Quarkonium properties at T>0 from lattice NRQCD and pNRQCD

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References:

S.Kim, P. Petreczky, A.R., JHEP 1811 (2018) 088

P. Petreczky, A.R., J. Weber, NPA982 (2019) 735 & in preparation

D. Lafferty, A.R. in preparation

Motivation: Quarkonium in HIC

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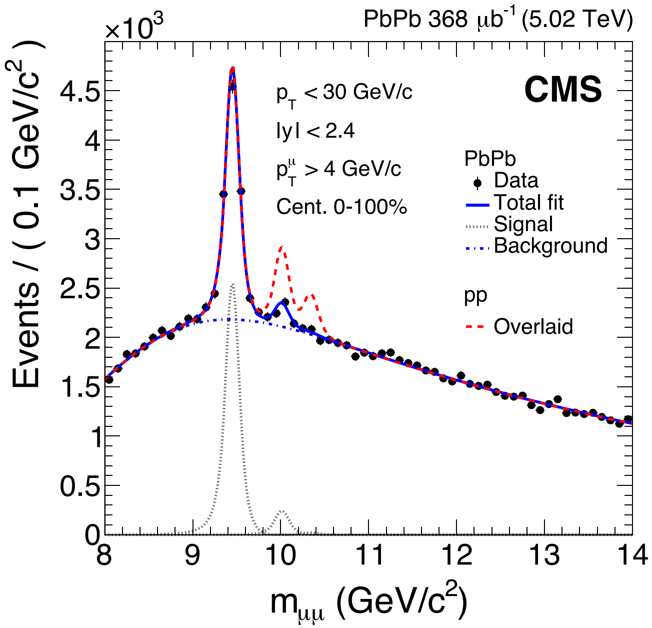
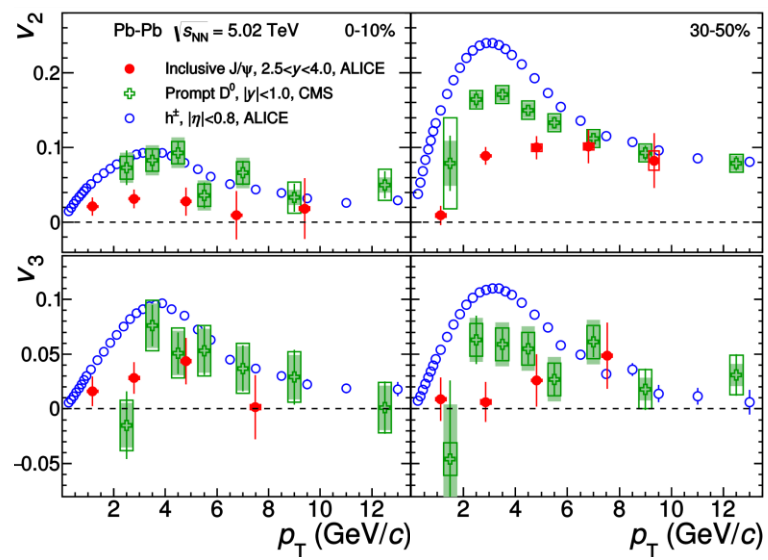
- Heavy quarkonium: **Precision probes** of the QGP



[CMS collaboration] PRL120 (2018) 142301

Bottomonium: a non-equilibrium probe of the full QGP evolution



[ALICE collaboration] JHEP 1902 (2019) 012

Charmonium: a partially equilibrated probe, sensitive to the late stages

- Here idealized setting: properties of **quarkonium in a static medium**

For recent work on open-quantum-system dynamics see e.g. N. Brambilla et.al. arXiv:1903.08063 and PRD97 (2018) 074009
J.P. Blaizot, M. Escobedo JHEP 1806 (2018) 034
S. Kajimoto et.al. Phys.Rev. D97 (2018) no.1, 014003

Equilibrium strategy

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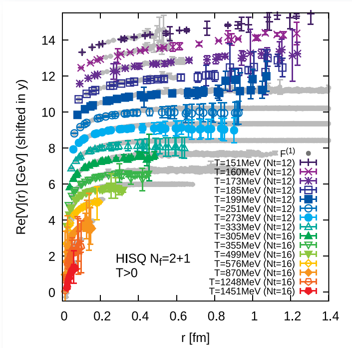
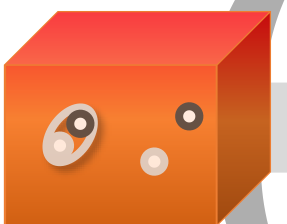


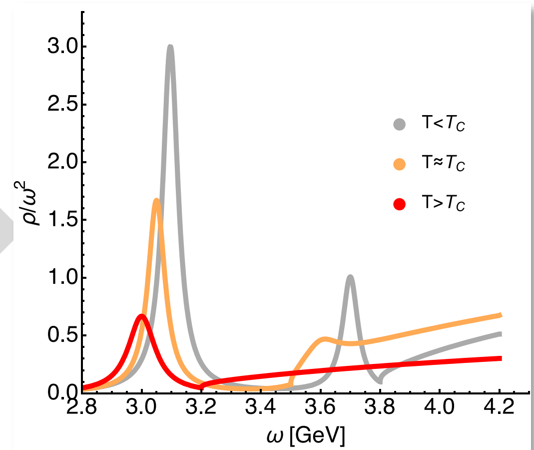
II. Via pNRQCD potential from the lattice QCD Wilson loop

(see e.g. P. Petreczky, A.R., J. Weber, NPA982 (2019) 735
see also Y.Burnier, O.Kaczmarek, A.R. JHEP 1512 (2015) 101)

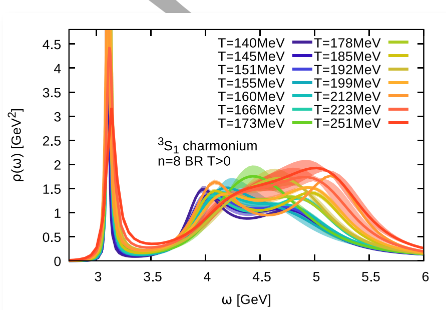


Assume full kinetic themalization of QQ
&
Static medium from lattice QCD

In-medium meson spectra



schematic depiction



I. Direct reconstruction of lattice meson spectra using lattice NRQCD

(see e.g. S.Kim, P. Petreczky, A.R., JHEP 1811 (2018) 088
see also FASTSUM JHEP 1407 (2014) 097)

Lattice NRQCD

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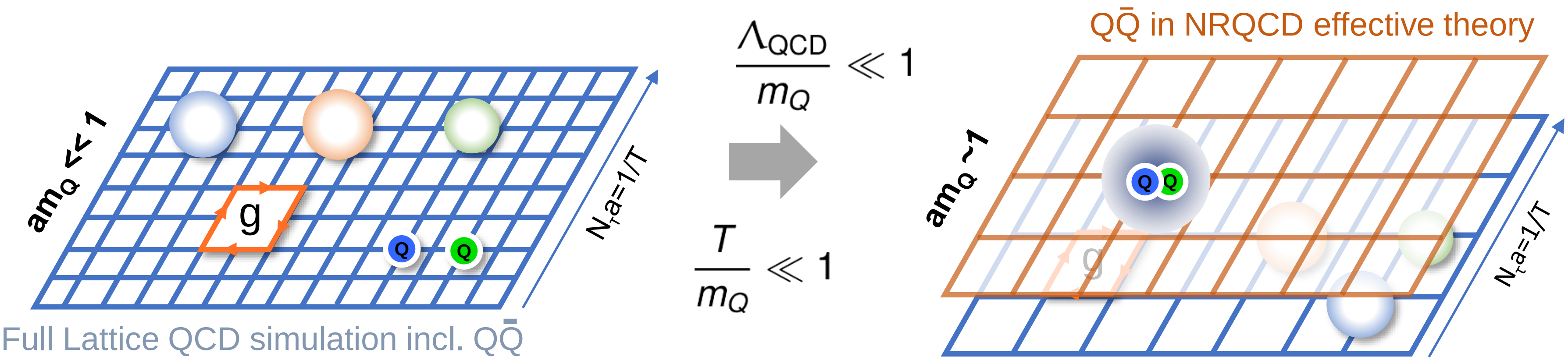
- Exploit separation of scales to treat heavy quarks non-relativistically



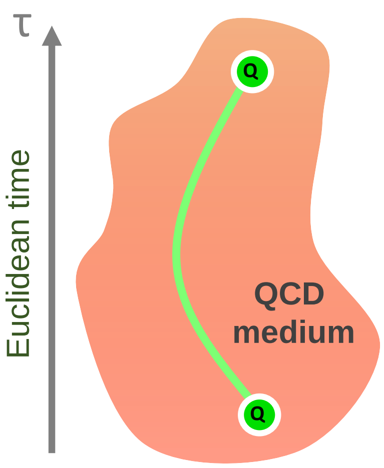
Full Lattice QCD simulation incl. $Q\bar{Q}$ (still too costly)

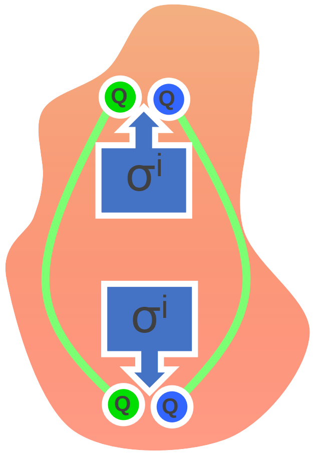
Lattice QCD simulation without $Q\bar{Q}$

- **Lattice Non-Relativistic QCD** (NRQCD) well established at T=0, applicable at T>0
  - no modeling, systematic expansion of QCD action in $1/m_Q a$, includes $v\neq 0$ contributions
    Thacker, Lepage Phys.Rev. D43 (1991) 196-208
  - our implementation uses $O(v^4)$, i.e. $O(1/(m_Q a)^3)$ and leading order Wilson coefficients
- **Realistic** & **high statistics** simulations of the QCD medium by HotQCD
  HotQCD PRD85 (2012) 054503, PRD90 (2014) 094503
  - **m_π=161MeV** T=0 N_τ=32-64 T>0 N_τ =12
    - T= [**140 – 407**] MeV $m_b a$= [2.759 – 1.559] Lepage n_b=4
    - T= [140 – 251] MeV **$m_c a$**= [0.757 – 0.427] Lepage n_c=8

