Calculation of quarkonium fragmentation functions

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Outline

I. Introduction

II. Fragmentation functions in NRQCD

III. Resummation in soft gluon factorization

IV. Summary

Properties of heavy quarkonium

- **A non-relativistic QCD system:** $\alpha_s(mv) \sim v$

  Charmonium: $\mathrm{m}{\sim}1.3\mathrm{Ge}V,\ v^2 \approx 0.3$

  Bottomonium: $m{\sim}4.5GeV,\ v^2 \approx 0.1$

- **Multiple well-separated scales :**

  Quark mass: M
  Momentum: Mv
  Energy: Mv^2

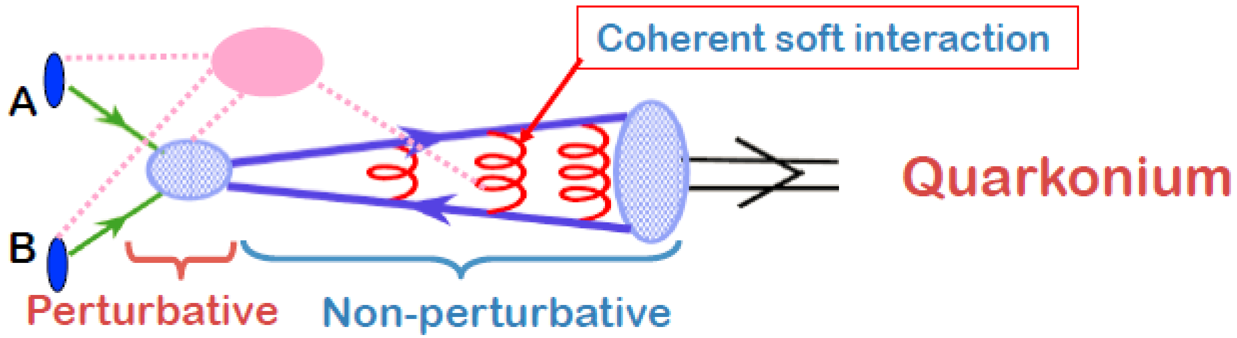
  M >> Mv >> $Mv^2 {\sim} \Lambda_{\mathrm{QCD}}$

- **Involving both perturbative and nonperturbative physics**

- **Production: ideal to understand hadronization, to study QGP**

Space-time picture for production

- Hadronization followed by production of an off-shell heavy quark pair



$\Delta r \sim \frac{1}{2m}, \quad \frac{1}{m\,v}, \quad \frac{1}{m\,v^2}$

- Time scale for producing heavy quark pair: $\frac{1}{2\,m}$
- Time scale for expansion: $\frac{1}{m\,v}$
- Time scale for forming bound state: $\frac{1}{m\,v^2}$

Approximation

➢ On-shell pair + hadronization

$$\sigma_{AB\to H+X} = \sum_n \int_n d\Gamma_{(Q\bar{Q})_n} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{(Q\bar{Q})_n}}\right] F_{(Q\bar{Q})_n\to H}(p_Q, p_{\bar{Q}}, P_H)$$

- **Different assumptions/treatments on how the heavy quark pair becomes a heavy quarkonium: different factorization methods**

- **Factorization needs justification, hard**

Theories for quarkonium production

- **Color Singlet Model:** Einhorn, Ellis (1975), Chang (1980) ...

  Only pair with the correct quantum number contributes

- **Color Evaporation Model:** Fritzsch (1977), Halzen (1977) ...

  Any pair contributes, probabilities for different pairs are the same

- **NRQCD factorization:** Bodwin, Braaten, Lepage, 9407339

  Any pair contributes, but with different probabilities for different pairs

- **Soft Gluon Factorization:** YQM, Chao, 1703.08402

  Any pair contributes, with different probability, pair has different momentum from quarkonia

SGF: a "TMD" version of NRQCD, can resum large logs in NRQCD

Theories for different kinematics

- **High p_T region:**

  Single-parton fragmentation

  Braaten, Cheung, Yuan, 9302307 ,
  Braaten, Yuan, 9303205 ,
  …

  Double-parton fragmentation

  Kang, Qiu, Sterman, 1109.1520
  Fleming, Leibovich, Mehen, Rothstein 1207.2578
  Kang, YQM, Qiu, Sterman, 1401.0923

- **Low p_T region:**

  Kang, YQM, Venugopalan, 1309.7337,
  …
  (See Kazuhiro Watanabe's and Tomasz Stebel's talks)

  Combine with Color glass condensate EFT

- **End point region:**

  Beneke, Rothstein, Wise, 9705286,
  …
  (See Xiaohui Liu's talk)

  Shape functions

Why study high p_T?

