Importance of electron diffusion in bulk-matter tests of the Pauli Exclusion Principle

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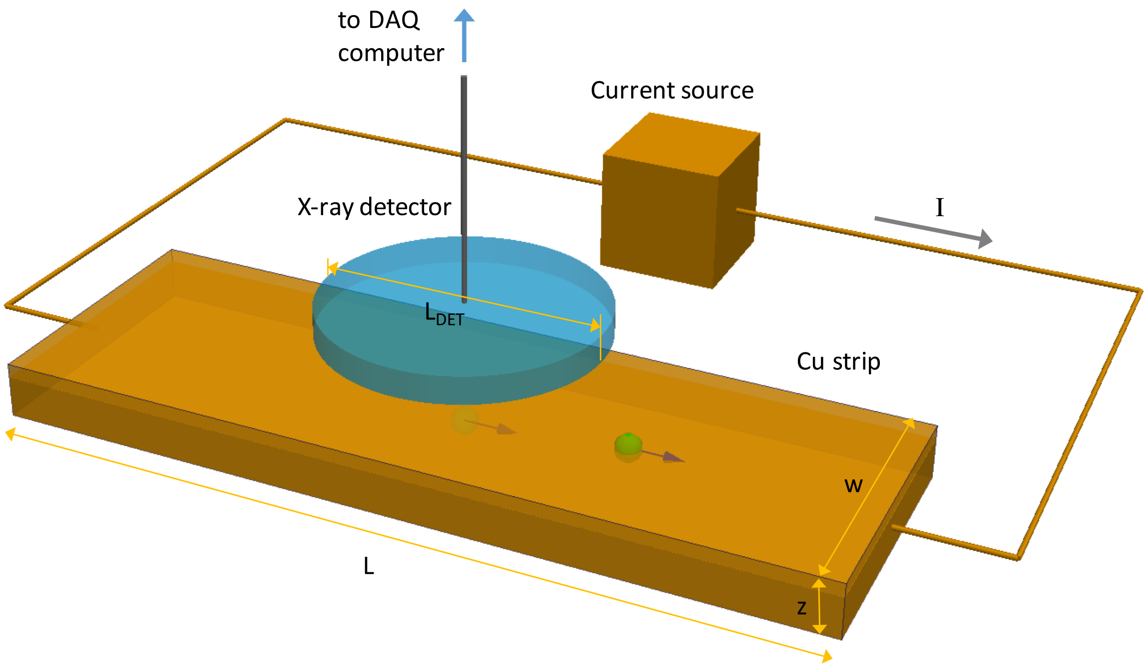
The VIP experiment (and its upgraded version, VIP-2) uses the Ramberg and Snow (RS) method to search for violations of the Pauli Exclusion Principle in the Gran Sasso underground laboratory.

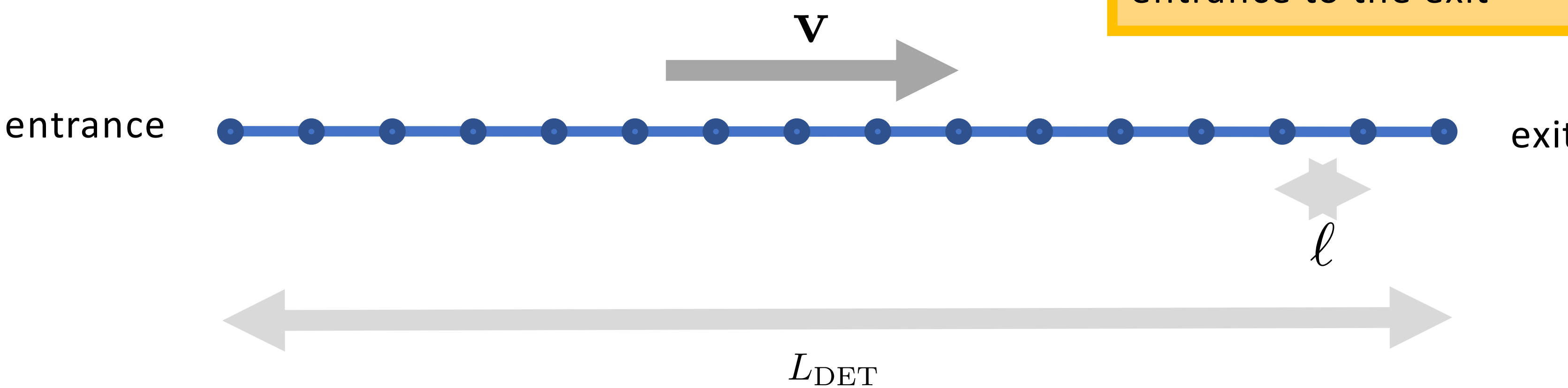
The RS method consists in feeding a copper conductor with a high DC current, so that the large number of newly-injected conduction electrons can interact with the copper atoms and possibly cascade electromagnetically to an already occupied atomic ground state if their wavefunction has the wrong symmetry with respect to the atomic electrons, emitting characteristic X-rays as they do so.

