SPARC LAB

Dottorato in Fisica degli Acceleratori
Laboratorio di Acceleratori
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Transverse Diagnostics: beam size and emittance

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Introduction

- Diagnostics is the "organ of sense" of an accelerator
  - Instrumentation for daily check
    - profile measurements, charge, beam position, …
  - Instrumentation for commissioning and accelerator development
    - emittance and bunch length measurements (and more and more)

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Motivation

- Particle beam properties in the transverse phase space are characterized by the **transverse beam emittance**
  - **Key parameter** both for light sources (**spectral brilliance**) and colliders (**luminosity**)

Measure for phase space density of photon flux

$$B = \frac{\#photons}{[sec][mm^2][mrad^2][0.1\%BW]}$$

connection to machine parameters

$$B \propto \frac{I}{\varepsilon_x \varepsilon_y} [\mathrm{A}/(\mathrm{m}^*\mathrm{mrad})^2]$$

Measure for the collider performance

$$\dot{N} = L\sigma$$

connection to machine parameters

$$L \propto \frac{I_1 I_2}{\varepsilon}$$

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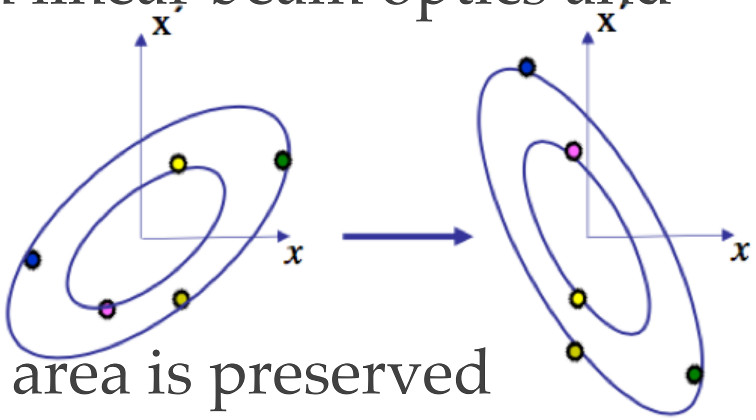
Transverse Emittance

- The transverse emittance is the projected phase space area
  - **Liouville's theorem**: transverse emittance is conserved in linear beam optics and under linear forces
    - any particle moves on an ellipse in phase space (x,x')
    - ellipse rotates in magnets and shears along drifts, but area is preserved



- Propagation along the accelerator implies the knowledge of the magnet structure => beam optics
  - Transformation from initial (i) and final (f) location

Single particle transformation

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

Transformation of optical functions

$$\begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix}_f = \begin{pmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -R_{11}R_{21} & 1+R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{pmatrix} \cdot \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix}_i$$

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Transverse Emittance

- Propagation along the accelerator implies the knowledge of the magnet structure => beam optics
  - Transformation from initial (i) and final (f) location

Single particle transformation

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \underbrace{\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}}_{R} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

$$R_{drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$x_f = x_i + L x'_i$$

$$x'_f = x'_i$$

$$R_{quad} = \begin{pmatrix} 1 & 0 \\ \pm 1/f & 1 \end{pmatrix}$$

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Transverse Emittance Ellipse

- The transverse emittance is described either in the form of an **ellipse equation** via the Courant-Snyder or **Twiss parameters** as

$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

- or as **statistical definition**

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}$$

  - **characterization of beam charge distribution by its 2nd statistical moments**

rms size

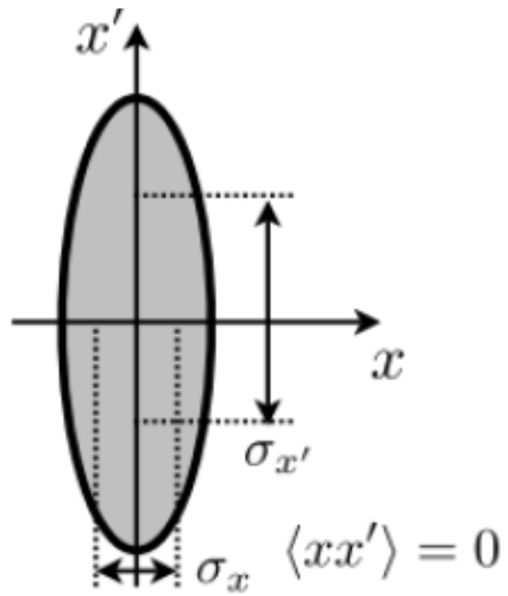
$$\sigma_x^2(z) = \langle x^2 \rangle = \frac{1}{N_e} \sum_j x_j^2$$

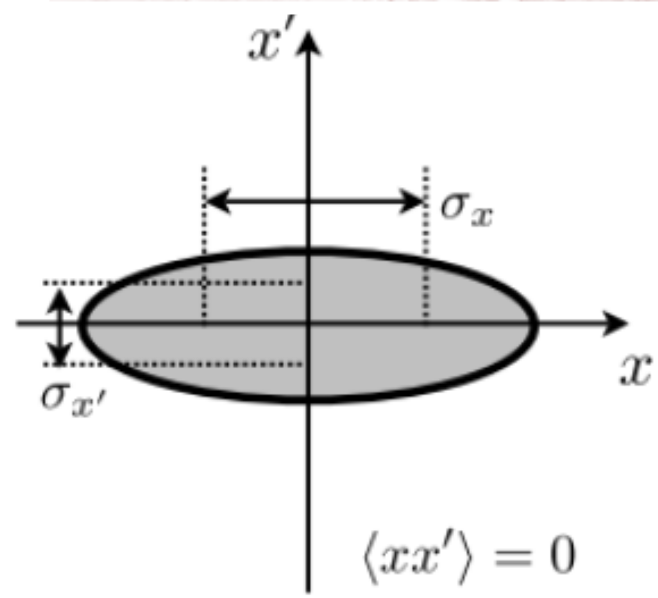
rms divergence

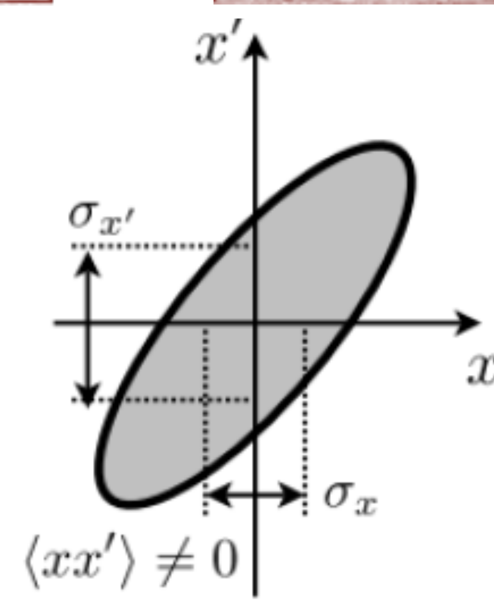
$$\sigma_{x'}^2(z) = \langle x'^2 \rangle = \frac{1}{N_e} \sum_j x_j'^2$$

