PNRQCD FACTORIZATION OF ELECTROMAGNETIC QUARKONIUM PRODUCTION

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OUTLINE

- Exclusive electromagnetic production of quarkonia and precision tests of nonrelativistic EFTs
- Relativistic corrections and quarkonium Fock states
- Calculation of NRQCD LDMEs in pNRQCD
- Improved prediction of $e^+e^- \to \chi_{cJ} + \gamma$ cross section

EM QUARKONIUM PRODUCTION

- Exclusive electromagnetic production provide good tests of nonrelativistic EFT description of quarkonia.
  - Same long-distance matrix elements (LDMEs) appear in cross section and in electromagnetic decays.
  - Have been measured in lepton collider experiments, experimental precision will continue to improve.
- Improving theoretical precision requires not only the calculation of short-distance coefficients to higher orders, but also the inclusion of LDMEs of higher orders in v.

EM QUARKONIUM PRODUCTION

- LDMEs beyond leading order in v include operators that probe the ***color-octet*** Fock states of quarkonia.

$$|H\rangle = O(1)|Q\bar{Q}\rangle + O(v)|Q\bar{Q}g\rangle + O(v^2)|Q\bar{Q}gg\rangle + \cdots$$

- Hence, precision studies of EM quarkonium processes can probe ***color octet production of quarkonia***.
- Generally, LDMEs beyond leading order in v are not well determined.
- In potential NRQCD, LDMEs simplify into wave functions and universal gluonic correlators, which can be computed in potential models and Lattice QCD.

EM PRODUCTION OF χ_{cJ}

- The process $e^+e^- \to \chi_{cJ} + \gamma$ have been proposed to be measured in lepton colliders.
  - LO in α_s and v : **HSC, J. Lee, C. Yu, PRD78 (2008) 074022**
  - NLO QCD correction at LO in v
    W.-L. Sang and Y.-Q. Chen, PRD81 (2010) 034028
    D. Li, Z.-G. He, K.-T. Chao, PRD80 (2009) 114014
  - Partial order-v^2 and order-$\alpha_s v^2$ corrections
    Y.-J. Li, G.-Z. Xu, K.-Y. Liu, Y.-J. Zhang, JHEP01 (2014) 022
    K.-T. Chao, Z.-G. He, D. Li, C. Meng, arXiv:1310.8597
    G.-Z. Xu, Y.-J. Li, K.-Y. Liu, Y.-J. Zhang, JHEP10 (2014) 71
  - Complete order-v^2 correction at LO in α_s including color-octet production **Brambilla, Chen, Jia, Shtabovenko, Vairo, PRD97 (2018) 096001**
- The cross section for χ_{c1} have been measured by Belle.

NRQCD MATRIX ELEMENTS

- Up to relative order v^2 accuracy,

$$\sigma = \frac{F_1(^3P_J)}{m^4}\langle \text{vac}|\mathcal{O}_1^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle + \frac{G_1(^3P_J)}{m^6}\langle \text{vac}|\mathcal{P}_1^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle + \frac{T_8(^3P_J)}{m^5}\langle \text{vac}|\mathcal{T}_8^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle$$

Brambilla, Chen, Jia, Shtabovenko, Vairo, PRD97 (2018) 096001

- At leading order in v, one LDME contributes to the cross section. For J=0,

$$\langle \text{vac}|\mathcal{O}_1^{\chi_{c0}}(^3P_0)|\text{vac}\rangle = \frac{1}{3}\langle \text{vac}|\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})\chi|\chi_{c0}\rangle\langle\chi_{c0}|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})\psi|\text{vac}\rangle$$

- This LDME is known at leading order in v as $\frac{3}{2\pi}N_c|R'(0)|^2$.

