

# Dark Matter and Dark Energy from Mimetic Gravity

Alexander Ganz

Dipartimento di Fisica e Astronomia "Galileo Galilei"  
Università di Padova

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A. Ganz, P. Karmakar, S. Matarrese, D. Sorokin (in progress)  
N. Bartolo, A. Ganz, P. Karmakar, S. Matarrese (in progress)

# Motivation

## Problem

- Dark sector of the universe

## Mimetic matter

- Unified description of dark matter and dark energy
  - One constrained scalar field
- ⇒ New phenomenology

# Mimetic Matter (1)

- Starting from the Einstein-Hilbert action [Chamseddine & Mukhanov \(2013\)](#)

$$S = \frac{1}{2} \int d^4x \sqrt{-g} R(g_{\mu\nu}) + S_m(g_{\mu\nu}, \psi)$$

- Non-invertible conformal transformation

$$g_{\mu\nu} = -(\tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi) \tilde{g}_{\mu\nu} \equiv \tilde{X} \tilde{g}_{\mu\nu}$$

$\Rightarrow$  Conformal invariant theory

$$\tilde{g}_{\mu\nu} \rightarrow \Omega(x) \tilde{g}_{\mu\nu}, \quad \tilde{X} \rightarrow \Omega^{-1}(x) \tilde{X}$$

- Transforming the action

$$S = \frac{1}{2} \int d^4x \sqrt{-\tilde{g}} \left( \tilde{X} \tilde{R} + \frac{3}{2} \tilde{X}^{-1} \tilde{g}^{\mu\nu} \partial_\mu \tilde{X} \partial_\nu \tilde{X} - 3 \square \tilde{X} \right) + S_m(\tilde{g}_{\mu\nu}, \psi)$$

# Mimetic Matter (2)

- Fixing the Conformal gauge degree of freedom  $\tilde{X} = 1$  Barvinsky (2014), Golovnev (2013)

$$S = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{1}{2} \tilde{R} - \lambda (\tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + 1) \right] + S_m(\tilde{g}_{\mu\nu}, \psi)$$

## Equations of Motion

$$g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + 1 = 0$$

$$G_{\mu\nu} - T_{\mu\nu} = \underbrace{(-G + T)}_{2\lambda} \partial_\mu \varphi \partial_\nu \varphi$$

$$\partial_\mu (\sqrt{-g} (-G + T) g^{\mu\nu} \partial_\nu \varphi) = 0$$

- The trace  $\lambda$  mimics the CDM density

$$T_{\mu\nu}^{\text{dm}} = \rho_{\text{dm}} u_\mu u_\nu \equiv (-G + T) \partial_\mu \varphi \partial_\nu \varphi$$

# Generalization of Mimetic Gravity

- Starting from a general scalar-tensor theory [Chamseddine et al. \(2014\)](#),  
[Arroja et al. \(2015\)](#), [Arroja et al. \(2016\)](#)

$$S = \int d^4x \sqrt{-g} \left[ \mathcal{L}(g_{\mu\nu}, \varphi) - \lambda (g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + 1) \right] + S_m(g_{\mu\nu}, \psi)$$

## Equations of Motion

$$g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + 1 = 0$$

$$E_{\mu\nu} + T_{\mu\nu} = -(E + T) \partial_\mu \varphi \partial_\nu \varphi$$

$$\partial_\mu (\sqrt{-g} (E + T) g^{\mu\nu} \partial_\nu \varphi) + \Omega_\varphi = 0$$

- $\mathcal{L}(g_{\mu\nu})$  can be any Lagrangian for instance Horndeski model
- Possible to model any background

# Linear Perturbations around FRW (1)

- Evolution equation for mimetic Horndeski model [Arroja et al. 2016](#)

$$\Phi'' + C_1 \Phi' + C_2 \Phi = 0$$

- No Laplacian term  $\Rightarrow$  sound speed  $c_s^2 = 0$
- Mimetic cubic Horndeski model

$$\mathcal{L} = \underbrace{K(\varphi, X) - G_3(\varphi, X)\square\varphi}_{\sim V(\varphi)} + \frac{1}{2}R$$

- Same evolution and Poisson equation as for CDM and a perfect dark energy fluid [Arroja et al. \(2017\)](#)

$$\Phi' + \mathcal{H}\Phi + \frac{a^2}{2M_{pl}^2} \left[ \bar{\rho}_{CDM}v_\varphi + (\bar{\rho}_{DE} + \bar{p}_{DE})v_\varphi + \sum_{f=r,b} (\bar{\rho}_f + \bar{p}_f)v_f \right] = 0$$

$$\Delta\Phi = 4\pi G_{\text{eff}} a^2 \delta\rho$$

# Linear Perturbations around FRW (2)

- Restricting to  $\Lambda$ CDM background ( $V(\varphi) = \Lambda$ )  $\Rightarrow$  behavior of standard  $\Lambda$ CDM at linear level
- Direct coupling between scalar field and curvature

$$\mathcal{L} = V(\varphi) + f(\varphi)R$$

- Gravitational slip

$$\gamma = \frac{\Psi}{\Phi} = 1 + \frac{\ln(f)'}{\ln(\delta\varphi)'}$$

- Different evolution and Poisson equation
- $\Rightarrow$  Deviations from a standard  $\Lambda$ CDM model at linear level

# Instabilities

- Non-linear Hamiltonian for mimetic matter [Chaichian et al. \(2014\)](#)

$$\mathcal{H} = \mathcal{H}_{\text{gr}} + p_\varphi \sqrt{1 + h^{ij} \partial_i \varphi \partial_j \varphi}$$

⇒ Stability condition

$$\lambda = \frac{p_\varphi}{\sqrt{h} \sqrt{1 + h^{ij} \partial_i \varphi \partial_j \varphi}} > 0$$

- Physical interpretation: positive dark matter density
- Generalization for mimetic scalar-tensor models

# Summary

- Mimetic gravity offers a unified description for CDM and DE
- Stable solutions require  $\lambda > 0$
- At linear level one can model exact the same behavior as standard  $\Lambda$ CDM
- Deviations enforces direct coupling between curvature and the scalar field  $f(\varphi)R$
- Future plans: Constraining the parameter space

# Backup

- Generalization to non-invertible disformal transformation [Deruelle, Rua \(2014\)](#), [Arroja et al. \(2015\)](#)

$$g_{\mu\nu} = A(\varphi, \tilde{X})\tilde{g}_{\mu\nu} + B(\varphi, \tilde{X})\partial_\mu\varphi\partial_\nu\varphi$$
$$B(\varphi, \tilde{X}) = \frac{A(\varphi, \tilde{X})}{\tilde{X}} + b(\varphi)$$

- Choice of  $A(\varphi, \tilde{X}) = \tilde{X}$  and  $b(\varphi) = -1$  leads to non-invertible conformal transformation
- Equivalence to the non-invertible conformal transformation [Takahashi, Kobayashi \(2017\)](#), [Langlois et al. \(2018\)](#)