In order to evaluate the probability of finding such non-Paulian configurations, we need a model for the capture of the "wrong symmetry" electrons.

Ramberg and Snow provided such a model in their original experimental proposal in Phys. Lett. 238 (1990) 438.



The RS model of electron transfer under the detector: a linear series of steps from the entrance to the exit



number of “fresh” electrons
injected into copper strip

capture probability
in the 2P state

probability of non-
Paulian transition

number of electron-
atom scatterings

$$N_X \geq \frac{1}{2}\beta^2 N_{\mathrm{new}} \times \frac{1}{10} N_{\mathrm{int}} \times (\text{geometric factor})$$

detector length

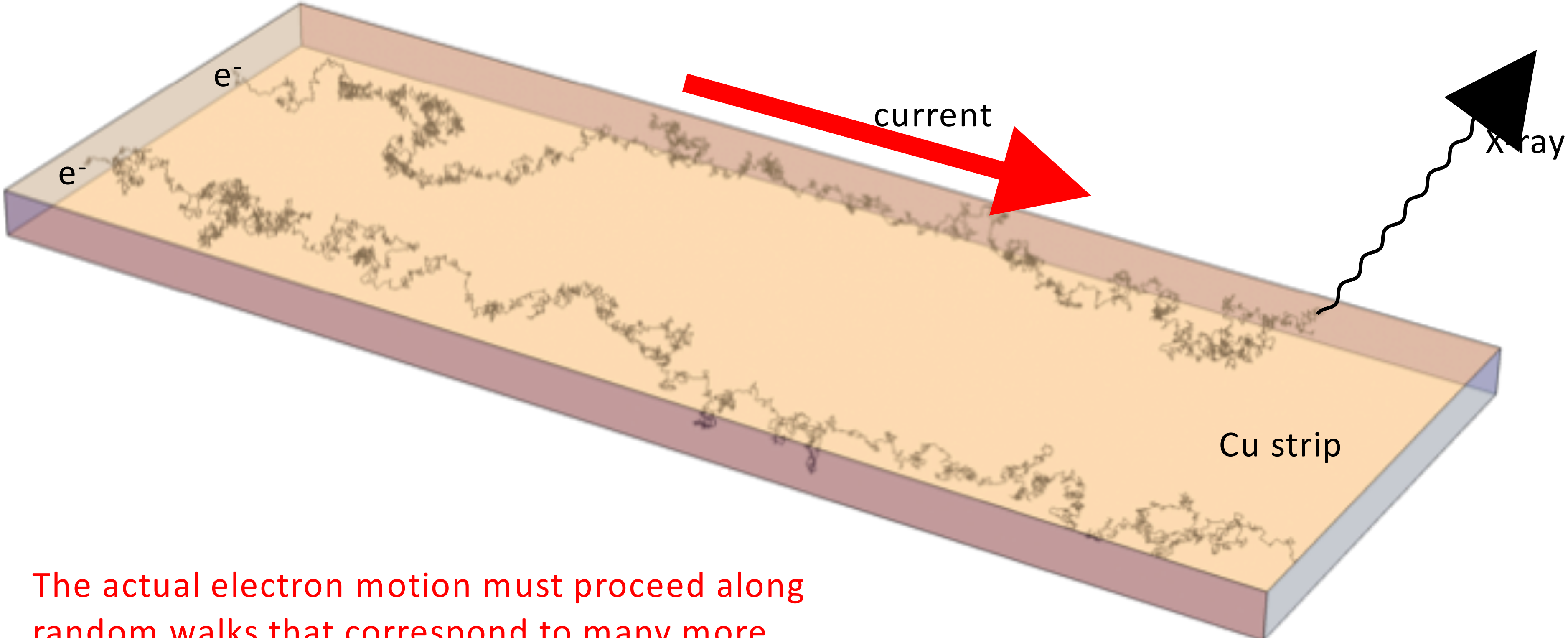
$$= \frac{\beta^2 \left(\sum I \Delta t\right) L_{\mathrm{DET}}}{e \ell \rho z \sigma} \times \frac{1}{8}\pi \times \frac{1}{20}$$

X-ray cross-section

mean free path

density

strip thickness



The actual electron motion must proceed along random walks that correspond to many more scatterings than then naive RS estimate. Can we incorporate this in a new estimate of the violation parameter?

If one tackles this problem in a naive way, a seemingly absurd situation soon arises.