- **Theoretically:**

  Factorization is more reliable in this region;
  have confident to compare data with theory

- **Experimentally:**

  Many data, reduction of theoretical uncertainty is badly needed

- NLO: most processes are available
- NNLO: some relatively simple processes are available; other processes are still very hard

See Hua-Sheng Shao's talk

For recent works:
Feng, Jia, Sang, 1505.02665
Feng, Jia, Sang, 1902.11288

- **Study high p_T in fragmentation framework can go beyond the current NLO calculation**

  Concentrate on single-parton fragmentation in this talk

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Single-parton FFs

Collins, Soper, Sterman, *Adv. Ser. Direct. High Energy Phys.* **5** (1989) 1-91

- **QCD Collinear Factorization**

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\hat{\sigma}_{A+B\to i+X}\left(\frac{p_T}{z},\mu\right) \otimes D_{i\to H}(z,\mu) + \mathcal{O}\left(\frac{1}{p_T^2}\right)$$

- **DGLAP evolution**

Gribov, Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 438-450
Altarelli, Parisi, *Nucl. Phys.* **B126** (1977) 298-318
Dokshitzer, *Sov. Phys. JETP* **46** (1977) 641-653

$$\mu\frac{d}{d\mu}D_{g\to H}(z,\mu) = \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ig}\left(\frac{z}{\xi};\alpha_s(\mu)\right) D_{i\to H}(\xi,\mu)$$

- **NRQCD factorization of FFs**

$$D_{i\to H}(z,\mu_0) = \sum_n d_n(z,\mu_0,\mu_f)\langle\mathcal{O}^H(n)\rangle$$

  - **LDMEs: potential model, experimental data**
  - **SDCs: perturbatively calculable**
    - **Parton fragments into a free $Q\bar{Q}$ pair**
    - **Calculate Feynman diagrams**

Precious results

- **Up to α_s^2-order**

Braaten, Cheung, Yuan, 9302307 …
For a summary see:
YQM, Qiu, Zhang, *Phys.Rev.* **D89** (2014) 094029
YQM, Qiu, Zhang, *JHEP* **06** (2015) 021

- **α_s^3-order**
  - $g \to Q\bar{Q}({}^3S_1^{[1]}) + gg$
    - **Numerical**

      Braaten, Yuan, *Phys. Rev. Lett.* **71** (1993) 1673-1676
      Braaten, Yuan, *Phys. Rev.* **D52** (1995) 1125-1171
    - **Analytical**

      Zhang, YQM, Chen, Chao, *Phys. Rev.* **D96** (2017) 094016
  - $g \to Q\bar{Q}({}^1P_1^{[1]}) + gg$
    - **Analytical**

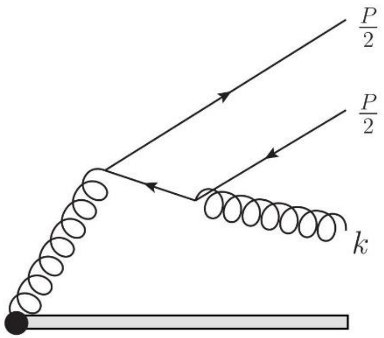
      Sun, Jia, Liu, Zhu, *Phys. Rev.* **D98** (2018) 014039
  - $g \to Q\bar{Q}({}^1S_0^{[1]}) + X$ **(NLO)**
    - **Numerical**

      Artoisenet, Braaten, *JHEP.* **04** (2015) 121

LO Calculation

Zhang, Wang, Liu, YQM, Meng, Chao, *JHEP 1904 (2019) 116*

- **Study of $g \to Q\bar{Q}({}^1S_0^{[1,8]}) + X$**
  - Important for η_c and J/ψ production
- **Feynman diagram at LO**



- **Results**

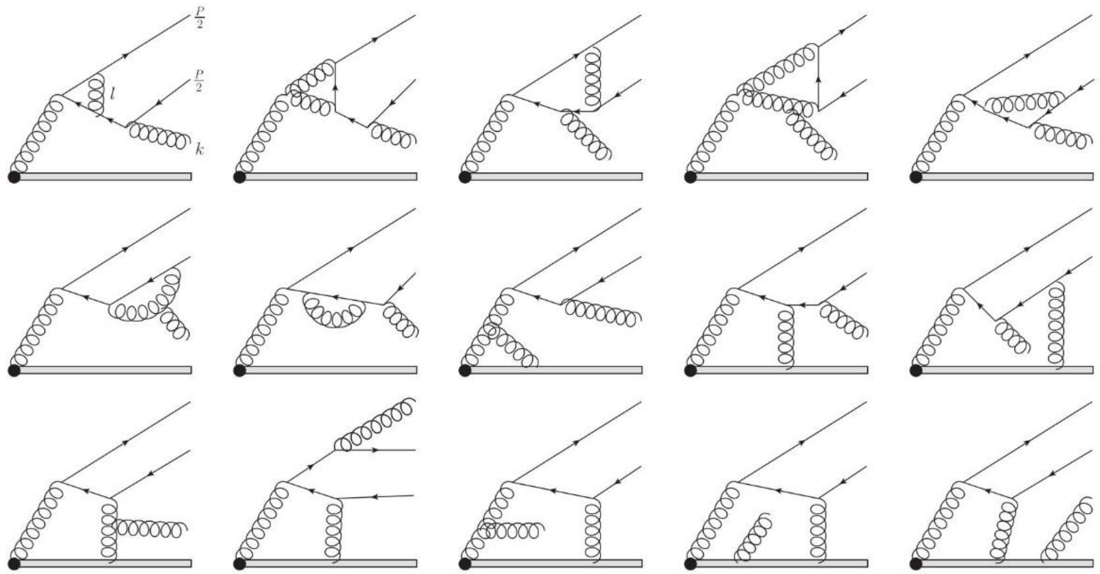
$$d_{\rm LO}^{[1]}(z) = \frac{\alpha_s^2}{2(1-\epsilon)N_c m_Q^3}\left(\frac{\pi\mu_r^2}{m_Q^2}\right)^{\epsilon} d_{\rm LO}(z),$$

