From B-physics anomalies to model building and direct searches



Genova, 9/05/2018

Outline

- Introduction
- Recap: B-physics anomalies
- Combined EFT fit of the anomalies
- Simplified models & direct searches of the mediators
- UV example: a composite Higgs model with scalar LQ.
 direct searches of other states

Introduction

The hierarchy problem of the EW scale suggests

The strong bounds from flavour physics require instead

 $\Lambda \lesssim \text{TeV}$ $\Lambda \gg \text{TeV}^*$ * for arbitrary flavour structure

To have NP at the TeV scale, the common lore suggested that it should be almost flavour diagonal (MFV-like).

Many theorists believed the LHC era would give:

Abundance of new resonances at the LHC!!!

Boring flavour physics...



Instead we ended up with:



No direct signal of new particles...



Exciting anomalies in flavour physics!!!



Data

Neutral-current anomalies



 $b \rightarrow s \ \mu^+ \ \mu^- \ vs. \ b \rightarrow s \ e^+ \ e^-$

The LHCb experimented:



ranching ratios Bachad production



Lepton Flavour Universality ratios

$$R(K^{(*)}) = \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \to K^{(*)}e^+e^-)}$$

Clean SM prediction



Neutral-current anomalies $C_{0}^{\mu} \stackrel{\text{New } F}{=}$

 $C_{\mathrm{q}}^{\mathrm{SM}} \approx -C_{10}^{\mathrm{SM}} \approx 4.2$

The SM contribution is aligned along the effective operator

- → all deviations are consistent
- \rightarrow 4 6 σ deviation in global fits
- → Best fit with NP in LH current

$$\mathcal{L} \supset \frac{c_i}{\Lambda^2} (\bar{s}_L \gamma^{\alpha} b_L) (\bar{\mu}_L \gamma_{\alpha} \mu_L) + h.c.$$

What is the scale of NP?

No suppression: $c_i = 1 \longrightarrow \Lambda \sim 31 \text{ TeV}$

MFV or U(2): $c_i = V_{ts} \longrightarrow \Lambda \sim 6 \text{ TeV}$

Loop + MFV: $c_i = V_{ts}/4\pi \rightarrow \Lambda \sim 0.5 \text{ TeV}$

the relevant $\Delta C_{9}^{\mu} = -\Delta C_{10}^{\mu} = -0.61 \pm 0.12$ taining Λ_{μ} i.e. from tl of new phy direct sear

 $(\bar{s}_L \gamma^{\nu} b_L) (\bar{\mu}_L \gamma_{\nu} \mu_{\nu} c_0^{\mu} = Using this$

some assu

Adding SM gauge invariance: New Physics in at least one of the peretocian

 $C_T (\bar{Q}_L^i \gamma_\mu \sigma^a Q_L^j) (\bar{L}_L^\alpha \gamma^\mu \sigma^a L_L^\beta) + C_S (\bar{Q}_L^i \gamma_\mu Q_L^j) (\bar{L}_L^\alpha \gamma^\mu h j g h - energe$ fects shou

Charged-current anomalies



b to c transition in τv final state.

Tree-level SM process with Vcb suppression.

LFU ratio to reduce QCD uncertainties



$$R(D^{(*)}) \equiv \frac{\mathcal{B}(B^0 \to D^{(*)+} \tau \nu)}{\mathcal{B}(B^0 \to D^{(*)+} \ell \nu)},$$
$$\ell = \mu, e$$

All results since 2012 consistently above SM prediction

 $R_{D^{(*)}} \equiv R(D^{(*)})/R(D^{(*)})_{\rm SM} = 1.234 \pm 0.052$

While μ /e universality tested at % level.

~ 20% enhancement from the SM $\sim 4\sigma$ from the SM

Charged-current anomalies



The tree-level SM contribution is mediated by:

$$\mathcal{H}_{\rm SM} = \frac{4G_F}{\sqrt{2}} V_{cb} (\bar{c}_L \gamma_\mu b_L) (\bar{\tau}_L \gamma^\mu \nu_\tau) + h.c.$$

Freytsis, Ligeti, Ruderman 2015

$$\mathcal{L}_{\rm BSM} = \frac{2c}{\Lambda^2} (\bar{c}_L \gamma_\mu b_L) (\bar{\tau}_L \gamma^\mu \nu_\tau) + h.c.$$

What is the scale of NP?

 $\rightarrow \Lambda \sim 3.7 \text{ TeV}$ $c_i = 1$ MFV or U(2): $c_i = V_{cb} \rightarrow \Lambda \sim 0.7 \text{ TeV}$ Loop + MFV: $c_i = V_{cb}/4\pi \rightarrow \Lambda \sim 0.2 \text{ TeV}$

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Adding SU(2)_L gauge invariance: New Physics in this operator



To summarise



Strongest effect in third generation fermions, smaller for second, negligible for first. Very similar to SM fermion masses pattern!

Best SMEFT operators to fit the anomalies



Flavour Universality

Since fermions from different generations have same gauge quantum numbers, gauge interactions in the SM are *accidentally* flavour-universal.

$$\mathcal{L}_{\text{gauge}} = i \sum_{j=1}^{3} \sum_{q,u,d,\ell,e} \bar{\psi}_{j} \not{\!\!\!D} \psi_{j} \quad \text{Accidental global symmetry} \quad \text{U}(3)^{5}$$

$$\mathcal{L}_{\text{gauge}} = i \sum_{3}^{3} \sum_{q,u,d,\ell,e} \bar{\psi}_{j} \not{\!\!\!D} \psi_{j}$$

The only other fermionic interactions in the SM (Yukawa interactions) $x = \bar{q}_L Y_u u_R H \delta m p \bar{l} g H d k \bar{\ell} h \& e g m metry \Delta d k a b g h lity maximally!$

$$\mathcal{L}_{\text{Yuk}} = \bar{q}_L Y_u u_R H^* + \bar{d}_L Y_d d_R H + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H = \bar{\ell}_L Y_e e_R H \qquad Y_{u,d,e} \approx \text{diag}(\mathbf{u}_R H) + \bar{\ell}_L Y_e e_R H = \bar{\ell}_L Y_e e_R H = \bar{\ell}_L Y_e e_R H + \bar{\ell}_L Y_e e_R H = \bar{\ell}_L Y_e e_R H + \bar{\ell}_L Y_e e_R H +$$

 $\begin{array}{l} \underset{\mu}{\overset{\text{Since}}{\approx}} y_{\mu} \ll y_{\tau} \sim \underset{\mu}{\overset{10^{-2}}{\approx}} \underset{\mu}{\overset{\text{in most high-energy SM processes}}{\overset{\text{SM processes}}{\overset{\overset{\text{SM processes}}{\overset{\text{SM processes}$

BSM interactions are expected to violate this.

