Breakdown of NRQCD Factorization in Processes Involving Two Quarkonia and its Cure

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History of factorization

- In CSM, uncancelled IR divergences in decays of $P$-wave states at LO and in relativistic corrections to decays of $S$-wave states at NLO [Barbieri et al. 76]; also for $L > 1$ [Belanger et al. 81].

- In NRQCD factorization formalism, IR divergences absorbed into CO LDMEs [BBL 95].

- Still uncancelled IR divergences in exclusive decay $\chi_{cJ} \to l^+ l^-$ [Yang et al. 04] and double charmium production in $e^+ e^-$ annihilation [Bodwin et al. 08].

- In exclusive processes, IR divergence appears in loop integration of virtual corrections. They either disappear in limits $m_c/m_b, m_c/\sqrt{s} \to 0$ or can be eliminated in pNRQCD [Beneke et al. 09].
New type of IR divergence in $\Upsilon \rightarrow$ charmonium + X

- In NRQCD factorization, for $H = \eta_c, J/\psi, \chi_{cJ}$, ...

\[
\Gamma(\Upsilon \rightarrow H + X) = \sum_{m,n} \hat{\Gamma}(b\bar{b}(m) \rightarrow c\bar{c}(n) + X) \langle \Upsilon | \mathcal{O}(m) | \Upsilon \rangle \langle \mathcal{O}^H(n) \rangle
\]

- In single $P$-wave case, if either $m$ or $n$ is $P$ wave, IR divergence can be absorbed into NRQCD LDMEs.
- But not if $m$ and $n$ are both $P$ wave! Consider e.g. $b\bar{b}(3P[^8]_{J_b}) \rightarrow c\bar{c}(3P[^1]_{J_c}) + gg$.
\[ b\bar{b}(^3P_{J_b}^{[8]}) \rightarrow c\bar{c}(^3P_{J_c}^{[1]}) + gg \]

- Calculate SDCs directly via covariant spin project method and do phase space integration for soft region analytically.
- Divide divergent results into 3 parts,
  \[ \Gamma_{\text{div}}(J_c) = \Gamma_{1}^{\text{div}} + 9\Gamma_{2}^{\text{div}}(J_c) + \Gamma_{3}^{\text{div}}(J_c), \]
  where

\[ \hat{\Gamma}_1^{\text{div}} = \frac{-8\alpha_s}{27\pi m_c^2} \frac{1}{\epsilon_{\text{IR}}} \times \frac{5\pi^2\alpha_s^3(3r^4 + 2r^2 + 7)}{72m_b^7r^3(1 - r^2)}, \]

\[ \hat{\Gamma}_2^{\text{div}}(J_c) = \frac{-5\alpha_s}{9\pi m_b^2} \frac{1}{\epsilon_{\text{IR}}} \times \begin{cases} \frac{\pi^2\alpha_s^3(1-3r^2)^2}{81m_b^7r^3(1-r^2)}, & J_c = 0, \\ \frac{2\pi^2\alpha_s^3(r^2+1)}{81m_b^7r^3(1-r^2)}, & J_c = 1, \\ \frac{2\pi^2\alpha_s^3(6r^4+3r^2+1)}{405m_b^7r^3(1-r^2)}, & J_c = 2, \end{cases} \]
\[ b \bar{b}(^3 P_{J_b}^{[8]}) \rightarrow c \bar{c}(^3 P_{J_c}^{[1]}) + gg \]\[ \text{and} \]
\[
\hat{\Gamma}^\text{div}_3(0) = -\frac{10\pi\alpha_s^4}{81 m_b^9 r^3 (1 - r^2)^4 \epsilon_{IR}} \times (3r^4 - 10r^2 + 3) (r^4 - 4r^2 \ln r - 1),
\]
\[
\hat{\Gamma}^\text{div}_3(1) = \frac{10\pi\alpha_s^4}{81 m_b^9 r^3 (1 - r^2)^4 \epsilon_{IR}} \left[ -r^6 + 9r^4 - 7r^2 + 4r^2 (r^4 - 3r^2 - 2) \ln r - 1 \right],
\]
\[
\hat{\Gamma}^\text{div}_3(2) = \frac{2\pi\alpha_s^4}{81 m_b^9 r^3 (1 - r^2)^4 \epsilon_{IR}} \left[ 6r^8 + 23r^6 - 27r^4 + r^2 - 4r^4 (9r^2 + 11) \ln r - 3 \right],
\]

