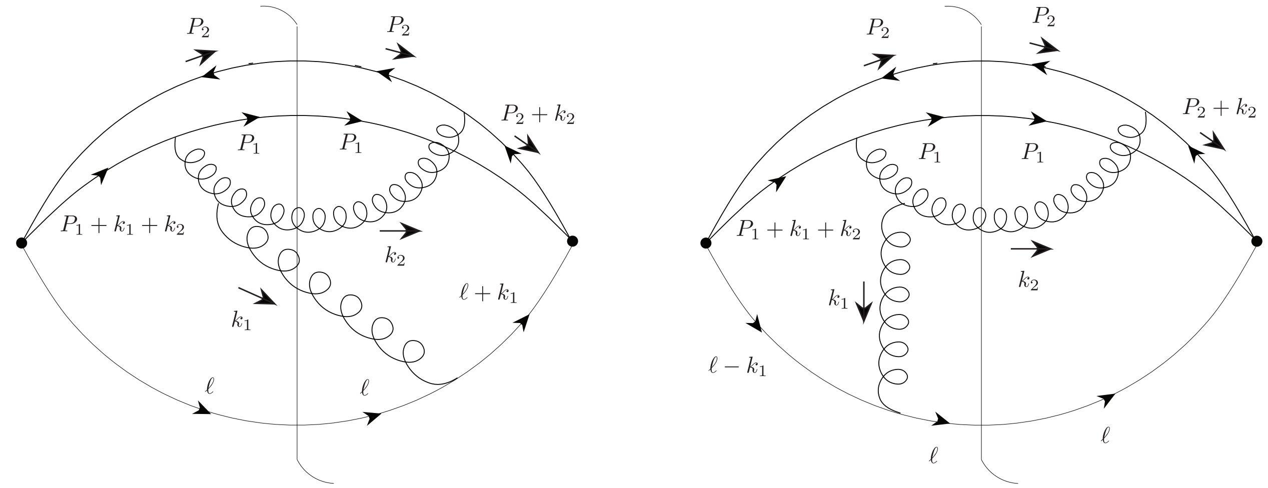
- The resulting IR poles are

Preliminary

$$\mathcal{D} \equiv 2\mathrm{Re}\left(\sum_{i,j} \mathcal{D}^{P_iP_j}\right)_{\text{color}} = \frac{\alpha_s^2}{4\epsilon_{\mathrm{IR}}}\left[\frac{N_c(N_c^2-1)}{4}\right]\left[1-\frac{a\log\left(a+\sqrt{a^2-1}\right)}{\sqrt{a^2-1}}\right]$$

discrepancy with NQS's result (NQS, PRD, 2005)

Diagram $\mathcal{E}$



$\mathcal{E}_1^{P_1 P_2}$ $\mathcal{E}_2^{P_1 P_2}$

color factor

$$C_{\mathcal{E}} = -\frac{N_c(N_c^2-1)}{4}$$

$$\mathcal{E} \equiv 2\mathrm{Re}\left(\sum_{i,j} \mathcal{E}^{P_i P_j}\right)_{\mathrm{color}} = \frac{\alpha_s^2}{4\epsilon_{\mathrm{IR}}}\left[-\frac{N_c(N_c^2-1)}{4}\right]\left[1 - \frac{a\log\left(a+\sqrt{a^2-1}\right)}{\sqrt{a^2-1}}\right]$$

agrees with NQS's result (NQS, PRD, 2006)

- In computing the above IR poles, we encountered very complicated intermediate expressions involving hundreds of dilogs and trilogs that cancel in the end. The analytic proof for the cancellation is accomplished through the use of many dilog and trilog identities, one of them is

$$\mathrm{Li}_2(u) + \mathrm{Li}_2(v) - \mathrm{Li}_2(uv) - \mathrm{Li}_2\left(\frac{u-uv}{1-uv}\right) - \mathrm{Li}_2\left(\frac{v-uv}{1-uv}\right) = \log\left(\frac{1-u}{1-uv}\right)\log\left(\frac{1-v}{1-uv}\right)$$

which is essential for $1/\epsilon^2$ pole cancellation

Discussions

- IR poles from the non-topologically factorized two-loop diagrams:

$$\mathcal{A} = 0 \qquad \mathcal{B} = O(\epsilon^0)$$

$$\mathcal{C} = \left[\frac{\tilde{\mu}^2 P_1^2 (2c)^2}{\Lambda^4}\right]^{\epsilon} \left[\frac{1}{\pi^2 \epsilon_{\mathrm{UV}}^2} + \frac{7}{4}\right]_{k_1} \frac{\alpha_s^2}{4\epsilon_{\mathrm{IR}}} \left[\frac{N_c(N_c^2-1)}{4}\right] \left[1 - \frac{a \log\left(a + \sqrt{a^2-1}\right)}{\sqrt{a^2-1}}\right]$$

$$\mathcal{D} = \frac{\alpha_s^2}{4\epsilon_{\mathrm{IR}}} \left[\frac{N_c(N_c^2-1)}{4}\right] \left[1 - \frac{a \log\left(a + \sqrt{a^2-1}\right)}{\sqrt{a^2-1}}\right]$$

$$\mathcal{E} = \frac{\alpha_s^2}{4\epsilon_{\mathrm{IR}}} \left[-\frac{N_c(N_c^2-1)}{4}\right] \left[1 - \frac{a \log\left(a + \sqrt{a^2-1}\right)}{\sqrt{a^2-1}}\right]$$

- All IR poles are independent of the direction of ℓ note that $a = \frac{P_1 \cdot P_2}{P_1^2} = \frac{1+v^2}{1-v^2}$
- The velocity dependent factor is $\left[1 - \frac{a \log\left(a + \sqrt{a^2-1}\right)}{\sqrt{a^2-1}}\right] = -\frac{4}{3}v^2 - \frac{8}{15}v^4 + O(v^6)$
- The inherently two-loop contributions to LDMEs are from $\mathcal{D}$ and $\mathcal{E}$ and they cancel: $\mathcal{D} + \mathcal{E} = O(\epsilon^0)$ Preliminary

Conclusions

- Our covariant method was intended to be an independent approach and expected to give simpler calculations than the light-cone integrations
- However, the covariant method is more complicated because, in contrast with the light-cone method, cancellation of real-real diagrams against part of real-virtual diagrams, and additional cancellation of poles from some of the residues of light-cone variable integrations through symmetry cannot be made manifest
- Work still in progress on understanding the simplicity of the final result and the discrepancy with NQS results for $\mathcal{D}$

Thank you!

Backup slides

NRQCD and Fragmentation function

- Assuming the NRQCD factorization of the fragmentation function, we can write the fragmentation function as

Braaten, Yuan, PRL, 1993
Braaten, Yuan, PRD, 1995

$$D[g \to H](z, \mu_\Lambda) = \sum_n d_{g \to Q\bar{Q}[n]}(z) \langle \mathcal{O}_n^H \rangle$$

where the conventional NRQCD LDMEs are given by

$$\langle \mathcal{O}_n^H \rangle = \langle 0 | \chi^\dagger \kappa_n \psi \mathcal{P}_{H(P)} \psi^\dagger \kappa'_n \chi | 0 \rangle$$

κ_n, κ'_n : Pauli and color matches

- $d_{g \to Q\bar{Q}[n]}(z)$ is the fragmentation short-distance coefficients, and since they are independent of hadronic states H, they can be obtained from the matching of the free $Q\bar{Q}$ fragmentation functions:

$$\underbrace{D[g \to Q\bar{Q}](z, \mu_\Lambda)}_{\text{full QCD}} = \sum_n d_{g \to Q\bar{Q}[n]}(z) \underbrace{\langle \mathcal{O}_n^{Q\bar{Q}} \rangle}_{\text{NRQCD}}$$