New Physics in 3rd generation

In many motivated models:

New Physics

3rd generation fermions

Top quark has biggest mass

K* meson

Z boson

W boson

ū

B meson

biggest coupling to the Higgs. Maybe 3rd family is particularly related to EW scale dynamics. e.g.: top partners.

Rare B decays are very sensitive probes of such New Physics scenarios

Suppressed in the SM by loop factor, GIM, and small mixing angles



Future Prospects Albrecht et al 1709.10308

Experimental Timeline Milestone 2018 2021 2017 2019 2020 Q1 Q2 Q3 Q4 Belle II ~ 5 ab-1 Start of Data taking period LHCb ~ 8 fb⁻¹ Run 2 Milestone I 2022 $202\overline{3}$ 2024 2025 2026 Q1 Q2 Q3 Q4 Belle II ~ 50 ab-1 | End of Data taking period LHCb ~ 22 fb⁻¹ Run 3 Milestone III 2030 2027 2028 2029 Q1 Q2 Q3 Q4 Q1 Q2 Q3 Q4 Q1 Q2 Q3 Q4 Q1 Q2 Q3 Q4 LHCb Run 4 ~ 50 fb⁻

+ very precise measurements on many other related observables.

In just a few years we will know if these are genuine NP signals or not.

Charged-current



Neutral-current

Assuming present central value, LHCb will measure R(K) and R(K*) at >5 σ by Milestone I (2020), ~15 σ at Milestone III (2030).

Also Belle-II will reach 7-8o by Milestone II (2025).

Let us assume these anomalies are due to New Physics.

Can we find at least one consistent solution?

SM EFT fit

Our EFT framework

 $Q_L^i = \begin{pmatrix} V_{ji}^* u_L^j \\ d_L^i \end{pmatrix}$

 $\frac{1}{v^2}\lambda_{ij}^q\lambda_{\alpha\beta}^\ell \left[C_T \ (\bar{Q}_L^i\gamma_\mu\sigma^a Q_L^j)(\bar{L}_L^\alpha\gamma^\mu\sigma^a L_L^\beta) + C_S \ (\bar{Q}_L^i\gamma_\mu Q_L^j)(\bar{L}_L^\alpha\gamma^\mu L_L^\beta) \right]$

- · Large effect in 3rd gen.
- Smaller effects in light fermions.
- 3-2 mixing of O(CKM)

All this is automatic if the EFT enjoys an approximate

 $U(2)_{Q} \times U(2)_{L}$

flavor symmetry minimally broken

\

$$\lambda^{9} \sim \begin{pmatrix} \circ & \circ & \lambda_{bs} & V_{ub} \\ \circ & \lambda_{ss} & \lambda_{bs} \\ \lambda_{bs} & V_{ub} & \lambda_{bs} & \mathbf{1} \end{pmatrix} \qquad \lambda_{ss} \sim O(V_{ts}) \\ \lambda_{ss} & V_{ub} & \lambda_{bs} & \mathbf{1} \end{pmatrix} \qquad \lambda_{ss} \sim O(\lambda_{ss}^{2})$$

$$\begin{pmatrix} \circ & \circ & \circ \\ & & \\ &$$

Only 5 free parameters in the fit

 C_{T} , C_{S} , $\lambda^{q}_{bs} \sim O(V_{ts})$, $\lambda^{\ell}_{\mu\mu} \sim O(\lambda^{\ell}_{\tau\mu})^{2}$,

λℓ_{TH}

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Challenge: to fit R(D^(*))

The low-energy operator $(\bar{c}_L\gamma_\mu b_L)(\bar{\tau}_L\gamma^\mu\nu_\tau)$ receives two contributions:



If the **32** term (λ^{q}_{bs}) is negligible, to fit the anomaly one needs $C_T \sim 0.12$

Assuming a tree-level mediator (required for having such a large effect): $C_T \sim g_X^2 \frac{v^2}{M_X^2} \longrightarrow M_X \sim 700 \text{ GeV} \text{ (for } g_X \sim 1\text{)}$

Challenge: to fit R(D^(*))

 $C_{T} \sim 0.12$

High-pT



RGE effects and EWPT



$$\sim \frac{3y_t^2}{16\pi^2} \log \frac{M_X^2}{m_t^2} \frac{C_T}{v^2} (H^{\dagger} \sigma^a i \stackrel{\leftrightarrow}{D_{\mu}} H) (\bar{L}_L^3 \gamma^{\mu} \sigma^a L_L^3)$$

Problems in well measured (per-mille) $Z\tau\tau$ couplings at LEP-1 and LFU in τ decays.

Ferruglio, Paradisi, Pattori 2016-2017

Solution: 'large mixing'

Buttazzo, Greljo, Isidori, DM 2017

 $R_{D^{(*)}}^{\tau\ell} \approx 1 + 2C_T \left(1 - \lambda_{sb}^q \frac{V_{tb}^*}{V_{ts}^*} \right) = 1.237 \pm 0.053$

Allow the natural value $\lambda^{q}_{sb} \sim (few) \times IV_{ts}I$.





With $\lambda^{q}_{sb} \approx 3 |V_{ts}| C_T$ can be smaller by a factor of 4 $\rightarrow M_X$ larger by a factor of 2.

EWPT ($\propto C_{T,S}$) are crucial to select this region.