The average transit time under the detector is $\Delta t_T = L/v_d$

The drift speed is $v_d = \frac{1}{ne}\frac{I}{A}$

Each electron contributes on average $\Delta t_T/\tau$ scatterings

There are $N_{\rm scatt} = \Delta t_T I/e\tau$ scatterings per unit time

Therefore

$$N_{\rm scatt} = \frac{\Delta t_T I}{e\tau} = \frac{L}{e\tau v_d} I = \frac{nLA}{\tau}$$

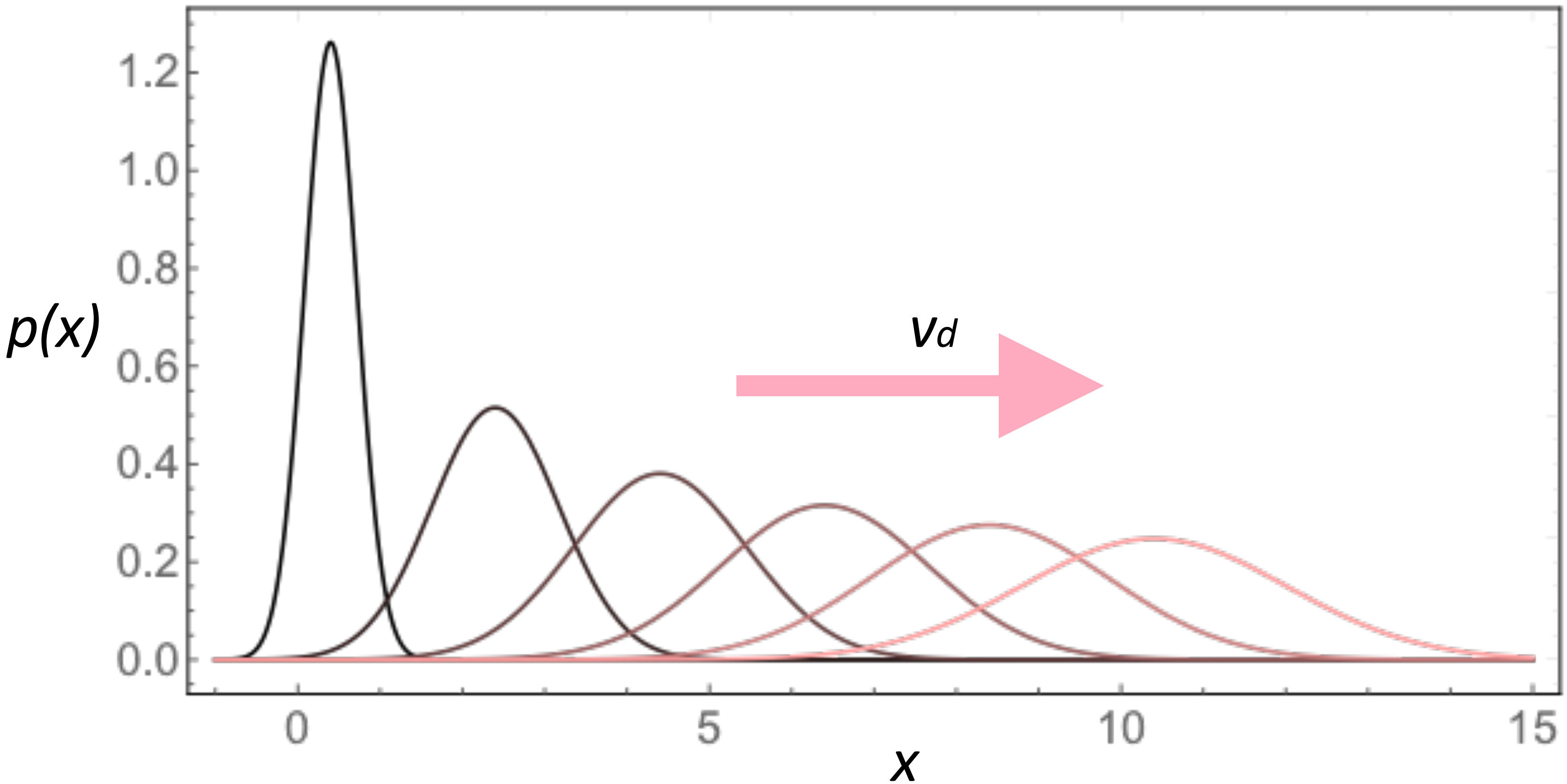
result does not depend on current !!!