For FASTSUM results see C. Alltons talk at 10:05h (better $m_Q a$, less realistic medium m_π)

NRQCD Euclidean correlators





Non-rel. propagator of a single heavy quark G

Davies, Thacker Phys.Rev. D45 (1992)

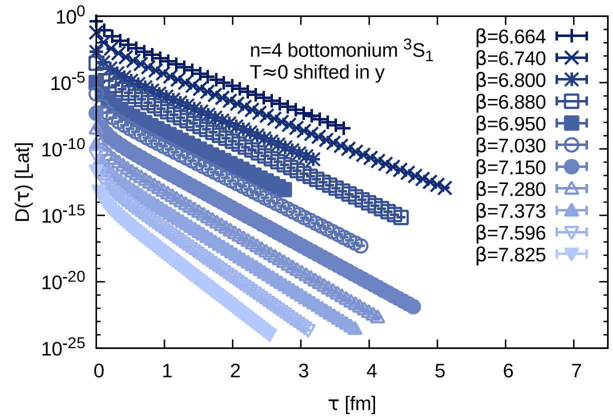
see talk on extended sources by R. Larsen at 10:30h today

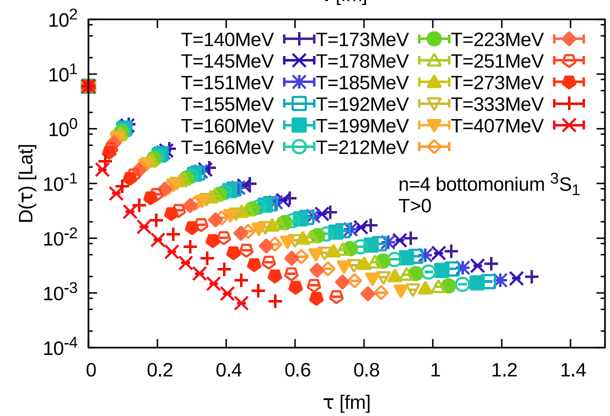
QQ propagator projected to a certain channel

„correlator of QQ wavefct.

$D_{J/\psi}(\tau) \triangleq \langle \psi_{J/\psi}(\tau)\psi^\dagger_{J/\psi}(0)\rangle$ "

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423





Euclidean correlation functions at T=0 and T>0

Bayesian spectral reconstruction

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- Inversion of Laplace transform required to obtain spectra from correlators

$$D(\tau_i) = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l \Delta\omega_l \qquad D(\tau) = \int_{-2M_Q}^{\infty} d\omega\, \exp[-\omega\tau]\, \rho(\omega)$$

1. N_ω parameters ρ_l >> N_τ datapoints
2. simulation input D_i has finite precision

- Bayes: regulate χ^2 fit (P[D|ρ]=exp[-L]) with prior information (P[ρ|I]=exp[S])

$$P[\rho|D,I] \propto P[D|\rho]\; P[\rho|I] \qquad \left.\frac{\delta}{\delta\rho} P[\rho|D,I]\right|_{\rho=\rho^{BR}} = 0$$

for standard MEM see e.g.
Asakawa, Hatsuda, Nakahara
Prog.Part.Nucl.Phys. 46 (2001) 459

  - Regularization affects the end result: convergence to unique result as N_τ -> ∞ dD/D ->0

Standard BR method (BR)

$$S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log\left[\frac{\rho}{m}\right]\right)$$

- Resolves narrow peaked structures with high accuracy
- Ringing in broad structures if reconstructed from small # of datapoints

„high gain – high noise“

Y. Burnier, A.R. PRL 111 (2013) 182003

Low ringing BR method (BR_l)

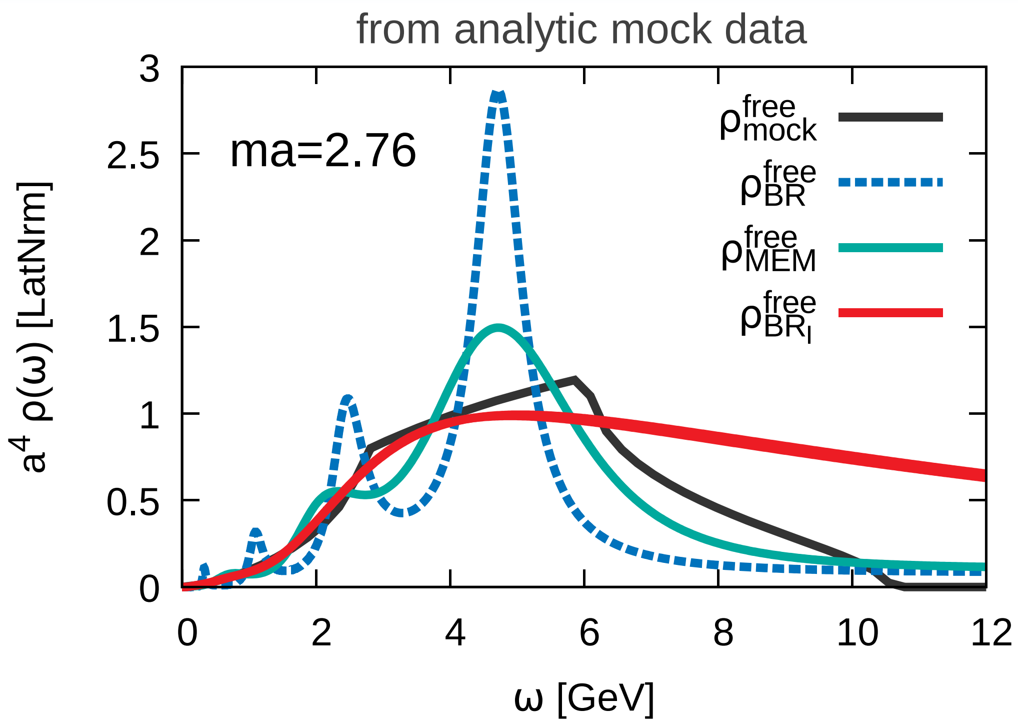
$$S_{BR_\ell} = \alpha \int d\omega \left(\kappa\left(\frac{\partial\rho}{\partial\omega}\right)^2 + 1 - \frac{\rho}{m} + \log\left[\frac{\rho}{m}\right]\right)$$

- Introduces penalty on arc length of reconstruction $(dL/dw)^2=1+(d\rho/dw)^2$
- Efficiently removes ringing but may lead to overestimated peak widths

„low gain – low noise“

c.f. C. Fsicher et.al. PRD98 (2018) 014009

Calibrating the smooth BR method



- Hyperparameter κ makes smoothing explicit (c.f. in MEM implicit in # datapoints)
- Use prior knowledge: free spectral functions known analytically
- In reconstruction from N_τ=12 data: κ=1 successfully suppresses ringing
- Use smooth BR to verify peak existence, standard BR for peak position etc.

Towards finite temperature spectra

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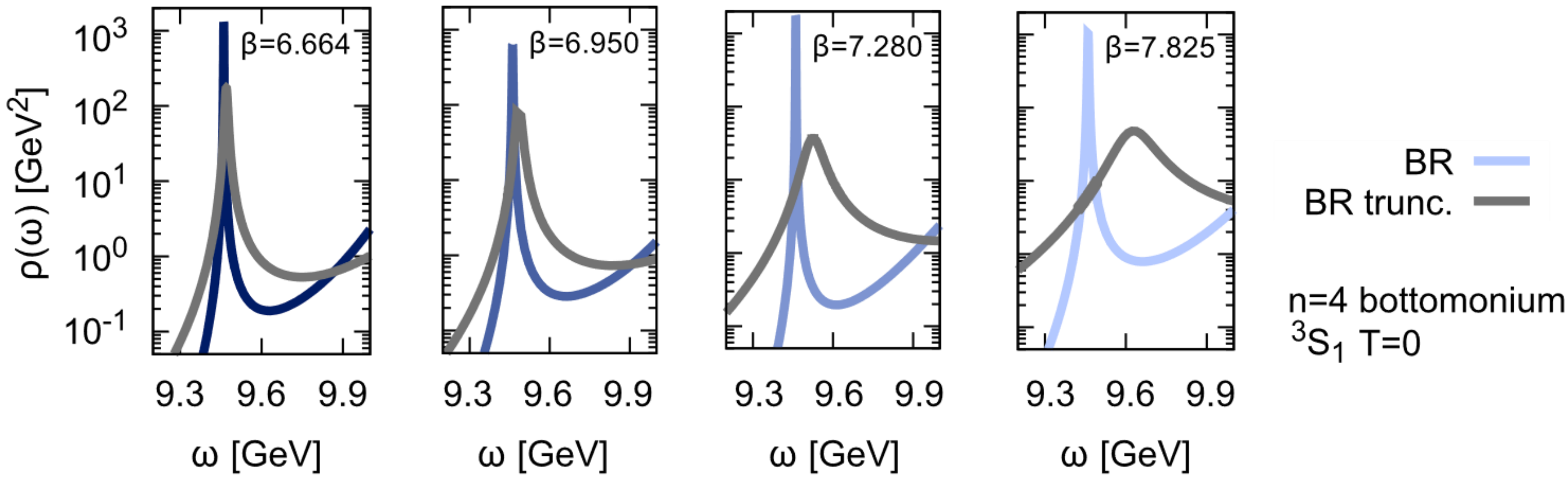
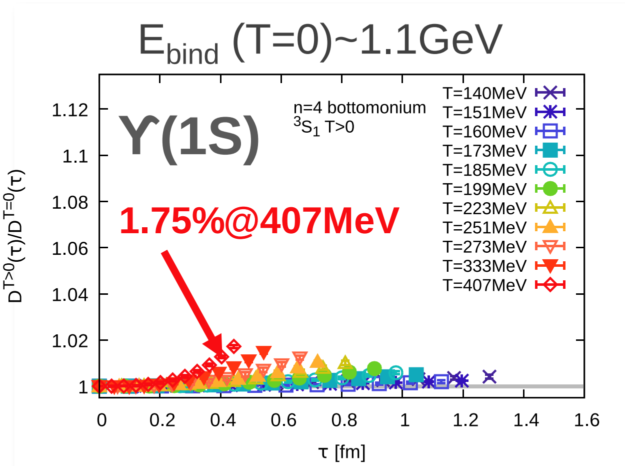
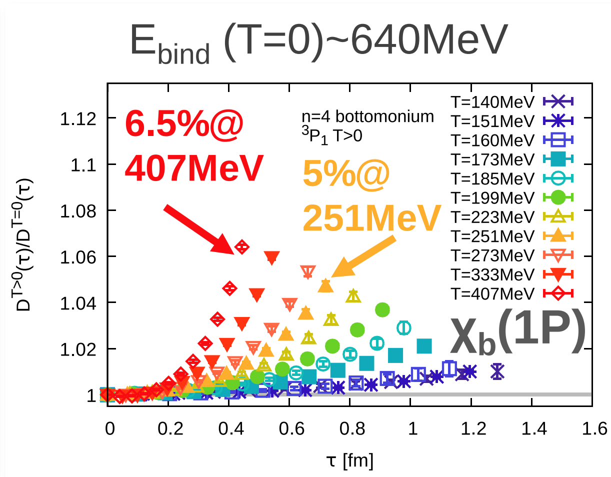