A posteriori, this also **solves the direct searches** problems: **allows heavier mediators**.

Other effects of large mixing

$$(C_T - C_S)\lambda_{bs}(\bar{b}_L\gamma_\mu s_L)(\bar{\nu}_\tau\gamma^\mu\nu_\tau)$$

This can generate too large corrections O(1) to $B \rightarrow K^* vv$

<u>Requires the singlet operator</u> with $C_T \sim C_S$





Also, depending on the UV model, there might be **problems with Bs mixing** (see later).

EFT Fit - Results

Buttazzo, Greljo, Isidori, DM 2017

 2σ

 3σ

-5

0

Observable	Experimental bound	Linearised expression
$R_{D^{(*)}}^{\tau\ell}$	1.237 ± 0.053	$1 + 2C_T (1 - \lambda_{sb}^q V_{tb}^* / V_{ts}^*) (1 - \lambda_{\mu\mu}^\ell / 2)$
$\Delta C_9^\mu = -\Delta C_{10}^\mu$	-0.61 ± 0.12 [36]	$-\frac{\pi}{\alpha_{\rm em}V_{tb}V_{ts}^*}\lambda_{\mu\mu}^\ell\lambda_{sb}^q(C_T+C_S)$
$R^{\mu e}_{b \to c} - 1$	0.00 ± 0.02	$2C_T(1-\lambda_{sb}^q V_{tb}^*/V_{ts}^*)\lambda_{\mu\mu}^\ell$
$B_{K^{(*)}\nu\bar\nu}$	0.0 ± 2.6	$1 + \frac{2}{3} \frac{\pi}{\alpha_{\rm em} V_{tb} V_{ts}^* C_{\nu}^{\rm SM}} (C_T - C_S) \lambda_{sb}^q (1 + \lambda_{\mu\mu}^\ell)$
$\delta g^Z_{ au_L}$	-0.0002 ± 0.0006	$0.033C_T - 0.043C_S$
$\delta g^Z_{ u_ au}$	-0.0040 ± 0.0021	$-0.033C_T - 0.043C_S$
$ g^W_ au/g^W_\ell $	1.00097 ± 0.00098	$1 - 0.084C_T$
$\mathcal{B}(au o 3\mu)$	$(0.0 \pm 0.6) \times 10^{-8}$	$2.5 \times 10^{-4} (C_S - C_T)^2 (\lambda_{\tau\mu}^{\ell})^2$



natural values of parameters

- small overall coefficient: higher NP scale
- no special alignment required
- R(K) can be easily fit by a suitable value of $\lambda_{\mu\mu} \sim 10^{-2}$
- $\lambda_{\tau\mu} \sim 0.1$ is OK for LFV bounds.





Simplified Models

Strong assumptions:

- both anomalies are due to the same mediator(s).
- LL operators give the leading contribution.

Tree-level mediators

Buttazzo, Greljo, Isidori, DM 2017



Colorless vectors

 $V_3 = W' = (\mathbf{1}, \mathbf{3}, 0),$ $V_1 = B' = (\mathbf{1}, \mathbf{1}, 0),$



The size of R(D) anomaly suggests a tree-level mediator.

 $C_T (\bar{Q}_L^i \gamma_\mu \sigma^a Q_L^j) (\bar{L}_L^\alpha \gamma^\mu \sigma^a L_L^\beta) + C_S (\bar{Q}_L^i \gamma_\mu Q_L^j) (\bar{L}_L^\alpha \gamma^\mu L_L^\beta)$

The Singlet and Triplet operators can be generated at the tree-level by:

Vector Leptoquarks

 $U_1 = (\mathbf{3}, \mathbf{1}, 2/3),$ $U_3 = (\mathbf{3}, \mathbf{3}, 2/3),$



Scalar Leptoquarks

 $S_1 = (\mathbf{\bar{3}}, \mathbf{1}, 1/3),$ $S_3 = (\mathbf{\bar{3}}, \mathbf{3}, 1/3),$



Tree-level mediators

Buttazzo, Greljo, Isidori, DM 2017

Each mediator generates a specific combination of the singlet and triplet operators:



Colorless vectors

 $W' = (\mathbf{1}, \mathbf{3}, 0),$ B' = (\mathbf{1}, \mathbf{1}, 0),

Vector Leptoquarks $U_1 = (\mathbf{3}, \mathbf{1}, 2/3),$ $U_3 = (\mathbf{3}, \mathbf{3}, 2/3),$

Scalar Leptoquarks $S_1 = (\mathbf{\bar{3}}, \mathbf{1}, 1/3),$ $S_3 = (\mathbf{\bar{3}}, \mathbf{3}, 1/3),$

Notable:

The **U₁ can fit perfectly** the anomalies as a single mediator.

Combinations of mediators are also OK





Scalar Leptoquarks $S_1 = (\bar{3}, 1, 1/3), S_3 = (\bar{3}, 3, 1/3)$

 $\mathcal{L} \supset g_1 \beta_{1 i \alpha} (\bar{Q}_L^{c i} \epsilon L_L^{\alpha}) S_1 + g_3 \beta_{3 i \alpha} (\bar{Q}_L^{c i} \epsilon \sigma^a L_L^{\alpha}) S_3^a + \text{h.c.}$

 $|\epsilon_{1,3}|^2 = C_{1,3} = v^2 |g_{1,3}|^2 / (4M_{S_{1,3}}^2) > 0$ $C_S = -C_1 - 3C_3,$ $C_T = C_1 - C_3$



- → The flavor structure of the two LQ has to be misaligned: $\beta_{1,s\tau} \approx -\beta_{3,s\tau} \approx (\text{few}) \times |V_{ts}|$
- \rightarrow Some residual tension at the ~1.5 σ level between ZTT and R(D)
- \rightarrow B_s-mixing is calculable and in tension with R(D):

 $\frac{(\Delta M_{B_s})^{S_1+S_3}}{(\Delta M_{B_s})^{\rm SM}} \approx 0.74 \left(\frac{m_{S_{1,3}}}{1 \text{ TeV}}\right)^2 \left(\frac{R_{D^{(*)}}/R_{D^{(*)}}^{\rm SM}-1}{0.23}\right)^2 \lesssim 10\%$

Requires a tuning with extra contributions at the ~10% level.