$$d_{\rm LO}^{[8]}(z) = \frac{\alpha_s^2(N_c^2-4)}{4(1-\epsilon)N_c(N_c^2-1)m_Q^3}\left(\frac{\pi\mu_r^2}{m_Q^2}\right)^{\epsilon} d_{\rm LO}(z)$$

$$d_{\rm LO}^{(0)}(z) = \lim_{\epsilon\to 0} d_{\rm LO}(z) = (3-2z)z + 2(1-z)\ln(1-z)$$

Virtual contribution

- **Feynman Diagrams**



- **Integrals to be calculated**

$$\frac{P\cdot n}{z^2}\int \frac{\mathrm{d}^D k}{(2\pi)^{D-1}}\frac{\mathrm{d}^D l}{(2\pi)^D}\delta_+(k^2)\delta\left(k\cdot n-\frac{1-z}{z}P\cdot n\right)\prod_i \frac{1}{F_i^{a_i}}$$

$$F_1 = k\cdot P\,,\qquad F_2 = 2\,k\cdot P+1\,,$$

$$F_3 = l^2\,,\qquad F_4 = (l+k)^2\,,\qquad F_5 = (l+P)^2\,,$$

$$F_6 = (l+\frac{P}{2})^2-\frac{1}{4}\,,\qquad F_7 = (l-\frac{P}{2})^2-\frac{1}{4}\,,\qquad F_8 = (l+k+\frac{P}{2})^2-\frac{1}{4}\,,$$

$$F_9 = (l+k+P)^2\,,\qquad F_{10} = l\cdot n\,.$$

IBP reduction

Chetyrkin, Tkachov, *Nucl. Phys.* **A15** (1981) 159-204

- **For a family of Feynman integrals**

$$F(a_1,\ldots,a_n) = \int\cdots\int \frac{\mathrm{d}^D l_1 \ldots \mathrm{d}^D l_h}{D_1^{a_1}\ldots D_n^{a_n}}$$

D_i : linear functions with respect to the scalar product of loop momenta and external momenta.

- **IBP relations (dimensional regularization)**

$$\int\cdots\int \mathrm{d}^D l_1 \ldots \mathrm{d}^D l_h \, p_j^\mu \frac{\partial}{\partial l_i^\mu} \frac{1}{D_1^{a_1}\ldots D_n^{a_n}} = 0$$

- **Linear relation between FIs**

$$\sum c_i F(a_1 + b_{i,1},\ldots,a_n + b_{i,n}) = 0 \qquad b_{i,j} \in \{-1,0,1\}$$

- **Reduce FIs to 66 master integrals (MIs)**

Translate to pure loop integrals

$$\frac{P\cdot n}{z^2}\int \frac{\mathrm{d}^D k}{(2\pi)^{D-1}}\frac{\mathrm{d}^D l}{(2\pi)^D}\delta_+(k^2)\delta\left(k\cdot n-\frac{1-z}{z}P\cdot n\right)\prod_i \frac{1}{F_i^{a_i}}$$

➢ **Using Cutkosky's cutting rule**

$$(2\pi)\delta(x)=\lim_{\eta\to 0}\left(\frac{i}{x+i\eta}-\frac{i}{x-i\eta}\right)$$

