# Breakdown of NRQCD Factorization in Processes Involving Two Quarkonia and its Cure

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# Outline

# History of factorization

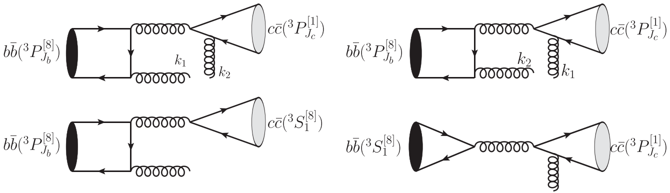
- In CSM, uncancelled IR divergences in decays of $P$-wave states at LO and in relativistic corrections to decays of $S$-wave states at NLO [Barbieri et al. 76]; also for $L > 1$ [Belanger et al. 81].
- In NRQCD factorization formalism, IR divergences absorbed into CO LDMEs [BBL 95].
- Still uncancelled IR divergences in exclusive decay $\chi_{cJ} \to l^+ l^-$ [Yang et al. 04] and double charmium production in $e^+ e^-$ annihilation [Bodwin et al. 08].
- In exclusive processes, IR divergence appears in loop integration of virtual corrections. They either disappear in limits $m_c/m_b, m_c/\sqrt{s} \to 0$ or can be eliminated in pNRQCD [Beneke et al. 09].

# New type of IR divergence in $\Upsilon \to$ charmonium + X

- In NRQCD factorization, for $H = \eta_c, J/\psi, \chi_{cJ}, \ldots$

$$\Gamma(\Upsilon \to H + X) = \sum_{m,n} \hat{\Gamma}(b\bar{b}(m) \to c\bar{c}(n) + X)\langle\Upsilon|\mathcal{O}(m)|\Upsilon\rangle\langle\mathcal{O}^H(n)\rangle$$

- In single $P$-wave case, if either $m$ or $n$ is $P$ wave, IR divergence can be absorbed into NRQCD LDMEs.
- But not if $m$ and $n$ are both $P$ wave! Consider e.g. $b\bar{b}({}^3P_{J_b}^{[8]}) \to c\bar{c}({}^3P_{J_c}^{[1]}) + gg$.



# $b\bar{b}({}^3P_{J_b}^{[8]}) \to c\bar{c}({}^3P_{J_c}^{[1]}) + gg$ I

- Calculate SDCs directly via covariant spin project method and do phase space integration for soft region analytically.
- Divide divergent results into 3 parts,
  $\Gamma^{\rm div}(J_c) = \Gamma_1^{\rm div} + 9\Gamma_2^{\rm div}(J_c) + \Gamma_3^{\rm div}(J_c)$, where

$$
\begin{aligned}
\hat{\Gamma}_1^{\rm div} &= \frac{-8\alpha_s}{27\pi m_c^2}\frac{1}{\epsilon_{\rm IR}} \times \frac{5\pi^2\alpha_s^3(3r^4+2r^2+7)}{72 m_b^7 r^3(1-r^2)}, \\
\hat{\Gamma}_2^{\rm div}(J_c) &= \frac{-5\alpha_s}{9\pi m_b^2}\frac{1}{\epsilon_{\rm IR}} \times \begin{cases} \frac{\pi^2\alpha_s^3\left(1-3r^2\right)^2}{81 m_b^7 r^3(1-r^2)}, & J_c = 0, \\ \frac{2\pi^2\alpha_s^3\left(r^2+1\right)}{81 m_b^7 r^3(1-r^2)}, & J_c = 1, \\ \frac{2\pi^2\alpha_s^3\left(6r^4+3r^2+1\right)}{405 m_b^7 r^3(1-r^2)}, & J_c = 2, \end{cases}
\end{aligned}
$$

- and

$$\hat{\Gamma}_3^{\rm div}(0) = -\frac{10\pi\alpha_s^4}{81 m_b^9 r^3 \left(1-r^2\right)^4 \epsilon_{\rm IR}} \times \left(3r^4 - 10r^2 + 3\right)\left(r^4 - 4r^2 \ln r - 1\right),$$

$$\hat{\Gamma}_3^{\rm div}(1) = \frac{10\pi\alpha_s^4}{81 m_b^9 r^3 \left(1-r^2\right)^4 \epsilon_{\rm IR}} \left[-r^6 + 9r^4 - 7r^2 + 4r^2\left(r^4 - 3r^2 - 2\right)\ln r - 1\right],$$

$$\hat{\Gamma}_3^{\rm div}(2) = \frac{2\pi\alpha_s^4}{81 m_b^9 r^3 \left(1-r^2\right)^4 \epsilon_{\rm IR}} \left[6r^8 + 23r^6 - 27r^4 + r^2 - 4r^4\left(9r^2 + 11\right)\ln r - 3\right],$$

with $r = m_c/m_b$.