NRQCD MATRIX ELEMENTS

- At relative order v^2 , two more LDMEs appear. For $J=0$,

$$\langle \text{vac}|\mathcal{P}_1^{\chi_{c0}}(^3P_0)|\text{vac}\rangle = \frac{1}{6}\langle \text{vac}|\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\chi|\chi_{c0}\rangle\langle\chi_{c0}|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})\psi|\text{vac}\rangle + \text{c.c.}$$

$$\langle \text{vac}|\mathcal{T}_8^{\chi_{c0}}(^3P_0)|\text{vac}\rangle = -\frac{1}{3}\langle \text{vac}|\psi^\dagger(ig\boldsymbol{E}\cdot\boldsymbol{\sigma})\chi|\chi_{c0}\rangle\langle\chi_{c0}|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})\psi|\text{vac}\rangle + \text{c.c.}$$

Brambilla, Chen, Jia, Shtabovenko, Vairo, PRD97 (2018) 096001

- The LDME $\langle \text{vac}|\mathcal{T}_8^{\chi_{c0}}(^3P_0)|\text{vac}\rangle$ receives contribution from the $|Q\bar{Q}g\rangle$ Fock state.
  Therefore calculation of this ***color-octet*** LDME requires knowledge of the $|Q\bar{Q}g\rangle$ Fock state.

NRQCD MATRIX ELEMENTS

- Computation of LDMEs in pNRQCD beyond LO in v requires calculation of corrections to the quarkonium state, which include the color octet Fock states.
- In general, this can be done as a formal expansion in $1/m$. NRQCD Hamiltonian :

$$H = \int d^3x \, \frac{1}{2}(\boldsymbol{E}^a \cdot \boldsymbol{E}^a + \boldsymbol{B}^a \cdot \boldsymbol{B}^a) + \frac{1}{m}\left[-\frac{1}{2} \int d^3x \, \psi^\dagger (\boldsymbol{D}^2 + g c_F \boldsymbol{\sigma} \cdot \boldsymbol{B}) \psi + \frac{1}{2} \int d^3x \, \chi^\dagger (\boldsymbol{D}^2 + g c_F \boldsymbol{\sigma} \cdot \boldsymbol{B}) \chi \right] + O(1/m^2)$$

Brambilla, Pineda, Soto, Vairo, PRD63 (2001) 014023
Pineda and Vairo, PRD63 (2001) 054007

QM PERTURBATION THEORY IN NRQCD

- Corrections to the quarkonium state can be computed using the corrections to the eigenstates of the leading-order Hamiltonian.

Brambilla, Pineda, Soto, Vairo, PRD63 (2001) 014023

- Leading-order Hamiltonian : $H^{(0)} = \int d^3x \, \frac{1}{2}(\boldsymbol{E}^a \cdot \boldsymbol{E}^a + \boldsymbol{B}^a \cdot \boldsymbol{B}^a)$
- Eigenstates : $|n\rangle \equiv |n; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ encode the gluon content at heavy quark and antiquark positions $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$., with eigenenergies $E_n \equiv E_n(\boldsymbol{x}_1, \boldsymbol{x}_2)$, and $|\underline{n}\rangle \equiv \psi^\dagger(\boldsymbol{x}_1)\chi(\boldsymbol{x}_2)|n\rangle$
- Perturbation : $\frac{1}{2m}\int d^3x \, \psi^\dagger(\boldsymbol{D}^2 + g c_F \boldsymbol{\sigma} \cdot \boldsymbol{B})\psi + \text{c.c.}$

Contributes to $|Q\bar{Q}g\rangle$ Fock state through orbital angular momentum flip

Contributes to $|Q\bar{Q}g\rangle$ Fock state through spin-flip interaction

QM PERTURBATION THEORY IN NRQCD

- First order correction to the eigenstate (spin-independent part)