To find a solution, let's turn to the diffusion equation with drift (advection-diffusion equation)

$$\frac{\partial p}{\partial t} = D\nabla^2 p - \mathbf{v}_d \cdot \nabla p$$

It is not difficult to show that in the 1D case its Green's function is

$$p(x|t) = \frac{1}{\sqrt{2\pi Dt}} \exp\left\{-\frac{[x - v_d t]^2}{2Dt}\right\},$$


1.2
1.0
0.8
p(x)
0.6
0.4
0.2
0.0
v_d
0
5
10
15
x

The probability of finding one electron in the target at time *t* is given by the integral

$$P(t) = \int_0^L p(x|t)dx$$
$$= \int_0^L \frac{1}{\sqrt{2\pi Dt}} \exp\left(-\frac{(x - v_d t)^2}{2Dt}\right) dx$$

where the target spans the *x*-interval (0,*L*).

Using the standard definition of the error function

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt,$$

we find

$$P(t) = \frac{1}{2}\left[\mathrm{erf}\left((L - v_d t)/\sqrt{2Dt}\right) - \mathrm{erf}\left(-v_d t/\sqrt{2Dt}\right)\right]$$

Then, the number of scatterings that are observed **in the time inteval *(t,t+dt)*** is just

$$P(t)dt/\tau$$

Since the number of scatterings in the time interval (t, t+dt) is

$$P(t)dt/\tau$$

the total number of scatterings observed for this single electron is

$$S(\Delta t_M, I) = \frac{1}{\tau} \int_0^{\Delta t_M} P(t)dt,$$

which depends both on the time interval and on the current, by way of the dependence

$$v_d(I) = I/newz$$

According to the RS analysis, in a fraction $\beta^2/20$ of all scatterings electrons can be captured, so that the number of produced **X-rays per electron** is

$$N_X = \frac{\beta^2}{20} S(\Delta t_M, I) = \frac{\beta^2}{20\tau} \int_0^{\Delta t_M} P(t)dt$$

In particular, for a very long (infinite) measurement time

$$\begin{aligned} S_\infty(I) = S(\infty, I) &= \frac{1}{\tau}\int_0^\infty P(t)dt \\ &= \frac{1}{2\tau}\int_0^\infty \left[\mathrm{erf}\left((L - v_d t)/\sqrt{2Dt}\right) - \mathrm{erf}\left(-v_d t/\sqrt{2Dt}\right)\right] dt \\ &= \frac{1}{\tau}\int_0^\infty dt \int_0^L \frac{1}{\sqrt{2\pi Dt}} \exp\left(-\frac{(x - v_d t)^2}{2Dt}\right) dx. \end{aligned}$$

Then, using the special integral

$$\int_0^\infty dt \int_0^1 \frac{1}{\sqrt{2\pi Dt}} \exp\left(-\frac{(x-t)^2}{2Dt}\right) dx = 1,$$

we find

$$S_\infty(I) = \frac{L}{v_d \tau} = \frac{\Delta t_T}{\tau},$$

and this result is the same as that found in the previous section with the very naive model, so that even in this better defined context there is no dependence on current.

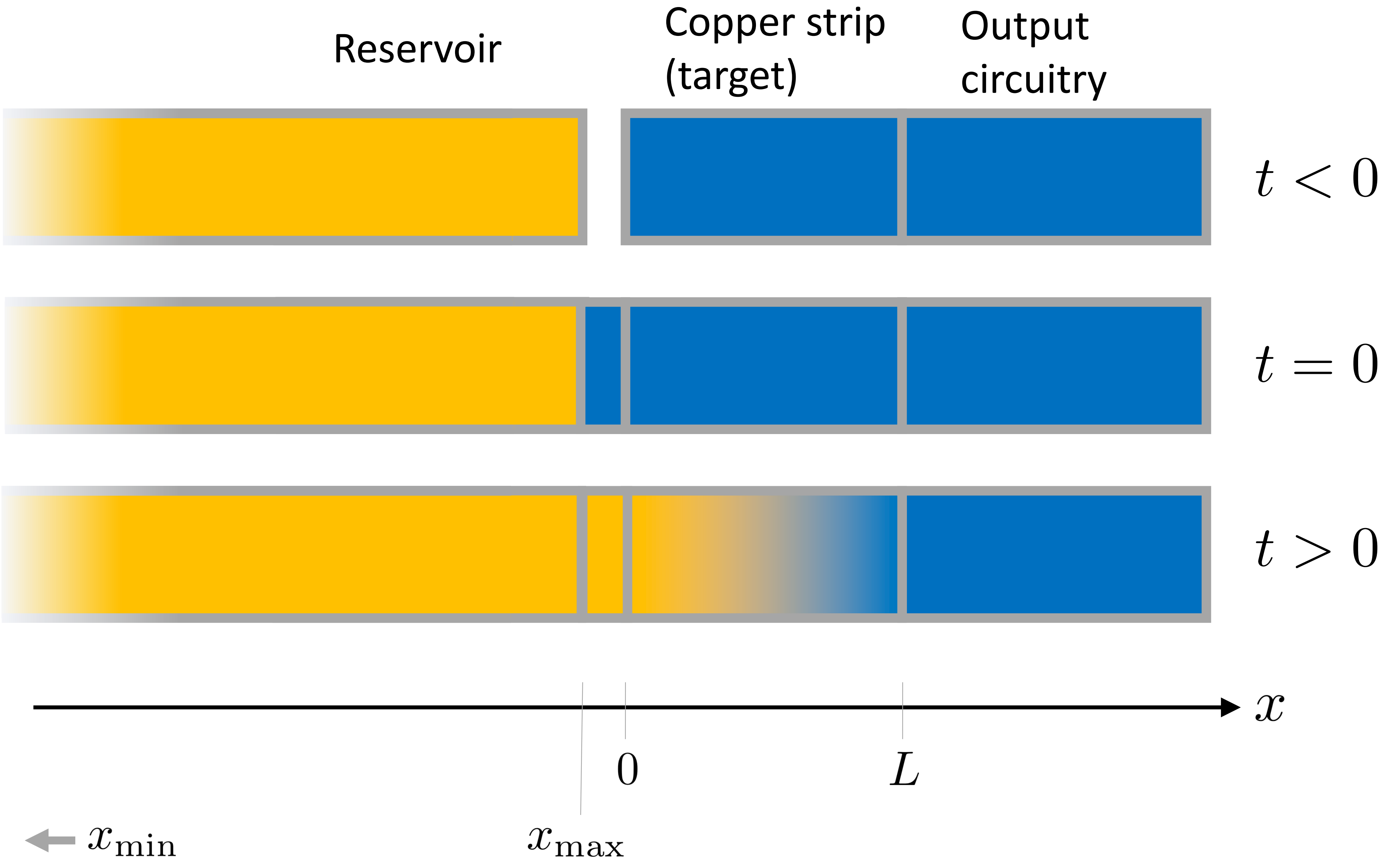
However, the model also underscores the importance of the time development of the signal.