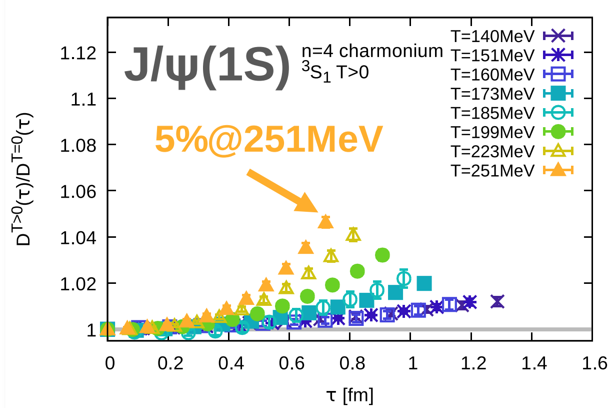
- “High-gain” BR method resolves T=0 ground state very well from N_T=48-64 points
- How does accuracy suffer from limited Euclidean extent at T>0 (N_T=12) ?

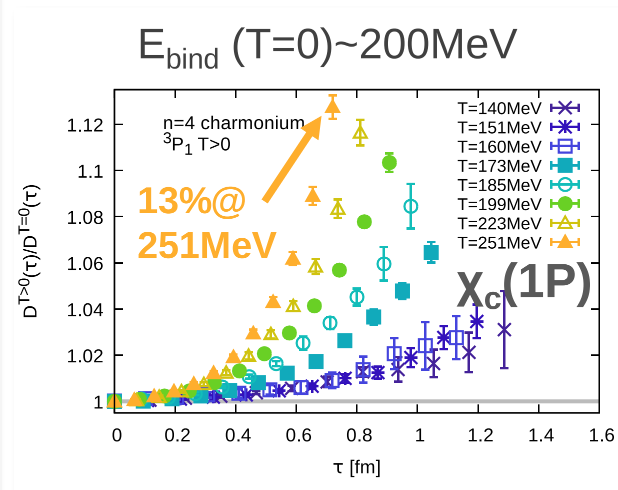
→ Systematic shift of peaks to higher frequencies, as well as broadening. needs to be accounted for when analyzing T>0 spectra

Correlator ratios









- Significant upward bent in the QGP phase
- Already in hadronic phase deviations from unity

- Overall in-medium modification hierarchically ordered with vacuum binding energy

S-wave bottomonium melting at T>0

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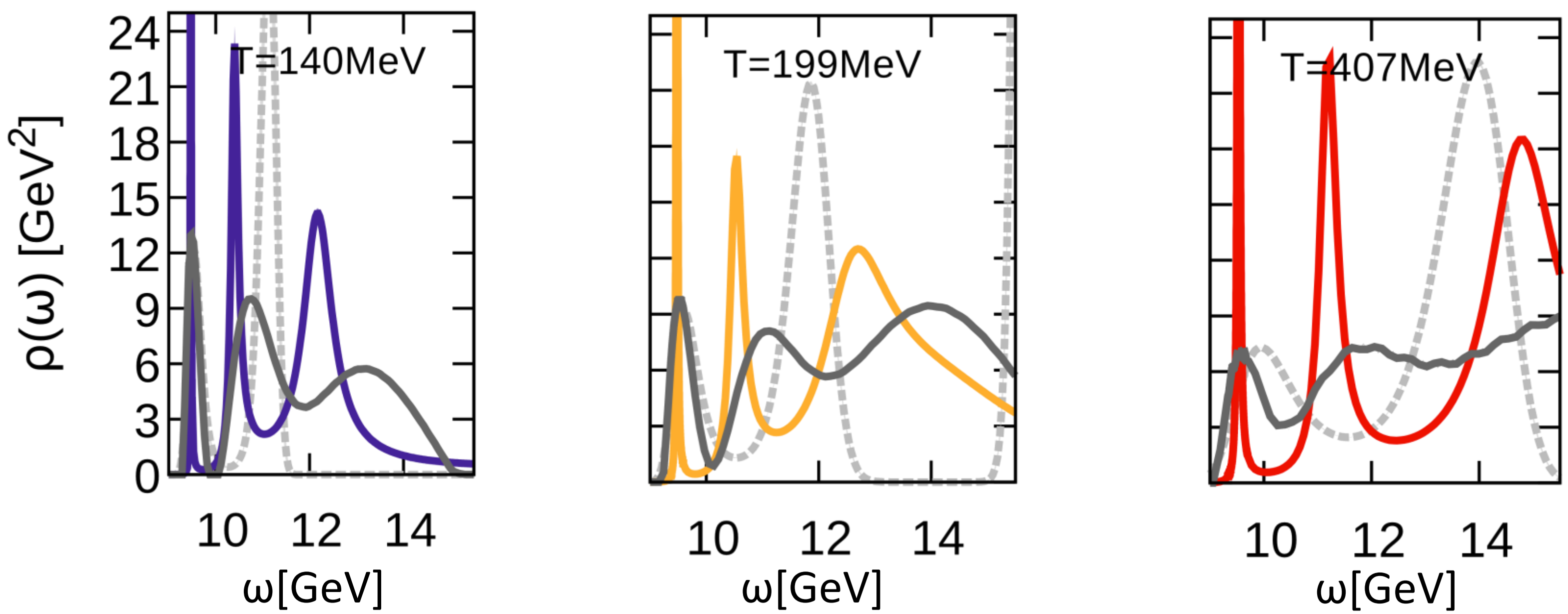
- Naïve definition of melting temperatures: disappearance of peak structures



- Three methods: BR (colored), smooth BR (gray solid) & MEM (gray dashed)
- Very similar results for MEM and smooth BR for ground state strength

All three methods show consistently **GS peak remnant at T=407 MeV**

Bottomonium P-wave melting at T>0

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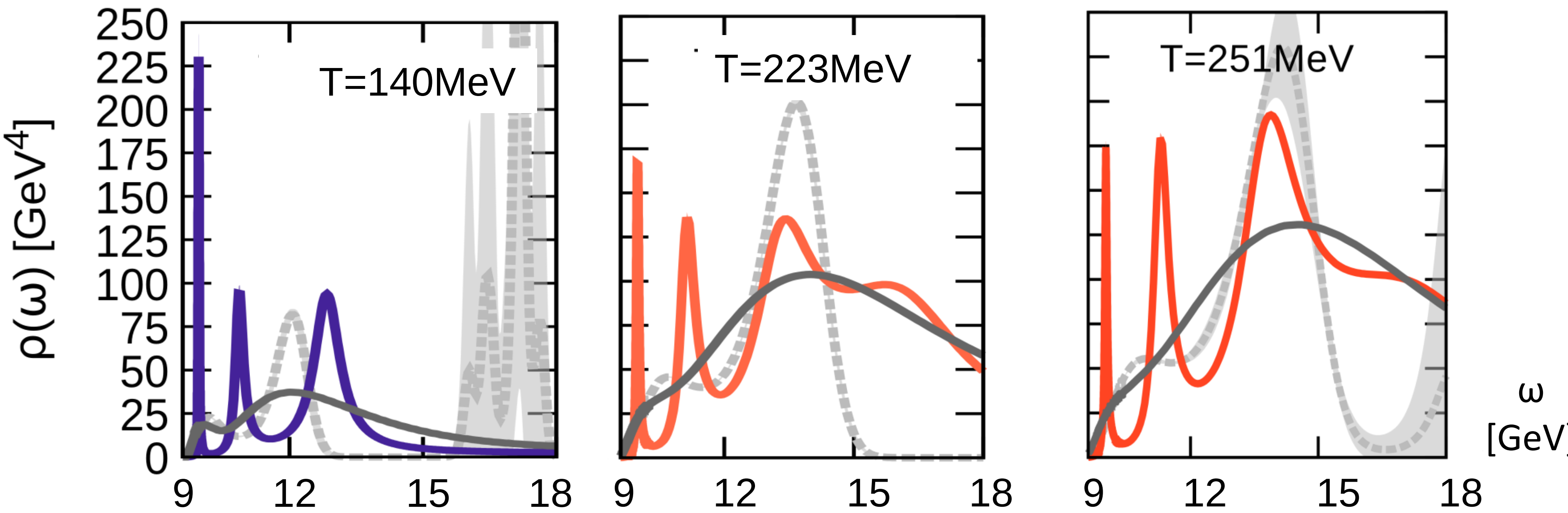

