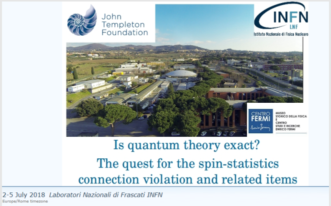


Dynamics of Quantum Correlations in Gaussian Open System

Aurelian Isar

Department of Theoretical Physics
National Institute of Physics and Nuclear Engineering
Bucharest-Magurele, Romania
isar@theory.nipne.ro

Outline

- possibility of E + Gaussian q. discord generation in a s. of 2 coupled bosonic modes in a common thermal env.
- initial state of the subs is taken of Gaussian form and the evolution in the th. of OSs based on CP q. dyn. semig assures preservation in time of Gaussian form of the state
- evolution of QE and GQD in terms of covariance matrix for a Gaussian input state (logarithmic negativity - degree of QE)
- initial separable STS: E generation may take place, for definite values of squeezing parameter, average photon no., T, dissipation const. and of the strength of interaction between the 2 modes; after its generation - temporary suppressions and revivals of E
- initial entangled STS: ESD takes place for all T of the thermal bath; temporary revivals and suppressions of E
- initial uni-modal SS: generation of GQD takes place
- initial separable STS: generation of GQ Steering - possible
- limit of large times

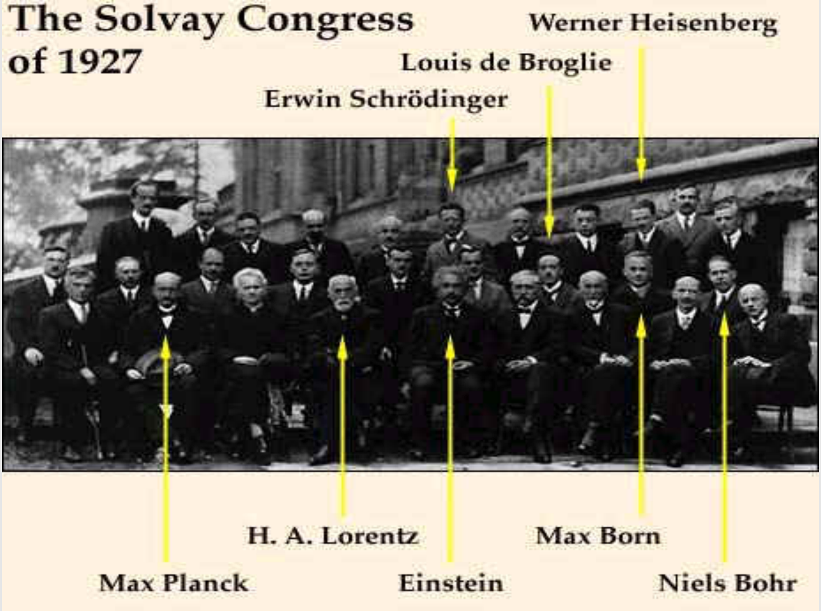
History (1)

- Quantum Entanglement - Schrödinger (1935): I would not call [entanglement] one, but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.
- Einstein: We believe in the possibility of a theory which is able to give a complete description of reality, the laws of which establish relations between the things themselves and not merely between their probabilities ... God does not play dice. The more success the quantum theory has the sillier it looks.
- Bohr: Einstein, don't tell God what to do! Those who are not shocked when they first come across quantum mechanics cannot possibly have understood it.
- Heisenberg: We have to remember that what we observe is not nature itself but nature exposed to our method of questioning. I, at any rate, am convinced that He is not playing at dice.

History (2)

- Schrödinger: I do not like it, and I am sorry I ever had anything to do with it. Had I known that we were not going to get rid of this damned quantum jumping, I never would have involved myself in this business!
- de Broglie: Electrons should not be considered simply as particles, but that frequency must be assigned to them also.
- Feynman: No, you're not going to be able to understand it . . . You see, my physics students don't understand it either. That is because I don't understand it. Nobody does... The theory of quantum electrodynamics describes Nature as absurd from the point of view of common sense. And it agrees fully with an experiment. So I hope that you can accept Nature as She is – absurd.

Solvey Congress



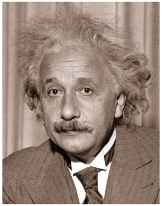
EPR paper 1935

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.



While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.

History (3)

- 1935: Einstein, Podolsky and Rosen (EPR): can q. mechanical description of physical reality be considered complete? an entangled wave f. does not describe physical reality completely
- element of physical reality: if, without in any way disturbing a s., we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresp. to this physical quantity (suff., not nec. cond. to define an element of reality)
- completeness: in a complete theory there is an element corresponding to each element of reality
- locality: the real factual situation of the s. A is indep. of what is done with the s. B, which is spatially separated from the former
- EPR paradox - "spooky action at a distance": the mysterious long-range correlations between 2 widely separated particles (non-local correlations between observations arbitrarily far)

History (4)

- local hidden variables (LHV) (supplementary parameters): in order to restore locality and completeness to QM
- Bohr strongly opposed this conclusion: QM description is complete, you cannot add anything to it $\rightarrow$ a debate for many decades - intense debate between Bohr and Einstein without much attention from a majority of physicists
- QM accumulates success in understanding nature: structure and properties of matter, light, and their interaction (atoms, molecules, absorption, spontaneous emission, solid properties, superconductivity, superfluidity, elementary particles) + new concepts leading to revolutionary inventions: transistor (later: laser, integrated circuits)
- no disagreement on the validity of quantum predictions, only on its interpretation
  - Bell's theorem (1964) has allowed us to settle the debate

Entanglement + Bell Theorem

- central to EPR paper is an entangled state: notion of entanglement - introduced by Schrödinger to describe a situation in which ”maximal knowledge of a total s. does not necessarily include total knowledge of all its parts, not even when these are fully separated from each other and at the moment are not influencing each other at all.”
- understanding of q. entanglement: information in a composite s. resides more in the correlations than in properties of individuals
- 1964 - Bell: local realism imposes experimentally constraints on the statistical measurements of separated ss. (Bell inequalities) - are satisfied for all class. correlations, but would possibly be violated if there were q. correlations present (can be violated by the predictions of QM); Bell contribution: consider correlations predicted for 3 spin measurements not at right angles but at an arbitrary angle θ; he was able to prove that correlations predicted by QM are larger than could be obtained from any LHV th.

Bell Inequalities - class.