- **Change $1/D_j$ back to $\delta(D_j)$ after IBP reduction**

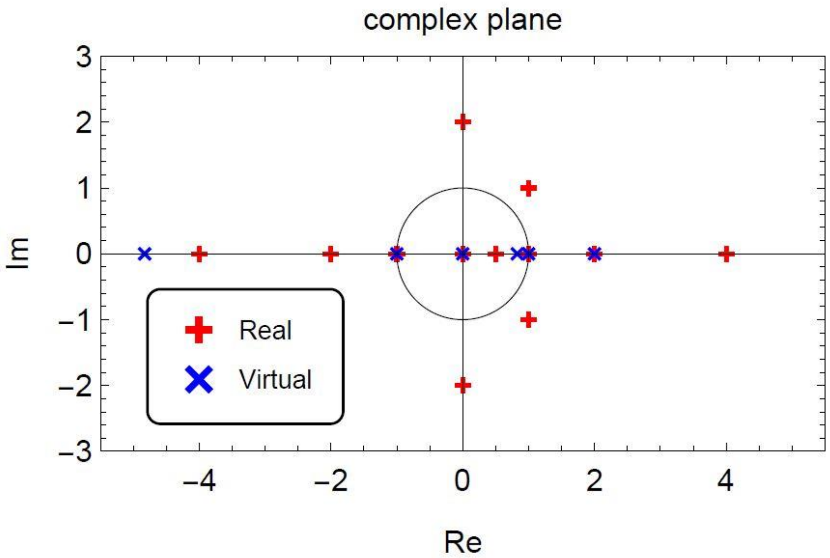
Calculation of MIs

- **IBP can set up differential equations (DEs) for MIs**

$$\frac{\mathrm{d}\boldsymbol{I}(\epsilon,z)}{\mathrm{d}z} = A(\epsilon,z)\boldsymbol{I}(\epsilon,z)$$

Kotikov (1991)

- **Singularities in DEs:** $0, 2(\sqrt{2}-1), 1/2, 1$



- **Asymptotic expansions**

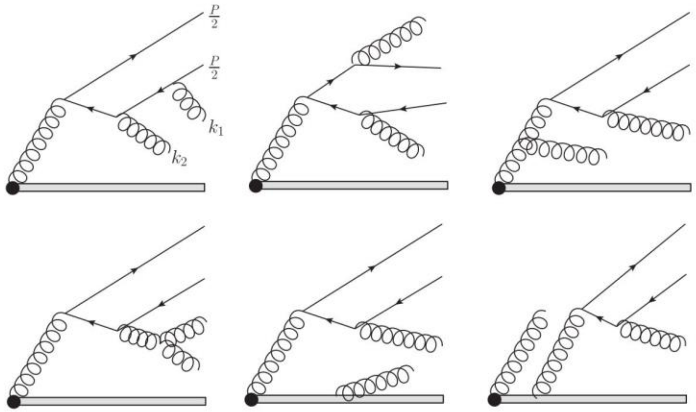
$$I_k(z,\epsilon)|_{z_0} = \sum_s \sum_{i=0}^{n_s} (z-z_0)^s \ln^i(z-z_0) \sum_{j=0}^{\infty} I_k^{s\,i\,j}(\epsilon)(z-z_0)^j$$

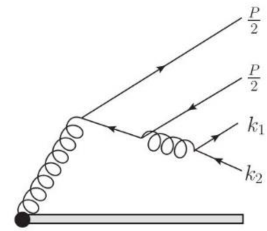
Calculation at a boundary

Liu, YQM, Wang, *Phys. Lett.* **B779** (2018) 353-357
Liu, YQM, Phys.Rev. D99 (2019) no.7, 071501

- **Calculation at any z based series representation**
  - Add $i\eta$ in all denominators containing loop momentum l
  - Set up new DEs w.r.t η
  - Calculate the boundaries of new MIs at $\eta \to \infty$ (trivial)
  - Obtain new MIs at $\eta \to 0$ with the new DEs
  - Use these values as boundaries for previous DEs
- **Very high precision can be obtained**

Real contribution





$$\frac{P\cdot n}{2z^2}\int\frac{\mathrm{d}^D k_1}{(2\pi)^{D-1}}\frac{\mathrm{d}^D k_2}{(2\pi)^{D-1}}\delta_+(k_1^2)\delta_+(k_2^2)\delta\left(k_1\cdot n+k_2\cdot n-\frac{1-z}{z}P\cdot n\right)\prod_i\frac{1}{E_i^{a_i}}$$

$$E_1 = k_1\cdot k_2\,,\qquad E_2 = k_1\cdot P\,,\qquad E_3 = k_2\cdot P\,,$$
$$E_4 = 2\,k_1\cdot P+1\,,\qquad E_5 = 2\,k_2\cdot P+1\,,$$
$$E_6 = 2\,k_1\cdot k_2+k_1\cdot P+k_2\cdot P\,,$$
$$E_7 = 2\,k_1\cdot k_2+2\,k_1\cdot P+2\,k_2\cdot P+1\,,$$
$$E_8 = k_1\cdot n\,,\qquad E_9 = k_1\cdot n+P\cdot n\,,$$
$$E_{10} = k_2\cdot n\,,\qquad E_{11} = k_2\cdot n+P\cdot n\,.$$

$$E_{12} = k_1^2\,,$$
$$E_{13} = k_2^2\,,$$
$$E_{14} = k_1\cdot n+k_2\cdot n-\frac{1-z}{z}P\cdot n$$