- Three methods: BR (colored), smooth BR (gray solid) & MEM (gray dashed)
  - Ringing artifacts fully absent in smooth BR method at higher frequencies
  - Smooth BR shows disappearance of GS peak similar to MEM at T~223MeV
  - We now understand: standard BR method GS peak at T=251MeV is ringing
- Our updated melting T are lower than before and move closer to FASTSUM

In-medium mass shifts

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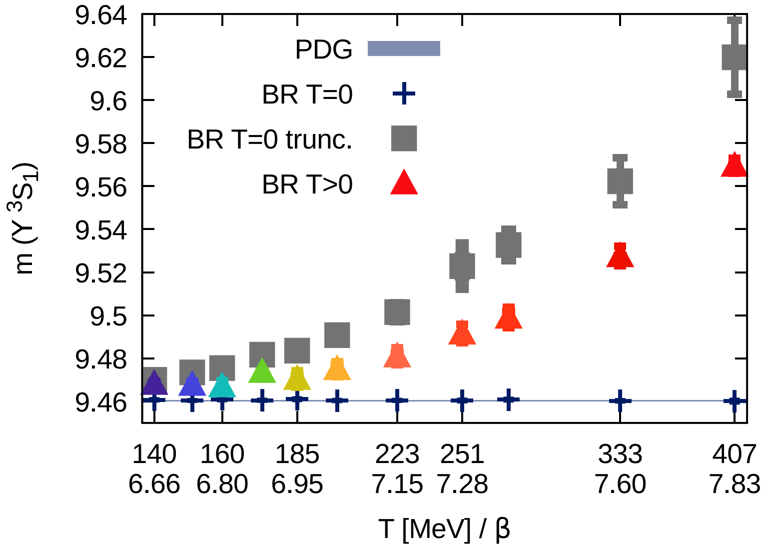
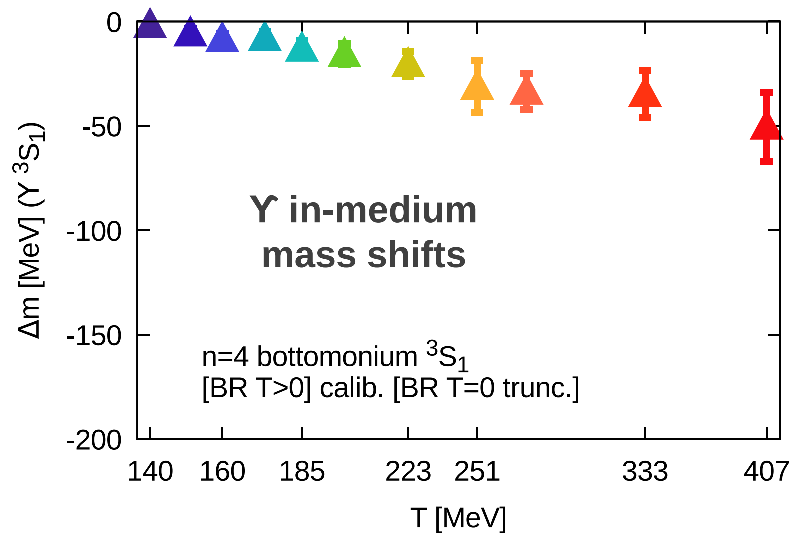
- Crucial ingredient: establish the correct baseline to interpret the in-medium masses



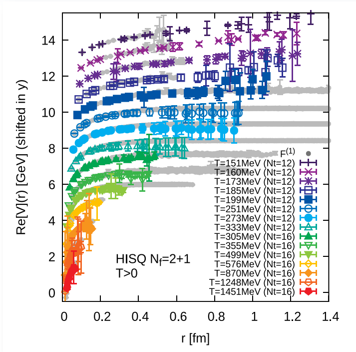


- Truncated T=0 reconstruction shows artificial shift to higher frequencies (gray sq.)

- In-medium shifts at T=140MeV very close to truncated results (no in-medium mod.)

- At higher temperatures masses lie clearly below the baseline
  (compatible with non-perturbative potential based computations (pNRQCD))

Equilibrium strategy



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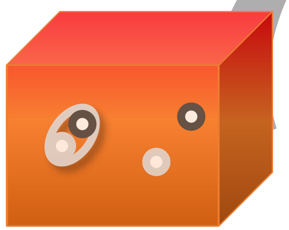

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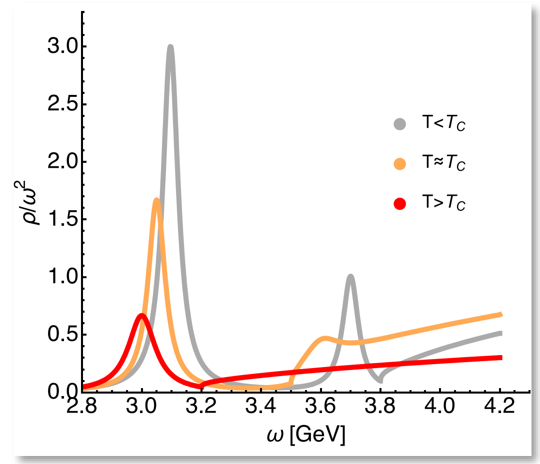

schematic depiction

The real-time interquark potential

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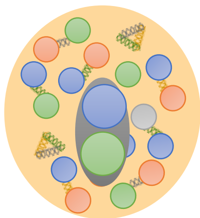
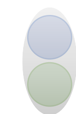
- Exploit $\frac{T}{m_Q} \ll 1,\ \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically

Relativistic T>0 field theory



Brambilla et.al. Rev.Mod.Phys. 77 (2005) 1423

QCD	NRQCD	pNRQCD
Dirac fields	Pauli fields	Singlet/Octet
$Q(x), \bar{Q}(x)$	$\psi, \psi^\dagger, \chi, \chi^\dagger$	$\psi_S(R,t), \psi_O(R,t)$



Quantum mechanics

$$i\partial_t \langle \psi_s(t)\psi_s(0)\rangle = \left(V^{QCD}(R) + \mathcal{O}(m_Q^{-1}) + \Theta(R,t)\right)\langle \psi_s(t)\psi_s(0)\rangle$$

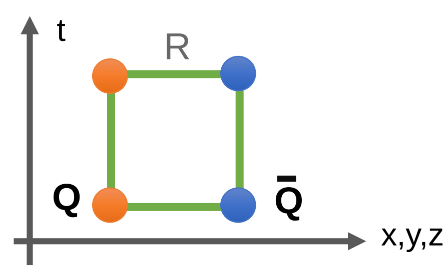
- Matching to underlying QCD in the infinite mass limit: Wilson loop

$$\langle \psi_S(R,t)\psi_S^*(R,0)\rangle_{\text{pNRQCD}} \equiv W_\square(R,t) = \left\langle \text{Tr}\left[\exp\left(-ig\int_\square dx^\mu A_\mu(x)\right)\right]\right\rangle_{\text{QCD}}$$

- Wilson loop: potential emerges at late times

$$V(R) = \lim_{t\to\infty} \frac{i\partial_t W_\square(R,t)}{W_\square(R,t)} \in \mathbb{C}$$

Im[V]: Laine et al. JHEP03 (2007) 054;
Beraudo et. al. NPA 806:312,2008
Brambilla et.al. PRD 78 (2008) 014017



- In this form: Minkowski time quantities and not directly accessible on the lattice

Non-perturbative evaluation of V(R)

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- How to connect to the Euclidean domain: **spectral functions**

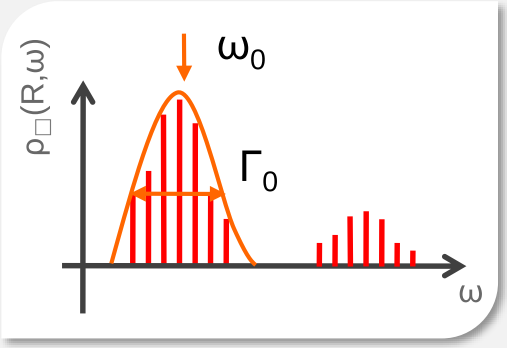
A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega) \quad \Leftrightarrow \quad W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau} \rho_\square(R,\omega)$$

Spectral Decomposition

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t} \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega)}$$

A.R., T.Hatsuda & S.Sasaki PoS LAT2009 (2009) 162



well defined V(R) if low lying Breit-Wigner present in Wilson loop spectral function

For technical details see Y.B., A.R. PRD86 (2012) 051503

$$V(R) = \omega_0(R) - i\Gamma_0(R)$$

Spectral Reconstruction

- In case of usual ΔW/W=10^{-2} statistical uncertainty in $W_\square$: **Bayesian inference**

incorporate prior information to regularize the inversion task (BR method)

- In case of **small ΔW/W<10^{-3}** statistical uncertainty in $W_\square$ also **Pade approximation**

exploit the analyticity of the Wilson correlator to extract spectra

Extracting the potential

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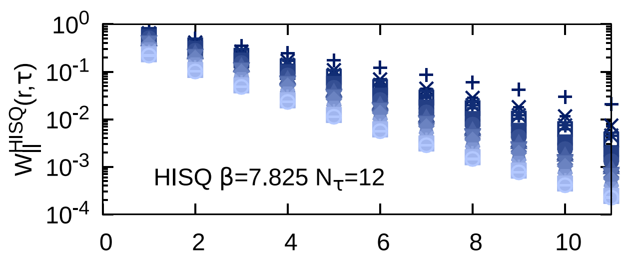
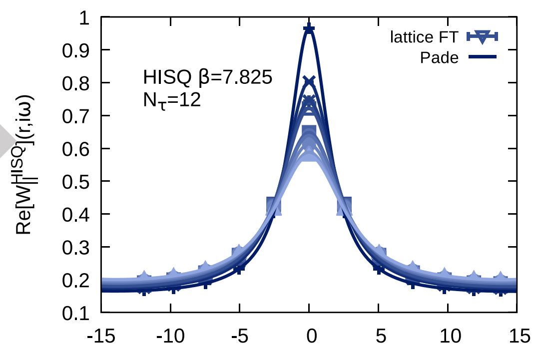
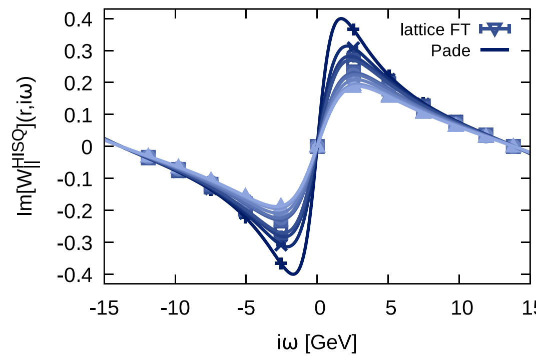
- Example: Pade based reconstructions at β=7.825 T=407MeV N_τ=12