with \( r = m_c/m_b \).
\[ b\bar{b}(3P_{J_{b}}^{[8]}) \rightarrow c\bar{c}(3P_{J_{c}}^{[1]}) + gg \]

- \( \Gamma_{1}^{\text{div}} \) is from soft gluon emitted and absorbed by same charm (anti)quark, can be absorbed by NLO QCD corrections to \( \langle \mathcal{O}\chi_{cJ}(3S_{1}^{[8]}) \rangle \) in \( b\bar{b}(3P_{J_{b}}^{[8]}) \rightarrow c\bar{c}(3S_{1}^{[8]}) + g \).

- \( \Gamma_{2}^{\text{div}}(J_{c}) \) is from soft gluon emitted and absorbed by same bottom (anti)quark, can be absorbed by NLO QCD corrections to \( \langle \gamma|\mathcal{O}(3S_{1}^{[8]})|\gamma \rangle \) in \( b\bar{b}(3S_{1}^{[8]}) \rightarrow c\bar{c}(3P_{J_{c}}^{[1]}) + g \).

- However, \( \Gamma_{3}^{\text{div}}(J_{c}) \) is from soft gluon emitted and absorbed by different heavy quarks. There are no LDMEs to describe such effects yet!

Unlike for exclusive \( P \)-wave production, \( \Gamma_{3}^{\text{div}}(J_{c}) \) do not vanish for \( r \rightarrow 0 \)!
More general case

- $\Gamma^{\text{div}}_3(J_c)$ is due to interference of diagrams with soft-gluon emission by $P$-wave $b\bar{b}$ and $c\bar{c}$ Fock states, which can appear in NRQCD treatment of any inclusive bottomonium decay to charmonium. NRQCD factorization will break down for any such process at some order of $v_c^2$ and $v_b^2$.

- In particular, for $\chi_{bJ} \rightarrow \chi_{cJ} + X$ production this happens already at LO in $v_b^2$ and $v_c^2$.

What will happen for double charmonium production at hadron collider?
\[ e^+ e^- \rightarrow c\bar{c}(3P_{J_1}^{[8]}) + c\bar{c}(3P_{J_2}^{[1]}) + g \]

- For prompt double \( J/\psi \) hadroproduction, soft-gluon emission starts at NLO in \( \alpha_s \), e.g. \( gg \rightarrow c\bar{c}(3P_{J_1}^{[8]}) + c\bar{c}(3P_{J_2}^{[8]}) + g \).
- There will be additional IR divergences to be cancelled by virtual corrections. \( \rightsquigarrow \) Difficult to calculate.
- Alternatively, consider \( J/\psi + \chi_c J \) production in \( e^+ e^- \) annihilation through \( e^+ e^- \rightarrow c\bar{c}(3P_{J_1}^{[8]}) + c\bar{c}(3P_{J_2}^{[1]}) + g \) for illustration:
\[ e^+e^- \rightarrow c\bar{c}(^3P_{J_1}^{[8]}) + c\bar{c}(^3P_{J_2}^{[1]}) + g \parallel \]

- Again, 3 sources of infrared divergences:
  - Squared amplitude where gluon is attached to \( c\bar{c}(^3P_{J_2}^{[1]}) \): \( \sigma_{1\text{div}} \)
  - Squared amplitude where gluon attached to \( c\bar{c}(^3P_{J_1}^{[8]}) \): \( \sigma_{2\text{div}}(J_2) \)
  - Interference between these two amplitude parts: \( \sigma_{3\text{div}}(J_2) \)