- arguments of EPR for realism + locality (hidden variables) are incompatible with QM
- violation of Bell inequalities is one method to identify entanglement
- original Bell inequalities are not suitable for realistic experimental verification; one of the most common form of Bell inequalities is Clauser-Horne-Shimony-Holt (CHSH) inequality (1969) for 2 qubit s.
  $-2 \leq E(a_1, b_1) + E(a_1, b_2) + E(a_2, b_1) - E(a_2, b_2) \leq 2$
  $E(a_i, b_j)$ - correlation of measurements between a_i and b_j for the 2 ss.
- classically, $E(a_i, b_j) = a_i b_j \rightarrow$
  $-2 \leq a_1(b_1 + b_2) + a_2(b_1 - b_2) \leq 2$
  this inequality is always valid under class. th.:
  since $a_i, b_j = \pm 1$, either $b_1 + b_2 \pm 2$, or $b_1 + b_2 = 0$
  $b_1 + b_2 = 0 \rightarrow b_1 - b_2 = \pm 2 \rightarrow a_1(b_1 + b_2) + a_2(b_1 - b_2) = \pm 2$
  $b_1 + b_2 = \pm 2 \rightarrow b_1 - b_2 = 0 \rightarrow a_1(b_1 + b_2) + a_2(b_1 - b_2) = \pm 2$

Bell Inequalities - q.

- q. mechanically, $E(a_i, b_j) =< \psi|\overrightarrow{\sigma} \cdot \hat{n}_{a_i} \otimes \overrightarrow{\sigma} \cdot \hat{n}_{b_j}|\psi >$
  - for maximally entangled state $|\psi >= (|01 > -|10 >)/\sqrt{2}$
  $E(a_i, b_j) = -\cos\theta_{a_i b_j} = -\cos(\theta_i^a - \theta_j^b)$
  - for angles $\theta_1^a = \pi/2, \theta_2^a = 0, \theta_1^b = \pi/4, \theta_2^b = 3\pi/4$
  $E_{11}(\theta_1^a, \theta_1^b) = -1/\sqrt{2}, E_{12}(\theta_1^a, \theta_2^b) = -1/\sqrt{2}, E_{21}(\theta_2^a, \theta_1^b) = -1/\sqrt{2}, E_{22}(\theta_2^a, \theta_2^b) = 1/\sqrt{2}$
  $E_{11} + E_{12} + E_{21} - E_{22} = -2\sqrt{2}$ and CHSH inequality is violated
- Bell's inequalities are violated by certain q. predictions
- any LHV theory must satisfy Bell's inequalities (CHSH ineq. (Clauser, Horne, Shimony, Holt, 1969))
- CONFLICT between QM and Einstein's world view (local realism based on relativity)!
- Bell's theorem demonstrates a quantitative incompatibility between the local realist world view (a la Einstein) - which is constrained by Bell's inequalities, and q. predictions for pairs of entangled particles – which violate Bell's ineqs.

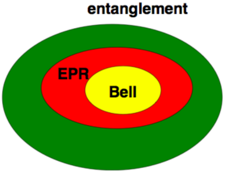
Experiments

- Bell's inequalities apply to all correlations that can be described within classical physics (mechanics, electrodynamics) and to most of the situations which are described within q. physics (except EPR correlations)
- possibility to complete QM according to Einstein ideas is no longer a matter of taste (of interpretation) $\rightarrow$ it has turned into an experimental question
- three generations of experiments from 1972- USA, France, Switzerland
- 1982 - Aspect experiment: 2 detectors were placed 13 m apart and a container of excited calcium atoms midway between them; spin states of two entangled photons
  1997 - Gisin: two detectors were placed 11 km apart
  - rule out local hidden variables
- are Bell inequalities violated by all pure entangled states?
- recent developments: Gisin + others
  Gisin theorem (1991, 1992): every pure bipartite entangled state in two dimensions violates the CHSH inequality

Results of experiments

- fantastic experiments – Bell inequalities violated $\rightarrow$ results in agreement with QM in experiments closer and closer to the Gedanken-Experiment
- Einstein's local realism is untenable $\rightarrow$ failure of local realism
- quantum non-locality - Quantum holism
- properties of a pair of entangled particles are more than the addition of the individual properties of the constituents of the pairs (even space like separated)
- entanglement = global property (no faster than light transmission of a "utilizable" signal)
- QM goes often against our classical intuition
- E - most intrinsic q. feature, and Bell's inequality violation its most striking consequence; q. correlations are more powerful than class. corrs

Nonlocality Hierarchy



- Not all entanglement is the same
- Classification of entanglement

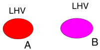
Entanglement $\Rightarrow$ *failure of quantum separability*

$$\rho = \sum_R P_R \rho_A^R \rho_B^R$$

$$\Longrightarrow \quad P(x_\theta^A, x_\phi^B) = \int_R \rho(\lambda)\ P_Q(x^A|\theta,\lambda) P_Q(x^B|\phi,\lambda)\ d\lambda$$

Bell's nonlocality: failure of local hidden variables (LHV)

ie *hidden variable separability*



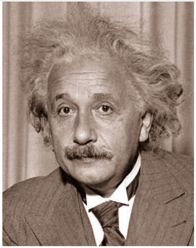
$$P(x_\theta^A, x_\phi^B) = \int_R \rho(\lambda)\ P(x^A|\theta,\lambda) P(x^B|\phi,\lambda)\ d\lambda$$

The concept of steering

$$P(x_\theta^A, x_\phi^B) = \int_R \rho(\lambda)\; P\,(x^A|\theta,\lambda)\,P_Q(x^B|\phi,\lambda)\; d\lambda$$

EPR steering iff this model fails

Wiseman, Jones, Doherty, PRL 2007;



Steering

Concept introduced in Schrodinger's famous reply to EPR paradox, 1935

Generalised EPR paradox for different measurements

Alice appears to "**steer**" Bob's state from **distant** site

EPR argument

Alice can infer Bob's outcomes: x **and** p

Local realism implies "elements of reality" for Bob

If these "elements of reality" inconsistent with a quantum state *then*

Quantum Mechanics is incomplete

[1] H. M. Wiseman, S. J. Jones, A. C. Doherty, Phys. Rev. Lett. **98**, 140402 (2007)

Quantum Information Science (1)

- Entanglement: a physical resource for q. information, like energy or mass: from Quantum Superposition
- Entanglement can be quantified and can be used to gain insight into q. information processing, and into other physical processes; the understanding of the extraordinary properties of entanglement and its generalization to more than two particles has triggered a new research field: QI
- Fundamental physics - decoherence; quantum $\rightarrow$ classical; ultimate control over ”large” systems
- Quantum cryptography - secure key distribution; superdense coding
- Quantum communcation - teleportation; linking separated quantum systems (”q. network”)
- Quantum computation - quantum algorithms; simulating other quantum systems; error correction
- Quantum metrology - measurements beyond the classical limit; non-invasive measurements; measurements on quantum systems; q. sensing; q. imaging