P. Petreczky, A.R. J. Weber in preparation



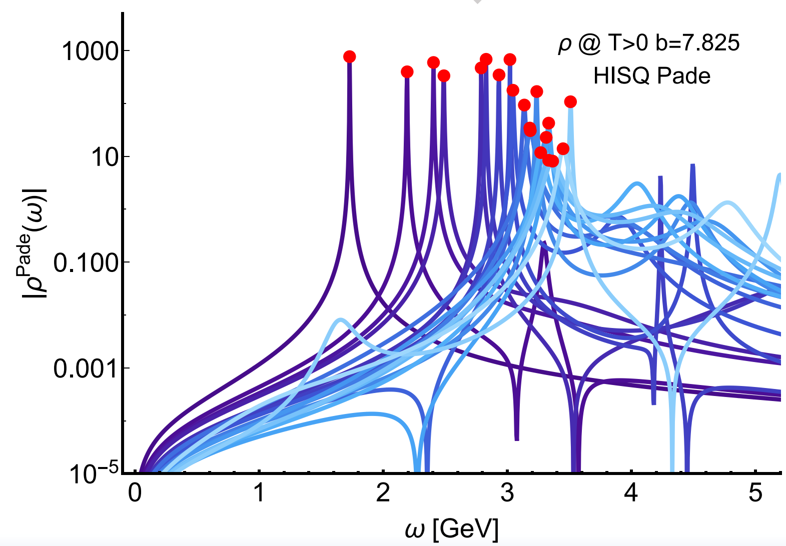
DFT





$$\rho(\omega) \approx -\frac{1}{\pi}\mathrm{Im}[R_{N_\tau}(\omega)]$$

- Always find well defined lowest peak: potential picture appears viable
- Beware of Pade artifacts besides peak: e.g. positivity violation, spikes
- Not yet conclusive since also reasonable χ^2 for model fit where additional low lying structures compensate change in dominant peak



Latest results on the lattice potential

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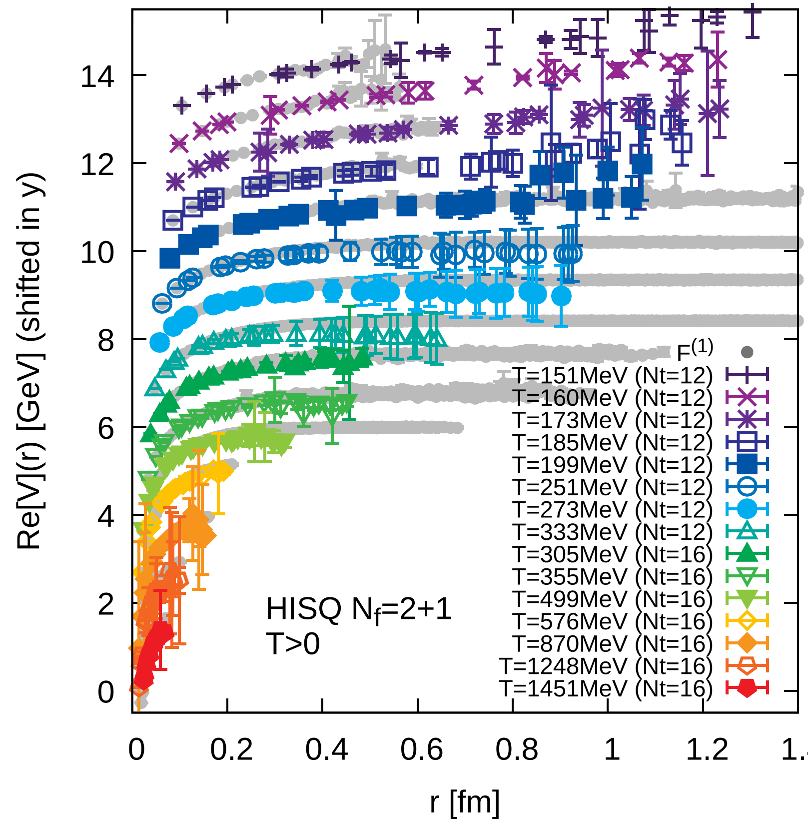
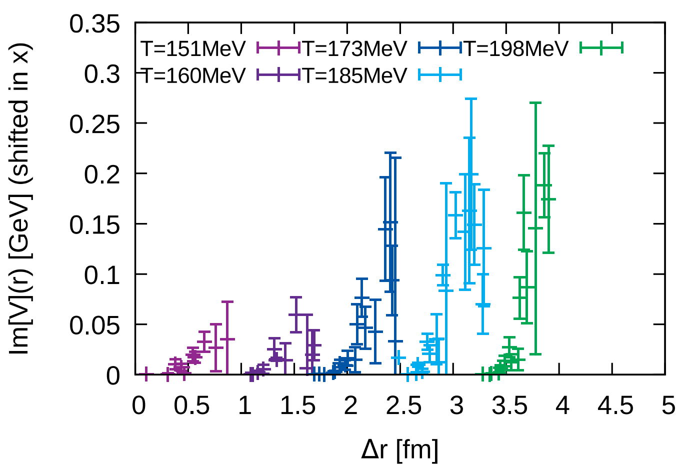
- Lattices with dynamical u,d,s quarks (HISQ action, HotQCD & TUMQCD)

A. Bazavov et.al. PRD97 (2018) 014510, HotQCD PRD90 (2014) 094503

  - realistic m_π~161MeV (T=151-1451MeV)
  - fixed box (N_s=48 - **N_τ=12, N_τ=16**) & very **high statistics** 4000-9000 realizations
  - Pade based extraction for Re[V] possible



P. Petreczky, A.R., J. Weber, NPA982 (2019) 735



- Smooth transition from Cornell @ T=0 to Debye screened @ T>T_C
- Finite Im[V] above T_c present

An improved Gauss law approach

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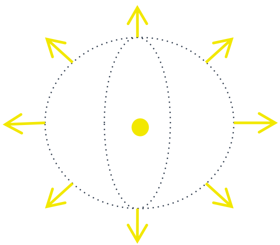
- For use in phenomenology applications: analytic expression for Re[V] and Im[V]



$$V_{Q\bar{Q}}^{T=0}(R) = V_C(R) + V_S(R) = -\frac{\alpha_S}{r} + \sigma r + c$$

$$\mathcal{G}_a[V(R)] = \vec{\nabla}\left(\frac{\vec{\nabla}V(R)}{R^{a+1}}\right) = -4\pi q \delta^{(3)}(\vec{R})$$

Strategy:

α_s,σ and c are vacuum prop. and do not change with T

$$V(R) = aqR^a$$

Coulombic: a=-1 q=α_s

$$\vec{\nabla}\left(\vec{\nabla}V_C(R)\right) = -4\pi\alpha_S\delta(\vec{R})$$

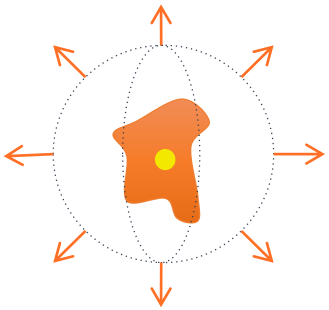
String-like: a=+1 q=σ

$$\vec{\nabla}\left(\frac{\vec{\nabla}V_S(R)}{R^2}\right) = -4\pi\sigma\delta(\vec{R})$$

V. V. Dixit, Mod. Phys. Lett. A 5, 227 (1990)

- Immerse non-perturbative charge in weak coupling HTL medium: permittivity ε

original idea: Y.Burnier, A.R. Phys.Lett. B753 (2016) 232 improved derivation D.Lafferty and A.R. in preparation



$$V^{med}(\mathbf{p}) = V^{vac}(\mathbf{p})/\epsilon(\mathbf{p}) \qquad \epsilon^{-1}(\vec{p}, m_D) = \frac{p^2}{p^2+m_D^2} - i\pi T\frac{pm_D^2}{(p^2+m_D^2)^2}$$

$$\mathcal{G}_a[V^{med}(\mathbf{r})] = \mathcal{G}_a\int d^3y\left(V^{vac}(\mathbf{r}-\mathbf{y})\epsilon^{-1}(\mathbf{y})\right) = 4\pi q\epsilon^{-1}(\mathbf{r}, m_D)$$

- 3 vacuum parameters and 1 temperature dependent m_D fix both Re[V] and Im[V].