- **Similar except a rapidity regulator is introduced**

Final results

$$b_0 = \frac{11N_c - 2n_f}{6}$$

$$d_{\rm LO}^{(0)}(z) = \lim_{\epsilon\to 0} d_{\rm LO}(z) = (3-2z)z + 2(1-z)\ln(1-z)$$

$$d_{\rm NLO}^{[1]}(z) = \frac{\alpha_s^3}{2\pi N_c m_Q^3} \times \left(d^{[1]}(z) + \ln\left(\frac{\mu_r^2}{4m_Q^2}\right) b_0\, d_{\rm LO}^{(0)}(z) + \ln\left(\frac{\mu_f^2}{4m_Q^2}\right) f(z) \right),$$

$$d_{\rm NLO}^{[8]}(z) = \frac{\alpha_s^3 (N_c^2-4)}{4\pi N_c (N_c^2-1) m_Q^3} \times \left(d^{[8]}(z) + \ln\left(\frac{\mu_r^2}{4m_Q^2}\right) b_0\, d_{\rm LO}^{(0)}(z) + \ln\left(\frac{\mu_f^2}{4m_Q^2}\right) f(z) \right),$$

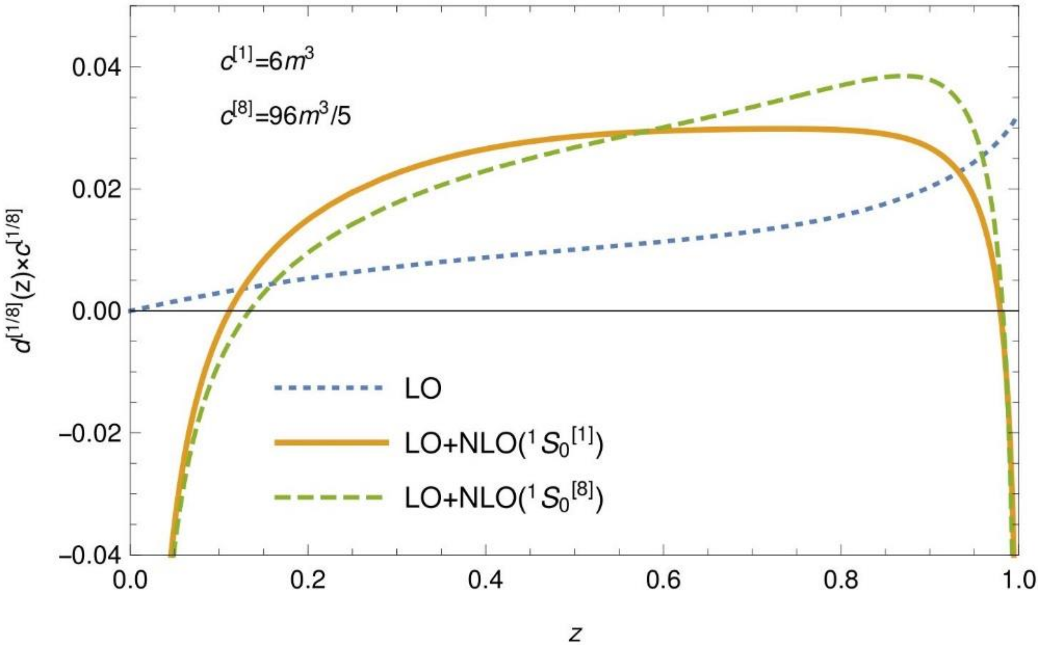
$$f(z) = -\frac{n_f}{6} d_{\rm LO}^{(0)}(z) + N_c \Bigg(-2(z+2)\mathrm{Li}_2(z) - 2(z-1)\ln^2(1-z) + 2(z-1)\ln(z)\ln(1-z) + (z-4)z\ln(z) - \frac{(2z+1)\left(9z^2-5z-6\right)\ln(1-z)}{6z} + \frac{46z^3 + \left(8\pi^2-3\right)z^2 + 4\left(\pi^2-9\right)z + 4}{12z} \Bigg),$$

$$d^{[1/8]}(z) = \begin{cases} \frac{N_c}{2z} \sum_{i=0}^{2}\sum_{j=0}^{\infty} \ln^i z\,(2z)^j \left(A_{ij}^f n_f + A_{ij}^{[1/8]} N_c + \frac{A_{ij}^N}{N_c} \right), & \text{for } 0 < z < \frac{1}{4} \\ \sum_{j=0}^{\infty} (2z-1)^j \left(B_j^f n_f + B_j^{[1/8]} N_c + \frac{B_j^N}{N_c} \right), & \text{for } \frac{1}{4} \le z \le \frac{3}{4} \\ \sum_{i=0}^{3}\sum_{j=0}^{\infty} \ln^i(1-z)(2-2z)^j \left(C_{ij}^f n_f + C_{ij}^{[1/8]} N_c + \frac{C_{ij}^N}{N_c} \right), & \text{for } \frac{3}{4} < z < 1 \end{cases}$$