correlation

$$\langle x x' \rangle = \frac{1}{N_e} \sum_j x_j x_j'$$







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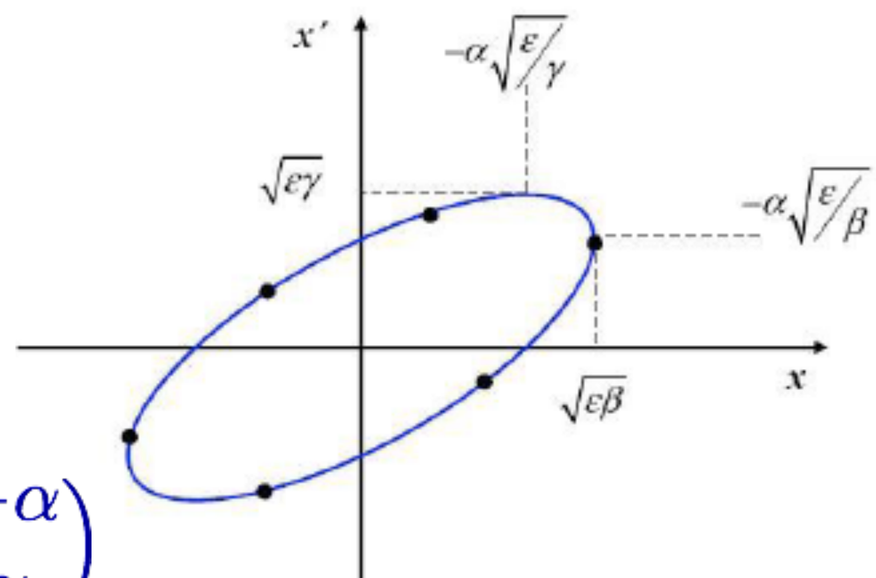
Emittance and Beam Matrix

- The emittance itself is not directly measured
- The measurable quantities are the projections onto both axes, i.e. beam profile or beam divergence
- Beam matrix based schemes, e.g. **Twiss parameters** or mapping of the phase space
  - exploit the transfer properties of the beam matrix

Let assume uncoupled motion: 2D sub-space



Beam matrix

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\varepsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11}\Sigma_{22} - \Sigma_{12}^2}$$

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Beam Matrix based Measurements

The emittance is determined by measurement of 3 matrix elements.

The observable is the rms beam size $\sigma = \sqrt{\Sigma_{11}} = \sqrt{\varepsilon\beta} = \sqrt{\langle x^2 \rangle}$

Σ_{12} and Σ_{22} must be inferred from beam profiles taken under various transport conditions,

therefore the knowledge of transport matrix R is required

$$\Sigma^b = R\Sigma^a R^T \qquad R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

Since $\Sigma(s)$ → determination of Σ required at same location.

Beam size measurements for **at least 3 different matrix elements** are required in order to solve for the 3 independent unknown parameters: ε, β and α.

$$\Sigma^f_{11} = R^2_{11}\Sigma^i_{11} + 2R_{11}R_{12}\Sigma^i_{12} + R^2_{12}\Sigma^i_{22}$$

measurement: profiles

known: transport optics

deduced: beam matrix elements at initial location

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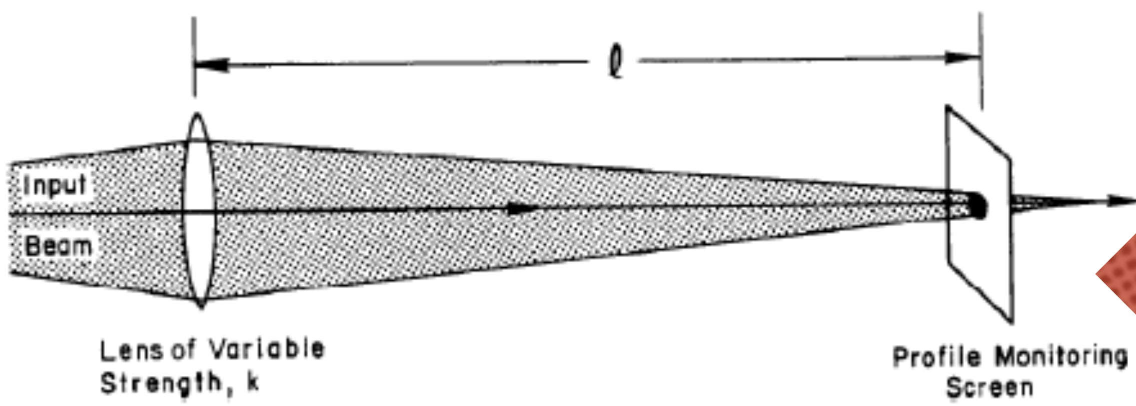
Beam Matrix based Measurements

„quadrupole scan" method

- use of variable quadrupole strengths

→ change quadrupole settings and measure beam size in profile monitor located downstream



in thin lens approximation

Σ_{11} depends quadratically on quadrupole field strength

Q (f = 1/K) **S** (drift space)