Q. Correlations (3)

- 3 famous types of nonlocality
  1. Entanglement: failure of quantum Separability (Failure of Local Quantum State (LQS) model)
  2. Bells nonlocality: failure of local hidden variables (LHV) state model (Failure of Local Realism)
  3. EPR Steering nonlocality: Concept introduced in Schrödingers famous reply to EPR paradox, 1935 (Failure of Hybrid LHV-LQS model)
- a hierarchy of different q. corrs - in order of decreasing strength: Bell nonlocality, EPR steering, q. entanglement, discord-type

Quantum discord (1)

- a signature of quantumness in correlated states (including separable)
- can be revealed by the necessary disturbance due to any local measurement

- if there is at least one local measurement we can perform without affecting the state $\rightarrow$ quantum-classical (or classically correlated)
- otherwise - with quantum discord
- classical correlations - those remaining after a minimally disturbing local measurement
- quantum correlations - those destroyed by a minimally disturbing local measurement

Open systems

- the simplest dynamics for an OS which describes an irreversible process: semigroup of transformations introducing a preferred direction in time (characteristics for dissipative processes)
- in GKLS axiomatic formalism of introducing dissipation in quantum mechanics, the usual von Neumann-Liouville eq. ruling the time evolution of closed q. ss is replaced by the following Markovian master eq. (GKLS) for the density operator $\rho(t)$ in the Schrödinger rep.:

$$\frac{d\Phi_t(\rho)}{dt} = L(\Phi_t(\rho))$$

- Φ_t - the dynamical semigroup describing the irreversible time evolution of the open system and L is the infinitesimal generator of Φ_t
- fundamental properties are fulfilled (positivity, unitarity, Hermiticity)

Markovian master equation

- in axiomatic formalism based on CP q. dyn. semigs, irreversible time evolution of an OS (that incorporates the dissipative and noisy effects due to the environment) is described by Kossakowski-Lindblad Markovian master eq. for the density operator (Schrödinger rep.)

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H,\rho(t)] + \frac{1}{2\hbar}\sum_j (2V_j\rho(t)V_j^\dagger - \{\rho(t), V_j^\dagger V_j\}_+)$$

- H - Hamiltonian of the q. OS
- $V_j, V_j^\dagger$ - operators defined on the Hilbert space of H (model the interaction of OS with the env.)

- the semigroup dynamics of the density operator which must hold for a quantum Markov process is valid only for the weak-coupling regime, with the damping λ typically obeying the inequality $\lambda \ll \omega_0$, where ω_0 is the lowest frequency typical of reversible motions

Complete positivity and entanglement

- positivity property guarantees the physical consistency of evolving states of single systems, while complete positivity prevents inconsistencies in entangled composite systems
- therefore the existence of entangled states makes the request of complete positivity necessary
- the positivity of the states of the compound system will be preserved only if the dyn. semig. of the subs. is completely positive

Operators

- q. dyn. semigs that preserve in time Gaussian form of the states: H - polyn. of second degree in coordinates x, y and momenta p_x, p_y of the 2 q. OS and V_j, $V_j^\dagger$ - polyns. of first degree in canonical observables ($j = 1, 2, 3, 4$):

$$V_j = a_{xj} p_x + a_{yj} p_y + b_{xj} x + b_{yj} y$$

- Hamiltonian of 2 identical coupled modes:

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{m\omega^2}{2}(x^2 + y^2) + qxy$$

- dyn. semig. implies positivity of the matrix formed by the scalar products of the vectors $\mathbf{a}_x, \mathbf{a}_y, \mathbf{b}_x, \mathbf{b}_y$ (their entries are the components $a_{xj}, a_{yj}, b_{xj}, b_{yj}$, resp.)

Equations of motion

- bimodal covariance matrix

$$\sigma(t) = \begin{pmatrix} \sigma_{xx} & \sigma_{xp_x} & \sigma_{xy} & \sigma_{xp_y} \\ \sigma_{xp_x} & \sigma_{p_xp_x} & \sigma_{yp_x} & \sigma_{p_xp_y} \\ \sigma_{xy} & \sigma_{yp_x} & \sigma_{yy} & \sigma_{yp_y} \\ \sigma_{xp_y} & \sigma_{p_xp_y} & \sigma_{yp_y} & \sigma_{p_yp_y} \end{pmatrix}$$

$$\frac{d\sigma}{dt} = Y\sigma + \sigma Y^{\mathrm{T}} + 2D, \quad Y = \begin{pmatrix} -\lambda & 1/m & 0 & 0 \\ -m\omega^2 & -\lambda & -q & 0 \\ 0 & 0 & -\lambda & 1/m \\ -q & 0 & -m\omega^2 & -\lambda \end{pmatrix}$$

D - matrix of diffusion coefficients

$$D = \begin{pmatrix} D_{xx} & D_{xp_x} & D_{xy} & D_{xp_y} \\ D_{xp_x} & D_{p_xp_x} & D_{yp_x} & D_{p_xp_y} \\ D_{xy} & D_{yp_x} & D_{yy} & D_{yp_y} \\ D_{xp_y} & D_{p_xp_y} & D_{yp_y} & D_{p_yp_y} \end{pmatrix}$$

Time-dependent solution

$$\sigma(t) = \begin{pmatrix} \sigma_{xx} & \sigma_{xp_x} & \sigma_{xy} & \sigma_{xp_y} \\ \sigma_{xp_x} & \sigma_{p_xp_x} & \sigma_{yp_x} & \sigma_{p_xp_y} \\ \sigma_{xy} & \sigma_{yp_x} & \sigma_{yy} & \sigma_{yp_y} \\ \sigma_{xp_y} & \sigma_{p_xp_y} & \sigma_{yp_y} & \sigma_{p_yp_y} \end{pmatrix}$$

$$\sigma(t) = M(t)(\sigma(0) - \sigma(\infty))M^{\mathrm{T}}(t) + \sigma(\infty),$$

$M(t) = \exp(tY)$, $\lim_{t\to\infty} M(t) = 0$ (Y must only have eigenvalues with negative real parts)

$$Y\sigma(\infty) + \sigma(\infty)Y^{\mathrm{T}} = -2D$$

Covariance matrix

Two-mode Gaussian state is entirely specified by its covariance matrix σ, which is a real, symmetric and positive matrix

$$\sigma = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix}$$

(A, B and C are 2×2 matrices)

- for environments inducing asymptotic Gibbs state

$$m\omega D_{xx} = \frac{1}{m\omega} D_{p_x p_x} = m\omega D_{yy} = \frac{1}{m\omega} D_{p_y p_y} = \frac{\lambda}{2} \coth \frac{\hbar\omega}{2kT},$$
$$D_{xp_x} = D_{yp_y} = D_{xy} = D_{p_x p_y} = D_{xp_y} = D_{yp_x} = 0$$

- then we have equal unimodal covariance matrices $A = B$ and symmetric entanglement matrix C
- Gaussian states with $\det C \geq 0$ are separable states, but for $\det C < 0$, it may be possible that the states are entangled