Gauss-law solution to Re[V] & Im[V]

- We find an interesting connection to the classic Karsch-Mehr-Satz result:

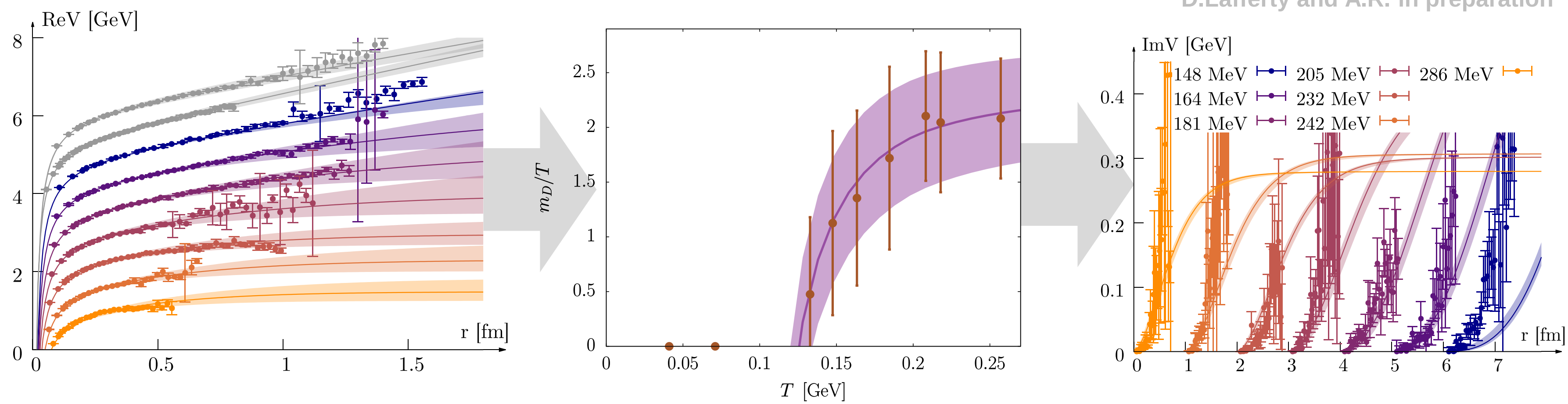
$$Re[V^{med}](r) = \frac{2\sigma}{m_D}\left(1 - e^{-m_D r}\right) - \sigma r e^{-m_D r} - \frac{\alpha_s}{r} e^{-m_D r}$$

$$V^{KMS}(r) = \frac{\sigma}{m_D}\left(1 - e^{-m_D r}\right) \quad + \quad V^{entropic} = \frac{\partial}{\partial T} V^{KMS}(r,T)$$

c.f. e.g. H. Satz, EPJC75 (2015) 193 and Guo et.al. arXiv:1806.04376

- Explicit and closed expressions for Im[V] are also obtained
- Gauss-Law result allows to fit the lattice data even in the non-perturbative regime

D.Lafferty and A.R. in preparation



Spectral functions from the potential

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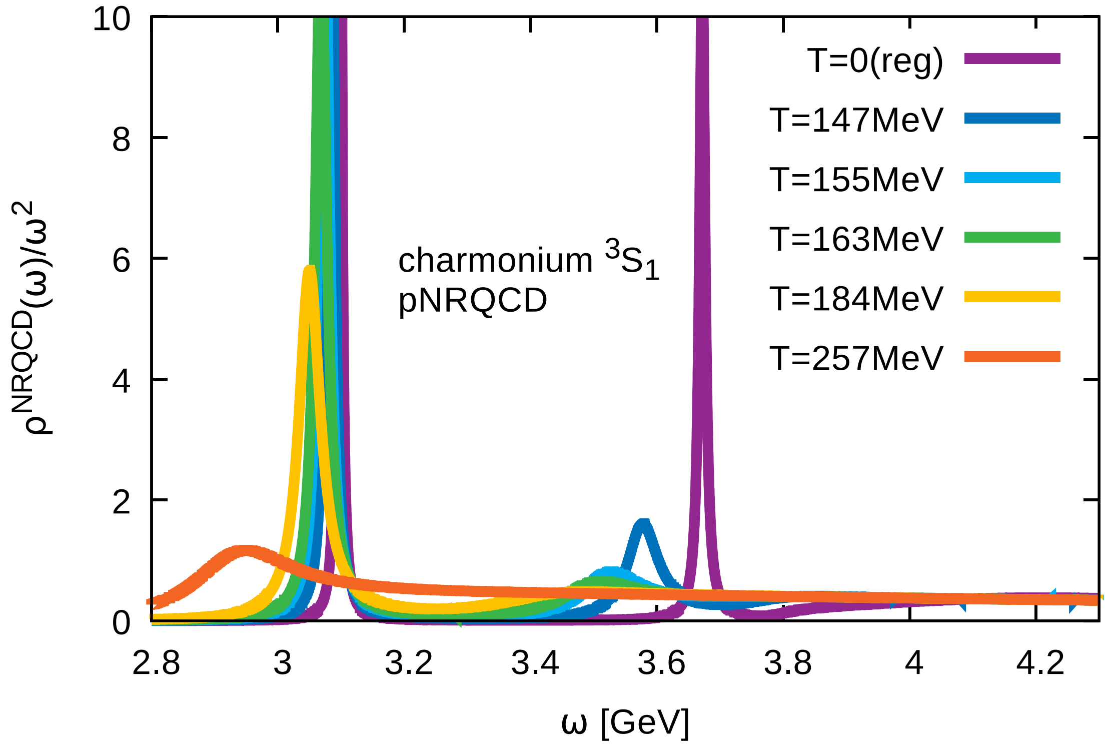

Y.Burnier, O.Kaczmarek, A.R.
JHEP 1512 (2015) 101

- Lattice pNRQCD based in-medium spectra: shift to **lower masses** and broadening
- We find behavior **consistent with lattice NRQCD** extracted spectral functions

Conclusion

- Lattice QCD and EFT are powerful tools to elucidate T>0 quarkonium
- Direct reconstruction of in-medium spectra from lattice NRQCD
  - **Use of multiple Bayesian methods** provide better control over systematics
  - Convergence of **melting T's** from different methods and groups
  - Determination of **negative in-medium mass shifts** consistent with pNRQCD
- Extracting the nonperturbative pNRQCD T>0 static interquark potential
  - Exploring a **Pade based** extraction of the in-medium **heavy quark potential**
  - Close to first determination of complex V(R) on lattices with **realistic** $\mathbf{m_\pi}$
  - Lattice vetted Gauss-law **parametrization** with $\mathbf{m_D}$ **single T dep. parameter**

Grazie per l'attenzione - Thank you for your attention

Mass splittings at T=0

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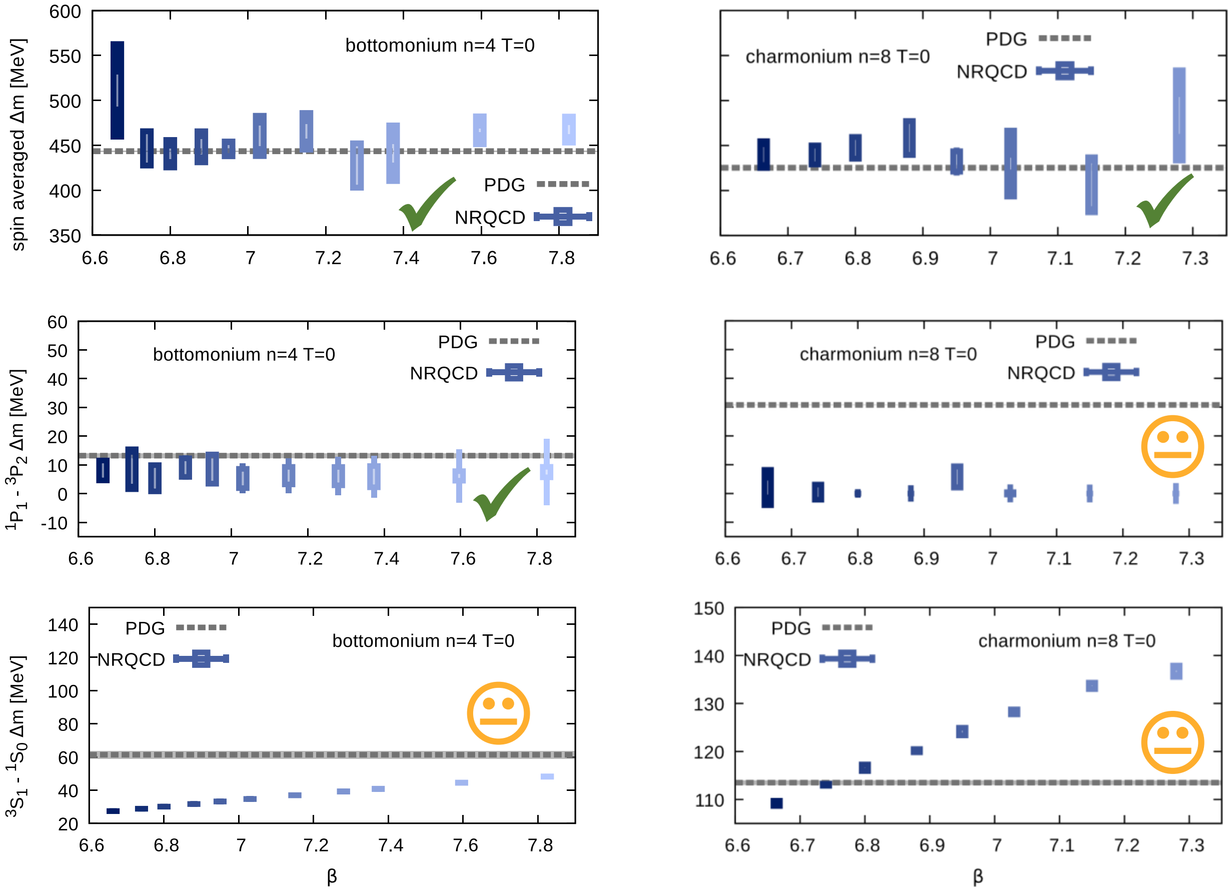
- With T>0 spectra goal, no T=0 specific NRQCD improvements: Accuracy?



Spin weighted difference between S- and P-wave (c.f. potential model: dep. only on central pot.)