Direct Searches

[D.M. 1803.10972]

$$S_1 = (\bar{\mathbf{3}}, \mathbf{1}, 1/3), \quad S_3 = (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$$

 $S_{1,-\frac{1}{3}} = (s_{3,-\frac{4}{3}}, s_{3,-\frac{1}{3}}, s_{3,-\frac{2}{3}}) \sim 3 \text{ of SU}(3)_c$

For LHC only the interactions with the third generation are relevant:

$$\mathcal{L}_{LQ} = g_1 s_{1,-\frac{1}{3}}^{\dagger} \left(\bar{t}_L^c \tau_L - \bar{b}_L^c \nu_\tau \right) + g_3 s_{3,-\frac{1}{3}}^{\dagger} \left(-\bar{t}_L^c \tau_L - \bar{b}_L^c \nu_\tau \right) + h.c.$$

+ $\sqrt{2}g_3 \left(s_{3,\frac{2}{3}}^{\dagger} \bar{t}_L^c \nu_\tau - s_{3,-\frac{4}{3}}^{\dagger} \bar{b}_L^c \tau_L \right) + h.c. ,$

QCD pair production

single production

off-shell



 σ depends only on m_S









For high masses: $\sigma \propto (|g_{1,3}|^2 / m_S^2)^2 \propto (C_{1,3})^2$

 $\sigma \propto |g_{1,3}|^2$ More sensitive at high masses

Direct Searches

[D.M. 1803.10972]



Tree-level mediators for B anomalies

Colorless Vectors

Scalar Leptoquarks

Vector Leptoquark



Killed either by B_s mixing or direct searches.



UN completions

$$\begin{split} \mathcal{L} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &+ i \bar{\Psi} D \!\!\!/ \psi \\ &+ D_{\mu} \Phi^{\dagger} D^{\mu} \Phi - V(\Phi) \\ &+ \bar{\Psi}_L \hat{Y} \Phi \Psi_R + h.c. \end{split}$$



bottom

V

τ-neutrino



• Elementary LQ gauge boson [See Marzia's talk]

Di Luzio, Greljo, Nardecchia 2017; Calibbi, Crivellin, Li 2017; Bordone, Cornella, Fuentes-Martin, Isidori 2017

• Elementary W', Z' gauge bosons

Cline, Camalich 2017, Megias, Quiros, Salas, Panico [in 5D] 2017

• Composite W', Z' resonances Buttazzo, Greljo, Isidori, D.M. 2016

Vector LQ U₁ = (**3**, **1**, 2/3)

The U₁ LQ gives the best fit of the anomalies with smallest number of param.

The massive vector of Pati-Salam has same quantum numbers as U₁ LQ.

2 classes of UV completions

Gauge boson

Di Luzio, Greljo, Nardecchia 2017; Calibbi, Crivellin, Li 2017; Bordone, Cornella, Fuentes-Martin, Isidori 2017 Barbieri, Isidori, Pattori, Senia 2015; Barbieri, Murphy, Senia 2016; Buttazzo, Greljo, Isidori, D.M. 2017; Barbieri, Tesi 2017

Composite vector

Both also predict massive color-octed and massive Z', which couple to SM fermions.

It is crucial to avoid large tree-level FCNC effects and direct searches limits on these.

- All these constraint point to strong gauge couplings, at the limit of perturbativity.
- Doesn't address the EW hiearchy problem.

The theory is fully strongly coupled at that scale: no calculability

The issue is:

 $m_{VLO} \sim \Lambda$

No parametric splitting between LQ and other states

Composite Scalar LQ

One would like a mass splitting between the B anomalies mediator and other states: Z', heavy gluons, etc..

Scalar LQ as pseudo-Goldstone boson



Like between pions and ρ mesons in QCD.

 $m_{SLQ} \ll \Lambda$

Requirements:



Μ

f

Higgs

 $\Lambda \sim g_{\rho} f \sim 10 \text{ TeV}$

other resonances

Gap

m_{pNGB} ~ 1.5 TeV

Flavor-mediators

Fundamental description of the strong-sector QCD-like



Higgs boson and the two scalar LQ as Goldstones

Custod

Custodial symmetry

Fermionic Composite Higgs

Buttazzo, Greljo, Isidori, D.M. 2017; D.M. 1803.10972

Gauge group:

 $SU(N_{HC}) \times SU(3)_c \times SU(2)_w \times U(1)_Y$ "HyperColor"

		$SU(N_{HC})$	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_w$	$\mathrm{U}(1)_Y$
Extra HC Dirac fermions:	Ψ_L	N_{HC}	1	2	Y_L
	Ψ_N	$\mathbf{N}_{\mathbf{HC}}$	1	1	$Y_L + 1/2$
	Ψ_E	$\mathbf{N}_{\mathbf{HC}}$	1	1	$Y_L - 1/2$
	Ψ_Q	$\mathbf{N}_{\mathbf{HC}}$	3	2	$Y_L - 1/3$

 $SU(N_{HC})$ confines at $\Lambda_{HC} \sim 10~TeV$

In absence of SM gauging, the strong sector has a global symmetry



$$G = SU(10)_{L} \times SU(10)_{R} \times U(1)_{V}$$
$$\langle \bar{\Psi}_{i} \Psi_{j} \rangle = -B_{0} f^{2} \delta_{ij} \int f \sim 1 \text{TeV}$$
$$H = SU(10)_{V} \times U(1)_{V}$$