# $b\bar{b}({}^3P_{J_b}^{[8]}) \to c\bar{c}({}^3P_{J_c}^{[1]}) + gg$ III

- $\Gamma_1^{\rm div}$ is from soft gluon emitted and absorbed by same charm (anti)quark, can be absorbed by NLO QCD corrections to $\langle \mathcal{O}^{\chi_{cJ}}({}^3S_1^{[8]})\rangle$ in $b\bar{b}({}^3P_{J_b}^{[8]}) \to c\bar{c}({}^3S_1^{[8]}) + g$.
- $\Gamma_2^{\rm div}(J_c)$ is from soft gluon emitted and absorbed by same bottom (anti)quark, can be absorbed by NLO QCD corrections to $\langle \Upsilon|\mathcal{O}({}^3S_1^{[8]})|\Upsilon\rangle$ in $b\bar{b}({}^3S_1^{[8]}) \to c\bar{c}({}^3P_{J_c}^{[1]}) + g$.
- However, $\Gamma_3^{\rm div}(J_c)$ is from soft gluon emitted and absorbed by different heavy quarks. There are no LDMEs to describe such effects yet!

Unlike for exclusive $P$-wave production, $\Gamma_3^{\rm div}(J_c)$ do not vanish for $r \to 0$!

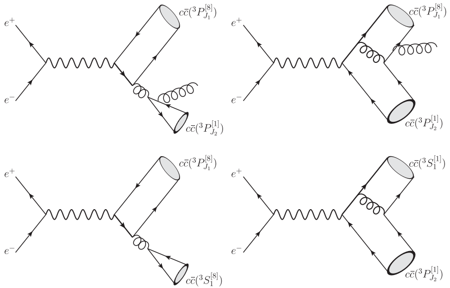
# More general case

- $\Gamma_3^{\rm div}(J_c)$ is due to interference of diagrams with soft-gluon emission by $P$-wave $b\bar{b}$ and $c\bar{c}$ Fock states, which can appear in NRQCD treatment of any inclusive bottomonium decay to charmonium. NRQCD factorization will break down for any such process at some order of $v_c^2$ and $v_b^2$.
- In particular, for $\chi_{bJ} \to \chi_{cJ} + X$ production this happens already at LO in $v_b^2$ and $v_c^2$.

What will happen for double charmonium production at hadron collider?

# $e^+e^- \to c\bar{c}(^3P_{J_1}^{[8]}) + c\bar{c}(^3P_{J_2}^{[1]}) + g$ I

- For prompt double $J/\psi$ hadroproduction, soft-gluon emission starts at NLO in $\alpha_s$, e.g. $gg \to c\bar{c}(^3P_{J_1}^{[8]}) + c\bar{c}(^3P_{J_2}^{[8]}) + g$.
- There will be additional IR divergences to be cancelled by virtual corrections. $\rightsquigarrow$ Difficult to calculate.
- Alternatively, consider $J/\psi + \chi_{cJ}$ production in $e^+e^-$ annihilation through $e^+e^- \to c\bar{c}(^3P_{J_1}^{[8]}) + c\bar{c}(^3P_{J_2}^{[1]}) + g$ for illustration:



# $e^+e^- \to c\bar{c}(^3P_{J_1}^{[8]}) + c\bar{c}(^3P_{J_2}^{[1]}) + g$ II

- Again, 3 sources of infrared divergences:
  - Squared amplitude where gluon is attached to $c\bar{c}(^3P_{J_2}^{[1]})$: $\sigma_1^{\rm div}$
  - Squared amplitude where gluon attached to $c\bar{c}(^3P_{J_1}^{[8]})$: $\sigma_2^{\rm div}(J_2)$
  - Interference between these two amplitude parts: $\sigma_3^{\rm div}(J_2)$
- Total result:

$$\sigma^{\rm div} = \sigma_1^{\rm div} + 9\sigma_2^{\rm div}(J_2) + \sigma_3^{\rm div}(J_2)$$

where

$$\hat{\sigma}_1^{\rm div} = -\frac{8\alpha_s}{27\pi m_c^2}\frac{1}{\epsilon_{\rm IR}}\frac{2^{10}\pi^3\alpha^2\alpha_s^2 S}{729 s^5 r^6} \times$$
$$(864r^{10} - 144r^8 - 1568r^6 + 1224r^4 - 130r^2 + 27),$$

# $e^+e^- \to c\bar{c}({}^3P_{J_1}^{[8]}) + c\bar{c}({}^3P_{J_2}^{[1]}) + g$ III

$$\hat{\sigma}_2^{\rm div}(J_2) = -\frac{4\alpha_s}{3\pi m_c^2}\frac{1}{\epsilon_{\rm IR}} \times \frac{2^{18}\pi^3\alpha^2\alpha_s^2 S}{19683 s^5 r^4}$$
$$\begin{cases} (144r^8 + 152r^6 - 428r^4 + 182r^2 + 1), & J_2 = 0, \\ 8(18r^6 + 13r^4 - 12r^2 + 2), & J_2 = 1, \\ \frac{2}{5}(360r^8 + 308r^6 - 188r^4 + 20r^2 + 1), & J_2 = 2, \end{cases}$$

$$\hat{\sigma}_3^{\rm div}(0) = \frac{2^{19}\pi^2\alpha^2\alpha_s^3}{3^8 s^6 r^4 \epsilon_{\rm IR}} \times \left[\left(144r^8 + 184r^6 - 504r^4 + 170r^2 + 33\right) S + 8\left(72r^{10} + 56r^8 - 284r^6 + 149r^4 + r^2\right) T\right],$$

# $e^+e^- \to c\bar{c}({}^3P_J^{[8]}) + c\bar{c}({}^3P_J^{[1]}) + g$ IV

$$\hat{\sigma}_3^{\rm div}(1) = \frac{2^{19}\pi^2\alpha^2\alpha_s^3}{3^8 s^6 r^2 \epsilon_{\rm IR}} \times \left[\left(144r^6 + 28r^4 - 176r^2 + 43\right) S + \left(576r^{10} - 176r^8 - 792r^6 + 424r^4 - 48r^2\right) T\right],$$

$$\hat{\sigma}_3^{\rm div}(2) = \frac{2^{19}\pi^2\alpha^2\alpha_s^3}{5\cdot 3^8 s^6 r^4 \epsilon_{\rm IR}} \left[\left(720r^8 + 452r^6 - 696r^4 + 7r^2 - 15\right) S + \left(2880r^{10} + 368r^8 - 3560r^6 + 1856r^4 - 56r^2\right) T\right],$$

where $r = 2m_c/\sqrt{s}$, $S = \sqrt{1-4r^2}$, and $T = \tanh^{-1} S$.

# $e^+e^- \to c\bar{c}({}^3P_J^{[8]}) + c\bar{c}({}^3P_J^{[1]}) + g$ V

- $\sigma_1^{\mathrm{div}}$ is cancelled upon including NLO QCD corrections to $\langle \mathcal{O}^{\chi_{cJ}}({}^3S_1^{[8]})\rangle$ in $e^+e^- \to c\bar{c}({}^3P_{J_1}^{[8]}) + c\bar{c}({}^3S_1^{[8]})$.
- $\sigma_2^{\mathrm{div}}(J_2)$ is cancelled upon including NLO QCD corrections to $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle$ in $e^+e^- \to c\bar{c}({}^3S_1^{[1]}) + c\bar{c}({}^3P_{J_2}^{[1]})$.
- $\hat{\sigma}_3^{\mathrm{div}}(J_2)$ is left!
- Behaviors for $r \to 0$: $\sigma_1^{\mathrm{div}} \propto \frac{1}{r^7}$, $\sigma_2^{\mathrm{div}} \propto \frac{1}{r^5}$, $\sigma_3^{\mathrm{div}} \propto \frac{1}{r^4}$.
- Although new types of singularies will not disappear, they are less singular for $r \to 0$.