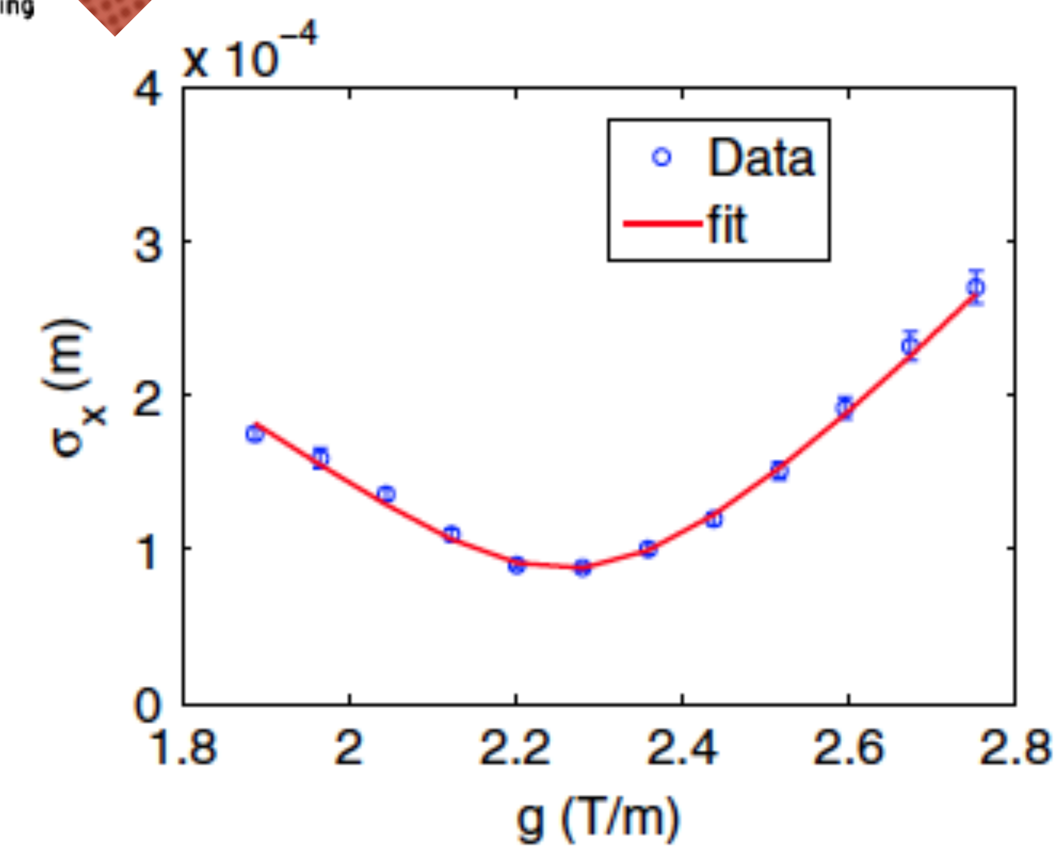
quadrupole transfer matrix

$$Q=\begin{pmatrix} 1 & 0 \\ \pm 1/f & 1 \end{pmatrix}$$

drift space transfer matrix

$$S=\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

$\rightarrow$ $\mathbf{R = SQ}$



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Emittance measurement at the waist point

For the thin lens approximation we can evaluate the
emittance by only two points

In focal plane (L=f):

$$\Sigma_{11}^{f} = R_{11}^{2}\Sigma_{11}^{i} + 2R_{11}R_{12}\Sigma_{12}^{i} + R_{12}^{2}\Sigma_{22}^{i}$$

0 0

At the same time this point is the waist of the beam:

$$\Sigma_{12}^{f} = 0$$

After that You already know two out of three coefficients, thus to find thir
d we can simply use one more point (basically any point):

$$\Sigma_{11}^{f} = R_{11}^{2}\Sigma_{11}^{i} + 2R_{11}R_{12}\Sigma_{12}^{i} + R_{12}^{2}\Sigma_{22}^{i}$$

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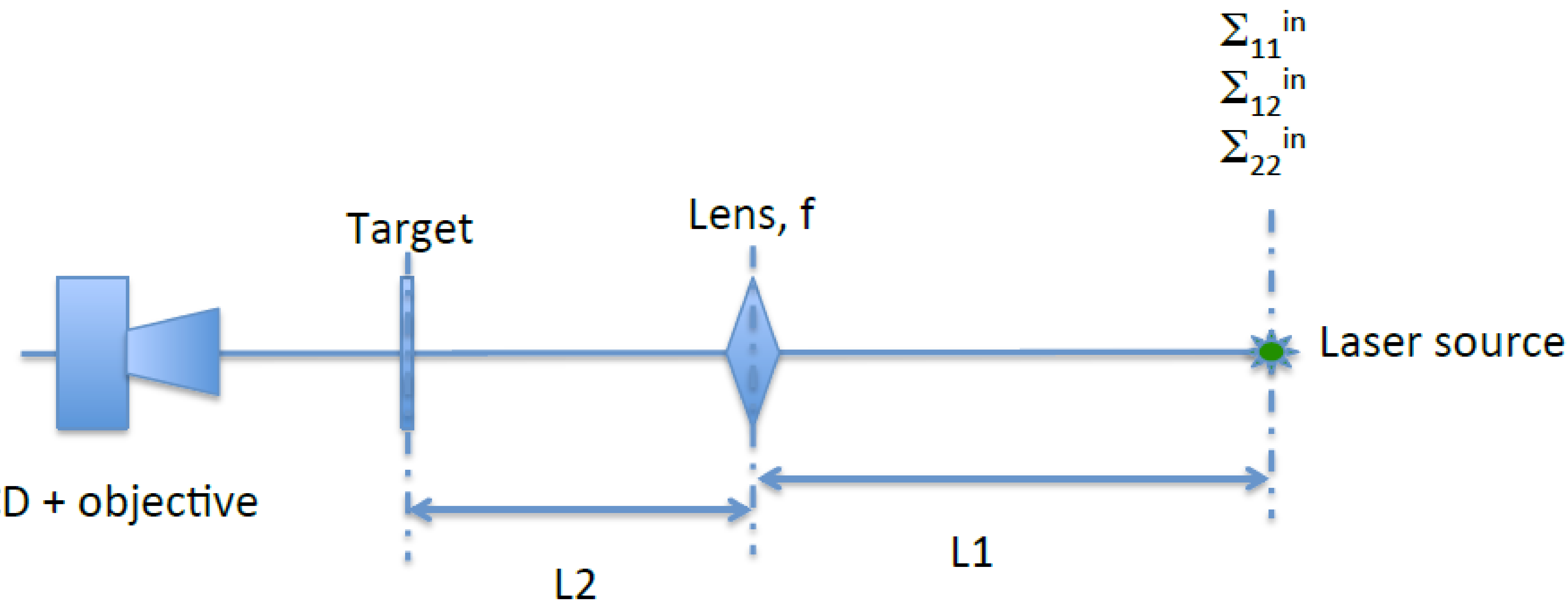
Transport matrix



$$R_{tr} = R(L_2) \cdot R(f) \cdot R(L_1)$$

Emittance of the perfect Gaussian laser beam

$$\epsilon_{gauss} = \frac{\lambda}{4\pi}$$

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"Ciò che dobbiamo imparare a fare, lo impariamo facendolo."