Goldstone Bosons

D.M. 1803.10972

 $\mathbf{G} = \mathbf{SU}(10)_{\mathrm{L}} \times \mathbf{SU}(10)_{\mathrm{R}} \times \mathbf{U}(1)_{\mathrm{V}} \longrightarrow \mathbf{H} = \mathbf{SU}(10)_{\mathrm{V}} \times \mathbf{U}(1)_{\mathrm{V}}$ JJ JJ Like QCD pions, the pNGB are composite states of HC-fermion bilinears: In terms of SM representations Two Higgs doublets: $H_{1,2} \sim (1,2)_{1/2}$ Singlet and Triplet LQ: $S_1 \sim (3,1)_{-1/3} + S_1 \sim (3,3)_{-1/3}$ Three singlets: $\eta_{1,2,3} \sim (1,1)_0$ Other electroweak states: $\omega \sim (1,1)_1 + \Pi_{L,Q} \sim (1,3)_0$ $R_2 \sim (\mathbf{3,2})_{1/6} + T_2 \sim (\mathbf{3,2})_{-5/6}$ Other coloured states: $\tilde{\pi}_1 \sim (8,1)_0 + \tilde{\pi}_3 \sim (8,3)_0$

> For energies $E \ll \Lambda_{HC}$ the theory is described by a weakly coupled effective chiral Lagrangian. Structure driven by the symmetries and spurions.

Yukawas & LQ couplings

Coupling with SM fermions from 4-Fermi operators

$$\mathcal{L}_{4-\text{Fermi}} \sim \frac{c_{\psi\Psi}}{\Lambda_t^2} \bar{\psi}_{\text{SM}} \psi_{\text{SM}} \bar{\Psi} \Psi \xrightarrow{E \lesssim \Lambda_{HC}} \sim y_{\psi\phi} \, \bar{\psi}_{\text{SM}} \psi_{\text{SM}} \phi + \dots$$

 $\Lambda_t \gtrsim \Lambda_{HC}$

SM Yukawas + LQ couplings

A new sector responsible for these operators is necessary (as Extended Technicolor)

An approximate SU(2)⁵ flavor symmetry protects from unwanted flavor violation $G_F = SU(2)_q \times SU(2)_u \times SU(2)_d \times SU(2)_l \times SU(2)_e$ minimally broken by these spurions: $\Delta Y_u = (\mathbf{2}, \mathbf{\bar{2}}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \quad \Delta Y_d = (\mathbf{2}, \mathbf{1}, \mathbf{\bar{2}}, \mathbf{1}, \mathbf{1}), \quad \Delta Y_e = (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{\bar{2}})$ $V_q = (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \quad V_l = (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1})$ Good structure to fit the flavour anomalies!

Imposing conservation of B & L such that the proton is stable, automatically allows only the Higgses Yukawas and LQ couplings.

Higgs Yukawas

UV effective Lagrangian:

$$\mathcal{L}_F \supset \frac{1}{\Lambda_t^2} \left(\bar{u}_R c_{1,u}^{\dagger} q_L + \bar{q}_L c_{1,d} d_R \epsilon + \bar{l}_L c_{1,e} e_R \epsilon \right) \left(\bar{\Psi}_L \gamma_5 \Psi_N \right) + \frac{1}{\Lambda_t^2} \left(\bar{u}_R c_{2,u}^{\dagger} q_L \epsilon + \bar{q}_L c_{2,d} d_R + \bar{l}_L c_{2,e} e_R \right) \left(\bar{\Psi}_E \gamma_5 \Psi_L \right) + h.c.$$

At low energy:

$$\begin{aligned} \mathcal{L}_{\text{Yuk}}^{\text{eff}} = & \frac{f}{2} \left(\bar{u}_R \tilde{y}_{1,u}^{\dagger} q_L^{\beta} \epsilon^{\beta \alpha} + \bar{q}_L^{\alpha} \tilde{y}_{1,d} d_R + \bar{l}_L^{\alpha} \tilde{y}_{1,e} e_R \right) \operatorname{Tr}[\Delta_{H_1}^{\alpha} (U - U^{\dagger})] + \\ & + \frac{f}{2} \left(\bar{u}_R \tilde{y}_{2,u}^{\dagger} q_L^{\beta} \epsilon^{\beta \alpha} + \bar{q}_L^{\alpha} \tilde{y}_{2,d} d_R + \bar{l}_L^{\alpha} \tilde{y}_{2,e} e_R \right) \operatorname{Tr}[\Delta_{H_2}^{\alpha} (U - U^{\dagger})] + h.c. \end{aligned}$$

The spurion gives the Higgses as leading terms: $\operatorname{Tr}[\Delta_{H_{1,2}}^{\alpha}(U-U^{\dagger})] = i \frac{2\sqrt{2}}{f} H_{1,2}^{\alpha} + \mathcal{O}(\phi^2/f^2)$

Fermion masses:
$$m_f = f \sin \theta (\tilde{y}_{1,f} - \tilde{y}_{2,f}) = \frac{v}{\sqrt{2}} (\tilde{y}_{1,f} - \tilde{y}_{2,f}) \equiv \frac{v}{\sqrt{2}} y_f$$

The Yukawa matrices of the two Higgses need to be identical to avoid flavour-violating couplings and custodial symmetry-breaking effects

LQ couplings

UV effective Lagrangian:

$$\mathcal{L}_F \supset \frac{1}{\Lambda_t^2} \left[\left(\bar{q}_L^c c_{1,ql} \epsilon l_L + \bar{e}_R^c c_{1,eu} u_R \right) \left(\bar{\Psi}_Q \gamma_5 \Psi_L \right) + \left(\bar{q}_L^c c_{3,ql} \epsilon \sigma^A l_L \right) \left(\bar{\Psi}_Q \gamma_5 \sigma^A \Psi_L \right) \right] + h.c.$$
$$\bar{\Psi}_{i,L} \Psi_{j,R} \rightarrow -B_0 f^2 U(\phi)_{ji} , \qquad \bar{\Psi}_{i,R} \Psi_{j,L} \rightarrow -B_0 f^2 U^{\dagger}(\phi)_{ji}$$