Brambilla, Pineda, Soto, Vairo, PRD63 (2001) 014023

$$
\begin{aligned}
|\underline{n}\rangle^{(1)} = &-\sum_{k\neq n} |\underline{k}\rangle^{(0)} \Bigg[\boxed{\frac{{}^{(0)}\langle k|g\boldsymbol{E}_1|n\rangle^{(0)}}{(E_n^{(0)}-E_k^{(0)})^2} \cdot \overleftarrow{\boldsymbol{\nabla}}_1} - \frac{1}{2}\frac{{}^{(0)}\langle k|[\boldsymbol{D}_1\cdot, g\boldsymbol{E}_1]|n\rangle^{(0)}}{(E_n^{(0)}-E_k^{(0)})^2} \\
&+2(\boldsymbol{\nabla}_1 E_n^{(0)}) \cdot \frac{{}^{(0)}\langle k|g\boldsymbol{E}_1|n\rangle^{(0)}}{(E_n^{(0)}-E_k^{(0)})^3} + \sum_{j\neq n} \frac{{}^{(0)}\langle k|g\boldsymbol{E}_1|j\rangle^{(0)} \cdot {}^{(0)}\langle j|g\boldsymbol{E}_1|n\rangle^{(0)}}{(E_n^{(0)}-E_k^{(0)})^2(E_n^{(0)}-E_j^{(0)})} \Bigg] \\
&+\sum_{k\neq n} |\underline{k}\rangle^{(0)} \Bigg[\boxed{\frac{{}^{(0)}\langle k|g\boldsymbol{E}_2^T|n\rangle^{(0)}}{(E_n^{(0)}-E_k^{(0)})^2} \cdot \overleftarrow{\boldsymbol{\nabla}}_2} - \frac{1}{2}\frac{{}^{(0)}\langle k|[\boldsymbol{D}_{c2}\cdot, g\boldsymbol{E}_2^T]|n\rangle^{(0)}}{(E_n^{(0)}-E_k^{(0)})^2} \\
&+2(\boldsymbol{\nabla}_2 E_n^{(0)}) \cdot \frac{{}^{(0)}\langle k|g\boldsymbol{E}_2^T|n\rangle^{(0)}}{(E_n^{(0)}-E_k^{(0)})^3} + \sum_{j\neq n} \frac{{}^{(0)}\langle k|g\boldsymbol{E}_2^T|j\rangle^{(0)} \cdot {}^{(0)}\langle j|g\boldsymbol{E}_2^T|n\rangle^{(0)}}{(E_n^{(0)}-E_k^{(0)})^2(E_n^{(0)}-E_j^{(0)})} \Bigg]
\end{aligned}
$$

POTENTIAL NRQCD

- We work in the strong-coupling regime where $mv \gtrsim \Lambda_{\rm QCD} \gg mv^2$, and integrate out scales above mv^2.

  The weak-coupling regime ($mv \gg mv^2 \gtrsim \Lambda_{\rm QCD}$) leads to a different version of pNRQCD, which is appropriate for systems with small radii such as the Υ.

- The degrees of freedom are color-singlet fields $S(\boldsymbol{r},\boldsymbol{R},t)$ that depend on the relative coordinate $\boldsymbol{r}=\boldsymbol{x}_1-\boldsymbol{x}_2$ and the center-of momentum coordinate $\boldsymbol{R}=(\boldsymbol{x}_1+\boldsymbol{x}_2)/2$ of the $Q\bar{Q}$.

$$\mathcal{L}_{\rm pNRQCD} = {\rm tr}[S^\dagger (i\partial_0 - h) S]$$

- The pNRQCD Hamiltonian h is found by matching pNRQCD to NRQCD. Matching condition :

$$h(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2) = E_0(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2)$$

Energy of the ground state $|\underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$

Pineda and Soto, NPB Proc. Suppl. 64 (1998) 428
Brambilla, Pineda, Soto, Vairo, NPB566 (2000) 275

POTENTIAL NRQCD

- At leading order in v, we obtain a Schrödinger-like picture of quark and antiquark interacting through a potential.
- Eigenstates $|njls\rangle$ of the pNRQCD Hamiltonian with principal quantum number n, total, orbital, and spin angular momentum quantum numbers j, l, s give the wave functions $\langle \boldsymbol{r}|njls\rangle$, which diagonalize the real part of the Hamiltonian with eigenvalues given by the binding energy.
- The wave functions can be found by solving Schrödinger-like equations with QCD potentials.