– *Aristotele*

Final goal of the experience

- **Measurement of the emittance of the laser beam**
  - transverse beam size
    - definition of the imaging system
      - CCD+objective
        - calibration of the CCD
          - magnification
      - resolution (USAF target)
      - Depth of Field
      - region of confusion (Siemens target)

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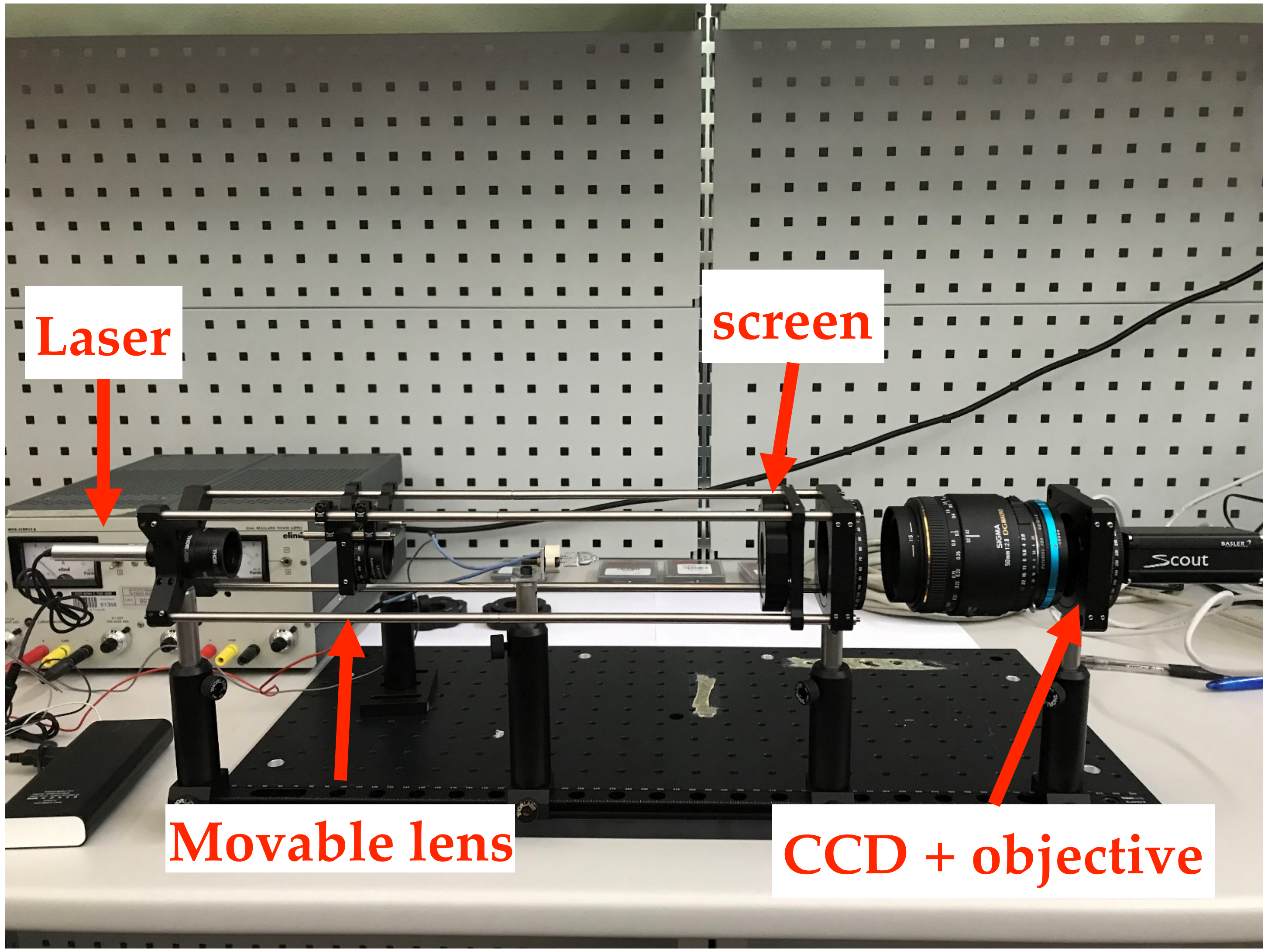
Experimental Setup

Test setup



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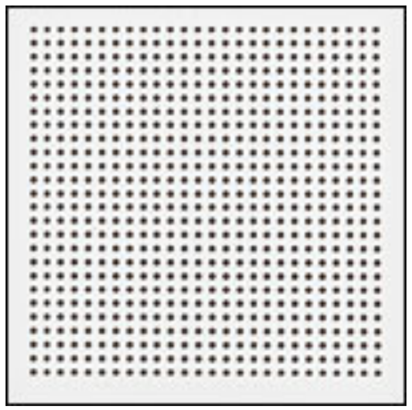
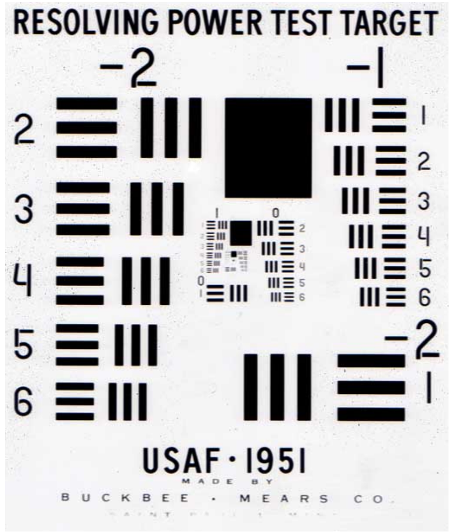
Experimental setup

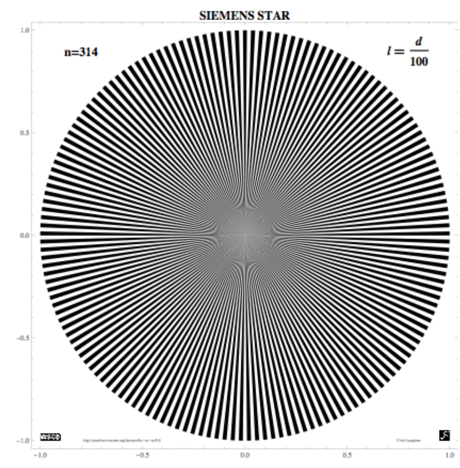
Item	LDM635
Wavelength, Typical	531.9 nm
Wavelength, Min/Max	625 - 645 nm
Beam Diameter	3.5 mm
Power	4.5 mW