Logarithmic negativity (1)

- for Gaussian states, the measures of entanglement of bipartite systems are based on some invariants constructed from the elements of the covariance matrix - logarithmic negativity
- for a Gaussian density operator, logarithmic negativity is completely defined by the symplectic spectrum of the partial transpose of the covariance matrix
- $E_N = \max\{0, -\log_2 2\tilde{\nu}_-\}$, where $\tilde{\nu}_-$ is the smallest of the two symplectic eigenvalues of the partial transpose $\tilde{\sigma}$ of the 2-mode covariance matrix σ:

$$2\tilde{\nu}_{\mp}^2 = \tilde{\Delta} \mp \sqrt{\tilde{\Delta}^2 - 4\det\sigma}$$

- symplectic invariant (seralian) $\tilde{\Delta} = \det A + \det B - 2\det C$

Logarithmic negativity (2)

We apply the measure of entanglement based on negative eigenvalues of the partial transpose of the subsystem density matrix. In case of the Gaussian density operator, the negativity is completely defined by the symplectic spectrum of the partial transpose of the covariance matrix.

- **Logarithmic negativity** $E_N = -\frac{1}{2}\log_2[4f(\sigma)]$,

$$f(\sigma) = \frac{1}{2}(\det A + \det B)$$

$$- \det C - \sqrt{\left[\frac{1}{2}(\det A + \det B) - \det C\right]^2 - \det \sigma}$$

determines the strength of entanglement for $E_N > 0$; if $E_N \leq 0$, then the state is separable

Entangled initial states (1)

- initial Gaussian state: 2-mode STS, with CM

$$\sigma_{st}(0) = \begin{pmatrix} a & 0 & c & 0 \\ 0 & a & 0 & -c \\ c & 0 & b & 0 \\ 0 & -c & 0 & b \end{pmatrix},$$

$$a = n_1 \cosh^2 r + n_2 \sinh^2 r + \frac{1}{2} \cosh 2r,$$
$$b = n_1 \sinh^2 r + n_2 \cosh^2 r + \frac{1}{2} \cosh 2r,$$
$$c = \frac{1}{2}(n_1 + n_2 + 1) \sinh 2r,$$

  n_1, n_2 : average no. of thermal photons; r : squeezing parameter; $n_1 = 0$ and $n_2 = 0 \to$ CM of the 2-mode SVS
- a 2-mode STS is entangled when the $r > r_s$, where

$$\cosh^2 r_s = \frac{(n_1 + 1)(n_2 + 1)}{n_1 + n_2 + 1}$$

Entangled initial state (2)

- for all T, at certain finite moment of time, which depends on T, $E_N(t)$ becomes 0 and the state becomes separable - so-called phenomenon of entanglement sudden death; it is in contrast to the q. decoherence, during which the loss of q. coherence is usually gradual
- dissipation favorizes the phenomenon of entanglement sudden death – with increasing the dissipation parameter λ, entanglement suppression happens earlier
- dynamics of entanglement of the 2 os depends strongly on the initial states and the coefficients describing the interaction of the system with the thermal environment (dissipation constant and temperature)

Asymptotic covariance matrix

- while in the case of independent bosonic modes, the form of the coefficients would determine an asymptotic product Gibbs state describing a thermal equilibrium of the two modes with the thermal bath at temperature T, in the present model with coupled bosonic modes, the asymptotic state does not have anymore the form of a product state:

$$\sigma(\infty) = \frac{C}{4(L^2 - q^2)} \times$$

$$\times \begin{pmatrix} 2L^2 - q^2 & \lambda q^2 & -Lq & -\lambda Lq \\ \lambda q^2 & 2L^2 + (\lambda^2 - 2)q^2 & -\lambda Lq & q(L - q^2) \\ -Lq & -\lambda Lq & 2L^2 - q^2 & \lambda q^2 \\ -\lambda Lq & q(L - q^2) & \lambda q^2 & 2L^2 + (\lambda^2 - 2)q^2 \end{pmatrix}$$

$\omega_1 = \omega_2 \equiv \omega$, $C \equiv \coth(\omega/2kT)$, $L \equiv 1 + \lambda^2$

Asymptotic logarithmic negativity

- logarithmic negativity in the limit of large times:

$$E(\infty) = -\frac{1}{2}\log_2\{\frac{\coth^2\frac{\omega}{2kT}}{4}[4 + \frac{3(1+\lambda^2)q^2}{(1+\lambda^2)^2 - q^2}$$
$$-\frac{q}{(1+\lambda^2)^2 - q^2}\sqrt{16(1+\lambda^2)^3 + 8(-1+\lambda^4)q^2 + q^4}]\}$$

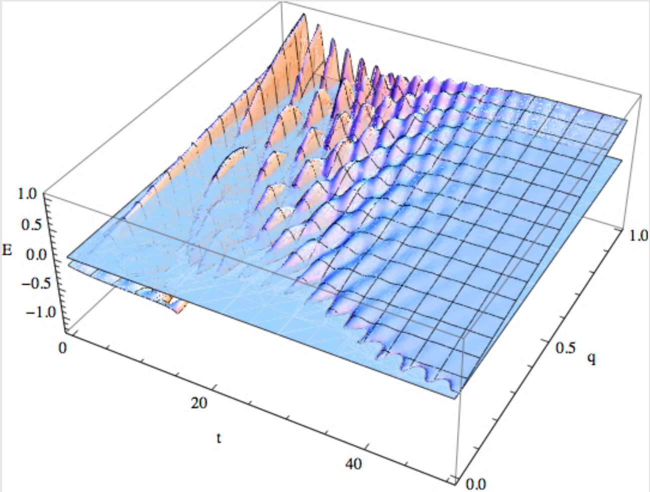


Figure: Logarithmic negativity E versus time t and interaction strength q for an initial separable squeezed thermal state with squeezing parameter $r = 0.5$, average photon numbers $n_1 = 0.5$, $n_2 = 1$, dissipation constant $\lambda = 0.08$, and temperature $T = 0$ of the thermal environment ($\hbar = 1$).

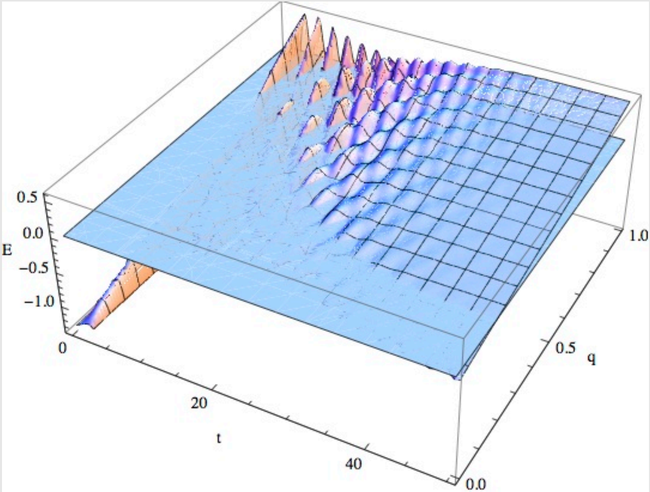


Figure: Same as in Fig. 1, for an initial separable squeezed thermal state with squeezing parameter $r = 0.3$, average photon numbers $n_1 = 1$, $n_2 = 1$, dissipation constant $\lambda = 0.08$, and $\frac{1}{2}\coth\frac{\omega}{2kT} = 1.1$.

