Crawling Technicolor

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(based on arXiv:1803.08513 with R. Crewther and L. Tunstall)

Outline

- Motivation
- Scale invariance and its breaking
- Technicolor around IR fixed points: walking vs crawling
- EFT analysis
- Tests of Crawling TC
- Conclusions

Motivation

- Old idea: EWSB from a new strong confining interaction at the TeV scale. Flagship: Technicolor. [Weinberg'76; Susskind'79]

- Originally, technicolor was a higgsless theory, in analogy with QCD. Nowadays ruled out.

- *However*, a light scalar can be present, typically as a pseudo-NG boson. Explicit breaking at the EW scale will give it a mass, but protected by the Goldstone symmetry from effects at the dynamical scale and above. The mechanism is viable but not many realizations of it.

1. Composite Higgs models: [e.g., Agashe et al'04]
   (i) hard to make them realistic (fermions);
   (ii) typical v/f deviations from the SM are rather constrained.

2. Walking TC with dilatonic extensions: [e.g., Appelquist et al'10]
   (i) large number of technipions;
   (ii) hard to prove whether a light scalar actually exists.

- Can one circumvent some of these objections?

Scale invariance and its breaking

- A scale transformation on coordinates, $x^\mu \to e^\lambda x^\mu$, induces the scale transformation on fields:

$$\varphi(x) \to e^{\lambda d}\varphi(e^\lambda x)$$

  d (canonical dimension) depends on the field. Infinitesimally,

$$\delta\varphi(x) = (d + x^\mu \partial_\mu)\varphi(x)$$

- The associated Noether current is the dilatation current:

$$\mathcal{D}_\mu(x) = x^\nu \theta_{\mu\nu}(x)$$

  where $\theta_{\mu\nu}$ is **symmetric**, **gauge invariant** and **improved**.

- For a general operator $\mathcal{O}$ in a Lagrangian, one finds

$$\delta\mathcal{O} = \partial_\mu(x^\mu \mathcal{O}) + (d_{\mathcal{O}} - 4)\mathcal{O}$$

  and

$$\partial_\mu \mathcal{D}^\mu(x) = \theta^\mu_\mu(x) = \delta\mathcal{L} = \sum_j (d_j - 4)\mathcal{O}_j$$

  The current is conserved as long as operators have overall $d = 4$, i.e., if no scales are present.

Scale invariance and its breaking

Example:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \sum_j \bar{\psi}_j (i\gamma^\mu D_\mu - m_j)\psi_j$$

- Classically, one finds that

$$\theta^\mu_\mu(x) = \sum_j m_j \bar{\psi}_j \psi_j$$

  signalling explicit breaking by (mass) scales.

- At the quantum level, we know that g_s is scale-dependent. Even in the absence of masses, the Lagrangian has a separate source of breaking:

$$\theta^\mu_\mu = \beta(g)\frac{\delta \mathcal{L}}{\delta g} = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^{\mu\nu a}$$

  referred to as the **trace anomaly**.

- Anomalous breaking can be avoided at fixed points, i.e., points where $\beta(\alpha_s^*) = 0$. At those points scale symmetry is exact.

Dilatons in the QCD spectrum?

- The existence of nontrivial (infrared) fixed points in QCD is speculative, but they are not excluded and actually very interesting, theoretically and phenomenologically.

- Conventional view: no IR fixed point, since hadronic masses should arise from

$$M_N \sim \langle N|\theta^\mu_\mu|N\rangle$$

  and therefore $\beta(\alpha_s)$ has to be sizeable:

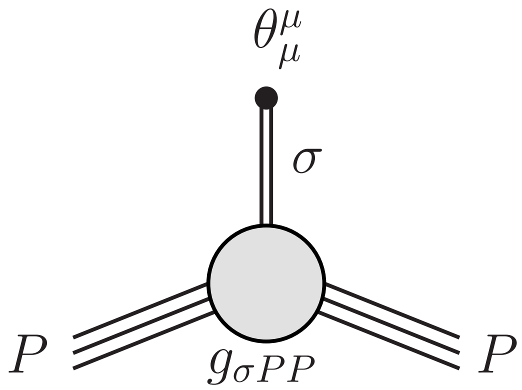
$$\theta^\mu_\mu = \left\{ \frac{\beta(\alpha_s)}{4\alpha_s} G^A_{\mu\nu} G^{A\mu\nu} + \big(1 + \gamma_m(\alpha_s)\big) \sum_q m_q \bar{q} q \right\}$$