To clearly define timing, we need to modify the usual RS experimental scheme.

- **Initially the copper strip is separated from the reservoir of "new" electrons that have never been close to the atoms in the target.**

- **At time t=0 the reservoir is put in contact with the target, and electrons can flow from the source towards the output circuitry (t>0).**

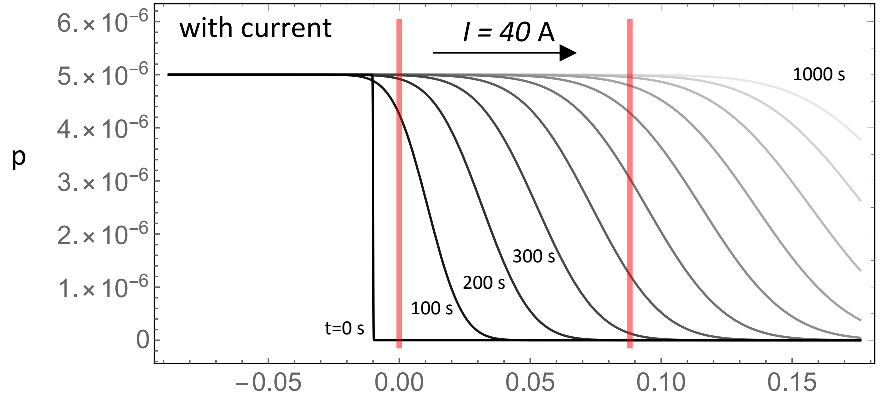
- **As electrons traverse the target – if there is a violation of the Pauli Exclusion Principle – they can be captured by the atoms and cascade electromagnetically to the fully occupied ground level.**


Reservoir
Copper strip
(target)
Output
circuitry
$t < 0$
$t = 0$
$t > 0$
x
0
L
$x_{\min}$
$x_{\max}$

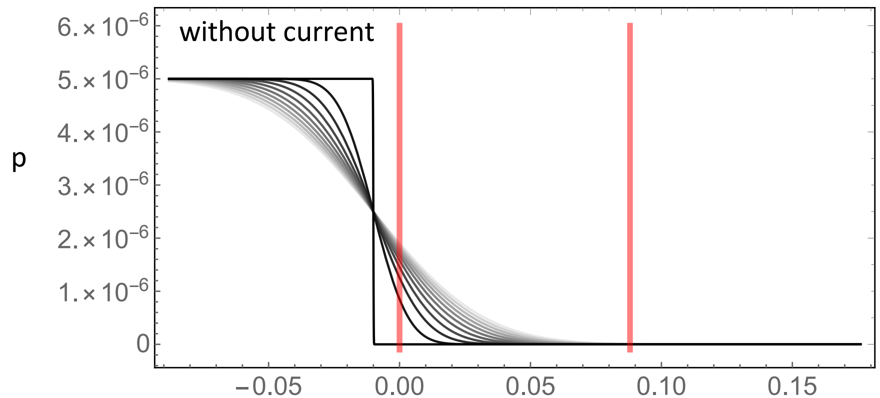
The distribution of the "new" electrons in the reservoir can be described by a uniform distribution of electrons all along the reservoir.