- Total result:
  \[
  \sigma^{\text{div}} = \sigma_{1\text{div}} + 9\sigma_{2\text{div}}(J_2) + \sigma_{3\text{div}}(J_2)
  \]
  
  where

  \[
  \hat{\sigma}^{\text{div}}_1 = -\frac{8\alpha_s}{27\pi m_c^2} \frac{1}{\epsilon_{\text{IR}}} \frac{2^{10}\pi^3 \alpha_s^2}{729s^5r^6} \times
  
  (864r^{10} - 144r^8 - 1568r^6 + 1224r^4 - 130r^2 + 27),
  \]
\[ e^+ e^- \rightarrow c \bar{c}(^{3}P_{J_1}) + c \bar{c}(^{3}P_{J_2}) + g \]

\[
\hat{\sigma}_2^{\text{div}}(J_2) = -\frac{4\alpha_s}{3\pi m_c^2} \frac{1}{\epsilon_{\text{IR}}} \times \frac{2^{18} \pi^3 \alpha^2 \alpha_s^2 S}{19683 s^5 r^4}
\]

\[
\begin{cases}
(144r^8 + 152r^6 - 428r^4 + 182r^2 + 1), & J_2 = 0, \\
8(18r^6 + 13r^4 - 12r^2 + 2), & J_2 = 1, \\
\frac{2}{5}(360r^8 + 308r^6 - 188r^4 + 20r^2 + 1), & J_2 = 2,
\end{cases}
\]

\[
\hat{\sigma}_3^{\text{div}}(0) = \frac{2^{19} \pi^2 \alpha^2 \alpha_s^3}{3^8 s^6 r^4 \epsilon_{\text{IR}}} \times [(144r^8 + 184r^6 - 504r^4 + 170r^2 + 33) S
\]

\[+8 \left(72r^{10} + 56r^8 - 284r^6 + 149r^4 + r^2\right) T],
\]
\begin{equation}
\hat{\sigma}_3^{\text{div}}(1) = \frac{2^{19} \pi^2 \alpha^2 \alpha_s^3}{3^8 s^6 r^2 \epsilon_{\text{IR}}} \times \left[ (144r^6 + 28r^4 - 176r^2 + 43) S \\
+ (576r^{10} - 176r^8 - 792r^6 + 424r^4 - 48r^2) T \right],
\end{equation}

\begin{equation}
\hat{\sigma}_3^{\text{div}}(2) = \frac{2^{19} \pi^2 \alpha^2 \alpha_s^3}{5 \cdot 3^8 s^6 r^4 \epsilon_{\text{IR}}} \left[ (720r^8 + 452r^6 - 696r^4 + 7r^2 - 15) S \\
+ (2880r^{10} + 368r^8 - 3560r^6 + 1856r^4 - 56r^2) T \right],
\end{equation}

where $r = 2m_c/\sqrt{s}$, $S = \sqrt{1 - 4r^2}$, and $T = \tanh^{-1} S$. 

\[ e^+ e^- \rightarrow c \bar{c} \left( ^3P_j^{[8]} \right) + c \bar{c} \left( ^3P_j^{[1]} \right) + g \text{ IV} \]
\[ e^+ e^- \rightarrow c\bar{c}(^3P_j^{[8]}) + c\bar{c}(^3P_j^{[1]}) + g \, V \]

- \( \sigma_1^{\text{div}} \) is cancelled upon including NLO QCD corrections to
  \( \langle O_{\chi c J}(^3S_1^{[8]}) \rangle \) in \( e^+ e^- \rightarrow c\bar{c}(^3P_{J_1}^{[8]}) + c\bar{c}(^3S_1^{[8]}) \).

- \( \sigma_2^{\text{div}}(J_2) \) is cancelled upon including NLO QCD corrections to
  \( \langle O_{J/\psi}(^3S_1^{[1]}) \rangle \) in \( e^+ e^- \rightarrow c\bar{c}(^3S_1^{[1]}) + c\bar{c}(^3P_{J_2}^{[1]}) \).