Pineda and Soto, NPB Proc. Suppl. 64 (1998) 428
Brambilla, Pineda, Soto, Vairo, NPB566 (2000) 275

NRQCD LDMES IN PNRQCD

- Now NRQCD LDMEs can be computed in terms of pNRQCD matrix elements. For an NRQCD operator $\mathcal{O}$ with a quarkonium state $|H\rangle$ with momentum $\boldsymbol{P}=0$,

$$\langle H|\mathcal{O}|H\rangle = \frac{1}{\langle \boldsymbol{P}=0|\boldsymbol{P}=0\rangle}\int d^3r d^3r' d^3R d^3R' \langle \boldsymbol{P}=0|\boldsymbol{R}\rangle\langle njls|\boldsymbol{r}\rangle$$
$$\times\langle \underline{0};\boldsymbol{x}_1,\boldsymbol{x}_2|\int d^3\xi \mathcal{O}(\xi)|\underline{0};\boldsymbol{x}_1',\boldsymbol{x}_2'\rangle$$
$$\times\langle \boldsymbol{R}'|\boldsymbol{P}=0\rangle\langle \boldsymbol{r}'|njls\rangle$$

Brambilla, Eiras, Pineda, Soto, Vairo, PRD67 (2003) 034018

- Since gluonic excitations are integrated out in pNRQCD, only the ground state $|\underline{0};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle$ appears.

NRQCD LDMES IN PNRQCD

- The matrix element $\langle \underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2 | \int d^3\xi \mathcal{O}(\xi) | \underline{0}; \boldsymbol{x}_1', \boldsymbol{x}_2' \rangle$ yields, for the leading order LDME at leading order in $1/m$,

$$^{(0)}\langle \underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2 | \int d^3\xi \left[\frac{1}{3} \chi^\dagger (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) \psi | \text{vac} \rangle \langle \text{vac} | \psi^\dagger (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) \chi \right] (\boldsymbol{z}) | \underline{0}; \boldsymbol{x}_1', \boldsymbol{x}_2' \rangle^{(0)}$$
$$= -\frac{N_c}{3} \boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \boldsymbol{\sigma} \delta^{(3)}(\boldsymbol{r}) \boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \boldsymbol{\sigma} \delta^{(3)}(\boldsymbol{x}_1 - \boldsymbol{x}_1') \delta^{(3)}(\boldsymbol{x}_2 - \boldsymbol{x}_2')$$

- In the NRQCD LDME, derivatives act on the wave functions and yield first derivative of the P-wave wave function at the origin.

Brambilla, Eiras, Pineda, Soto, Vairo, PRD67 (2003) 034018

NRQCD LDMES IN PNRQCD

- The color-octet LDME yields

$$\sum_{k\neq 0}\left(\boldsymbol{\nabla}_1\cdot\frac{{}^{(0)}\langle 0|g\boldsymbol{E}_1|k\rangle^{(0)}}{(E_0^{(0)}-E_k^{(0)})^2}-\boldsymbol{\nabla}_2\cdot\frac{{}^{(0)}\langle 0|g\boldsymbol{E}_2^T|k\rangle^{(0)}}{(E_0^{(0)}-E_k^{(0)})^2}\right)\langle\underline{k};\boldsymbol{x}_1,\boldsymbol{x}_2|$$

$$^{(1)}\langle\underline{0};\boldsymbol{x}_1,\boldsymbol{x}_2|\int d^3\xi\left[-\frac{1}{3}\chi^\dagger(ig\boldsymbol{E}\cdot\boldsymbol{\sigma})\psi|\text{vac}\rangle\langle\text{vac}|\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})\chi\right](\boldsymbol{z})|\underline{0};\boldsymbol{x}_1',\boldsymbol{x}_2'\rangle^{(0)}$$

$$=-\frac{2}{3}\frac{N_c}{3}\boldsymbol{\nabla}_{\boldsymbol{r}}\cdot\boldsymbol{\sigma}\mathcal{E}_1\delta^{(3)}(\boldsymbol{r})\boldsymbol{\nabla}_{\boldsymbol{r}}\cdot\boldsymbol{\sigma}\delta^{(3)}(\boldsymbol{x}_1-\boldsymbol{x}_1')\delta^{(3)}(\boldsymbol{x}_2-\boldsymbol{x}_2')$$