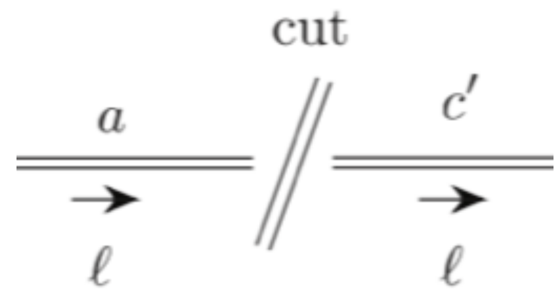
full QCD

NRQCD

$\boxed{\text{Matching}}$

sharing identical IR structures

Convergent factor



- The eikonal line propagator of the fragmentation function on the cut is proportional to the +-momentum conserving delta function:

$$\delta\left[\ell \cdot (k - P_1 - P_2 - \Sigma_i k_i)\right]$$

- Integrating delta function over all allowed values of $\ell \cdot (P_1 + P_2)$, we obtain

$$\theta\left[\ell \cdot (k - \Sigma_i k_i)\right]$$

- And integrating over all value k with an IR conserving weight function,

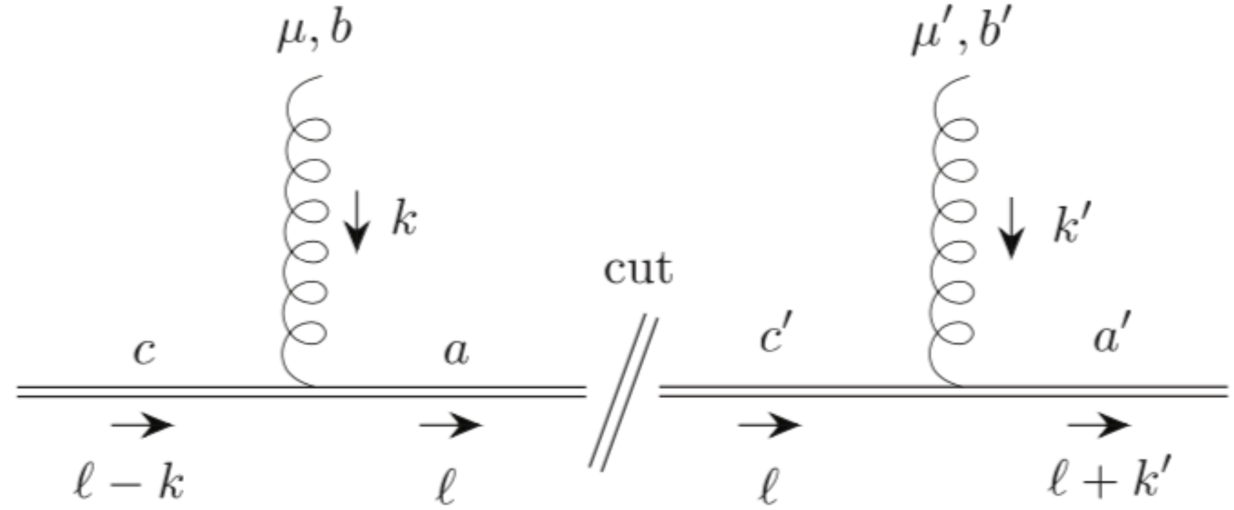
$$w(\ell \cdot k) \equiv \frac{\Lambda^2}{\left[(\ell \cdot k) + \Lambda^2\right]^2}$$

we obtain

$$\int_0^\infty d(\ell \cdot k) w(\ell \cdot k)\, \theta\left[\ell \cdot (k - \Sigma_i k_i)\right] = \frac{\Lambda^2}{\ell \cdot (\Sigma_i k_i) + \Lambda^2}$$