- \( \hat{\sigma}_3^{\text{div}}(J_2) \) is left!

- Behaviors for \( r \rightarrow 0 \): \( \sigma_1^{\text{div}} \propto \frac{1}{r^7}, \sigma_2^{\text{div}} \propto \frac{1}{r^5}, \sigma_3^{\text{div}} \propto \frac{1}{r^4} \).

- Although new types of singularities will not disappear, they are less singular for \( r \rightarrow 0 \).
New type of IR divergence originates from interference of diagrams where soft gluon is attached to different $Q\bar{Q}$ states, independent of initial state and not requiring the quark pairs to have same flavor.

We predict that this will also appear in NLO QCD corrections for double $J/\psi$ and $J/\psi + \Upsilon$ hadroproduction.

However, structure of new IR divergence can be more complicated because more channels are involved.

For $gg \rightarrow c\bar{c}(3P_{Jc}^{[8]}) + b\bar{b}(3P_{Jb}^{[8]}) + g$, there will be 4 pairings:

$c\bar{c}(3S_1^{[8]}) + b\bar{b}(3P_{Jb}^{[8]})$, $c\bar{c}(3S_1^{[1]}) + b\bar{b}(3P_{Jb}^{[8]})$, $c\bar{c}(3P_{Jc}^{[8]}) + b\bar{b}(3S_1^{[1]})$, and $c\bar{c}(3P_{Jc}^{[8]}) + b\bar{b}(3S_1^{[8]})$, which lead to more interference terms.
Basic idea to solve the problem I

- Recall that factorization implies complete separation of perturbative and nonperturbative effects.
- First find a way to separate IR singular terms like $\Gamma_3^{\text{div}}(J_c)$ and $\sigma_3^{\text{div}}(J_2)$ into contributions from hard- and soft-scale regimes.
- 2 pairs cannot be at rest simultaneously. $\leadsto$ Use covariant form of NRQCD Lagrangian:

$$\mathcal{L}^{\text{LO}}_{\text{NRQCD}} = \bar{\psi}_v \left[ i v \cdot D + \frac{(iD^\mu)(iD_{\perp \mu})}{2m} \right] \psi_v + \bar{\chi}_v \left[ -i v \cdot D + \frac{(iD^\mu)(iD_{\perp \mu})}{2m} \right] \chi_v,$$

where $v^\mu = p^\mu / m$ with $m$ heavy-quark mass and $p^\mu$ $Q\bar{Q}$ momentum.
Basic idea to solve the problem II

- $\psi_v$ and $\chi_v$ are nonrelativistic heavy-quark and -antiquark 4-component spinor fields satisfying $\not\!\!\! v \psi_v = \psi_v$ and $\not\!\!\! v \chi_v = -\chi_v$.
- $\perp$ component of vector $a^\mu$ is defined as $a_\perp^\mu = a^\mu - \nu^\mu \nu \cdot a$.
- Creation and annihilation of heavy-quark pair surely take place at short distance. 1-loop correction diagrams:

Interference effect is described in last 2 panels.
To describe vertex, introduce new operators:

\[ \psi_{b,v_1} \mathcal{K}^{\mu_1 \nu_1} T^{a_1} \bar{\chi}_{b,v_1} \psi_{c,v_2} \gamma^\nu_2 T^{a_2} \chi_{c,v_2}, \]

\[ \psi_{b,v_1} \gamma_\Sigma T^a \bar{\chi}_{b,v_1} \psi_{c,v_2} \mathcal{K}^{\mu_2 \nu_2} \chi_{c,v_2}, \]

and charge conjugates for \( b\bar{b}(3P_{[8]}^J) \rightarrow c\bar{c}(3P_{[1]}^J) + gg \) and \( \bar{c}c(3P_{[1]}^J) + g. \)