- **A,B,C: calculated to 160-digit precision**

Comparison

- **Numerical results**

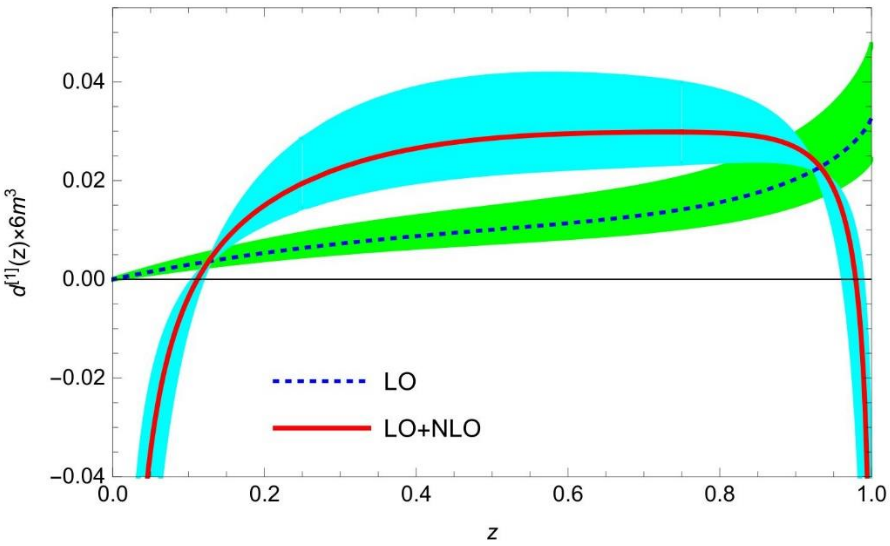


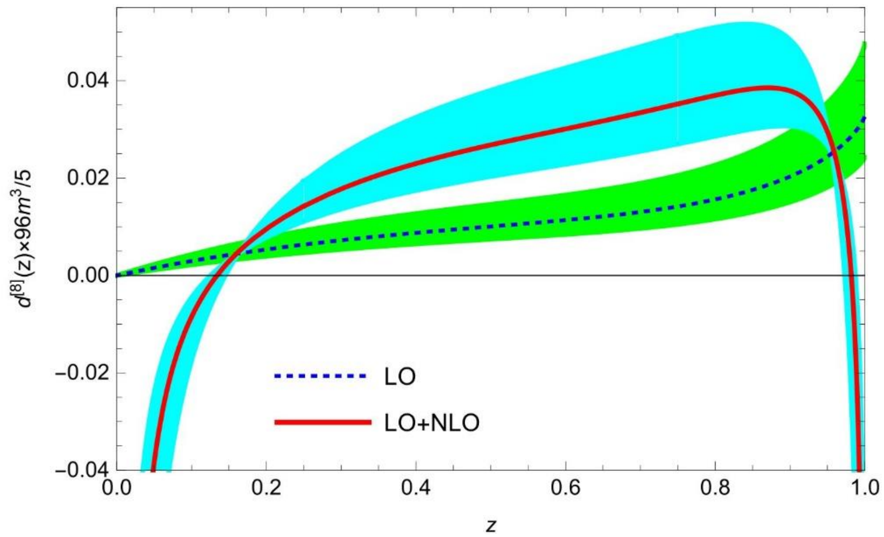
  - Large correction

[1] Artoisenet, Braaten, *JHEP* **01** (2019) 227
[2] Feng, Jia, arXiv: 1810.04138

- **Agree with Refs [1,2] (pure numerical results)**

Renormalization scale





μ_r **change from** m_b **to** $4m_b$

- **Theoretical uncertainty is still large**

Cross sections

- **Estimate the effect for cross sections**

$$\int_0^1 \mathrm{d}z\, z^n (d_{\mathrm{LO}}^{[1/8]}(z) + d_{\mathrm{NLO}}^{[1/8]}(z)) * c^{[1/8]}$$

- **For 2nd,4th,6th moments:**

State	SDCs$*c^{[1/8]}$	z^2	z^4	z^6
	LO ($\times 10^{-3}$)	5.55116944444	3.85331761905	2.99519856859
${}^1S_0^{[1]}$	LO+NLO ($\times 10^{-3}$)	7.54577896198	3.90413390636	2.31890675630
	K-factor	1.35931339108	1.01318767159	0.774208021001
${}^1S_0^{[8]}$	LO+NLO ($\times 10^{-3}$)	8.94021475091	4.99398540596	3.12511443983
	K-factor	1.61051015293	1.29602225917	1.04337471064

- **K-factor are moderate**
  - Cancellation between middle and high z

Divergences at the endpoints

- **Divergence at $z \to 0$**
  - Leading behavior: $1/z$
  - Partonic hard part behaves as $z^n (n \geq 4)$
  - Small z region has negligible contributions to physical cross sections
- **Divergence at $z \to 1$**
  - Leading behavior: $\ln^2(1-z)$, important contribution
  - Perturbative calculation expansion is not good
  - Resummation is needed