# Impact on double quarkonium hadroproduction

- New type of IR divergence originates from interference of diagrams where soft gluon is attached to different $Q\bar{Q}$ states, independent of initial state and not requring the quark pairs to have same flavor.
- We predict that this will also appear in NLO QCD corrections for double $J/\psi$ and $J/\psi + \Upsilon$ hadroproduction.
- However, structure of new IR divergence can be more complicated because more channels are involved.
- For $gg \to c\bar{c}({}^3P_{J_c}^{[8]}) + b\bar{b}({}^3P_{J_b}^{[8]}) + g$, there will be 4 pairings $c\bar{c}({}^3S_1^{[8]}) + b\bar{b}({}^3P_{J_b}^{[8]})$, $c\bar{c}({}^3S_1^{[1]}) + b\bar{b}({}^3P_{J_b}^{[8]})$, $c\bar{c}({}^3P_{J_c}^{[8]}) + b\bar{b}({}^3S_1^{[1]})$, and $c\bar{c}({}^3P_{J_c}^{[8]}) + b\bar{b}({}^3S_1^{[8]})$, which lead to more interference terms.

# Basic idea to sovle the problem I

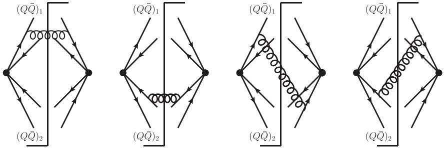
- Recall that factorization implies complete speration of perturbative and nonperturbative effects.
- First find a way to separate IR sigular terms like $\Gamma_3^{\mathrm{div}}(J_c)$ and $\sigma_3^{\mathrm{div}}(J_2)$ into contributions from hard- and soft-scale regimes.
- 2 pairs cannot be at rest simultaneously. $\rightsquigarrow$ Use covariant form of NRQCD Lagrangian:

$$\begin{aligned}\mathcal{L}_{\mathrm{NRQCD}}^{\mathrm{LO}} &= \bar{\psi}_v \left[ iv\cdot D + \frac{(iD_\top^\mu)(iD_{\top\mu})}{2m} \right] \psi_v \\ &+ \bar{\chi}_v \left[ -iv\cdot D + \frac{(iD_\top^\mu)(iD_{\top\mu})}{2m} \right] \chi_v ,\end{aligned}$$

where $v^\mu = p^\mu/m$ with $m$ heavy-quark mass and $p^\mu$ $Q\bar{Q}$ momentum.

# Basic idea to sovle the problem II

- $\psi_v$ and $\chi_v$ are nonrelativistic heavy-quark and -antiquark 4-component spinor fields satisfying $\not{v}\psi_v = \psi_v$ and $\not{v}\chi_v = -\chi_v$.
- $\perp$ component of vector $a^\mu$ is defined as $a_\perp^\mu = a^\mu - v^\mu v \cdot a$.
- Creation and annihilation of heavy-quark pair surely take place at short distance. 1-loop correction diagrams:




- Interference effect is described in last 2 panels.