Item	LDM635
CCD camera	Basler scout scA640-74gm
resolution	659×494 px
pixel size	9.9×9.9 μm
Size of the matrix	6.5×4.9 mm

- In the same position of the screen characterize:
  - **calibration**
    - dot grid target (**spacing: 0.5 mm**)
  - **resolution**
    - USAF 1951-target
  - **focusing**
    - Siemens star (n = 314, l=d/100)







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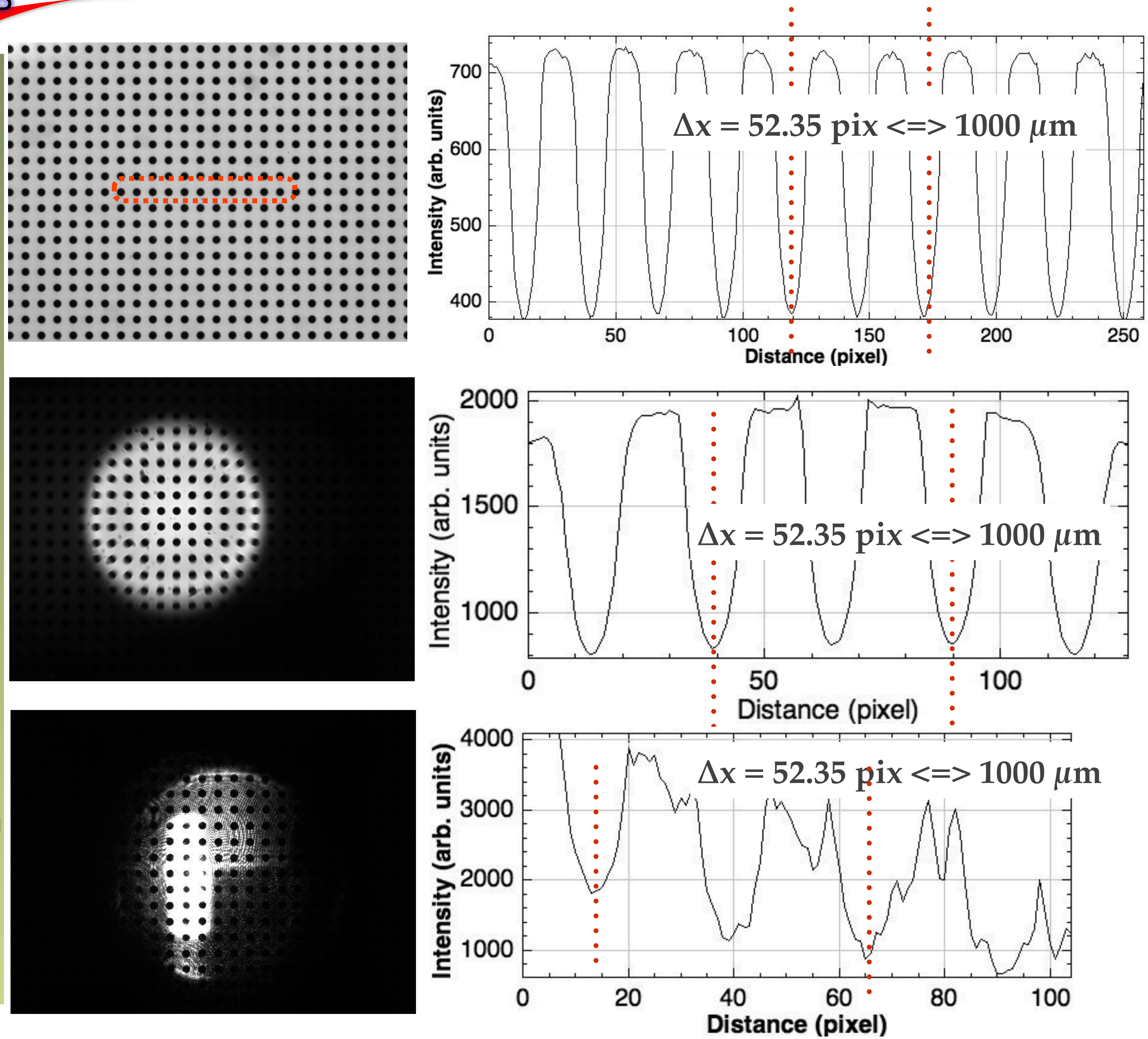
Dot grid target

The importance of uniform illumination



19.1 μm/pix

19.69 μm/pix

19.43 μm/pix

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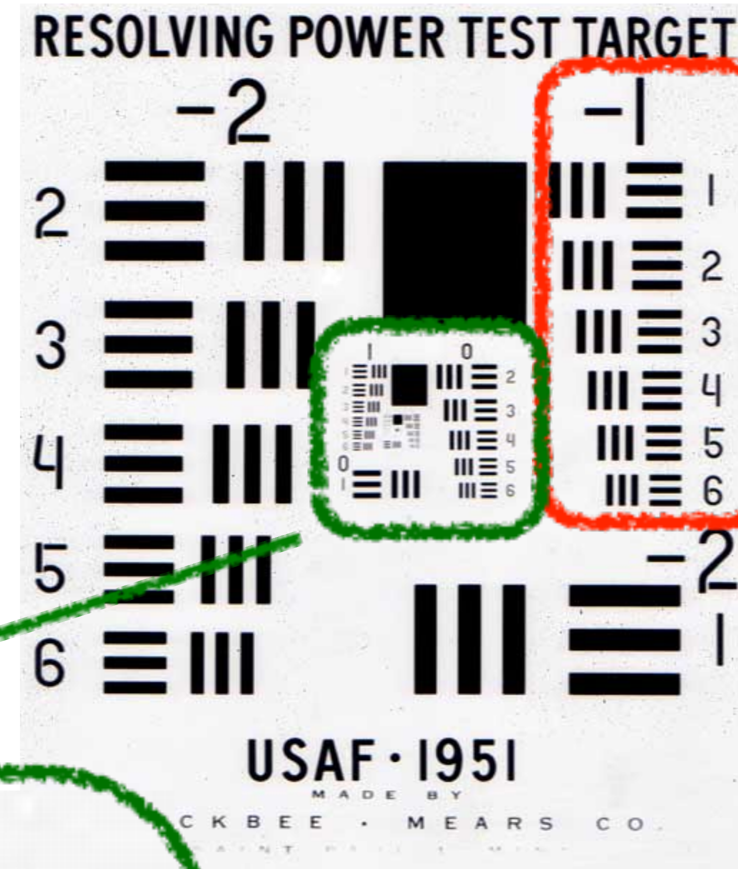
USAF 1951-target

The targets consist of "groups" of 6 "elements" each. The group numbers at the top of the group, the element numbers are located at the sides of the groups.
Each element consists of three horizontal and three vertical bars.