Gluonic correlator : $$\mathcal{E}_n=(-i)^{n+1}n!\sum_{k\neq 0}\frac{{}^{(0)}\langle 0|g\boldsymbol{E}^i|k\rangle^{(0)}\,{}^{(0)}\langle k|g\boldsymbol{E}^j|0\rangle^{(0)}}{(E_k^{(0)}-E_0^{(0)})^{n+1}}$$

$$=\frac{T_F}{N_c}\int_0^\infty dt\,t^n\langle\text{vac}|g\boldsymbol{E}^{i,a}(t)\Phi_{ab}g\boldsymbol{E}^{i,b}(0)|\text{vac}\rangle$$

Wilson line

NRQCD LDMES IN PNRQCD

- For the color-singlet operator at relative order v^2,

$$
{}^{(0)}\langle \underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2 | \int d^3\xi \left[\frac{1}{6} \chi^\dagger (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) \psi |\text{vac}\rangle \langle \text{vac}| \psi^\dagger (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}})^2 \chi + \text{H.c.} \right] (\boldsymbol{z}) | \underline{0}; \boldsymbol{x}_1', \boldsymbol{x}_2' \rangle^{(0)}
$$

$$
= -\frac{N_c}{3} \boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \boldsymbol{\sigma} \delta^{(3)}(\boldsymbol{r}) \left(\boldsymbol{\nabla}_{\boldsymbol{r}}^2 + \frac{5}{3} \mathcal{E}_1 \right) \boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \boldsymbol{\sigma} \delta^{(3)}(\boldsymbol{x}_1 - \boldsymbol{x}_1') \delta^{(3)}(\boldsymbol{x}_2 - \boldsymbol{x}_2')
$$

Gluonic correlator :
$$
\mathcal{E}_n = (-i)^{n+1} n! \sum_{k \neq 0} \frac{{}^{(0)}\langle 0 | g \boldsymbol{E}^i | k \rangle^{(0)} \, {}^{(0)}\langle k | g \boldsymbol{E}^j | 0 \rangle^{(0)}}{(E_k^{(0)} - E_0^{(0)})^{n+1}}
$$

$$
= \frac{T_F}{N_c} \int_0^\infty dt \, t^n \langle \text{vac} | g \boldsymbol{E}^{i,a}(t) \Phi_{ab} g \boldsymbol{E}^{i,b}(0) | \text{vac} \rangle
$$

Wilson line

NRQCD LDMES IN PNRQCD

- Results in dimensional regularization (J=0,1,2)

$$\langle \text{vac}|\mathcal{O}_1^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle = \frac{3}{2\pi}N_c|R'(0)|^2\left[1+\frac{2}{3}\frac{i\mathcal{E}_2}{m}-\frac{1}{9}\left(\frac{\mathcal{E}_2}{m}\right)^2+O(v^2)\right]$$

$$\langle \text{vac}|\mathcal{T}_8^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle = -\frac{3}{2\pi}N_c|R'(0)|^2\left(\frac{4}{3}\frac{\mathcal{E}_1}{m}+O(v^3)\right)$$

NEW

$$\langle \text{vac}|\mathcal{P}_1^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle = \frac{3}{2\pi}N_c|R'(0)|^2\left(mE_B-\frac{2}{3}\frac{\mathcal{E}_1}{m}+O(v^3)\right)$$

NEW

Binding energy

- In $\langle \text{vac}|\mathcal{P}_1^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle$, the pNRQCD Hamiltonian was used to rewrite higher derivatives of wave function at the origin with the binding energy.

GLUONIC CORRELATORS

- Gluonic correlators are purely gluonic quantities that involve only the scale $\Lambda_{\rm QCD}$. $$\mathcal{E}_n = \frac{T_F}{N_c} \int_0^\infty dt\, t^n \langle \text{vac} | g\boldsymbol{E}^{i,a}(t) \Phi_{ab} g\boldsymbol{E}^{i,b}(0) | \text{vac} \rangle$$
- They are universal objects that do not depend on the heavy quark flavor or on specific heavy quarkonium states.
- Correlators can be computed in Lattice QCD, but we need to convert them to $\overline{\rm MS}$ scheme to obtain LDMEs in $\overline{\rm MS}$.
- Same correlators appear in LDMEs of different quarkonium states; computation of the correlators can greatly simplify determination of LDMEs of many quarkonium states.