# Basic idea to sovle the problem III

- To describe vertex, introduce new operators: $\psi_{b,v_1}\mathcal{K}^{\mu_1\nu_1}T^{a_1}\bar{\chi}_{b,v_1}\bar{\psi}_{c,v_2}\gamma_\top^{\nu_2}T^{a_2}\chi_{c,v_2}$, $\psi_{b,v_1}\gamma_\top^{\nu_1}T^{a}\bar{\chi}_{b,v_1}\bar{\psi}_{c,v_2}\mathcal{K}_{J_c}^{\mu_2\nu_2}\chi_{c,v_2}$ and charge conjugates for $b\bar{b}(^3P_{J_b}^{[8]}) \to c\bar{c}(^3P_{J_c}^{[1]}) + gg$ and $\bar{\psi}_{c,v_1}\mathcal{K}^{\mu_1\nu_1}T^{a_1}\chi_{c,v_1}\bar{\psi}_{c,v_2}\gamma_\top^{\nu_2}T^{a_2}\chi_{c,v_2}$, $\bar{\psi}_{c,v_1}\gamma_\top^{\nu_1}\chi_{c,v_1}\bar{\psi}_{c,v_2}\mathcal{K}_{J_2}^{\mu_2\nu_2}\chi_{c,v_2}$ and charge conjugatates for $e^+e^- \to c\bar{c}(^3P_{J_1}^{[8]}) + c\bar{c}(^3P_{J_2}^{[1]}) + g$.
- $\mathcal{K}$s are defined as $\mathcal{K}_0^{\mu\nu} = \frac{g^{\mu\nu} - v^\mu v^\nu}{\sqrt{3}}(-\frac{i}{2}\overleftrightarrow{\not{D}}_\top)$, $\mathcal{K}_1^{\mu\nu} = \frac{-i}{2}(\overleftrightarrow{\not{D}}_\top^{[\mu}\gamma_\top^{\nu]})$, $\mathcal{K}_2^{\mu\nu} = \frac{-i}{2}(\overleftrightarrow{\not{D}}_\top^{(\mu}\gamma_\top^{\nu)})$, $\mathcal{K}^{\mu\nu} = \frac{-i}{2}(\overleftrightarrow{\not{D}}_\top^{\mu}\gamma_\top^{\nu})$, with $a^{[\mu}b^{\nu]} = \frac{1}{2}(a^\mu b^\nu - a^\nu b^\mu)$, $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu) - \frac{g^{\mu\nu} - v^\mu v^\nu}{3}a \cdot b$.

# One loop corrections of soft gluon

- Derive Feynman rules from Lagrangian and calculate loop integrals. E.g. the third panel:

$$I = -ig_s^2\mu^{4-D}\int \frac{d^D l}{(2\pi)^D}\,\frac{\left[v_1 + \frac{(2q_1+l)_\top}{2m_1}\right]\cdot\left[v_2 + \frac{(2q_2+l)_\top}{2m_2}\right]}{l^2\left[l\cdot v_1 + \frac{(l+q_1)_\top^2}{2m_1}\right]\left[l\cdot v_2 + \frac{(l+q_2)_\top^2}{2m_2}\right]}.$$

- Although numerically $m_b \approx 3m_c$, assume that $m_c \gg m_b v_b$ to ensure that nonrelativistic approximation still applies to charmonium.
- Expand integrand in series of $1/m_i$ and drop terms of order $1/m_i^2$ and higher.

# One loop corrections of soft gluon

- We get

$$
\begin{aligned}
I &= I_0 + \frac{\alpha_s \mu^{4-D}}{\pi m_1 m_2} \left( \frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}} \right) \\
&\times \left[ \frac{\ln(\omega + \sqrt{\omega^2 - 1}) - \omega \sqrt{\omega^2 - 1}}{2(\omega^2 - 1)^{3/2}} q_1 \cdot q_2 \right. \\
&+ \left. \frac{(\omega^2 + 2)\sqrt{\omega^2 - 1} - 3\omega \ln(\sqrt{\omega^2 - 1} + \omega)}{2(\omega^2 - 1)^{5/2}} (v_1 \cdot q_2)(v_2 \cdot q_1) \right],
\end{aligned}
$$

where $\omega = v_1 \cdot v_2$

- $I_0$ disappears upon summation over all diagrams.

# One loop corrections of soft gluon

- Loop integrals are process independent, although results depend on $\omega$.
- UV divergence can be removed through operator renormalization.
- Multiply with corresponding SCDs and decompose tensor and color structures into basis of total-angular momentum and color states.
- IR-singular parts exactly match those in $\Gamma_3^{\mathrm{div}}(J_c)$ and $\sigma_3^{\mathrm{div}}(J_2)$!
- I.e. it is possible to construct gerneral factorization formalism within NRQCD to describe processes involving 2 or more heavy quarkonia.

# Summary and outlook

- We illustrated via 2 examples, $\Upsilon \to \chi_{cJ} + X$ and $e^+e^- \to J/\psi + \chi_{cJ} + X$, that there are uncancelled IR divergences in standard formulation of NRQCD factorization approach.
- We extended the conclusions to any subprocess involving 2 $P$-wave Fock states.
- We introduced new types of operators and showed that their NLO QCD corrections precisely reproduce the uncancelled IR divergences.
- Much more further work is needed to construct a generalized NRQCD factorization formalism and to investigate its phenomenological impact, especially for double quarkonia hadroproduction.

# Thank you!