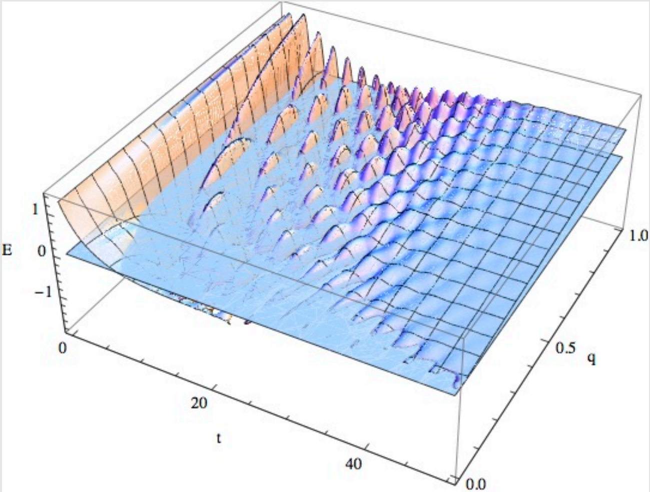


Figure: Same as in Fig. 1, for an initial entangled squeezed thermal state with squeezing parameter $r = 1$, average photon numbers $n_1 = 0.5$, $n_2 = 1$, dissipation constant $\lambda = 0.08$, and temperature $T = 0$ of the thermal environment.

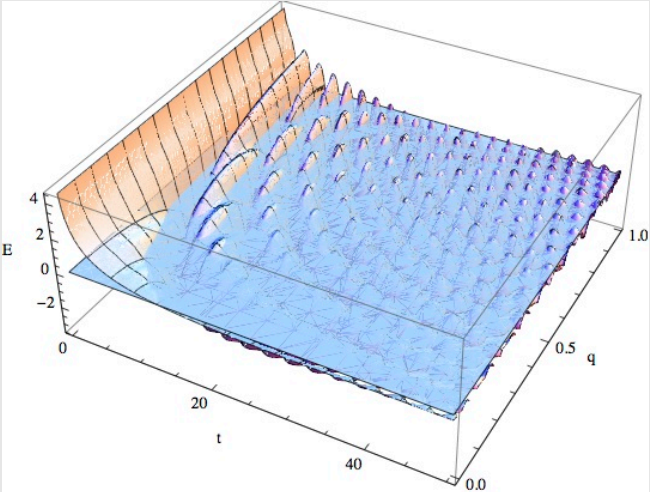


Figure: Same as in Fig. 1, for an initial entangled squeezed thermal state with squeezing parameter $r = 2$, average photon numbers $n_1 = 0.5$, $n_2 = 0.5$, dissipation constant $\lambda = 0.05$, and $\frac{1}{2} \coth \frac{\omega}{2kT} = 1.1$.

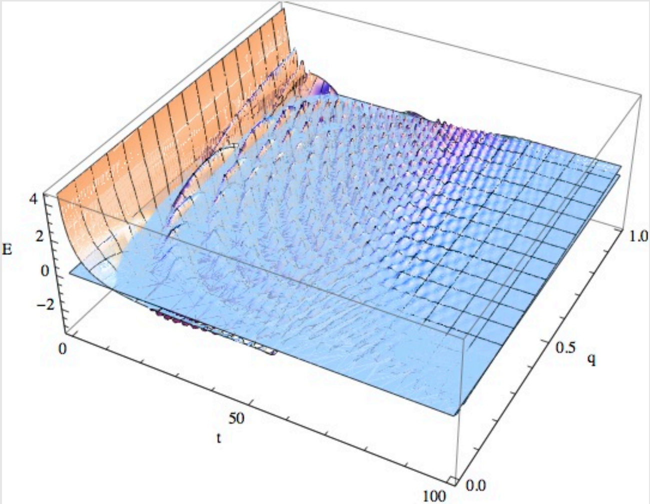


Figure: Same as in Fig. 4, for larger times.

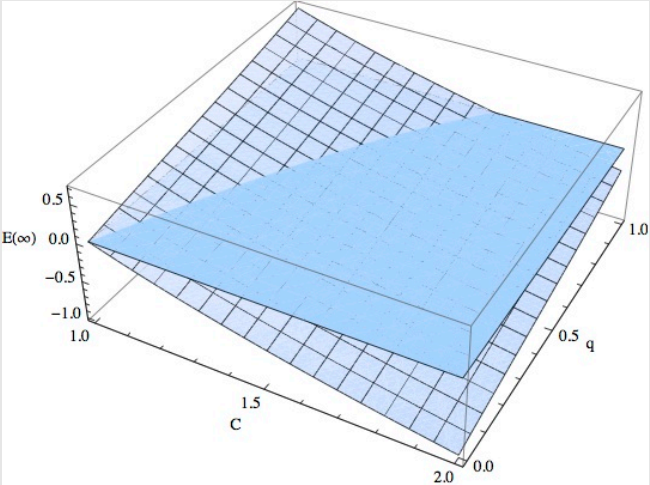


Figure: Asymptotic logarithmic negativity.

Quantum discord (1)

- QE does not describe all the non-classical properties of q. correlations – recent theoretical and experimental results indicate that some non-entangled mixed states can improve performance in some quantum computing tasks
- Zurek defined QD as a measure of q. correlations which includes entanglement of bipartite ss and it can also exist in separable states
- recently, an operational interpretation was given to QD in terms on consumption of entanglement in an extended quantum state merging protocol
- total amount of correlations contained in a q. state is given by the q. mutual information which is equal to the sum of the QD and classical correlations

Quantum discord (2)

- separability of q. states has often been described as a property synonymous with the classicality; however, recent studies have shown that separable states, usually considered as being classically correlated, might also contain q. correlations
- QD was introduced as a measure of all q. correlations in a bipartite state, including – but not restricted to – QE
- QD has been defined as the difference between 2 q. analogues of classically equivalent expression of the mutual information, which is a measure of total correlations in a q. state
- for pure entangled states QD coincides with the entropy of entanglement
- QD can be different from 0 also for some mixed separable states – correlations in such separable states with positive discord are an indicator of quantumness

Gaussian QD (1)