- Alternative: an IR fixed point exists, at which scale symmetry gets spontaneously broken. If scale invariance is hidden, a dilaton appears in the spectrum and scales can be generated. PCDC then implies

$$\langle 0|\theta_{\mu\nu}|\sigma(q)\rangle = \frac{f_\sigma}{3}(q_\mu q_\nu - g_{\mu\nu} q^2)$$

  σ is the dilaton, a true dilaton if $f_\sigma \neq 0$ at the fixed point.

Dilatons in the QCD spectrum?



- The hadronic mass is given by a Goldberger-Treiman-like relation:

$$M_N \sim \langle N|H|N\sigma\rangle \frac{1}{p^2} \langle \sigma|\theta^\mu_\mu|0\rangle \sim f_\sigma g_{\sigma NN}$$

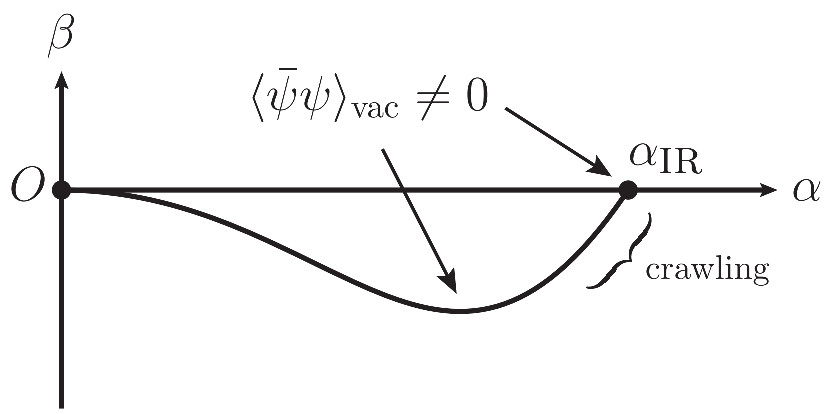
- Old but somehow forgotten wisdom: arbitrary scales can be generated close to an IRFP if scale symmetry is hidden. [Salam; Zumino; ...]
- What do we do with the dilaton? Even before QCD, speculation that a light scalar could play its role. [Carruthers'70]
- Recently, the idea has been revived, assuming that the $\sigma(500)$ is a dilaton. [Crewther et al'12] Chiral-scale perturbation theory has better convergence properties, gives an alternative explanation for the $\Delta I = \frac{1}{2}$ rule, but depends crucially on whether QCD has a nontrivial IRFP.

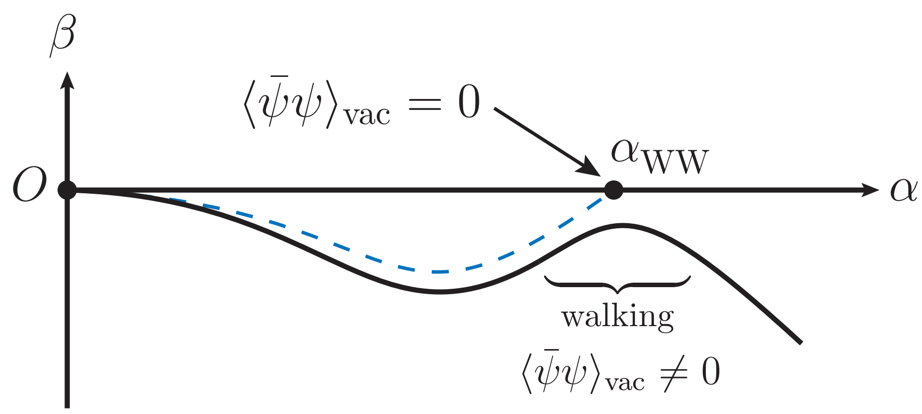
Crawling vs Walking TC

- Technicolor introduced as a scaled-up version of QCD to account for electroweak symmetry breaking.
- (Big) pro: nature has chosen dynamical symmetry breaking already;
- (Big) con: it is hard to account for every single aspect (flavor, etc.) of the SM.
- Technicolor v1.0 was put in serious trouble by EWPO. Subsequent sophistications of the theory, mostly to address flavor issues:

$$\mathbf{TC} \quad \rightarrow \quad \mathbf{ETC} \quad \rightarrow \quad \mathbf{Walking\ (E)TC}$$

- The discovery of the Higgs ruled out Higgsless TC, but Walking TC models might contain light scalars.