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Large logs at NLO

- **Take** $g \to Q\bar{Q}({}^{\mathbf{3}}S_{\mathbf{1}}^{[8]}) + X$ **as an example**

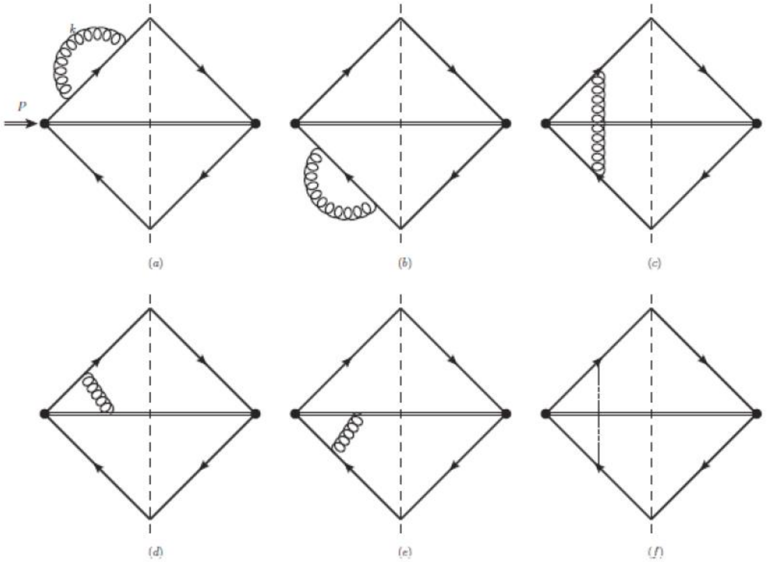
  Other FFs, like $g \to Q\bar{Q}({}^{1}S_{0}^{[1,8]}) + X$**, can be dealt with similarly**

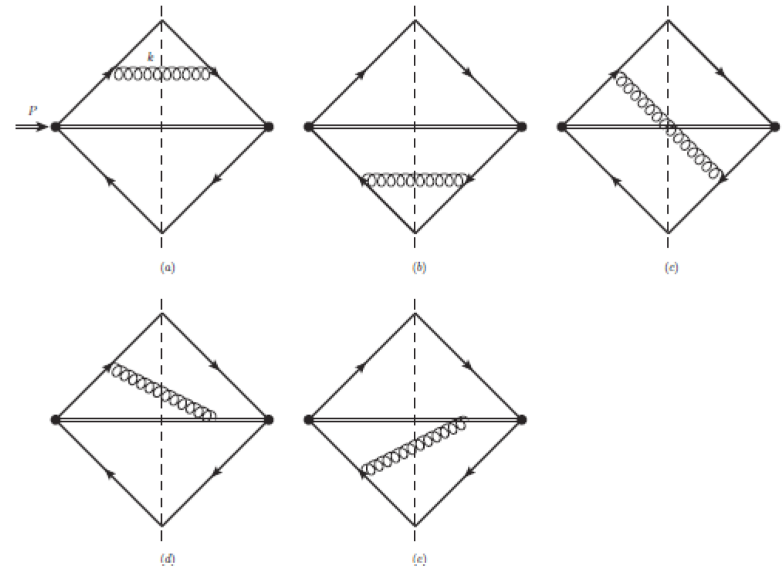
- **NRQCD** $D_{i\to H}(z,\mu_0) = \sum_n d_n(z,\mu_0,\mu_f)\langle \mathcal{O}^H(n)\rangle$

$$\frac{8\pi\alpha_s}{M_H^3}\delta(1-z) + \frac{4\alpha_s^2 N_c}{M_H^3}\left[A(\mu)\delta(1-z) + \frac{1}{N_c}\mathcal{P}_{gg}(z)\left(\ln\left(\frac{\mu^2}{M_H^2}\right) - 1\right)\right.$$
$$\left. + \frac{2(1-z)}{z} - \frac{4(1-z+z^2)^2}{z}\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{1}{N_c^2}\frac{\pi^2}{2\Delta}\delta(1-z)\right].$$

- **SGF** $D_{g\to H}(z, P_H, m_Q, \mu) = \int_z^1 \frac{dx}{x}\hat{d}_{11}(\hat{z}, P_H/x, \mu, \mu_f)\hat{F}_{11}^H(x, P_H, m_Q^2, \mu_f)$