Gaussian QD of a general 2-mode Gaussian state ρ_{12} is QD where conditional entropy is restricted to generalized Gaussian positive operator valued measurements (POVM) on the mode 2; in terms of symplectic invariants (symmetry between modes 1 and 2 is broken) (Gerardo Adesso & Animesh Datta)

$$D = f(\sqrt{\beta}) - f(\nu_-) - f(\nu_+) + f(\sqrt{\varepsilon})$$

$$f(x) = \frac{x+1}{2}\log\frac{x+1}{2} - \frac{x-1}{2}\log\frac{x-1}{2}$$

$$\varepsilon = \begin{cases} \dfrac{2\gamma^2 + (\beta-1)(\delta-\alpha) + 2|\gamma|\sqrt{\gamma^2 + (\beta-1)(\delta-\alpha)}}{(\beta-1)^2}, \\ \quad \text{if } (\delta-\alpha\beta)^2 \le (\beta+1)\gamma^2(\alpha+\delta) \\ \\ \dfrac{\alpha\beta - \gamma^2 + \delta - \sqrt{\gamma^4 + (\delta-\alpha\beta)^2 - 2\gamma^2(\delta+\alpha\beta)}}{2\beta}, \\ \quad \text{otherwise} \end{cases}$$

Gaussian QD (2)

$$\alpha = 4\det A, \quad \beta = 4\det B, \quad \gamma = 4\det C, \quad \delta = 16\det\sigma$$

$\nu_{\mp}$ are the symplectic eigenvalues of the state, given by

$$2\nu_{\mp}^2 = \Delta \mp \sqrt{\Delta^2 - 4\det\sigma}$$

$\Delta = \det A + \det B + 2\det C$
Gaussian QD only depends on $|\det C|$, i.e., entangled ($\det C < 0$) and separable states are treated on equal footing

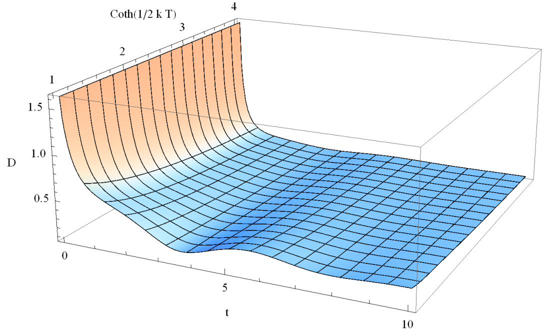


Figure: Gaussian Quantum Discord.

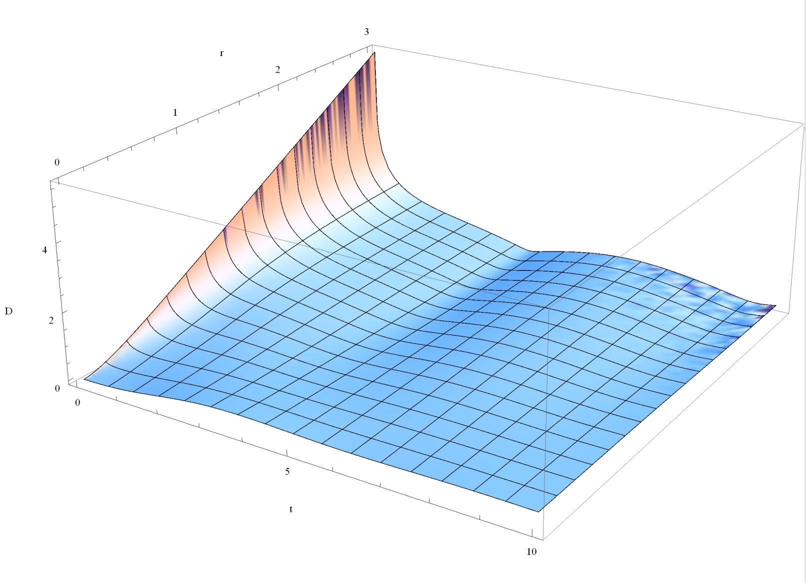


Figure: Gaussian Quantum Discord.

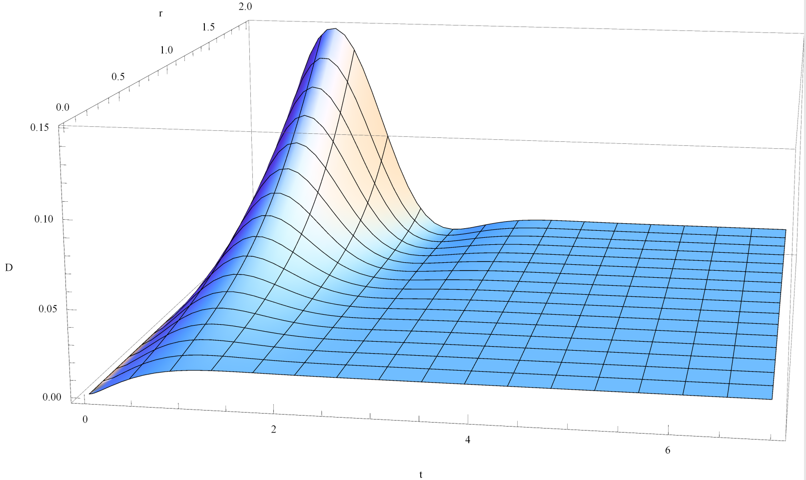


Figure: Gaussian Quantum Discord.

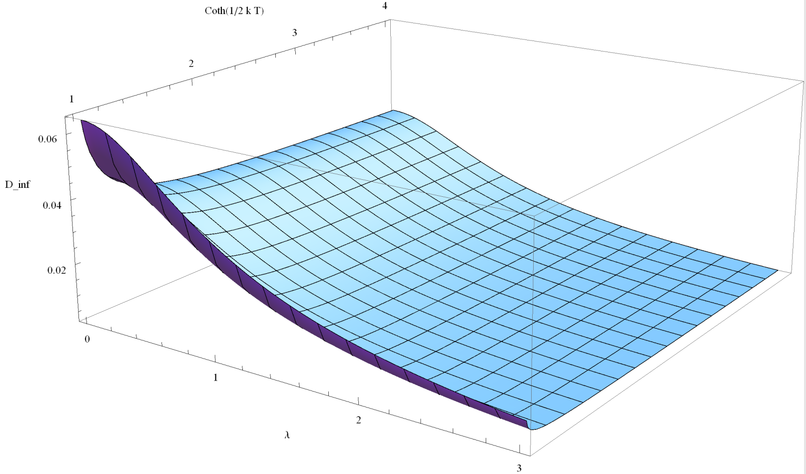


Figure: Gaussian Quantum Discord.