Crawling vs Walking TC

WALKING TC:

- Walking TC has a fixed point, but if phenomenologically viable, dynamics have to turn the RG flow. Otherwise, no scales get generated. Very hard to find models with such RG flow.

- Phase transition: confinement, a light scalon and a large chiral condensate in WTC appear in the subcritical region $N_f < N_f^c$ but disappear inside the conformal window.

- In order to generate a conformal window, a large N_f is needed. This results in a large number of technipions.

- Unsure about the presence of light scalars.

CRAWLING TC:

- Scales can exist at α_{IR}. A dilaton is present (guaranteed by symmetry).

- No conformal window is needed: small N_f is perfectly valid. Technipions are not a problem. No phase transition needed.

- Genuine nonperturbative effect. The chiral condensate that breaks EW symmetry is also a scale condensate.

However, dilatons are subtler than pions...

WW vs NG modes of scale invariance

- Take the Callan-Symanzik (CS) equation for an operator $\mathcal{O}$ in a theory with massless matter:

$$\left\{\mu\frac{\partial}{\partial\mu}+\beta(\alpha)\frac{\partial}{\partial\alpha}+\gamma_{\mathcal{O}}(\alpha)\right\}\langle 0|\mathcal{O}(0)|0\rangle = 0$$

- $\beta(\alpha)\frac{\partial}{\partial\alpha}$ is equivalent to a G^2 insertion, which is related to θ^μ_μ. Therefore,

$$\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\mathcal{O}}(\alpha)\right\}\langle 0|\mathcal{O}(0)|0\rangle = -i\lim_{q\to 0}\int d^4x\, e^{iq\cdot x}\,\mathrm{T}\langle 0|\theta^\mu_\mu(x)\mathcal{O}(0)|0\rangle$$

- WW mode:

$$\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\mathcal{O}}(\alpha_{WW})\right\}\langle 0|\mathcal{O}(0)|0\rangle_{\mathrm{WW}} = 0$$

- NG mode:

$$\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\mathcal{O}}(\alpha_{NG})\right\}\langle 0|\mathcal{O}(0)|0\rangle_{\mathrm{NG}} = f_\sigma\langle\sigma(0)|\mathcal{O}(0)|0\rangle_{\mathrm{NG}} = d_{\mathcal{O}}\langle 0|\mathcal{O}(0)|0\rangle_{\mathrm{NG}}$$

- Hyperscaling vs soft-dilaton theorems.

Scalons vs dilatons

- Manifest vs hidden symmetry.
- No scales vs scales at the fixed point.
- Perturbative phenomena (explicit breaking through deformations, pert. th.) vs nonperturbative physics (scale condensates).
- In both cases, scale symmetry broken by the trace anomaly:

$$\partial^\mu \mathcal{D}_\mu = \theta^\mu_\mu = \frac{\beta(\alpha)}{4\alpha}\Big\{\hat{G}^A_{\mu\nu}\hat{G}^{A\mu\nu} - \big\langle \hat{G}^A_{\mu\nu}\hat{G}^{A\mu\nu}\big\rangle_{\text{vac}}\Big\}$$

- Key distinction: decoupling vs nondecoupling. Light scalars in WW mode might exist but not protected by symmetry (no genuine dilaton):

$$m_\sigma^2 f_\sigma = -\langle 0|\theta^\mu_\mu|\sigma\rangle = O(\alpha - \alpha^*)$$

  Compare:

$f_\sigma \sim 0$ for $\alpha \sim \alpha_{\text{WW}}$	m_σ arbitrary
$f_\sigma \to$ constant $\neq 0$ as $\alpha \to \alpha_{\text{IR}}$	m_σ small

- Examples of scalons: scalars with Coleman-Weinberg potentials, dilatonic WTC, etc.