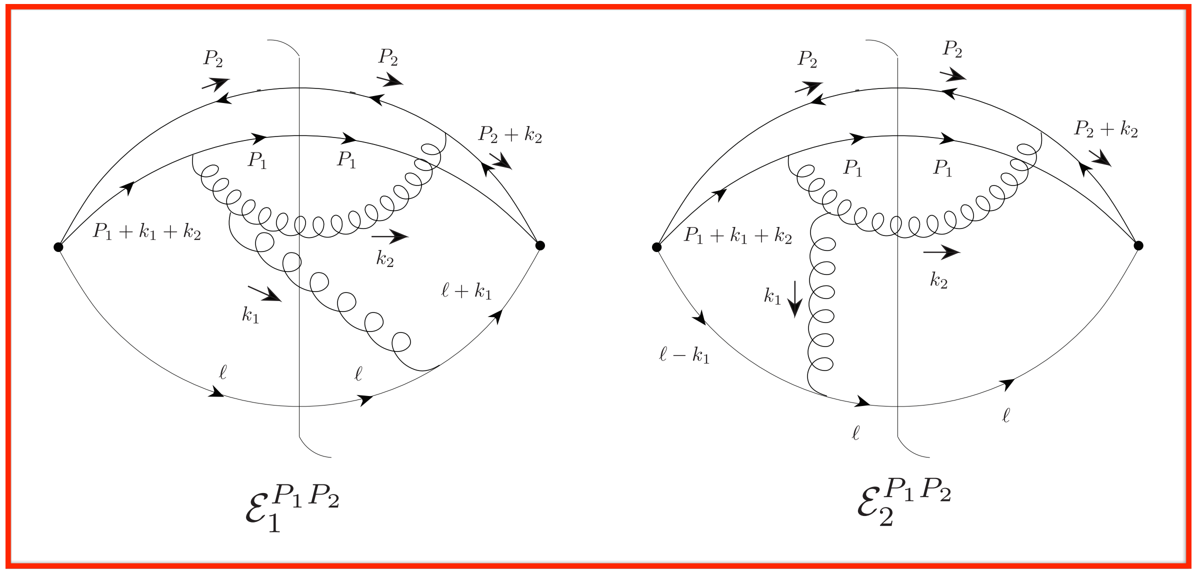
- This convergent factor works as a UV cutoff and allows us to make manifestly covariant phase-space integration

Feynman rules for eikonal line



	propagator	vertex	Cut line propagator
LHS of cut	$\dfrac{-i}{\ell \cdot (-k) + i\varepsilon}$,	$+g_s \ell^\mu f^{abc}$	$2\pi\delta\left[\ell \cdot (P_1 + P_2 - \sum_i k_i)\right]$
RHS of cut	$\left[\dfrac{-i}{\ell \cdot k' + i\varepsilon}\right]^*$,	$-g_s \ell^{\mu'} f^{a'b'c'}$	

Two-loop non-Abelian diagrams



- Rearranging numerator terms due to triple-gluon vertex,

$$\mathcal{E}_1^{P_1P_2} = \mathcal{E}_{10} + \mathcal{E}'_{10} + 8g_s^4 P_1^\mu P_2^\nu \ell^\rho \Big\{ (g_{\alpha\rho}g_{\mu\nu} + 2g_{\alpha\mu}g_{\nu\rho} - 2g_{\alpha\nu}g_{\mu\rho})\,\mathcal{E}_{11}^\alpha + (2g_{\alpha\rho}g_{\mu\nu})\,\mathcal{E}_{12}^\alpha \Big\},$$

$$\mathcal{E}_2^{P_1P_2} = \mathcal{E}_{20} + \mathcal{E}'_{20} + 8g_s^4 P_1^\mu P_2^\nu \ell^\rho \Big\{ (g_{\alpha\rho}g_{\mu\nu} + 2g_{\alpha\mu}g_{\nu\rho} - 2g_{\alpha\nu}g_{\mu\rho})\,\mathcal{E}_{21}^\alpha + (2g_{\alpha\rho}g_{\mu\nu})\,\mathcal{E}_{22}^\alpha \Big\},$$

depend only on P_1 or P_2 canceled by P_1P_1 or P_2P_2 diagrams

- $\mathcal{E}_{ij}^\alpha$ is the part of the integral $\mathcal{E}_i^{P_1P_2}$ whose integrand proportional to k_j^α
- The contributions proportional to ℓ^α vanish:

$$P_1^\mu P_2^\nu \ell^\rho\,(g_{\alpha\rho}g_{\mu\nu} + 2g_{\alpha\mu}g_{\nu\rho} - 2g_{\alpha\nu}g_{\mu\rho})\,\ell^\alpha = 0 \quad \text{and} \quad P_1^\mu P_2^\nu \ell^\rho\,(2g_{\alpha\rho}g_{\mu\nu})\,\ell^\alpha = 0.$$