This means that for a single electron the probability density function of being located at position *x* at time *t* is

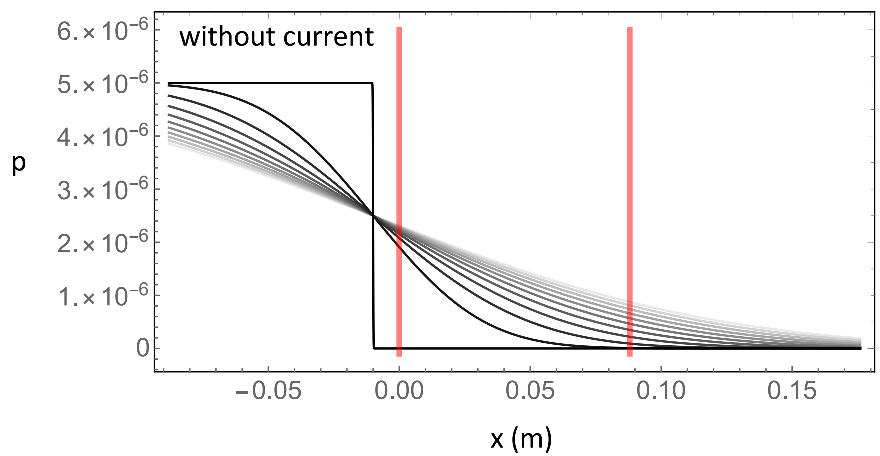
$$
\begin{aligned}
p(x,t|x_{\min},x_{\max},v_d) &= \int_{x_{\min}}^{x_{\max}} p(x,t|x_0,v_d)p(x_0)dx_0 \\
&= \frac{1}{x_{\max}-x_{\min}} \int_{x_{\min}}^{x_{\max}} \frac{1}{\sqrt{2\pi Dt}} \exp\left(-\frac{(x-x_0-v_d t)^2}{2Dt}\right) dx_0 \\
&= \frac{1}{2(x_{\max}-x_{\min})}\left[\mathrm{erf}\left(\frac{x-x_{\min}-v_d t}{\sqrt{2Dt}}\right)-\mathrm{erf}\left(\frac{x-x_{\max}-v_d t}{\sqrt{2Dt}}\right)\right]
\end{aligned}
$$



40 A current at times 0 s to 1000 s



no current at times 0 s to 1000 s



no current at times 0 s to 10000 s

The total number of "new" electrons associated with this reservoir is obviously

$$nA(x_{\max} - x_{\min})$$

and the number of "new" electrons in any slice of thickness Δx at position x and time t is

$$\begin{aligned} nA(x_{\max} - x_{\min}) p(x, t | x_{\min}, x_{\max}, v_d) \Delta x = \\ = nA\Delta x \left[\operatorname{erf}\left(\frac{x - x_{\min} - v_d t}{\sqrt{2Dt}} \right) - \operatorname{erf}\left(\frac{x - x_{\max} - v_d t}{\sqrt{2Dt}} \right) \right] \end{aligned}$$

Finally, the expected number of anomalous X-rays per unit time at time *t* is

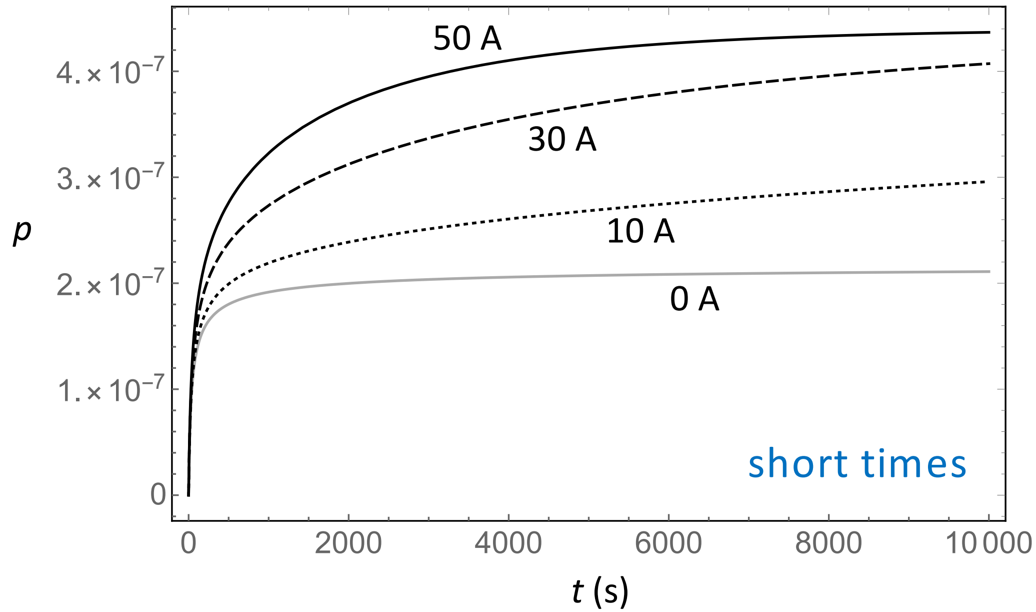
$$\frac{\beta^2}{2} r \nu n A \int_0^L \left[\operatorname{erf}\left(\frac{x - x_{\min} - v_d t}{\sqrt{2Dt}} \right) - \operatorname{erf}\left(\frac{x - x_{\max} - v_d t}{\sqrt{2Dt}} \right) \right] dx$$