NRQCD LDMES IN PNRQCD

- Even though the correlators yet to be measured accurately, we can still estimate the LDMEs using EM decay rates $\Gamma(\chi_{cJ} \to \gamma\gamma)$ (J=0,2) and potential models.
- Using the Cornell potential to compute the wavefunction at the origin, we obtain

$$\begin{aligned}
\langle \mathrm{vac}|\mathcal{O}_1^{\chi_{cJ}}(^3P_J)|\mathrm{vac}\rangle &= \quad (0.072 \pm 0.036)\ \mathrm{GeV}^5 \\
\langle \mathrm{vac}|\mathcal{T}_8^{\chi_{cJ}}(^3P_J)|\mathrm{vac}\rangle &= -(0.109 \pm 0.068)\ \mathrm{GeV}^6 \\
\langle \mathrm{vac}|\mathcal{P}_1^{\chi_{cJ}}(^3P_J)|\mathrm{vac}\rangle &= \quad (0.012 \pm 0.068)\ \mathrm{GeV}^7
\end{aligned}$$

Preliminary

- Compared to estimates simply based on power counting, the uncertainties reduce mildly.

EM CROSS SECTIONS

- The LDMEs we obtained allow us to make improved prediction of cross sections at $\sqrt{s} = 10.6\,\mathrm{GeV}$:

$$\sigma(e^+e^- \to \chi_{c0} + \gamma) = 1.3 \pm 0.7\,\mathrm{fb}$$
$$\sigma(e^+e^- \to \chi_{c1} + \gamma) = 22.2 \pm 4.0\,\mathrm{fb}$$
$$\sigma(e^+e^- \to \chi_{c2} + \gamma) = 8.0 \pm 1.3\,\mathrm{fb}$$

Preliminary

- The χ_{c1} cross section agrees with recent Belle measurement within uncertainties.

$$\sigma(e^+e^- \to \chi_{c1} + \gamma) = 17.3^{+4.2}_{-3.9} \pm 1.7\,\mathrm{fb}$$

Belle, PRD98 (2018) 092015

SUMMARY AND OUTLOOK

- Exclusive electromagnetic production of heavy quarkonia provide precision tests of nonrelativistic EFTs, including color octet production of heavy quarkonia.
- Using pNRQCD, we computed χ_{cJ} LDMEs of higher orders in v, that include effect of the color-octet Fock state of the P-wave quarkonia.
- We made improved prediction of $\sigma(e^+e^- \to \chi_{cJ} + \gamma)$.
- This approach may be useful in understanding the properties of NRQCD LDMEs and the NRQCD color octet mechanism in inclusive hadroproduction.

BACKUP

GLUONIC CORRELATORS

- Gluonic correlators are purely gluonic quantities that do not involve heavy quarks.

$$\mathcal{E}_n = \frac{T_F}{N_c} \int_0^\infty dt\, t^n \langle \text{vac} | g\boldsymbol{E}^{i,a}(t) \Phi_{ab} g\boldsymbol{E}^{i,b}(0) | \text{vac} \rangle$$

- In perturbative QCD, the correlators are scaless, and hence, vanish in dimensional regularization.
- $\mathcal{E}_1$ and $\mathcal{E}_2$ are quadratically and linearly power UV divergent, respectively.
- The correlators can be measured in Lattice QCD.

GREMM-KAPUSTIN RELATION

- For any vacuum-to-quarkonium matrix element of an NRQCD operator $\mathcal{O}$,

$$\underbrace{E_B}_{\text{Binding energy}} \langle H|\mathcal{O}|\text{vac}\rangle = \langle H|[H,\mathcal{O}]|\text{vac}\rangle$$

- This leads to the following Gremm-Kapustin relation

$$\langle \text{vac}|\mathcal{P}_1^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle = mE_B\langle \text{vac}|\mathcal{O}_1^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle - \frac{m}{2}\langle \text{vac}|\mathcal{T}_8^{\chi_{cJ}}(^3P_J)|\text{vac}\rangle$$

- The pNRQCD expressions for the LDMEs automatically satisfy the above relation.