Group "-1" with the elements "1-6"

Group 1: element 5 is still resolved; element 6 can't be resolved (the bars blur)

The size of the square:

S=2.5/value from the table [mm]

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USAF 1951-target

Element	Group Number									
	-2	-1	0	1	2	3	4	5	6	7
1	0.250	0.500	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00
2	0.280	0.561	1.12	2.24	4.49	8.98	17.95	36.0	71.8	144.0
3	0.315	0.630	1.26	2.52	5.04	10.10	20.16	40.3	80.6	161.0
4	0.353	0.707	1.41	2.83	5.66	11.30	22.62	45.3	90.5	181.0
5	0.397	0.793	1.59	3.17	6.35	12.70	25.39	50.8	102.0	203.0
6	0.445	0.891	1.78	3.56	7.13	14.30	28.50	57.0	114.0	228.0

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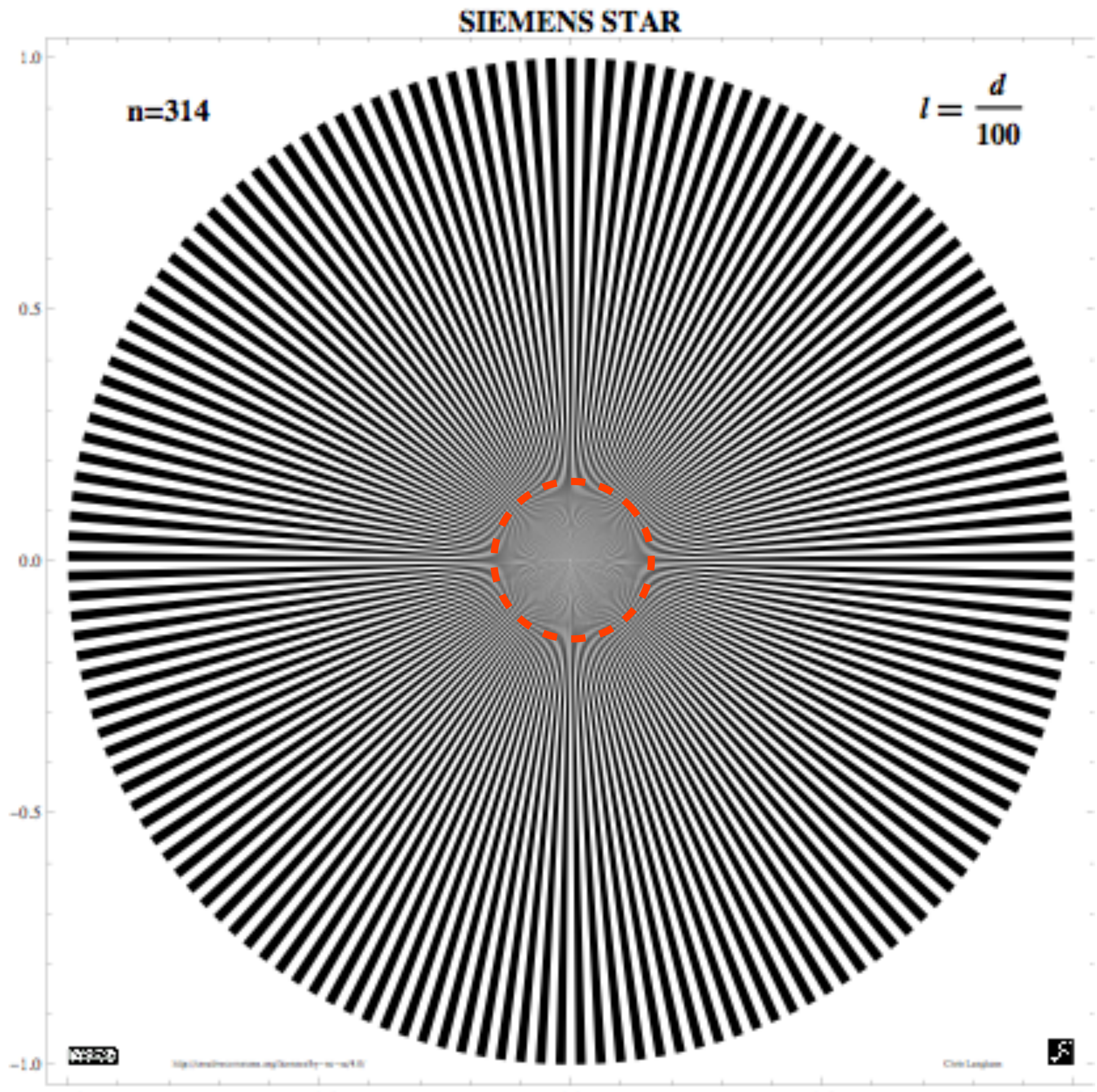
Siemens star

- It consists of alternating black and white thin "pie shaped" segments: moving towards the center of the star, the lines get closer and closer together.
  - The higher the resolution of the system generating the star pattern, the closer to the center of the star they will appear to merge.