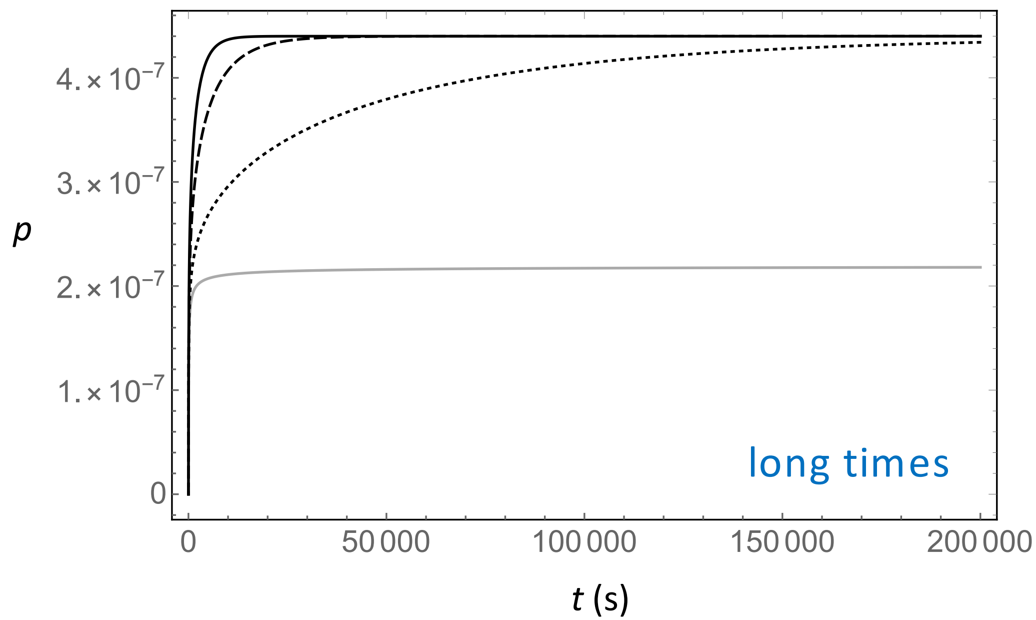
where *r* is the radiative capture probability per scattering and ν is the scattering rate, and the total number of emitted X-rays during the data taking time Δt_M is

$$\frac{\beta^2}{2} r \nu n A \int_0^{\Delta t_M} \int_0^L \left[\operatorname{erf}\left(\frac{x - x_{\min} - v_d t}{\sqrt{2Dt}} \right) - \operatorname{erf}\left(\frac{x - x_{\max} - v_d t}{\sqrt{2Dt}} \right) \right] dx\, dt$$

Time development of the X-ray signal according to

$$\frac{\beta^2}{2} r \nu n A \int_0^L \left[\mathrm{erf}\left(\frac{x - x_{\mathrm{min}} - v_d t}{\sqrt{2Dt}} \right) - \mathrm{erf}\left(\frac{x - x_{\mathrm{max}} - v_d t}{\sqrt{2Dt}} \right) \right] dx$$





In the previous slides we have considered a diffusion model without reassessing the numbers that are to be used to find the actual bound on the validity of the Pauli Exclusion Principle.

In particular, we have postponed a discussion of the scattering frequency ν. If we were to replicate the RS idea that important scatterings are those that contribute to conduction, then $\nu=1/\tau$ and the total number of scatterings during the traversal time is $\Delta t_{\mathrm{T}}/\tau$, which is about 1.7×10^{16} in VIP and 4×10^{14} in VIP-2.

Using the original RS proposal (the straight path) would lead to about 2.2×10^{6} scatterings in VIP and to about 1.8×10^{6} scatterings in VIP-2.