Crawling TC

Alternative to WTC with the following ingredients:

- New interactions (e.g. $SU(3)$) strong at the TeV scale.
- An infrared fixed point (IRFP) exists, α_{IR}. Scale-invariant limit exists, but hidden.
- At α_{IR}, Higgs as a (massless) dilaton. In the physical region $0 \leq \alpha \leq \alpha_{IR}$, the Higgs gets a (small) mass.
- *Both* electroweak and scale symmetries spontaneously broken by the same object, e.g. a chiral condensate:

$$\langle \bar{\psi}\psi \rangle_{\mathrm{TC}} \neq 0$$

- To avoid technipions, $SU(2) \times SU(2)$ global symmetry (not ruled out!).

Two important results from the Callan-Symanzik NG-mode equation

- Consider the CS equation for a RG-invariant amplitude $\mathcal{A}$:

$$\left\{\mu\frac{\partial}{\partial\mu}+\beta(\alpha)\frac{\partial}{\partial\alpha}\right\}\mathcal{A}=0$$

- Apply $\alpha\partial/\partial\alpha$:

$$\left\{\mu\frac{\partial}{\partial\mu}+\beta(\alpha)\frac{\partial}{\partial\alpha}+\beta'(\alpha)-\frac{\beta(\alpha)}{\alpha}\right\}\alpha\frac{\partial\mathcal{A}}{\partial\alpha}=0$$

- But $\alpha\partial\mathcal{A}/\partial\alpha$ is a $\hat{G}^2$ insertion, so

$$\left\{\mu\frac{\partial}{\partial\mu}+\beta(\alpha)\frac{\partial}{\partial\alpha}+\gamma_{\hat{G}^2}(\alpha)\right\}\mathcal{A}_{\hat{G}^2}=0$$

  and the anomalous dimension of $\hat{G}^2$ can be read off:

$$\gamma_{\hat{G}^2}(\alpha)=\beta'(\alpha)-\frac{\beta(\alpha)}{\alpha}$$

- The breaking of scale invariance due to the anomaly is driven by

$$\gamma_{\hat{G}^2}(\alpha_{IR})=\beta'(\alpha_{IR})$$

Two important results from the Callan-Symanzik NG-mode equation

- Take the CS equation for $\hat{G}^2$ itself in the NG mode:

$$\frac{\beta(\alpha)}{4\alpha}\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\hat{G}^2}(\alpha)\right\}\left\langle\hat{G}^2\right\rangle_{\text{vac}}=f_\sigma\left\langle\sigma\middle|\theta^\mu_\mu\middle|\text{vac}\right\rangle$$

- For an IR expansion in the physical region, $\epsilon=\alpha_{IR}-\alpha\gtrsim 0$,

$$\frac{\beta(\alpha)}{4\alpha}\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\hat{G}^2}(\alpha_{IR})\right\}\left\langle\hat{G}^2\right\rangle_{\text{vac}}=-\frac{\epsilon\beta'(4+\beta')}{4\alpha_{IR}}\left\langle\hat{G}^2\right\rangle_{\text{vac}}+O(\epsilon^2)=-m_\sigma^2f_\sigma^2$$

- Prediction for the mass:

$$\boxed{m_\sigma^2=\frac{\epsilon\beta'(4+\beta')}{4\alpha_{\text{IR}}f_\sigma^2}\left\langle\hat{G}^2\right\rangle_{\text{vac}}+O(\epsilon^2)}$$

- The formula can be easily generalized if technifermion masses are present:

$$m_\sigma^2f_\sigma^2=\frac{\epsilon\beta'(4+\beta')}{4\alpha_{IR}}\langle\hat{G}^2\rangle_{\text{vac}}-(3-\gamma_m)(1+\gamma_m)m_\psi\langle\bar{\psi}\psi\rangle_{\text{vac}}+O(\epsilon^2,\epsilon m_\psi,m_\psi^2)$$