At low energy it becomes:

$$\mathcal{L}_{LQ}^{\text{eff}} = i \frac{f}{4} \left(g_1 \bar{q}_L^{c,a} \beta_1 \epsilon l_L + g_1^u \bar{e}_R^c \beta_1^u u_R^a \right) \operatorname{Tr} [\Delta_{S_1}^a (U - U^{\dagger})] + h.c. \\ + i \frac{f}{4} \left(g_3 \bar{q}_L^{c,a} \beta_3 \epsilon \sigma^A l_L \right) \operatorname{Tr} [\Delta_{S_3}^{A,a} (U - U^{\dagger})] + h.c. = \\ = -g_1 \beta_{1,i\alpha} (\bar{q}_L^{c\,i} \epsilon l_L^\alpha) S_1 - g_1^u (\beta_1^u)_{\alpha i}^T (\bar{e}_R^{c\,\alpha} u_R^i) S_1 - g_3 \beta_{3,i\alpha} (\bar{q}_L^{c\,i} \epsilon \sigma^A l_L^\alpha) S_3^A + h.c. + \mathcal{O}(\phi^2)$$
Flavour structure:
$$\beta_{1,3} \sim \left(\begin{array}{c} V_q^* V_l^{\dagger} & V_q^* \\ V_l^{\dagger} & 1 \end{array} \right) \qquad \beta_1^u \sim \left(\begin{array}{c} 0 & (V_q^{\dagger} \Delta Y_u)^T \\ V_l^{\dagger} \Delta Y_e & 1 \end{array} \right)$$

enurione

The coupling of S₁ to RH fermions induces an m_t -enhanced contribution to $\tau \rightarrow \mu \gamma$.

Requires $g_1^u \lesssim 10^{-2}g_1$

Introducing an extra approximate $U(1)_e$ symmetry for the RH leptons to protect the τ Yukawa would give:

 $g_1^u/g_1 \sim y_\tau/y_t \sim 10^{-2}$

Scalar Potential

The pNGB potential arises at 1-loop from all the explicit breaking terms



The gauge contribution is positive and is larger for colored states. EW charges give subleading corrections.

$$\begin{split} \Delta m_{\omega}^2 &\approx (0.05\Lambda_{HC})^2 , \quad \Delta m_{H_{1,2}}^2 \approx (0.08\Lambda_{HC})^2 , \quad \Delta m_{\Pi_{L,Q}}^2 \approx (0.13\Lambda_{HC})^2 , \quad \sim \mathbf{1} \text{ of } \mathrm{SU}(3)_{\mathrm{c}} \\ \Delta m_{S_1}^2 &\approx (0.17\Lambda_{HC})^2 , \quad \Delta m_{S_3}^2 \approx (0.21\Lambda_{HC})^2 . \quad \Delta m_{\tilde{R}_2,T_2}^2 \approx (0.19\Lambda_{HC})^2 . \quad \sim \mathbf{3} \text{ of } \mathrm{SU}(3)_{\mathrm{c}} \\ \Delta m_{\tilde{\pi}_1}^2 &\approx (0.26\Lambda_{HC})^2 , \quad \Delta m_{\tilde{\pi}_3}^2 \approx (0.28\Lambda_{HC})^2 , \quad \sim \mathbf{8} \text{ of } \mathrm{SU}(3)_{\mathrm{c}} \end{split}$$

 $\Lambda_{HC} \gtrsim 10 \text{ TeV}$

Scalar Potential

The pNGB potential arises at 1-loop from all the explicit breaking terms



Tuning to get EWSB as in usual Composite Higgs models:

$$m_{H_{1,2}}^2 \approx 2B_0(m_L + m_E) + \Delta m_{\text{gauge}}^2 + \Delta m_{\text{Yuk}}^2 < 0 \qquad \xi \equiv \frac{v^2}{f^2} = 2\sin^2\frac{v_h}{\sqrt{2}f} \leq 10\%$$

From the structure of the potential and the expressions for the various terms I get

$$m_h^2 = (C_t - C_g)f^2\xi \sim N_c c_t m_t^2 - 3c_w m_W^2$$

The deviations in Higgs couplings and the EWPT are similar to most Composite Higgs models.

Spectrum

valence	irrep.	valence	irrep.
$H_1 \sim i\sigma^2(\bar{\Psi}_L \Psi_N)$	$({f 1},{f 2})_{1/2}$	$H_2 \sim (\bar{\Psi}_E \Psi_L)$	$({f 1},{f 2})_{1/2}$
$S_1 \sim (\bar{\Psi}_Q \Psi_L)$	$(ar{3}, m{1})_{1/3}$	$S_3 \sim (\bar{\Psi}_Q \sigma^a \Psi_L)$	$(ar{3}, m{3})_{1/3}$
$\omega^{\pm} \sim (\bar{\Psi}_N \Psi_E)$	$({f 1},{f 1})_{-1}$	$\Pi_L \sim (\bar{\Psi}_L \sigma^a \Psi_L)$	$(1,3)_0$
$\tilde{R}_2 \sim (\bar{\Psi}_E \Psi_Q)$	$({f 3},{f 2})_{1/6}$	$T_2 \sim (\bar{\Psi}_Q \Psi_N)$	$(ar{3},f{2})_{5/6}$
$\tilde{\pi}_1 \sim (\bar{\Psi}_Q T^A \Psi_Q)$	$({f 8},{f 1})_0$	$\tilde{\pi}_3 \sim (\bar{\Psi}_Q T^A \sigma^a \Psi_Q)$	$({f 8},{f 3})_0$
$\Pi_Q \sim (\bar{\Psi}_Q \sigma^a \Psi_Q)$	$(1,3)_0$	$\eta_i \sim 3 \times c_i^a (\bar{\Psi}_a \Psi_a)$	$(1,1)_0$

Using the structure of the potential from the explicit breaking terms and the NDA estimates I get