Thus, a proper consideration of the electrons paths leads to amplification by a factor of about 10^{10} in VIP and more than 10^{8} in VIP-2.

New estimate of "scattering frequency" ("frequency of close encounters")

The problem with the scattering frequency ν is that the estimate provided by RS is not actually relevant to the test of the Pauli Exclusion Principle.

Indeed, the scatterings that are considered in the RS paper are mostly electron-phonon scattering (in addition to other lattice irregularities like dopants, lattice dislocations, etc.), and have nothing to do with the actual electron capture process.

Here we replace these electron scatterings with the ``close encounters'' with individual atoms, and give a rough estimate of their frequency starting from the electron wavelength.

$$\lambda_e = h/m_e v \approx h/m_e v_F \approx 6.1 \times 10^{-10}\ \mathrm{m}$$

The radiative capture probability per close encounter can be estimated from the measured width of the naturally occurring Kα line complex G ≈ 2.73 eV (weighted value for the whole Kα complex in Cu), and from the transit time

$$\lambda_e / v_F \approx 5.3 \times 10^{-16} \text{ s}$$

A rough approximation for *r* in Cu yields

$$r \approx (\hbar / \Gamma) \times (\lambda_e / v_F) \approx 1.6 > 1$$

this means that we can assume that every close encounter leads to a capture with probability that is only limited by the Pauli violating probability $\beta^2 / 2$

Now, using the computed electron wavelength and the electron density in copper we find the average distance between close encounters with atoms

$$\ell \approx 1/n\pi(\lambda/2)^2 \approx 41 \text{ pm}$$

and the corresponding mean time between close encounters

$$\ell/v_F \approx 3.5 \times 10^{-17} \text{ s}$$

This means that with a 40 A current and a mean traversal time of 420 s (as in VIP), there are on average at least 1.2×10^{19} close encounters, instead of the $\approx 2.3 \times 10^{6}$ scatterings that can be computed from the RS approach, based on a straight path below the detector. Then the bound that is obtained using the viewpoint exposed here is better than the bound found with the RS approach by a factor 5.2×10^{12}. In the case of VIP-2 (I = 100 A) with a mean traversal time of about 10 s, there are on average at least 2.8×10^{17} close encounters, and the bound improves by a factor 1.6×10^{11}. If we take the more recent VIP-2 experiment this translates into a bound on the violation parameter:

$$\beta^2/2 < 2.6 \times 10^{-40}$$

The violation parameter can be understood in the context of non-relativistic quantum mechanics either with a model of small violations like that of Ignatiev and Kuzmin, or in the framework of electrons with a mixed symmetry state like in the paper by Rahal and Campa, and in non-relativistic quantum field theory with a model like the quons proposed by Greenberg.

In previous work, we always considered our data either in the framework of the model of Ignatiev and Kuzmin or Greenberg's quons: however these models share common difficulties, highlighted in the past by A. B. Govorkov.

Rahal and Campa introduced the violation parameter as a normalized count of the configurations of all the electrons in the universe where a given subset of electrons commute, so that the global electron-wave function has only an approximate antisymmetric character.

Non-relativistic quantum mechanics allows such a mixed symmetry wavefunction, which is instead ruled out by the spin-statistics theorem, **therefore within such an interpretation an RS-like experiment is a direct test of the spin-statistics connection.**

It is also interesting to remark that taking the estimate of the observable mass of the whole universe,

$$M \approx 1.6 \times 10^{55} \text{ g}$$

(similar to many other recent estimates)ssuming that it is nearly all composed of hydrogen atoms, we find that the total number of electrons in the observable universe is about 10^{79}

Then, as a consequence, the bound from VIP-2 means that less than 10^{39} electron pairs in the universe can actually have a wrong symmetry pairing.

By analyzing the motion of the electrons in the context of a classical random walk we have seen that for long times there is no difference in the rate of the anomalous X-rays produced by different DC currents flowing in the target of the VIP and VIP-2 experiments, while the time required to achieve this X-ray rate depends strongly on the current.

The extreme case of the reservoir connected with no current produces a final X-ray rate that is just half of the case with current because there is no drift, and half of the electrons moves into the target, while the other half moves away from it (this result is strictly true only for an infinitely long reservoir).

We also found that a large current may be detrimental to the experiment, because it reduces the actual number of interactions between electrons in the target as they move from the entrance to the exit.

This leads to some important considerations:

- There should be an optimal value of the current, such that the X-ray rate reaches the stationary value soon enough, but is not so large that the electrons do no have the time to wander around and interact with as many electrons as possible in the target.

- The time dependence can potentially be used to the experiment's advantage, as it modulates the X-ray rate. In this way we could use powerful signal analysis techniques to further reduce the effect of the (unmodulated) background.