How to build scale-invariant theories of NG mode

- Old wisdom (60s-70s), starting with Salam et al.; Wess et al; Ellis.
- Given a Lagrangian operator $\mathcal{O}$, promote it to

$$\begin{aligned}\mathcal{O} \ \rightarrow \ \mathcal{O}_\sigma &= \mathcal{O} \times \left\{ c_{\mathcal{O}} e^{(4-d_{\mathcal{O}})\sigma/f_\sigma} + (1 - c_{\mathcal{O}}) e^{(4-d_{\mathcal{O}}+\beta')\sigma/f_\sigma} \right\} \\ &= c_{\mathcal{O}} \mathcal{O}_{\mathrm{inv}} + (1 - c_{\mathcal{O}}) \mathcal{O}_{\beta'}\end{aligned}$$

- $\mathcal{O}_{\mathrm{inv}}$ has dimension 4 (scale-invariant part), while $\mathcal{O}_{\beta'}$ has dimension $4 + \beta'$ (explicit scale breaking by the trace anomaly near α_{IR})
- $c_{\mathcal{O}} = 1 + O(\epsilon)$, implied by

$$\theta^\mu_{\ \mu}\Big|_{\mathrm{eff}} = \sum_j \big(d_j - 4\big) \big\{ \mathcal{O}^j_\sigma - \big\langle \mathcal{O}^j_\sigma \big\rangle_{\mathsf{vac}} \big\} = \beta' \sum_j (1 - c_{\mathcal{O}^j}) \big\{ \mathcal{O}^j_{\beta'} - \langle \mathcal{O}^j_{\beta'} \rangle_{\mathrm{vac}} \big\}$$

which vanishes in the scale-invariant limit.

Effective Field Theory

- General description valid below the TeV scale, which is the natural cutoff of the EFT:

$$\Lambda \sim 4\pi f_\sigma \sim 4\pi v$$

  Agnostic about the explicit UV theory.

- Assume minimal setup: $SU(2)_L \times SU(2)_R$ global symmetry, spontaneously broken to give 3 Goldstones; Higgs as a dilaton; only SM fields.

- Main advantage of the effective Lagrangian formalism: radiative corrections are easily computed; contact can be made with the SM Lagrangian.

- Final result can be constructed from higgsless EFT with conformal weights, with Goldstones of electroweak and scale symmetry breaking parametrized by

$$U = e^{i\varphi^a \tau^a / v}; \qquad \chi_c = f_\sigma e^{\sigma / f_\sigma}$$

Effective Field Theory

- LO EFT:

$$\mathcal{L}_{\rm LO} = \frac{1}{2}e^{2\sigma/f_\sigma}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma) - \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$+ \bar{q}_L i\not{D} q_L + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + \bar{\ell}_L i\not{D} \ell_L + \bar{e}_R i\not{D} e_R$$
$$+ \frac{v^2}{4}\mathrm{tr}(D_\mu U D^\mu U^\dagger)e^{2\sigma/f_\sigma} - v\Big\{\bar{q}_L \hat{Y}_u U \mathcal{U}_R + \bar{q}_L \hat{Y}_d U \mathcal{D}_R + \bar{\ell}_L \hat{Y}_e U \mathcal{E}_R + \mathrm{h.c.}\Big\}e^{\sigma/f_\sigma}$$

- Power counting: LO Lagrangian dictated by homogeneity in chiral dimensions $[\mathcal{L}_{\rm LO}] = 2$:

$$[G_\mu, W_\mu, B_\mu, \sigma, \phi^a] = 0\ , \quad [\psi] = \tfrac{1}{2}\ , \quad [g_s, g_w, g'_w, \hat{Y}_{u,d,e}, \partial_\mu] = 1\ , \quad [M_\sigma^2 \sim \epsilon] = 2$$

- Subleading terms given by

$$\mathcal{L}_{\rm EFT} = \sum_{\ell \geqslant 0} \mathcal{L}_{{\rm N}^\ell{\rm LO}} \text{ with } \left[\mathcal{L}_{{\rm N}^\ell{\rm LO}}\right] = 2\ell + 2$$

Effective Field Theory

- Dilaton potential: a scale-invariant term $\sim e^{4\sigma/F_\sigma}$ is not allowed (no minimum).