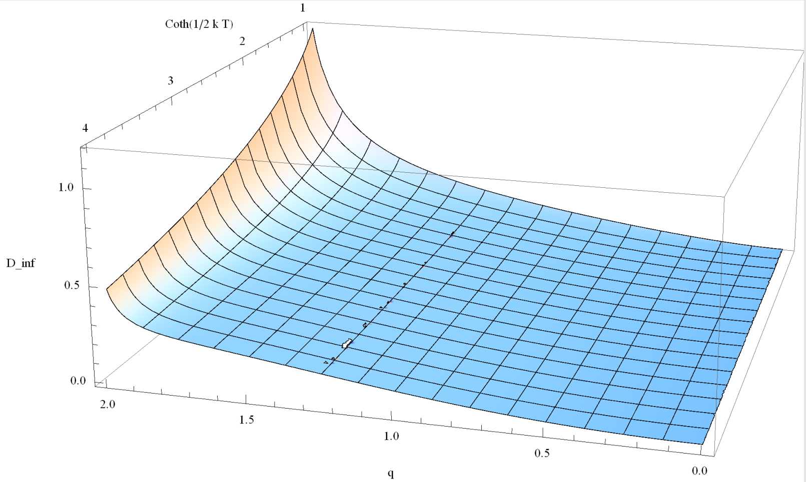


Figure: Gaussian Quantum Discord

EPR q. steering

- steering is a type of q. nonlocality first identified in the EPR paper, which is distinct from both nonseparability and Bell nonlocality (a type of q. corrs intermediate between E and nonlocality); to infer the steerability between two parties is equivalent with verifying the shared entanglement distribution by an untrusted party: Alice has to convince Bob (who does not trust Alice) that the state they share is entangled, by performing local measurements and classical communications;
- Alice performs a local measurement on her s., which makes it possible to steer Bob's local state depending on her choice of measurement settings (possibility for Alice to remotely prepare Bob's s. in different states depending on her own local measurements)
- behaviour of discord is strongly related to steering; for symmetric states, if the states are highly discordant, they are also highly steerable; a state is always steerable provided the discord exceeds a certain threshold

Gaussian q. steering

- captures the EPR paradox and quantifies to which extent Bob's mode can be steered by Alice Gaussian measurements on her mode in a 2-mode entangled Gaussian state
- in the case of bipartite Gaussian states a suitable measure of steering from Alice to Bob has been proposed using Gaussian measurements, which is easily computable for an arbitrary no. of modes, and has a particularly simple form when the steered party has one mode: $G^{A->B}(\gamma) = \max\{0, \frac{1}{2}\ln\frac{\det A}{4\det\gamma}\} = \max\{0, S(A) - S(\gamma)\}$, S : Renyi-2 entropy; for Gaussian states $S = \frac{1}{2}\ln(16\det\gamma)$

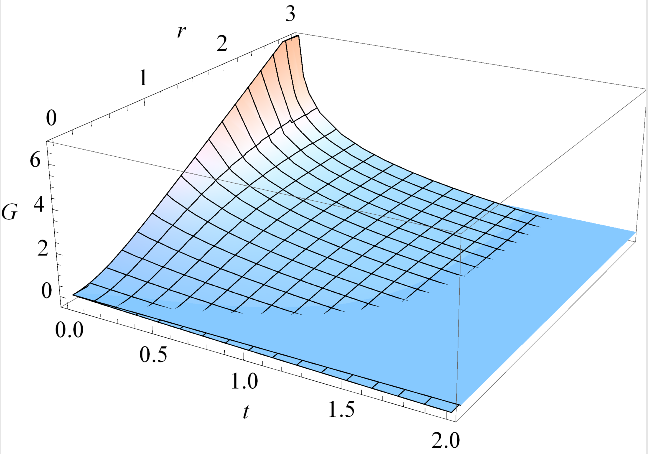


Figure: Gaussian quantum steering G versus time t and squeezing parameter r for an initial squeezed vacuum state in a thermal environment with temperature $C \equiv \coth(\omega/2kT) = 2$, dissipation parameter $\lambda = 0.1$ and $\omega = 1$ ($\omega_1 = \omega_2$).

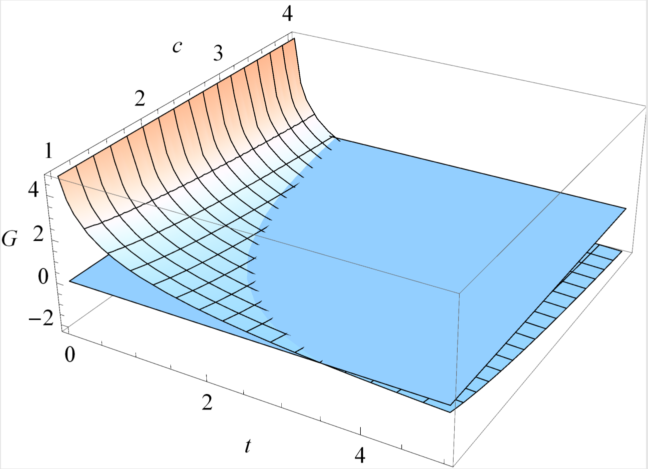


Figure: Gaussian quantum steering G versus time t and temperature $C \equiv \coth \frac{\omega}{2kT}$ for an initial squeezed vacuum state for $r = 2$, $\lambda = 0.1$ and $\omega = 1$.

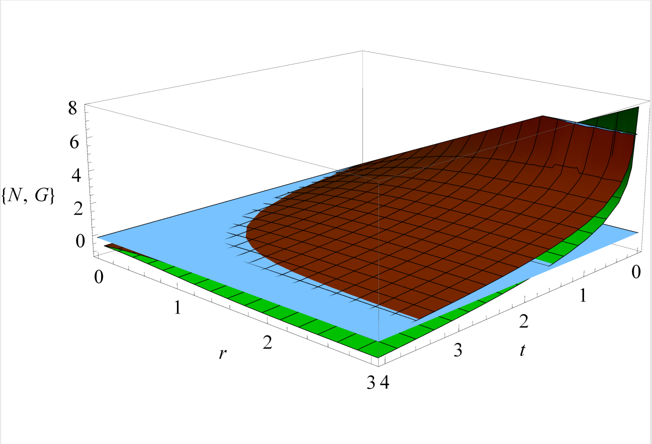


Figure: Gaussian quantum steering G (green plot) and logarithmic negativity N (red plot) versus time t and squeezing parameter r for an initial squeezed vacuum state, with $C \equiv \coth(\omega/2kT) = 2$, $\lambda = 0.1$ and $\omega = 1$.