circle of confusion

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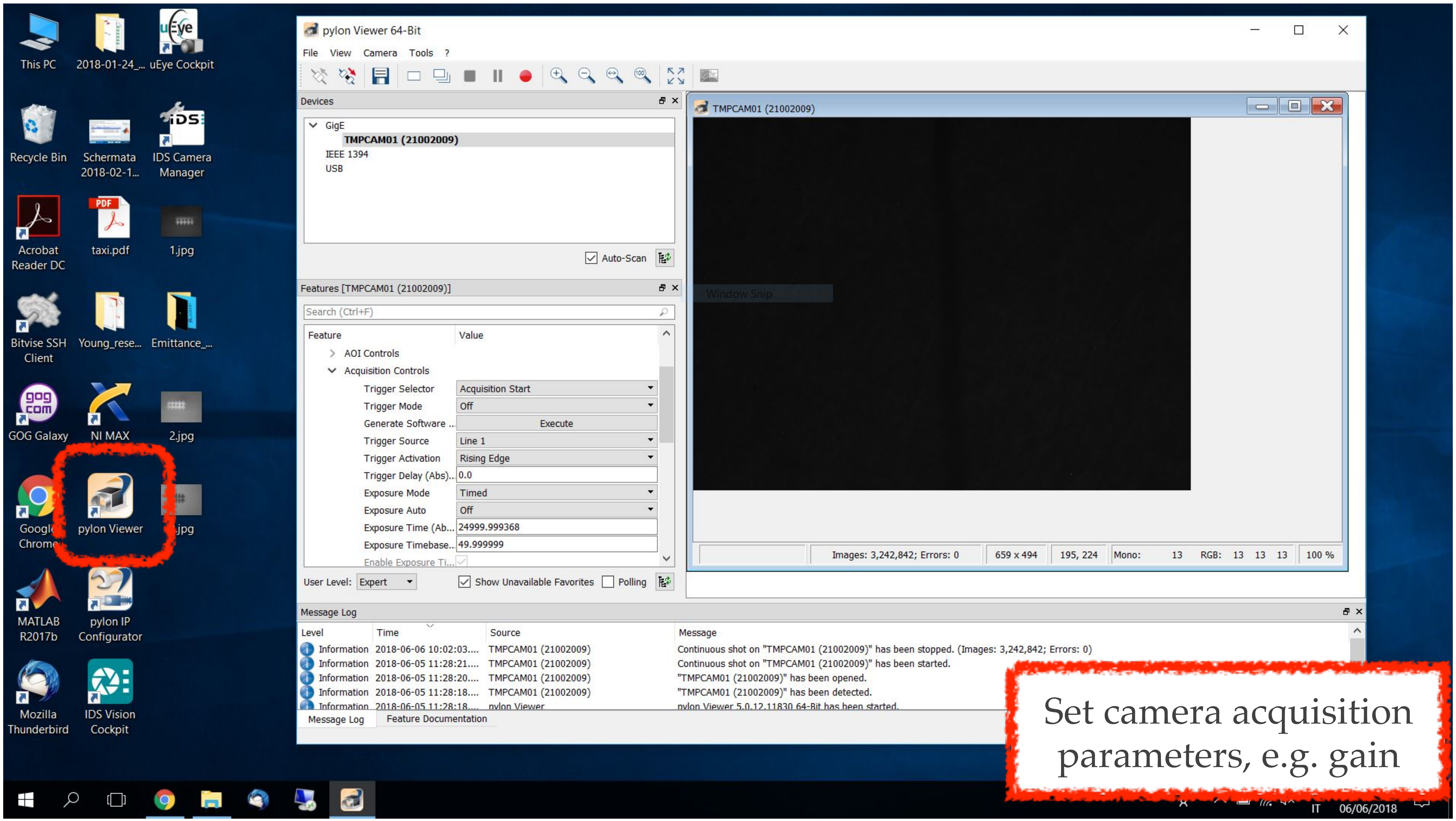
CCD readout



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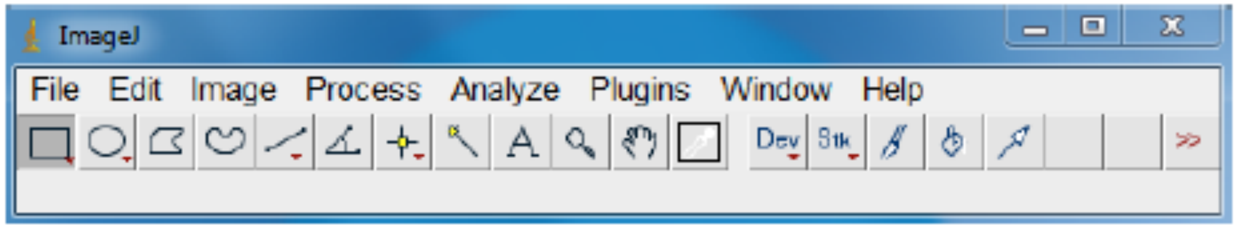
ImageJ Introduction

press icon access to start panel

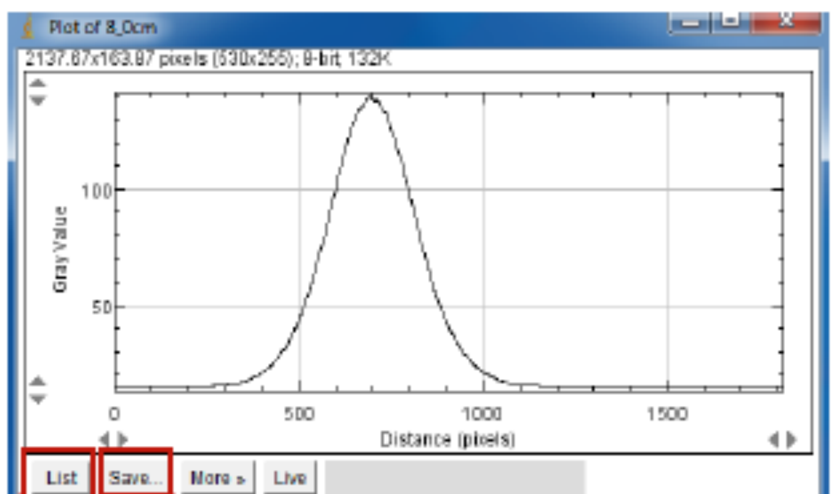


load image file → File → Open (Shortcut: Ctrl + O)

select ROI: in start panel: select left button (below "File"), usually already pre-selected

then with left mouse button: draw rectangular ROI

plot horizontal projection → Analyze → Plot Profile (Shortcut: Ctrl + k)



list data points

save data as Excel sheet (required for profile fitting)

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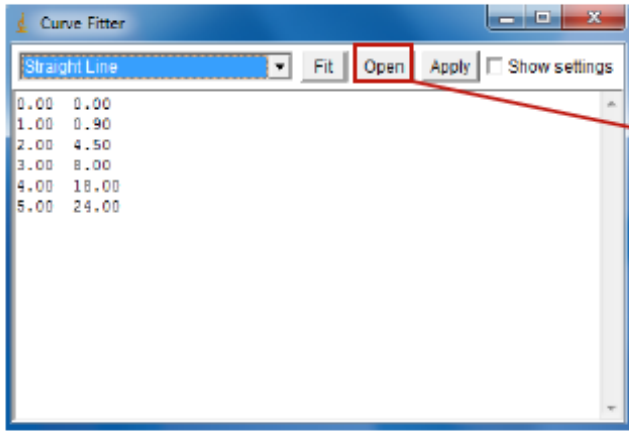
ImageJ Introduction