- $V(\sigma)$ has to be of first order in ϵ:

$$V(\sigma) = c_{1V} e^{4\sigma/F_\sigma} + c_{2V} e^{(4+\beta')\sigma/F_\sigma}\,; \quad c_{1V}, c_{2V} = O(\epsilon)$$

- Assume $c_{1V} < 0$ and $c_{2V} > 0$ (minimum exists) and $\langle\sigma\rangle_{\text{vac}} = 0$ (convenient). c_{1V}, c_{2V} determined by first and second derivatives (no tadpole condition and m_σ^2).

- Dilaton potential in closed form:

$$V(\sigma) = \frac{m_\sigma^2 f_\sigma^2}{\beta'}\left[-\frac{1}{4} e^{4\sigma/f_\sigma} + \frac{1}{4+\beta'} e^{(4+\beta')\sigma/f_\sigma}\right]$$

- The Higgs potential is entirely fixed by the existence of an IRFP with scale symmetry broken by the (gluonic) trace anomaly.

- As expected for Goldstones, interactions with any power of σ.

Effective Field Theory

- Simplification: perform the field redefinition

$$h = \int_0^\sigma e^{\sigma'/f_\sigma} d\sigma' = f_\sigma (e^{\sigma/f_\sigma} - 1)\,, \qquad h \geq -f_\sigma$$

which brings the dilaton kinetic term into canonical form.

- The LO Lagrangian takes the form:

$$\begin{aligned}\mathcal{L}_{\rm LO} &= \frac{1}{2}(\partial h)^2 - V(h) - \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \bar{q}_L i\not{D} q_L + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + \bar{\ell}_L i\not{D} \ell_L + \bar{e}_R i\not{D} e_R + \frac{v^2}{4}\mathrm{tr}(D_\mu U D^\mu U^\dagger)\left(1 + \frac{h}{f_\sigma}\right)^2 \\ &- v\left\{\bar{q}_L \hat{Y}_u U \mathcal{U}_R + \bar{q}_L \hat{Y}_d U \mathcal{D}_R + \bar{\ell}_L \hat{Y}_e U \mathcal{E}_R + \mathrm{h.c.}\right\}\left(1 + \frac{h}{f_\sigma}\right)\end{aligned}$$

with

$$V(h) = \frac{m_\sigma^2 f_\sigma^2}{\beta'}\left[-\frac{1}{4}\left(1 + \frac{h}{f_\sigma}\right)^4 + \frac{1}{4+\beta'}\left(1 + \frac{h}{f_\sigma}\right)^{4+\beta'}\right]$$

Phenomenological highlights

- Higgs couplings of the LO Lagrangian behave like the SM with $v \to f_\sigma$.

- In Crawling TC no significant deviations expected: electroweak and scale invariance get spontaneously broken by the same condensate, so $v \sim f_\sigma$. (Compare with Goldstone Higgs of internal symmetries)

- Higgs self-interactions differ. They are affected by β':

$$V(h) - V(0) = m_\sigma^2 f_\sigma^2 \left\{ \frac{1}{2}\left(\frac{h}{f_\sigma}\right)^2 + \frac{5+\beta'}{3!}\left(\frac{h}{f_\sigma}\right)^3 + \frac{11+\beta'(\beta'+6)}{4!}\left(\frac{h}{f_\sigma}\right)^4 + O(h^5) \right\}$$

- Even with small β', triple vertex at least twice as big. However, β' nonperturbative, so not necessarily small.

- At the LHC: constraints on β' from Higgs double production. Challenging but feasible.

Dilaton mass (again)

- An expression of the dilaton mass can be found from the CS equation for $\hat{G}^2$:

$$\frac{\beta(\alpha)}{4\alpha}\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\hat{G}^2}(\alpha)\right\}\left\langle\hat{G}^2\right\rangle_{\text{vac}}=F_\sigma\left\langle\sigma\middle|\theta^\mu_\mu\middle|\text{vac}\right\rangle$$

with

$$\gamma_{\hat{G}^2}(\alpha)=\beta'(\alpha)-\frac{\beta(\alpha)}{\alpha}$$

- Alternatively, from the EFT by matching the trace anomaly at the fundamental and EFT levels:

$$\theta^\mu_\mu\big|_{\text{eff}}=-\frac{M_\sigma^2F_\sigma^2}{4+\beta'}\left\{\left(1+\frac{h}{F_\sigma}\right)^{4+\beta'}-1\right\};\qquad \theta^\mu_\mu=-\frac{\epsilon\beta'}{4\alpha_{\text{IR}}}\left\{\hat{G}^2-\langle\hat{G}^2\rangle_{\text{vac}}\right\}+O(\epsilon^2)$$