P-wave $^1P_1$ $^3P_2$ splitting (c.f. potential model: spin-orbit coupling)

S-wave splitting (c.f. potential model: HFS)

Max 35MeV deviation. (HFS: NRQCD $O(v^6)$ and $O(\alpha_S v^4)$)

- Reasonable agreement but no competition with high-prec. T=0 NRQCD

Mock data tests with HTL & Pade (II)

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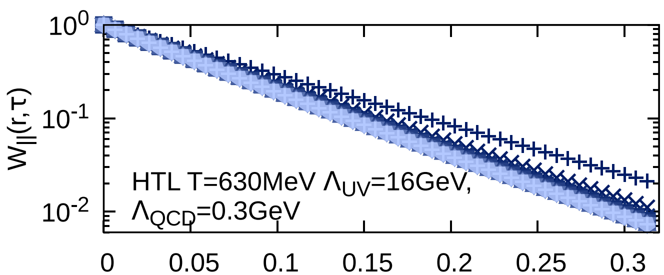
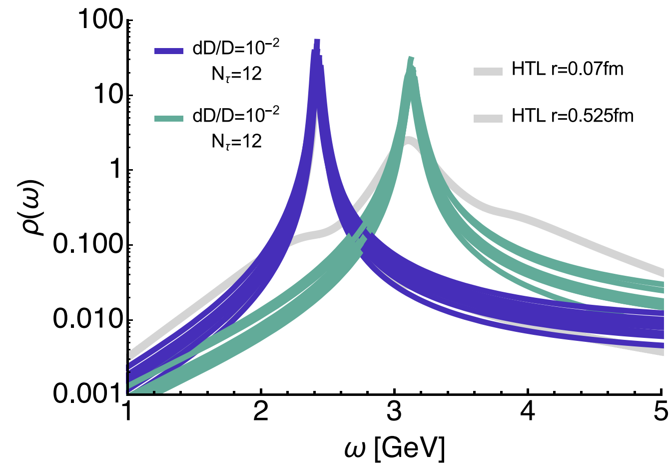
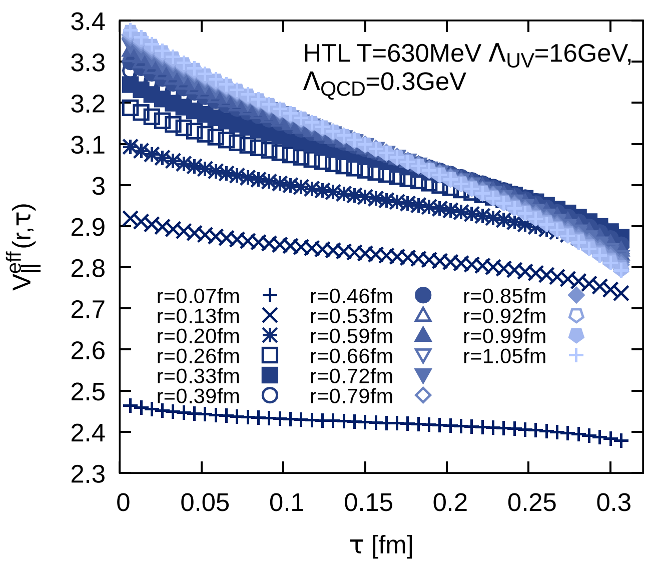
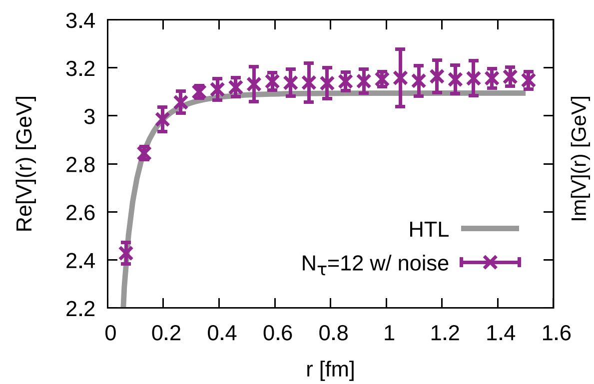
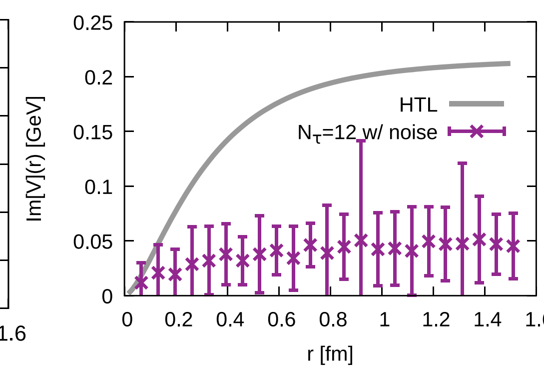
- Perform mock data analysis with resummed perturbative Wilson correlators (HTL)









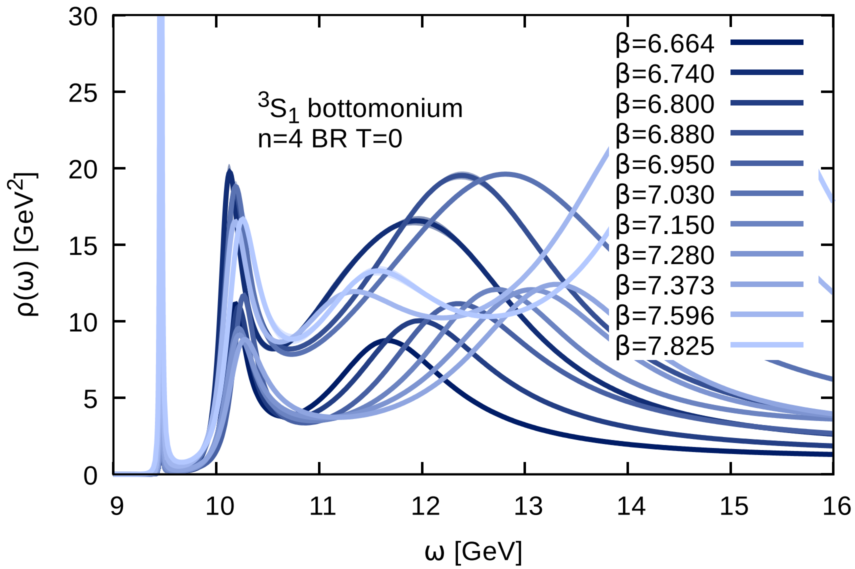


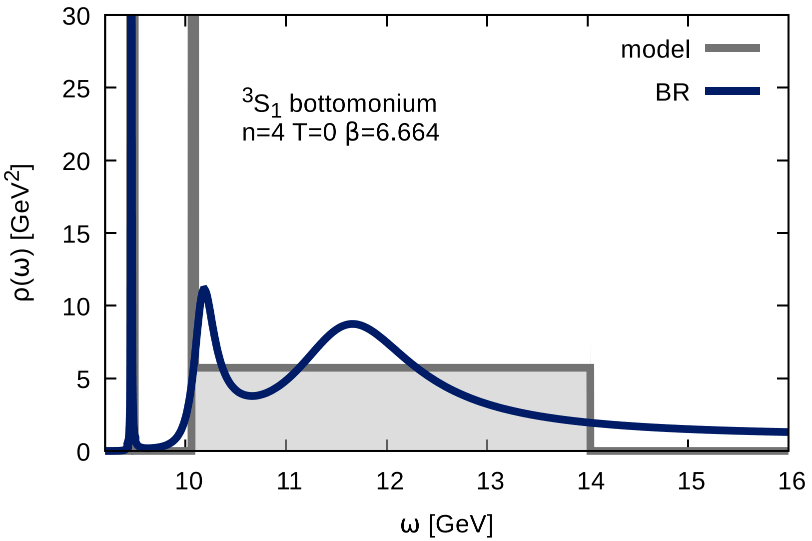
No plateau in effective potential $V_{eff}=-\log[W_i/W_{i+1}]$: large spectral widths

At $N_\tau=12$, errors of $dD/D=10^{-2}$ detrimental to Im[V] but Re[V] well reconstructed

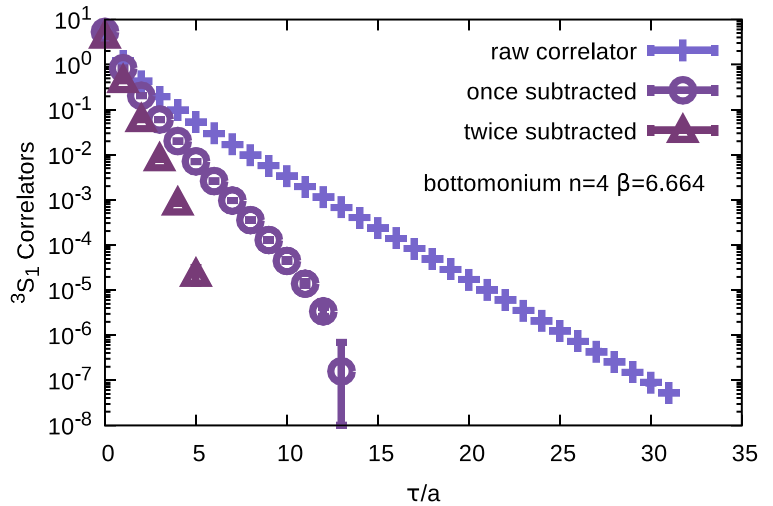
HTL correlators: Y.Burnier, A.R. PRD87 (2013) 114019

Information content at T=0





- With a hypothetically perfect extraction, how much can we learn from the data?
- Offer alternative interpretation of correlator: two peaks and a box. Fits correlator $\chi^2/N_\tau \approx 1$
- After subtracting two peaks, only small # of relevant points remain.



How to improve reconstructions?