The lightest pNGBs are the singlets. Some pNGB have anomalous couplings to gauge bosons:

$$\mathcal{L}_{\rm WZW} \supset -\frac{g_{\beta}g_{\gamma}}{16\pi^2} \frac{\phi^{\alpha}}{f} 2N_{HC} A_{\beta\gamma}^{\phi^{\alpha}} F_{\mu\nu}^{\beta} \widetilde{F}^{\gamma\mu\nu}$$

$$\frac{A_{\beta\gamma}^{\phi^{\alpha}}}{\eta_1} \frac{g_1^2}{Y_L} \frac{g_2^2}{g_2^2} \frac{g_3^2}{g_3^2} \frac{g_1g_2}{g_1g_3} \frac{g_2g_3}{g_2g_3}$$

$$\frac{\eta_1}{\eta_2} \frac{Y_L}{1-\frac{1}{4\sqrt{2}}} \frac{1}{4\sqrt{2}} 0 0 0 0 0 0$$

$$\frac{\eta_3}{\eta_3} \frac{1+48Y_L}{12\sqrt{30}} -\frac{\sqrt{3}}{4\sqrt{10}} \frac{-\frac{1}{\sqrt{30}}}{g_4\sqrt{10}} 0 0 0 0 0$$

$$\frac{\eta_3}{\tilde{\pi}_1} \frac{0}{0} 0 \frac{d^{\alpha\beta\gamma}/(2\sqrt{2})}{d^{\alpha\beta\gamma}/(2\sqrt{2})} \frac{1}{\sqrt{2}} (Y_L - \frac{1}{3})} 0$$

$$\frac{\eta_L}{\eta_Q} 0 0 0 \frac{\sqrt{3}}{2} (Y_L - \frac{1}{3}) 0 0$$

$$Can be produced in gg-fusion!$$

Singlet η_3



Other pNGBs



Other pNGBs



The other pNGBs can be pair-produced but do not decay directly to SM particles. They can decay via



None of them is expected to be observable at the LHC (too heavy or only EW couplings).

The other resonances have masses at the $\Lambda \sim 4\pi f > 10_{S_l}$ TeV scale $S_l = \frac{S_l}{\Pi_L - T_2 - \tilde{\pi}_l} = \frac{\rho_{\rm r...}}{\Lambda_{\rm HC} - \Lambda_{\rm t}}$

 \boldsymbol{L}

Composite pNGB Scalar LQ



UV models often point to interesting collider signatures not directly related to the anomalies: scalar singlets in this case. Heavy Z', G' in vector LQ models.

Summary



 $\frac{1}{v^2}\lambda^q_{ij}\lambda^\ell_{\alpha\beta}\left[C_T \ (\bar{Q}^i_L\gamma_\mu\sigma^a Q^j_L)(\bar{L}^\alpha_L\gamma^\mu\sigma^a L^\beta_L) + C_S \ (\bar{Q}^i_L\gamma_\mu Q^j_L)(\bar{L}^\alpha_L\gamma^\mu L^\beta_L)\right]$



First attempts on UV models for both vector and scalar LQ.

Scalar LQ can be naturally lighter than other mediators of flavour effects if they arise as pseudo-NGB of a strongly coupled sector. Allows to address also the Higgs hierarchy problem.

Thank you!

Backup

U(2) flavour symmetry

Keeping only the third-generation Yukawa couplings, the SM enjoys an approximate SU(2)⁵ flavor symmetry

 One can assume this is
 $\Delta Y_u = (2, \overline{2}, 1, 1, 1)$, $\Delta Y_d = (2, 1, \overline{2}, 1, 1)$, $\Delta Y_e = (1, 1, 1, 2, \overline{2})$

 minimally broken by the spurions:
 $V_q = (2, 1, 1, 1, 1)$, $V_l = (1, 1, 1, 2, 1)$

 $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \text{The Yukawa matrices} \\ \text{get this structure:} \quad Y_{u,d} \approx y_u \begin{pmatrix} \Delta \\ y_t \\ 0 \end{pmatrix} \begin{pmatrix} \nabla \\ y_t \\ 10 \end{pmatrix} \quad \frac{1}{1} \end{pmatrix} \begin{pmatrix} \Delta \\ v_q \\ 1 \end{pmatrix} \begin{pmatrix} 2, 2 \\ y_d \\ \sim y_b \\ 0 \end{pmatrix} , \quad y_e \sim y_\tau \begin{pmatrix} \Delta Y_e & V_l \\ 0 & 1 \end{pmatrix}$

The doublet spurions regulate the mixing of the third generation with the lighter ones:

$$V_q = a_q \begin{pmatrix} V_{td}^* \\ V_{ts}^* \end{pmatrix} \qquad V_l \approx \begin{pmatrix} 0 \\ \lambda_{\tau\mu} \end{pmatrix}$$

CKM unknowns

Problems from RG effects

Feruglio, Paradisi, Pattori [1606.00524]

Problems with LFU in \tau decays at 1-loop



$$R_{\tau}^{\tau/\ell_{1,2}} = \frac{\mathcal{B}(\tau \to \ell_{2,1}\nu\bar{\nu})_{\exp}/\mathcal{B}(\tau \to \ell_{2,1}\nu\bar{\nu})_{\mathrm{SM}}}{\mathcal{B}(\mu \to e\nu\bar{\nu})_{\exp}/\mathcal{B}(\mu \to e\nu\bar{\nu})_{\mathrm{SM}}}$$

$$R_{\tau}^{\tau/\mu} = 1.0022 \pm 0.0030$$

$$R_{\tau}^{\tau/e} = 1.0060 \pm 0.0030$$

$$R_{\tau}^{\tau/e} = 1.0060 \pm 0.0030$$

$$R_{\tau}^{\tau/e} = 0,017 \pm 0,023$$

$$R_{\tau}^{\tau/e} = 0,13 \pm 0,03$$

Need a ~ 20% tuning with some other contribution (from 1-loop or direct from another dim-6 operator)



τ→μγ & (g-2)_μ

The S1 LQ in general couples to both LH and RH fermions:

$$\mathcal{L}_{S_1} \supset \overline{t}^c \left[g_1 \beta_{1,b\alpha} P_L + g_1^u \beta_{1,t\alpha}^u P_R \right] \ell^\alpha S_1 + h.c.$$

This induces an mt-enhanced contribution to $\tau \rightarrow \mu \gamma$ and $(g-2)_{\mu}$