- End result:

$$\boxed{M_\sigma^2=\frac{\epsilon\beta'(4+\beta')}{4\alpha_{\text{IR}}F_\sigma^2}\left\langle\hat{G}^2\right\rangle_{\text{vac}}+O(\epsilon^2)}$$

Testing CTC on the lattice

Different possibilities:

- Freezing of α outside the conformal window in the deep infrared. Hard but a large window for N_f.

- Light scalar mass, $m_\sigma^2 \propto m_\psi$ of the form

$$\tilde{m}_\sigma^2 = \frac{\epsilon\beta'(4+\beta')}{4\tilde{f}_\sigma^2}\langle \hat{G}^2\rangle_{\text{vac}} - (3-\gamma_m)(1+\gamma_m)m_\psi \frac{\langle\bar{\psi}\psi\rangle_{\text{vac}}}{\tilde{f}_\sigma^2} + O(\epsilon^2, \epsilon m_\psi, m_\psi^2)$$

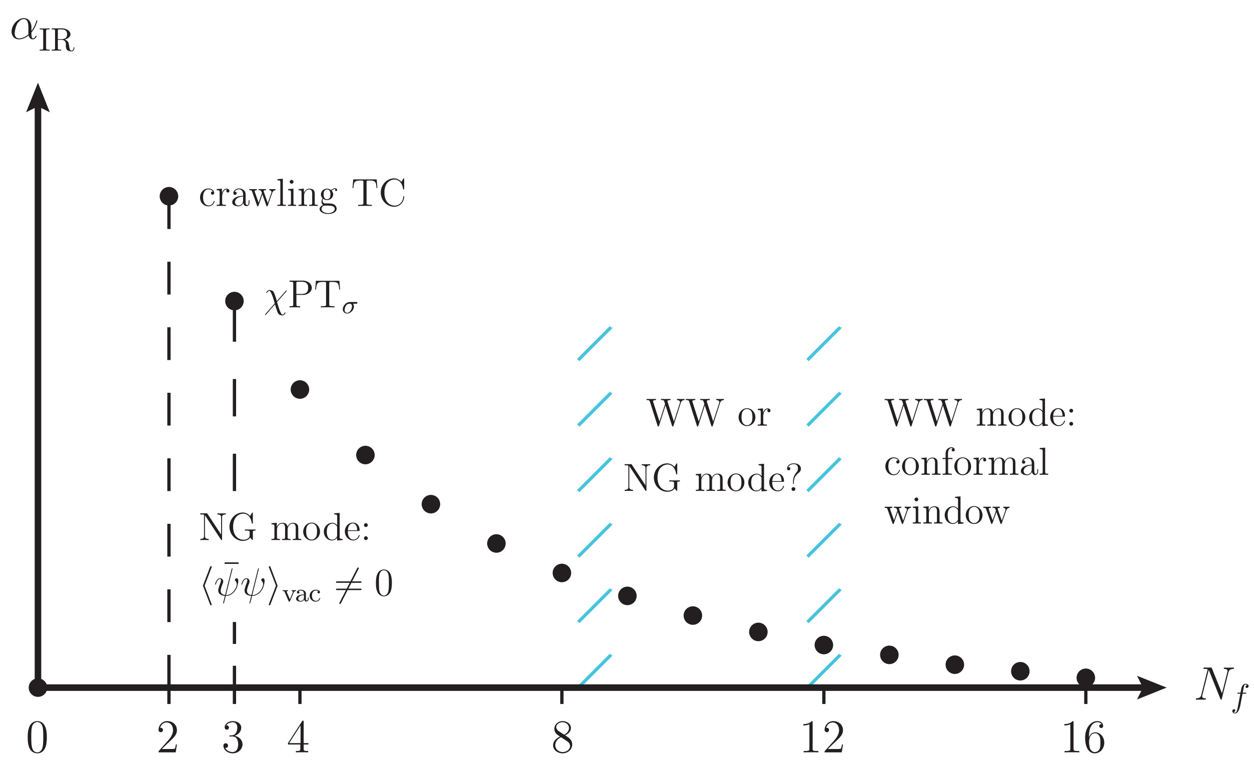
- Promising candidates: $N_f = 8$ (triplet fermions) [Aoki et al'14]; $N_f = 2$ (sextet fermions) [Fodor et al'14]. Search methods have to be adapted: hyperscaling relations do not hold, soft-dilaton theorems do.