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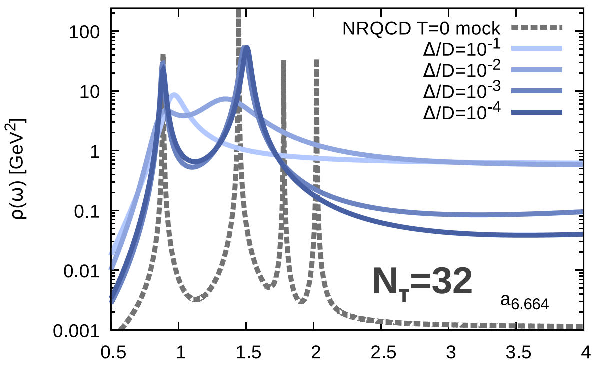
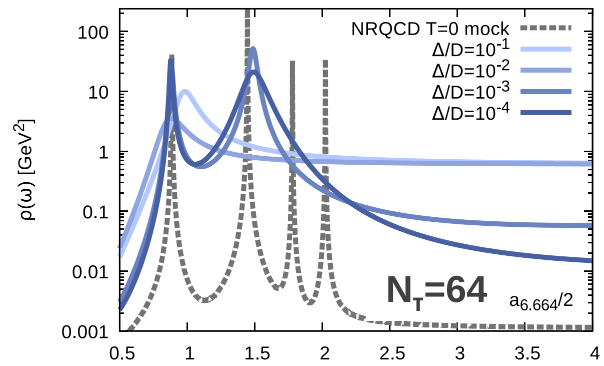
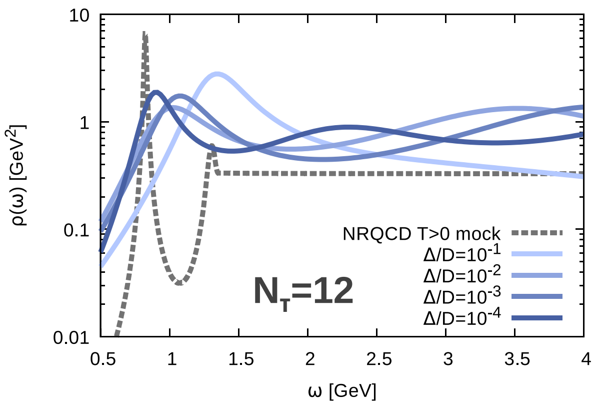
Mock Test

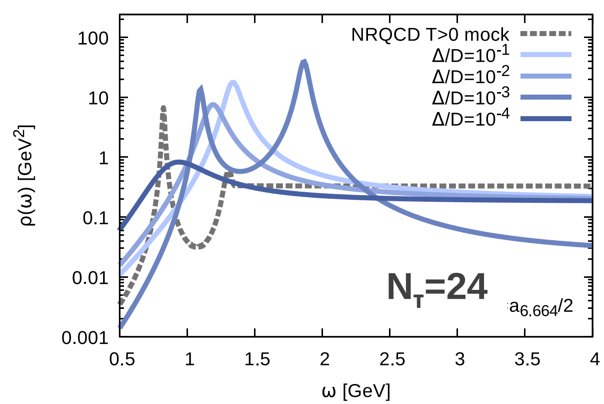




- Towards the continuum: no significant improvement of bound state reconstruction

Mock Test





- With e.g. anisotropic lattices, the continuum will be better under control
- Progress needs new ideas: e.g. full QCD multilevel algorithm.

Charmonium P-wave melting at T>0

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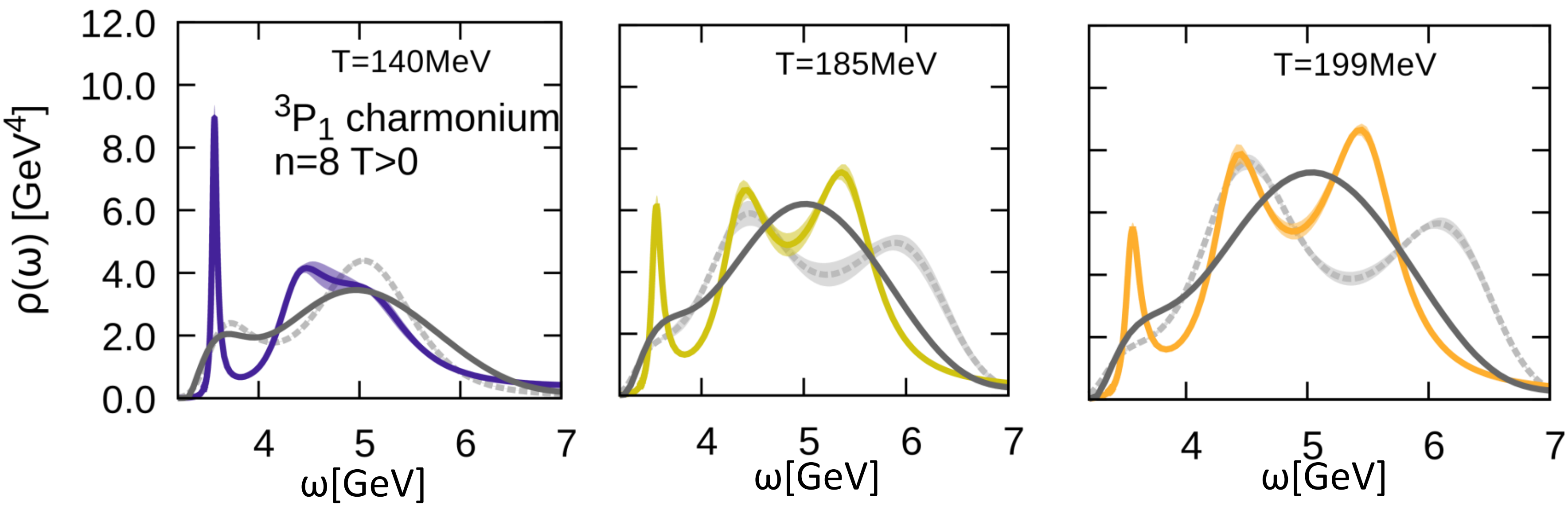


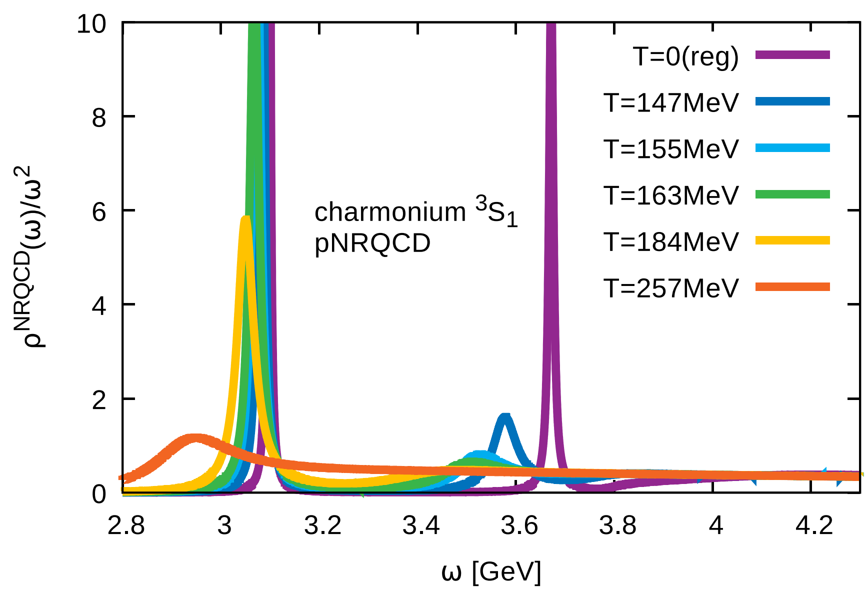
- Three methods: BR (colored), smooth BR (gray solid) & MEM (gray dashed)

At T<185MeV: all methods show remnant structure (threshold enhancement?)

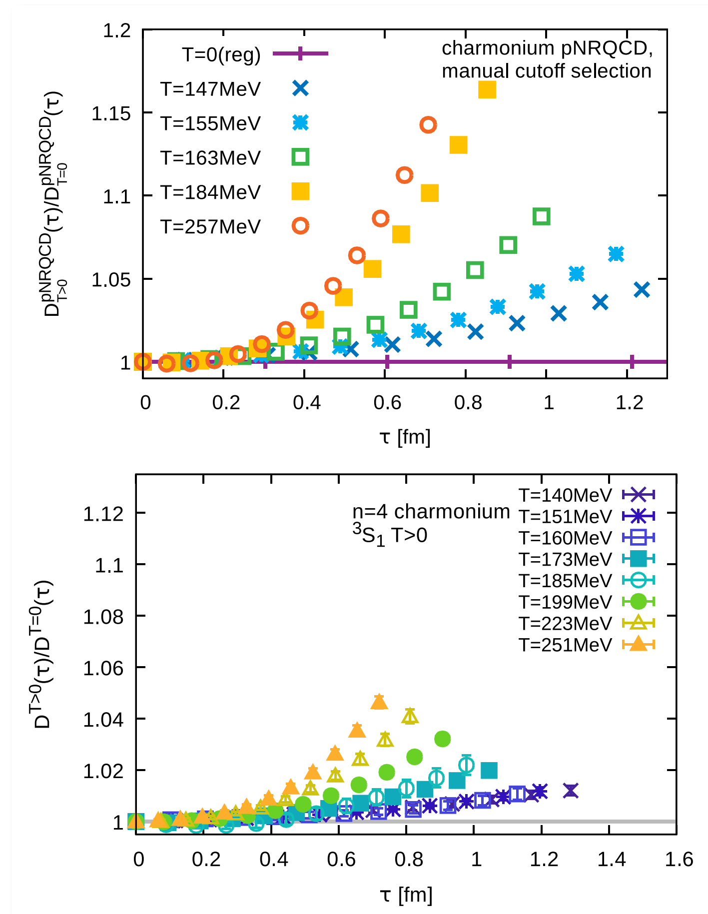
For T=185MeV: only original BR shows peak (amplitude higher than next structure)

For T>185MeV: lowest peak in original BR smaller than next, most likely ringing

Spectral functions from the potential



Correlator ratio approximated from the pNRQCD spectral function



- Continuum corrected pNRQCD spectral functions: shift to lower m and broadening
- Translated into correlator ratios: qualitatively **consistent with lattice NRQCD**