\( \mathcal{K}s \) are defined as

\[ \mathcal{K}_0^{\mu \nu} = \frac{g^{\mu \nu} - \nu^{\mu} \nu^{\nu}}{\sqrt{3}} \left( -\frac{i}{2} \mathcal{D}_f \right), \]

\[ \mathcal{K}_1^{\mu \nu} = -\frac{i}{2} \left( \mathcal{D}_f \left[ \frac{\mu}{\Sigma} \frac{\nu}{\Sigma} \right] \right), \]

\[ \mathcal{K}_2^{\mu \nu} = -\frac{i}{2} \left( \mathcal{D}_f \left[ \frac{\mu}{\Sigma} \frac{\nu}{\Sigma} \right] \right), \]

\[ \mathcal{K}_{\mu \nu} = - \frac{i}{2} \left( \mathcal{D}_f \left[ \frac{\mu}{\Sigma} \frac{\nu}{\Sigma} \right] \right), \]

with

\[ a^{[\mu \nu]} = \frac{1}{2} (a^{\mu \nu} - a^{\nu \mu}), \]

\[ a^{(\mu \nu)} = \frac{1}{2} (a^{\mu \nu} + a^{\nu \mu}) - \frac{g^{\mu \nu} - \nu^{\mu} \nu^{\nu}}{3} a \cdot b. \]
One loop corrections of soft gluon

- Derive Feynman rules from Lagrangian and calculate loop integrals. E.g. the third panel:

\[
I = -ig_s^2 \mu^{4-D} \int \frac{d^D l}{(2\pi)^D} \frac{1}{2} \left[ \mathbf{v}_1 + \frac{(2q_1 + l)_\perp}{2m_1} \right] \cdot \left[ \mathbf{v}_2 + \frac{(2q_2 + l)_\perp}{2m_2} \right] \frac{1}{l \cdot \mathbf{v}_1 + \frac{(l+q_1)^2}{2m_1}} \frac{1}{l \cdot \mathbf{v}_2 + \frac{(l+q_2)^2}{2m_2}}.
\]

- Although numerically \( m_b \approx 3m_c \), assume that \( m_c \gg m_b v_b \) to ensure that nonrelativistic approximation still applies to charmonium.

- Expand integrand in series of \( 1/m_i \) and drop terms of order \( 1/m_i^2 \) and higher.
We get

\[ I = I_0 + \frac{\alpha_s \mu^{4-D}}{\pi m_1 m_2} \left( \frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right) \]

\[ \times \left[ \ln(\omega + \sqrt{\omega^2 - 1}) - \omega \sqrt{\omega^2 - 1} \right] \frac{q_1 \cdot q_2}{2(\omega^2 - 1)^{3/2}} \]

\[ + \frac{(\omega^2 + 2) \sqrt{\omega^2 - 1} - 3\omega \ln(\sqrt{\omega^2 - 1} + \omega)}{2(\omega^2 - 1)^{5/2}} (v_1 \cdot q_2)(v_2 \cdot q_1) \],

where \( \omega = v_1 \cdot v_2 \)

\( I_0 \) disappears upon summation over all diagrams.
Loop integrals are process independent, although results depend on $\omega$.

UV divergence can be removed through operator renormalization.

Multiply with corresponding SCDs and decompose tensor and color structures into basis of total-angular momentum and color states.

IR-singular parts exactly match those in $\Gamma_3^{\text{div}}(J_c)$ and $\sigma_3^{\text{div}}(J_2)$!

I.e. it is possible to construct general factorization formalism within NRQCD to describe processes involving 2 or more heavy quarkonia.
Summary and outlook

- We illustrated via 2 examples, $\Upsilon \rightarrow \chi_{cJ} + X$ and $e^+e^- \rightarrow J/\psi + \chi_{cJ} + X$, that there are uncancelled IR divergences in standard formulation of NRQCD factorization approach.
- We extended the conclusions to any subprocess involving 2 $P$-wave Fock states.
- We introduced new types of operators and showed that their NLO QCD corrections precisely reproduce the uncancelled IR divergences.
- Much more further work is needed to construct a generalized NRQCD factorization formalism and to investigate its phenomenological impact, especially for double quarkonia hadroproduction.

Thank you!