- f_σ from matrix element residue with dilatons, and γ_m from the soft-dilaton theorem:

$$3 - \gamma_m = f_\sigma \frac{\langle \sigma|\bar{\psi}\psi|0\rangle}{\langle 0|\bar{\psi}\psi|0\rangle} + O(m_\psi)$$

Testing CTC on the lattice

- Summary chart for lattice searches on IRFPs relevant for EWSB:



- Crawling TC opens up a broader range of phenomenologically relevant IRFPs.

Flavor aspects

- In ETC, generically there is a competition between FCNC suppression and fermion mass generation:

$$\mathcal{L}_{q_i \leftrightarrow q_j} = c_{ij}\big(g_X/M_X\big)^2 \bar{q}_{i\,L}\gamma^\mu \psi_{\mathrm{TC}\,R}\bar{\psi}_{\mathrm{TC}\,L}\gamma_\mu q_{j\,R} + \text{h.c.},$$
$$\mathcal{L}_{|\Delta S|=2} = c_{\Delta S=2}\big(g_X/M_X\big)^2 \bar{d}_L\gamma^\mu s_R \bar{s}_L\gamma_\mu d_R + \text{h.c.}$$

- Walking TC was proposed to ammeliorate the problem. If $\beta \sim 0$, then the RG evolution is not logarithmic but power law:

$$\big\langle \text{vac}\big|\bar{\psi}_{\mathrm{TC}\,L}\psi_{\mathrm{TC}\,R}\big|\text{vac}\big\rangle_{\mathrm{ETC}} = \big\langle \text{vac}\big|\bar{\psi}_{\mathrm{TC}\,L}\psi_{\mathrm{TC}\,R}\big|\text{vac}\big\rangle_{\mathrm{TC}} \exp\int_{\Lambda_v}^{\Lambda_V}\frac{d\mu}{\mu}\,\gamma_m\big(\alpha(\mu)\big) \approx \Lambda_v^3\left(\frac{\Lambda_V}{\Lambda_v}\right)^{\gamma_m^*}$$

- Crawling TC has a similar mechanism, if one assumes that β' is sufficiently small (delayed asymptotic freedom). Then power-law scaling is also possible:

$$\exp\int_{m_h}^{M_{\text{max}}}\frac{d\mu}{\mu}\,\gamma_m\big(\alpha(\mu)\big) \approx \left(\frac{M_{\text{max}}}{m_h}\right)^{\gamma_m(\alpha_{IR})}$$

Conclusions

- CTC vs WTC: NG-mode vs WW-mode implementation of conformal breaking in dynamical EWSB. Crawling has no hierarchy problem, no large number of technipions and a similar mechanism to accommodate FCNCs and fermion masses.

- NG-mode IR fixed points for small N_f are not excluded: power-law scaling of Green's functions tests only WW-mode IRFPs.

- Phenomenologically interesting: no expected deviations for Higgs couplings to fermions and gauge bosons. Deviations in Higgs potential can be written down in closed form in terms of $\beta'(\alpha_{IR})$. Typically, couplings larger than the SM.

- CTC dilaton is a genuine Goldstone mode of hidden scale symmetry:

$$m_\sigma^2 = \frac{\epsilon\beta'(4+\beta')}{4\alpha_{\rm IR} f_\sigma^2} \left\langle \hat{G}^2 \right\rangle_{\rm vac} + O(\epsilon^2)$$

- Tests at the LHC (Higgs double production) and with lattice simulations ($f_\sigma, \gamma_m, \beta', \langle \hat{G}^2 \rangle$).