SDCs in SGF





➢ **Results: No plus functions (large logs)**

Chen, YQM, In preparation

$$\hat{d}_{\mathbf{11}}^{(LO)}(x,2E,\mu,\mu_f) = \frac{\pi\alpha_s}{(N_c^2-1)(D-1)E^3}\delta(1-x),$$

$$\hat{d}_{\mathbf{11}}^{(NLO)}(x,2E,\mu,\mu_f) = \frac{\alpha_s^2 N_c}{2(N_c^2-1)(D-1)E^3}\left[\frac{1}{2}\delta(1-x)\left(2A(\mu)+\ln^2\left(\frac{\mu_f^2 e^{-1}}{4E^2}\right)+\frac{\pi^2}{6}-1\right)+\frac{1}{N_c}\mathcal{P}_{gg}(x)\ln\left(\frac{\mu^2}{\mu_f^2}\right)\right.$$
$$+\left(\frac{2(1-x)}{x}+x(4-x+3x^2)+\frac{\beta_0}{N_c}\delta(1-x)\right)\ln\left(\frac{\mu_f^2 e^{-1}}{4E^2}\right)+\frac{2(1-x)}{x}+x^2(1-x)$$
$$\left.-\frac{\ln(1-x)}{x}(4-4x+8x^2-2x^3+6x^4)\right]. \qquad ($$

Distributions in SGF

➢Renormalization equations in SGF

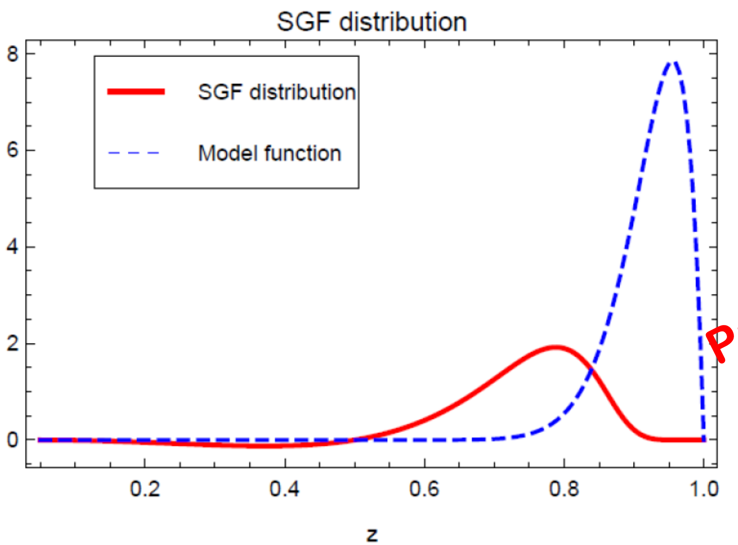
$$\frac{d}{d\ln\mu_f}\hat{F}^H_{11}(z,M_H,\mu_f)=\int_z^1\frac{dx}{x}\mathcal{K}(z,x,M_H,\mu_f)\hat{F}^H_{11}(x,M_H,\mu_f)$$

$$\mathcal{K}(z,x,M_H,\mu_f)=\Gamma_{\rm cusp}(\alpha_s)\left[\frac{\hat{z}}{(1-\hat{z})_+}+\frac{1-\hat{z}}{2\hat{z}}-\ln\frac{x\mu_f}{M_H}\delta(1-\hat{z})\right]-\gamma^S(\alpha_s)\delta(1-\hat{z})$$

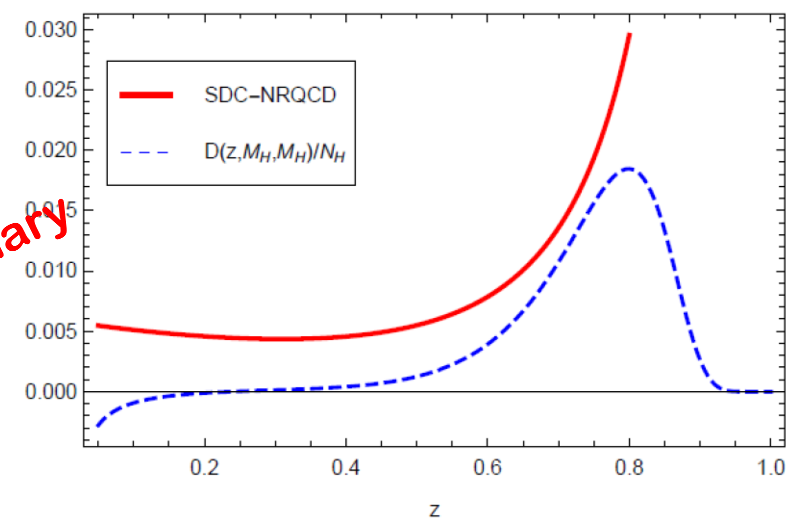
➢Resummed FFs

$$\tilde{F}^H_{11}(y,M_H,\mu_f)=U(y,M_H,\mu_f,\mu_0)\tilde{F}^H_{11}(y,M_H,\mu_0),$$

Chen, YQM, In preparation



Preliminary



Summary

- **A systematic method to calculate high order quarkonium FFs is constructed**
  - Applicable for both NRQCD and SGF; both single-parton and double-parton
  - (Almost) analytical result
- **Large corrections for** $g \to Q\bar{Q}({}^1S_0^{[1,8]}) + X$
- **Large logarithms are resumed in SGF**
  - The results may have important impact on phenomenology

Thank you!