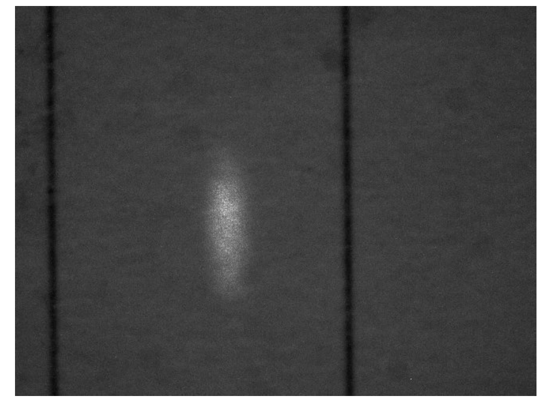
profile fitting → Analyze → Tools → Curve Fitting…

- load profile data:



load data file

Signal - Background



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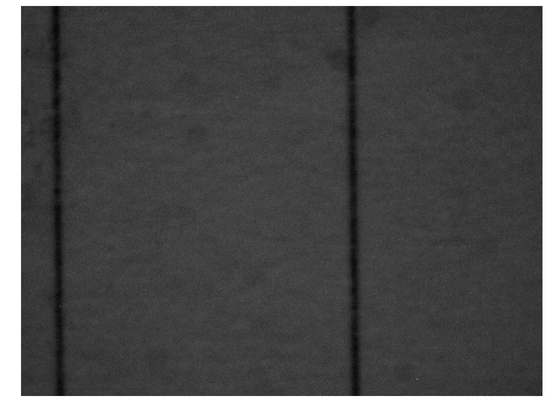

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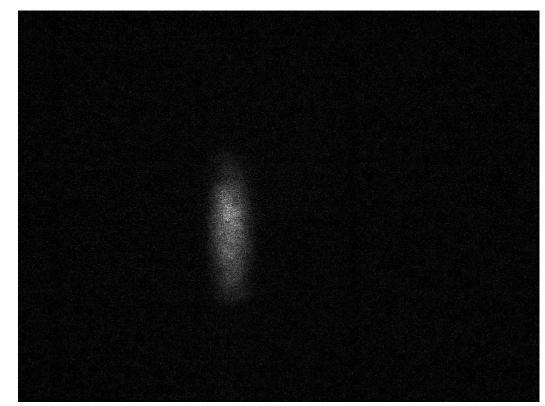
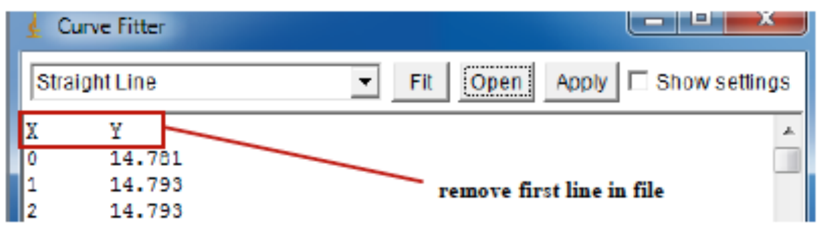
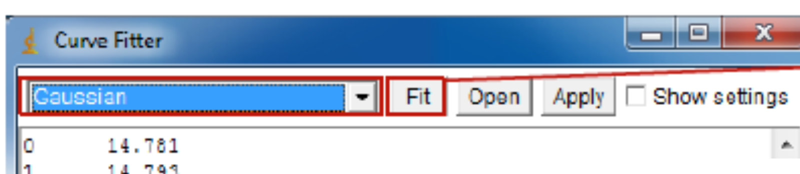


- delete bad data:



remove first line in file

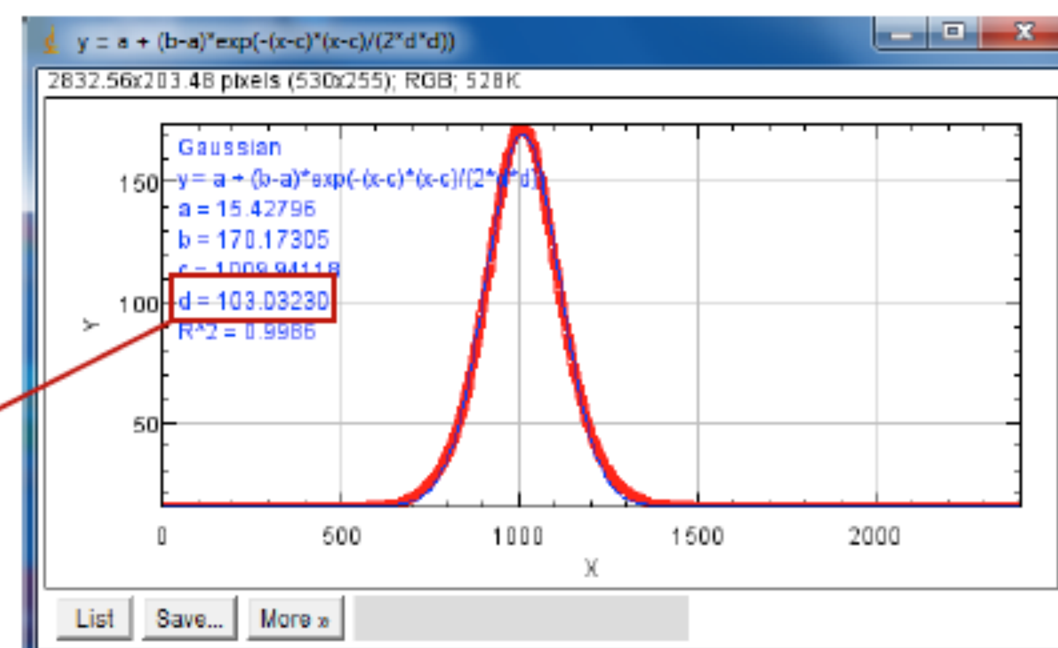
- select fit function:



fit profile data

$$y = a + (b-a)\cdot e^{-\frac{(x-c)^2}{2d^2}}$$

fit results:



1 σ-width (in pixel)

Acknowledgements

- This experience and part of the slides material have been freely taken from Gero Kube (DESY, Hamburg) as prepared for the EDIT2015 School. Other material comes from Zhirong Huang (SLAC) at the S^3EPB 2013, YouTube (*A Simple Guide to Depth of Field* by Dylan Bennett) and Optowiki (http://www.optowiki.info/)

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