Requires $g_1^u \lesssim 10^{-2}g_1$

Introducing an extra approximate $U(1)_e$ symmetry for the RH leptons to protect the τ Yukawa would give:

$$g_1^u/g_1 \sim y_\tau/y_t \sim 10^{-2}$$

 $\delta a_{\mu} \approx (7.9 \times 10^{-11}) \times \frac{\epsilon_{1}^{u}}{10^{-3}} \frac{\epsilon_{1}}{0.1} \frac{\beta_{1,b\mu}}{0.1} \frac{\beta_{1,b\mu}^{u}}{0.1} \xrightarrow{\beta_{1,b\mu}^{u}}{0.1} \quad \text{too small to fit the anomaly} \quad (\delta a_{\mu})_{exp} = (2.8 \pm 0.9) \times 10^{-9}$



Limits and prospects from pair production of 3rd gen. LQ

Gauge Vector LQ

Di Luzio, Greljo, Nardecchia 2017; Calibbi, Crivellin, Li 2017; Bordone, Cornella, Fuentes-Martin, Isidori 2017

The U₁ LQ gives the best fit of the anomalies with smallest number of param.

The massive vector of Pati-Salam has same quantum numbers as $U_1 LQ$.

 $SU(4)_{PS} \times U(1)' \rightarrow SU(3)_c \times U(1)_Y \qquad U_1 = (3, 1, 2/3)$

But in PS $M_{\cup} \ge 10^3$ TeV due to coupling with light generations.

Need to add extra (vectorlike) fermions or extra gauge bosons (or both) so that it mainly couples only to third generation.

 $SU(4) \times SU(3)' \times SU(2)_L \times U(1)' \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ Di Luzio, Greljo, Nardecchia 2017

There are also massive color-octed and massive Z'. Crucial to avoid large FCNC effects and direct searches limits. All these constraint point to strong gauge couplings, at the limit of perturbativity.

Doesn't address the EW hiearchy problem.

Field	d SU(4)	SU(3)'	$SU(2)_L$	U(1)'
$q_L^{\prime i}$	1	3	2	1/6
$u_R^{\prime i}$	1	3	1	2/3
$d_R^{\prime i}$	1	3	1	-1/3
$\ell_L^{\prime i}$	1	1	2	-1/2
$e_R^{\prime i}$	1	1	1	-1
$ \Psi_L^i$	4	1	2	0
$ \Psi_R^i$	4	1	2	0

Composite Models

Vector LQ as Composite Resonance

Λ ~ g_ρ f ~ 10 TeV other resonances Flavor-mediators

Μ

The vector LQ can be considered as a composite resonance of a new strongly coupled sector, of which the Higgs arises as a pNGB.

Barbieri, Isidori, Pattori, Senia 2015; Barbieri, Murphy, Senia 2016; Buttazzo, Greljo, Isidori, D.M. 2017; Barbieri, Tesi 2017

 $m_{VLQ} \sim \Lambda$

The theory is fully strongly coupled at that scale: no calculability

Higgs (pNGB)

 Many other vector resonances (Z', G') are expected with similar mass and with same flavour-violating couplings: expect very strong bounds from tree-level contribution to Bs mixing.

B and L conservation

I assign a combination of B and L, F+ = 3B + L, to the HC fermions such that the Higgs Yukawas and LQ couplings are allowed:

 $(\bar{q}_L u_R + \bar{d}_R q_L + \bar{e}_R l_L)(\bar{\Psi}_N \Psi_L) , \qquad (\bar{q}_L u_R + \bar{d}_R q_L + \bar{e}_R l_L)(\bar{\Psi}_L \Psi_E)$ $(\bar{q}_L^c l_L + \bar{e}_R^c u_R)(\bar{\Psi}_Q \Psi_L) , \qquad (\bar{q}_L^c \sigma^a l_L)(\bar{\Psi}_Q \sigma^a \Psi_L) ,$ $F_+(\Psi_L) = F_+(\Psi_N) = F_+(\Psi_E) = F_L , \qquad F_+(\Psi_Q) = F_L + 2$

These operators are then automatically forbidden

 $(\bar{q}_L^c q_L + \bar{u}_R^c d_R)(\bar{\Psi}_L \Psi_Q) , \qquad (\bar{d}_R l_L)(\bar{\Psi}_E \Psi_Q) , \qquad (\bar{l}_L^c l_L)(\bar{\Psi}_E \Psi_N)$

EWSB and Higgs mass

Better to change basis in the two Higgs doublets:

so that only one Higgs takes a vev

$$\tilde{H}_1 = \left(G^+, \frac{v_h + h + iG^0}{\sqrt{2}}\right)^T ,$$

'eaten NGB' and light Higgs couples linearly to fermions and SM gauge bosons

$$H_1 = \frac{i\tilde{H}_1 + \tilde{H}_2}{\sqrt{2}}$$
, $H_2 = \frac{-i\tilde{H}_1 + \tilde{H}_2}{\sqrt{2}}$

$$\tilde{H}_2 = \left(H^+, \frac{h_2 + iA_0}{\sqrt{2}}\right)^T$$

Heavy Higgs no linear couplings to SM

Effective potential for the light Higgs vev:

$$V(\theta) = -C_m f^4 \cos \theta - C_g f^4 \cos 2\theta - 2C_t f^4 \sin^2 \theta \qquad \theta = v_h / \sqrt{2}f$$

$$C_m = \frac{2B_0}{f^2} (m_E + m_L) , \quad C_g = \frac{3\Lambda_{HC}^2}{16\pi^2 f^2} \left(\frac{3}{4}c_w g_w^2 + \frac{1}{4}c_Y g_Y^2\right) , \quad C_t = \frac{N_c y_t^2 c_t \Lambda_{HC}^2}{16\pi^2 f^2}$$

$$\frac{v^2}{f^2} \equiv \xi = 2\sin^2\theta_{\min} = 2 - \frac{C_m^2}{8(C_t - C_g)^2} \qquad m_h^2 = (C_t - C_g)f^2\xi \sim N_c c_t m_t^2 - 3c_w m_W^2$$