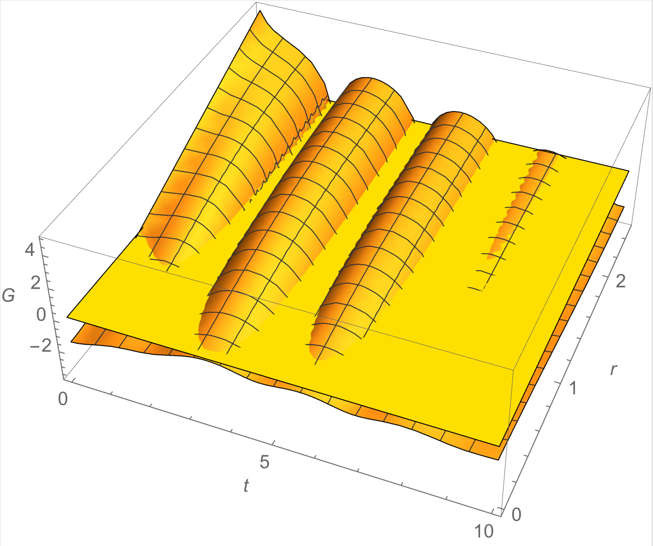


Figure: Gaussian quantum steering G versus time t and squeezing parameter r for an initial separable squeezed thermal state with $n_1 = 0.5, n_2 = 1$ in a thermal environment with temperature $T = 0$, dissipation parameter $\lambda = 0.03$, strength of interaction $q = 0.9$ and

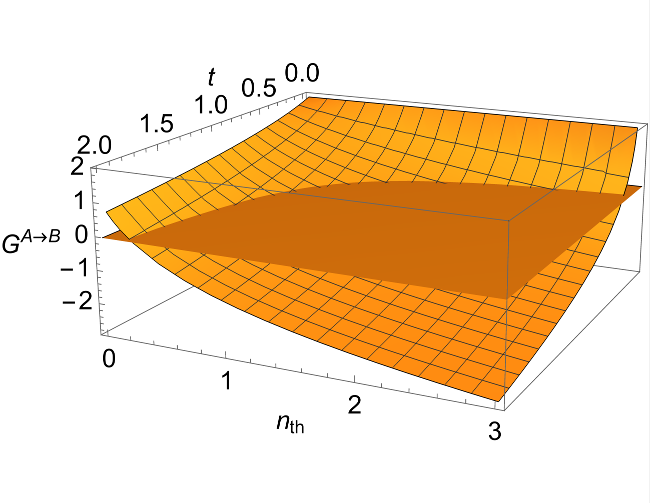


Figure: Gaussian quantum steering G versus time t and thermal photon number n_{th}.

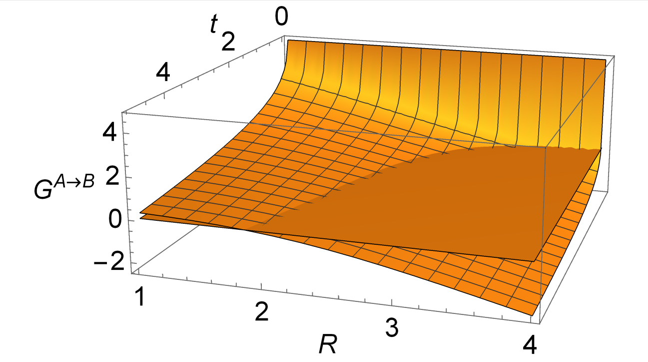


Figure: Gaussian quantum steering G versus time t and squeezing parameter R of the squeezed thermal environment.

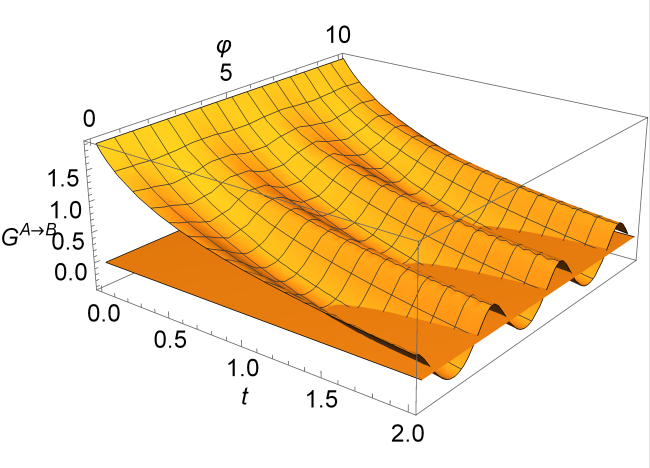


Figure: Gaussian quantum steering G versus time t and phase φ of the squeezed thermal environment.

Gaussian quantum steering

- an initial squeezed vacuum state ($n_1 = n_2 = 0$) is always steerable for $r > 0$; while Gaussian steering is increasing with squeezing parameter r, the thermal noise and dissipation destroy the steerability between the two parts
- compared to the GQD, which is decreasing asymptotically in time, the Gaussian quantum steering suffers a sudden death behaviour like quantum entanglement
- we described the time evolution of a measure that quantifies steerability for arbitrary bipartite Gaussian states in a system consisting of two bosonic modes embedded in a common thermal environment.
- we study Gaussian quantum steering in terms of the covariance matrix under the influence of noise and dissipation and find that the interaction with the environment destroys the steerability between the two parts

Conclusions (1)

- in the framework of th. of OS based on CP q. dyn. semigs - possibility of E + GQD + GQS generation in a s. consisting of 2 interacting bosonic modes embedded in a common thermal env.
- we solved master eq. for 2 interacting modes interacting also with an env.
- initial state of the subs is taken of Gaussian form and the evolution under the q. dyn. semig assures the preservation in time of the Gaussian form of the state
- evolution of QE and GQD in terms of covariance matrix for a Gaussian input state (logarithmic negativity - degree of QE)
- initial entangled STS - E suppression (ESD) takes place for all T of the thermal bath; one can also observe temporary revivals and suppressions of E

Conclusions (2)

- initial separable STS - E generation may take place, for definite values of the parameters characterizing the initial state of the s. (squeezing parameter, average photon no.), coeffs describing interaction of the s. with reservoir (T, dissipation const.) and of the strength of interaction between the 2 modes; after its generation one can observe temporary suppressions and revivals of E
- limit of large t: s. evolves asymptotically to an equilibrium state which can be entangled or separable; the direct interaction between the 2 modes favours generation or preservation of the created E, while T of the thermal bath acts towards preventing generation of E, or suppressing it once it was created - competition between these 2 factors determines the final state of being separable or entangled
- GQD + Gaussian q. steering

  1. A. Isar, Open Sys. Inf. Dynamics **23**, 1650007 (2016)
  2. A. Isar, T. Mihaescu, Eur. Phys. J. D **71**, 144 (2017)
  3. T. Mihaescu, A. Isar, Eur. Phys. J. D **72**, 104 (2018)

Figure: QIT group - Department of Theoretical Physics, National Institute of Physics and Nuclear Engineering, Bucharest-Magurele
Cristian Ivan, Iulia Ghiu, A.I., Tatiana Mihaescu, Irina Dumitru, Diana Dragomirescu
Serban Suciu, Marian Boromiza

Thank You!